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A NOTE ON THE VELOCITY DISTRIBUTION AND BOTTOM STRESS IN A WIND-DRIVEN WATER CURRENT SYSTEM¹

By

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ABSTRACT

The shear stress of the wind on a water surface is sometimes found by measuring the slope of the surface in a lake or wind tunnel. The stress on the bottom of the lake due to the return water current is an error term which has been variously estimated. An experimental method of determining the error is described, and the results indicate that it is but some $1\frac{1}{2}\%$ of the wind stress.

One way of finding the aerodynamic stress of wind on a water surface τ_s is to observe the slope α of the surface along the direction of the wind. The wind causes both the slope and also a drift current towards the lee side in the upper layers of water; consequently, along the bottom of an enclosed lake or sea a return current exists which brings the water back again to the windward side and exerts on the bottom a shear stress τ_b . These two stresses act on the water in the same direction and are balanced by the hydrostatic force due to the rise in water level downwind. Thus, $\tau_s + \tau_b = g\rho\alpha h$, where h is the mean depth of water. It will be seen, therefore, that a measurement of α cannot alone be used to find τ_s , an important oceanographic quantity.

If it can be assumed that the flow throughout the water is laminar, the ratio τ_b/τ_s can be found by analytical methods. These are fully worked by Hellstrom (1941) and Keulegan (1951), where it is found that $\tau_b = 0.50 \tau_s$. However, the water-flows in a lake or sea or even in a laboratory wind-wave tunnel are turbulent in all practical cases, and τ_b/τ_s may therefore be different from the laminar flow value. Hellstrom uses values of 0.15 to 0.80 depending on the depth of the water, and they are based on Boussinesq's theory for turbulent flow. Keulegan uses a purely arbitrary constant value of 0.25 for his wind tunnel experiments. Francis (1951), in some wind tunnel experiments of a similar nature, took measurements of the mean velocity of the return water current, and by applying the usual drag formula $\tau_b = c\rho u^2$ (u = water speed, c = drag coefficient), he found that τ_b never exceeded 3% of τ_s . In this last case, a conservatively high value of c

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was used in order to allow empirically for the oscillatory motion imposed by the wind-generated surface waves on the much slower mean velocity. It was thought that the turbulence added by the oscillations might increase the drag.

An attempt was made to measure τ_b directly in the small wind-wave tunnel mentioned above while the water currents were being wholly generated by wind. A flat smooth plate was arranged just to float

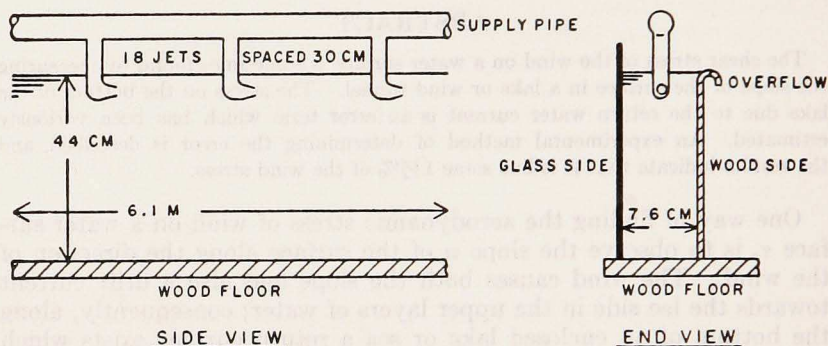


Figure 1. Tank used in experiments.

in the surface of a pool of heavy fluid at the bottom of the tunnel. The plate was constrained by a spring system so that the shear stress on the plate could be measured by its deflection. The experiment was a failure because the wave motions caused the plate to oscillate badly; not even a mean position could be determined. Better success has now been achieved with another experimental technique which uses horizontal jets of water to apply to the water surface the momentum that was applied before by the wind. This artifice ensures a surface without waves and water-flows in the tank that are noticeably steadier. The total amount of water in the tank is kept constant by a uniform overflow all along the top of one side of the tank; thus the jets add momentum but not mass to the water already there. Below the upper few centimetres there is no motion caused by the water overflow. A slight correction is made because of the finite size of the jets, since the momentum is really applied to a thin layer at the surface, making the effective surface for shear somewhat below the actual surface.

Fig. 1 shows the tank, which is 6.1 m long, 12.7 cm wide, and 44 cm deep from bottom to overflow. Eighteen jets, spaced 30 cm apart, were supplied with sea water through a 5 cm pipe parallel to the tank length and above the water level. The jet outlets, just in the surface of the water, were of an oval, flattened shape, the total cross-sectional

area of all the jets being 10.7 cm^2 . The shear stress measuring-device is shown in Fig. 2. Its design is based upon the equipment used by Sheppard (1946) for measuring stress in the atmosphere. The circular moving plate is brass, 7.6 cm in diameter, and is of an inverted dish shape. It floats on carbon tetrachloride (density 1.70 gm cm^{-3}) in a shallow dish with only 5 mm clearance all round. A piece of small

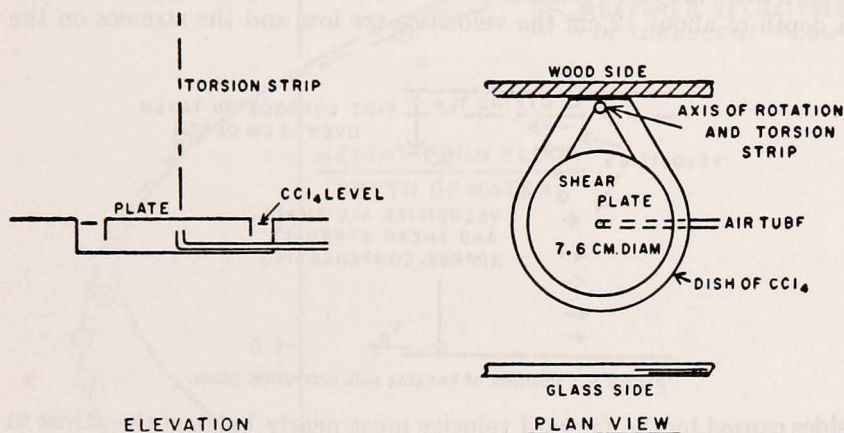


Figure 2. Detail of shear plate.

bore tube allows air to be blown into the underside of the plate so that its buoyancy may be adjusted with some precision to place the top of the plate, the surface of the CCl_4 , and the top of the circumscribing dish all in one plane. A false floor 1.2 cm thick is laid in the tank so that the apparatus does not project into the flow. The plate is pivoted on a vertical axis at the side of the tank; and the spring restraint is a vertical torsion strip 2.1 mm by 0.3 mm by 17 cm long. It is clamped at its upper end clear of the water, with an adjustment for the no-load position of the plate. The position of the plate is observed by a travelling telescope (cathetometer) which views a mark on the periphery through the glass side of the tank. The torsion constant of the apparatus was measured by putting weights in a paper pan on a fine cotton thread attached to the plate; the resulting calibration was linear, with negligible scatter for load increasing and decreasing.

The jet discharge Q through the total orifice area a was calibrated against the pressure in the supply pipe by collecting the water in a large tank. The momentum supplied by the jets per second (the available thrust force) is therefore $\rho Q^2/a$. Not all of this is available for driving the water currents; some of it is counterbalanced by the backward thrust of the further end of the tank where the high velocity

stream strikes; some of it is counterbalanced by the thrust of the water on the backs of the jet pipes which must project into the fast surface stream; and some is counterbalanced by the shear stress of the fast upper stream on the sides of the tank. The last correction is necessary only because of the limited width of the tank in the shallow zone where there are fast velocities solely due to the dispersing jets. Below a depth of about 12 cm the velocities are low and the stresses on the

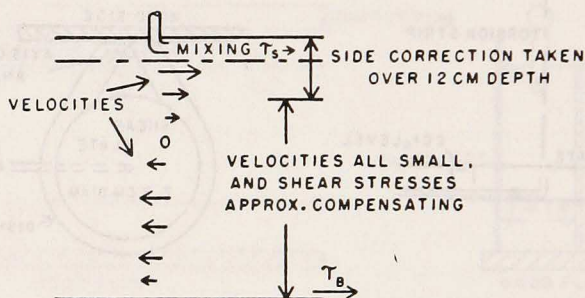


Figure 3. Position of hanging side correction plate.

sides caused by the forward velocity must nearly balance the stress in the opposite direction caused by the backward return current below (see Fig. 3). The correction terms were all measured directly, the first by placing a false flat end plate on a pendulum and measuring the deflection; the second by mounting a jet pipe on a pendulum and immersing to the appropriate depth; and the third by suspending similarly a smooth flat plate 90 cm long and 12 cm deep parallel to, and 0.4 cm away from, the glass side of the tank.

The observations for two different water-flows are summarized below.

TABLE OF FORCES ON WATER IN TANK OF PLAN AREA 7720 cm^2

Water flow ($\text{cm}^3 \cdot \text{sec}^{-1}$)	$\rho Q^2/a$ Input		Net input			Net	Measured	τ_b/τ_s (%)
	by jets ($\text{dynes} \times 10^3$)	Corrections End Side	Jets	force ($\text{dynes} \times 10^3$)	τ_s ($\text{dynes} \cdot \text{cm}^{-2}$)	τ_b ($\text{dynes} \cdot \text{cm}^{-2}$)		
2230	480	98 72	54	256	33.1	0.46	1.4	
2900	800	173 98	82	447	57.9	0.77	1.3	

τ_b is the average of measurements at two places on the floor of the tank.

There was still a little oscillation of the shear plate, due presumably to large scale turbulence in the tank, but the mean deflection of the plate could be easily estimated. Lower jet speeds, however, gave such small deflections of the plate that the results are not significant. It will be seen that the bottom stress is small compared with the top

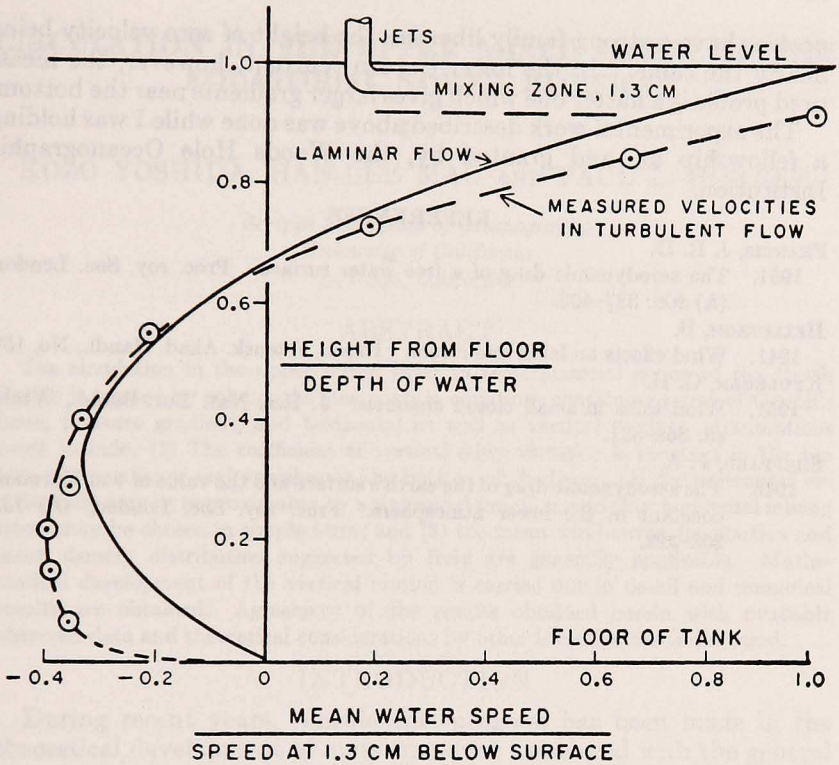


Figure 4. Comparison between theoretical and measured velocity profiles (measured velocity at 1.3 cm below surface = 26.3 cm sec^{-1}).

stress and can be neglected in most cases, for in practice the measurement of slope α is fraught with errors that are greater than this correction.

Velocity profile. The distribution of mean velocity was determined at one jet speed by a total head tube traversed on the centreline of the tank. The static pressure was taken at a tapping in the side of the tank in the plane of the traverse. The pressures were measured by a micrometer point in a glass stilling well.

The profile is shown in Fig. 4 together with the parabolic one for laminar flow. Both profiles are defined by nondimensional parameters, the reference velocity being that at 1.3 cm below the water surface. A reference velocity at or nearer the surface would be atypical of the drift system as a whole, for the occasional high speed jet gives a very nonuniform flow there. It will be seen that the two

profiles have a strong family likeness, the height of zero velocity being nearly the same. In the lower (return) current, however, the measured profile is a flatter one which gives larger gradients near the bottom.

The experimental work described above was done while I was holding a fellowship at, and granted by, the Woods Hole Oceanographic Institution.

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