

YALE PEABODY MUSEUM

P.O. BOX 208118 | NEW HAVEN CT 06520-8118 USA | PEABODY.YALE. EDU

JOURNAL OF MARINE RESEARCH

The *Journal of Marine Research*, one of the oldest journals in American marine science, published important peer-reviewed original research on a broad array of topics in physical, biological, and chemical oceanography vital to the academic oceanographic community in the long and rich tradition of the Sears Foundation for Marine Research at Yale University.

An archive of all issues from 1937 to 2021 (Volume 1–79) are available through EliScholar, a digital platform for scholarly publishing provided by Yale University Library at <https://elischolar.library.yale.edu/>.

Requests for permission to clear rights for use of this content should be directed to the authors, their estates, or other representatives. The *Journal of Marine Research* has no contact information beyond the affiliations listed in the published articles. We ask that you provide attribution to the *Journal of Marine Research*.

Yale University provides access to these materials for educational and research purposes only. Copyright or other proprietary rights to content contained in this document may be held by individuals or entities other than, or in addition to, Yale University. You are solely responsible for determining the ownership of the copyright, and for obtaining permission for your intended use. Yale University makes no warranty that your distribution, reproduction, or other use of these materials will not infringe the rights of third parties.



This work is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 4.0 International License.
<https://creativecommons.org/licenses/by-nc-sa/4.0/>



THE ACTION OF VARIABLE WIND STRESSES ON A STRATIFIED OCEAN^{1, 2}

BY

G. VERONIS AND HENRY STOMMEL

Institute for Advanced Study, Princeton, N. J.

ABSTRACT

The forced response of a horizontally infinite ocean on a rotating earth to a transient wind system is investigated. In an equilibrium state, the ocean consists of two homogeneous layers of fluid of finite depth, the fluid in the lower layer being slightly denser than that in the upper. The wind system is periodic in space, and its time-dependence is periodic or is given by a step function.

Little energy goes into the inertio-gravity waves for winds that blow longer than a half pendulum day; most of the energy appears as geostrophically balanced Rossby (planetary) waves. Wind systems with periods from a day to a year excite motions which are in part barotropic and in part baroclinic. Winds of long period (100 years or more) excite the (internal) baroclinic Rossby mode, in which the currents are confined to the upper layer. The applicability and oceanographic implications of the model are discussed quantitatively.

1. INTRODUCTION

This paper treats the response of a stratified ocean to time-dependent wind stress systems of various dimensions. The model of the ocean is infinite horizontally, but the wind systems are of finite dimensions.

We anticipate that the density stratification of the ocean will play an important role in the response of ocean to wind. To account for the effect of density variation, a highly idealized structure is introduced, viz., a two-layer ocean with each layer homogeneous in density. This structure is supposed to correspond roughly to the two masses of water above and below the main thermocline in the real ocean.

In accordance with the theoretical framework for the dynamics of

¹ Work was sponsored by the Office of Naval Research under Contract No. N-6-ORI-139, Task Order I. Reproduction in whole or in part is permitted for any purpose of the U. S. government.

² Contribution No. 859 of the Woods Hole Oceanographic Institution.

Note added in proof. In a recent publication (Veronis, 1956) it was pointed out that this article in the *Journal of Marine Research* contained a wrong conclusion. The mistake referred to has been corrected in this article subsequent to publication of the Veronis (1956) article in *Deep Sea Research*.

currents in central oceanic areas as given by Sverdrup (1947) and Reid (1948), we will neglect horizontal friction due to large scale lateral turbulent processes.

Cases where the effects of coastal boundaries are involved are not considered because of the complexity of the general problem. It is not generally possible therefore to apply the results of this paper to the real ocean in an enclosed basin in the same satisfactory way that theories of the mean steady-state circulation (e. g. Munk, 1950) have been applied. Application of theoretical results must be limited to types of motion (or initial stages of certain types) in which boundaries or coasts do not exert an immediate influence.

Essentially, what we have sought is an understanding of how the two-layer ocean responds to changes in the applied wind. Is the wind-induced velocity (a) confined to the upper layer, (b) nearly the same throughout the depth of the ocean, or (c) a combination of a and b? Correspondingly, how does the interface between the two layers respond in relation to the free surface? (a) Does it respond in such fashion as to keep the horizontal pressure gradients (and hence velocities) negligible in the deep layer? (b) Does it remain essentially unaffected, hence permitting the horizontal pressure gradients to be nearly equal in both layers? (c) Or does it respond as a combination of a and b?³ From a practical oceanographic point of view, a clear picture as to which of these alternative modes of motion is probable is of great importance in any attempt to relate changes of oceanic circulations to changes in the applied wind stress.

The surface wind-drift currents within the top 100 m are almost always in equilibrium with the wind stress and are measured directly by discrepancies in navigation of ships, while currents in deeper portions of the ocean are inferred from dynamic computations only. It is well known that the latter method fails to detect the barotropic component of velocity.

Despite the fact that there are no satisfactory *direct* measurements of velocity distribution in deep waters beneath the main thermocline, it is quite obvious from a study of the distribution of certain properties such as salinity and dissolved oxygen that these deep waters do not move with the same high mean velocity of the water masses above the main thermocline. Therefore, so far as the mean circulation is concerned, it is certain that the time-mean velocity is essentially baroclinic, and one may be sure that any theory which results in a barotropic mean circulation has some essential physical feature lacking. Barotropic mean currents are excluded by observation; but

³ Motion of the type (a) will be called baroclinic or internal; motion of the type (b) will be referred to as barotropic or external.

time-periodic barotropic currents are permissible provided they do not involve horizontal displacements large enough to confuse the picture of the mean distribution of deep ocean properties. For example, ordinary tidal currents are barotropic, but, since the amplitude of the periodic horizontal displacement associated with them is of the order of one kilometer, they produce unobservably small periodic variations in the mean distribution of deep water properties at any fixed point. It is necessary to examine the theoretical implications of any time-variable barotropic flow which might result from the passage of storms, from prolonged disturbances to the circulation, or even from seasonal variations in wind. If the theoretical result requires barotropic modes with periodic horizontal displacements of the deep water of more than, say, 300 km, then it seems safe to say that such a result is in conflict with observation.

Finally, one further test by observation should be mentioned. Barotropic currents set up by a storm might involve a permanent displacement of the deep water in addition to the periodic displacements. The passage of a number of storms, more or less at random, would then result in deep water particles executing horizontal "random walks." This type of mixing process can then be expressed in terms of a coefficient of eddy diffusivity. If theoretically predicted motions of the deep water layers imply an eddy coefficient of much more than $10^8 \text{ cm}^2 \text{ sec}^{-1}$ (Sverdrup, 1939), they are in conflict with observational material.

The history of previous theoretical work on this subject goes back to Rossby (1938) who, in a pioneer theoretical investigation, came to the conclusion that the ocean responds barotropically to variable applied wind stress. Because this result was so surprising and since it might lead to contradictions with observation in some one of the ways mentioned above, Rossby re-examined his results by means of several different models. In addition to the simple case of two homogeneous layers with an abrupt density discontinuity, he studied a case where the density stratification was continuous in the lower layer; and he even considered the effect of placing another thick homogeneous layer beneath the layer of constant stability. Despite these elaborations he was unable to escape the disconcerting conclusion that currents produced by changes in wind stress are almost independent of depth and are not affected significantly by density stratification.

Since Rossby's conclusion is independent of frequency of applied stress (except that it must be less than the inertial frequency), it seems that one must infer that even seasonal fluctuations of winds produce barotropic changes in the circulation, although in this case the boundaries must certainly have some influence on the circulation.

However, if the boundary conditions are *linear*, then they would change only the horizontal distribution of the velocity field, not the fact that velocity is independent of depth. Finally, it is impossible, on the basis of Rossby's model, to account for the fact that the observed mean wind-induced circulation is confined to the upper layer of the ocean. In the latter case, one can point out that bottom friction, which is not included in Rossby's model, might well be important in preventing the mean wind-induced circulation from extending to the bottom, but it could hardly be effective on a time scale such as that of a storm. Despite difficulties in interpreting Rossby's model for long-period wind variations, in which a completely naïve application of the model leads to results that contradict observation, the theoretical conclusions regarding short-period wind variations seemed quite inescapable.

Rossby's model involves a uniform Coriolis parameter. It is important to see what effect the introduction of a *variable* Coriolis parameter would have on Rossby's conclusions, on the grounds that the variable Coriolis parameter is a vital feature of the steady-state wind-driven circulation theories (Sverdrup, Reid, Stommel, Munk, et al.). Its presence in the vorticity equation might modify the response of the ocean to a variable wind system.

A number of other theoretical studies on the time-response of the ocean have been published in the intervening time, and it may be helpful to the reader to see explicitly the relation of the present study to some of the previous ones. A brief summary of the essential characters of these studies is given in Table I.

The theories of Sverdrup (1947), Reid (1948), Stommel (1948), and Munk (1950) are steady-state theories of the circulation set up by a steady wind system. They provide the justification for neglecting horizontal frictional forces in the interior of the ocean.

The theories of Ichiye (1951) and of Veronis and Morgan (1955) are concerned with changes in the total circulation in an enclosed basin due to variations in wind. The papers deal with a homogeneous ocean.

A recent study by Groves (1954) shows some of the features produced in the Rossby (1938) type of model in the presence of a coast, but since the influence of boundaries is not included in the present study, there will be no further occasion to refer to the Groves paper.

An equation for predicting the change in depth of the seasonal thermocline, in terms of the applied wind-stress field alone, has recently been published (Freeman, 1954); this is based on the assumption that the induced velocity field is baroclinic. In view of the fact that no reason has been formed to dismiss Rossby's 1939 demonstration that

TABLE I.

	<i>Bound- aries</i>	<i>Variation of Coriolis parameter</i>	<i>Time vari- ability of wind</i>	<i>Stratifi- cation</i>
Sverdrup, 1947	+ (one)	+	-	-
Stommel, 1948	+	+	-	-
Reid, 1948	+ (one)	+	-	+
Stockmann, 1946	+	+	-	+
Munk, 1950; and other similar theories	+	+	-	-
Ichiye, T., 1951	+	+	+	-
Veronis & Morgan, 1955	+	+	+	-
Rossby, 1938	- ‡	-	+	+
Cahn, 1945	- ‡	-	+	-
Bolin, 1953	- ‡	-	+	+
Groves, 1954	+ (one)	-	+	+
Charney, 1955	+ (two) †	+ †	+	+
Freeman, 1954	-	-	+	-*
Present study	-	+	+	+

‡ No fixed boundaries, but initial stream restricted in extent.

† Charney does consider variation of Coriolis parameter with latitude in one portion of his study, but not in the model with boundaries.

* In this case the response of the interface is specified, not determined.

short-period response must be barotropic, such an assumption seems unsubstantiable.

During the latter half of 1954, when J. G. Charney (1955) was preparing a contribution for The Convocation held at Woods Hole in connection with the dedication of the new U. S. Navy Laboratory of Oceanography at Woods Hole, we had an opportunity to read his manuscript and to discuss it at length with him; we are indebted to Dr. Charney for many valuable suggestions. His paper uses the quasigeostrophic approximation throughout. Starting with Rossby's two-layer model, he studies the effects of coastal boundaries and of the variation of Coriolis parameter with latitude. The presence of a coastal boundary tends to favor the baroclinic mode (even in the absence of the variation of Coriolis parameter with latitude), a result which may be of fundamental importance in future theoretical studies of such coastal currents or of the Gulf Stream.

Finally, mention should be made of two papers, by Cahn (1945) and Bolin (1953), in which particular attention is paid to the dispersal of energy from an initially concentrated current system by means of inertio-gravitational waves. Whereas the present study does include gravitational and inertial waves, it cannot produce many of the

interesting features of the Cahn and Bolin models. This shortcoming arises from the periodic spatial distribution of applied wind stress in the present study.

2. THE VORTICITY EQUATIONS IN NORMAL MODES

The model which will be considered in the subsequent analysis consists of a horizontally infinite rotating ocean which is of constant depth in the equilibrium state. The density stratification, an idealization of the observed (continuous) density distribution of the ocean, consists of two layers of homogeneous water separated by a surface of discontinuity which corresponds to the mean thermocline. It is assumed that the motions are quasi-hydrostatic and that no momentum is transmitted across the interface by friction. If the motions are small, the equations may be linearized. Thus

$$\frac{\partial u_1}{\partial t} - f v_1 = -g \frac{\partial \eta_1}{\partial x} + \frac{\tau'}{D_1}, \quad (2.1)$$

$$\frac{\partial v_1}{\partial t} + f u_1 = -g \frac{\partial \eta_1}{\partial y} + \frac{\tau}{D_1}, \quad (2.2)$$

$$\frac{\partial}{\partial t} (\eta_1 - \eta_2) + D_1 \left(\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} \right) = 0, \quad (2.3)$$

$$\frac{\partial u_2}{\partial t} - f v_2 = -g \left[a \frac{\partial \eta_1}{\partial x} + b \frac{\partial \eta_2}{\partial x} \right], \quad (2.4)$$

$$\frac{\partial v_2}{\partial t} + f u_2 = -g \left[a \frac{\partial \eta_1}{\partial y} + b \frac{\partial \eta_2}{\partial y} \right], \quad (2.5)$$

$$\frac{\partial \eta_2}{\partial t} + D_2 \left(\frac{\partial u_2}{\partial x} + \frac{\partial v_2}{\partial y} \right) = 0, \quad (2.6)$$

where x (directed eastward) and y (directed northward) are the horizontal co-ordinates; t is time; u_i and v_i ($i = 1, 2$) are the velocities in the x and y directions respectively; D_1 is the equilibrium thickness of the upper layer;⁴ η_1 is the deviation of the free surface from its equilibrium position; g is gravity; $f = 2 \Omega \sin$ (latitude) is the Coriolis parameter; Ω is the angular speed of the earth's rotation; $a = \rho_1/\rho_2$, $b = (\rho_2 - \rho_1)/\rho_2$; ρ_1 is the density of the upper layer; and τ' and τ are stresses in the x and y directions, respectively, exerted on the ocean surface by a wind. Charney (1955) has shown in some detail that it is justifiable to treat the wind stress, which is, of course, actually applied

⁴ The terms with subscript 2 define similar quantities in the lower layer.

at top surface, as a body force distributed evenly with depth over the entire top layer.

If

$$\eta = \eta_1, \phi = \eta_1 - \eta_2 \quad (2.7)$$

are substituted above, one finds

$$\frac{\partial u_1}{\partial t} - f v_1 = -g \frac{\partial \eta}{\partial x} + \frac{\tau'}{D_1}, \quad (2.8)$$

$$\frac{\partial v_1}{\partial t} + f u_1 = -g \frac{\partial \eta}{\partial y} + \frac{\tau}{D_1}, \quad (2.9)$$

$$\frac{\partial \phi}{\partial t} + D_1 \left(\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} \right) = 0, \quad (2.10)$$

$$\frac{\partial u_2}{\partial t} - f v_2 = -g \left[\frac{\partial \eta}{\partial x} - b \frac{\partial \phi}{\partial x} \right], \quad (2.11)$$

$$\frac{\partial v_2}{\partial t} + f u_2 = -g \left[\frac{\partial \eta}{\partial y} - b \frac{\partial \phi}{\partial y} \right], \quad (2.12)$$

$$\frac{\partial}{\partial t} (\eta - \phi) + D_2 \left(\frac{\partial u_2}{\partial x} + \frac{\partial v_2}{\partial y} \right) = 0. \quad (2.13)$$

In the subsequent analysis we propose to take account of the variability of the Coriolis parameter by writing $df/dy = \beta = \text{constant}$ whenever f appears in differentiated form. Otherwise f will be assumed constant. Cross differentiating (2.8) and (2.9) and (2.11) and (2.12), we obtain the vorticity equations

$$\frac{\partial \zeta_1}{\partial t} - \frac{f}{D_1} \frac{\partial \phi}{\partial t} + \beta v_1 = \frac{1}{D_1} \left(\frac{\partial \tau}{\partial x} - \frac{\partial \tau'}{\partial y} \right), \quad (2.14)$$

$$\frac{\partial \zeta_2}{\partial t} - \frac{f}{D_2} \frac{\partial}{\partial t} (\eta - \phi) + \beta v_2 = 0, \quad (2.15)$$

where $\zeta_1 = \partial v_1/\partial x - \partial u_1/\partial y$ and $\zeta_2 = \partial v_2/\partial x - \partial u_2/\partial y$ are the relative vorticities of the upper and lower layers respectively.

At this point it is assumed that the motions are independent of the y co-ordinate. The effect of β is thereby incorporated into the equations of the simple one-dimensional model. Care must be exercised, however, in the choice of equations which describe the system; otherwise the trivial and uninteresting condition $\beta \equiv 0$ results. Thus, in eliminating all but a single dependent variable, one must not include the curl operation, since now $\partial/\partial y \equiv 0$, and the only possible consequence is $\beta \equiv 0$.

The only set of equations which is consistent with the above remarks is:

$$\frac{\partial^2 v_1}{\partial x \partial t} - \frac{f}{D_1} \frac{\partial \phi}{\partial t} + \beta v_1 = \frac{1}{D_1} \frac{\partial \tau}{\partial x}, \quad (2.16)$$

$$\frac{\partial u_1}{\partial t} - f v_1 = -g \frac{\partial \eta}{\partial x}, \quad (2.17)$$

$$\frac{\partial \phi}{\partial t} + D_1 \frac{\partial u_1}{\partial x} = 0, \quad (2.18)$$

$$\frac{\partial^2 v_2}{\partial x \partial t} - \frac{f}{D_2} \frac{\partial}{\partial t} (\eta - \phi) + \beta v_2 = 0, \quad (2.19)$$

$$\frac{\partial u_2}{\partial t} - f v_2 = -g \left[\frac{\partial \eta}{\partial x} - b \frac{\partial \phi}{\partial x} \right], \quad (2.20)$$

$$\frac{\partial}{\partial t} (\eta - \phi) + D_2 \frac{\partial u_2}{\partial x} = 0 \quad (2.21)$$

Eliminating u_1 from (2.17) and (2.18) and substituting the value of v_1 in (2.16) we find for the upper layer

$$r \lambda^2 \eta_{zzzt} - \frac{\phi_{zttt}}{f^2} - \phi_{zt} + r \beta \lambda^2 \eta_{zz} - \frac{\beta}{f^2} \phi_{tt} = \frac{\tau_{zz}}{f}, \quad (2.22)$$

where $r = D_1/D_2$ and $\lambda^2 = gD_2/f^2$; λ is the quantity Rossby called the radius of deformation.

A similar procedure in the last three equations results in the following equation for the lower layer

$$\lambda^2 (\eta - b \phi)_{zzzt} - \frac{(\eta - \phi)_{zttt}}{f^2} - (\eta - \phi)_{zt} + \beta \lambda^2 (\eta - b \phi)_{zz} - \frac{\beta}{f^2} (\eta - \phi)_{tt} = 0 \quad (2.23)$$

The two equations (2.22) and (2.23) may be used for a detailed study of the response of η and ϕ to the variable wind stress τ , but from an analytical point of view it is much simpler to introduce normal modes, which enable one to separate the two dependent variables in the two vorticity equations without increasing the order of the differential form. Equation (2.23), multiplied by the arbitrary constant α , is added to (2.22) to obtain

$$\lambda^2 \left[(r + \alpha) \eta - \alpha b \phi \right]_{zzzt} - \frac{1}{f^2} \left[(1 - \alpha) \phi + \alpha \eta \right]_{zttt} - \left[(1 - \alpha) \phi + \alpha \eta \right]_{zt} + \beta \lambda^2 \left[(r + \alpha) \eta - \alpha b \phi \right]_{zz} - \frac{\beta}{f^2} \left[(1 - \alpha) \phi + \alpha \eta \right]_{tt} = \frac{\tau_{zz}}{f} \quad (2.24)$$

The conditions

$$(1 + \alpha) \phi + \alpha \eta = R \quad \text{and} \quad (r + \alpha) \eta - \alpha b \phi = kR, \quad (2.25)$$

where k is a constant of proportionality, are sufficient to reduce the equations to a single variable. The above conditions determine α and k by the necessity that

$$k = -\frac{\alpha b}{1 - \alpha} = \frac{r + \alpha}{\alpha} \quad (2.26)$$

Therefore

$$\alpha = \frac{1 - r \pm \sqrt{(1 + r)^2 - 4rb}}{2(1 - b)},$$

and

$$\alpha_1 \approx \frac{1 + r - rb}{(1 + r)(1 - b)}, \quad \alpha_2 \approx \frac{-r(1 + r - b)}{(1 + r)(1 - b)}, \quad (2.27)$$

$$k_1 \approx 1 + r, \quad k_2 \approx \frac{rb}{1 + r}.$$

The two values of R (two normal modes corresponding to the two values of α) are

$$R_i = (1 - \alpha_i) \phi_i + \alpha_i \eta \quad (i = 1, 2). \quad (2.28)$$

Thus (2.24) may be written

$$\lambda^2 k_i R_{izzzt} - \frac{1}{f^2} R_{izttt} - R_{izt} + \beta \lambda^2 k_i R_{izz} - \frac{\beta}{f^2} R_{itt} = \frac{\tau_{zz}}{f} \quad (2.29)$$

These will be referred to as the equations of normal modes. Once the normal modes have been determined, it is possible to obtain other more primitive quantities, such as elevation of the free surface and interface, as well as the velocities in each layer from definitions (2.27), (2.28) and equations (2.17), (2.18), (2.20), (2.21); e.g.,

$$\phi = \frac{\alpha_2 R_1 - \alpha_1 R_2}{\alpha_2 - \alpha_1}, \quad (2.30)$$

$$\eta = \frac{(\alpha_2 - 1) R_1 - (\alpha_1 - 1) R_2}{\alpha_2 - \alpha_1} \quad (2.31)$$

3. FREE WAVES

In subsequent sections we shall have occasion to refer to various wave motions which are induced by the action of a transient wind stress on the sea surface. It is appropriate therefore to consider the free waves which are inherent in the normal mode equations and to study thereby the behavior of the solutions. A clear picture of the free wave motions is also desirable because it helps in visualizing certain resonance phenomena which occur in cases where the wind stress is periodic in time.

Consider a normal mode of the form

$$R_i = S_i \sin (lx + \omega_i t) \quad , \quad (3.1)$$

where $S_i = \text{const.}$ and where l is the wave number associated with the wave length L , i.e. $l = 2\pi/L$. The assumed form of the normal mode is therefore a wave progressing in the negative x direction when the frequency ω_i is positive. Substituting R_i into (2.29), we find that the equations reduce to a pair of frequency equations of the form

$$\left(\frac{\omega_i}{f}\right)^3 - \frac{\beta}{fl} \left(\frac{\omega_i}{f}\right)^2 + (1 + \mu_i^2) \frac{\omega_i}{f} + \frac{\beta \mu_i^2}{lf} = 0 \quad (i = 1, 2) \quad , \quad (3.2)$$

where $\mu_i^2 = k_i l^2 \lambda^2$. The solutions of the algebraic equations (3.2) give the frequency ω_i in terms of the wave number l .

From an oceanographic point of view, only a limited range of values of l is of interest, and it will be demonstrated that the approximate roots of the cubic equation within this range of values of l are:

$$\omega_{i1} = \frac{\beta \mu_i^2}{l \gamma_i} \quad , \quad (3.3)$$

$$\omega_{i2} = \frac{\beta}{2l\gamma_i} + \sqrt{\left(\frac{\beta}{2\gamma_i l}\right)^2 + \frac{4\beta^2 \mu_i^2}{l^2 \gamma_i^2} + 4f^2 \gamma_i} \quad , \quad (3.4)$$

$$\omega_{i3} = \frac{\beta}{2l\gamma_i} - \sqrt{\left(\frac{\beta}{2\gamma_i l}\right)^2 + \frac{4\beta^2 \mu_i^2}{l^2 \gamma_i^2} + 4f^2 \gamma_i} \quad , \quad (3.5)$$

where $\gamma_i = 1 + \mu_i^2$. In order to indicate the degree of approximation involved in these expressions for the roots, the original cubic equation may be compared with the expression

$$\frac{1}{f^3} (\omega_i - \omega_{i1}) (\omega_i - \omega_{i2}) (\omega_i - \omega_{i3}) = 0$$

or

$$\left(\frac{\omega_i}{f}\right)^3 - \frac{\beta}{fl} \left(\frac{\omega_i}{f}\right)^2 - (1 + \mu_i^2) \left(\frac{\omega_i}{f}\right) + \frac{\beta\mu_i^2}{fl} (1 + \epsilon_i) = 0,$$

where $\epsilon_i = \frac{\beta^2\mu_i^2}{l^2f^2\gamma_i^3}$. The original cubic frequency equation is distinguished from the approximate equation only by the absence of the term ϵ_i . In order to ascertain the limitations of the approximations, we consider an ocean model with the following properties

$$D_1 = 500 \text{ m}$$

$$\beta = 2 \times 10^{-11} \text{ m}^{-1} \text{ sec}^{-1}$$

$$D_2 = 3500 \text{ m}$$

$$f = 10^{-4} \text{ sec}^{-1}$$

$$r = 1/7$$

$$b = 2 \times 10^{-3}.$$

Then

$$\lambda^2 = 3.5 \times 10^{12} \text{ m}^2$$

$$\epsilon_1 \leq .16 \text{ for all } l$$

$$k_1 \approx 8/7$$

$$\epsilon_2 \leq 4 \times 10^{-6} \text{ for all } l.$$

$$k_2 \approx 2.5 \times 10^{-4}$$

The largest values of ϵ_i occur for $l = 0$, i.e., for an infinite wave length L . Since L will be restricted by $L \leq 12,000$ km, the practical upper limit of ϵ_1 is .02. Therefore the approximate roots are actually close to the real ones.

The phase velocity and frequency of these various waves are shown in Fig. 1 for the range of wave lengths $10 \text{ km} \leq L \leq 12,000 \text{ km}$.

The six wave motions may be separated into two categories. The waves with frequency ω_{11} and ω_{21} represent the geostrophically balanced current motions; ordinarily they contain the measurable flow in the ocean. The remaining four waves arise from a completely different source, viz., the unbalance between Coriolis and pressure forces. These inertio-gravitational motions will often be referred to as "transient" motions, since in a nonperiodic model their energy would be rapidly dispersed into the outlying regions (see Cahn, 1945).

Specifically, the waves with frequency ω_{11} and ω_{21} are barotropic and baroclinic Rossby waves, respectively, moving toward the west. They determine the external and internal geostrophic response of the ocean to an applied wind stress. For scales of motion most often encountered in the ocean, i.e. $L \geq 1,000$ km, the isostatic Rossby wave merges into a nondispersive type of wave.

For the smaller wave lengths ($L \leq 2,000$ km), the ω_{12} and ω_{13} waves are barotropic gravity waves of the familiar nondispersive type moving to the west and east respectively. For larger wave lengths, the waves become dispersive waves of the Sverdrup type (Sverdrup, 1926), although this characteristic of their behavior is practically unnoticeable except for long waves, $L \geq 6,000$ km. In

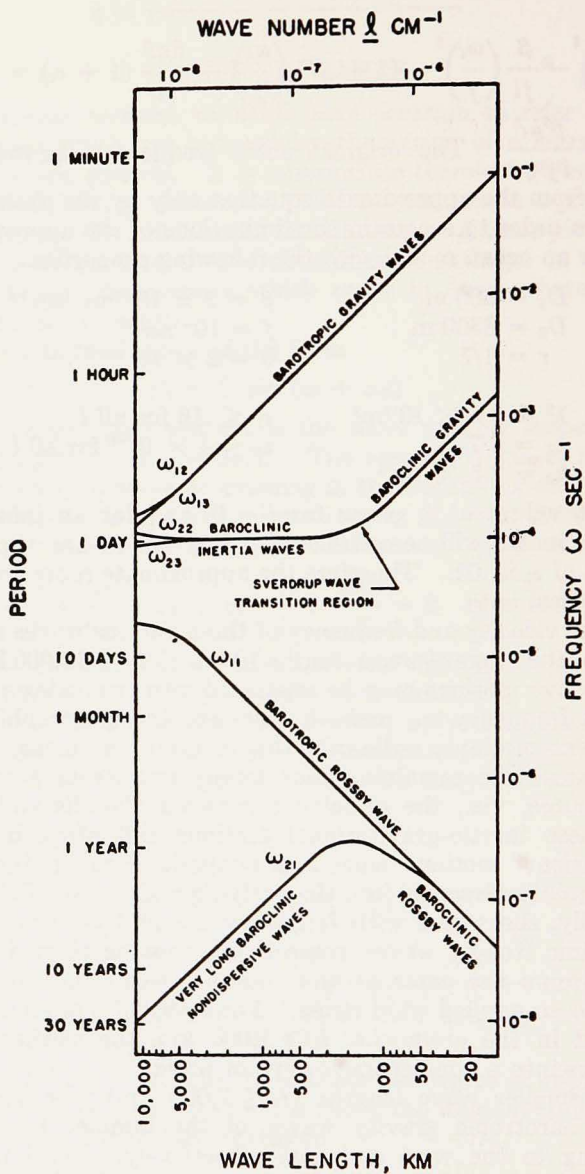


Figure 1

the latter range, both the earth's rotation and the variation of the Coriolis parameter affect the velocities of propagation; thus the westward moving waves ω_{12} move faster than long gravity waves and the eastward moving waves ω_{13} move more slowly.

The waves ω_{22} and ω_{23} are baroclinic waves moving to the west and east respectively. For extremely small scale motions ($L \leq 100$ km), they take the form of internal long gravity waves (speed $\sqrt{bgD_1}$). For somewhat larger wave lengths ($100 \text{ km} \leq L \leq 400 \text{ km}$) they are of the Sverdrup type. For the more realistic values $L > 400$ km they are essentially inertial waves with a period of a half pendulum day.

Since the properties of the various types of waves have been discussed in the literature they will not be considered here in any more detail. Baroclinic and barotropic long gravity, Sverdrup, and inertial waves are discussed in Proudman (1953). Barotropic Rossby waves are discussed by Rossby (1949) and Yeh (1949).

4. MOTIONS CAUSED BY MOVING WIND SYSTEM

As the simplest case of a time-dependent wind stress, consider the y component of wind stress to be of the form

$$\tau = W \sin (lx + \nu t).^5 \quad (4.1)$$

The modes excited by the present form of τ are

$$R_i = S_i \sin (lx + \nu t) \quad , \quad (4.2)$$

where frequency and wave number of the forced motion are the same as those of the stress field. The amplitudes S_i of the normal modes are determined from the vorticity equations (2.29) and are

$$S_i = \frac{Wl}{f^2 \mathfrak{D}_i(\nu, l)} \quad , \quad (4.3)$$

where

$$\mathfrak{D}_i(\nu, l) = \left(\frac{\nu}{f}\right)^3 - \frac{\beta}{lf} \left(\frac{\nu}{f}\right)^2 - (1 + \mu_i^2) \frac{\nu}{f} + \frac{\beta}{lf} \mu_i^2. \quad (4.4)$$

From S_i , the physical quantities may be determined immediately by means of relations

⁵ In this case, τ could be interpreted as wind stress caused by (slowly) moving large-scale wind systems such as those obtained from five-day-mean weather charts. Reasonable values for the frequency ν and for the wave-number l are $\nu = -5 \times 10^{-8} \text{ sec}^{-1}$ and $l = 10^{-6} \text{ m}^{-1}$. These correspond to a period of about two weeks and to a wave-length of 6,000 km. The system moves toward the east and ν is therefore negative. W , the amplitude of stress, is constant.

$$\begin{aligned}
 \phi' &= \frac{rS_1 + S_2}{1 + r} & u_1' &= -\frac{\nu}{D_1 l} \phi' \\
 \eta_1' &= S_1 + \frac{b}{(1 + r)^2} S_2 & u_2' &= -\frac{\nu}{D_2 l} \eta_2' \\
 \eta_2' &= \frac{S_1 - S_2}{1 + r} & v_1' &= \frac{\nu}{f} u_1 + \frac{gl}{f} \eta_1' \\
 v_2' &= \frac{\nu}{f} u_2' + \frac{gl}{f} (\eta_1' - b\phi'),
 \end{aligned} \tag{4.5}$$

where

$$\begin{aligned}
 (\phi, \eta_1, \eta_2, u_1, u_2) &= (\phi', \eta_1', \eta_2', u_1', u_2') \sin(lx + \nu t) \\
 (v_1, v_2) &= (v_1', v_2') \cos(lx + \nu t).
 \end{aligned} \tag{4.6}$$

The exact forms for the solutions will not be written here since they will not be used.

It is apparent that resonance will occur in the present system where the denominator $\mathfrak{D}_i(\nu, l)$ vanishes. Specifically, the resonant frequencies correspond to those of the various free waves (Section 3). An eastward moving wind system can excite resonance only for the internal and barotropic inertio-gravity waves since there are no eastward moving Rossby waves. Since natural wind systems rarely move to the west, there is no point in considering the cases $\nu > 0$.

In footnote 4 we noted that the present form of the wind stress could correspond to slowly moving patterns of ridges and troughs in the atmosphere. The frequencies of these atmospheric motions correspond to periods of the order of several weeks. If such low frequencies are assumed, the quantities $\mathfrak{D}_i(\nu, l)$ are given approximately by

$$\begin{aligned}
 \mathfrak{D}_1(\nu, l) &= - (1 + \mu_1^2) \frac{\nu}{f} + \frac{\beta}{lf} \mu_1^2 \\
 \mathfrak{D}_2(\nu, l) &= - (1 + \mu_2^2) \frac{\nu}{f} + \frac{\beta}{lf} \mu_2^2,
 \end{aligned} \tag{4.7}$$

where ν/f is negative.

Consider the case where $l = 10^{-6} \text{m}^{-1}$ (corresponding to a wave length of approximately 6,000 km). If we use here the values of the physical quantities D_1, D_2 , etc. which were prescribed in Section 3, then $\mu_1^2 = 4$ and $\mu_2^2 = .88 \times 10^{-3}$. It is apparent, therefore, that the $\mathfrak{D}_i(\nu, l)$ given by (4.7) can be further simplified for certain ranges of ν/f . Thus

$$\left. \begin{array}{l} \text{Range 1: } 10^{-2} \leq -\frac{\nu}{f} \leq 10^{-1} \\ \text{or } 70 \text{ days} \geq T \geq 7 \text{ days} \end{array} \right\} \text{yields } \left\{ \begin{array}{l} \mathfrak{D}_1(\nu, l) = -(1 + \mu_1^2) \frac{\nu}{f} + \frac{\beta}{lf} \mu_1^2 \\ \mathfrak{D}_2(\nu, l) = -\frac{\nu}{f} \end{array} \right. ,$$

$$\left. \begin{array}{l} \text{Range 2: } 2 \times 10^{-3} \leq -\frac{\nu}{f} < 10^{-2} \\ \text{or } 1 \text{ year} \geq T > 70 \text{ days} \end{array} \right\} \text{yields } \left\{ \begin{array}{l} \mathfrak{D}_1(\nu, l) = \frac{\beta}{lf} \mu_1^2 \\ \mathfrak{D}_2(\nu, l) = -\frac{\nu}{f} \end{array} \right. , \quad (4.8)$$

$$\left. \begin{array}{l} \text{Range 3: } 0 \leq -\frac{\nu}{f} \leq 2 \times 10^{-5} \\ \text{or } T \geq 100 \text{ years} \end{array} \right\} \text{yields } \left\{ \begin{array}{l} \mathfrak{D}_1(\nu, l) = \frac{\beta}{lf} \mu_1^2 \\ \mathfrak{D}_2(\nu, l) = \frac{\beta}{lf} \mu_2^2 \end{array} \right. ,$$

where $T = -2\pi/\nu$ is the period.

The first range, corresponding to periods which are considerably longer than those of the inertio-gravitational motions, is still comparable to the period of the barotropic Rossby wave; this initial range contains magnitudes that are comparable to observed phenomena in the atmosphere (see footnote 4). Range 2 represents periods which are much longer than the barotropic but shorter than the baroclinic Rossby wave period. The final range exceeds the internal Rossby wave period by at least a factor of 10.

In Range 1, the crosswind velocity component in the upper layer u_1 is simply the Ekman frictional wind-drift velocity directed to the right of the applied wind stress. It is independent of period. This is depicted in Fig. 2, which contains the amplitudes of the four velocities as well as the two surface heights for Range 1, with the magnitude of the wind-stress $W = 1 \text{ cm}^2/\text{sec}^2$. Since the crosswind velocity in the upper layer is independent of period, its total effect on the mass distribution is increased with increasing period. Thus the divergences are larger and the thermocline is displaced further. Fig. 2 shows that the thermocline response is linearly proportional to the period. The free surface deviation is small in this range (less than 2 cm), and for larger periods in the range it is also proportional to the period. The crosswind velocity in the lower layer u_2 tends to compensate for the divergence caused by u_1 in the upper layer. Since it is a function of the free surface deviation as well as of the thermocline response, it is not exactly constant in this range.

The downwind velocities, mainly geostrophic in the large period end of this range, are proportional to period there. It is interesting to note that, for the small periods, the accelerative (nongeostrophic)

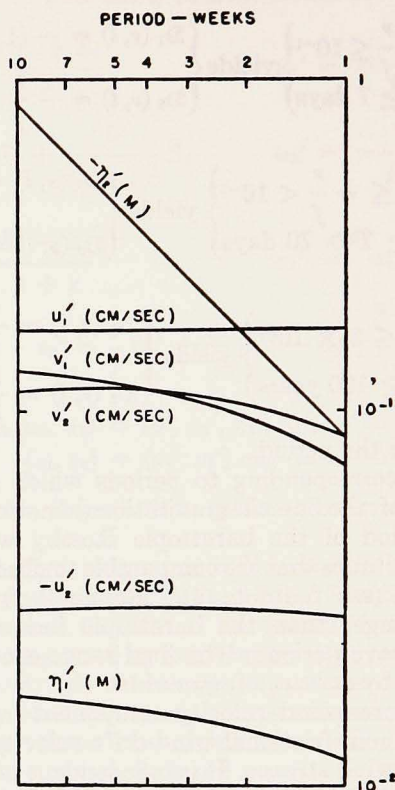


Figure 2

effects which tend to retard the motion are quite prominent, representing roughly 20% of the contribution to the downstream velocity. Since these effects are considerably stronger in the upper layer (where wind stress is a direct cause of the unbalanced state), the downwind velocity of the upper layer is actually smaller than that of the lower layer. The accelerative effects become negligible for a period longer than four weeks. It is evident also that the principal response of the system throughout Range 1 is barotropic. Near the large period end of the range, internal effects become important.

The periods contained in Range 2 represent the seasonal to yearly variations in the wind stress systems. Although boundary effects must certainly become important for such long periods, the results of the present investigation yield some features which are of interest and which will therefore be given here.

The solutions of the two modes are

$$S_1 = \frac{Wf}{\beta g(D_1 + D_2)}, \quad S_2 = -\frac{Wl}{f^2(\nu/f)}, \quad (4.9)$$

and the physical variables become (approximately)

$$\eta_1' = \frac{Wf}{\beta g(1+r)D_2} - \frac{Wlb}{(1+r)^2 f^2(\nu/f)}, \quad (4.10)$$

$$\eta_2' = \frac{Wl}{(1+r)f^2(\nu/f)}, \quad (4.11)$$

$$u_1' = \frac{W}{(1+r)D_1 f}, \quad (4.12)$$

$$u_2' = -\frac{W}{(1+r)D_2 f}, \quad (4.13)$$

$$v_1' = \frac{Wgl}{(1+r)f} \left[\frac{f}{\beta g D_2} - \frac{bl}{(1+r)f^2(\nu/f)} \right], \quad (4.14)$$

$$v_2' = \frac{Wl}{(1+r)\beta D_2} \quad (4.15)$$

When interpreting results for the present range, one must keep in mind the fact that Range 2 corresponds to periods which are considerably longer than the barotropic Rossby wave period but shorter than the baroclinic Rossby wave period. However, in η_1' and v_1' the effect of the internal Rossby wave period appears. The longer the period, the larger the effect of this internal wave.

The following facts are evident from an inspection of (4.10) to (4.15):

(a) The crosswind velocity component in the upper layer is simply the Ekman frictional wind-drift current and is independent of the period. In the lower layer there is a compensating (baroclinic) counterflow.

(b) The thermocline response is directly proportional to the period.

(c) The downwind lower layer velocity, essentially constant, is unaffected by variations in the period.

The free surface deviation η_1' and the downwind upper layer velocity v_1' are dependent on the period. In each case the variable has a constant value upon which is superposed the baroclinic Rossby wave effect (proportional to period). The velocity v_1' exceeds the lower

layer velocity v_2' by the amount $-\frac{bg l^2 W}{(1+r)f^2(\nu/f)} = 1.53 \times 10^{-4}$

(f/ν) cm sec⁻¹. For the longest period in this range, i.e., $T \approx$ one year, this excess value is approximately half of the constant value

$\frac{Wl}{(1+r)D_2\beta}$. The baroclinic effect has therefore become quite important for this extreme value.

For Range 3, the periods are much longer than the internal Rossby wave period. Both D_1 and D_2 and consequently all of the coefficients of the physical quantities are independent of frequency. The flow in this range is now a quasisteady flow. Thus

$$\begin{aligned}n_1' &= \frac{1}{r} \frac{Wl^2}{\beta f \mu_1^2}, \\ \eta_2' &= \frac{1}{rb(1+r)} \frac{Wl^2}{\beta f \mu_1^2}, \\ v_2' &= u_1' = u_2' = 0, \\ v_1' &= \frac{Wl}{\beta D_1}\end{aligned}$$

The velocities are all contained in the upper layer. The total transport Wl/β is equal to that given by Munk (1950) for the steady-state model $\nu/f \rightarrow 0$. In this range of long period variations the flow is geostrophic.

5. MOTIONS CAUSED BY STORMS

We shall consider next a simple transient model, i.e., one in which all of the wave motions discussed in Section 3 are present. It is assumed that the ocean is initially at rest and that a wind stress begins to act on the ocean surface at time $t = 0$. The y -component of the stress is

$$\begin{aligned}\tau &= 0, & t < 0, \\ \tau &= W \cos lx, & t > 0.\end{aligned}\tag{5.1}$$

The effect of a storm on oceanic motions can be investigated with the present model by the simple expedient of adding the new condition $\tau = 0$ for $t > t_0$, where t_0 is, say, four or five days. A more realistic wind stress resulting from the passage of a storm is a moving wave which starts at $t = 0$, i.e.

$$\begin{aligned}\tau &= 0, & t < 0, \\ \tau &= W \cos (lx + \nu t), & t > 0.\end{aligned}\tag{5.2}$$

Except for possible resonance effects, the progressive storm, as opposed to a fixed storm, reduces the importance of the inertio-gravitational motions. However, since we are primarily interested in the qualitative effect of the initial unbalance and since the moving storm adds nothing new to the qualitative behavior, we shall confine our attention to the wind stress defined by (5.1).

The solution of the normal mode equations (2.29), with the wind stress τ given by (5.1), is⁶

$$R_1 = Wlf \left\{ \frac{\cos lx - \cos (lx + \omega_{11}t)}{\frac{\beta}{l} f^2 \mu_1^2} + \frac{\cos (lx + \omega_{12}t)}{2f^3 [1 + \mu_1^2]^{3/2}} - \frac{\cos (lx + \omega_{13}t)}{2f^3 [1 + \mu_1^2]^{3/2}} \right\}, \quad (5.3)$$

$$R_2 = Wlf \left\{ \frac{\cos lx - \cos (lx + \omega_{21}t)}{\frac{\beta}{l} f^2 \mu_2^2} + \frac{\cos (lx + \omega_{22}t)}{2f^3 (1 + \mu_2^2)^{3/2}} - \frac{\cos (lx + \omega_{23}t)}{2f^3 (1 + \mu_2^2)^{3/2}} \right\}, \quad (5.4)$$

where

$$\begin{aligned} \omega_{11} &= \frac{\beta \mu_1^2}{l(1 + \mu_1^2)}, & \omega_{12} &= \frac{\beta \mu_2^2}{l(1 + \mu_2^2)}, \\ \omega_{12} &= f\sqrt{1 + \mu_1^2} + \frac{\beta}{2l(1 + \mu_1^2)}, & \omega_{22} &= f\sqrt{1 + \mu_2^2} \\ & & & + \frac{\beta}{2l(1 + \mu_2^2)}, \quad (5.5) \\ \omega_{13} &= -f\sqrt{1 + \mu_1^2} + \frac{\beta}{2l(1 + \mu_1^2)}, & \omega_{23} &= -f\sqrt{1 + \mu_2^2} \\ & & & + \frac{\beta}{2l(1 + \mu_2^2)}. \end{aligned}$$

The velocities and surface heights can now be determined from (2.30), (2.31), (2.8), (2.10), (2.11), (2.13)

⁶ The solution is approximate, terms of order $\beta^2 k_i^2 \lambda^3 / f^2 \gamma_i^3$ having been considered small when compared to unity. This restriction is the same as the one used in Section 3 for the approximate roots of the frequency equation. The simpler forms for the frequencies, as given below, are also subject to the same approximation within the range of wave lengths of interest.

$$\phi = \frac{Wlf}{r+1} \left\{ \frac{l}{\beta f^2} \left[\frac{r}{\mu_1^2} + \frac{1}{\mu_2^2} \right] \cos lx - \frac{lr \cos (lx + \omega_{11}t)}{\beta f^2 \mu_1^2} - \frac{l \cos (lx + \omega_{21}t)}{\beta f^2 \mu_2^2} + \frac{r}{2f^2(1 + \mu_1^2)^{3/2}} \left[\cos (lx + \omega_{12}t) - \cos (lx + \omega_{13}t) \right] + \frac{1}{2f^2(1 + \mu_2^2)^{3/2}} \left[\cos (lx + \omega_{22}t) - \cos (lx + \omega_{23}t) \right] \right\}, \quad (5.6)$$

$$\eta = \eta_1 = Wl \left\{ \frac{l \cos lx}{\beta f r (r+1) \mu_1^2} - \frac{l \cos (lx + \omega_{11}t)}{\beta f \mu_1^2} - \frac{l \cos (lx + \omega_{21}t)}{\beta r f \mu_2^2} + \frac{1}{2f^2(1 + \mu_1^2)^{3/2}} \left[\cos (lx + \omega_{12}t) - \cos (lx + \omega_{13}t) \right] + \frac{1}{2f^2(1+r)^2(1 + \mu_2^2)^{3/2}} \left[\cos (lx + \omega_{22}t) - \cos (lx + \omega_{23}t) \right] \right\}, \quad (5.7)$$

$$\eta_2 = \frac{Wla}{1+r} \left\{ \frac{l}{\beta f} \left(\frac{1}{\mu_1^2} - \frac{1}{\mu_2^2} \right) \cos lx - \frac{l \cos (lx + \omega_{11}t)}{\beta f \mu_1^2} + \frac{l \cos (lx + \omega_{21}t)}{\beta f \mu_2^2} + \frac{\cos (lx + \omega_{12}t) - \cos (lx + \omega_{13}t)}{2f^2(1 + \mu_1^2)^{3/2}} - \frac{\cos (lx + \omega_{22}t) - \cos (lx + \omega_{23}t)}{2f^2(1 + \mu_2^2)^{3/2}} \right\}, \quad (5.8)$$

$$u_1 = -\frac{W}{D_1(1+r)} \left\{ -\frac{l\omega_{11} r \cos (lx + \omega_{11}t)}{\beta f \mu_1^2} - \frac{l\omega_{21} \cos (lx + \omega_{21}t)}{\beta f \mu_2^2} + \frac{r}{2f^2(1 + \mu_1^2)^{3/2}} \left[\omega_{12} \cos (lx + \omega_{12}t) - \omega_{13} \cos (lx + \omega_{13}t) \right] + \frac{1}{2f^2(1 + \mu_2^2)^{3/2}} \left[\omega_{22} \cos (lx + \omega_{22}t) - \omega_{23} \cos (lx + \omega_{23}t) \right] \right\}, \quad (5.9)$$

$$u_2 = -\frac{Wa}{D_2(1+r)} \left\{ -\frac{l\omega_{11} \cos (lx + \omega_{11}t)}{\beta f \mu_1^2} + \frac{l\omega_{21} \cos (lx + \omega_{21}t)}{\beta f \mu_2^2} + \frac{\omega_{12} \cos (lx + \omega_{12}t) - \omega_{13} \cos (lx + \omega_{13}t)}{2f^2(1 + \mu_1^2)} - \frac{\omega_{22} \cos (lx + \omega_{22}t) - \omega_{23} \cos (lx + \omega_{23}t)}{2f^2(1 + \mu_2^2)^{3/2}} \right\}, \quad (5.10)$$

$$\begin{aligned}
v_1 = & -\frac{W}{(1+r)D_1} \left\{ \frac{\omega_{11}^2 r l \sin(lx + \omega_{11}t)}{\beta f^2 \mu_1^2} + \frac{l \omega_{21}^2 \sin(lx + \omega_{21}t)}{\beta f^2 \mu_2^2} \right. \\
& + \frac{r}{2f^3(1 + \mu_1^2)^{3/2}} \left[-\omega_{12}^2 \sin(lx + \omega_{12}t) + \omega_{13}^2 \sin(lx + \omega_{13}t) \right] \\
& + \frac{1}{2f^3(1 + \mu_2^2)^{3/2}} \left[-\omega_{22}^2 \sin(lx + \omega_{22}t) + \omega_{23}^2 \sin(lx + \omega_{23}t) \right] \left. \right\} \\
& + Wl^2 g \left\{ \frac{-l(1+r) \sin lx}{\beta f^2 r \mu_1^2} + \frac{l \sin(lx + \omega_{11}t)}{\beta f^2 \mu_1^2} + \frac{l \sin(lx + \omega_{21}t)}{\beta f^2 r \mu_1^2} \right. \\
& - \frac{1}{2f^3(1 + \mu_1^2)^{3/2}} \left[\sin(lx + \omega_{12}t) - \sin(lx + \omega_{13}t) \right] \\
& \left. - \frac{b}{2f^3(1+r)^2(1 + \mu_2^2)^{3/2}} \left[\sin(lx + \omega_{22}t) - \sin(lx + \omega_{23}t) \right] \right\} , \tag{5.11}
\end{aligned}$$

$$\begin{aligned}
v_2 = & -\frac{Wa}{(1+r)D_2} \left\{ \frac{l \omega_{11}^2 \sin(lx + \omega_{11}t)}{\beta f^2 \mu_1^2} - \frac{l \omega_{21}^2 \sin(lx + \omega_{21}t)}{\beta f^2 \mu_2^2} \right. \\
& - \frac{\omega_{12}^2 \sin(lx + \omega_{12}t) - \omega_{13}^2 \sin(lx + \omega_{13}t)}{2f^3(1 + \mu_1^2)^{3/2}} \\
& \left. + \frac{\omega_{22}^2 \sin(lx + \omega_{22}t) - \omega_{23}^2 \sin(lx + \omega_{23}t)}{2f^3(1 + \mu_2^2)^{3/2}} \right\} \\
& + Wl^2 g \left\{ \frac{-l \sin(lx + \omega_{21}t) + l \sin(lx + \omega_{11}t)}{\beta f^2 \mu_1^2} \right. \\
& - \frac{\sin(lx + \omega_{12}t) - \sin(lx + \omega_{13}t)}{2f^3(1 + \mu_1^2)^{3/2}} \\
& \left. + \frac{br [\sin(lx + \omega_{22}t) - \sin(lx + \omega_{23}t)]}{2f^3(1+r)^2(1 + \mu_2^2)^{3/2}} \right\} . \tag{5.12}
\end{aligned}$$

From the above form of the solutions it can be seen that the various contributions consist of a stationary meridional flow and/or the six progressive waves discussed in Section 3. Comparison of the upper and lower layer velocities shows that amplitudes of the barotropic waves in the lower layer are essentially equal to those of the upper layer. The amplitude of the internal wave contributions of the lower layer differ by the factor $-r$. The stationary wave is not present in the lower layer since it is a direct consequence of wind action. With these facts in mind, only upper layer velocities and

surface heights will be discussed in the remaining portion of this section.

As stated earlier, the waves with second subscript equal to unity are the geostrophically balanced Rossby waves (see also Charney, 1955). Representing the geostrophic current motions, they are the features of flow which are ordinarily measurable. The remaining four terms are accelerative effects and have periods of less than a half pendulum day. Thus, for studies of motions with a sufficiently large time scale (say, two weeks or longer), the Rossby waves represent the essential form of the solution.

TABLE II.

Values of ω_i for three values of L .

	$L = 200$ km	$L = 2000$ km	$L = 8000$ km
ω_{11}	6.32×10^{-7} sec $^{-1}$	6.17×10^{-6} sec $^{-1}$	1.81×10^{-5} sec $^{-1}$
ω_{12}	6.22×10^{-3} sec $^{-1}$	6.3×10^{-4} sec $^{-1}$	1.91×10^{-4} sec $^{-1}$
ω_{13}	-6.22×10^{-3} sec $^{-1}$	-6.3×10^{-4} sec $^{-1}$	-1.83×10^{-4} sec $^{-1}$
ω_{21}	2.95×10^{-7} sec $^{-1}$	5.53×10^{-8} sec $^{-1}$	1.38×10^{-8} sec $^{-1}$
ω_{22}	1.37×10^{-4} sec $^{-1}$	1.03×10^{-4} sec $^{-1}$	1.13×10^{-4} sec $^{-1}$
ω_{23}	-1.37×10^{-4} sec $^{-1}$	$-.97 \times 10^{-4}$ sec $^{-1}$	$-.87 \times 10^{-4}$ sec $^{-1}$

The coefficients of the various wave contributions of η_1 , η_2 and u_1 . The three (vertical) values given for each coefficient are for $L = 200$, 2000, and 8000 km.

Coefficient of	η_1 (in m)	η_2 (in m)	u_1 (in cm/sec)
	.3	-150	—
$\cos lx$.3	-150	—
	.3	-150	—
	.0375	3.3×10^{-2}	1.9×10^{-6}
$\cos(\omega_{11}t + lx)$.0375	3.3×10^{-2}	1.8×10^{-3}
	.0375	3.3×10^{-2}	2.1×10^{-3}
	.2625	150	.276
$\cos(\omega_{21}t + lx)$.2625	150	.54
	.2625	150	.153
	1.8×10^{-6}	1.65×10^{-6}	-9.2×10^{-6}
$\cos(\omega_{12}t + lx)$	1.8×10^{-6}	1.6×10^{-4}	-9×10^{-4}
	1.8×10^{-6}	-1.6×10^{-4}	-1.1×10^{-3}
	-1.8×10^{-6}	-1.65×10^{-6}	-9.2×10^{-6}
$\cos(\omega_{13}t + lx)$	-1.8×10^{-6}	-1.6×10^{-4}	-9×10^{-4}
	-1.8×10^{-6}	-1.6×10^{-4}	-1.1×10^{-3}
	2.8×10^{-4}	1.6×10^{-1}	-.11
$\cos(\omega_{22}t + lx)$	7.3×10^{-5}	4.1×10^{-2}	-.27
	1.8×10^{-5}	1.0×10^{-2}	-.30
	-2.8×10^{-4}	-1.6×10^{-1}	-.11
$\cos(\omega_{23}t + lx)$	-7.3×10^{-5}	-4.1×10^{-2}	-.27
	-1.8×10^{-5}	-1.0×10^{-2}	-.23

TABLE II. (Continued)

The coefficients of the various wave contributions of v_1 . The first column represents the $\partial u_1/\partial t$ (or acceleration) contribution to the v_1 term, the second column gives the geostrophic or $g \partial \eta/\partial x$ contribution to v_1 .

Coefficient of:	v_1 (in cm/sec) ($\partial u_1/\partial t$ contribution)	v_1 (in cm/sec) ($g \partial \eta/\partial x$ contribution)
	—	-96
$\sin lx$	—	- 9.6
	—	- 2.4
	-1.2×10^{-7}	12
$\sin(\omega_{11}t + lx)$	-1.1×10^{-4}	1.2
	$- 4 \times 10^{-3}$.3
	-8.4×10^{-4}	84
$\sin(\omega_{21}t + lx)$	$- 3 \times 10^{-4}$	8.4
	-7.2×10^{-3}	2.1
	5.7×10^{-4}	- .6 $\times 10^{-4}$
$\sin(\omega_{12}t + lx)$	5.7×10^{-3}	- 5.7 $\times 10^{-3}$
	2.1×10^{-2}	- 1.4 $\times 10^{-2}$
	-5.7×10^{-4}	6 $\times 10^{-4}$
$\sin(\omega_{13}t + lx)$	-5.7×10^{-3}	5.7 $\times 10^{-3}$
	-1.9×10^{-2}	1.4 $\times 10^{-2}$
	.19	- 9 $\times 10^{-3}$
$\sin(\omega_{22}t + lx)$.28	- 2.1 $\times 10^{-3}$
	.34	- 1.4 $\times 10^{-4}$
	-.19	9 $\times 10^{-3}$
$\sin(\omega_{23}t + lx)$	-.25	2.1 $\times 10^{-3}$
	-.20	1.4 $\times 10^{-4}$

Table II gives compilations of the amplitudes of the various wave motions for three separate values of l corresponding to wave lengths of 200, 2000 and 8000 km. The 200 km wave length does not represent any actual wind-driven motions, but it does give a picture of the type of motion which can be expected for such a scale length. The other two cases, i.e., wave lengths of 2,000 and 8,000 km, represent the practical scale limits of wind-driven oceanic motions. The numerical values shown in Table II are based on the physical properties of the ocean (D_1 , D_2 , b , etc.) prescribed in Section 3. The wind-stress amplitude, taken as 3 dynes/cm², corresponds to wind velocities commonly found in storms.

In interpreting the results of Table II, it is necessary to consider the frequencies of the various waves. For example, in the values for surface heights η_1 , η_2 and downstream velocity v_1 , the amplitudes of the internal Rossby wave and of the standing wave are much larger than any of the other individual amplitudes. However, these two amplitudes differ in sign and, because of the low frequency of the baroclinic

Rossby wave, their net effect is small (comparable in magnitude to the external Rossby wave amplitude) for quite a long period of time. On the other hand, the barotropic Rossby wave, which has a considerably smaller amplitude, actually attains its maximum value after just a few days. For practical purposes, therefore, the geostrophic response of the ocean is a combination of the barotropic and baroclinic modes.

In order to extend the results to storms of finite duration, it is necessary to subtract from the above form of the solution a similar contribution with t replaced by $t - t_0$, where t_0 is the length of time during which the wind acts. From the values of the frequencies it is then clear that the internal Rossby wave does not attain its maximum amplitude for storms of, say, five days duration. The remaining terms execute at least a fourth of one oscillation in that time—that is, their maximum values are attained.

Note that the barotropic long gravity wave amplitude in the velocity terms is always at least an order of magnitude smaller than that of the baroclinic inertial wave. In the lower layer the internal amplitudes are reduced by a factor of $-r = -1/7$ and the two terms may become comparable.

We have separated the magnitudes of the various waves of the downstream velocity v_1 into two sections that correspond to the contributions from the acceleration term $\partial u_1/\partial t$ and the pressure term $g \partial \eta/\partial x$ in the first equation of motion:

$$v_1 = \frac{1}{f} \left(\frac{\partial u_1}{\partial t} + g \frac{\partial \eta}{\partial x} \right)$$

The purpose of this separation is to compare the relative effects of acceleration and pressure in building up the downwind velocity. As one would expect, the pressure terms are the dominant forces in producing the barotropic and internal Rossby waves, since these motions are essentially geostrophically balanced. However, the high frequency inertio-gravitational motions present an entirely different picture. These motions result primarily from acceleration effects.

Separation of acceleration and pressure terms has resulted in a rather surprising characteristic of the baroclinic inertio-gravitational motions. Rossby (1938) presented an intuitive picture of inertial motions in a homogeneous ocean. He suggested that an unbalanced stream is shifted to the right (in the northern hemisphere) of its downstream direction and that its inertia carries it beyond the equilibrium position to a point where the pressure gradients caused by the resultant

piling up of mass force it back again. The stream thus oscillates about its equilibrium position with a period of a half pendulum day. This intuitive picture was confirmed by the quantitative results of Cahn (1945). In a note following Cahn's paper, Starr (1945) suggested that the extension to a stratified system was relatively simple—that one can extend the results to a baroclinic ocean with relatively minor changes. Inspection of the internal inertial wave amplitudes for v_1 in Table I shows that such an extension is unjustified. It is seen that the Coriolis force is almost completely balanced by the acceleration terms and that neither the free surface nor the interface changes much in this contribution to the flow. *Thus the transient internal motion is purely an inertial motion* of the type such as a bead may execute when constrained to move on a rotating frictionless sphere.

At the outset of the investigation we had the impression that a possible indication of distant storms would be a measure of the change of the interface and that the period of oscillation would be roughly a half pendulum day. However, unless other effects, such as sloping bottom or resonance, contribute to amplification of the interface changes, this quantity would be quite unmeasurable. The velocities⁷ resulting from such storms at distant regions are also quite small. Possibly the only noticeable effect would be the amplitude of the barotropic Rossby wave contribution to the free surface height. Perhaps such measurements could be made with tide gauges on isolated oceanic islands.

As stated earlier, the "permanent" current velocity is a combination of the barotropic and internal Rossby wave contributions to the downwind velocity. Rossby's conclusion, viz., that changes in the winds must produce motions which extend to the ocean bottom, is verified here. These velocities are so small, however, that they are comparable only to the slow deep-water drift which results from large sources of heating and cooling. For example, a storm with a scale of 2,000 km, applying a wind stress of 3 dynes/cm² to the sea surface, gives rise to a maximum velocity of approximately 1 cm/sec in the deep layers of the ocean (result of barotropic Rossby wave contribution). The maximum value is attained in a period of three days. Thus the maximum possible mixing length attributable to such a storm is approximately 3×10^5 cm. This leads to a coefficient of lateral eddy diffusivity considerably smaller (3×10^6 cm²sec⁻¹) than the one assumed by Sverdrup (1939), viz. 10^8 cm²sec⁻¹. His justification for using such a value is found in the following quotation from his paper (Sverdrup,

⁷Frictionally wind-driven surface velocities would be measurable. However, these are in the direct vicinity of the storm area.

1939: 196): "Rossby (1938) has, however, shown that changes in the winds over the ocean may produce currents reaching from the surface to the greatest depths. All such motions will be directed at random and Rossby's conclusions, therefore, can be interpreted as showing that changing wind conditions may produce variable currents in the deep water, but no flow in a definite direction. If this is the case, a large scale lateral mixing may take place." His reasoning is probably based on the following excerpt from Rossby (1938: 262): "However, since the large scale atmospheric wind systems which drive the ocean circulation change from day to day and from season to season, it is permissible to state with a reasonable degree of assurance, that it is entirely inappropriate to consider the homogeneous bottom water as inert beyond the slow thermal circulation maintained by antarctic cooling."

Both of these statements are qualitatively correct. However, from quantitative considerations these statements appear to be misleading. The deep lateral mixing due to storms is probably negligible on the basis of the present investigation.

A few remarks should probably be made concerning the effect of the change of the Coriolis parameter with latitude. An investigation of a model with constant Coriolis parameter reveals that the geostrophically balanced downwind velocities must increase linearly with time. The essential effect of β is to keep the velocities finite (in the form of moving waves). The two solutions become comparable for sufficiently small values of t (less than a day) where the sinusoidal time-dependent Rossby wave terms can be linearized. In none of the inertio-gravitational wave frequencies does the β term contribute an effect of more than two percent. Therefore the effect of β on inertial oscillations is essentially negligible, whereas its effect on the geostrophically balanced motions is such as to alter the solution completely (from a linear function of time to a moving wave form) for time scales larger than the inertial period.

An important question which has been raised by various authors in the past concerns the amount of energy absorbed by the inertial oscillations when a transient wind blows over the ocean. This question may be partially answered by the present model, although a complete investigation would necessarily embrace many more features than those included here. By restricting our attention to the initial stages of the motion, say, to the first day or so, we may investigate the case of $\beta = 0$ as a good approximation to the present model. The total energy E of the flow between the limits $-L \leq x \leq L$ and per unit thickness in the y direction may be derived from a combination of equations (2.1) to (2.6) (with $\beta = 0$)

$$D_1 \frac{\rho_1}{2} \iint_0^L \frac{\partial}{\partial t} \left(u_1^2 + v_1^2 + \frac{g\eta_1^2}{D_1} \right) dydt + \frac{D_2\rho_2}{2} \iint_0^L \frac{\partial}{\partial t} \left(u_2^2 + v_2^2 + \frac{\Delta\rho g}{D_2} \eta_2^2 \right) dydt = \rho_1 \iint_0^L \tau v_1 dydt.$$

The term on the left represents the total (kinetic and potential) energy of the flow. The right-hand term, the energy input, can be evaluated directly

$$\rho_1 \iint_{-L}^L \tau v_1 dt dy = \frac{W^2 \rho_1 L}{D_1} \left\{ \frac{f^2 D_2}{D p_1^4} (1 - \cos p_1 t) + \frac{f^2 D_1}{D p^4} (1 - \cos p t) + \frac{1}{2} \left[1 - \frac{f^2 p_2^2}{p^2 p_1^2} \right] t^2 \right\},$$

where $p^2 = f^2 + gDl^2$, $p_1^2 = f^2 + \frac{bgD_1 D_2 l^2}{D}$, $p_2^2 = f^2 + gD_2 l^2$, and $D = D_1 + D_2$. The two sinusoidal terms represent the input energy which is absorbed by the inertio-gravitational motions; the remaining term (t^2) is the geostrophic current energy. For a wave length of 2,000 km, the term $\frac{f^2 D_1}{D p^4} (1 - \cos p t)$ (corresponding to the barotropic inertio-gravitational motions) is negligible as compared to either of the other terms. Hence a comparison of inertio-gravitational energy with geostrophic current energy is afforded by the ratio

$$\frac{E_i}{E_g} = \frac{\frac{f^2 D_2}{D p_1^4} (1 - \cos p_1 t)}{\frac{1}{2} [1 - f^2 p_2^2 / p^2 p_1^2] t^2}.$$

Time t in this expression is to be interpreted as the time during which energy is added to the system by a wind stress. If the length of time is considerably shorter than a half pendulum day, most of the energy is absorbed by transient oscillatory motions. For times longer than a half pendulum day, the maximum amount of energy is contained in the geostrophic current field. These facts are shown in Fig. 3, which gives the ratio E_i/E_g as a function of time of wind duration for $t < 1$ pendulum day. The graph cannot be carried any

further as a function of time because linearization of the Rossby waves is no longer valid. The tendency, however, is clearly indicated even for the short period shown in Fig. 3.

The case which we have chosen is conducive to strong inertial oscillations. If the wind system were moving with a speed of 10 m/sec, the reverse action of the wind after a day or so would have a destruc-

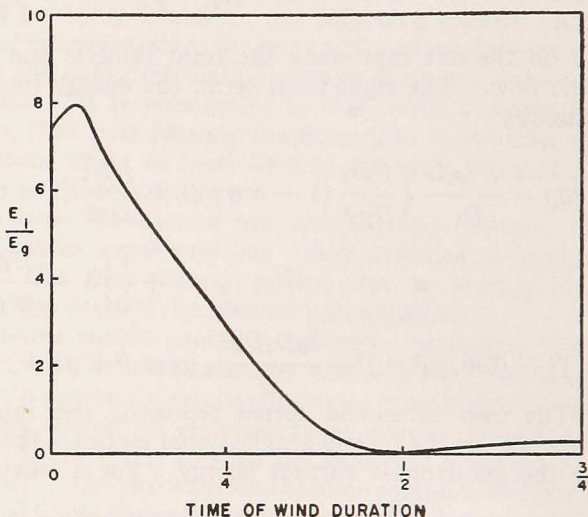


Figure 3

tive influence on the inertio-gravitational motions. However, the transient motions will dominate in the initial period ($< 1/2$ pendulum day) regardless of whether the wind system is moving, unless the frequency of the moving system is unrealistically large. Therefore, for motions with a sufficiently large time scale (> 1 pendulum day), the flow may be considered as essentially quasigeostrophic (Charney, 1955).

6. APPLICABILITY OF THE MODEL TO THE OCEAN

There are several questions concerning the applicability of the present model to the ocean which merit particular consideration: (1) To what extent are variable motions in the ocean independent of the presence of boundaries? (2) How justifiable is the neglect of friction? (3) Are other forces such as atmospheric pressure important in setting up oceanic currents? and (4) What are the effects of bottom topography on the induced motions?

(1) The disturbance set up by an instantaneous change in wind is propagated toward the boundaries in the form of waves. The length of time required for the influence of the boundaries to become evident at an interior point of the ocean therefore depends upon the time required for a wave to move from that point to the boundary. Since the wave velocities differ for each mode of motion, strict application of the model is limited by the time it takes the fastest wave (barotropic gravity wave) to reach the coast (a matter of a few hours). However, there are two reasons for assuming that the essential results of this model would not be affected by coastal influences for a considerably longer period. First, there is no interaction between the various modes in the interior of the ocean and there is no reason to suppose that there is any interaction between the transient and Rossby modes at the coast. Therefore, so far as the balanced motion is concerned, the model may be used in the interior of the oceans for the time it takes a barotropic Rossby wave to reach the coast (a matter of a few days). Second, the amount of energy contained in the transient motions is considerably smaller than that in the balanced motions. The reflection of this energy from the coasts would exert a relatively negligible influence on any observable motions. It seems likely, therefore, that the geostrophically balanced motion of the real ocean should respond to a three or five day storm much like the theoretical model—that is, during the “active phase” or life time of a central oceanic storm, the effect of coastal boundaries should be negligible.

It is clear that, for periods longer than a week, the barotropic Rossby mode of motion may be strongly influenced by the coasts. The length of time over which the internal Rossby mode is unaffected by the coasts could be as much as several years were it not for the uncertainty as to whether the two Rossby modes can interact at the coasts. There is a possibility that the proper boundary conditions are nonlinear in a coastal current such as the Gulf Stream, in which case there might be an interaction between the barotropic and baroclinic Rossby modes. This is the reason why we hesitate to extend the discussion of the internal Rossby mode to wind fluctuations of a seasonal time scale.

If there is, in fact, no interaction between Rossby modes at the coasts, this model leads us to the conclusion that equilibrium adjustment of the main thermocline in the central ocean does not occur even in response to wind variations with periods as long as a year.

(2) The effect of friction is neglected in the model under discussion, and it is important to form some estimate of the frictional force. For bottom friction, Proudman (1953) gives the frictional force on the ocean

bottom as $F = -k \rho u^2$, where $k = 2 \times 10^{-3}$. The bottom velocities involved in the models under discussion are less than those due to deep sea tides. The amplitude of the tidal velocity U may be regarded as purely periodic, in which case the resultant frictional force due to a long period deep sea velocity component u is simply $F = -2k U u$. If the bottom layer is of depth D , then the time constant K of the frictional damping of the lower layer velocity is $K = kU/D$. Taking representative values $k = 2 \times 10^{-3}$, $U = 2 \text{ cm sec}^{-1}$, $D = 4 \times 10^5 \text{ cm}$, one calculates $K = 10^{-8} \text{ sec}^{-2}$. Thus the length of time necessary for destruction of the velocity of the bottom layer by bottom friction is approximately three years.

The time constant involved in lateral friction is $K' = Al^2$, where l is the wave number and A the coefficient of lateral eddy viscosity. For the largest scale phenomena in the deep sea (Sverdrup, 1939), it appears that a largest possible limit of A is $10^8 \text{ cm}^2 \text{ sec}^{-1}$. Taking $l = 1/2 \times 10^{-8} \text{ cm}^{-1}$, the time constant becomes $K' = 0.25 \times 10^{-8}$. Thus lateral friction of the most intense sort can destroy motion in these models in the bottom layer only after 12 years. The conclusion is that the frictionless models are justified only for periods up to about one year, and that physically they should include friction explicitly for longer periods.

(3) There are other surface stresses which may affect the response of the ocean also. These are the variations of atmospheric pressure and the divergence of the wind stress. (The latter term is approximately an order of magnitude smaller than the curl of the wind stress.) Neither of these driving terms contributes to the current motions, but they tend only to increase the transient motions in the ocean. For this reason they have not been considered, although any thorough study of inertial oscillations would necessarily entail a detailed study of these effects.

(4) The effect of bottom topography has been neglected altogether. Perhaps there is no other justification for excluding this feature than the fact that the analysis is quite complex when it is included. It is conceivable that its effect may be important, particularly for the stratified ocean.

7. SUMMARY AND CONCLUSIONS

We have investigated the response of an infinite rotating ocean of finite depth to a transient wind stress applied at the sea surface. The density structure, an idealization of the observed stratification, consists of two layers of homogeneous water. The variation β of the Coriolis parameter f with latitude is included in the equations by taking $\beta = df/dy = \text{constant}$; elsewhere, f is assumed to be constant.

By using normal modes, the equations of motion of a two layer system can be reduced to a vorticity equation in a single variable whose highest order derivative is of the same order as that in the homogeneous system. The vorticity equation contains two separate types of motion corresponding to the two modes of oscillation in the system. In the barotropic case the motion is uniform vertically. The second type, baroclinic motion, occurs when the lower layer responds in such a manner as to offset the pressure gradient caused by deviations of the free surface from its equilibrium position.

Substitution of a moving wave solution in the homogeneous vorticity equation yields a frequency equation which shows that six different types of waves may be excited in the system. Two of the waves, the barotropic and baroclinic Rossby waves, represent the motions which are in geostrophic balance. The remaining four waves (two barotropic and two baroclinic) represent inertio-gravitational motions which result from incomplete balance between pressure and Coriolis forces. Behavior of the latter waves is dependent on both the Coriolis parameter and the static parameters (depth, density, gravity) of the system. Generally, however, the barotropic waves are pure gravity waves travelling with speed \sqrt{gD} , and the internal waves are pure inertial waves with a period of a half pendulum day, $2\pi/f$.

Two types of wind-driven motions are treated. The forced motions caused by a moving wave system with a scale comparable to atmospheric patterns as given by five-day mean charts are investigated by assuming a driving force of the type $W \sin(lx + \nu t)$, with $W = 1 \text{ cm}^2 \text{ sec}^{-1}$. The resultant motions may be studied by considering three separate ranges of period of the driving wind.

For periods of one to seven weeks (comparable to the barotropic Rossby wave period), the ocean responds principally as a homogeneous body of water. The baroclinic effects increase in importance with longer period.

For longer periods (up to a year, thus lying between the barotropic and baroclinic Rossby wave period) the motion is partly barotropic although the effects of baroclinity are more prominent. The upper layer downwind velocity is approximately half barotropic-half baroclinic when the wind period is a year for the scale chosen. The other variables are effected less by the baroclinity.

When the wind period is long (at least 100 years) the motion is purely baroclinic. The flow is geostrophic and the total transport is equal to that given by Munk for the steady-state model.

The second type of wind-driven motions treated here is that caused by a nonmoving wind which varies sinusoidally in space and which

begins impulsively at some time. In this case all of the possible free wave motions of the system occur, and one can study the relative importance of the inertio-gravitational and geostrophically balanced motions.

When the wind acts for a period of time comparable to the life of a storm, the geostrophically balanced motion is partially external and partially internal. The precise partition of the energy between the two modes depends on the scale of the motion. The thermocline therefore responds to the action of the wind stress and may be measurable (of the order of 10 to 20 m) when the wind blows for three days or more. The currents set up in the deep layers are small and are probably no stronger than the slow thermal currents resulting from antarctic cooling. Consequently, as a result of storms there is probably no mechanism for the formation of large-scale lateral mixing in the oceanic stratosphere.

The isostatic inertio-gravitational motions are much larger than the barotropic inertio-gravitational motions. However, they are not accompanied by any measurable movement of the thermocline; instead, they are purely horizontal inertial motions. In a stratified ocean, therefore, the observed inertial oscillation is not accompanied by pressure gradients resulting from excess piling up of water. Instead, it is mainly a lateral movement with an oscillation caused by the earth's rotation only.

From the present simple analysis, the essential effect of β in a transient problem is to amplify the baroclinic mode for large periods.

A simple investigation of the relative amounts of energy contained in the inertio-gravitational and geostrophically balanced flow indicates that, so long as the wind blows for more than a half pendulum day, most of the energy is contained in the geostrophically balanced flow. For winds of shorter duration the energy is expended in inertio-gravitational motions.

BIBLIOGRAPHY

BOLIN, B.

1953. The adjustment of a non-balanced velocity field towards geostrophic equilibrium in a stratified fluid. *Tellus*, 5 (3): 373-385.

CAHN, A.

1945. An investigation of the free oscillations of a simple current system. *J. Meteor.*, 2 (2): 113-119.

CHARNEY, J. G.

1955. The generation of ocean currents by wind. *J. Mar. Res.*, 14 (4): 477-498.

FREEMAN, J. C.

1954. Note on a prediction equation for the surface layer of a two-layer ocean. *Trans. Amer. geophys. Un.*, 35 (4): 585-587.

GROVES, G. W.

1954. Dynamic model of a two-layer ocean under influence of atmosphere and coastal boundary. Tech. Rept. No. 54-22, Scripps Institution of Oceanography, 27 pp.

ICHIYE, T.

1951. On the variation of oceanic circulation. *Oceanogr. Mag.*, 3 (2): 79-82.

MUNK, W. H.

1950. On the wind-driven ocean circulation. *J. Meteorol.*, 7 (2): 79-93.

PROUDMAN, J.

1953. *Dynamical Oceanography*. London Methuen, London. 409 pp.

REID, R. O.

1948. The equatorial currents of the eastern Pacific as maintained by the stress of the wind. *J. Mar. Res.*, 7 (2): 74-99.

ROSSBY, C. G.

1938. On the mutual adjustment of pressure and velocity distribution in certain simple current systems, II. *J. Mar. Res.*, 1: 239-263.
1949. On the dispersion of planetary waves in a barotropic atmosphere. *Tellus*, 1: 54-58.

STARR, J.

1945. Note on inertial oscillations. *J. Meteorol.*, 2 (2): 120-122.

STOCKMANN, W. B.

1946. Equations of the field of integrated currents caused by the wind in a non-homogeneous ocean. *Doklady Akad. Nauk. SSSR*, 54 (5): 403-406.

STOMMEL, HENRY

1948. The westward intensification of wind-driven ocean currents. *Trans. Amer. geophys. Un.*, 29 (2): 202-206.

SVERDRUP, H. U.

1926. Dynamic of tides on the North Siberian shelf. *Geofys. Publ.*, 4 (5): 75 pp.
1939. Lateral mixing in the deep water of the South Atlantic Ocean. *J. Mar. Res.*, 2 (3): 195-207.
1947. Wind-driven currents in a baroclinic ocean; with application to the equatorial currents of the eastern Pacific. *Proc. Nat. Acad. Sci.*, 33 (11): 318-326.

VERONIS, G.

1956. Partition of energy between geostrophic and non-geostrophic oceanic motions. *Deep-Sea Res.*, 3 (3): 157-177.

VERONIS, G. AND G. W. MORGAN

1955. A study of the time-dependent wind-driven ocean circulation. *Tellus*, 7 (2): 232-242.

YEH, T.

1949. On energy dispersion in the atmosphere. *J. Meteor.*, 6 (1): 1-16.