EXAMPLES OF THE POSSIBLE ROLE OF INERTIA AND STRATIFICATION IN THE DYNAMICS OF THE GULF STREAM SYSTEM[1]

By

HENRY STOMMEL

Woods Hole Oceanographic Institution

ABSTRACT

An hypothesis is offered to explain certain major features of the Florida Current: (1) the large axial gradient of vorticity, (2) the fact that seasonal fluctuations in the Miami-Cat Key tide gauge difference are twice the amplitude of those in the Key West-Havana tide gauge difference, and (3) the fact that the Havana-Cat Key difference is at a maximum during the period of minimum flow—the opposite of what might ordinarily be expected. An elementary perturbation theory of meanders in a wide stratified current is presented and its possible application to Gulf Stream meanders is discussed.

1. *Introduction.* Recent successful oceanographic studies of the steady-state wind-driven ocean circulation (Munk, 1950; Hidaka, 1949) do not require inclusion of the effects of density stratification or of inertia terms. Transient or periodic phenomena in general involve both inertia and stratification explicitly, and it seems to this author that results obtained from theoretical consideration of homogeneous models may omit important features. For example, Ichiye (1951) has obtained an interesting solution for the wind-driven ocean circulation due to a periodic wind-stress acting on a homogeneous ocean. In a stratified ocean, however, the mass movements which must accompany changes in the circulation are many hundredfold those occurring in a homogeneous ocean, and changes in potential energy are correspondingly greater. This raises the question as to whether or not the time-constants which characterize the response of a stratified ocean to a periodic wind-stress will be the same as those obtained by Ichiye's solution. This paper does not attempt to answer this difficult question. Instead, it treats two problems, the mass distribution of the Florida Current and the stability and size of Gulf Stream meanders, in which stratification and inertia play essential roles.

2. *Florida Current Tide and Current Data.* Numerous studies of tide gauge data have been made (Montgomery, 1938, 1941; Iselin, 1940; Hela, 1952) to obtain information about seasonal fluctuations

[1] Contribution No. 645 from the Woods Hole Oceanographic Institution.

in the cross-stream slope of the sea surface, and, by the geostrophic equation, to compute seasonal fluctuations of the mean surface velocity of the Florida Current. In addition to the tide gauge data already published for Key West, Miami, and Cat Key, tide gauge data for Havana are now available for the first time. These data are obtained from a gauge installed by the U. S. Coast and Geodetic Survey in 1946. Table I shows the monthly means of sea level (in feet)

TABLE I. MONTHLY MEANS OF SEA LEVEL (FEET), EACH REFERRED TO SEPARATE LOCAL DATUM

Month	*Cat Key 1938–early 1941*	*Key West 1930–1948*	*Havana 1947–1951*	*Miami Beach 1935–1944*
Jan.	3.75	4.88	4.82	3.34
Feb.	3.77	4.83	4.80	3.34
Mar.	3.82	4.82	4.92	3.22
Apr.	3.90	4.93	4.95	3.34
May	3.94	5.01	5.10	3.48
Jun.	3.98	5.03	5.09	3.41
Jul.	4.07	5.03	5.06	3.33
Aug.	4.18	5.17	5.26	3.46
Sep.	4.29	5.37	5.37	3.79
Oct.	4.18	5.53	5.47	4.07
Nov.	3.94	5.33	5.06	3.84
Dec.	3.84	5.06	4.94	3.74

measured at these stations, averaged over different sets of years, uncorrected for atmospheric pressure, and each referred to a different datum. A leveling survey has been made between Key West and Miami (Montgomery, 1941), but obviously it is impossible to connect Havana or Cat Key to the same datum by ordinary means. Therefore we cannot obtain true differences in sea level across the Straits, but we can obtain fluctuations in the differences of sea level at two stations, as shown in Fig. 1. At all four stations there is a maximum sea level in September-October which is apparently caused by the summer heating of the water. This average rise does not appear in the differences.

It is interesting to note that the fluctuation in the Miami-Cat Key differences is about twice that of the Key West-Havana differences. The maximum cross-stream slopes occur during July when the flow is strongest (Fuglister, 1951), at which time the maximum downstream slope from Key West to Miami also occurs. These relations of slopes to surface velocity are quite what might be expected from the most elementary considerations of Bernoulli's equation and the geostrophic

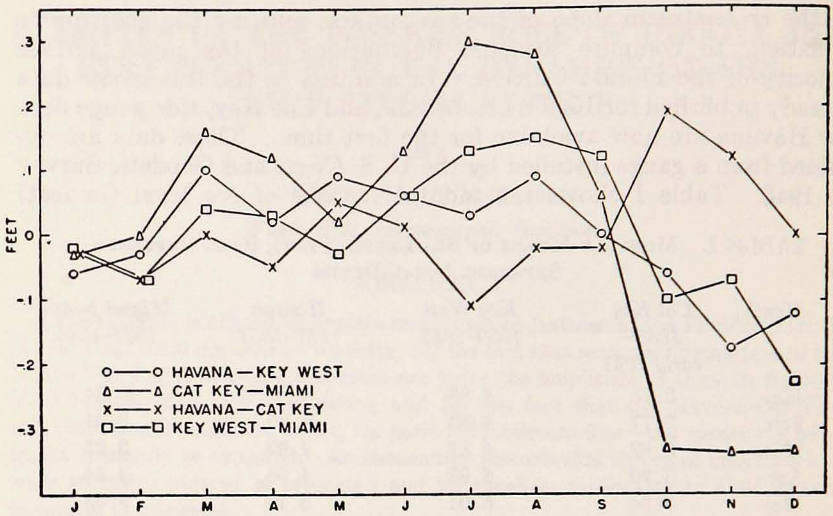

Figure 1. Monthly mean tide gauge differences for various pairs of stations, adjusted so that the annual mean of the differences vanishes.

equation. But at first sight it is rather startling that the fluctuations of the differences at Miami-Cat Key are larger than those at Key West-Havana and that the downstream slope between Havana and Cat Key is smaller in July rather than larger, hence an explanation is required.

Another noteworthy feature of the Florida Current is the extensive region of anticyclonic vorticity in the section off Miami. So far as direct velocity measurements from an anchored vessel are concerned, the data are scanty. The only observations at different depths are those of Pillsbury (1890), and these show a rapid decrease of velocity with depth. The axis of maximum surface velocity is not in the center but is displaced toward Miami; and this feature has been verified from many crossings of the Current off Miami by Murray (1952). Fig. 3 shows measurements with towed electrodes during one of these crossings, but it should be emphasized that towed electrodes are apt to give particularly misleading readings of velocity in the Florida Straits. Nevertheless, the general fact is that a wide zone of anticyclonic vorticity (of approximately $-[0.5 \pm 0.1]f$, where f is the local Coriolis parameter) seems to be established and also requires explanation. It is impossible, of course, for the variation of the Coriolis parameter with latitude to cause such shear over so short a distance as the length of the Florida Straits.

The channel at Miami is only about half as deep and half as wide as that at Key West, as shown in Fig. 2. The change in depth can

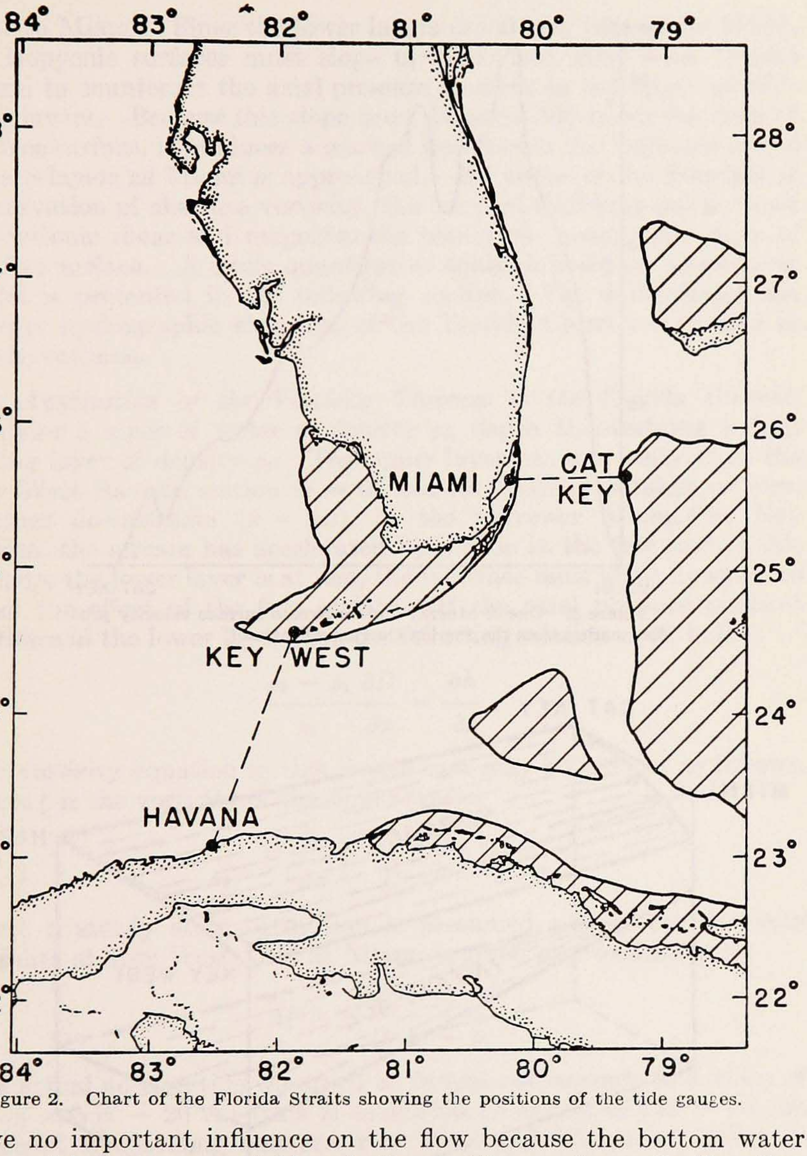

Figure 2. Chart of the Florida Straits showing the positions of the tide gauges.

have no important influence on the flow because the bottom water does not have an appreciable velocity at either section. On the other hand, the narrowing of the channel at Miami is important hydrographically because the water, in order to pass through it, is accelerated, and this requires a small drop in the level of the free surface from Key

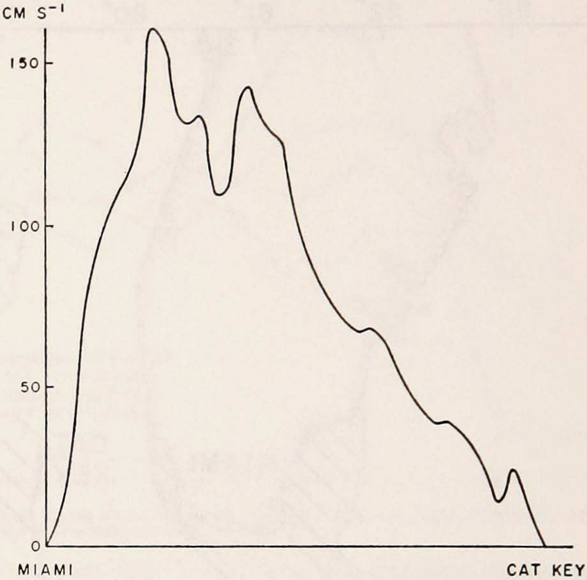

Figure 3. One of Murray's cross-stream surface velocity profiles made across the Florida Current off Miami.

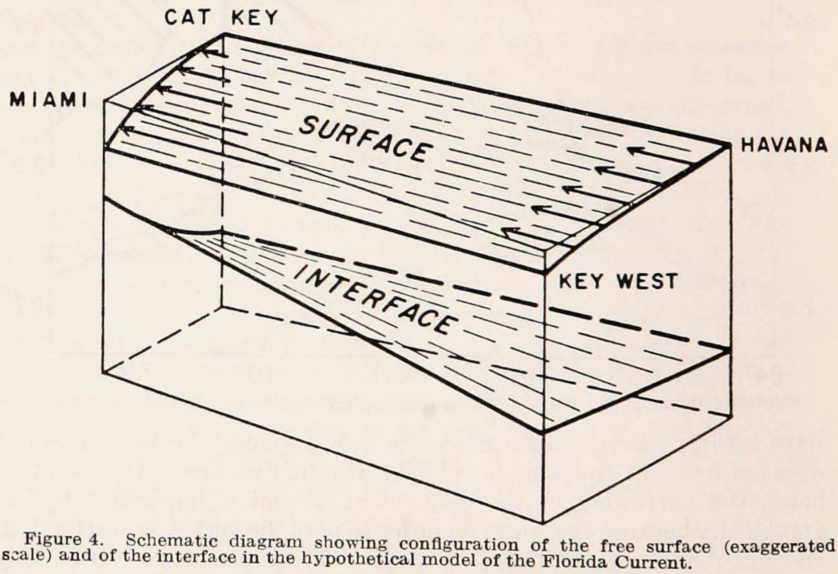

Figure 4. Schematic diagram showing configuration of the free surface (exaggerated scale) and of the interface in the hypothetical model of the Florida Current.

West to Miami. Since the lower layers are at rest (except for tides), the isopycnic surfaces must slope upward from Key West toward Miami to counteract the axial pressure gradient in the upper accelerating layers. Because this slope must be some 500 times the drop of the free surface, it produces a marked decrease in the thickness of the surface layers as Miami is approached. By virtue of the principle of conservation of absolute vorticity, this vertical shrinking produces an anticyclonic shear and magnifies the transverse geostrophic slope of the free surface. A crude quantitative analysis based on a two-layer model is presented in the following section. Fig. 4 illustrates the average hydrographic structure of the Florida Current envisaged in this hypothesis.

3. *Application of the Vorticity Theorem to the Florida Current.* Consider a layer of water of density ρ_1, depth D, overlying a deep resting layer of density ρ_2. The upper layer has a velocity U at the Key West-Havana section ($x = 0$) and its vorticity is taken as zero. Further downstream ($x = \Delta x$), at the narrower Miami-Cat Key section, the stream has accelerated by a drop in the free surface Δh. Because the lower layer is at rest, the interface must slope upwards to offset the effect of the free surface. If the axial pressure gradient vanishes in the lower layer then the following relation must hold:

$$\frac{\rho_2 - \rho_1}{\rho_2} \frac{\partial D}{\partial x} = \frac{\partial h}{\partial x} \, .$$

The vorticity equation in this simple case may be written as follows, where ζ is the vorticity of the upper layer:

$$U \frac{\partial}{\partial x} \left(\frac{\zeta + f}{D} \right) = 0 \, ,$$

where a steady state of motion is presumed. Since the vorticity vanishes at Key West, that at Miami is given approximately by

$$\Delta \zeta = f \frac{\Delta h}{D} \frac{\rho_2}{\rho_2 - \rho_1} \, .$$

The actual acceleration observed at Miami corresponds to a value of Δh of about $-$ 20 cm (this is somewhat in excess of the $-$ 4.9 cm obtained by leveling between Key West and Miami, according to Montgomery, 1941). The density difference of the two layers is approximately $(\rho_2 - \rho_1)/\rho_2 \cong 2 \times 10^{-3}$. The initial depth D of the current is roughly 250 m, which yields a value of the vorticity increase between Key West and Miami of $\Delta \zeta \cong - 0.4 f$, which corresponds within the limit of observational error to the observed value given above.

If the high anticyclonic vorticity at Miami is actually due to the vertical shrinking of the upper layers, then it should be possible to test this hypothesis by careful hydrographic observations on the two sections to try to detect the average rise of about 100 m in the isopycnic surface at 100–300 m depth. It will be difficult and tedious to measure this hypothetical rise because it may be partly masked by the large cross-stream slope of the isopycnic surfaces and by variations in depth due to tides. Present data are insufficient to attempt a satisfactory test.

4. *Gulf Stream Meanders.* The Gulf Stream meanders observed on the Multiple Ship Survey of June 1950 (Fuglister and Worthington, 1951) have inspired several theoretical studies. It has been suggested that they are analogous to the waves in the atmospheric jet stream, but both their small scale (100–400 km) and small velocities of propagation suggest that the variation of the Coriolis parameter with latitude is not a dominant factor in their dynamics. Haurwitz and Panofsky (1950) have constructed several models of currents in a homogeneous ocean with cross-stream velocity profiles similar to those observed in the true Gulf Stream and have carried out an intricate perturbation analysis to show the existence of unstable waves with reasonable velocities of propagation. These waves are a result of the shearing instability of the Stream.

It seemed to the writer that, if stratification were taken into account, certain types of meanders might exist in which horizontal divergence was dynamically important. In order to illustrate such a system, meanders in a wide current were studied. To simplify the analysis, the realistic cross-stream velocity profiles of Haurwitz and Panofsky were abandoned. The density stratification of the real ocean is approximated by a two-layer system such as that used in Section 3.

5. *A Simple Meander Theory for a Wide Current in a Stratified Ocean.* Let us suppose that the lower layer is sufficiently deep so that the horizontal pressure gradients vanish in it at all times. In the undisturbed state a steady current U flows in the x-direction in the upper layer. Associated with this current is a cross-stream pressure gradient of the following form:

$$fU = -g\frac{\partial h}{\partial y} = -g\frac{\partial D}{\partial y}\frac{\Delta\rho}{\rho}.$$

We now suppose that small perturbations, u, v, the velocity components, and h, the elevation of the free surface, occur, and that these quantities are independent of y, the cross-stream co-ordinate. The perturbation equations may be written in the form:

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)u - fv = -g\frac{\partial h}{\partial x}, \quad (1)$$

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)v + fu = 0, \quad (2)$$

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\left(\frac{\rho}{\Delta\rho} + 1\right)h + D\frac{\partial u}{\partial x} + v\frac{\partial D}{\partial y} = 0. \quad (3)$$

If the perturbations are all in the form $e^{i(kx-\sigma t)}$, we obtain the following frequency equation, taking f = constant:

$$k^2U^2(1-p)^3 - \left[f^2 + gk^2\frac{\Delta\rho}{\rho}D\right](1-p) + f^2 = 0,$$

where $p = \dfrac{c}{U} = \dfrac{\sigma}{kU} = p' + ip''$, the real part ($p'$) of which is the ratio of the velocity of propagation of the wave to the velocity of the current, and the imaginary part (p'') of which gives the instability of the wave motion. For the particular range of parameters involved, no one of these terms is small compared to the others. It is convenient to rewrite this equation in the form:

$$y^3 + 2 = \P y,$$

where $\P = 2\left(\dfrac{f^2}{2k^2U^2}\right)^{\frac{1}{3}}\left(\dfrac{f^2 + gk^2\frac{\Delta\rho}{\rho}D}{f^2}\right)$ and $y = \left(\dfrac{f^2}{2k^2U^2}\right)^{-\frac{1}{3}}(1-p)$.

The roots of this equation are all real provided $\P \geqslant 3$, in which case there are three types of stable waves present. If $\P < 3$, there is a region of unstable waves. These are of most interest to us because they are the only ones that are likely to grow large enough to be noticed on a ship survey. Examination of the coefficients of the frequency equation reveals that for $U^2 < g\dfrac{\Delta\rho}{\rho}D$, all waves are stable. At $U^2 = g\dfrac{\Delta\rho}{\rho}D$, a single wave number given by $k = f/(\sqrt{2}\,U)$ becomes "just unstable," whereas all other wave numbers are stable. For slightly larger values of U^2 there is a narrow range of wave numbers about $k = f/(\sqrt{2}\,U)$ in which waves are unstable. In the critical case of marginal stability the "just unstable" wave is stationary.

One objection to the application of this model to the meanders observed in the Gulf Stream is that the real Stream is not very wide. A more sophisticated theory would include lateral boundaries to the Stream and would provide for resting layers of water on each flank beyond the boundaries. Also a perturbation theory such as this applies only to waves of infinitesimal amplitude, whereas meanders often grow to large amplitude. Therefore it is important to regard this treatment of meanders as merely indicative of the possible role of divergence terms in the meandering of a stratified current. Their physical reality must be tested by actual observation of the depth of the current in the crests and troughs of meanders.

6. *Application to the Actual Gulf Stream.* Rossby (1951) has shown that velocity of the Gulf Stream does in fact approach the critical value $\sqrt{g \frac{\Delta\rho}{\rho} D}$. In a steady state one might expect the Stream to become progressively shallower downstream, gradually approaching the critical condition. Because of the paucity of hydrographic sections of the Stream in any one year or season it is necessary to construct a composite series of sections in order to determine whether or not there is a noticeable change in the depth of the Stream along its axis. A number of sections, all made in early June of several years, have been assembled and recomputed: (1) one at Hatteras at about 74° W; (2) two more on the Montauk Point to Bermuda line surveyed in Iselin's (1940) studies; (3) one at 58° W; and (4) two Ice Patrol sections along the 50° W meridian. The geostrophic transports at different depths were computed. In order to exhibit any changes in the depth of the Stream, the percentage of the total transport below certain selected depths was computed. The results are given in Table II; it is clear that there is no striking change in the depth of the Stream as indicated by these sections. The inference to be drawn from this is that the Stream is near to the critical velocity almost everywhere from Cape Hatteras to the tail of the Grand Banks and that unstable meanders of a single wavelength might be expected anywhere.

We may ask ourselves what the size of the meanders predicted by our two-layer meander theory might be expected to be. A surface layer 200 m thick moving at 200 cm sec^{-1} and having a density difference ratio of $\Delta\rho/\rho = 2 \times 10^{-3}$ is critical. The wavelength of the "just unstable" perturbation corresponding to this choice of parameters is 180 km. All other wavelengths are stable and do not grow. It is remarkable that this wavelength corresponds closely to that of the large stationary meander which was observed during growth and detachment into an eddy (Fuglister and Worthington, 1951).

During the process of growth this observed meander diverted momentum at right angles to the mean direction of the Stream; from mixing-length ideas we may calculate the order of magnitude of the local gross coefficient of lateral eddy viscosity due to this eddy: $A = U\,2\pi/k$, where k is wavelength of the detaching meander. Substituting numerical values above, we obtain $A = 3.6 \times 10^9 \text{cm}^2\text{sec}^{-1}$, a value much in excess of that used by Munk (1950) in his wind-driven ocean circulation theory. If it is recalled that Munk assumed a uniform viscosity over the entire length of the Gulf Stream System and if it is assumed that there is usually but one detaching eddy at a time, then the discrepancy is not so glaring. The above value of A acts on only 90 km of the Stream, but if it is averaged over the entire length of 4,500 km, a value of 7×10^7 cm^2 sec^{-1} is obtained which agrees rather well with Munk's value 5×10^7. It is inferred, therefore, that the meanders alone produce sufficiently large scale lateral mixing processes for Munk's theory of the climatological mean Gulf Stream.

TABLE II. Percentage of Total Transport Below Selected Depths at Different Hydrographic Sections Along the Gulf Stream (AT. = Atlantis; IP. = Ice Patrol)

Depth (m)	*74° W AT. 4570–4565*	*68° W AT. 2871–2867*	*68° W AT. 3058–3054*	*58° W AT. 2620–2616*	*50° W IP. 2715–2719*	*50° W IP. 4175–4184*
0	100	100	100	100	100	100
50	94	92	92	92	91	92
100	88	85	84	84	82	84
200	74	71	69	69	65	69
300	61	58	55	56	50	56
400	49	46	43	44	37	45
600	30	38	23	26	19	28
900	12	13	7	9	6	12
1200	4	7	2	3	2	4
1600	1	1	0	1	0	1
2000	0	0	0	0	0	0

Since a thin filament of anomalously fresh water preserves its integrity all along the Gulf Stream proper (Ford, Longard, and Banks, 1952), it seems that the influence of mixing processes of a scale smaller than meanders must be negligible. The physical explanation of the process which determines the width and vertical structure of the instantaneous Gulf Stream is quite obscure.

In concluding, it is important to emphasize that the meander theory presented here is not complete or proven. It is merely suggestive of a type of wave-motion which may possibly dominate the dynamics of meanders. It is interesting to note that if the energy of meanders is absorbed from the potential energy of the cross-stream mass distribution instead of from the kinetic energy of the flow, the meanders are then a part of the thermohaline circulation of the ocean.

ACKNOWLEDGMENT

The writer wishes to express his indebtedness to Dr. Arnold Arons for many stimulating discussions of the subject matter of Section 2, to Mr. Sainsbury Strack for discussions of Section 5, to Commander Floyd Soule for advice concerning Section 6, and to the Office of Naval Research for financial assistance.

REFERENCES

FORD, W. L., J. R. LONGARD, AND R. E. BANKS
1952. On the nature, occurrence and origin of cold low salinity water along the edge of the Gulf Stream. J. Mar. Res., *11* (3): 281–293.

FUGLISTER, F. C.
1951. Annual variations in current speeds in the Gulf Stream System. J. Mar. Res., *10* (1): 119–127.

FUGLISTER, F. C. AND L. V. WORTHINGTON
1951. Some results of a multiple ship survey of the Gulf Stream. Tellus, *3* (1): 1–14.

HAURWITZ, B. AND H. A. PANOFSKY
1950. Stability and meandering of the Gulf Stream. Trans. Amer. geophys. Un., *31* (5): 723–731.

HELA, I.
1952. The fluctuations of the Florida Current. Bull. mar. Sci. Gulf and Caribbean, *1* (4): 241–248.

HIDAKA, KOJI
1949. Mass transport in ocean currents and lateral mixing. Geophys. Notes, Tokyo, *2* (3): 1–4.

ICHIYE, T.
1951. On the variation of oceanic circulation. (1st paper). Oceanogr. Mag., Tokyo, *3* (2): 79–82.

ISELIN, C. O'D.
1940. Preliminary report on long-period variations in the transport of the Gulf Stream System. Pap. phys. Oceanogr. Meteorol., *8* (1): 1–40.

MONTGOMERY, R. B.
1938. Fluctuations in monthly sea level on eastern U. S. coast as related to dynamics of western North Atlantic Ocean. J. Mar. Res., *1* (2): 165–185.
1941. Sea level difference between Key West and Miami, Florida. J. Mar. Res., *4* (1): 32–37.
1941. Transport of the Florida Current off Habana. J. Mar. Res., *4* (3): 198–220.

MUNK, W. H.
1950. On the wind-driven ocean circulation. J. Meteor., *7* (2): 79–93.
MURRAY, K. M.
1952. Short period fluctuations of the Florida Current from geomagnetic electrokinetograph observations. Bull. Mar. Sci. Gulf and Caribbean, *2* (1): 360–375.
PILLSBURY, J. E.
1890. The Gulf Stream . . . Report of the Superintendent, U. S. Coast and Geodetic Survey, for 1890, Appendix No. 10: 459–620.
ROSSBY, C.-G.
1951. On the vertical and horizontal concentration of momentum in air and ocean currents. Tellus, *3* (1): 15–27.
WÜST, G.
1924. Florida- und Antillenstrom . . . Veröff. Inst. f. Meereskunde, Berlin, N. F. A, *12:* 1–48.