THE DYNAMIC STRUCTURE OF A COASTAL PLAIN ESTUARY[1]

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ABSTRACT

The time mean equations of motion applicable to a coastal plain estuary are developed and discussed. Time series observations of temperature, salinity, and current velocity from two sections in the James River are utilized to obtain the mean lateral and longitudinal components of the relative pressure field, the Coriolis force, and the field accelerations. Using appropriate boundary conditions, the equations of motion are then solved for the nonadvective or eddy flux of momentum. As a consequence of this solution, the depths of the pressure surfaces which are level relative to the longitudinal and lateral coordinates are found. The variations with depth of the longitudinal and lateral components of the pressure force are discussed, and the relative importance of the various terms in the equations of motion are evaluated.

INTRODUCTION

Notable advances in our understanding of the dynamics of estuarine circulation have been made by Stommel (1951) and Cameron (1951). It has not been possible thus far to solve the general hydrodynamic equations completely, hence it was necessary that these investigators make certain assumptions as to the relative importance of the various processes governing the dynamic structure of the estuary being investigated. Both Stommel and Cameron treated the relatively deep fiord estuary. Analysis of chemical and physical observations in Chesapeake Bay and its tributary estuaries shows that the existing treatments of the dynamics and mixing do not adequately describe conditions in this particular type of estuary.

In order to evaluate the relative importance of the various processes involved in controlling the dynamic structure in an estuary, extensive field observations have been combined with certain basic hydrodynamic equations. The procedure employed involves the treatment of the hydrodynamic equations in such a manner that unknown terms can be determined from observations of the mean distribution of tem-

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perature, salinity, and current velocity. The purpose of such an investigation is to lay a firm foundation for a more complete attack on the dynamics of estuarine circulation.

Extensive field studies of the distribution of chemical and physical properties have been under way in Chesapeake Bay and its tributary estuaries for the last several years. These investigations reveal that the general features of the salinity distribution and of the circulation pattern are similar among the several major estuaries which comprise the Chesapeake Bay system. During the summer of 1950 an intensive investigation was undertaken in the James River estuary; a summary discussion of the general features of salinity and current observations in the Chesapeake Bay system, with particular emphasis on the detailed study of the James River, has been reported (Pritchard, 1952).

Data collected from the James River estuary were employed in a detailed analysis of the salt balance in a coastal plain estuary. This study (Pritchard, 1954) showed that when time mean values of the various parameters over several tidal cycles are considered, the two most important terms involved in maintaining the salinity distribution are the mean horizontal advection and the nonadvective vertical flux of salt; the nonadvective horizontal flux of salt was found to be small. Use will be made of this latter fact in the present analysis.

We will consider a left-handed coordinate system with its origin in fresh water at the head of the estuary. The coordinate axes will be designated as x_i, the x_1-axis being directed longitudinally along the central axis of the estuary toward the ocean. The x_2-axis is directed vertically downward, the x_3-axis laterally toward the right-hand shore of the estuary.

THE LONGITUDINAL COMPONENT OF THE EQUATION OF MOTION

The purpose of this study is to investigate the relative magnitudes of the various terms which comprise the equation of motion as applied to a coastal plain estuary. We will first consider the x_1-component, which, for instantaneous values of the various parameters, may be written as

$$(1)\quad \frac{\partial v_1}{\partial t} + v_1 \frac{\partial v_1}{\partial x_1} + v_2 \frac{\partial v_1}{\partial x_2} + v_3 \frac{\partial v_1}{\partial x_3} = -\alpha\, \partial p/\partial x_1 - f v_3 + \alpha \left(\frac{\partial P_{11}'}{\partial x_1} + \frac{\partial P_{12}'}{\partial x_2} + \frac{\partial P_{13}'}{\partial x_3} \right).$$

Here f designates the Coriolis parameter, p the hydrostatic pressure, and P_{ij}' the appropriate components of the molecular stress related to the motion. It is assumed here that the Coriolis term related to the vertical motion is sufficiently small to be neglected.

THE MEAN LONGITUDINAL COMPONENT OF THE EQUATION OF MOTION

The instantaneous velocity v_i may be considered to be composed of three terms:

(a) A time mean velocity $\overline{v_i}$, obtained by averaging over one or more tidal cycles. This mean velocity has components along the longitudinal (x_1) and vertical (x_2) axis, but the lateral component is assumed to be negligible.

(b) An oscillatory tidal velocity U_1 which is directed up and down the longitudinal axis of the estuary. This term is assumed to be a function of x_1 only and to vary according to the simple harmonic function $U_1 = U_0 \cos \varphi$, where U_0 is the amplitude of the tidal current and φ represents the sum of an angular time argument of tidal period plus a phase angle. Both the phase angle φ and the amplitude U_0 may be functions of the longitudinal coordinate.

(c) A velocity deviation v_i', which results from those turbulent fluctuations which have a time scale smaller than the period employed in the averaging process. The period of averaging used here is one tidal cycle.

The three components of the instantaneous velocity are thus given by

$$v_1 = \bar{v}_1 + U_1 + v_1' \, ; \qquad v_2 = \bar{v}_2 + v_2' \, ; \qquad v_3 = v_3' \, . \tag{2}$$

Our next step is to introduce the velocity terms from eq. (2) into eq. (1) and to take the time mean over one or more tidal cycles of this latter equation. Before doing this, however, we note that:

(a) Since $U_1 = U_0 \cos \varphi$, a time mean over one or more tidal cycles results in $\overline{U_1} = 0$. Hence any of the cross products involving $\overline{U_1}$ will not appear in the resulting mean equation of motion.

(b) A field acceleration term will result from $\overline{U_1^2}$, since the mean value of $U_0^2 \cos^2 \varphi$ over a tidal cycle is $\frac{1}{2} U_0^2$.

(c) There is no reason to expect a correlation between the velocity deviations v_1', v_2', or v_3' on the one hand and the tidal component U_1 on the other, hence terms of the type $U_1 v_i'$ will not appear in the mean equation of motion. (There may be a correlation between the root mean square velocity deviations and U_1, since the magnitude of the turbulent velocities may be directly affected by the magnitude of the tidal velocities.)

(d) The mean value of the x_1-component of the pressure force $- < \alpha \dfrac{\partial p}{\partial x_1} >$ may be determined from the distribution of temperature and salinity, providing the depth of a level pressure surface is known. If we designate $- < \alpha \dfrac{\partial p}{\partial x_1} >_0$ as the x_1-component of the pressure force at any depth x_2 relative to the x_1-component of the pressure force at the surface, and if the actual longitudinal component of the pressure force at the surface is designated as b_0, then $- <\alpha \dfrac{\partial p}{\partial x_1}> = - <\alpha \dfrac{\partial p}{\partial x_1}>_0 + b_0$. Note that $- <\alpha\, \partial p/\partial x_1>_0$ can be determined from the distribution of temperature and salinity, while b_0 remains as yet an undetermined constant with respect to the vertical.

(e) Because the vertical gradient of the horizontal mean velocity is much larger than other components of the velocity gradient, the only molecular stress term which will be of any importance in the mean equation is $<P_{12}'>$. This term can be expressed by $\mu \dfrac{\partial \bar{v}_1}{\partial x_2}$, where μ is the molecular viscosity; hence it is seen that $<P_{12}'>$ is of significant size only near surface and near bottom, where the mean velocity gradient may approach significant magnitudes. Both $<P_{12}'>$ and $<v_2'v_1'>$ represent momentum transports due to velocity deviations, one on a molecular scale, the other on a macro-scale. In what follows, $< P_{12}'>$ will be included with $< v_2'v_1'>$, and at the boundaries (surface and bottom) $< v_2'v_1'>$ will take on the appropriate value representing the total flux of momentum across the boundaries. Within the interior of the fluid, only the so-called eddy processes need be considered.

Under these conditions, the time mean of eq. (1) over one or more tidal cycles is given for mean steady state by

$$\text{(3)}\quad \bar{v}_1 \frac{\partial \bar{v}_1}{\partial x_1} + \bar{v}_2 \frac{\partial \bar{v}_1}{\partial x_2} + U_0 \frac{\partial U_0}{\partial x_1} = - <\alpha \frac{\partial p}{\partial x_1}>_0 + b_0 - \frac{\partial}{\partial x_1} <v_1'v_1'> - \frac{\partial}{\partial x_2} <v_2'v_1'> - \frac{\partial}{\partial x_3} <v_3'v_1'>.$$

THE LATERAL COMPONENTS OF THE EQUATION OF MOTION

The x_3 component of the equation of motion, for instantaneous values of the various parameters, may be expressed as

$$(4)\quad \frac{\partial v_3}{\partial t} + v_1 \frac{\partial v_3}{\partial x_1} + v_2 \frac{\partial v_3}{\partial x_2} + v_3 \frac{\partial v_3}{\partial x_3} = -\alpha \frac{\partial p}{\partial x_3} + f\, v_1 + \alpha \left(\frac{\partial P_{31}'}{\partial x_1} + \frac{\partial P_{32}'}{\partial x_2} + \frac{\partial P_{33}'}{\partial x_3} \right).$$

Before taking the time mean of this equation we note that

(a) The mean values of the molecular stress terms at the longitudinal axis of the estuary are zero, since the assumption is made that, along the longitudinal axis, $\bar{v}_3 = \frac{\partial \bar{v}_2}{\partial x_3} = \frac{\partial \bar{v}_1}{\partial x_3} = 0.$

(b) Since $\bar{v}_3 = 0$, all the mean field acceleration terms vanish.

(c) As in the longitudinal component, the mean value of the x_3-component of the pressure force may be expressed as the x_3-component of the pressure force relative to that at the surface, which can be computed from the distribution of temperature and salinity, plus a constant with respect to the vertical, representing the actual x_3-component of the pressure force at the surface.

Under mean steady state conditions, the time mean of eq. (4) over one or more tidal cycles then becomes

$$(5)\quad 0 = -<\alpha \frac{\partial p}{\partial x_3}>_0 + b_0' + f\, \bar{v}_1 - \frac{\partial}{\partial x_1} <v_1'v_3'> - \frac{\partial}{\partial x_2} <v_2'v_3'> - \frac{\partial}{\partial x_3} <v_3'v_3'>.$$

THE MEAN EQUATIONS OF MOTION AS APPLIED TO THE JAMES RIVER ESTUARY

In a study of the salt balance in the James River estuary (Pritchard 1954), the horizontal component of the turbulent flux of salt was shown to be negligible compared to the horizontal advection and the vertical component of the turbulent flux of salt. If we assume by analogy that the horizontal component of the turbulent flux of momentum is also negligible compared to the other terms in the equation of motion, then the terms $\frac{\partial}{\partial x_1} <v_1'v_1'>$, $\frac{\partial}{\partial x_1} <v_1'v_3'>$, $\frac{\partial}{\partial x_3} <v_3'v_3'>$, and $\dfrac{\partial}{\partial x_3} <v_3'v_1'>$ will drop from eqs. (3) and (5). Under these conditions we have

$$(6)\quad \bar{v}_1 \frac{\partial \bar{v}_1}{\partial x_1} + \bar{v}_2 \frac{\partial \bar{v}_1}{\partial x_2} + U_0 \frac{\partial U_0}{\partial x_1} = - <\alpha \frac{\partial p}{\partial x_1}>_0 + b_0 - \frac{\partial}{\partial x_2} <v_2'v_1'>,$$

and

$$(7)\qquad 0 = - <\alpha \frac{\partial p}{\partial x_3}>_0 + b_0' + f\,\bar{v}_1 - \frac{\partial}{\partial x_2} <v_2'v_3'>.$$

Solving (6) for $< v_2'v_1'>$ and (7) for $<v_2'v_3'>$, we have

$$(8)\quad <v_2'v_1'> = - \left[\int\left(\bar{v}_1 \frac{\partial \bar{v}_1}{\partial x_1} + \bar{v}_2 \frac{\partial \bar{v}_1}{\partial x_2} + U_0 \frac{\partial U_0}{\partial x_1} + <\alpha \frac{\partial p}{\partial x_1}>_0 - b_0\right) dx_2 + C_1\right],$$

and

$$(9)\qquad <v_2'v_3'> = \left[\int\left(f\,\bar{v}_1 - <\alpha \frac{\partial p}{\partial x_3}>_0 + b_0'\right) dx_2 + C_2\right].$$

Assume for the moment that velocity data are available in sufficient detail to evaluate both the mean acceleration terms and the term involving the amplitude of the tidal velocity, $U_0 \dfrac{\partial U_0}{\partial x_1}$. From temperature and salinity data the relative pressure field could also be determined, leaving an unknown constant representing the absolute mean pressure force at the surface as an unknown quantity in each equation. There is also a second unknown constant, the constant of integration, in each equation. These constants can be evaluated providing the two boundary values for $< v_2'v_1'>$ and $<v_2'v_3'>$ are known.

At the surface and at the bottom, $\rho <v_2'v_1'>$ and $\rho <v_2'v_3'>$ will be equal to the net flux of momentum across these boundaries. During the period of study, the vector mean of the wind was close to zero, hence there was no appreciable momentum flux across the surface. The net longitudinal velocity at the bottom would lead to a net momentum flux across this interface directed along the x_1-axis. Hence $\rho <v_2'v_1'>$ would be equal to this momentum flux, or stress, at the bottom. Our best estimate of the boundary values of $<v_2'v_3'>$ is that it equals zero at the bottom as well as at the surface.

In a recent paper Lesser (1951) has shown that velocity measurements near the bottom in the open ocean satisfy the Prandtl-von

Karman boundary layer theory for a "rough" boundary. If z represents the vertical distance up from the bottom and v the magnitude of the mean horizontal velocity at the point z in the boundary layer, then the stress at the boundary is given by

$$\tau_h = \rho K_0^2 V_*^2 \bigg/ \left[\ln \frac{z + z_0}{z_0} \right]^2, \tag{10}$$

where k_0 is von Karman's constant and z_0 is the roughness parameter.

According to Lesser's analysis, the boundary layer within which the logarithmic velocity distribution characteristic of the Prandtl-von Karman boundary layer theory occurs is approximately one meter thick. Lesser has determined values of z_0 for three different types of bottom sediment. For a mud bottom the value of z_0, based on Lesser's measurements, was 0.02 cm, for a sand-gravel bottom 0.13 cm, and for a sand-mud bottom 0.16 cm. Lesser attributes the high value for the sand-mud bottom to the possible presence of ripples.

Taking $z_0 = 0.02$ cm as representative of the character of the mud bottom found in the central channel of the James River, it is possible to evaluate the boundary value of $< v_2'v_1' >$ from a single mean velocity measurement within the boundary layer, using eq. (10), since at the bottom $< v_2'v_1' > = \alpha\tau_h$.

Data from two sections in the James River have been employed in the evaluation of eqs. (8) and (9). Typical plots of $<v_2'v_1'>$ and of $<v_2'v_3'>$ vs. depth are given in Fig. 1. The determination of the constant b_0 in each equation also provides for evaluation of the depth of the level pressure surface, and, in fact, of the actual distribution of the pressure field.

These determinations reveal that a pressure surface which is level in regard to the longitudinal coordinate exists at near mid-depth. Above this depth, at which $\partial\bar{p}/\partial x_1 = 0$, the pressure surfaces slope downward from the river toward the sea. Below this reference depth the pressure surfaces slope in the opposite direction.

Based on an evaluation of eq. (9) for the James River estuary, there is also some mid-depth where $\partial\bar{p}/\partial x_3 = 0$, but it is not necessarily the same reference depth as that found from the longitudinal equation. Above this reference depth the pressure surfaces slope downward from the right side of the estuary (facing the mouth) to the left side. Below this reference depth the lateral slope is in the opposite direction.

In the James River estuary there is a net horizontal velocity directed down-estuary in an upper layer and up-estuary in a lower

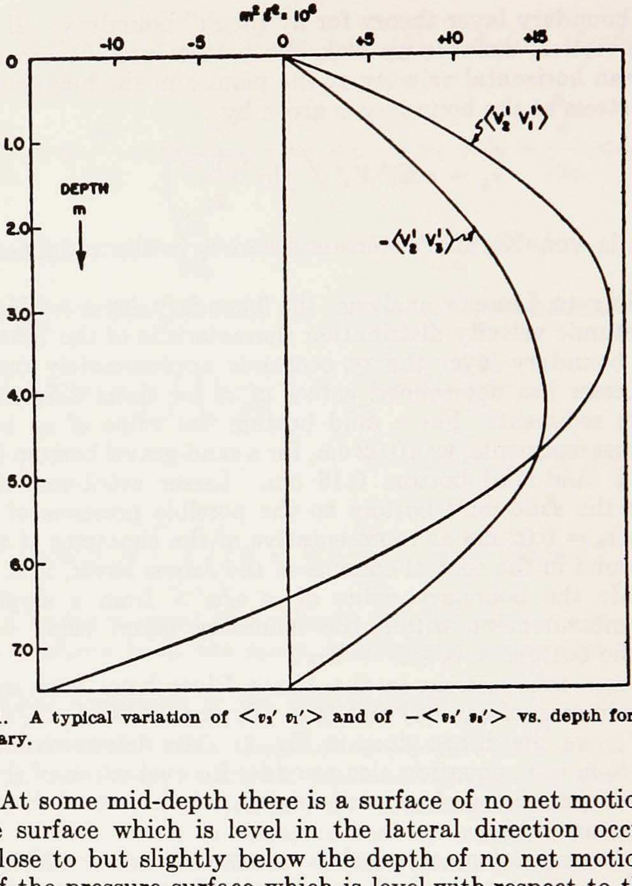

Figure 1. A typical variation of $<v_2' v_1'>$ and of $-<v_2' v_3'>$ vs. depth for the James River estuary.

layer. At some mid-depth there is a surface of no net motion. The pressure surface which is level in the lateral direction occurs at a depth close to but slightly below the depth of no net motion. The depth of the pressure surface which is level with respect to the longitudinal coordinate may be above or below the depth of no net motion, the departure depending primarily on the sign and magnitude of the term $U_0 \dfrac{\partial U_0}{\partial x_1}$. Table I gives the observed depths of no net motion, as well as computed depths at which $\partial \bar{p}/\partial x_1 = 0$ and $\partial \bar{p}/\partial x_3 = 0$.

These computations also allow a determination of the relative importance of the various terms which comprise the equation of motion. In the case of the longitudinal component, the pressure force is primarily balanced by the eddy frictional term $\dfrac{\partial}{\partial x_2} <v_2' v_1'>$. However, a significant portion of the pressure force may be balanced by the

TABLE I.

Station	Period	Bottom Depth	$\bar{v}_1$	Depth (m) of Zero value for: $<\alpha \frac{\partial p}{\partial x_1}>$	$<\alpha \frac{\partial p}{\partial x_3}>$	$\left(<\alpha \frac{\partial p}{\partial x_1}> + U_0 \frac{\partial U_0}{\partial x_1}\right)$
J-11	I	7.0	2.5	4.6	—	3.0
	II	7.0	2.5	5.4	—	2.7
	III	7.0	2.6	5.1	—	3.1
J-17	I	7.5	2.9	4.1	3.6	3.2
	II	7.5	2.9	3.9	3.3	3.0
	III	7.5	3.0	3.9	—	3.1
	IV	7.5	3.0	—	3.0	—

field change in the amplitude of the tidal velocity, $U_0 \frac{\partial U_0}{\partial x_1}$. When this latter term is small, the depth at which $\frac{\partial \bar{p}}{\partial x_1} = 0$ is nearly the same as the depth of no net motion. When the tidal velocity amplitude increases downstream, then the depth at which $\frac{\partial \bar{p}}{\partial x_1} = 0$ is greater than the depth of no net motion. If the tidal velocity amplitude decreases downstream, then the depth at which $\frac{\partial \bar{p}}{\partial x_1} = 0$ is less than the depth of no net motion. The field acceleration terms related to the mean motion are relatively small.

In the lateral component of the equation of motion, the Coriolis force resulting from the net horizontal motion is balanced primarily by the lateral pressure force. Of secondary importance is the eddy

TABLE II. Variation with Depth of the Terms in the Longitudinal Component of the Mean Equation of Motion—For a Typical Section in the James River Expressed in $ms^{-2} \times 10^6$

Depth (meters)	$\bar{v}_1 \frac{\partial \bar{v}_1}{\partial x_1}$	$\bar{v}_2 \frac{\partial \bar{v}_1}{\partial x_2}$	$U_0 \frac{\partial U_0}{\partial x_1}$	$-<\alpha \frac{\partial p}{\partial x_1}>$	$\frac{\partial}{\partial x_2}<v_2'v_1'>$
0	0.03	0.01	2.97	14.30	11.29
1	0.12	0.09	2.97	10.56	7.38
2	0.08	0.28	2.97	6.83	3.50
3	0.01	0.42	2.97	3.10	−0.30
4	0.09	0.20	2.97	−0.64	−3.90
5	0.11	0.04	2.97	−4.38	−7.50
6	−0.04	0.01	2.97	−8.09	−11.03
7	−0.04	0.02	2.97	−11.77	−14.72

frictional term, $\frac{\partial}{\partial x_2} <v_2'v_3'>$. The depth at which $\partial \bar{p}/\partial x_3 = 0$ is nearly the same as the depth of no net motion.

Tables II and III give the variation with depth of the various terms in the longitudinal and lateral components of the equation of motion for a typical section in the James River.

TABLE III. VARIATION WITH DEPTH OF THE TERMS IN THE LATERAL COMPONENT OF THE MEAN EQUATION OF MOTION—FOR A TYPICAL SECTION IN THE JAMES RIVER EXPRESSED IN $MS^{-2} \times 10^6$

Depth (meters)	$-<\alpha \frac{\partial p}{\partial x_3}>$	$f\bar{v}_1$	$\frac{\partial}{\partial x_2}<v_2'v_3'>$
0	−16.49	10.18	−6.31
1	−11.64	6.23	−5.41
2	−6.26	3.07	−3.19
3	−0.53	−1.23	−1.76
4	+5.08	−4.92	+0.16
5	+9.73	−6.15	+3.58
6	+12.63	−6.67	+5.96
7	+13.29	−5.97	+7.32

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