THE GENERATION OF OCEANIC CURRENTS BY WIND[1]

By

J. G. CHARNEY

Institute for Advanced Study
Princeton, New Jersey

1. *Introduction.* In 1937 and 1938, C.-G. Rossby published results of investigations into the manner in which velocity and pressure adjust themselves in ocean currents. He showed that an initially unbalanced rectilinear current would execute transverse inertial oscillations in the process of creating the pressure field that is necessary to balance the Coriolis forces associated with its momentum. The energy which flows into these oscillations depends on the rapidity with which the currents are built up, i. e., on the extent of the unbalance. Since large-scale atmospheric wind systems have characteristic periods large compared to the half pendulum day whereas inertio-gravitational oscillations have periods less than a half pendulum day, one would expect the bulk of energy to flow directly into the balanced current. It appeared desirable, therefore, to develop a formalism that is specifically applicable to the study of such large-scale balanced motions. In 1947 and 1948 the writer developed just such a formalism for treating large-scale atmospheric disturbances. It is the purpose of the present article to show how an analogous treatment may be applied to large-scale oceanic currents. Specifically, it is proposed to again consider Rossby's problem, but this time with the explicit assumption that wind stresses impart momentum slowly enough for transverse horizontal and vertical motions to create the Coriolian mass adjustment almost instantaneously.[2]

[1] This paper is based on a lecture entitled "Waves and Currents in Atmosphere and Ocean" delivered at the Oceanographic Convocation at Woods Hole on June 24, 1954. Part of the work was done at the Woods Hole Oceanographic Institution, where the author was the Woods Hole Oceanographic Associates Lecturer for 1954, and part at the Institute for Advanced Study under contract N6ori Task Order I with the Office of Naval Research and the Geophysics Research Directorate, Air Force Cambridge Research Center. Reproduction of this article in whole or in part is permitted for any purpose by the United States Government.

[2] The conditions under which this assumption holds will be given in a forthcoming article by Veronis and Stommel (1956). There it is shown that, under ordinary circumstances, negligible inertio-gravitational oscillations of large scale are created by the atmospheric wind systems.

The exclusion of inertio-gravitational oscillations greatly simplifies the mathematical problem and permits analysis of more complicated systems. The formalism will be applied to the study of currents generated in an infinite double-layer ocean by the action of various distributions of wind stress. In part, some of Rossby's earlier findings will be substantiated; but, in illustration of the generality of the method, the formalism will be applied also to the case of an ocean with parallel boundaries to show the necessity, under certain conditions, for development of a narrow intense current near the boundaries with roughly the dimensions of the Gulf Stream. Finally, some attempt will be made to take into account the dynamical effects of the variation of the Coriolis parameter.

2. *The Balance Equations.* The relevant equations will be derived in somewhat more generality than is necessary for present use in order to bring out more fully the underlying assumptions. To do so, one must draw a fundamental distinction between the two types of large-scale motion that exist in the ocean. The two have widely differing time scales and interact only weakly with one another. They may be described broadly as "waves" and "currents," if one includes in the "wave" category not only surface waves but large-scale gravitational oscillations influenced by the earth's rotation, and, in the "current" category, what is ordinarily meant by transient or stationary ocean currents.

In order to formulate mathematically the distinction between the two categories, we define certain parameters that characterize the space and time scales of a given fluid motion. Let S be a characteristic horizontal dimension, U a characteristic horizontal particle velocity, and let Ω be the speed of the earth's rotation. Then the order of the horizontal acceleration per unit mass is U^2/S and the order of the Coriolis acceleration is ΩU. The nondimensional ratio of the two, $U/\Omega S$, will be called the Rossby number Ro, after Fultz (1951). If it is small, the flow is quasi-geostrophic. In this case one may show, by evaluating the order of the terms in the equation for the vertical vorticity component, that the ratio of horizontal divergence to vertical vorticity has the order Ro and is therefore small also. However, the essential part of the argument is that the divergence-vorticity ratio is small whether or not the flow is quasi-geostrophic, providing only that it is not a "wave" motion. In the case where the flow is nongeostrophic, we define H as a characteristic vertical dimension, $\Delta\rho$ as a characteristic vertical potential density difference, and (C, U) as the larger in magnitude of the phase or particle speeds, C or U. Then it may be shown that for nongeostrophic but quasi-hydrostatic motions the divergence-vorticity ratio becomes the reciprocal of the intrinsic Richardson number, $R_i = g\,(\Delta\rho/\rho)\,H \div (C,\ U)^2$, which measures the ratio of the square of the speed of long internal gravitational waves to $(C,\ U)^2$.[3] The significance of R_i is that it is of order unity or smaller for gravity "waves" but is large compared to unity for "currents." Consequently one may state as a general rule that the divergence-vorticity ratio is small for "currents" and for "currents" only. Although exceptions to this rule exist, they are rare and somewhat pathological.

From what has been said, the current motions are to be considered horizontally quasi-nondivergent. This is not to say, however, that they are altogether nondivergent; small horizontal divergences must occur in conjunction with transverse horizontal and vertical displacements in order to bring about the instantaneous mass adjustment. Thus we may write the horizontal velocity vector $\vec{V}$ as the sum of a nondivergent and an irrotational vector:

$$\vec{V} = \nabla\psi \times \vec{k} + \nabla\alpha \equiv \vec{V}_\psi + \vec{V}_\alpha \quad , \tag{1}$$

where ψ is the stream function for the nondivergent component of the motion, α the velocity potential for the irrotational component, $\vec{k}$ a unit vertical vector, and ∇ the horizontal gradient operator; and we assume that $\vec{k} \cdot \nabla \times \vec{V} \equiv \vec{k} \cdot \nabla \times \vec{V}_\psi \gg \nabla \cdot \vec{V} \equiv \nabla \cdot \vec{V}_\alpha$ but that $\nabla \cdot \vec{V}_\alpha$ is not to be ignored. With suitable boundary conditions on ψ and α, one may deduce the important relation $|\vec{V}_\psi| \gg |\vec{V}_\alpha|$.

The formal system of equations may now be derived as follows. We assume that the flow is incompressible and that the horizontal scale is large in comparison to the vertical scale. The motion is then quasi-hydrostatic, and the Eulerian equations become

$$\bar{\rho}\left(\frac{\partial \vec{V}}{\partial t} + \vec{V} \cdot \nabla \vec{V} + w\frac{\partial \vec{V}}{\partial z} + f\vec{k} \times \vec{V}\right) = -\nabla p + \bar{\rho}\vec{F} \quad , \tag{2}$$

$$\rho g = -\frac{\partial p}{\partial z} \quad , \tag{3}$$

$$\nabla \cdot \vec{V} + \frac{\partial w}{\partial z} = 0 \quad , \tag{4}$$

$$\frac{\partial \rho}{\partial t} + \vec{V} \cdot \nabla \rho + w\frac{\partial \rho}{\partial z} = 0 \quad . \tag{5}$$

[3] This relationship may be derived for simple linear systems by analytic means, and for general systems by a kind of scale analysis similar to that used in the theory of the boundary-layer in fluid dynamics (Charney, 1948).

Here z is the vertical co-ordinate, t time, p pressure, ρ density, w vertical velocity, f the Coriolis parameter 2 Ω sin (latitude), and $\vec{F}$ the external force. For simplicity, the purely kinematic effects of the earth's curvature have been ignored and, as a quite valid approximation in the oceans as well as in the atmosphere, the density ρ (x, y, z) in eq. (2) has been replaced by the standard value $\bar{\rho}$ (z).

We now replace the first vector equation by the two scalar equations obtained by taking its curl and divergence. The curl operation gives the vorticity equation

$$\left(\frac{\partial}{\partial t} + \vec{V} \cdot \nabla + w\frac{\partial}{\partial z}\right) Z = - Z \nabla \cdot \vec{V}_a - \vec{k} \cdot \nabla w \times \frac{\partial \vec{V}}{\partial z} + \vec{k} \cdot \nabla \times \vec{F} \quad , \quad (6)$$

in which Z is the vertical component of absolute vorticity,

$$Z = \vec{k} \cdot \nabla \times \vec{V}_\psi + f \equiv \nabla^2\psi + f \quad . \quad (7)$$

The divergence operation gives

$$\left(\frac{\partial}{\partial t} + \vec{V} \cdot \nabla + w\frac{\partial}{\partial z}\right) \nabla \cdot \vec{V} + \nabla w \cdot \frac{\partial \vec{V}}{\partial z} + \left(\frac{\partial u}{\partial x}\right)^2 + 2\frac{\partial u}{\partial y}\frac{\partial v}{\partial x} + \left(\frac{\partial v}{\partial y}\right)^2 - f\vec{k} \cdot \nabla \times \vec{V} - \vec{k} \cdot \vec{V} \times \nabla f = -\frac{1}{\bar{\rho}} \nabla^2 p + \nabla \cdot \vec{F} \quad ,$$

if x and y are respectively the eastwardly and northwardly directed horizontal cartesian co-ordinates and if u and v are the corresponding velocity components. Here we utilize the inequalities $\vec{k} \cdot \nabla \times \vec{V} \gg \nabla \cdot \vec{V}$ and $|\vec{V}_\psi| \gg |\vec{V}_a|$ in conjunction with the continuity eq. (4) to justify ignoring the first two terms on the left-hand side. Anticipating that $\vec{F}$ is the force of friction, which is important only in the layer of frictional influence, whose dynamics will be given special treatment in the next section, we may ignore it here. We then obtain the reduced form

$$f\nabla^2\psi + \nabla\psi \cdot \nabla f + 2\left[\frac{\partial^2\psi}{\partial x^2}\frac{\partial^2\psi}{\partial y^2} - \left(\frac{\partial^2\psi}{\partial x \partial y}\right)^2\right] = \frac{1}{\bar{\rho}} \nabla^2 p \quad , \quad (8)$$

which states a direct relationship between the field of motion (expressed by the stream-function ψ) and the field of pressure. This is the *condition of balance.* If $\vec{V}$ is expressed in terms of ψ and α by (1), eqs. (3–8) suffice to determine the five dependent variables p, ρ, ψ, α, and w. We call them the *balance equations.* Essentially, the only change has been the replacement of the horizontal divergence equation by the balance condition (8).

The balance condition has been obtained independently by Fjortoft (unpublished work) as a necessary condition that the nondivergent part of the motion be stable with respect to horizontally divergent perturbations. We know of course that the oceanic motions *are* stable for the most part, for otherwise one should expect a spontaneous generation of "wave" motions from "current" motions.

The balance condition may be regarded as a generalization of the geostrophic approximation. If the inertial forces are small compared with the Coriolis force, the nonlinear term in ψ drops out and the resulting equation

$$f\nabla^2\psi + \nabla\psi \cdot \nabla f = \frac{1}{\bar{\rho}} \nabla^2 p$$

may be integrated to give

$$f\nabla\psi = \vec{V}_\psi \times \vec{k} = \frac{1}{\bar{\rho}} \nabla p \quad .$$

Solving for $\vec{V}_\psi$, we obtain

$$\vec{V}_\psi \cong \vec{V}_g \equiv \frac{\vec{k}}{\bar{\rho} f} \times \nabla p \cong k \times \nabla \left(\frac{p}{\bar{\rho} f}\right) \quad , \tag{9}$$

which is simply the geostrophic approximation.

If the intrinsic Rossby number is small everywhere, the quasi-geostrophic equations may be further simplified. The terms involving w in eq. (6) become small and may be ignored; the undifferentiated Z may be replaced by f; and $\vec{V}$ may be replaced in equations (5) and (6) by $\vec{V}_g$. If w is then eliminated between eqs. (4), (5) and (6), one obtains

$$\left(\frac{\partial}{\partial t} + \vec{V}_g \cdot \nabla\right) Z_g + f \frac{\partial}{\partial z}\left[\left(\frac{\partial \rho}{\partial z}\right)^{-1}\left(\frac{\partial}{\partial t} + \vec{V}_g \cdot \nabla\right)\rho\right] = \vec{k} \cdot \nabla \times \vec{F} \, , \tag{10}$$

which, when ρ and $\vec{V}_g$ are expressed in terms of p by means of (3) and (9), is alone sufficient to determine the motion.

3. *Specialization of Quasi-geostrophic Equations to Single- and Double-layer Oceans.* We shall consider oceans consisting of one or two homogeneous layers. If the flow is quasi-geostrophic as well as quasi-hydrostatic, it can be shown that the motion must be independent of height in each layer. The quasi-geostrophic equations may then be specialized in the following manner: The continuity eq. (4) is first integrated to give

$$\frac{\partial h}{\partial t} + \vec{V}_g \cdot \nabla h = -h\nabla \cdot \vec{V}_a = -h\nabla^2\alpha \quad , \tag{11}$$

where h is the thickness of the layer. We next substitute in (6) and use the geostrophic approximation $\vec{V}_\psi \cong \vec{V}_g$. Redefining the averaged force $\frac{1}{h}\int_0^h \vec{F}\, dz$ by $\vec{F}$, we obtain, after some combination of terms,

$$\left[\frac{\partial}{\partial t} + \vec{V}_g \cdot \nabla\right]\frac{Z_g}{h} = \frac{1}{h}\vec{k} \cdot \nabla \times \vec{F}. \tag{12}$$

In a single-layer homogeneous ocean with a free surface, pressure is related to thickness through the hydrostatic equation

$$p = \rho\, g(h - z) + \text{constant}. \tag{13}$$

Hence $\vec{V}_g = (g/f)\,\vec{k} \times \nabla h$, $Z_g = (g/f)\nabla^2 h + f$, and eqs. (11) and (12) suffice to determine the two dependent variables h and α. We note that if $\vec{F} = 0$, eq. (12) asserts the conservation of the "potential vorticity" Z_g/h.

In the two-layer case, let $\vec{V}$, h, ρ and p be the horizontal velocity, depth, density, and pressure respectively of the upper layer and denote corresponding quantities in the lower layer by primes. The hydrostatic conditions become

$$\begin{aligned} p &= g\rho(h + h' - z) + \text{constant} \\ p' &= g\rho'(\varepsilon h + h' - z) + \text{constant} \quad , \end{aligned} \tag{14}$$

where ε is the density ratio ρ/ρ'; the geostrophic velocities and vorticities are defined by

$$\begin{aligned} V_g &= (g/f)\,\vec{k} \times \nabla\,(h + h') \\ Z_g &= (g/f)\,\nabla^2\,(h + h') + f \end{aligned} \tag{15}$$

$$\begin{aligned} V_g' &= (g/f)\,\vec{k} \times \nabla\,(\varepsilon h + h') \\ Z_g' &= (g/f)\,\nabla^2\,(\varepsilon h + h') + f \quad . \end{aligned} \tag{16}$$

Eqs. (15) and (16) combined with (11) and (12), written for both primed and unprimed quantities, are then the governing set of equations.

4. *The Wind Force.* When the wind begins to blow over the surface of the ocean, momentum is transferred through the surface into deeper water by a complex frictional process. If the wind stress is uniform horizontally, then, as Ekman (1905) showed, the frictional force varies in magnitude and direction with depth, and the water particles oscillate inertially with varying phase and amplitude. On the other hand, if the wind is not uniform, or if rigid boundaries exist, the frictionally driven currents are divergent and produce pressure changes which lead to the establishment of permanent currents. Since we are not concerned here with such inertial oscillations as might exist—their energy will be small in any case—we must consider the effect of wind stress on the balanced part of the flow. This effect may be incorporated into the balance equations in the following manner:

Consider a continuously stratified ocean. If the surface wind stress is horizontally nonuniform, the horizontal divergence of the wind drift current within the layer of frictional influence far exceeds the divergence of the balanced part of the flow. Hence one may, with good approximation, attribute all divergence to the wind drift current. To this divergence corresponds a difference in vertical velocity along the vertical between the surface and the depth at which frictional influences become negligible. On the assumption that this depth D is small compared with the vertical scale of the balanced currents, the frictional divergence may be introduced as a boundary effect in the balance equations for the continuously stratified ocean. To show this, let the vertical velocity at the surface be w_0 and the vertical velocity at depth D be w_1. Then, if $\vec{V}_f$ is the drift current, we have

$$w_1 - w_0 = \int_{-D}^{0} \nabla \cdot \vec{V}_f \, dz \quad .$$

Ignoring inertial oscillations we may assume that the total Coriolis force acting on the drift current is exactly balanced by the surface stress $\vec{\tau}$, i.e., $\int_{-D}^{0} f\vec{k} \times \vec{V}_f \rho dz = \vec{\tau}$. In this formula the vertical variation of ρ may be ignored and one may divide both sides by ρf. Doing so and taking the curl of both sides, we obtain

$$\int_{-D}^{0} \nabla \cdot \vec{V}_f \, dz = \vec{k} \cdot \nabla \times (\vec{\tau}/\rho f) \quad .$$

The vertical velocity w_0 at the surface is the individual derivative of the height of the free surface ζ. Since depth D is relatively small, we may ignore the variations of density in the friction layer and write $d\zeta = dp_1/\rho_1 g$, where $_1$ denotes quantities at depth D. Hence

$$w_o = \frac{1}{\rho_1 g}\frac{dp_1}{dt} \equiv \frac{1}{\rho_1 g}\left(\frac{\partial}{\partial t} + \vec{V}_g \cdot \nabla\right) p_1 \,.$$

The vertical velocity w_1 at level D is given by eq. (5) with $\vec{V}$ evaluated geostrophically. Solving for w_1 we get

$$w_1 = -\left(\frac{\partial \rho}{\partial z}\right)_1^{-1}\left(\frac{\partial}{\partial t} + \vec{V}_g \cdot \nabla\right)\rho_1 = -\frac{1}{g}\left(\frac{\partial \rho}{\partial z}\right)_1^{-1}\left(\frac{\partial}{\partial t} + \vec{V}_g \cdot \nabla\right)\left(\frac{\partial p}{\partial z}\right)_1 .$$

Combining the last four equations, we obtain the required condition

$$\left(\frac{\partial}{\partial t} + \vec{V}_g \cdot \nabla\right)\left(\frac{\partial p}{\partial z}\right)_1 + m\left(\frac{\partial}{\partial t} + \vec{V}_g \cdot \nabla\right) p_1 = g m \vec{k} \cdot \nabla \times \left(\frac{\vec{\tau}}{f}\right) ,$$

where $m = -(\rho^{-1}\, \partial\rho/\partial z)_1$. Since D is small, this equation may be assumed to apply approximately at the surface. The usefulness of this expression lies in the fact that it converts the complicated Ekman-spiral microstructure of the frictional layer into a simple boundary condition on pressure.

In the case where wind-induced friction acts in the upper of two homogeneous layers, the effect may be directly incorporated by subtracting the integrated frictional divergence through the entire upper layer from the individual depth change of this layer due to divergence of the balanced flow. Thus in eq. (11) we merely subtract the term $\vec{k} \cdot \nabla \times (\vec{\tau}/\rho f)$ from the left-hand side:

$$\left(\frac{\partial}{\partial t} + \vec{V}_g \cdot \nabla\right) h = -\, h \nabla \cdot \vec{V} - \vec{k} \cdot \nabla \times \left(\frac{\vec{\tau}}{\rho f}\right) .$$

Proceeding as before in the derivation of eq. (12), we obtain

$$\left(\frac{\partial}{\partial t} + \vec{V}_g \cdot \nabla\right)\frac{Z_g}{h} = \frac{Z_g}{h}\vec{k} \cdot \nabla \times \left(\frac{\vec{\tau}}{\rho f}\right) ,$$

from which it follows that the body force $\vec{F}$ in eq. (12) may be identified with $\vec{\tau}/\rho h$, providing only that one may ignore the variations of h and f on the right-hand side of the above equation and that Z_g may be replaced by f. The latter approximation is always valid under quasi-geostrophic conditions, and the former is also permissible when one considers how little is known about actual surface stress distributions.

The above method of incorporating wind stress into the balance equations is similar to a method used by Charney and Eliassen (1949) for dealing with the effects of surface friction on atmospheric motions. By introducing the wind stress distribution as a boundary condition on pressure, or, in the case of the homogeneous upper layer, as a body force, it is possible to investigate the large-scale features of the wind-driven circulation without including in complete detail all the small-scale complexities of the surface frictional layer.

5. *Generation of Wind Currents in a Single-layer Ocean.* Now consider an infinite homogeneous ocean of constant depth H which is initially at rest. Beginning at time $t = 0$, a certain amount of y-momentum is imparted uniformly to an infinite strip of width 2a. What is the final equilibrium state? Rossby (1938) considered the case where the momentum is added impulsively. In the present analysis we allow the wind stress a finite time in which to act and we assume that the current is constantly in a state of quasi-geostrophic equilibrium. On the assumption that the motions are small, the appropriate equation for dealing with this situation is the linearized form of (12). Ignoring effects from the variation of f with latitude, we have simply

$$\left(\frac{\partial^2}{\partial x^2} - \frac{1}{\lambda^2}\right)\frac{\partial \eta}{\partial t} = \frac{f}{g}\frac{\partial F}{\partial x} \quad , \quad F = \begin{cases} B \text{ for } |x| \leq a \\ 0 \text{ for } |x| > a \quad , \end{cases} \tag{17}$$

where η is the perturbation height of the ocean and λ is Rossby's "radius of deformation" $(gH/f^2)^{\frac{1}{2}}$. The solution of this equation, subject to the conditions that $\eta = 0$ at $t = 0$ and that η be finite at $x = \infty$, is found to be

$$\eta = \frac{\lambda f B t}{g}\begin{cases} e^{-a/\lambda} \sinh (x/\lambda) & , \quad |x| \leq a \\ \text{sign } x e^{-|x|/\lambda} \sinh (a/\lambda) & , \quad |x| > a \end{cases} \tag{18}$$

and is represented schematically in Fig. 1.

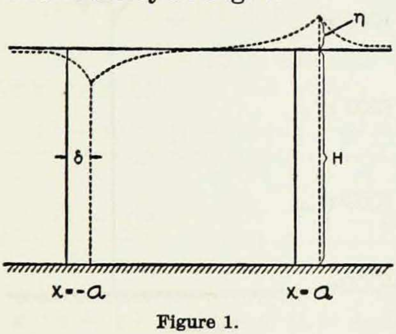

Figure 1.

The (infinitesimal) displacement δ of the central current to the right may be calculated from the transverse velocity u at $x = \pm a$. This is obtained from the continuity eq. (11) which, in the present instance, is

$$\frac{\partial \eta}{\partial t} = - H \frac{\partial u}{\partial x} \cdot \tag{19}$$

We find that $u(\pm a) = (B/2f)\,[1 + \exp(-2a/\lambda)]$. Since this quantity is independent of time, δ is merely $u(\pm a)t$. The velocity of the balanced current is obtained from $v = (g/f)\partial\eta/\partial x$,

$$v = Bt \begin{cases} e^{-a/\lambda}\cosh(x/\lambda) & , \quad |x| \leq a \\ -e^{-|x|/\lambda}\sinh(a/\lambda) & , \quad |x| > a \quad . \end{cases} \tag{20}$$

If the force B and the time during which it acts are varied in such a way that Bt remains constant, the resulting motion remains unchanged, since that part of the motion which depends on the rate at which momentum is added (the inertial oscillation) has been ignored. The final equilibrium state corresponding to a given addition of momentum is essentially the same as that in Rossby's model. The principal difference is that the energy imparted is here less by just the amount that is radiated away as gravitational wave energy in Rossby's model.

6. *Generation of Wind Currents in a Double-layer Ocean.* Of greater interest is the manner in which currents are created in a stratified ocean where there is the possibility of internal mass adjustment. Here we simplify the analysis by replacing the continuously stratified ocean by two homogeneous layers with the surface of discontinuity at the center of the mean thermocline as in Fig. 2.

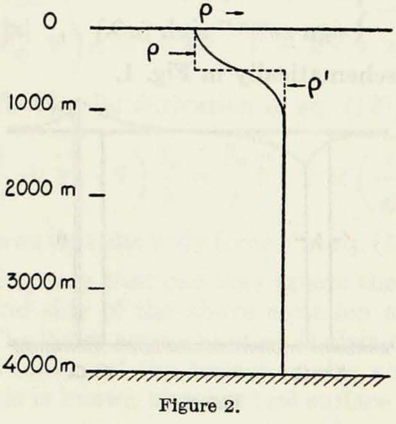

Figure 2.

To simulate actual conditions in the North Atlantic, we take the depths of the upper and lower layers to be 500m and 3500m respectively and the density difference $\rho' - \rho$ to be 2×10^{-3}gm cm^{-3}. Again, assuming small motions, we may disregard second order terms; thus eqs. (12), (15) and (16) may be combined to give

$$\begin{cases} \nabla^2 \dfrac{\partial}{\partial t}(\eta + \eta') - \dfrac{1}{\lambda^2}\dfrac{\partial \eta}{\partial t} + \beta \dfrac{\partial}{\partial x}(\eta + \eta') = \dfrac{f}{g}\vec{k} \cdot \nabla \times \vec{F} \ , \\ \nabla^2 \dfrac{\partial}{\partial t}(\epsilon\eta + \eta') - \dfrac{1}{\lambda'^2}\dfrac{\partial \eta'}{\partial t} + \beta \dfrac{\partial}{\partial x}(\epsilon\eta + \eta') = \dfrac{f}{g}\vec{k} \cdot \nabla \times \vec{F'} \ , \end{cases} \tag{21}$$

where η and η' are the perturbations of depths h and h' respectively, $\lambda = (gH/f^2)^{\frac{1}{2}}$ and $\lambda' = (gH'/f^2)^{\frac{1}{2}}$ are the radii of deformation corresponding to the upper and lower depths H and H', and $\beta = df/dy$. For simplicity, β is assumed constant.

In accordance with the idea that most of the frictional force acts in the upper hundred meters or so, we assume that no force acts in the lower layer and that there is no frictional transfer of momentum across the interface. Any motion created in the lower layer must therefore occur in response to pressure forces transmitted from above.

It will be convenient to separate the motion into its two normal modes: a barotropic, or external, mode in which the current velocities in both layers are the same and a baroclinic, or internal, mode in which the current velocities in each layer are inversely proportional to their depths. This separation is performed as follows: We multiply the first of eqs. (21) by σ and add to the second, choosing σ so that the linear combinations $\sigma(\eta + \eta') + \epsilon\eta + \eta'$ and $\sigma\eta/\lambda^2 + \eta'/\lambda'^2$ are proportional. Denoting the ratio H/H' by r, we find that σ must satisfy the equation

$$\sigma^2 + (1 - r)\,\sigma - r\epsilon = 0 \tag{22}$$

whose roots are

$$\sigma_1, \sigma_2 = -\,(1 - r)/2 \pm [(1 + r)^2/4 - r(1 - \epsilon)]^{\frac{1}{2}}.$$

Since $(1 - \epsilon) = (\rho' - \rho)/\rho' = 2 \times 10^{-3}$, whereas $(1 + r)^2/4 > \frac{1}{4}$, we may approximate these roots by

$$\sigma_1, \sigma_2 = -\,\frac{(1 - r)}{2} \pm \frac{(1 + r)}{2}\left[1 - \frac{2r\,(1 - \epsilon)}{(1 + r)^2}\right]$$

$$= r - \frac{(1 - \epsilon)r}{1 + r} \ , \ -1 + \frac{(1 - \epsilon)r}{1 + r}. \tag{23}$$

Defining

$$R_i = \sigma_i\,(\eta + \eta') + \epsilon\eta + \eta' \ (i = 1, 2) \ , \tag{24}$$

we obtain the required decomposition,

$$\nabla^2 \frac{\partial R_i}{\partial t} - \frac{\alpha_i^2}{\lambda^2}\frac{\partial R_i}{\partial t} + \beta \frac{\partial R_i}{\partial x} = \sigma_i \frac{f}{g} \vec{k} \cdot \nabla \times \vec{F} \,, \tag{25}$$

where

$$\begin{cases} \alpha_1^2 = \dfrac{1}{1-\varepsilon}\left(1 - \dfrac{r\varepsilon}{\sigma_1}\right) \approx \dfrac{r}{1+r} \,, \\ \alpha_2^2 = \dfrac{1}{1-\varepsilon}\left(1 - \dfrac{r\varepsilon}{\sigma_2}\right) \approx \dfrac{1+r}{1-\varepsilon}. \end{cases} \tag{26}$$

Once the R_1 have been found, $\eta + \eta'$ and $\varepsilon\eta + \eta'$ may be obtained by solving (24). Thus

$$\begin{cases} \eta + \eta' = \dfrac{R_1 - R_2}{\sigma_1 - \sigma_2} \approx \dfrac{R_1 - R_2}{1+r} \,, \\ \varepsilon\eta + \eta' = \dfrac{\sigma_1 R_2 - \sigma_2 R_1}{\sigma_1 - \sigma_2} \approx \dfrac{R_1 - rR_2}{1+r}. \end{cases} \tag{27}$$

It is evident that R_1 corresponds to the barotropic mode, since, for this component of motion, the geostrophic velocities $(g/f)\, \vec{k} \times \nabla\, (\eta + \eta')$ and $(g/f)\, \vec{k} \times \nabla\, (\varepsilon\eta + \eta')$ are equal. Likewise we see that the upper and lower velocities corresponding to R_2 are in the ratio $1/r$. This mode will be called the baroclinic, or internal, mode and is characterized by the large deformations of the interface needed to reduce the pressure gradient in the lower layer. The slope of the interface is of the order of one thousand times that of the free surface.

Let us assume now that a uniform wind stress acts on the infinite strip, $|x| \leq a$, producing a body force B in the y-direction and acting only in the upper layer. Disregarding the β-effect, we find, in complete analogy with the single layer case,

$$R_i = \frac{\sigma_i \lambda f B t}{\alpha_i g} \begin{cases} e^{-\alpha_i a/\lambda} \sinh(\alpha_i x/\lambda) & , \ |x| \leq a \,, \\ \text{sign } x \; e^{-\alpha_i |x|/\lambda} \sinh(\alpha_i a/\lambda) & , \ |x| > a \,. \end{cases} \tag{28}$$

The corresponding y-velocity components are

$$v_i = \frac{g}{f}\frac{\partial R_i}{\partial x} = \sigma_i B t \begin{cases} e^{-\alpha_i a/\lambda} \cosh(\alpha_i x/\lambda) & , \ |x| \leq a \\ -e^{-\alpha_i |x|/\lambda} \sinh(\alpha_i a/\lambda) & , \ |x| > a \,. \end{cases} \tag{29}$$

The upper layer velocities corresponding to each of the modes R_1 and R_2 are shown in Fig. 3 for the half-widths a = 100km and a = 1000km. It is seen that the baroclinic mode is predominant over a large part of

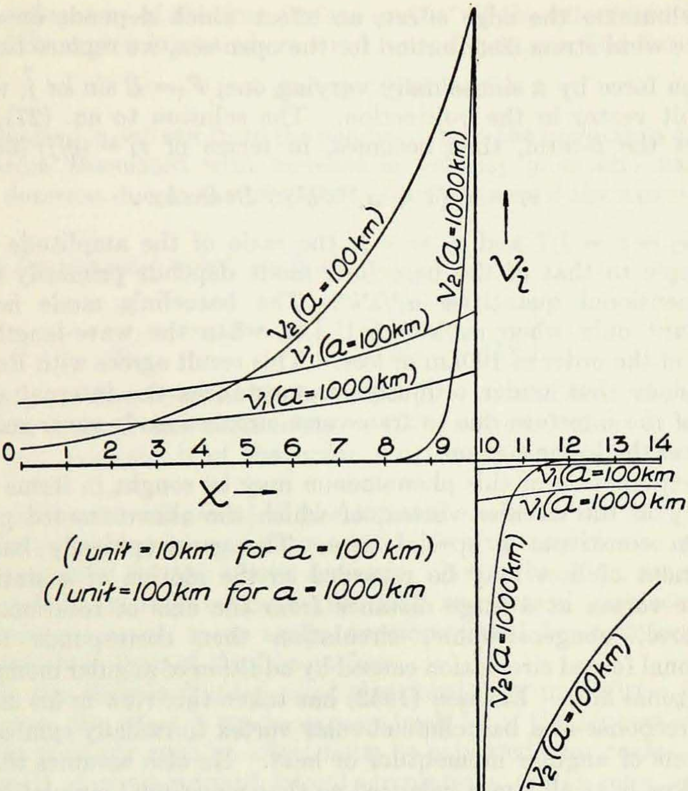

Figure 3

the current region for the smaller width whereas for the larger width the barotropic mode dominates except near the edges.

To estimate the orders of magnitude of the currents generated, we assume that the surface stress τ is one dyne cm^{-2} and f is 10^{-4} sec^{-1}. The body force then becomes $B = \tau/\rho H = 1.73$ cm sec^{-1} per day. The maximum currents and countercurrents occur at the edges. After one year, max $v_1 = 0.9$ m sec^{-1} for $a = 100$km and 0.6 m sec^{-1} for $a = 1000$km and max $v_2 = 3.2$ m sec^{-1} for either $a = 100$km or $a = 1000$km.

To eliminate the edge effect, an effect which depends on an unrealistic wind stress distribution for the open sea, we replace the step-function force by a sinusoidally varying one, $\vec{F} = B \sin kx \, \vec{j}$, where $\vec{j}$ is a unit vector in the y-direction. The solution to eq. (27), again without the β-term, then becomes, in terms of $v_i = (g/f)\,(\partial R_i/\partial x)$,

$$v_i = \sigma_i \,(1 + \alpha_i^2/\lambda^2 k^2)^{-1}\, Bt \sin kx \,. \tag{30}$$

Since $\sigma_1 \approx r = 1/7$ and $\sigma_2 \approx -1$, the ratio of the amplitude of the barotropic to that of the baroclinic mode depends primarily on the nondimensional quantities $\alpha_i^2/\lambda^2 k^2$. The baroclinic mode becomes important only when $\alpha_2^2/\lambda^2 k^2 \lesssim 1$, i.e., when the wave-length $L = 2\pi/k$ is of the order of 100km or less. This result agrees with Rossby's conclusions that under ordinary circumstances the internal adjustment of the interface due to transverse circulations is never sufficient to cancel the lower current.

An explanation of this phenomenon may be sought in terms of the stability of the circular vortex, of which the above treated parallel currents constitute a special case. The geostrophically balanced component of flow may be regarded as the motion of a stationary circular vortex at a large distance from the axis of rotation. The transverse, nongeostrophic, circulation then corresponds to the meridional forced circulation caused by addition of angular momentum to the zonal flow. Eliassen (1952) has taken this view in his analysis of the response of a baroclinic circular vortex to radially symmetrical injections of angular momentum or heat. He also assumes that the zonal flow is at all times balanced so that meridional circulations are necessary in order to maintain the balance. He finds that the meridional motion then takes place mainly in the direction of smallest stability. The stabilizing forces in the present instance are gravitational stability of the free surface and of the interface and inertial stability due to the earth's rotation. The first two act to resist vertical displacement and the last acts to resist horizontal displacements. As the lateral extent of the wind force (determined by a in the first problem and by L in the second) decreases, the vertical component of the transverse circulation increases relative to its horizontal component. Since gravity resists internal displacements of the interface much less than it resists displacements of the free surface, it is natural to expect the baroclinic modes to predominate for sufficiently small a or L.

Another explanation of this effect follows from the requirement that the angular momentum in the lower layer remain constant in the absence of external torques. Simplifying matters by supposing that the slopes m of the free surface and m' of the interface are constant in the region $|x| \leq a$ where wind force B acts, we find that

$$\varepsilon m + (1 - \varepsilon)m' + (a^2/3\lambda'^2)m' = 0 \ .$$

This equation is derived from the condition that the increase in angular momentum associated with increase in velocity is exactly balanced by the decrease due to the transport of water toward the axis of rotation.

Solving for the ratio m/m', we find

$$\frac{m}{m'} = - (1 - \varepsilon)\left[1 - \frac{a^2}{3(1 - \varepsilon)\lambda^{12}}\right] \ ,$$

which shows that m/m' is of the order $(1 - \varepsilon)$ and that the flow is baroclinic only if the ratio $a^2/(1 - \varepsilon)\lambda'^2$, corresponding to $\alpha_2{}^2/\lambda^2 k^2$ in the last example, is of the order 1 or less, a confirmation of our earlier result.

The curious development of the internal mode at the edge of the limited area over which a uniform wind is blowing may also be qualitatively explained by the above analysis. Near the edge, transverse horizontal displacements must give way to vertical displacements, and these are much more easily accommodated by a deformation of the interface than of the free surface.

While the sharply limited wind distribution is unrealistic for the open ocean, the effect it produces may itself be of the highest importance; for precisely such an effect must be expected near coasts where, as a result of wind-induced lateral circulations, water piles up or is removed, and where large deformations of the thermocline occur. One may speculate that the intense jet-like currents found on the western shores of the major oceans, such as the Gulf Stream and the Kuroshio, are due to a related phenomenon. We shall investigate this possibility further in the next section.

7. *The Coastal Jet.* Temperature profiles through the Gulf Stream (Worthington, 1953) reveal a marked baroclinicity. Profiles near 69° W, 38° N, measured in October, show an isotherm near the center of the thermocline sloping from say 800 m depth at the seaward edge of the Gulf Stream to nearly the surface at its center. It seems certain that a theory that would explain this phenomenon would also go far toward explaining the Gulf Stream itself; and it is difficult to avoid the conclusion that the theory must deal with an essentially baroclinic ocean. The most attractive of the existing theories, those of Stommel (1948) and Munk (1950), account for many of the *mean* features of the ocean circulations, including the mean westward

intensification, but since they deal only with a homogeneous ocean or with the total mass transport in a vertical column, they do not allow for effects of baroclinicity. Further, in Munk's theory, the detailed structure of the western current depends in an essential way on a lateral eddy viscosity whose magnitude requires eddies of the size of the Gulf Stream meanders themselves. Such a theory cannot be expected to apply to the individual jet.

The calculations to be presented below are not intended as a theory. They are meant to be suggestive only. However, they do have the advantage of referring to an actual, not a mean, jet-stream, and they take explicit account of the baroclinicity.

The system to be considered first is an infinite double-layer ocean confined between two parallel walls, $x = \pm a$. Assuming that the ocean is initially at rest, we inquire into the motion created by a wind blowing parallel to the walls and varying sinusoidally from one wall to the other according to the law $G = -B \sin (\pi x/2a)\vec{j}$. It should be emphasized that the system under consideration is essentially transient, since we postulate no dissipation of energy, without which no steady state can be reached. A complete analysis would contain a more realistic wind stress acting in an enclosed ocean and would include a dissipation mechanism. The point of view adopted here is that a mechanism which will bring the thermocline to the surface in the vicinity of coasts must continue to operate even after various modifying factors are brought into play.

If the β-effect is ignored, the equations of motion (25) take the form:

$$\left(\frac{\partial^2}{\partial x^2} - \frac{\alpha_i^2}{\lambda^2}\right)\frac{\partial \overline{R}_i}{\partial t} = -\sigma_i \cos kx \quad , \tag{31}$$

with $k \equiv \pi/2a$ and with $R_i = (kf\,B/g)\overline{R}_i$. The boundary condition is obtained by integrating the linearized continuity equations

$$\frac{\partial \eta}{\partial t} = -H\frac{\partial u}{\partial x} \quad , \quad \frac{\partial \eta'}{\partial t} = -H'\frac{\partial u'}{\partial x}$$

with respect to x. Since u and u' vanish at $x = \pm a$, we get

$$\frac{\partial}{\partial t}\int_{-a}^{a} \eta dx = \frac{\partial}{\partial t}\int_{-a}^{a} \eta' dx = 0 \quad ,$$

or, in terms of $\overline{R}_i$,

$$\frac{\partial}{\partial t}\int_{-a}^{a} \overline{R}_i dx = 0 \quad . \tag{32}$$

The solution of the system (31) and (32) is

$$\overline{R}_i = \frac{\sigma_i t}{k^2 + \alpha_i^2/\lambda^2}\left[\cos kx - \frac{\alpha_i \cosh(\alpha_i x/\lambda)}{\lambda k \sinh(\alpha_i a/\lambda)}\right], \tag{33}$$

which gives, for the velocities of the normal modes,

$$v_i = \frac{g}{f}\frac{\partial R_i}{\partial x} = -\frac{\sigma_i k^2 Bt}{k^2 + \alpha_i^2/\lambda^2}\left[\sin kx + \frac{\alpha_i^2}{\lambda^2 k^2}\frac{\sinh(\alpha_i x/\lambda)}{\sinh(\alpha_i a/\lambda)}\right]. \tag{34}$$

The first term in parentheses corresponds to the directly forced motion, in phase with the wind force; the second term corresponds to the motion produced by the Coriolian divergence of water from the coasts. We note by comparison with eq. (29) that this latter effect is analogous to that already found in the case where the wind stress is uniform.

For the baroclinic mode, $\alpha_i^2 >> \lambda^2 k^2$ for widths comparable to ocean widths. Hence, near the coasts, $v_2 \approx \pm\, \sigma_2\, Bte^{-\alpha_2(a-|x|)}$ and is very small elsewhere. The maximum velocity of the barotropic mode is $\sigma_1 Bt$ and diminishes slowly toward the center. At the coastal boundaries the velocity of the baroclinic mode is greater than $\sigma_1 Bt$ by the factor $-\dfrac{\sigma_2}{\sigma_1} \approx \dfrac{1}{r} = 7$. The distance from the coast beyond which the baroclinic velocity becomes small is $\lambda/\alpha_2 = \left[\dfrac{g(1-\varepsilon)H}{f^2(1+r)}\right]^{\frac{1}{2}}$, or about 30 km. If B corresponds to a wind stress of 1 dyne cm^{-2}, the upper layer velocity at each coast would become 2 m sec^{-1} after about four months. In this time the interface at the coast would rise 220 m above its undisturbed depth.

So far we have considered only cases where the wind blows parallel to the shores of an infinite ocean strip. If a wind begins to blow at right angles, say with a sinusoidal variation in the y-direction, no divergence *away* from the coasts will take place. Instead, piling up of water will occur in troughs and ridges at right angles to the coasts. Only later, as a consequence of pressure forces along the coast resulting from the initial divergence, will velocities parallel to the coasts be generated. In the absence of such asymmetries as might be caused by variation of the Coriolis parameter, this mechanism turns out to be not as effective as the parallel wind action in generating intense coastal currents.

To calculate the effect of a normal wind, we assume a wind force of the form $F = B \cos \mu y\, \vec{i}$, where $\vec{i}$ is a unit vector pointing westward, beginning to act at $t = 0$. Eq. (25) then becomes

$$\nabla^2 \frac{\partial R_i}{\partial t} - \frac{\alpha_i^2}{\lambda^2} \frac{\partial R_i}{\partial t} = - \frac{\sigma_i f \mu}{g} B \sin \mu\, y\,. \tag{35}$$

The vanishing of the normal velocity components u and u' at the boundaries is expressed by the geostrophic conditions

$$\frac{\partial R_i}{\partial y} = 0\quad . \tag{36}$$

If $\gamma_i^2 = \mu^2 + \alpha_i^2/\lambda^2$ and $\omega_i = f\mu/\gamma_i$, the solution of the system (35) and (36) may be written

$$R_i = \frac{\sigma_i \mu f B t}{g\, \gamma_i^2}\left(1 - \frac{\cosh \gamma_i x}{\cosh \gamma_i a}\right) \sin \mu y \tag{37}$$

or, in terms of the y-velocities of the normal modes,

$$v_i = - \frac{\sigma_i \mu B t}{\gamma_i} \frac{\sinh \gamma_i x}{\cosh \gamma_i a} \sin \mu y \quad . \tag{38}$$

Taking $\mu = \dfrac{2\pi}{4000}$ km^{-1} we find for the ratio of baroclinic to barotropic velocity at a point on the coast where the wind force is a maximum

$$- \frac{\sigma_2}{\sigma_1} \frac{\gamma_1}{\gamma_2} \approx \frac{\mu \lambda}{r \alpha_2} \approx 0.37\quad .$$

It would seem, therefore, that the wind stress must have components which are parallel to the coasts or which have a very large curl (large μ) if sharp coastal currents of appreciable strength are to be created.

8. *The Effect of the Variation of the Coriolis Parameter.* In the various examples treated, the barotropic (or external) mode of motion was found to be predominant except near coasts; that is to say, the current velocities in the open sea showed no tendency to decrease with depth, as they are observed to do. This result confirms the findings of Rossby, who likewise was unable to account dynamically for a rapid decrease of velocity with depth. However, Stommel has suggested that if one were to take into account the variation of the Coriolis parameter, this result might be altered. The following considerations will show that such indeed is the case if the ocean circulations are produced by transient wind systems moving with sufficiently large period. The physical explanation is that the variation of f permits self-propagating disturbances, and the impressed force system may approach resonance with the free baroclinic oscillatory mode.

To demonstrate this effect, we consider the forced oscillations produced by a traveling wind system exerting the force $F = B \sin (kx - \nu t) \sin \mu y \vec{j}$. The resulting motion, given by solution of eq. (25), is

$$R_i = \frac{\sigma_i f B}{\beta g} \frac{\sin (kx - \nu t)}{(1 - \nu/\nu_i)} \sin \mu y \quad , \tag{39}$$

where β is df/dy, and where

$$\nu_i = - \frac{k\beta}{k^2 + \mu^2 + \alpha_i^2/\lambda^2} \tag{40}$$

is the frequency of the Rossby wave corresponding to the ith mode. If $k = \frac{2\pi}{4000}$ km^{-1} and $\mu = \frac{\pi}{4000}$ km^{-1} we find for the periods $2\pi/\nu$: 9 days in the barotropic case and 8 years in the baroclinic case. Resonance occurs only if the wind system moves toward the west, an unusual occurrence. However, as $2\pi/\nu$ becomes of the order of or larger than the baroclinic period of 8 years, the baroclinic amplitude begins to predominate, and in the limit, $2\pi/\nu = 0$, it exceeds the barotropic by the factor $- \sigma_2/\sigma_1 \approx 7$.

If, instead of a forced disturbance, we consider the disturbance produced from rest by the sudden action of the stationary wind force $F = B \sin kx \sin \mu y \vec{j}$, we find that

$$R_i = \frac{\sigma_i f B}{\beta g} \left[\sin kx - \sin (kx - \nu_i t) \right] \sin \mu y \quad , \tag{41}$$

where the first term in the bracketed expression corresponds to the forced disturbance and the second term to the free disturbance. It is seen that, in both, the baroclinic mode is larger than the barotropic in the ratio $- \sigma_2/\sigma_1 \approx 1/r = 7$.

Unfortunately the foregoing results cannot be applied directly to the circulation produced in an enclosed ocean by a stationary wind field. It is not known how the resonance considerations become modified by the inclusion of boundaries. This problem is still under investigation. However, judging from the central role played by the planetary vorticity effect in Stommel's and Munk's theories of the wind-driven ocean circulation, it seems certain that any successful treatment of the planetary ocean circulation must eventually take it into account. Thus for example Stommel finds that the variation of the Coriolis parameter is responsible for a general westward intensification of the ocean circulation in a rectangular ocean. In the present case, where the circulation is produced from rest by wind action, one should expect to find the same effect. But here it has been shown that a reduction of the horizontal extent of a current is accompanied by an intensification of its baroclinic component near the coast. Hence the β-effect combined with the development of the baroclinic mode should lead to a strengthening of the sharp coastal current on the western side and a weakening on the eastern side.

9. *Summary and Conclusion.* On the assumption that both stationary and transient ocean currents produced by the action of large-scale atmospheric wind systems are in a state of quasi-nondivergent balance between pressure and velocity, a system of equations has been deduced which has the virtue of applying only to current motions to the exclusion of background inertio-gravitational wave "noise." In these equations the cross-current circulations, needed to restore the pressure-velocity balance when it is disturbed by wind action, were treated as forced motions. A method has been given whereby the complex Ekman-spiral action of the wind-induced forces in the layer of frictional influence is replaced by a simple boundary condition in the case of a continuously stratified atmosphere or by a body force in the case of an ocean consisting of superimposed homogeneous layers. Application of the combined formalism to the behavior of a double-layer ocean under the influence of wind stresses has led to the result that, in the absence of effects due to the variation of the Coriolis parameter, uniform currents extending all the way to the ocean bottom are created. This result confirms some earlier work of Rossby.

There was, however, an important exception. In the immediate vicinity of coasts, large deformations of the interface were found to occur because of the action of gravitational stabilizing forces. These deformations counteract the establishment of deep currents in such a way that the flow is confined primarily to the upper thin-layer. The width of the resultant coastal currents is of the order of 30 km and therefore it corresponds roughly to the observed widths of the Gulf Stream and Kuroshio. In order that such currents may form, the wind must have either a component parallel to the shore or an unusually large curl. (This last result may be changed when the variation of the Coriolis parameter is taken into account, for it will have the effect of reducing the extent of the western current and thereby, as was shown previously, accentuating still further the narrow baroclinic coastal current.)

In accordance with a suggestion of Stommel, a preliminary examination was made of modifications caused by the variation in the Coriolis parameter. It was found that in open oceans the variation gives rise to currents that are confined mainly to the upper layer if the wind stress system is horizontally nonuniform and moves very slowly, i.e., if it oscillates with a period of the order of or greater than 8 years. It has not been shown, however, to what extent these results carry over to enclosed oceans.

To account for the steady-state ocean circulation, one must evidently include a dissipation mechanism. The writer has attempted only to show how, in certain circumstances, the motion produced from rest by the action of a wind stress on the surface of a *baroclinic* ocean will generate intense coastal currents. Obviously the next step is to deal with an enclosed ocean with a variable Coriolis parameter and to take dissipation into account. However, even then it is not obvious how a dissipation mechanism will be introduced. There is some evidence that much of the total energy dissipation in the North Atlantic occurs as a result of Gulf Stream *meandering*. If this is so, then the circulation is intrinsically nonsteady and perhaps could best be approached as an initial value problem.

Finally, it should be pointed out that the explanation which has been given for the coastal currents has depended in an essential way on the transient character of the flow. One must postulate that after intense currents have been developed they will be maintained by mass advection in a manner that can be accounted for by only inclusion of nonlinear terms in the continuity and vorticity equations. One may surmise that mass transports produced by the action of wind stress in the open ocean will be successfully explained by a theory very close to that of Munk—perhaps involving some density stratification—and that the sharp coastal currents will be found to be a dynamic response of stratified coastal waters to these transports.

Acknowledgment. The author wishes to thank the administration of the Woods Hole Oceanographic Institution for its hospitality in extending to him its facilities during several visits, and he would like particularly to acknowledge his indebtedness to Henry Stommel of that Institution for many informative and helpful discussions. Thanks are also due H. Stommel and G. Veronis for a critical reading of his manuscript.

REFERENCES

Charney, J. G.

1947. The dynamics of long waves in a baroclinic westerly current. J. Meteorol., *4:* 135–162.

1948. On the scale of atmospheric motions. Geofys. Publ., *17:* 1–17.

Charney, J. G. and A. Eliassen

1949. A numerical method for predicting the perturbations of the middle latitude westerlies. Tellus, 1: 38–54.

Ekman, W.
1905. On the influence of the earth's rotation on ocean currents. Ark. Mat. Astr. Fys., *2*.

Eliassen, A.
1952. Slow thermally or frictionally controlled meridional circulations in a circular vortex. Astro. Norv., *5:* 19–60.

Fultz, D.
1951. Non-dimensional equations and modeling criteria for the atmosphere. J. Meteorol., *8:* 262–267.

Munk, W. H.
1950. On the wind-driven ocean circulation. J. Meteorol., *7:* 79–93.

Rossby, C. G.
1937. On the mutual adjustment of pressure and velocity distributions in certain simple current systems, Part I. J. Mar. Res., *1:* 15–28.
1938. On the mutual adjustment of pressure and velocity distributions in certain simple current systems, Part II. J. Mar. Res., *1:* 239–263.

Stommel, Henry
1948. The westward intensification of wind-driven ocean currents. Trans. Amer. geophys. Un., *29:* 202–206.

Veronis, G. and Henry Stommel
1956. The action of variable wind stresses on a stratified ocean. J. Mar. Res., *15* (1).

Worthington, L. V.
1953. Three detailed cross-sections of the Gulf Stream. Tech. Rep. Woods Hole Oceanogr. Inst., Ref. No. 53-85: 1–7 (Unpublished).