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ON THE ANTARCTIC CIRCUMPOLAR CURRENT[1]

By

KOJI HIDAKA[2] AND MIZUKI TSUCHIYA

ABSTRACT

In 1951 Munk and Palmén discussed the dynamics of the Antarctic Circumpolar Current and found that they could explain the total mass transport of this current by assuming a far larger value for the coefficient of lateral mixing than had been obtained for other parts of the oceans. An attempt to solve the same problem in terms of the current velocity itself reveals that the slope of the sea surface and the vertical distribution of current velocity could be satisfactorily explained if the very high value which had been anticipated by Munk and Palmén were assumed for the lateral mixing coefficient. It was found also that the distributions of surface slope and velocity are not practically influenced by lateral mixing and are inconsistent with actual observations so long as the coefficient of lateral mixing remains less than 10^8g cm^{-1} sec^{-1}.

INTRODUCTION

In 1951 Munk and Palmén published a paper on the dynamics of the Antarctic Circumpolar Current in which they showed that the mass transport in the Antarctic Ocean is inversely proportional to the coefficient of horizontal mixing and that the computed transport is one hundred times that observed. They attributed the discrepancy between the computed and observed values to the effect of submarine ridges, and therefore they suggested that appreciable motion must occur at the bottom.

The conclusion that in a zonal ocean the mass transport produced by a planetary wind system is inversely proportional to the coefficient of horizontal mixing confirms a result previously obtained by Hidaka (1950). He found that a value of 10^9g cm^{-1} sec^{-1} for the horizontal mixing coefficient is most plausible for explaining the actual surface slope of the ocean. Since both of these papers employ mass transport as the dependent variable, neither one allows a determination of the vertical velocity distribution. Recently Hidaka and Takano (1952) solved the problem in terms of the current velocity itself.

In the present paper, a similar solution in terms of the current velocity is applied to the Antarctic Circumpolar Current; and the

pattern of water movement in the Antarctic Ocean, particularly with reference to the influence of horizontal mixing, is studied.

THEORY

Taking x- and y-axis positive eastward and northward respectively and z-axis vertically downward, the equations of motion are

$$\frac{A_v}{\rho}\frac{\partial^2 u}{\partial z^2} + \frac{A_h}{\rho}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + 2\omega \sin\phi \cdot v = \frac{1}{\rho}\frac{\partial p}{\partial x},$$

$$\frac{A_v}{\rho}\frac{\partial^2 v}{\partial z^2} + \frac{A_h}{\rho}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - 2\omega \sin\phi \cdot u = \frac{1}{\rho}\frac{\partial p}{\partial y},$$

where u, v are the components of the current velocity in x- and y-directions, ρ the density, p the pressure, A_h and A_v the coefficients of horizontal and vertical mixing, ω the angular velocity of the earth, and ϕ the geographical latitude.

If we assume that the prevailing winds blow in the E-W direction only, u, v, p and the surface elevation ζ will be independent of x. In this case, putting $W = u + iv$, the above equations may be combined as follows:

$$\frac{A_v}{\rho}\frac{\partial^2 W}{\partial z^2} + \frac{A_h}{\rho}\frac{\partial^2 W}{\partial y^2} - 2\omega i \sin\phi W = ig\frac{\partial\zeta}{\partial y}, \tag{1}$$

where g is the acceleration of gravity.   The equation of continuity is

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{2}$$

where w is the vertical component of velocity.

As boundary conditions, we have

$$W = 0 \begin{cases} \text{at } z = h \text{ (the bottom)} & (3) \\ \text{at } y = \pm l \text{ (45° S and 70° S)} & (4) \end{cases}$$

$$A_v\frac{\partial W}{\partial z} + \tau = 0 \text{ at } z = 0 \text{ (the surface)} \tag{5}$$

where τ is the wind stress in x-direction.

Now we write

$$W = \sum_{s=1}^{\infty} w_s(y) \cos\frac{(2s-1)\pi z}{2h}, \tag{6}$$

where

$$w_s(y) = \frac{2}{h}\int_0^h W(\lambda) \cos\frac{(2s-1)\pi\lambda}{2h}\, d\lambda, \tag{7}$$

and by Stokes' method we write

$$\frac{\partial^2 W}{\partial z^2} = \frac{2}{h} \sum_{s=1}^{\infty} \cos \frac{(2s-1)\,\pi z}{2h} \int_0^h \frac{\partial^2 W}{\partial \lambda^2} \cos \frac{(2s-1)\,\pi \lambda}{2h}\, d\lambda \,. \quad (8)$$

Using (3), (5), (7), we have

$$\frac{2}{h} \int_0^h \frac{\partial^2 W}{\partial \lambda^2} \cos \frac{(2s-1)\,\pi \lambda}{2h}\, d\lambda = \left[\frac{2}{h}\,\frac{\tau}{A_v} - \frac{(2s-1)^2 \pi^2}{4h^2}\, w_s\,(y) \right] ; \quad (9)$$

then (8) becomes

$$\frac{\partial^2 W}{\partial z^2} = \sum_{s=1}^{\infty} \left[\frac{2}{h}\,\frac{\tau}{A_v} - \frac{(2s-1)^2 \pi^2}{4h^2}\, w_s\,(y) \right] \cos \frac{(2s-1)\,\pi z}{2h}\,. \quad (10)$$

Since $1 = \dfrac{4}{\pi} \displaystyle\sum_{s=1}^{\infty} \dfrac{(-1)^{s+1}}{(2s-1)} \cos \dfrac{(2s-1)\,\pi z}{2h}$, substitutions of (6) and

(10) into (1) give

$$A_h \frac{d^2 w_s}{dy^2} - \left(2\omega\rho i \sin\phi + \frac{(2s-1)^2 \pi^2}{4h^2}\, Av \right) w_s$$
$$= \frac{4}{\pi}\,\frac{(-1)^{s+1}\rho g i}{(2s-1)}\,\frac{\partial \zeta}{\partial y} - \frac{2}{h}\,\tau \,. \quad (11)$$

Let

$$w_s\,(y) = \sum_{m=1}^{\infty} D_{ms} \sin \frac{m\pi\,(l+y)}{2l}\,, \quad (12)$$

$$\frac{d\zeta}{dy} = \sum_{m=1}^{\infty} p_m \sin \frac{m\pi\,(l+y)}{2l}\,, \quad (13)$$

$$\tau\,(y) = \sum_{m=1}^{\infty} \tau_m \sin \frac{m\pi\,(l+y)}{2l}\,, \quad (14)$$

and since

$$\sin\phi \sin \frac{m\pi\,(l+y)}{2l} = \sum_{j=1}^{\infty} E_m{}^j \sin \frac{j\pi\,(l+y)}{2l}\,,$$

where

$$
E_m{}^j =
\begin{cases}
\dfrac{l}{R}\sin\phi_0 \sin\dfrac{l}{R}\left[\dfrac{1}{\left(\dfrac{l}{R}\right)^2-\left(m-j\right)^2\dfrac{\pi^2}{4}} - \dfrac{1}{\left(\dfrac{l}{R}\right)^2-\left(m+j\right)^2\dfrac{\pi^2}{4}}\right] \\
\hspace{7cm}\text{for } m+j\text{: even,} \\[4pt]
\dfrac{l}{R}\cos\phi_0 \cos\dfrac{l}{R}\left[\dfrac{1}{\left(\dfrac{l}{R}\right)^2-\left(m-j\right)^2\dfrac{\pi^2}{4}} - \dfrac{1}{\left(\dfrac{l}{R}\right)^2-\left(m+j\right)^2\dfrac{\pi^2}{4}}\right] \\
\hspace{7cm}\text{for } m+j\text{: odd,}
\end{cases}
$$

and ϕ_0 is the latitude of the origin and R is the radius of the earth, (11) then becomes

$$
-\left(\frac{m^2\pi^2}{4l^2}A_h + \frac{(2s-1)^2\pi^2}{4h^2}A_v\right)D_{ms} - 2\omega i\rho\sum_j D_{js}E_j{}^m
$$
$$
= \frac{4}{\pi}\frac{(-1)^{s+1}\rho g i}{(2s-1)}p_m - \frac{2}{h}\tau_m , \tag{15}
$$
$$
m = 1, 2, \ldots\ldots\ldots\ldots
$$
$$
s = 1, 2, \ldots\ldots\ldots .
$$

Solving equations (15), the values of D_{ms} are represented as polynomials of τ_m and p_m.

Now integration of (2) with respect to Z gives

$$
\int_0^h v\, dz = 0 . \tag{16}
$$

From (16), the values of p_m are determined as functions of τ_m so D_{ms} can be represented by τ_m only.

Then, from (6) and (12), the current velocity is given by

$$
W = \sum_{m,\,s} D_{ms}\sin\frac{m\pi\,(l+y)}{2l}\cos\frac{(2s-1)\pi z}{2h} . \tag{17}
$$

NUMERICAL COMPUTATIONS

We assume that the density is constant throughout the sea water and equal to 1, and that $\tau = 2$ dyne cm^{-2}; then

$$
\begin{aligned}
\tau_1 &= 2.546, & \tau_2 &= 0, \\
\tau_3 &= 0.849, & \tau_4 &= 0, \\
\tau_5 &= 0.509, & \tau_6 &= 0, \\
&\ldots\ldots\ldots & &\ldots\ldots\ldots
\end{aligned}
$$

With $\omega = 7.29 \times 10^{-5}$sec^{-1}, $g = 980$ cm sec^{-2}, $h = 4000$ m, $A_v = 2 \times 10^3$g cm^{-1} sec^{-1}, $A_h = 10^8$g cm^{-1} sec^{-1}, the values of D_{ms}, when m and s are less than 6 and 8 respectively, are computed and shown in Table I. Here practical computations $E_m{}^i$ other than $E_m{}^{m-1}$, $E_m{}^m$, $E_m{}^{m+1}$ can be neglected.

The surface elevation is given by

$$\zeta = -1202 \cos \frac{\pi\,(l+y)}{2l} - 32 \cos \frac{2\pi\,(l+y)}{2l} - 79 \cos \frac{3\pi\,(l+y)}{2l}$$
$$- 17 \cos \frac{5\pi\,(l+y)}{2l},$$

and the total mass transport M is given by

$$M = \int_{-l}^{l} \int_0^h \rho u \, dy \, dz = 8.1 \times 10^{15} \text{g sec}^{-1}.$$

Similarly, for $A_h = 10^{10}$g cm^{-1} sec^{-1} conputations are made. The values of D_{ms} are given in Table II. And the surface elevation and the total mass transport are as given by

$$\zeta = -148 \cos \frac{\pi\,(l+y)}{2l} - 3.9 \cos \frac{2\pi\,(l+y)}{2l} - 2.0 \cos \frac{3\pi\,(l+y)}{2l},$$

$$M = 9.3 \times 10^{14} \text{g sec}^{-1}.$$

In both cases we assume $A_v = 2 \times 10^3$g cm^{-1} sec^{-1}, although this value is a little larger than that of other oceans. The magnitude of A_v, however, is not very important in numerical computations. Further, when A_h is less than 10^8g cm^{-1} sec^{-1}, the computed values are scarcely affected by its variation.

TABLE I. VALUES OF D_{ms} FOR $A_h = 10^8$G CM^{-1} SEC^{-1}

s/m	1	2	3	4	5	6
1	$138.3 + .28i$	$-1.00 - .02i$	$27.49 + .08i$	$.00 - .01i$	$9.92 + .04i$	$-.01 + .00i$
2	$-46.08 + .43i$	$.24 - .03i$	$-9.16 + .14i$	$.01 - .01i$	$-3.31 + .08i$	$.00 + .00i$
3	$27.64 + .15i$	$-.17 - .01i$	$5.48 + .07i$	$-.01 - .01i$	$1.98 + .05i$	$.00 + .00i$
4	$-19.79 + .57i$	$.18 - .04i$	$-3.93 + .16i$	$.00 - .01i$	$-1.42 + .08i$	$.02 + .00i$
5	$15.39 + .02i$	$-.09 + .00i$	$3.08 + .05i$	$-.01 - .01i$	$1.11 + .04i$	$.00 + .00i$
6	$-12.53 + .71i$	$-.06 - .05i$	$-2.53 + .19i$	$-.01 - .02i$	$-.91 + .10i$	$.00 + .00i$
7	$10.66 - .12i$	$-.12 + .00i$	$2.09 + .02i$	$.00 - .01i$	$.76 + .02i$	$.00 - .02i$
8	$-9.20 + .85i$	$.09 - .06i$	$-1.79 + .21i$	$.00 + .01i$	$-.66 + .10i$	$.00 + .01i$

TABLE II. VALUES OF D_{ms} FOR $A_h = 10^{10}$G CM^{-1} SEC^{-1}

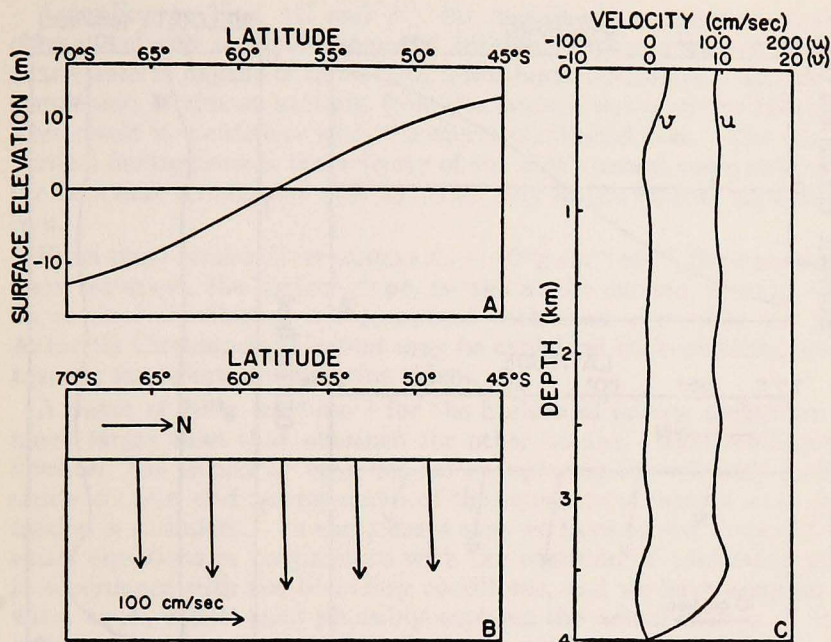| s/m	1	2	3	4	5	6
1	$16.97 + .15i$	$.01 - .01i$	$.68 + .05i$	$.01 - .01i$	$.16 + .02i$	$.00 + .00i$
2	$-5.67 + .40i$	$.00 - .03i$	$-.22 + .13i$	$-.01 - .01i$	$-.03 + .07i$	$.00 + .00i$
3	$3.40 + .27i$	$.00 - .02i$	$.14 + .10i$	$.00 - .01i$	$.05 + .05i$	$.00 + .00i$
4	$-2.41 + .38i$	$.01 - .03i$	$-.09 + .12i$	$-.01 - .01i$	$.00 + .07i$	$.00 + .00i$
5	$1.90 + .27i$	$.01 - .02i$	$.09 + .10i$	$.00 - .01i$	$.03 + .06i$	$.00 + .00i$
6	$-1.53 + .39i$	$.00 - .03i$	$-.05 + .12i$	$-.01 - .01i$	$.01 + .06i$	$.00 + .00i$
7	$1.32 + .26i$	$.00 - .02i$	$.06 + .10i$	$.03 - .01i$	$.03 + .06i$	$.00 + .00i$
8	$-1.11 + .39i$	$.00 - .03i$	$-.02 + .12i$	$-.04 - .01i$	$.01 + .06i$	$.00 + .00i$

Figure 1.   Wind current and surface elevation in the Antarctic ocean for $A_h = 10^8$ C. G. S.
A. Surface elevation.   B. Vector diagram of surface currents.   C. Vertical distribution of
the current velocity along the median line (57° 30′ S). u and v are taken positive eastward
and northward respectively, and in fig. 1C the scale for v is magnified ten times that of u.

CONCLUSIONS

Summarizing the above computed results, the following conclusions
are obtained.

(1) When the coefficient of horizontal mixing is less than 10^8g
cm^{-1} sec^{-1}, the pattern of water movement is practically independent
of the magnitude of A_h.   This result agrees with that of Hidaka and
Takano (1952).

(2) Total Mass Transport.   The computed total mass transport is
8.1×10^{15}g sec^{-1} for $A_h = 10^8$g cm^{-1} sec^{-1} and 9.3×10^{14}g sec^{-1} for
$A_h = 10^{10}$g cm^{-1} sec^{-1}.   This is consistent with the result that the mass
transport in a zonal ocean is inversely proportional to A_h (Hidaka,
1950; Munk and Palmén, 1951).   The actual mass transport in the
Antarctic Ocean is at least 1.5×10^{14}g sec^{-1}.   Even if we assume
$A_h = 10^{10}$g cm^{-1} sec^{-1}, the computed value seems to be a little larger
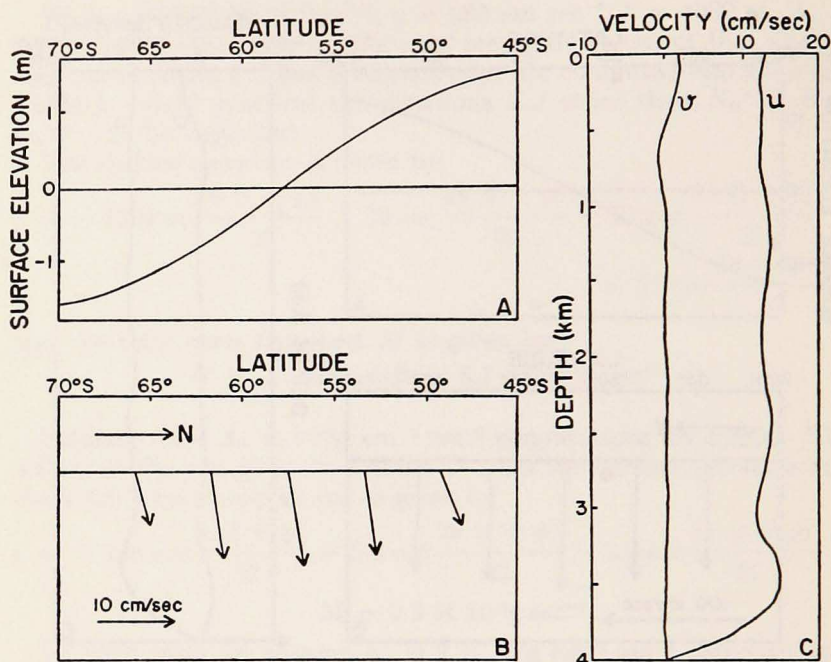than that observed.

Figure 2. Wind current and surface elevation in the Antarctic ocean for $A_h = 10^{10}$ C. G. S. A. Surface elevation. B. Vector diagram of surface currents. C. Vertical distribution of the current velocity along the median line (57° 30′ S). u and v are taken positive eastward and northward respectively.

(3) Surface Slope. In Figs. 1A and 2A, the surface slope is illustrated. Its range for $A_h = 10^8$g cm^{-1} sec^{-1} gives a large value of about 25 m, but for $A_h = 10^{10}$g cm^{-1} sec^{-1} the range is only 3 m. Extrapolation of the surface slope into the Antarctic region, as given by Defant's chart of absolute topography, gives a value of about 2–3 m per 2000 km. Therefore the computed value of the slope for $A_h = 10^{10}$g cm^{-1} sec^{-1} may be reasonable.

(4) Surface Current. In Figs. 1B and 2B, the surface currents are illustrated. For $A_h = 10^{10}$g cm^{-1} sec^{-1} the directions and magnitudes of the currents are in good agreement with those observed.

(5) Vertical Distribution of the Current. The fact that the Antarctic Circumpolar Current reaches considerable depth has been pointed out by several authors. In 1941, Sverdrup discussed qualitatively the influence of the bottom topography on surface currents and explained that the Antarctic Circumpolar Current is sufficiently deep so that very sharp leftward deflections of surface currents are observed over deep submarine ridges near Drake Passage.

According to Figs. 1C and 2C, the meridional component of the current velocity is small compared with the E-W component and is nearly zero in depths of more than a few hundred meters. The E-W component is almost uniform from the surface down to the bottom. This result is consistent with the above-mentioned fact. The slight vertical fluctuations in the velocity of the E-W current come out from the fact that we adopted several terms only in the Fourier expansion in u.

From these results, if we assume $A_h = 10^{10}$g cm^{-1} sec^{-1}, the computed mass transport, the surface slope, as well as the current velocity and its vertical distribution are consistent with those observed, and the Antarctic Circumpolar Current may be explained without taking into account the effect of submarine ridges.

A value of 10^{10}g cm^{-1} sec^{-1} for the horizontal mixing coefficient is much larger than that obtained for other oceans. This coefficient, however, has ordinarily been defined as replacing the viscosity coefficients in the x- and y-components of the equations of motion when the motion is turbulent. In the present case we have solved these differential equations in conjunction with the equation of continuity and in accordance with the boundary conditions, and we have assigned a value to A_h which most plausibly explains the actual motion of the water. Therefore, so large a value may not be unnatural if we can affirm the existence of exceptionally great horizontal mixing in the Antarctic Region.

ACKNOWLEDGMENT

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