INTERFERENCE PHENOMENA IN THE TIDES OF THE WOODS HOLE REGION[1]

By

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ABSTRACT

The relations of mean range of tide, time of high water, and time of slack water along Vineyard and Nantucket Sounds may be attributed to the interference of tidal waves entering the sounds from their opposite ends. Interference of the semidiurnal constituents is maximal in the neighborhood of Woods Hole where the great reduction of these constituents, relative to the diurnal constituents, results in a pronounced diurnal inequality in the range of successive tides at appropriate phases of the moon. The tide gauge record at Woods Hole is distorted in form as a result of the development of shallow water harmonics combined with the reduction of the fundamental semidiurnal constituent by interference. The inequality in the duration of flood and ebb currents in Woods Hole Passage may be attributed to the effect of these harmonics on the tide levels in Great Harbor.

In discussing the complicated system of tides in Nantucket and Vineyard Sounds in 1855, Bache stated that "observations already made pointed to the interference of the several tide-waves having access to this space as the source of the seeming irregularities." In the present paper it will be shown that a number of characteristics of the tides in this region can be explained by interference between two systems of waves which enter Vineyard Sound, one directly from the ocean, the other from the Gulf of Maine via Nantucket Sound. The characteristics in question are: (1) The relation of time of high water and slack water and the mean range of tide along Vineyard and Nantucket Sounds; (2) The relative values of the diurnal and semidiurnal tidal constants at Woods Hole; (3) The specific form of the tide gauge curve at Woods Hole; and (4) The relative duration of flow eastward and westward in Woods Hole passage.

COTIDAL LINES

The simultaneous positions of the crest of high water, as the tidal wave approaches the New England Coast, were first drawn by Bache (1855) and were later amplified by Harris (1904). The cotidal lines for the region, modified to take account of the more complete information presently available (U. S. Coast and Geodetic Survey, 1951), are shown in Fig. 1.

[1] Contribution No. 636 from the Woods Hole Oceanographic Institution.

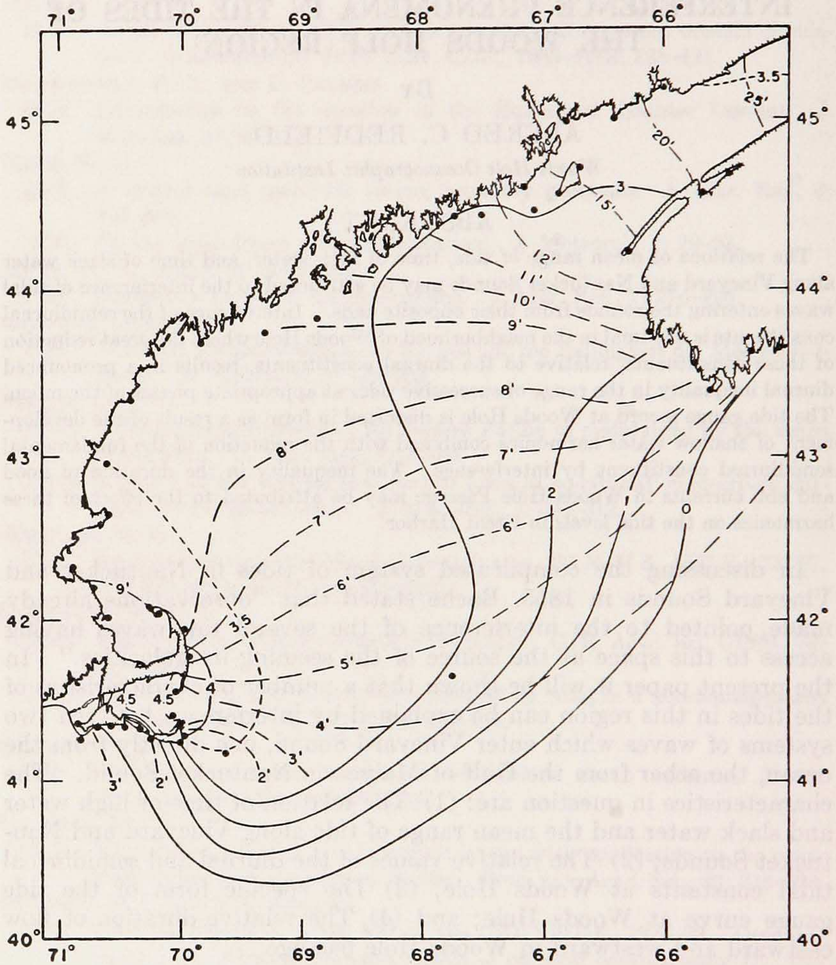

Figure 1. Cotidal and corange lines for the Gulf of Maine. Mean times of high water shown by continuous and broken lines in lunar hours relative to moon transit at Greenwich. Mean range of tide shown by dashed lines in feet.

High water occurs almost simultaneously at outlying points along the coast between Long Island, New York, and Nova Scotia. From offshore the crest of high water moves progressively along Vineyard Sound and enters Nantucket Sound from the west after a lapse of four hours. A similar movement into Nantucket Sound occurs through Muskeget Channel.

On entering the Gulf of Maine, the tidal wave is deflected to the left with the result that the delay in high water is greatest along the Massachusetts Coast. There the crest of high water moves southward and reaches the entrance to Nantucket Sound at Monomoy Point and Great Point about four hours after its occurrence on the outer coast.

Within Nantucket Sound high water is latest in an area bounded by Hyannisport, Dennisport, and Nantucket Harbor, where it occurs almost 4½ hours after high water at No Mans Land. In this region the wave that enters through Vineyard Sound and that from the Gulf of Maine meet from opposite directions.

THE TIME OF HIGH WATER, SLACK WATER, AND MEAN RANGE OF TIDE IN VINEYARD AND NANTUCKET SOUNDS

Cotidal and corange lines for Vineyard and Nantucket Sounds are shown in Figs. 2 and 3. Not only is the time of high water delayed as the tidal wave progresses along Vineyard Sound but the mean range of tide decreases from a value of 3.0 feet at Gay Head and No Mans Land to a minimum of 1.3 feet at Falmouth. Eastward of Falmouth the mean range increases to 3.7 feet at Monomoy Point. The interval by which slack water (current turns west) follows high water is about three hours in Vineyard Sound. Along the Falmouth shore, this interval decreases rapidly, and east of Succonessett slack water precedes high water by an interval which increases to about two hours at Monomoy Point.

Theory. These relations are superficially similar to those that would be expected by the interference of two progressive waves moving in opposite directions along the channel. They may be examined quantitatively by methods recently employed in the analysis of tidal phenomena in narrow embayments in which it is assumed that the reflection of the primary tidal wave by the barrier at the head of the embayment produces the mathematical equivalent to a second wave moving through the barrier in the opposite direction (Redfield, 1950). The assumptions necessary to the treatment are:

1. That irregularities in the cross section of the channel alter the velocity of the respective waves, thus distorting the geographical distribution of phase differences along the channel, but that they do not alter the fundamental relations of elevation, motion, and phase difference within the wave;
2. That damping is proportional to the phase change within the wave rather than to the distance traveled;
3. That the damping coefficient is constant along the length of the channel.

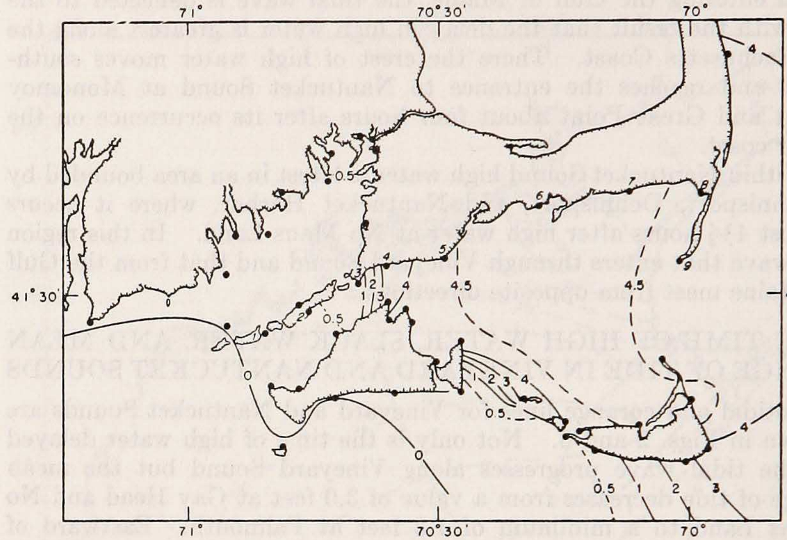

Figure 2. Cotidal lines for Vineyard and Nantucket Sounds. **Mean times of high water shown in lunar hours relative to moon's transit at Greenwich.**

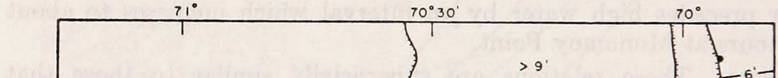

Figure 3. Corange lines for Vineyard and Nantucket Sounds, **showing mean range of tide in feet.**

In a strait separating an island from the mainland it may be expected that tidal waves will enter, meet, and pass in opposite directions. The following treatment should apply, provided the strait is sufficiently narrow to prevent the development of marked effects of the Coriolis force or the production of rotatory currents. It is evident that some point must exist where waves of like period are always in phase. The time of high water at this point of phase equality is taken as the origin of time. Letting σ be the change in phase per unit time, the time angle σt gives the phase at any point relative to the phase at the position of phase equality.

The position of phase equality is also taken as the origin from which phase differences due to position are measured. In a regular channel, phase differences may be expressed by kx, where k is the change of phase per unit of distance and x is the distance measured from the origin. Since phase changes in an irregular channel are not necessarily proportional to distance, x cannot be taken to represent distance in a geographic sense. However, k may be defined as a phase difference of 360° and x as that part of a cycle which a wave will complete in passing between any designated points. Consequently, kx denotes the phase difference due to position between any point and the position of phase equality.

Damping is assumed to be proportional to the phase change along the channel. The attenuation of each wave in its direction of progression is given by $e^{-\mu x}$, where the damping coefficient μ represents the attenuation per cycle and x is the fraction or multiple of the cycle completed.

To find the time of high water at any point. The elevation of the surface η at any time and place along the channel is given by the sum of the elevations of the respective waves, η_1 and η_2. If A and $(1 + f)$ A are the amplitudes of the respective waves at the position of phase equality,

$$\eta_1 = A \cos (\sigma t - kx)\, e^{-\mu x} \text{ and } \eta_2 = (1 + f)\, A \cos (\sigma t + kx)\, e^{\mu x},$$

η_2 being the elevation of the wave taken to be moving in the negative direction with respect to x.

Since $\eta = \eta_1 + \eta_2$,

$$\eta = A\, [\cos (\sigma t - kx)\, e^{-\mu x} + (1 + f) \cos (\sigma t + kx)\, e^{\mu x}]\,. \qquad (1)$$

High water occurs when $\delta\eta/\delta t = 0$ or when $\sin\, (\sigma t - kx)\, e^{-\mu x} + (1 + f) \sin\, (\sigma t + kx)\, e^{\mu x} = 0$, from which it follows that the local time angle of high water, σt_H, is given by

$$\tan^{-1}\left[-\tan kx \,\frac{(1 + f)\, e^{\mu x} - e^{-\mu x}}{(1 + f)\, e^{\mu x} + e^{-\mu x}} \right]$$

Letting F represent the term $\dfrac{(1+f)\,e^{\mu x} - e^{-\mu x}}{(1+f)\,e^{\mu x} + e^{-\mu x}}$, this becomes

$$\sigma t_H = \tan^{-1}[-\tan kx \cdot F]\,. \qquad (2)$$

To find the elevation at any point at high water. From (1) the elevation at any time and place may be written

$$\eta = A\,(\cos\sigma t\cos kx + \sin\sigma t\sin kx)\,e^{-\mu x} + (1+f)\,(\cos\sigma t\cos kx - \sin\sigma t\sin kx)\,e^{\mu x}.$$

At high water it follows from (2) that

$$\cos\sigma t_H = \frac{1}{\sqrt{1 + F^2\tan^2 kx}} \text{ and}$$

$$\sin\sigma t_H = \frac{-F\tan kx}{\sqrt{1 + F^2\tan^2 kx}}\,.$$

Consequently the elevation at high water η_H may be written

$$\eta_H = \qquad (3)$$

$$A\,\frac{(\cos kx - F\tan kx\sin kx)\,e^{-\mu x} + (1+f)\,(\cos kx + F\tan kx\sin kx)\,e^{\mu x}}{\sqrt{1 + F^2\tan^2 kx}\,.}$$

To find the time of slack water at any point. The velocity of the current due to the respective waves at any time and place is given by

$$u_1 = A\,\frac{b}{h}\left(\frac{1}{\sqrt{\mu^2 + k^2}}\right)e^{-\mu x}\cos(\sigma t - kx) \text{ and}$$

$$u_2 = -(1+f)\,A\,\frac{b}{h}\left(\frac{1}{\sqrt{\mu^2 + k^2}}\right)e^{\mu x}\cos(\sigma t + kx)\,,$$

where h is the depth and b the cross section of the channel.

At any point along the channel slack water occurs when $u_1 + u_2 = 0$, or when

$$e^{-\mu x}\cos(\sigma t - kx) - (1+f)\,e^{\mu x}\cos(\sigma t + kx) = 0, \text{ or}$$

$$\cos\sigma t\cos kx\,[e^{-\mu x} - (1+f)\,e^{\mu x}] + \sin\sigma t\sin kx\,[e^{-\mu x} + (1+f)\,e^{\mu x}] = 0\,.$$

Letting σt_s denote the local time angle of slack water, it follows that

$$\sigma t_s = \tan^{-1}\cdot\frac{F}{\tan kx}\,. \qquad (4)$$

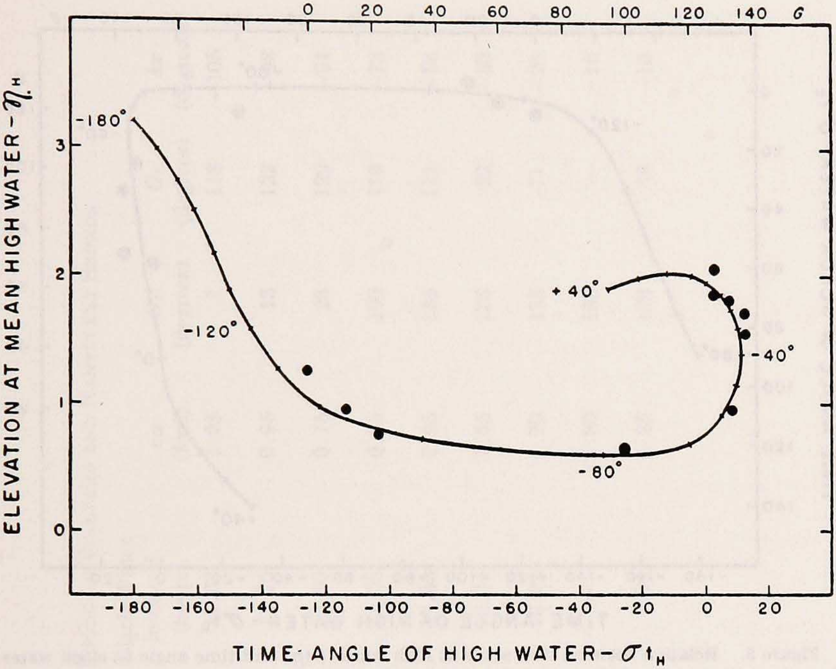

Figure 4. Relation between elevation at high water (η_H) and time angle at high water (σt_H) along Vineyard and Nantucket Sounds. Curve estimated from equations (2) and (3) taking $\mu = 3$, $f = 2$, and $A = 0.65$ feet. Points represent stations recorded in Table I.

Applications. In this treatment, effects of the earth's rotation and the deformation of the tidal wave by shallow water and damping are neglected. In its application to Vineyard and Nantucket Sounds, the influence of tidal flow through the Muskeget Channel is not taken into account. For this reason, attention is limited to stations along the northern side of the Channel where there is a relatively unbroken boundary.

To test the application of equations (2), (3), and (4) to tidal relations in Vineyard and Nantucket Sounds, data from the tide and current tables of the U. S. Coast and Geodetic Survey relative to nine stations along the northern coast of the sounds have been selected. In Table I, times of high water and slack water at these stations are expressed by the Greenwich epoch G which is related to σt by $G = \sigma t + K$, where K is the Greenwich epoch of high water at the point of phase equality. The elevation at high water, η_H, is given as equal to one-half the mean range of tide.

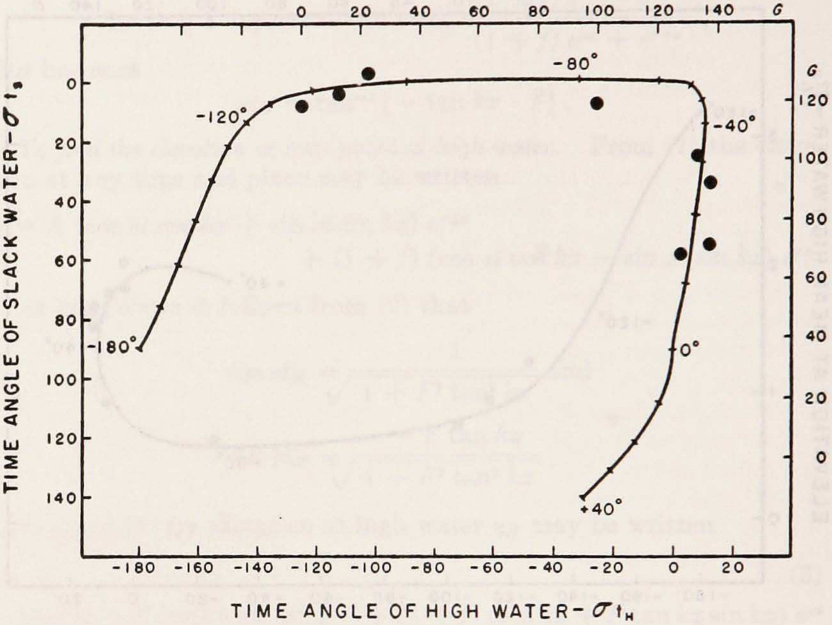

Figure 5. Relation between time angle at high water (σt_H) and time angle at slack water (σt_s) along Vineyard and Nantucket Sounds. Curve estimated from equations (3) and (4), taking μ = 3.0, f = 2, and A = 0.65 feet. Points represent stations recorded in Table I. Note: Time angles of slack water should have negative signs.

By assuming arbitrarily chosen values for the constants μ, A, and f, and solving equations (2) and (3) for the values of σt_H and η_H corresponding to a series of values for kx, a curve may be drawn which shows the unique relations of the time of high water and the elevation at high water determined by the constants chosen, as in Fig. 4.

Along the curve a scale of values of kx corresponding to each combination is marked. The relations σt_H and σt_S or η_H and σt_S may be treated similarly, the former combination being shown in Fig. 5.

Appropriate pairs of observed parameters may be plotted on such diagrams and tested for correspondence with the theoretical curves. If η_H is plotted as its logarithm, the "shape" of the curves is determined solely by μ and f, and the values of A and K are found by the vertical and horizontal displacements of the observed points required to secure the best fit. A final test of fit is obtained by comparing the values of kx which may be assigned to the positions of observation by the analysis of different pairs of parameters.

The curves shown in Figs. 4 and 5 are drawn with $\mu = 3$, $f = 2$, $A = 0.65$ feet, and $K = 126°$. The observed points are distributed

TABLE I. Tidal Data for North Shore of Vineyard and Nantucket Sounds

No.	Station*	Mean Range (feet)	High Water interval (hours)	Slack Water interval† (hours)	η_H (feet)	G_H (degrees)	G_S (degrees)	kx (degrees)
1	Quicks Hole	2.5	7.66	11.62	1.25	1	118	−105
2	Tarpaulin Cove	1.9	8.02	11.76	0.95	13	122	−98
3	Nobska Point	1.5	8.53	12.00	0.75	23	129	−94
4	Falmouth Heights	1.3	11.00	11.66	0.65	100	119	−78
5	Succonnessett Point	1.9	12.16	11.38	0.95	134	111	−58
6	Hyannisport	3.1	12.33	10.76	1.55	138	92	−30
7	Dennisport	3.4	12.35	10.02	1.70	138	71	−20
8	Stage Harbor	3.6	12.20	—	1.80	133	—	−16
9	Monomoy Point	3.7	12.00	9.94	1.85	128	68	−10

* Positions of stations are identified by number in Fig. 2.
† Slack before current turns west.

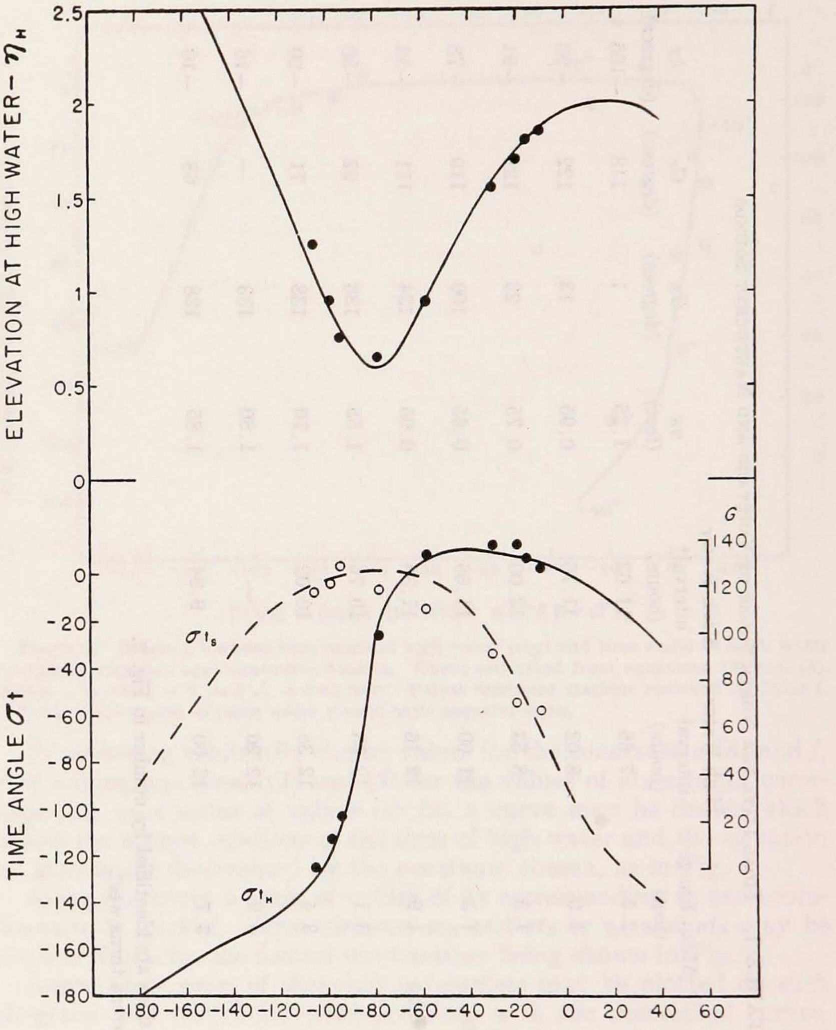

Figure 6. Curves showing the theoretically estimated relations of elevation at high water (ηH), time angle of high water (σt_H), and time angle of slack water (σt_s) to the phase difference along the channel. The plotted points represent observations at stations recorded in Table I.

closely along these curves. From the scales marked along the curves, approximate values of kx may be obtained for each position of observation and may be adjusted so as to reconcile the differences given by the two curves. The adjusted values, entered in Table I, represent the phase difference of the waves at each station relative to the position of phase equality. In Fig. 6 the estimated values of η_H, σt_H, and σt_S are plotted as curves against the corresponding values of kx. The positions of observation are spaced along the abscissa in accordance with the values of kx assigned to each, and the measured values of the several properties of the tide at each position are plotted for comparison with the theoretical curves. Fig. 6 shows that the observed values for mean range of tide, time of high water, and time of slack water are consistent with the values estimated from the several equations.

The values of the constants chosen to represent the properties of the tide in Vineyard and Nantucket Sounds have been found by trial to give a reasonably good description of the observations. They are not necessarily those which give the best fit in any statistical sense. The value of the damping coefficient μ is somewhat greater than that deduced from Long Island Sound and the Bay of Fundy, where μ was between 1 and 1.5 (Redfield, 1950). This is reasonable in view of the limited depth of Nantucket and Vineyard Sounds. The value of f is concordant with the facts that the general range of tide is much greater in the Gulf of Maine than it is along the outer coast south of New England. The position of phase equality appears to lie to the east of Monomoy Point. At this position it may be expected that the value of G for mean high water is 126°, that the elevation at high water due to the wave moving eastward through Vineyard and Nantucket Sounds is 0.65 feet, and that the elevation at high water due to the wave moving westward from the Gulf of Maine is 1.95 feet above mean sea level.

Along the southern shore of Vineyard and Nantucket Sounds the general trend of tidal phenomena is similar to that on the opposite coast. In general the ranges of tide are somewhat greater along the shore of Marthas Vineyard than are those at corresponding points on the mainland; this may be attributed to effects of the earth's rotation. Along the shore of Nantucket, Tuckernuck, and Muskeget Islands, the relations are presumably determined primarily by interference between the wave originating in the Gulf of Maine and that entering Muskeget Channel, which should produce similar but not necessarily the same relations as those found along the northern coasts of the sounds.

In view of the correspondence of theory and observation along the northern coast and the qualitative agreement along the southern shore, it is concluded that the relations between the time of high water, slack water, and mean range of tide along Vineyard and Nantucket Sounds are satisfactorily accounted for by the interference of tidal waves of unequal amplitude which traverse the sounds in opposite directions.

THE AMPLITUDE OF THE DIURNAL AND SEMIDIURNAL TIDAL CONSTITUENTS AT WOODS HOLE

The foregoing analysis is based on the mean values for range of tide, high water interval, and slack water interval. Since the tidal phenomena in the North Atlantic are dominated by the M_2 constituent, it may be assumed that the analysis applies to a single wave having a speed number of 29° per hour (Proudman and Doodson, 1924). Thus, the interference phenomena described are those which affect the semidiurnal constituents of the tide. Nobska Point, at the entrance to Great Harbor, Woods Hole, is situated about 1/4 wave length from the point of phase equality, and consequently the amplitude of the semidiurnal constituents is maximally reduced by interference in its neighborhood.

The diurnal waves that enter the sounds from opposite ends may be expected to be in phase at about the same position as the semidiurnal waves, since they move with similar velocity. However, the phase difference of the diurnal waves along the channel will be only half as great since the wave length is double. Consequently, the phase difference between Nobska Point and the position of phase equality will be approximately 1/8 wave length, and the reduction of the amplitude by interference will be much less than that in the case of the semidiurnal constituents.

The correctness of this reasoning is indicated by a comparison of the amplitude H of the principle tidal constituents at Woods Hole and at Atlantic City, which is on the outer coast where the local geographic conditions may be expected to have a minimal effect upon the ocean tide. The values of the principle constituents at these stations, based on analyses of complete records during the year 1940, are shown in Table II. With the exception of the small Q_1 component, the amplitudes at Woods Hole are reduced relative to those at Atlantic City. The proportional reduction of the semidiurnal constituents is substantially greater than that of the diurnal constituents. Taking the ratio of the sums of the amplitudes of each species to give a weighted expression, the combined amplitudes of the diurnal constituents at Woods Hole is 76% of those at Atlantic City, while the ratio for the semidiurnal constituents is 44%. The reduction of the semidiurnal constituents is thus more than twice that of the diurnal constituents, and this may be attributed to the interference phenomena associated with the topography of Vineyard and Nantucket Sounds.

The consequence of the unequal reduction of the diurnal and semidiurnal constituents at Woods Hole is to increase the diurnal inequality of the tidal range. This gives rise to a condition approaching the mixed tides characteristic of the Pacific Ocean, where great inequality exists between the elevation of successive high (and low) waters at the time of the moon's maximum semimonthly declination and where periods of equality follow when the moon is over the equator. This characteristic of the tide at Woods Hole has not impressed local observers because the range of tide there is small and is subject to relatively great variation from meteorological causes. However, it appears clearly from measurements of tidal records (see Fig. 7).

TABLE II. AMPLITUDE (H) OF THE PRINCIPLE CONSTITUENTS OF THE TIDE AT WOODS HOLE AND ATLANTIC CITY

	Constituent	Woods Hole	Atlantic City	Ratio
Diurnal	K_1	0.224	0.344	0.652
	O_1	0.203	0.236	0.854
	P_1	0.075	0.113	0.664
	Q_1	0.053	0.039	1.36
	Sum (C_1)	0.555	0.732	0.76
Semidiurnal	M_2	0.759	1.928	0.393
	S_2	0.200	0.393	0.509
	N_2	0.246	0.447	0.551
	K_2	0.043	0.096	0.448
	Sum (C_2)	1.248	2.864	0.44

The ratio of the sum of the diurnal constituents C_1 to the sum of the semidiurnal constituents C_2 serves as a ready index of the tendency for diurnal inequality of the tide at any place. The following tabulation compares this index for Woods Hole with that for positions on the outer coasts of the Atlantic (Atlantic City, N. J.) and Pacific (Claoquot, B. C.). There is added the index for Victoria, B. C., where the tide is almost completely diurnal owing to the suppression of the semidiurnal constituents by interference similar to that ascribed to the Woods Hole tide (Redfield, 1950).

		C_1	C_2	C_1/C_2
Atlantic City	1940	0.732	2.864	0.255
Woods Hole	1940	0.555	1.248	0.444
Claoquot	1939	2.63	5.11	0.516
Victoria	1939	4.15	1.88	2.22

The index for Woods Hole approaches closely that for the example chosen for the Pacific Ocean.

THE FORM OF THE TIDE GAUGE CURVE AT WOODS HOLE

Records of the elevation of the sea surface by the tide gauge at the Woods Hole Oceanographic Institution on Great Harbor show a striking departure from the usual sinusoidal curve. The rise of the

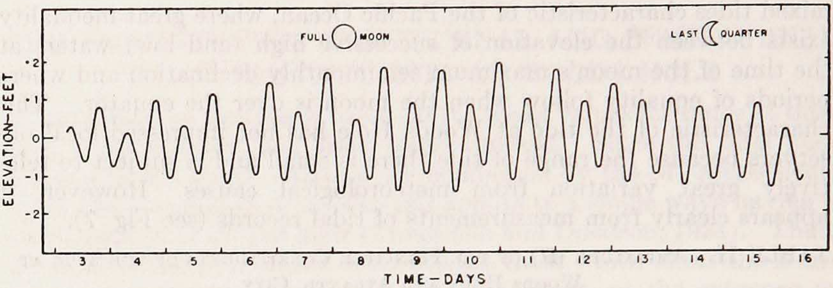

Figure 7. Diagram showing elevation of the tide at Woods Hole at successive high and low water during June 3–16, 1952. The form of the curve and the spacing of high and low waters are not represented exactly.

tide occupies about 5.4 hours, while 7.0 hours intervene between high water and the following low water. During the ebb, at about half tide, the fall is interrupted and the surface level remains constant or rises slightly for nearly an hour (see Fig. 8). The effect varies from day to day and appears to be most pronounced when the range of tide is relatively great.

The distortion of the record during the fall in tide level appears to be due to phenomena similar to those responsible for double high- and double low-waters which are discussed by Doodson and Warburg (1941) in relation to conditions near Southampton. A nearby case of double low-water is cited by Harris (1907) for Providence, Rhode Island. These phenomena in general are explained as follows. In shallow water tidal waves are distorted in such a way that they may

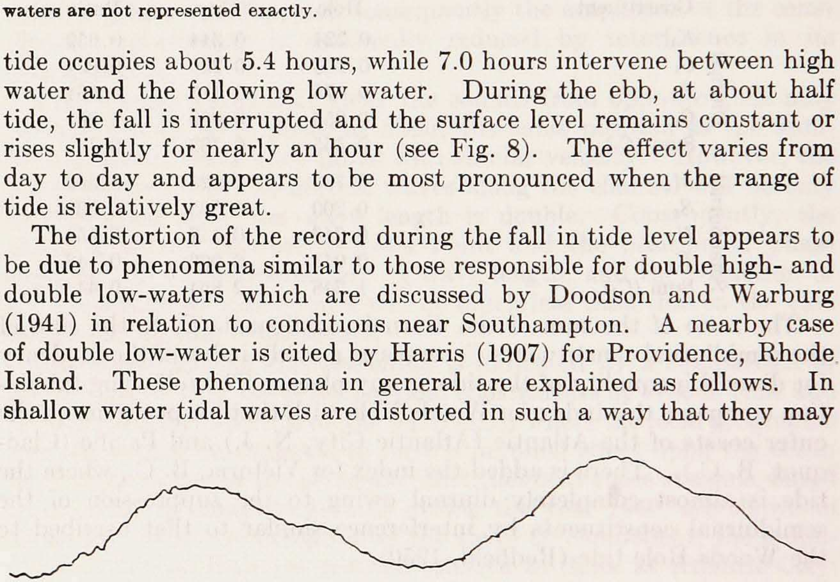

Figure 8. Record from tide gauge at Great Harbor, Woods Hole, September 30, 1952.

be described by the sum of a series of harmonics. At points where the fundamental constituent is suppressed by interference, certain of the harmonics are undisturbed or are reinforced, and thus they may stand out as recognizable distortions of the sinusoidal wave. The phase relations of these distortions depend on the topographic conditions which determine the interference patterns of the fundamental and its harmonics. Where the semidiurnal constituent M_2 is the principle constituent of the tide, the quarter diurnal constituent M_4 will be the harmonic largely responsible for the distortion of the curve. If σt represents the phase of the M_2 wave, the phase of the M_4 wave is given by $2\sigma t + \theta$. If the angle θ is 180°, double high-waters may result; if it is 0°, double low-waters are possible. If θ is 90°, or thereabout, the effect is to steepen the rise of tide and to delay its fall when half tide level is reached. This is apparently the condition in the present case, and this is the phase relation which may be expected when harmonics are generated by progressive waves that undergo distortion in shallow water. These distortions are sufficient to reverse the slope of the record only when interference reduces substantially the amplitude of the fundamental wave relative to the harmonic.

An exact harmonic analysis of the tide at Woods Hole has been made by the U. S. Coast and Geodetic Survey for the years 1933 and 1940. The constants for the M_2 and S_2 components and their harmonics for 1933 are as follows:

	H	k
M_2	0.703	254.7
M_3	0.011	176.0
M_4	0.162	74.6
M_6	0.052	337.8
M_8	0.009	329.0
S_2	0.185	253.2
S_4	0.017	70.0

The principle harmonic present, M_4, has an amplitude of 23% of its fundamental M_2. The phase displacement of the M_2 and M_4 components may be estimated from the values of k so that $\theta = 75°$. The M_6 component also has sufficient amplitude to affect the shape of the tide gauge record appreciably. A curve calculated from the M_2, M_4, and M_6 components shows distortion which is similar to that of the record but which is less pronounced than that usually observed.

A curve which is a reasonable facsimile of the record is shown in Fig. 9, Curve B. The constants used in constructing this curve are given on page 138 and are such that $\theta = 90°$ in the case of both the M_4 and M_6 constituents. The nature of the distortion is shown by a comparison of Curve B with Curve A, which is that for the M_2 constituent alone.

The harmonics present in the tide at Woods Hole, and their exaggeration relative to the fundamental waves, may be explained by the distortion of tidal waves by the shoal water of Vineyard and Nantucket Sounds. Wherever the harmonics may be generated, the suppression of the semidiurnal wave, which is required to make their presence evident by the distortion of the tide record, is explained by the interference of the semidiurnal waves which traverse the sounds in opposite directions.

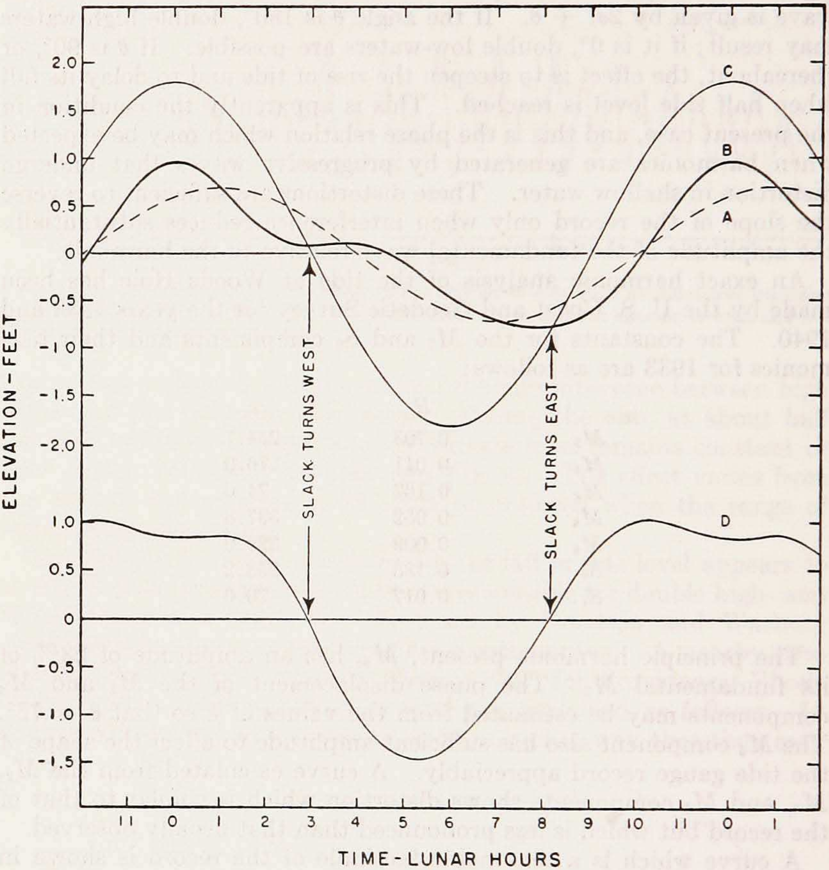

Figure 9. Tidal relations in Woods Hole Passage. Curve A: Elevation due to assumed M_2 constituent at Great Harbor. Curve B: Elevation of sea surface at Great Harbor resulting from sum of assumed M_2, M_4, and M_6 constituents. Curve C: Elevation at Uncatena Island represented by a single (M_2) constituent. Curve D: Hydrostatic head between Uncatena Island and Great Harbor. Time: Lunar hours relative to time of high water at Uncatena Island.

At Woods Hole, just ¼ wave length from the position of phase equality, the semidiurnal waves are maximally reduced by interference. If quarter-diurnal waves accompany these fundamentals at Woods Hole, they would be ½ their wave length from the position of phase equality and they would reinforce one another.

Harmonic analysis of the tide at a number of stations scattered about the area would be required to clarify the geographical origin of the phenomena. The existing information, however, is sufficient to indicate that the form of the tide record at Woods Hole is due primarily to the M_4 and M_6 constituents. The suppression of the fundamental semidiurnal components, required to develop the observed distortion of the record, is explained adequately by the interference of the fundamental waves which traverse the sound in opposite directions.

THE TIDAL CURRENT IN WOODS HOLE PASSAGE

The movement of water through Woods Hole Passage is due to a hydraulic current resulting from inequalities of range and time of tide in Buzzards Bay and Vineyard Sound. These inequalities arise from the topography of these tidal basins.

Buzzards Bay is a relatively short closed embayment in which reflection of the advancing tidal wave produces a tide of the standing wave type. High water is delayed less than 45 minutes between its mouth and its head. The mean range of tide increases as the head of the bay is approached, being 3.4 feet at Penikese Island and 4.1 feet at Onset.

Vineyard Sound is a part of a long system of channels in which the tide advances as a progressive wave. The wave arising in the Gulf of Maine, and traversing the sounds in the opposite direction, produces interference effects. The length of the channel is such that off Woods Hole Passage the range of tide is greatly reduced. Because of the high degree of damping, the west-moving wave is so reduced in amplitude that interference is incomplete and the progressive character of the tidal wave which enters Vineyard Sound remains evident.

At the eastern end of Woods Hole Passage, in Great Harbor, the mean range of tide is 1.8 feet as compared to 3.6 feet at the western end, at Uncatena Island. The time of high water at Great Harbor is only 5 minutes later than at Uncatena Island, but low water is 1.7 hours later. Evidently the major condition responsible for the hydraulic head is the difference in level which amounts to about 1.0 foot at high water and nearly 1.5 feet at low water.

A striking characteristic of the currents in Woods Hole Passage is the unequal duration of flow in opposite directions. The duration of the eastward current exceeds that of the westward current by about two hours. This characteristic may be explained as the result of the peculiar form of the tide curve in Great Harbor which appears to arise from the presence of the harmonics in the tidal complex.

If the mean elevations on either side of the Passage were determined solely by the relative amplitudes and times of the semidiurnal tidal constituents in Great Harbor and Buzzards Bay, the differences in elevation and the resulting flow would follow a symmetrical sinusoidal curve, and the duration of the eastward and westward currents would

TABLE III. COMPARISON OF TIDAL CHARACTERISTICS OF WOODS HOLE PASSAGE ESTIMATED FROM MATHEMATICAL MODEL AND PREDICTED BY THE TIDE AND CURRENT TABLES. *Lunar hours or feet.*

	Model	Tables	Difference
Uncatena Island HW	0	0	0
Great Harbor HW	0.1	0.1	0
Great Harbor LW	7.9	7.9	0
Current Turns West	2.9	2.6	+ 0.3
Current Turns East	8.2	7.6	+ 0.6
Current Flows West	5.3	5.0	+ 0.3
Current Flows East	6.7	7.0	− 0.3
Maximum Current West	5.6	5.7	− 0.1
Maximum Current East	11.0	10.8	+ 0.2
Mean Range:			
Uncatena	3.6	3.6	0
Great Harbor	1.73	1.8	0.07

	Maximum Head	Maximum Currents*
West	1.49 feet	3.6 knots
East	1.02 feet	2.7 knots
Ratio	1.46	1.33

* Current pole measurements (Haight, 1936)

be equal. The harmonics in Great Harbor produce an additional component of flow through the Passage. The time relations of this component of flow are such that it is eastward at both periods when slack water would result from the interaction of the semidiurnal components alone. In this way, the duration of the eastward current may be prolonged at the expense of that of the westward current.

Fig. 9 demonstrates this principle and approximates the observed conditions. The tidal elevations at Uncatena Island are represented by a single semidiurnal constituent, M_2, those at Great Harbor by the combination of a semidiurnal, M_2, a quarter diurnal, M_4, and a sixth diurnal constituent, M_6. The harmonic constants are as follows:

		H	k
Uncatena Island	M_2	1.8 feet	233°
Great Harbor	M_2	0.7 feet	263°
	M_4	0.3 feet	76°
	M_6	0.1 feet	339°

Note that the elevations due to the M_2 components, Curves A and C, are equal and that slack water would occur at hours 2.5 and 8.5 in the absence of the higher harmonics. At these times the elevations due to the combined harmonic constituents are negative, producing a hydrostatic head favoring flow eastward from Uncatena Island to

Great Harbor. Equal elevations occur 0.4 hours later during the falling tides and 0.3 hours earlier during the rising tides. This produces inequality in the duration of east and west flow.

The hydrostatic head between Uncatena Island and Great Harbor is shown in the lower part of Fig. 9. This curve resembles that which describes current measurements made by the U. S. Coast and Geodetic Survey off Devils Foot Island (Haight, 1936: fig. 32, Station 13035) in indicating that the westward current is not only shorter in duration but is also of greater maximum velocity. Table III shows how closely the characteristics of the tides in Woods Hole Passage, as represented by the mathematical model, agree with the predictions of the tide and current tables.

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SUMMARY

1. The relations of mean range of tide, time of high water, and time of slack water along Vineyard and Nantucket Sounds may be attributed to the interference of tidal waves entering the sounds from their opposite ends.

2. These waves are in phase at a position to the east of Monomoy Point. At this position the wave approaching from the Gulf of Maine has twice the amplitude of that which has traversed the sounds.

3. The damping coefficient of the tidal waves in the sounds is of the order of 3.0.

4. Interference of the semidiurnal constituents is maximal in the neighborhood of Woods Hole. The great reduction of the semidiurnal constituents relative to the diurnal constituents results in a pronounced diurnal inequality in the range of successive tides at appropriate phases of the moon.

5. The tide gauge record at Woods Hole is distorted in form as a result of the development of shallow water harmonics combined with the reduction of the fundamental semidiurnal constituent by interference.

6. The inequality in the duration of flood and ebb currents in Woods Hole Passage may be attributed to the effect of these harmonics on the tide levels in Great Harbor.

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