THE BATHYSTROPHIC STORM TIDE[1]

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ABSTRACT

The tides on a regular open coast change rapidly as a storm approaches, and the time history of this change is important in computing dangerously high tides in bays; in addition, this change influences the maximum wave height offshore. Simple physical assumptions are used (and justified) to define a "bathystrophic" flow, and it is shown mathematically that this flow is of great importance in the computation of storm tides. Using only the traditionally accepted formulae and constants for wind stress and bottom friction, examples of the time change of the tide have been calculated. These results agree with observations as well as can be expected. Deviations from observations may be due to an edge wave.

INTRODUCTION

The storm tide is the primary cause of death and property damage in a hurricane, and as such, the study and forecast of storm tides are among the most important practical problems in meteorology and oceanography. We submit, here, a simple theory of coastal storm tides and the resulting forecast method.

The storm tide is a function of time and of at least two space dimensions. Presumably the tide can be computed on an electronic computer by using the complete set of two-dimensional equations of motion and continuity.[2] Freeman and Baer (1957) have described a "method of wave derivatives" which is closely related to the physics of the problem and which can be carried out graphically. But these methods of computation are expensive and difficult to use as presently developed, therefore a simple approach to storm tides and a facile forecast method are called for.

[1] The research and preparation of this paper was supported entirely by Gulf Consultants as part of a program for attracting outside support for both theoretical and practical studies of storm tides and water levels.

[2] We might add this result of the present work: When we do the full problem on a computer we expect to compute deviations from the bathystrophic storm tide in addition to the bathystrophic storm tide itself.

Consideration of the significance of the various terms in the equations of motion and continuity and of certain physical assumptions leads to a quasi-one-dimensional concept of the large-scale and long-term changes in the coastal storm tide. This coastal storm tide makes a significant contribution to the storm tide in any basin that is connected with the sea on a continental shelf.

DISCUSSION

Assumption 1.[3] On an open coastline with regular depth contours there is no sustained transport toward shore across the contours. If there were, much larger land areas would be inundated than are observed. For example, a 20 knot wind blowing on shore for 10 hours, if uncompensated, would cover an area up to 60 miles inland with a flood six feet deep. This is computed from the formulae[4]

$$\frac{\partial F_x}{\partial t} = \frac{\tau_x}{\rho} = 12 \times 10^{-4} \text{ knot}^2 \text{ for a 20 kt wind;}$$

$$\int_0^{10} F_x \, dt = 6 \times 10^{-2} \text{ mi.}^3 \text{ per mile of coast}$$

$$= 6 \text{ ft. deep} \times 60 \text{ mi. inland.}$$

The equivalent also holds true for offshore flow. Thus the only sustained currents toward shore are insignificant.

This postulates, then, that a compensating sustained current does exist parallel to the depth contours, and we define it as the "bathystrophic" current. The concept is somewhat analogous to that of the "geostrophic" wind in the atmosphere.

Assumption 2. Divergence of the velocity field does not bring about significant changes in the height of the water surface. Consider a mound of water which is superimposed on a level sea with a shoaling bottom; the mound is leveled by the action of gravity. In this case the edge of the mound expands at speeds of the same order of magnitude as the wave speed. Stationary disturbances, maintained by divergence in the velocity field, must be in currents with speeds of the order of magnitude of the wave speed. The greatest current which could act to build up the water mound (from six fathoms depth out to sea) is less than five knots. All of the wave speeds at which the mound edges are diverging are greater than 14 knots. Thus the dissipation of such a mound proceeds at a faster rate than its potential building up.

[3] This first assumption of our development is further supported in the APPENDIX.

[4] See list of symbols for definitions.

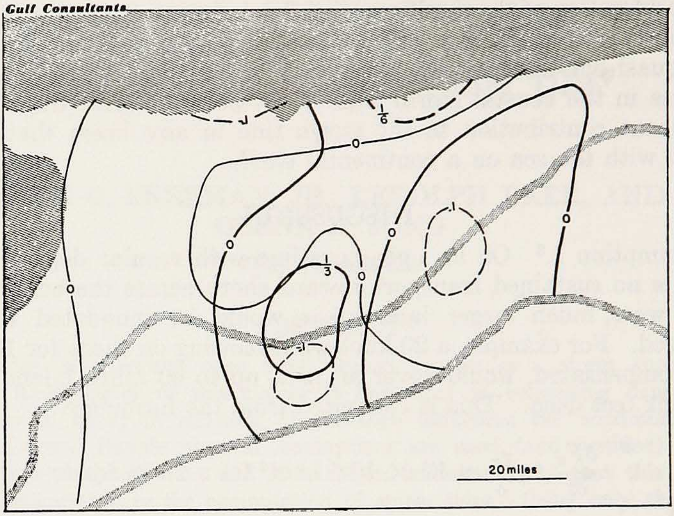

Figure 1. The heavy lines give the deviation of the wave speed that would build up if a divergence field acted for one hour without compensation; 1/3 is equivalent to about 2 feet of tide. The light lines give the deviation of the wave speed after the two dimensional equations of motion and continuity have been acting for ten minutes.

An example of the dissipation of the water heights built up by divergence is repeated here from Freeman and Baer (1956); see Fig. 1. A divergent current field is given and it is assumed that it works (uncompensated) for an hour to build up a height deviation in the water of about two feet ($1/3$ knot in diagram). This deviation is dissipated in 10 minutes to about 0.6 foot; further, the full computation shows that the current field becomes adjusted to discourage further build-up. In essence, we have assumed that a small amount of cross-contour transport develops to compensate for the divergence in the bathystrophic current; thus, the continuity equation need not be considered in our development.

Assumption 3. As we move in the y-direction the height changes are insignificant. A transport along the height contours can become very large without changing the height of the tide. (A transport normal to the depth contours causes changes in the height of the tide.) There is nothing equivalent to a barrier which allows water to pile up and give significant height differences in the y-direction.

Assumption 4. We assume that space derivatives of the current speed can be neglected with respect to the Coriolis parameter. We are discussing space scales of the order of 25 to 50 miles, and the maximum current we can expect is 5 knots. If a change in speed of 5 knots in 25 miles occurred, as an extreme example, we would have space derivatives of the current that are of the same order as the Coriolis parameter. In the normal situation, we assume that there is a change of the order of 1 knot in 50 miles; thus,

$$\frac{\partial v}{\partial y} = 0.02 \ll 0.25 = f, \text{ the Coriolis parameter.}$$

Bathystrophic Current and Storm Tides. As a consequence of these four assumptions, we have a quasi-one-dimensional (time-dependent) problem, and the solution can be found at each point for which the time history of the wind is known. The solution consists of determining the flux parallel to the contours as well as the derivative normal to the contours of height of the storm tide. In order to find the tide, it is necessary to integrate the derivative from deep water to the point for which the tide is desired. In our first computations we neglected the bottom friction term. This led to the results shown in Figs. 3 and 4 and to the logical absurdity that a sustained wind would give an ever-increasing storm tide. Thus it is shown that bottom friction must be included to allow computation of the falling storm tide.

Mathematical Development. We study the flow in a curved, orthogonal, right-hand coordinate system with the y-direction parallel to, and the x-direction normal to, the depth contours. The x-direction is positive toward shore.

The discussion will be facilitated by using the following symbols:

F_y = flux parallel to depth contours = $\int_{-h}^{\eta} u \, dz$

F_x = flux normal to depth contours = $\int_{-h}^{\eta} v \, dz$

u = current speed in x-direction

v = current speed in y-direction

t = time

z = vertical coordinate

g = acceleration of gravity

h = mean low water depth

η = deviation from mean low water depth

τ_x = wind stress in x-direction

τ_y = wind stress in y-direction

ρ = density of water
f = Coriolis parameter
k = bottom friction coefficient
τ_b = bottom stress

The assumptions discussed above may be expressed:

$$| F_x | \ll | F_y | \tag{1}$$

$$\left| \frac{\partial F_y}{\partial y} \right| \ll \left| \frac{\partial \eta}{\partial t} \right| \tag{2}$$

$$\left| gh \frac{\partial \eta}{\partial y} \right| \ll \left| \frac{\tau_y}{\rho} \right| \tag{3}$$

$$\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y} \ll f . \tag{4}$$

Using Assumption 4 and integrating the equations of motion for fluid flow in the z-direction, we obtain the following:

$$\frac{\partial F_x}{\partial t} = - gh \frac{\partial \eta}{\partial x} + f F_y + \frac{\tau_x}{\rho} - \frac{\tau_{bx}}{\rho} \tag{5}$$

$$\frac{\partial F_y}{\partial t} = - gh \frac{\partial \eta}{\partial y} - f F_x + \frac{\tau_y}{\rho} - \frac{\tau_{by}}{\rho}. \tag{6}$$

We now apply Assumptions 1 and 3 to obtain

$$gh \frac{\partial \eta}{\partial x} = f F_y + \frac{\tau_x}{\rho} - \frac{\tau_{bx}}{\rho} \tag{7}$$

$$\frac{\partial F_y}{\partial t} = \frac{\tau_y}{\rho} - \frac{\tau_{by}}{\rho} . \tag{8}$$

We use Manning's Formula as given by Linsley, *et al.* (1949) to find the bottom friction term. This gives us

$$\frac{\tau_{by}}{\rho} = \frac{k}{h^{7/3}} F_y^2 . \tag{9}$$

and

$$\frac{\tau_{bx}}{\rho} = 0 \tag{10}$$

by virtue of Assumption 1. Thus Equations (8) and (7) become

$$\frac{\partial F_y}{\partial t} = \frac{\tau_y}{\rho} - \frac{k}{h^{7/3}} F_y^2 \tag{11}$$

$$gh \frac{\partial \eta}{\partial x} = f F_y + \frac{\tau_x}{\rho}. \tag{12}$$

Equations (11) and (12) are expressed only in terms of the two unknowns, η and F_y. These are the prediction equations for the bathystrophic storm tide. At a given time the value of F_y is found by integrating (11) from the time when a significant wind began. Then η is found at any point by integrating (12) from deep water where η is assumed to be zero.

Note that we have not used the continuity equation. By virtue of Asumptions 1 and 2 we find that $\partial\eta/\partial t$, as given by the continuity equation, involves terms we neglect $\left(\text{i. e., } \frac{\partial F_x}{\partial x} \text{ and } \frac{\partial F_y}{\partial y}\right)$. Thus we may find η from the quasi-steady state conditions without using the continuity equation.

Some Samples of the Bathystrophic Storm Tide. Equations (11) and (12) have been integrated for several stations in Hurricane BAKER 1950 and for Hurricane EDNA 1954. The paths of these storms are shown in Fig. 2 and typical results are shown in Figs. 3 and 4. These results are interesting in view of the fact that only commonly-accepted values for the stress due to wind and bottom resistance were used. The wind fields were analyzed before the forecast was made.

In order to show that forecast values of the wind will be accurate enough to make good estimates of the storm tide, we increased the wind speeds by an arbitrary 30%. This gives a difference of about 30% in the maximum storm tide, as shown in Figs. 3 and 4. Thus the storm tide forecast is expected to have a linear correlation with the wind forecast.

The agreement of the bathystrophic storm tide with observations at Gulf Shores, Alabama (Fig. 3) is very good. When the pressure term is taken into account, 0.5 foot should be added to the value computed; this gives a computed total of 2.7 feet. The observed maximum value as shown in Fig. 3 is 3.3 feet (including astronomical tide).

At Atlantic City, New Jersey (Fig. 4) the bathystrophic storm tide seems to be a good approximation to the rising and falling tide, but the computed maximum increase over the "astronomical" tide does not occur. There seems to be a disturbance at Atlantic City which is not taken into account by this computation.

First it came to mind that there may have been an error in observing or in recording the observation at Atlantic City, but we were

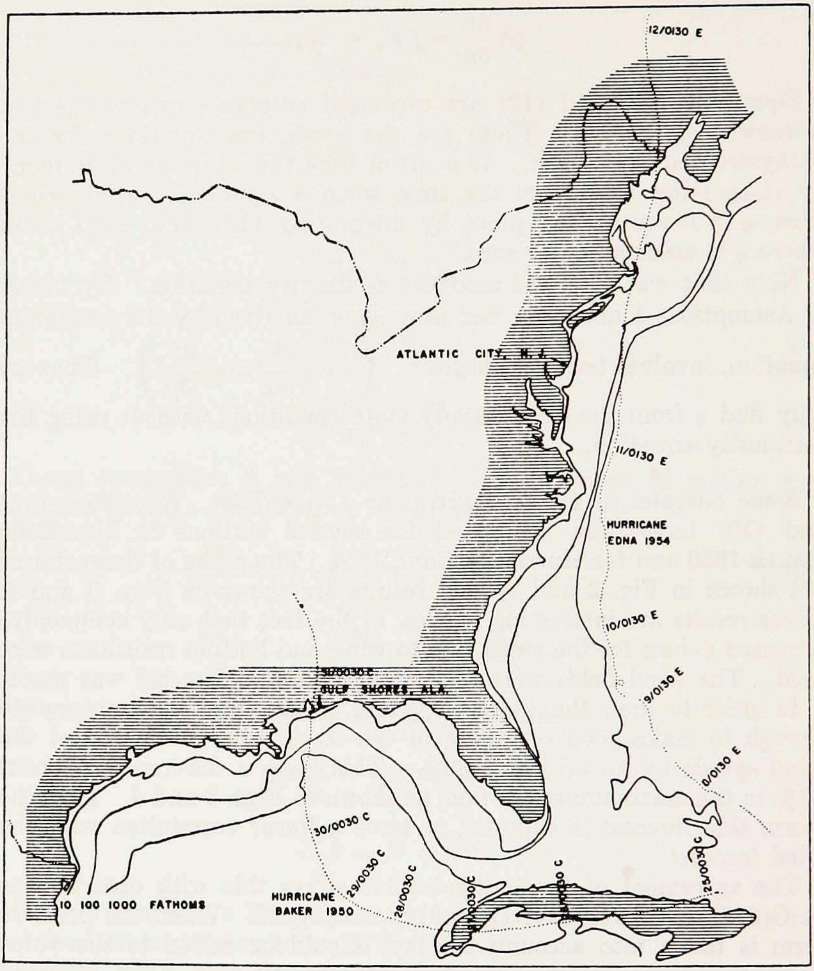

Figure 2. The bathymetry for the shores of the eastern United States and the paths of Hurricane Baker and Hurricane Edna.

assured by Mr. D. Lee Harris of the Weather Bureau that this was not the case. We suggest, then, the possibility that the water flows over a long low barrier or into a bay (the Delaware?) somewhere near Atlantic City and that this action limits the rise of the tide. This causes a "wave" of reduced tide rise to move past Atlantic City.[5]

[5] Many oceanographers will immediately think of edge waves. Munk (1956) has shown that the period of edge waves in this area conforms with the observed period.

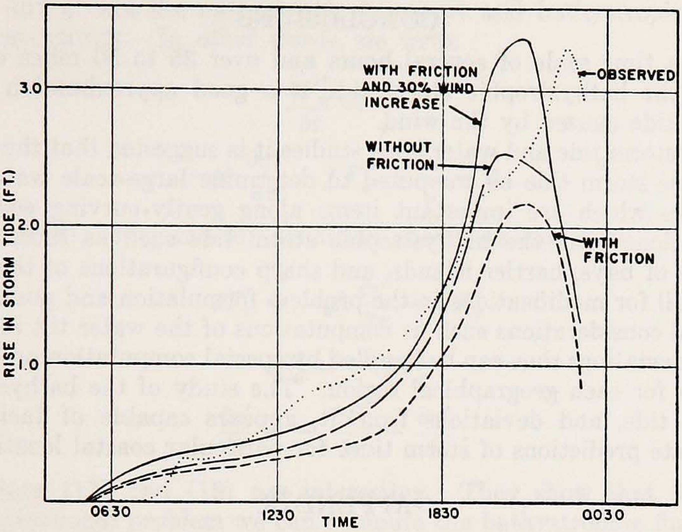

Figure 3. Gulf Shores, Alabama: Various computations of the storm tide during Hurricane Baker. The observed tide includes the astronomical tide at this station.

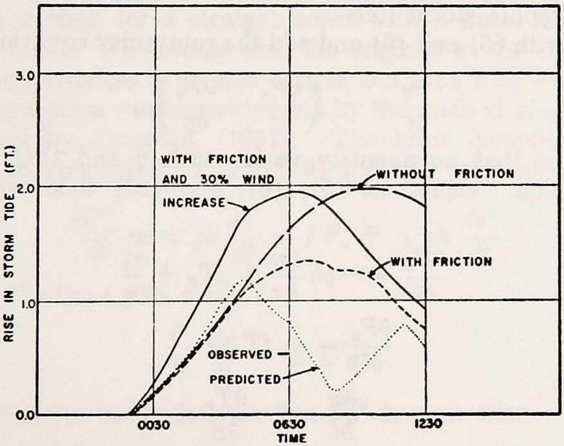

Figure 4. Atlantic City, New Jersey: Various computations of the storm tide during Hurricane Edna. The observed minus predicted tide shows the difference between the actual tide and U. S. Coast and Geodetic Survey computation of the ordinary tide.

A similar occurrence at Sandy Hook indicated a wave moving from Atlantic City northward.

CONCLUSIONS

On a time scale of several hours and over 25 to 50 miles of open coast the bathystrophic storm tide is a good approximation to the storm tide caused by the wind.

For storm tide and water level studies it is suggested that the bathystrophic storm tide be computed to determine large-scale water level changes which are important items along gently-curving coastlines. Deviations from the bathystrophic storm tide such as these due to effects of bays, barrier islands, and sharp configurations of the coastline call for modifications in the problem formulation and possibly for special considerations such as computations of the water tilt in a bay. Such deviations thus can be handled by special computation procedures set up for each geographical region. The study of the bathystrophic storm tide, and deviations from it, appears capable of facilitating accurate predictions of storm tides for particular coastal locations.

APPENDIX

It is possible to show by computation that F_x and changes in η due to variations in F_x remain small while the bathystrophic storm tide builds up. In other words, we can support Assumption 1 with a computation that is (in some ways) more general than the computation of the bathystrophic storm tide.

We start with (5) and (6) and add the continuity equation

$$\frac{\partial \eta}{\partial t} = -\left(\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y}\right) . \tag{13}$$

We assume that no quantity varies with y and that there is no bottom friction. Equations (5), (6), and (13) then reduce to the following:

$$\frac{\partial F_x}{\partial t} = -gh\frac{\partial u}{\partial x} + fT_y + \frac{\tau_x}{\rho} \tag{14}$$

$$\frac{\partial F_y}{\partial t} = -fT_x + \frac{\tau_y}{\rho} \tag{15}$$

$$\frac{\partial u}{\partial t} = -\frac{\partial T_x}{\partial x} . \tag{16}$$

Now we define two new sets of quantities:

$$F_x = F_x'$$

$$\eta = \eta^* + \eta'$$

$$F_y = F_y^* + F_y' .$$

F_y^* and η^* are the bathystropic transport and bathystropic storm tide respectively. In other words, we write

$$\frac{\partial F_y^*}{\partial t} = \frac{\tau_y}{\rho} \tag{17}$$

$$-gh\,\frac{\partial \eta^*}{\partial x} + f\,F_y^* + \frac{\tau_x}{\rho} = 0\,. \tag{18}$$

From (14), (16), and the definition above, we can write

$$\partial F_x' = -gh\,\frac{\partial \eta'}{\partial x} + f\,F_y' \tag{19}$$

$$\partial F_y' = -\,f\,F_x' \tag{20}$$

$$\frac{\partial \eta'}{\partial t} = -\,\frac{\partial F_x'}{\partial x} - \frac{\partial \eta^*}{\partial t}\,. \tag{21}$$

Equations (17) and (18) are interesting. They show that for this one-dimensional problem we can compute the bathystrophic flux, F_y^*, and the storm tide, η^*, for all time and then use the time history of η^* in (19) to (21) to compute F_x', F_y', and η'.

We can solve a particular problem for F_x', F_y', and η', and we have shown that η^* soon becomes (and stays) much larger than η'. This problem is worked for a straight coast with a linear bottom profile of slope 1/1,000. We assume that the origin is at the shore and that no significant changes in η occur farther out than $x = -\,45 \times 10^4$ ft.

The computation can be carried out by the method of characteristics as described by Freeman (1951). The basic computing formulae found by applying this method to (19) and (20) are that the conditions

$$\frac{dF_x'}{dt} \pm \sqrt{gh}\,\frac{d\eta'}{dt} = f\,F_x' \mp \sqrt{gh}\,\frac{d\eta^*}{dt} \tag{22}$$

hold along the lines with slope

$$\frac{dx}{dt} = \pm\,\sqrt{gh}\,. \tag{23}$$

We assume the bathystrophic flux F_y^* is such that

$$\frac{\partial \eta^*}{\partial t} = (1.85 \times 10^{-10}\,x + 8.3 \times 10^{-5})\ \text{kts.}$$

Starting with the water absolutely level and computing for two hours with $\partial\eta^*/\partial t$ as given above, we compute the time history at the shore, which is illustrated in Fig. 5.

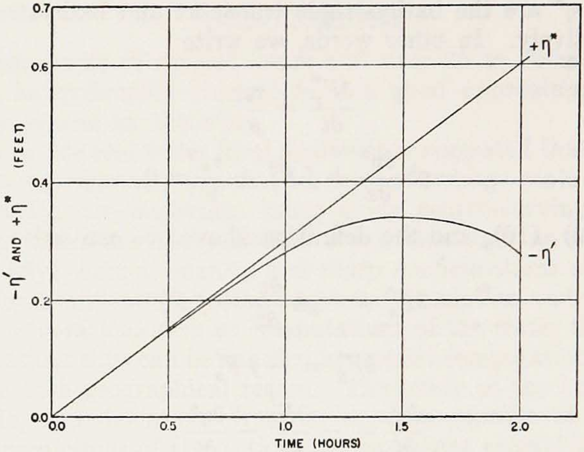

Figure 5. The bathystrophic storm tide, η^*, compared with the deviation from the bathystrophic storm tide η' as computed by the method of characteristics.

Note that in this time η^* becomes 0.59 feet at the shore and η' becomes —.30 feet. η^* is still increasing at the initial rate, but $|\eta'|$ is now decreasing. The plot of η' and η^* vs. time at other values of x show a similar behavior.

This study indicates that η' stays near the values reached after two hours while η^* increases indefinitely. Thus η' does not become significant, and F_x' and F_y' remain insignificant compared to F_y^*. Thus our first assumption is justified in this model which would have allowed it to be violated.

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