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ON THE EXCHANGE OF MOMENTUM BETWEEN THE ATMOSPHERE, THE OCEANS, AND THE SOLID EARTH

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There is general agreement now among oceanographers that the large ocean currents are essentially maintained by the stress exerted by planetary wind systems on ocean surfaces. Therefore it is of primary importance for oceanography to know the distribution of the wind stress on oceans and its variation in time.

The wind stress τ_0 has generally been expressed in terms of a resistance coefficient, γ^2, defined by

$$\tau_0 = \gamma^2 \rho_a V_0^2. \tag{1}$$

Here V_0 denotes the wind speed at a level near the surface (anemometer level) and ρ_a the density of the surface air. The resistance coefficient γ^2 is not a real constant, since it depends upon the roughness of the sea surface. Thus, in the case of a water surface, roughness depends partly on the waves formed by the wind. Different investigations have produced different values of the resistance coefficient, and an interesting summary of the present disagreement concerning the value of the resistance coefficient is found in a recent paper by Montgomery (1952), who summarizes the situation as follows: "The present evidence is so conflicting that at no wind speed is the resistance coefficient known confidently within its half value."

For oceanography it is probably less important than for other fields of study to have more or less exact measurements of the wind stress under given conditions; it is probably sufficient for oceanographers to know reasonably well the mean wind stress on large ocean areas and over extended periods of time. In the following paragraphs some of the methods which could be used for a computation of the global distribution of wind stress will be discussed.

The global wind system is characterized by the prevailing easterlies, roughly between Lat. 30° N and 30° S, and by the prevailing westerlies polewards from these latitudes. This large-scale planetary wind system results in a characteristic exchange of angular momentum between the earth, including the oceans and the atmosphere. In the belt of easterlies, the earth, due to the interfacial stress, exerts

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a torque on the atmosphere. The rotation of the atmosphere in the tropics is therefore speeded up and its westerly angular momentum increases. In the belts of westerlies, the interfacial stress acts in an opposite direction and the rotation of the atmosphere in those regions has a tendency to be reduced. The principle of conservation of angular momentum means that the total angular momentum of the solid earth, the hydrosphere, and the atmosphere combined remains constant. For periods of time ranging from a few days to seasons there could be observable changes in the angular momentum of the solid earth, the hydrosphere and the atmosphere taken separately; seasonal changes in this respect have been observed and have been found to be in agreement with the dynamic theories. However, those changes are of minor importance for our problem, and, on the whole, we can assume therefore that the angular momentum of the atmosphere, the oceans, and the solid earth taken separately remains constant for extended periods of time.

This principle means that there must be a continuous flux of angular momentum in the earth's atmosphere from the "source region" in the tropical and subtropical easterlies to the "sink regions" in the extratropical westerlies. The maximum meridional flux of angular momentum then occurs in the latitudes that separate the zone of easterlies from the belts of westerlies. Hence the total interfacial stress in the trade-wind belt multiplied by the distance from the earth's axis must equal the interfacial stress in the zones of westerlies multiplied by the distance from the earth's axis in these regions, but with opposite sign.

This principle can be used for a computation of mean surface stress in the principal wind belts. If it is possible to determine the poleward flux of angular momentum in the atmosphere across every latitude, these fluxes would also determine the mean surface stress in every singular latitudinal belt.

In recent years extensive investigations have been carried out in this field. Without going into detail I especially want to refer to the important work done on this subject at Massachusetts Institute of Technology by Starr and White (1952) and others, at the University of California at Los Angeles by Bjerknes (1951) and Mintz (1951), and in Australia by Priestley (1951). Due essentially to the results of this work, we know reasonably well the mean flux of angular momentum in the meridional direction as a function of latitude, and consequently we are also in a position to compute the mean surface stresses in selected latitudinal belts.

As an example, the fluxes of angular momentum in January and July across Lat. 30° N are presented in Fig. 1. The amount of

angular momentum carried through this latitude in January averages about 50×10^{25} g cm²sec⁻² and in July about 15×10^{25} g cm²sec⁻². According to the principle presented previously, these values also represent the angular momentum which is brought up into the atmosphere between Lat. 30° N and the equator and which is carried down to the earth north of 30° N. The corresponding angular momentum flux through Lat. 30° S is not so well known due to lack of meteorological data from the southern hemisphere. However, from available data, this flux could be estimated to be about 35×10^{25} units in January and about 55×10^{25} units in July. Therefore, the total torque in the trade-wind belt due to surface wind stress, in the most general meaning of the word, amounts to about 85×10^{25} g cm²sec⁻² in January and about 70×10^{25} g cm²sec⁻² in July. From
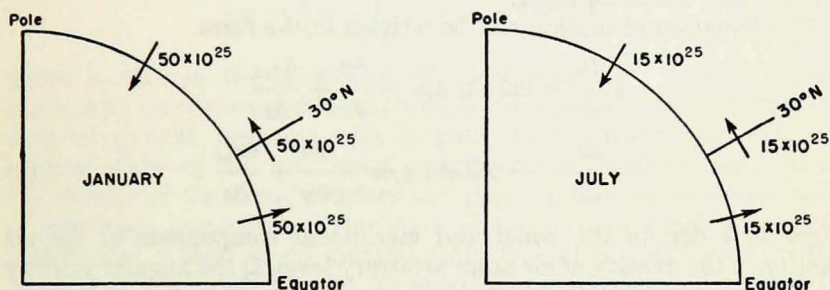

Figure 1. Schematic figure showing the mean flux of angular momentum across Lat. 30° N in July and January (essentially according to Mintz); unit g cm²sec⁻².

these values the mean surface wind stress in the whole trade-wind zone can be evaluated.

A part of this exchange of angular momentum represents a direct exchange between the atmosphere and the solid earth and another part represents an exchange between the atmosphere and the oceans. This latter part could be computed if the ratio of oceans and continents and, in addition, the mean surface stress in these different areas were known. Before we discuss this question, a few words should be said concerning the mechanism of the meridional flux of angular momentum in the atmosphere.

According to the investigations referred to previously, the meridional flux of angular momentum in the atmosphere across Lat. 30° N and 30° S is essentially performed by large-scale atmospheric disturbances of the type of long waves, cyclones and anticyclones, and so on. Consequently the flux can be considered as a result of different kinds of large-scale eddy stresses, "Reynolds stresses," between the tropical and extratropical atmosphere. These large-scale eddies are especially

effective in the middle and upper parts of the troposphere and in the tropopause region, as has been shown by Mintz (1951) and by Starr and White (1952), As a result of this, the momentum flux across the latitudes in question occurs essentially in the layers between 700 and 100 mb, or between 10,000 and 55,000 feet. It is of considerable interest that computations carried out for atmospheric layers so far removed from the sea surface can be used to determine wind stress on ocean surfaces.

The interfacial stress, however, can also be determined from wind observations at lower levels. As has already been mentioned, the conventional method is to use eq. (1). Another method, probably first used by Richardson (1920) and later by Rossby and Montgomery (1935) and several others, is based on the departure from geostrophic wind in the frictional layer.

The equation of motion can be written in the form

$$\rho \frac{du}{dt} = 2\Omega \sin \phi \rho v - \frac{\partial p}{\partial x} + \frac{\partial \tau_{xz}}{\partial z}$$

$$\rho \frac{dv}{dt} = - 2\Omega \sin \phi \rho u - \frac{\partial p}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} . \tag{2}$$

Here u, v denote the zonal and meridional components of the air motion, ρ the density of air at an arbitrary level, Ω the angular velocity of the earth's rotation, ϕ the latitude, and p the pressure; τ_{xz}, τ_{yz} denote the components of shearing stress in the direction of the axes. In these equations some small terms have been neglected. By integrating these equations, the difference in shearing stresses between the earth's surface and an arbitrary level H can be determined:

$$\tau_{xH} - \tau_{x0} = - 2\Omega \sin \phi \int_0^H \rho \, (v - v_g) \, dz + \int_0^H \rho \frac{du}{dt} \, dz$$

$$\tau_{yH} - \tau_{y0} = 2\Omega \sin \phi \int_0^H \rho \, (u - u_g) \, dz + \int_0^H \rho \frac{dv}{dt} \, dz . \tag{3}$$

Here τ_{xH}, τ_{x0} and τ_{yH}, τ_{y0} denote the components of the stresses at the level H and the surface; u_g, v_g denote the components of the geostrophic wind.

If we apply these equations to the mean motion of a total latitudinal ring in the lower atmosphere, then the terms containing the acceleration components become relatively small and can be neglected, as a

first approximation. In this case also the mean geostrophic meridional component, $\bar{v}_g$, vanishes. Thus we get

$$\bar{\tau}_{x0} = \bar{\tau}_{xH} + 2\Omega \sin \phi \int_0^H \rho \, \bar{v} \, dz$$

$$\bar{\tau}_{y0} = \bar{\tau}_{yH} - 2\Omega \sin \phi \int_0^H \rho \, (\bar{u} - \bar{u}_g) \, dz \, . \tag{4}$$

The bars denote mean values for a total ring of latitude.

The components of shearing stress in the atmosphere can be written

$$\tau_{xH} = \left(\mu \, \frac{\partial u}{\partial z} \right)_H \, , \quad \tau_{yH} = \left(\mu \, \frac{\partial v}{\partial z} \right)_H \, , \tag{5}$$

where μ denotes the coefficient of eddy viscosity. In the atmosphere, with its poleward-directed temperature gradient, the geostrophic west component increases with height. In the trade-wind belt the vertical shear of the real zonal component is therefore negative in the vicinity of the earth's surface and passes a level of no shear (level of maximum easterlies). The level of no vertical zonal shear, H_1, is then also a level of no vertical eddy flux of momentum. In the zones of surface westerlies, no corresponding level of zero stress exists.

If the integration in (4) is extended from the surface to the level H_1, the zonal component of the surface stress can be determined by the simple formula

$$\bar{\tau}_{x0} = 2\Omega \sin \phi \int_0^{H_1} \rho \, \bar{v} \, dz = 2\Omega \sin \phi \, [m_\phi]_0^{H_1} \, , \tag{6}$$

where $[m_\phi]_0^{H_1}$ denotes the mean meridional mass flow through the unit length of the parallel ϕ in the layer between the surface and the level of maximum east wind.

An attempt was made to compute the mean zonal and meridional wind components at different levels in the vicinity of the latitude of strongest easterlies (around Lat. 12° N in winter), and the stations used for that purpose are marked in Fig. 2. The complete results will not be presented here. However, a similar computation was made for the time period January–February 1952 for only eight of those stations which could be considered as real ocean stations. The result is presented graphically in Fig. 3, with a linear pressure scale as ordinate. The mean values of the u-components are marked by
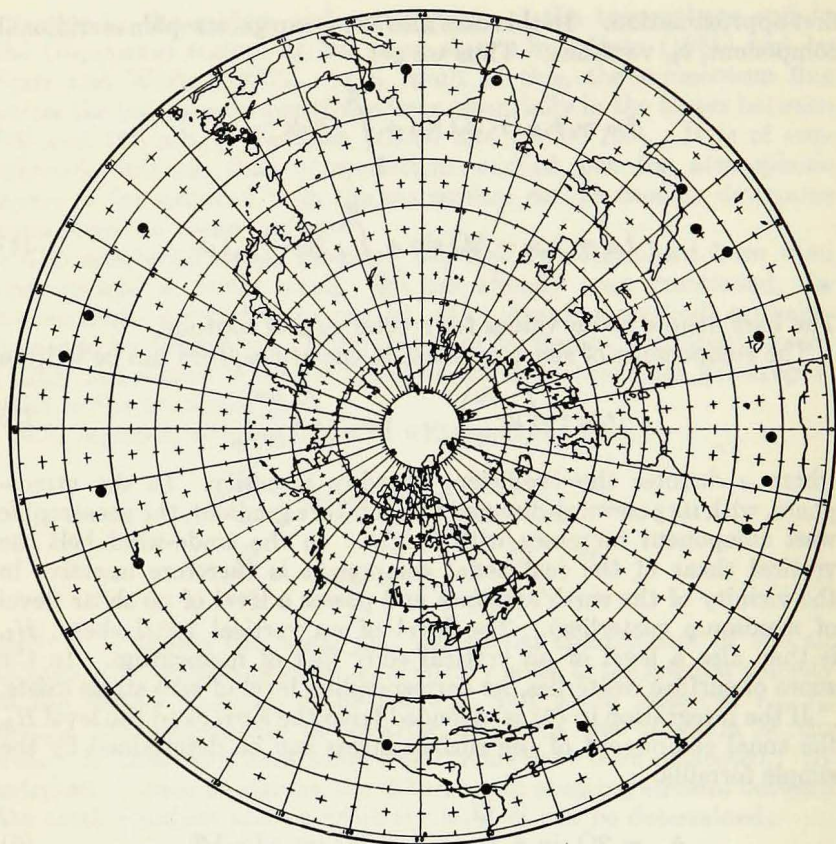
Figure 2. Circumpolar wind stations used to compute the mean distribution of the zonal and meridional wind components with height in the vicinity of Lat. 13° N.

crosses at the levels of wind observations, whereas the mean values of the v-components are marked by rings. The mean level of maximum east wind is 915 mb, and the mean level of maximum north wind is 957.5 mb. The components of surface stresses are proportional to the areas A and B, shaded in the diagram. For mean values of τ_{x0} and τ_{y0} we get the values -1.0 and -0.6 dynes/cm^2 respectively.

This example shows how the mean zonal and meridional components of surface stress could be determined for different latitudes if sufficiently good wind data were available. Unfortunately the network of upper wind stations is not dense enough over the ocean areas to permit a detailed computation of surface stresses on a global scale.
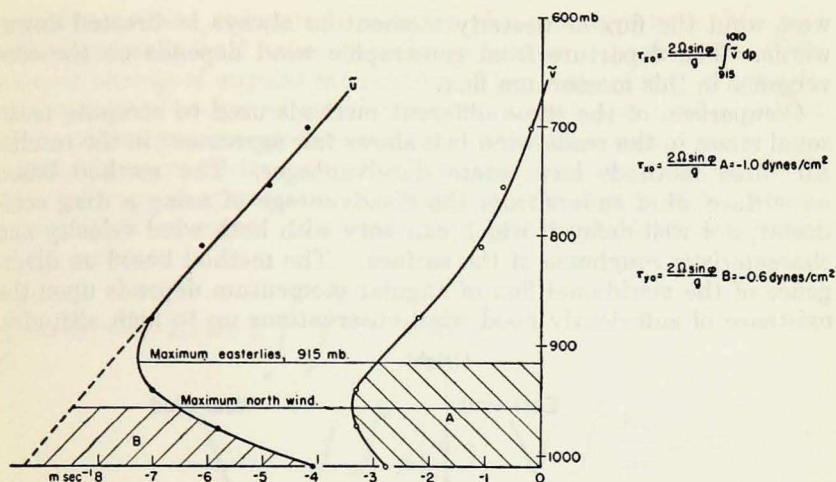
Figure 3. Mean distribution of the zonal and meridional wind components, with height, over eight selected ocean stations (linear pressure scale). The shaded areas A and B are proportional to the total ageostrophic meridional and zonal mass flow used for the computation of components of surface stress given. The dashed curve indicates the mean geostrophic zonal wind.

The above values are valid for January and February and for the mean Lat. 13° N, where the acceleration terms in (3) approximately vanish. They seem to be in fair agreement with the corresponding values of the stress computed by Priestley (1951) from surface wind data by using equation (1). Our values, however, are somewhat higher.

Since, in the zones of westerlies, no level of vanishing τ_{xH} exists in the lower atmosphere, the surface stress here cannot be computed directly from the departure from geostrophic wind in the frictional layer. This important difference between the zones of easterlies and the zones of westerlies was first pointed out by Sheppard, *et al.* (1952) and by Sheppard and Omar (1952). As can be seen from (4), the mean ageostrophic mass flow is proportional to the difference between surface stress and stress in an arbitrary level, not to the surface stress itself. This seems to be the most probable explanation of the fact that computations of surface stress from the departure of the wind from geostrophic flow generally have resulted in considerably smaller values than those computed in other ways (Montgomery, 1952).

Fig. 4 shows schematically the difference of momentum flux in cases of surface easterlies and westerlies. For an east wind, westerly momentum is transported upwards below the level of maximum easterlies and downwards above that level, whereas in the case of a

west wind the flux of westerly momentum always is directed downwards. The departure from geostrophic wind depends on the convergence in this momentum flux.

Comparison of the three different methods used to compute mean zonal stress in the trade-wind belt shows fair agreement in the results. All three methods have some disadvantages. The method based on surface wind suffers from the disadvantage of using a drag coefficient, not well defined, which can vary with both wind velocity and characteristic roughness of the surface. The method based on divergence of the meridional flux of angular momentum depends upon the existence of sufficiently good wind observations up to high altitudes.
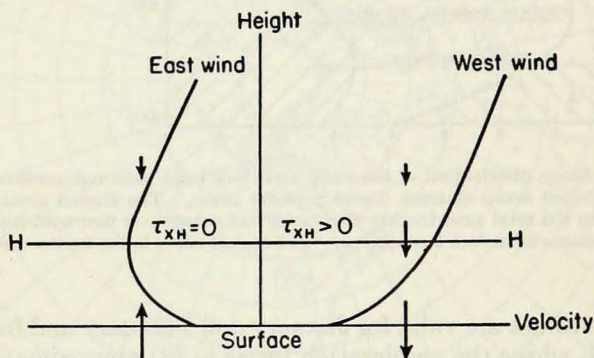

Figure 4. Schematic figure showing the difference in vertical eddy flux of momentum in the zones of easterlies and of westerlies. The curves represent the characteristic profiles of the zonal wind components with height; the arrows indicate the direction and amount of eddy flux of westerly momentum.

Also the third method, based on departure from geostrophic flow in the lower atmosphere, depends entirely upon the existence of a good network of wind stations evenly distributed over the oceans.

Priestley has determined from surface wind data that the mean zonal stress in January for the zone Equator $-30°$ N is about -0.5 dynes/cm². From values of the total flux of angular momentum in the atmosphere through Lat. 30° N, a mean zonal stress of about -0.60 dynes/cm² results. As shown previously, the same value was obtained from the mean meridional mass flow in the frictional layer. It is probable that the somewhat higher values computed from meridional flux of angular momentum and meridional mass flow are influenced by the stronger friction over continents than over oceans due partly to zonal pressure forces acting upon mountains.

From these values of mean zonal surface stress in the northern trade-wind belt, the corresponding mean stress in the zone of surface

westerlies can be computed by using the principle that the total surface torque in both zones must be in balance. In that case there is no net change of angular momentum of the atmosphere. However, for a complete budget of angular momentum we also need a budget for the exchange between the solid earth and the oceans.

From the previous discussion it follows that, over longer time periods, the total angular momentum of every latitudinal strip of the oceans must remain constant. This rule can be expressed approximately by the equation

$$\int_0^L \tau_{x0}\, dx = -\,2\Omega \sin\phi \int_0^L \int_0^D \rho_w v_w\, dz\, dx\; +$$

$$\text{(7)}$$

where the first integral denotes the total mass transport in meridional direction in the water layer influenced by the wind, the second integral is the total pressure acting in the zonal direction, and the third integral determines the stress on the lateral boundaries of the strip. The indices w denote that we are now dealing with movement in the water; D is the depth of the water layer considered, and L is the zonal width of the ocean. At depth D the frictional stress vanishes.

For an entire zonal strip of the ocean, the net meridional mass transport can be considered zero. If we neglect the meridional stresses, about which we know little, eq. (7) can be written

$$\text{(8)}$$

Eq. (8), which determines the zonal pressure gradient in the water as a function of the zonal wind stress in the absence of any bottom and lateral stress, has been used to determine the wind stress from the slope of the isobaric surfaces in the direction of the wind.

In a schematic rectangular ocean extending over the hemispheric belt of easterlies and the belts of westerlies, the slope of the ocean surface should have the characteristics shown in Fig. 5, if D is constant. Applied to real oceans, however, the influence of the currents induced by wind stress should also be considered. This problem has been studied by Sverdrup (1947) and in greater detail by Munk (1950).

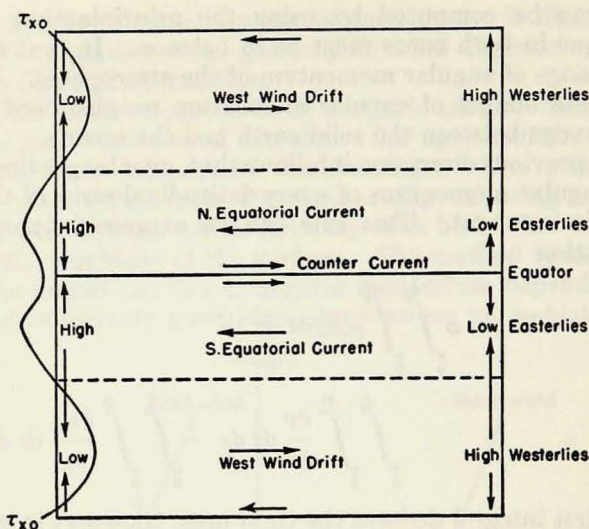In order to get an idea of the slope of the ocean surface that is

Figure 5. Schematic picture showing the zonal wind stress, the resulting height of the ocean surface, and the corresponding currents in a rectangular ocean. The influence of the currents upon the topography of the ocean surface is not considered in this figure.

necessary to compensate for the momentum transferred to the ocean from the atmosphere, we make the assumption that the ocean covers the whole northern trade-wind belt except for a continental barrier of no zonal extension along a given meridian. If the pressure gradient is constant in the whole layer with depth D, assumed to be 200 m, and if the mean wind stress is 0.5 dynes/cm², there would be a pressure difference of 10 cb between the eastern and western side of the barrier. This pressure difference corresponds approximately to a difference in height of 100 cm.

Considering the width of the three oceans in the northern trade-wind belt, in January the surface of the Atlantic Ocean would have a total slope of about 17 cm, the Pacific Ocean would have a slope of 47 cm, and the Indian Ocean one of 15 cm. In July the slope of the Atlantic and Pacific oceans would be considerably smaller, whereas the slope of the Indian Ocean north of the equator would have an opposite sign due to the southwest Monsoon. The real topography of the ocean surface, however, is much more complicated. As was mentioned before, the topography is determined not only by the wind stress but by the depth D and the circulation in the upper layer caused by the planetary wind system.

In computing these approximate values of the zonal component of

slope of ocean surfaces, the exchange of momentum due to lateral stresses has been neglected.   Lateral stresses, however, only exchange momentum between different parts of the oceans, and our purpose was to study only the mechanism of the exchange of momentum between oceans and continents.   For a complete theory it would be necessary to evaluate the net transfer of angular momentum between the atmosphere and all oceans and, considering the possible frictional transfer at the continental boundaries and at the bottom of oceans, determine the total pressure force.   This, however, is not possible at the present time.

In parts of the Antarctic Circumpolar Current no continental barriers exist.   Consequently here no zonal pressure force can act around the globe.   It is necessary, therefore, to consider the possible existence of a relatively strong zonal bottom current in this special part of the oceans, as Munk and Palmén have pointed out (1951).

Formula (8) has also been used for computation of the mean wind stress on a water surface.   When dealing with enclosed basins of larger seas or with lakes where the wind under favored conditions can be considered approximately constant, this formula probably gives the best values that can be achieved.   The integrated form,

$$\lambda \, \tau_0 = \rho_w g \, DS, \tag{9}$$

was first used by Ekman (1905) for the Baltic Sea and later by several others (Hela, 1948; Palmén and Laurila, 1938).   In (9), g is the acceleration of gravity, S denotes the slope of the free surface, and λ is a constant which depends upon the depth of the water and which approaches 1 if the depth is so large that bottom friction can be neglected.

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