A THEORY OF FOG FORMATION

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ABSTRACT

It is shown that just above a cold black-body surface the effect of radiation is to warm the air, whereas just above a warm surface the effect of radiation is to cool the air. Above a height of the order of one meter the signs of the radiative temperature change are reversed. Since, as Emmons and Montgomery (1947) have pointed out, fog formation usually requires net radiation from the air, it follows that, above a cold surface, fog forms only above a certain critical height whereas above a warm surface fog forms below a critical height. Other processes which produce condensation appear to be less important.

INTRODUCTION

It has been pointed out by Emmons and Montgomery (1947) that turbulence is equally effective in conducting moisture and heat from a nearly saturated atmosphere to a colder surface. They conclude that supersaturation and condensation can be achieved only "when there is further cooling by radiation directly from the air or when saturation is not required."

Intuition suggests that the air immediately above a cold black-body surface must be heated by radiation from the warmer atmosphere, whereas the warmer air farther from the cold surface may be cooled by interchange of radiation with the surface. Thus, it appears that fog cannot form immediately above a cold surface but that it may form at a distance above the surface. Conversely, it appears that fog may form in contact with a warm surface. It is the purpose of this paper to verify these intuitive conclusions and to discuss quantitatively the conditions under which fog can or cannot form.

DISTRIBUTION OF RADIATIVE TEMPERATURE CHANGE

The rate of temperature change at a point due to radiation may be developed from Schwarzchild's equation. Bruinenberg (1946) has expressed this rate of temperature change as the sum of integrals representing the upward and downward radiation. Following Brooks (1950), Bruinenberg's equation may be written for water vapor alone in the form

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$$\frac{\partial T}{\partial t} = \frac{q}{c_p}\left(\frac{p}{1000mb}\right)^{\frac{1}{2}}\left[\int_0^{w_1}\frac{\partial}{\partial w}(\sigma T^4)\,\frac{\partial \varepsilon}{\partial w}\,dw + \frac{\partial S_E}{\partial w} - \int_0^{\infty}\frac{\partial}{\partial w}(\sigma T^4)\,\frac{\partial \varepsilon}{\partial w}\,dw\right] \quad (1)$$

if it is assumed that $\partial\varepsilon/\partial w$ is independent of temperature and that both surface and atmosphere are horizontally homogeneous. The symbols have the following meanings: T, absolute temperature; t, time; q, specific humidity; c_p, specific heat at constant pressure; p, pressure; ρ, air density; σ, Stefan's constant; ε, emissivity of water vapor; w, optical path length of water vapor between the reference height, h, and the variable height, z $({}_h\int^z \rho q dz)$; w_1, w evaluated between the reference height and the top of the water vapor atmosphere; S_E, radiation flux due to water vapor at top of the water vapor atmosphere.

The effect of radiation absorbed and emitted by carbon dioxide is not considered explicitly in what follows; this appears to be justified because the error due to neglecting carbon dioxide probably is less than that due to the approximations introduced for simplicity.

Since w is a function of the single independent variable, z,

$$\frac{\partial T}{\partial w}\,dw = \frac{\partial T}{\partial z}\,dz\,.$$

Furthermore, if calculations are made only near sea level, (1) may be written

$$\frac{\partial T}{\partial t} = \frac{4\sigma q}{c_p}\left[\int_h^{\infty} T^3\,\frac{\partial \varepsilon}{\partial w}\,\frac{\partial T}{\partial z}\,dz + \frac{\partial S_E}{\partial w} + \int_h^{0} T^3\,\frac{\partial \varepsilon}{\partial w}\,\frac{\partial T}{\partial z}\,dz\right]. \quad (2)$$

The change of sign preceding the second integral is due to the fact that below the reference height $dw = -\rho q dz$.

The problem discussed here concerns temperature lapse rates of the order of degrees per meter very near the reference height. For this case, T^3 properly may be considered constant and $\partial S_E/\partial w$ may be neglected. Thus, (2) may be written

$$\frac{\partial T}{\partial t} = \frac{4\sigma q T^3}{c_p}\left[\int_h^{\infty}\frac{\partial \varepsilon}{\partial w}\,\frac{\partial T}{\partial z}\,dz - \int_0^{h}\frac{\partial \varepsilon}{\partial w}\,\frac{\partial T}{\partial z}\,dz\right]. \quad (3)$$

The rate of temperature change may be expressed as a function of height if $\partial\varepsilon/\partial w$ and $\partial T/\partial z$ are expressed as functions of height. By combining the theoretical result of the work by Ladenburg and Reiche (1913) for very short optical paths with the summarized

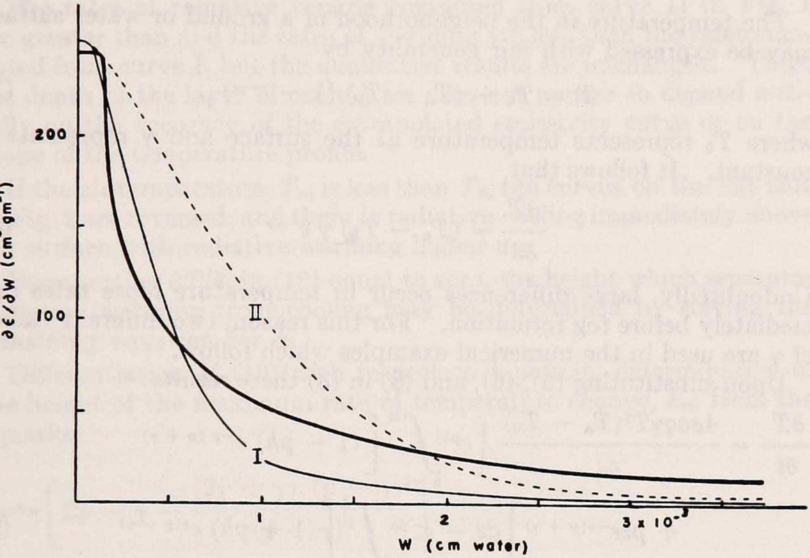

Figure 1. Slopes of the water vapor emissivity curve ($\partial\varepsilon/\partial w$) as a function of optical path length (w) determined from Ladenburg and Reiche (1913) and Brooks (1950) [heavy line] and computed from equation (4), using $a = 250$ cm² gm⁻¹, $b = 3950$ cm² gm⁻¹ [curve I] and $a = 250$ cm² gm⁻¹, $b = 2000$ cm² gm⁻¹ [curve II].

experimental data given by Brooks (1950), it is possible to represent the dependence of $\partial\varepsilon/\partial w$ on w as shown in Fig. 1 by the heavy line. This line may be approximated by the relation

$$\frac{\partial\varepsilon}{\partial w} = a\,(1 + bw)\, e^{-bw}\,, \tag{4}$$

where a and b are constants to be determined from the curve.[2] It is particularly important to fit the curve accurately in the region of very short path lengths. Contrasting curves computed from two values of b are shown in Fig. 1. The slope of the emissivity curve may be expressed as a function of height if the plausible assumptions of uniform density and specific humidity are made. Then (4) becomes for the region above the reference height

$$\left(\frac{\partial\varepsilon}{\partial w}\right)_{z>h} = a\,[1 + b\rho q(z - h)]\, e^{-b\rho q(z-h)} \tag{5}$$

and for the region below the reference height

$$\left(\frac{\partial\varepsilon}{\partial w}\right)_{z<h} = a\,[1 - b\rho q(z - h)]\, e^{-b\rho q(z-h)}\,. \tag{6}$$

[2] The form of equation (4) was suggested by Maurice Rattray.

The temperature in the neighborhood of a ground or water surface may be expressed with fair generality by

$$T = T_0 + (T_a - T_0)\,(1 - e^{-\gamma z})\,, \tag{7}$$

where T_0 represents temperature at the surface and γ represents a constant. It follows that

$$\frac{\partial T}{\partial z} = (T_a - T_0)\,\gamma\, e^{-\gamma z}\,. \tag{8}$$

Undoubtedly, large differences occur in temperature lapse rates immediately before fog formation. For this reason, two different values of γ are used in the numerical examples which follow.

Upon substituting (5), (6), and (8) in (3) there results

$$\frac{\partial T}{\partial t} = \frac{4\sigma a q \gamma T^3 (T_a - T_0)}{c_p} \Bigg\{ e^{ph} \int_h^\infty \Big[(1 - ph)\, e^{-z\,(p+\gamma)} + pze^{-z(p+\gamma)} \Big] dz - e^{-ph} \int_0^h \Big[(1 + ph)\, e^{z(p-\gamma)} - pze^{z(p-\gamma)} \Big] dz \Bigg\}, \tag{9}$$

where $b\rho q \equiv p$. Integration gives

$$\frac{\partial T}{\partial t} = A \left\{ e^{-h\gamma} \left[\frac{2p + \gamma}{(\gamma + p)^2} - \frac{2p - \gamma}{(p - \gamma)^2} \right] + e^{-ph} \left[\frac{ph}{p - \gamma} + \frac{2p - \gamma}{(p - \gamma)^2} \right] \right\}, \tag{10}$$

where the coefficient of the large bracket in (9) is represented by A.

The properties of equation (10) may be most easily understood by considering numerical examples. Fig. 2 shows the vertical distribution of $\partial T/\partial t$ computed from (10) by using the following values: a, 250 cm^{2}gm^{-1}; b, 3950 cm^{2}gm^{-1}; ρ, 1.25×10^{-3} gm cm^{-3}; q, 8 gm kg^{-1}; γ, 9.0×10^{-3} cm^{-1} (A), 1.6×10^{-3} cm^{-1} (B); T, 280 K; $T_a - T_0$, 10C; It is clear that the air immediately above the surface is warmed by radiation, whereas the air above 70 cm or so is cooled by radiation. The rate of radiative temperature change increases with increase in curvature of the temperature profile (increase in γ), but the approximate height of the boundary between the regions of cooling and warming is not affected much by change in γ. It should be understood that the rate of temperature change computed from (10) prevails only so long as the temperature profile remains essentially unchanged.

The rates of radiative cooling computed from curve II in Fig. 1 are greater than and the rates of warming are less than the rates computed from curve I, but the qualitative results are unchanged. Thus, the depth of the layer of cooling air does not appear to depend critically on the accuracy of the extrapolated emissivity curve or on the shape of the temperature profile.

If the air temperature, T_a, is less than T_0, the curves on the left side of Fig. 2 are reversed; and there is radiative cooling immediately above the surface with radiative warming higher up.

Upon setting $\partial T/\partial t$ in (10) equal to zero, the height which separates radiative warming from cooling may be determined by solving the remaining equation for h.

Differentiation of (10) with respect to h permits determination of the height of the maximum rate of temperature change, h_m, from the equation

$$\gamma e^{ph_m}\left[2p - \gamma - \frac{(2p+\gamma)\,(p-\gamma)^2}{(p+\gamma)^2}\right] + pe^{\gamma h_m}\left[2p - \gamma + (p-\gamma)\,(1 - ph_m)\right] = 0\,. \quad (11)$$

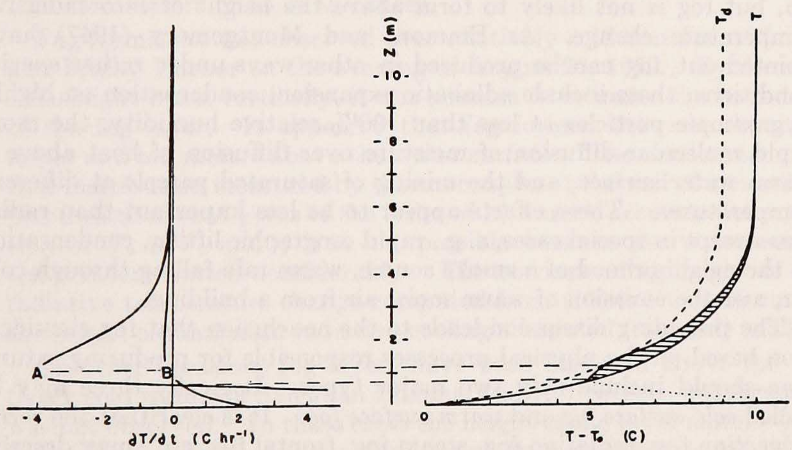

Figure. 2. *Left.* Vertical distribution of rate of radiative temperature change computed from equation (10), using $p = 3.95 \times 10^{-2}$ cm^{-1} and $\gamma = 9.0 \times 10^{-3}$ cm^{-1} (*A*), $\gamma = 1.6 \times 10^{-3}$ cm^{-1} (*B*). *Right.* Vertical temperature distribution computed from equation (7), using $\gamma = 9.0 \times 10^{-3}$ cm^{-1} (curve *T*), corresponding vertical distribution of dew point (curve T_d), and 15-minute radiational temperature change computed from equation (10). Dotted curve below 70 cm represents the fictitious temperature computed for this region.

THE FORMATION OF FOG

The preceding analysis of the rate of radiative temperature change near black-body surfaces indicates that fog can form only within a layer bounded on one side by the height of zero radiative temperature change. The processes which lead to fog formation above a cold moist surface are illustrated in Fig. 2. Turbulent eddies conduct heat and moisture from the air to the surface, with the result that the dew point temperature and air temperature approach one another near the surface. Immediately above the surface, radiative warming assures that the temperature does not fall to the dew point; but within the region of radiative cooling, the air temperature eventually reaches the dew point, as shown on the right side of Fig. 2. The radiation absorbed just above the surface results in only slight increase in temperature because this energy is conducted promptly to the cold surface.

Above a warm moist surface the processes which lead to fog formation are similar although the results are different. Turbulent eddies carry heat and moisture upward, with the result that the air temperature and dew point distributions are represented by a mirror image of the corresponding curves in Fig. 2 with labels reversed. Radiative cooling then occurs immediately above the surface, and radiative warming occurs above a height of the order of one meter. Thus, fog may form promptly at the surface and somewhat more slowly higher up, but fog is not likely to form above the height of zero radiative temperature change. As Emmons and Montgomery (1947) have pointed out, fog can be produced in other ways under rather special conditions; these include adiabatic expansion, condensation on highly hygroscopic particles at less than 100% relative humidity, the more rapid molecular diffusion of moisture over diffusion of heat above a warm water surface, and the mixing of saturated parcels at different temperatures. These effects appear to be less important than radiation except in special cases, e. g., rapid orographic lifting, condensation in the neighborhood of a smoke source, warm rain falling through cold air, and the emission of warm moist air from a building.

The preceding discussion leads to the conclusion that fog classification based on the physical processes responsible for producing saturation should include only two major types. Logically these may be called *cold surface fog* and *warm surface fog*. It is clear that the terms advection fog, radiation fog, steam fog, frontal fog, etc., may describe certain features of the weather situation; but they do not help very much in understanding the physical processes which bring about the fog.

After fog has formed, the physical processes are altered greatly. The fog acts essentially as a black-body radiating surface, and the latent heat released by condensation introduces a new factor into the problem. The subsequent temperature distribution and its effect on turbulent diffusion have been discussed by Fleagle, Parrott, and Barad (1952).

OBSERVATIONS

It is extremely difficult to make accurate observations which may be used to test the validity of the preceding theory. Horizontal homogeneity seldom can be assumed in accessible locations, and turbulent motions produce large fluctuations in the temperature distributions. However, several observations which are consistent with the theory have been made.

Observations made with thermocouples mounted on a four meter mast above a fresh water lake on San Juan Island early on the morning of August 19, 1952 indicate that the air was 10.4 C colder than the water and γ was about .02. The height of zero radiative temperature change computed from these data is about 25 cm, and the rate of radiative cooling at the water surface is about .15 C min^{-1}. Visual observation indicated that the fog formed only within a layer less than a meter in thickness and formed most rapidly at the water surface, which is in substantial agreement with the computations. Air currents then carried the fog upward so that eventually the fog occupied a layer ten meters or so in thickness.

Fog formation was observed over relatively cold San Juan Channel and Friday Harbor on the evening of August 26, 1952. By directing a flashlight beam vertically, it was possible to estimate the boundaries of the fog layer. It appeared that fog formed between about one meter and ten meters above the water surface. The vertical temperature distribution measured by thermocouples at the end of a cantilever pier indicated that γ was of the order of 10^{-2} cm^{-1}; accurate determination of $T_a - T_0$ and γ was not possible because the water surface temperature varied considerably. The corresponding height of zero radiative temperature change computed from (10) is about 40 cm and the height of maximum radiative cooling computed from (11) is 50 cm.

On other occasions fog layers have been observed above the relatively cold waters of San Juan Channel and Haro Strait, but formation was not observed. In these cases the height of the lower surface of the fog varied from place to place, but generally it was a meter or so above the water surface. On one occasion the lower surface of fog in a particular region was observed to slope down to the water surface, which suggests that the water in this region was warmer than the air.

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