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THE EXCHANGES OF FRESH AND SALT WATERS IN TIDAL ESTUARIES[1]

By

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ABSTRACT

An empirical theory is presented which describes the exchanges between various parts of an estuary as a result of tidal oscillations, and which permits the calculation of the average distribution of fresh and salt water within the estuary. The characteristics of the estuary used in the calculations are the mean range of tides, the river flow, and the topography, all readily available for most estuaries.

The calculations are shown to produce results which are similar to distributions observed in three very different estuaries. The theory will permit calculation of the changes in distribution of salinity and fresh water in any given estuary to be expected as a result of variation of river flow.

INTRODUCTION

In studies of the ecology of tidal estuaries the exchanges of fresh and salt water are of primary importance in determining the character of the environment. Thus, for example, the distribution of salinity and the extreme conditions to which sedentary organisms will be exposed are determined by these exchanges. The distribution of planktonic populations, of eggs spawned within the estuary, of pollutants introduced by cities or industries, in short, the distribution of any material dissolved or suspended in the water, is determined by the circulation of fresh and salt water and by the exchanges between various parts of the estuary. These exchanges, and the resulting distributions, are related in a regular way over short periods of time to the tidal oscillations and over longer periods of time to fluctuations of river flow. They are likewise related in more erratic ways to the effects of winds.

Although numerous estuarine studies have been made, the theoretical basis of the circulation is only slightly understood. Interpretation of survey data is improved by using the river water as a tracer to determine exchange ratios, flushing times and nontidal drifts (Tully, 1949; Ketchum, 1950; Ketchum, Redfield and Ayers, 1951).

In this paper an empirical theory is presented which describes the exchanges across various cross sections and which permits the calcu-

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lation of the resulting average distribution of fresh and salt water in an estuary. The exchanges of any other property of the water can be derived in a similar way. The data used in the calculations are tidal heights, river flow and the topography of the basin, all readily available on most important estuaries. The calculation is applied to three estuaries, and the hydrographic conditions which influence the results of the calculation are discussed.

SIMPLIFYING CONCEPTS

An estuary is defined as a region where river water mixes with, and measurably dilutes, sea water. The theory attempts to predict average conditions in successive volume segments of an estuary for a constant rate of river flow and the mean tidal range. Variations in the horizontal distribution of fresh water within a cross section, such as are commonly found as a result of Coriolis' forces, are neglected. The effects of vertical stratification will be discussed below.

The following simplifying concepts are involved:

1. *Steady State Distribution.* It is assumed that there is a steady state distribution of fresh and salt water within the estuary. This is obviously true for long periods of time, since estuaries are not becoming either fresher or more saline. At times of changing runoff of the rivers, however, the average distribution within the estuary changes, and the steady state distribution cannot be assumed.

In order to maintain a steady state the following conditions must be met at every complete cross section of the estuary:

a.) *Salt:* There must be no net exchange of salt during a tidal cycle across the section. This, of course, does not mean that the circulation of salt water is zero but that there is a balance between the inflow and outflow of sea water.

b.) *Fresh water:* During a tidal cycle there must move seaward a volume of fresh water equal to the volume introduced by the river in the same period of time.

2. *Inner End of Estuary.* The inner end of the estuary is defined as the section above which the volume required to raise the level of the water from low to high water mark is equal to the volume contributed by the river during a tidal cycle. There will be no net exchange of water through this section during a flood tide, since the entire rise of tide above the section is contributed by river flow. During the ebb tide there will be a loss through this section of one river flow per tide. Since there is no exchange on the flood tide the water above this section should be completely fresh. It will be noted that this is a

dynamic, not a geographic definition, since the boundary will move corresponding to changes in river flow.

3. *Segmentation of the Estuary.* The above paragraph defines a section which is the seaward boundary of the innermost volume segment of the tidal portion of the river. This is called Segment 0. Consecutive volume segments are defined so that the distance between their inner and outer boundaries is equal to the average excursion of a particle of water on the flooding tide. The average excursion is derived from the volume entering each part of the estuary on the flooding tide as well as the topography of the estuary. If the volume entering on the flooding tide were to act like a piston, displacing and pushing upstream its own volume of water from the next landward segment, the distance moved would be the average excursion of a particle of water on the flood tide in that part of the estuary. The segment so defined would contain, at high tide, a volume equal to that contained in the adjacent seaward segment at low tide. Consequently, along the length of the estuary each segment is so defined that the high tide volume in the landward one is equal to the low tide volume in the adjacent seaward one. Beginning with Segment 0, defined above, the entire estuary can thus be subdivided into a series of volume segments.

With R as the volume of river water introduced during a tidal cycle, P the local intertidal volume[2] and V the low tide volume of each segment, the following equations summarize these fundamental definitions:

1. The inner end of the estuary is defined as the section above which $P_0 = R$.
2. The limits of each successive volume segment are placed so that

$$\begin{aligned} V_1 &= V_0 + P_0, \\ V_2 &= V_1 + P_1 = V_0 + P_0 + P_1\,, \\ V_3 &= V_2 + P_2 = V_0 + P_0 + P_1 + P_2 \text{ etc.} \end{aligned}$$

It will be noted that the low tide volume in each segment equals the total tidal prism within the inner boundary of the segment, plus the low tide volume in Segment 0; that is,

$$V_n = V_0 + \Sigma_0^{n-1} P\,,$$

or, since $P_0 = R$: $$V_n = V_0 + R + \Sigma_1^{n-1} P\,.$$

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[2] The tidal prism is defined as the volume of water required to produce the observed rise in water level in the estuary on the flooding tide. The part of the total tidal prism in each segment at high tide is called the local intertidal volume.

THE EXCHANGE RATIO

The estuary is divided into volume segments defined as above, so that the limits of each segment show the average excursion of a particle of water on the flood tide. If it is assumed that the water within such a volume segment is completely mixed at high tide, the proportion of water removed on the ebb tide will be given by the ratio between the local intertidal volume and the high tide volume of the segment. This same proportion of river water and of anything dissolved or suspended in the water will be removed by the ebb tide. Thus, an exchange ratio (r) can be defined for each segment (n):

$$r_n = \frac{P_n}{P_n + V_n} .$$

The effect of incomplete mixing on the exchanges will be discussed below.

The river water present in each segment is a mixture of river water accumulated during many tidal cycles. Each segment receives on each tidal cycle a volume of river water (R) equal to the volume introduced by the river in the same period of time. If we take R_1 as the volume of river water arriving during the current tidal cycle, then the amount removed on the ebb tide will be rR_1, and the amount remaining behind will be $(1 - r)R_1$. This same proportion of river water (R_2) arriving on the previous tidal cycle was left behind by the previous ebb tide, and the amount of this river water (of age two tidal cycles) removed will be $r\,(1 - r)R_2$ and the amount of this water remaining, after being depleted by two successive ebb tides, will be $(1 - r)^2\,R_2$. The proportion of water of various tidal ages removed, and remaining behind, as a result of the exchanges on any given ebb tide may be summarized as follows:

Age in Tidal Cycles	*River Water Removed*	*River Water Remaining*
1	rR_1	$(1 - r)\,R_1$
2	$r\,(1 - r)\,R_2$	$(1 - r)^2\,R_2$
3	$r\,(1 - r)^2\,R_3$	$(1 - r)^3\,R_3$
m	$r\,(1 - r)^{m-1}\,R_m$	$(1 - r)^m\,R_m$;

here 1, 2, 3 . . . m represent the number of tides each portion of the river water has been within the segment considered.

If the river flow is constant, all values of R are equal and the steady state condition can be assumed to exist. The total volume (Q_n) of river water accumulated within any volume segment (n) of the estuary at high tide is then the sum of the remaining volumes given in the final column above. Since the equation is written for the high tide condition, one volume of the river flow which has not yet been depleted is also present; that is,

$$Q_n = R\{1 + (1 - r_n) + (1 - r_n)^2 + (1 - r_n)^3 + \dots + (1 - r_n)^m\} .$$

As m becomes large, this series will approach the limit

$$Q_n = R \frac{1 - (1 - r_n)^m}{r_n} .$$

Since $(1 - r)$ is always less than unity, $(1 - r)^m$ approaches zero after an infinite number of tidal cycles, and the sum can be reduced to

$$Q_n = \frac{R}{r_n} .$$

It will be remembered that one qualification of the steady state distribution was that the river water passing seaward through each cross section on each tidal cycle must equal the volume of river water (R) introduced during the same period of time. The above relationship states that the volume of river water moving seaward is the product of the exchange ratio and the accumulated volume of river water (rQ). It is readily seen that this product equals R, so that this criterion of steady state distribution is satisfied.

The exchange ratio is defined on the assumption of complete mixing of the volume of water in each segment at high tide. The average excursion of water during the flooding tide is presumed to set the upper limit of the length of the estuary over which complete mixing can be logically assumed. It is for this reason that the subdivision of the estuary is essential in the application of the theory.

Incomplete vertical mixing can be detected readily if salinity observations are available, since the water diluted by river effluent will be limited to the upper layers of the water column. Only the mixed volume of water should be considered in such a case, and the exchange ratio applicable would be

$$r'_n = \frac{P_n}{P_n + V_n} \times \frac{D}{H} ,$$

in which D is the average depth of the Segment n and H is the depth of the mixed layer. The exchange ratio so calculated will be greater, and the resulting accumulation of river water (Q) will be smaller than would be obtained if the entire depth were used. In such a case the segmentation of the estuary is also made using volumes computed to the mixed depth. The entire treatment is developed, therefore, as though the water below the mixed zone had no part in the tidal mixing process and could be replaced, without changing the distribution, by a false bottom.

Incomplete horizontal mixing will also modify the distributions to be expected. The effects are not incorporated into the theory since there are no independent criteria to evaluate their magnitude. The effects of incomplete horizontal mixing will be discussed qualitatively below.

EXCHANGES IN A MODEL ESTUARY

The exchanges in a theoretical estuary, rectangular in cross section, in which the depth at low tide is half of the high tide depth (i. e. $P = V$), will be calculated in order to indicate how the theory is applied. The results are presented in Table I. For every volume segment the exchange ratio is 0.5, and the accumulation of river water is $2R$. Taking R as unity, the volume enclosed in Segment 0 at high tide is 2, which is all river water—100% fresh. In Segment I, having a relative high tide volume of 4, half of the water is river water. On the ebb tide Segment I receives one volume (all of it fresh water) from the river (Segment 0) and loses 3 volumes ($P_0 + P_1$) at a concentration of 50% fresh water (1.5 volumes of river water) to Segment II. At low tide, therefore, Segment I contains 2 volumes, three-quarters of which is river water. On the flooding tide Segment I loses no water landward, since, above its inner limit, river flow is sufficient to provide the observed rise in water level. It receives from the adjacent seaward segment two volumes which contain 25% river water. The high tide content is thus returned to the original condition. The net result of the tidal oscillation has been to add to Segment I a volume of river water from the river, and to move an equal volume of river water from Segment I to Segment II. Similar treatments, shown for the river boundary and for the first four segments, can be expanded to the nth segment with the same net seaward transport of river water through every cross section. The fact that each segment returns to its original high tide content shows that the net exchange of salt during a tidal cycle is zero, and thus the steady state requirements are satisfied.

A basic assumption of the theory is that the mixing is complete within volume segments defined by the average excursion of a particle of water on the flooding tide. For each segment in Table I, complete mixing of the volumes of water added during the tidal cycle will produce mixed water of the same characteristics as the water present at high tide. In Segment I, for example, one volume of water is added on the ebb tide and two volumes on the flood tide. These three volumes contain a total of 1.5 volumes of river water. If they were mixed together the composition would be 50% fresh water—the same composition as the high tide content of the segment. However the mixing may be accomplished, the net effect of the mixing processes must be equivalent to the mixing of the volumes of water added to each segment during a tidal cycle if the theory is reliable.

APPLICATION TO NATURAL ESTUARIES

The total accumulation of river water has been calculated by the above method for Raritan River and Bay, New Jersey, for Alberni Inlet, B. C., and for Great Pond, near Falmouth, Massachusetts. These three estuaries are different in almost all characteristics. In all three, survey data are available which permit calculation of the fresh water content from the salinity distribution, so that the results of the survey and of the calculation can be compared. The method used, and described here for Raritan Bay, is similar for the other two locations.

Raritan River and Bay, New Jersey. A survey of Raritan River and Bay was made in December 1948, which permits comparison of the computed distribution of fresh and salt water with the observed distribution over a length of about 20 miles of the estuary. Raritan River and Bay have sandy or marshy shores, a maximum depth of 40 feet in dredged channels, and they vary in width from 0.1 to 7.0 miles. The surface area of the surveyed portion of the estuary was 1,670 million square feet, or about 45 square nautical miles. Since the mean range of tides is 5.5 feet, the total volume of the tidal prism is 9,200 million cubic feet. This is almost 300 times greater than the volume introduced by the river during a tidal cycle at the time of our survey.

In the Bay the average salinity for each of five cross sections was determined from the results obtained at 4 to 7 stations. These stations were occupied once each hour throughout the complete tidal cycle. The high tide data are used for comparison with the theory. In the river the results of a single station, occupied at high tide, was used to determine the average salinity across each section. The method used for calculating the distribution of fresh water from the salinity observations has been described by Ketchum, 1950.

The first step in the application of the theory is to determine the total low tide volume of the estuary enclosed by various cross sections. This is done by drawing numerous cross sections with the aid of a navigation chart and determining their cross-sectional area with a planimeter. The volume is computed by multiplying the distance

TABLE I. Net Exchanges in a Model Estuary as a Result of Tidal Oscillations and River Flow (all volumes expressed as multiples of river flow).

Segment	*0*		*1*		*2*		*3*		*4*	
Intertidal volume (P_n)	1		2		4		8		16	
High tide volume ($P_n + V_n$)	2		4		8		16		32	
Exchange ratio (r_n)	0.5		0.5		0.5		0.5		0.5	
River water content (Q_n)	2.0		2.0		2.0		2.0		2.0	
% Fresh	100		50		25		12.5		6.25	
EBB TIDE EXCHANGES	Vol.	Fresh	Vol.	Fresh	Vol.	Fresh	Vol.	Fresh	Vol.	Fresh
Landward	0	0	+1.0	+1.0	+3.0	+1.5	+ 7.0	+1.75	+15.0	+1.875
Seaward	−1.0	−1.0	−3.0	−1.5	−7.0	−1.75	−15.0	−1.875	−31.0	−1.9375
Net change	−1.0	−1.0	−2.0	−0.5	−4.0	−0.25	− 8.0	−0.125	−16.0	−0.0625
FLOOD TIDE EXCHANGES										
Landward	+1.0	+1.0	−0	−0	−2.0	−0.5	− 6.0	−0.75	−14.0	−0.875
Seaward	0	0	+2.0	+0.5	+6.0	+0.75	+14.0	+0.875	+30.0	+0.9375
Net change	+1.0	+1.0	+2.0	+0.5	+4.0	+0.25	+ 8.0	+0.125	+16.0	+0.0625

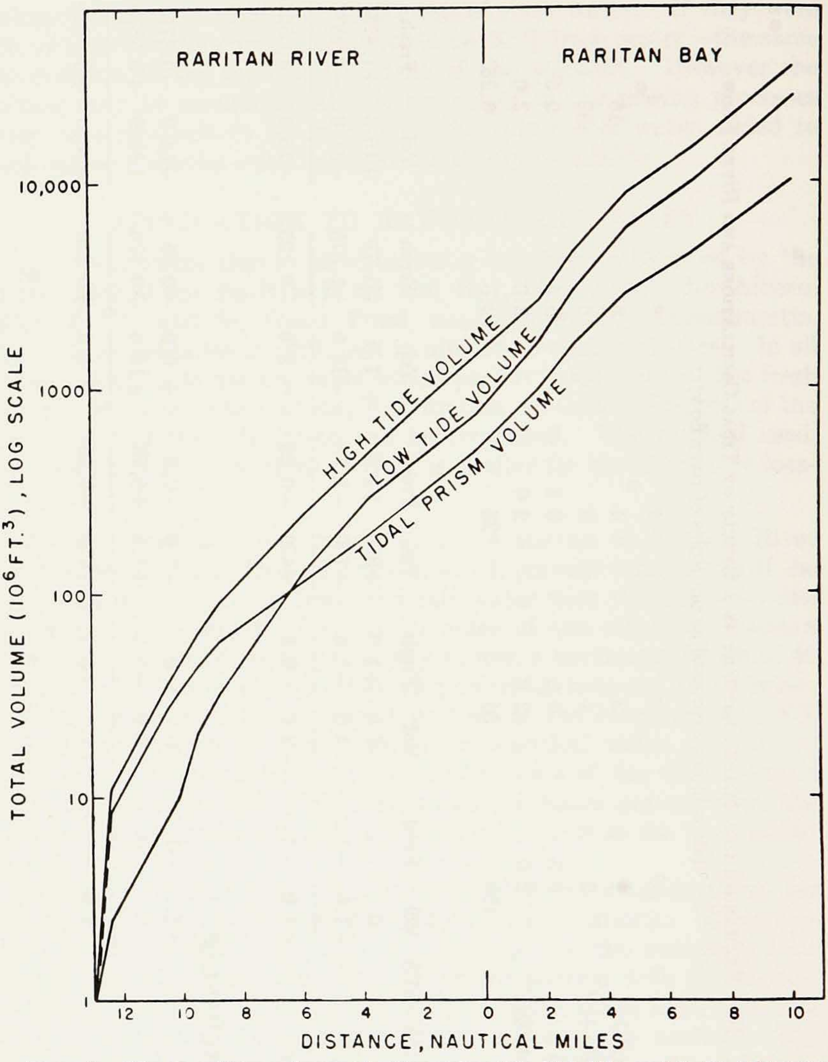

Figure 1. The cumulative volumes of water at low tide, high tide, and in the tidal prism enclosed by various cross sections in Raritan River and Bay. The river end of the estuary is at the left. (The semi-log scale is used for convenience only.)

between sections by the average of their cross-sectional areas. The cross sections drawn should be selected so that the change in width and depth of the estuary between sections is as uniform and gradual as possible. The sum of the volumes enclosed at low tide by each section is determined and plotted against the length of the estuary, as in Fig. 1.

The volume of the tidal prism is computed from the chart and from the tide tables using the mean range of tides. This volume is also plotted against the length of the estuary. The sum of these low tide and tidal prism volumes gives the high tide volume enclosed by each section, which is calculated and also plotted.

Determination of the segmentation of the estuary is illustrated in Table II for the average river flow that obtained during the survey conducted in December 1948 (33 x 10^6 ft.^{3}/tidal cycle). The distance at which the volume of tidal prism is equivalent to the river flow per tide (P_0) is read from the graph, and for the same distance the low tide (V_0) and high tide volumes ($P_0 + V_0$) are recorded. This defines Section 0 through which there is no net exchange on the flooding tide. The next section is located at the point where the enclosed low tide volume ($V_0 + V_1$) has increased by a volume equal to the high tide volume ($V_0 + P_0$) of the previous segment. The low tide, high tide and tidal prism volumes at this point, read from the various curves, are recorded. This process is repeated, placing each successive boundary where the increase in the low tide volume equals the high tide volume within the adjacent landward segment. The lengths of the segments enclosed by successive boundaries give the average excursion of a particle of water on the flooding tide.

TABLE II. Segmentation of Raritan River and Bay, Using a River Flow of 33 x 10^6 Ft.^{3}/Tidal Cycle (740 Ft.^{3}/Sec.)

	Segment	*Distance from River Mouth*	*Length of Segment*	*Cumulative Volumes* (10^6 *ft.*^{3})		
				Low Tide	*Tidal Prism*	*High Tide*
	(*n*)	*miles*	*miles*	$\Sigma_0^n V$	$\Sigma_0^n P$	$\Sigma_0^n (P + V)$
Raritan River	0	10.1	2.90	10	33	43
	I	7.8	2.30	53	75	128
	II	5.8	2.00	138	116	254
	III	4.1	1.70	264	180	444
	IV	2.2	1.90	454	305	759
	V	0.2	2.00	770	500	1,270
Raritan Bay	VI	1.1	1.30	1,280	800	2,080
	VII	2.2	1.10	2,090	1,190	3,280
	VIII	3.1	0.90	3,290	1,710	5,000
	IX	4.2	1.10	5,010	2,500	7,510
	X	5.5	1.30	7,520	3,500	11,020
	XI	7.1	1.60	11,030	4,960	15,990
	XII	8.3	1.20	16,000	6,600	22,600
	XIII	9.6	1.30	22,610	9,200	31,810

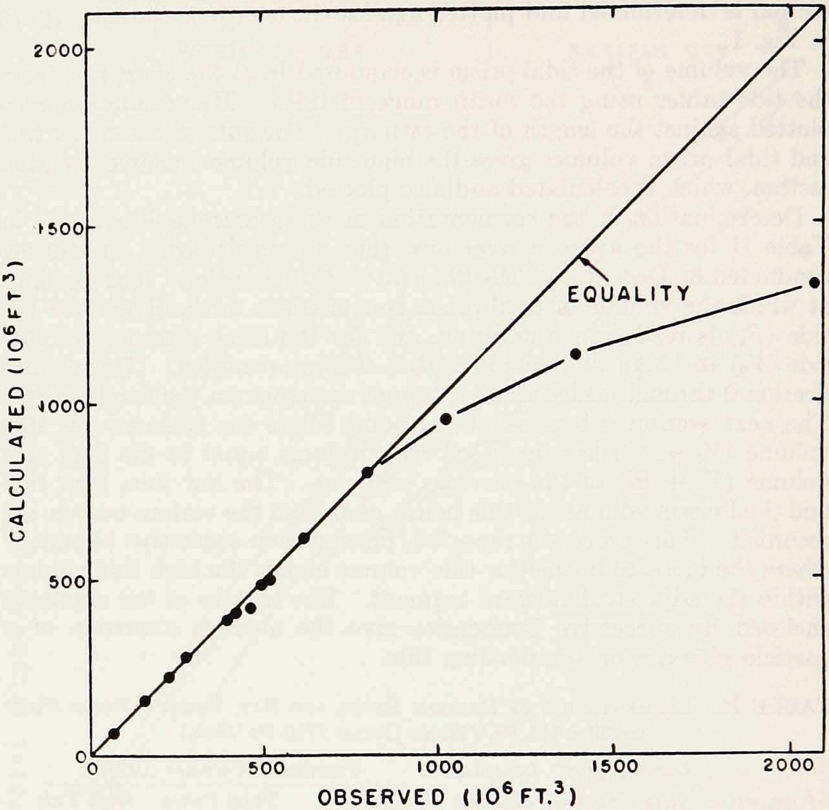

Figure 2. Comparison of the calculated accumulation of river water in Raritan River and Bay with the accumulation observed during a survey conducted in December 1948. The river flow was 33 x 10^6 ft.^{3}/tidal cycle (740 ft.^{3}/sec.).

The volumes within each segment, the tidal exchange ratios, and the calculated accumulation of river water are listed in Table III. The total volume of river water accumulated within the area has also been calculated from the salinity distribution observed during the hydrographic survey made in December 1948 (Ayers, Ketchum and Redfield, 1949). In Fig. 2 the accumulation of river water calculated by the theory described in this paper is compared to the accumulation observed during the survey. Excellent agreement between calculated and observed values was obtained within the eleven miles of the River and the first four miles of the Bay, where the total accumulated volume of river water was almost 800 million cubic feet. This volume of river water corresponds to 24 times the volume contributed by the river

TABLE III. CALCULATION OF THE ACCUMULATION OF RIVER WATER IN RARITAN RIVER AND BAY, USING A RIVER FLOW OF 33 x 10^6 FT.^{3}/TIDAL CYCLE (740 FT.^{3}/SEC.)

Segment	*Length of Segment*	*Local Volumes*		*Exchange Ratio*	*Accumulation*	
		Intertidal	*High Tide*			
		P_n	$P_n + V_n$		Q_n	$\Sigma_0^n Q$
(n)	*miles*	10^6 *ft.*^{3}	10^6 *ft.*^{3}	r	10^6 *ft.*^{3}	10^6 *ft.*^{3}
0	2.90	33	43	0.767	43	43
I	2.30	42	85	0.494	67	110
II	2.00	41	126	0.325	102	212
III	1.70	64	190	0.337	98	310
IV	1.90	125	315	0.397	83	393
V	2.00	195	510	0.382	86	479
VI	1.30	300	810	0.370	89	568
VII	1.10	390	1200	0.325	102	670
VIII	0.90	520	1720	0.302	109	779
IX	1.10	790	2510	0.315	105	884
X	1.30	1000	3510	0.285	116	1000
XI	1.60	1460	4970	0.294	112	1112
XII	1.20	1640	6610	0.248	133	1245
XIII	1.30	2600	9210	0.282	117	1362

during a tidal cycle, i. e., to about the first 12 days of transport. Beyond this, the accumulation of river water calculated from the theory is less than the observed volumes of river water.

From the calculated accumulation of river water in each volume segment of the estuary, the percentage of fresh water and the salinity this would produce have been calculated. In Fig. 3 these are compared with the observed distribution of salinity and fresh water.[3] It will be seen that the difference between the calculated values and the observed values of accumulated river water in the outer part of the Bay correspond to small salinity differences. Since the plot in Fig. 2 is a cumulative one, and since the volumes in the Bay are great, this small difference in the salinity value results in a comparatively large discrepancy in the calculated total volumes. It is possible that the discrepancy is the result of incomplete horizontal mixing, the effects of which will be discussed below. The seepage of ground water into the Bay may contribute to the discrepancy.

It has been stated above that the subdivision of the estuary defines

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[3] The source "sea water" which mixes with the Raritan River water is actually a mixture of Hudson River water and sea water which has a salinity of 27°/oo (cf. Ayers, Ketchum and Redfield, 1949). This is the reason the salinity approaches 27°/oo rather than a higher salinity as the percentage of Raritan River water approaches zero.

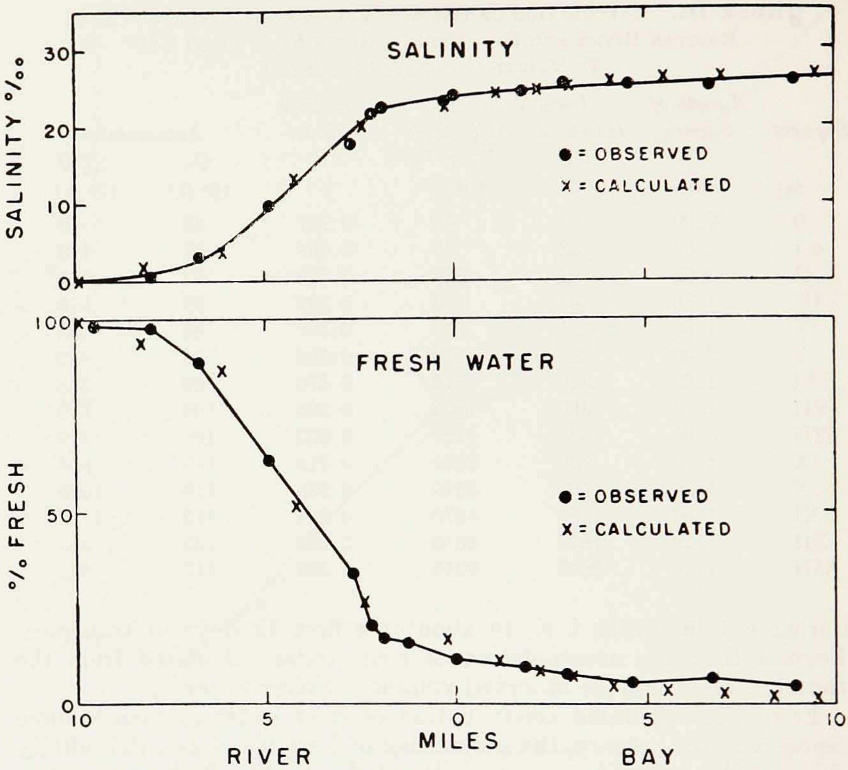

Figure 3. The observed and calculated distributions of salinity and fresh water in Raritan River and Bay in December 1948.

the average excursion of a particle of water on the flooding tide. A comparison of the distances between the boundaries in Table II with tidal currents measured by the Coast and Geodetic Survey (Marmer, 1935) is informative. Within the segments numbered X and XI, for example, the observed tidal currents at four stations at the center of the Bay produced transports which averaged 2.35 miles on the flooding tide.[4] The average velocity across the section would be approximately 0.7 of the velocity measured near the axis of the current. The average transport at this point would thus be about 1.65 miles per tide. The length of Segment X is 1.3 miles, of XI 1.6 miles. Measured currents at other locations within the Bay also gave trans-

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[4] These transports are computed on the assumption that the tidal current curve approximates a cosin curve and that the average current velocity is $2/\pi$ multiplied by the maximum velocity.

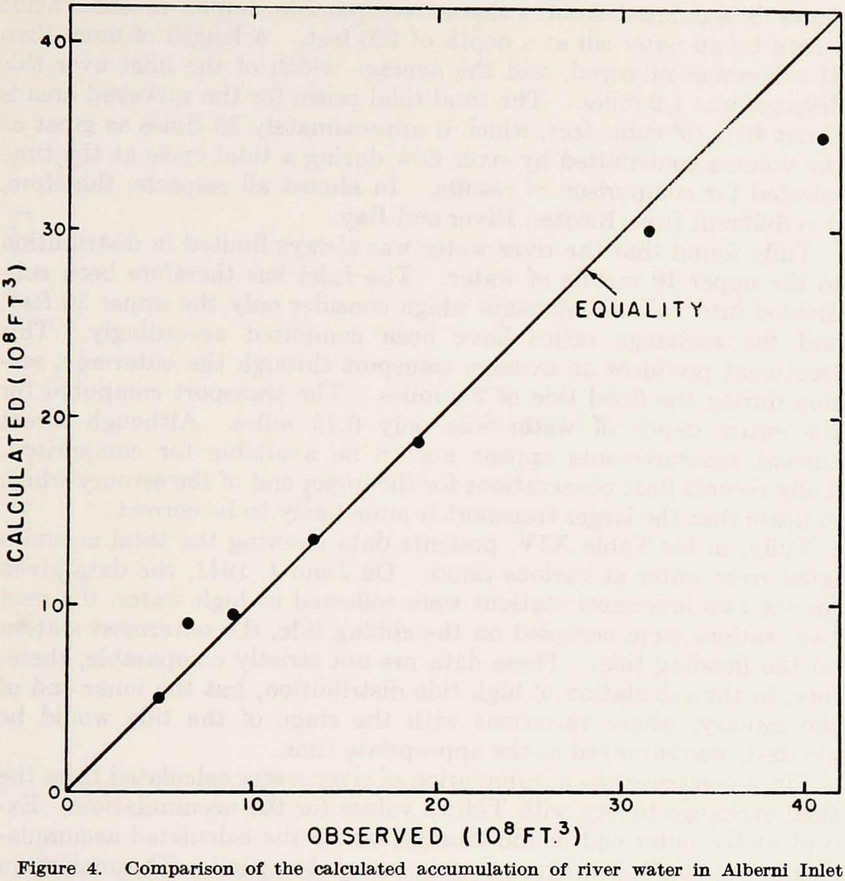

Figure 4. **Comparison of the calculated accumulation of river water in Alberni Inlet with the accumulation observed by Tully (1949) on June 4, 1941 when the river flow was 1.2 x 10^8 ft.^{3}/tidal cycle (2,740 ft.^{3}/sec.).**

ports which are approximately the same as the distances between boundaries calculated from the tidal displacement.

It may be concluded that the calculated distribution of fresh and salt water in Raritan River and Bay closely approximates the observed distribution for a length of about 15 miles of the estuary. The volume of river water within this part of the estuary corresponds to the volume introduced by the river during 24 tidal cycles.

Alberni Inlet, Vancouver Island, B. C. Tully (1949) has made extensive studies of the distribution of salinity and river water in Alberni Inlet. The Inlet is a typical fjord, having precipitous rocky sides with depths within the inlet as great as 1000 feet. This deep water is separated from water at comparable depths in the Pacific Ocean by an outer sill at a depth of 120 feet. A length of more than 21 miles was surveyed, and the average width of the inlet over this distance was 1.2 miles. The total tidal prism for the surveyed area is about 40 x 10^8 cubic feet, which is approximately 30 times as great as the volume contributed by river flow during a tidal cycle at the time selected for comparison of results. In almost all respects, therefore, it is different from Raritan River and Bay.

Tully found that the river water was always limited in distribution to the upper 10 meters of water. The Inlet has therefore been subdivided into volume segments which consider only the upper 30 feet, and the exchange ratios have been computed accordingly. This treatment produces an average transport through the outermost section during the flood tide of 2.5 miles. The transport computed for the entire depth of water was only 0.15 miles. Although direct current measurements appear not to be available for comparison, Tully records float observations for the upper end of the estuary which indicate that the larger transport is more likely to be correct.

Tully, in his Table XIV, presents data showing the total accumulated river water at various times. On June 4, 1941, the data given for the two innermost stations were collected at high water, the next two stations were occupied on the ebbing tide, the outermost station on the flooding tide. These data are not strictly comparable, therefore, to the calculation of high tide distribution, but the inner end of the estuary, where variations with the stage of the tide would be greatest, was surveyed at the appropriate time.

Fig. 4 compares the accumulation of river water calculated from the tidal exchange theory with Tully's values for the accumulation. Except at the outer end of the estuary, where the calculated accumulation is too small, the agreement is reasonably good. The maximum accumulation corresponds to nearly 34 times the contribution from the river during a tidal cycle.

Great Pond, Falmouth, Mass. Great Pond is a narrow, sandy embayment on the south shore of Cape Cod which receives the outflow of the Coonamessett River. The Pond is about a mile and a half in length, averages less than a quarter of a mile in width, and has a maximum depth of 9 feet at high tide. The mean range of tide is about 1.5 feet. The tidal prism volume for the surveyed part of the pond is 7.6 x 10^6 cubic feet, which is about 15 times as great as the volume introduced by the river during a tidal cycle at the time of the survey.

A survey of this pond was made in June 1950 when the river flow was half a million cubic feet per tide (10.7 $\text{ft.}^3/\text{sec.}$). The

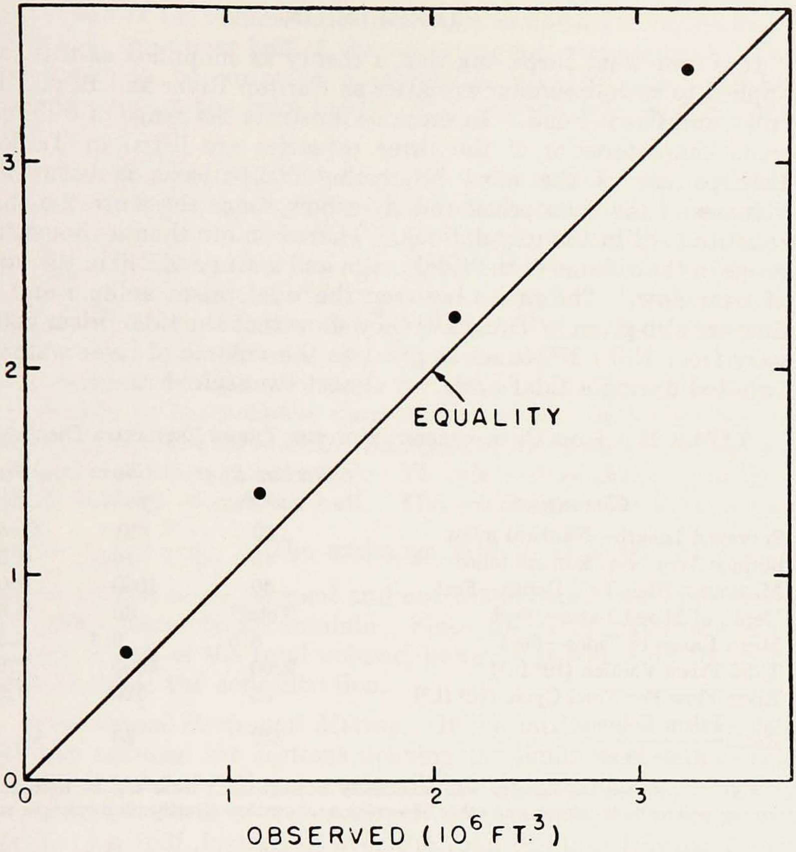

Figure 5. Comparison of the calculated accumulation of river water in Great Pond with the accumulation observed during a survey conducted on June 26, 1950 when the river flow was 0.5 x 10^6 ft.^{3}/tidal cycle (10.7 ft.^{3}/sec.).

salinity indicated that the river water was mainly restricted in depth to the upper half of the pond at high water. Consequently, in this pond, as in Alberni Inlet, the calculation must be made to the depth of the mixed layer in order to correct for incomplete vertical mixing.

The calculated volume of river water accumulated in the pond is compared to the volume deduced from the salinity distribution in Fig. 5. The agreement is good for the first mile of the estuary, which was as far as the calculation could be carried, since the next cross section would have been located beyond the limits covered by the survey. The total volume of accumulated river water was seven times as great as the volume contributed by the river during a tidal cycle.

DISCUSSION

It is somewhat surprising that a theory as simplified as this can be applied to such dissimilar estuaries as Raritan River and Bay, Alberni Inlet, and Great Pond. In order to illustrate the range of differences, some characteristics of the three estuaries are listed in Table IV. Perhaps one of the most interesting comparisons is between the volumes of the tidal prism and river flow, since these are the characteristics used in the calculations. There is more than a thousandfold range in the volume of the tidal prism and a range of 240 in the volume of river flow. The ratios between the tidal prism volume and river flow are also given in Table IV; they show that the tidal prism volumes vary from 15 to 279 times as great as the volume of river water contributed during a tidal cycle, an almost twentyfold range.

TABLE IV. Some Characteristics of the Three Estuaries Discussed

Characteristic	*Raritan River and Bay*	*Alberni Inlet*	*Great Pond*
Surveyed Length—Nautical miles	20	21	1.5
Surface Area—Sq. nautical miles	45	16	0.375
Maximum High Tide Depth—Feet	40	1000	9.0
Depth of Mixed Layer—Feet	Total*	30	3.5 − 5
Mean Range of Tides—Feet	5.5	6.4	1.5
Tidal Prism Volume (10^6 ft.^{3})	9200	4000	7.6
River Flow Per Tidal Cycle (10^6 ft.^{3})	33	120	0.5
Ratio $\frac{\text{Prism Volume}}{\text{River Flow}}$	279	33	15

* At each station the salinity was practically homogeneous from top to bottom. This survey was made in winter, and other observations show that stratification exists in summer.

The theory is based upon an assumption of complete mixing of water within various defined volume segments of the estuary at high tide. It will be useful to discuss, in a qualitative way, how incomplete mixing will influence the results of the calculation.

1. *Vertical Mixing.* The effects of incomplete vertical mixing have been discussed above but will be reviewed here. If the distribution of river water is restricted vertically to a surface layer, the actual tidal exchange will be greater than the exchange ratio calculated for the entire depth. A simple example will make this clear. If the mixed layer is half of the total depth at high tide, then the actual exchange will be $\frac{P}{0.5\,(P+V)}$. The exchange ratio (r) for half the depth would thus be twice as great as one calculated for the entire depth. The actual volume of river water accumulated within each segment (Q)

would be half of the volume which would be computed if the limitation of mixing to the upper half of the water column were ignored. This discrepancy can be corrected if salinity observations are available, and the correction has been used in both Alberni Inlet and in Great Pond.

2. *Transverse Horizontal Mixing.* It is frequently observed that the salinity contours in an estuary will indicate that, facing upstream, the salt water penetrates further on the right whereas the less saline water extends further seaward on the left of the estuary.[5] This is the result of the rotation of the earth. In such a distribution the mixing is obviously not complete in a transverse section across the estuary. A simple example will show the effect of this. If one postulates a distribution so that the estuary is divided into half longitudinally by the isohaline characteristic of pure sea water, one can assume that there is no seaward transport of river water through the right-hand half (facing upstream). Therefore all of the river water is escaping through the other half. The exchange ratio in this case would be $\frac{0.5\,P}{0.5\,(P+V)}$. The exchange ratio is thus the same as it would be for the entire segment and one would expect the same quantity of river water to accumulate. Since the river water would be contained in half of the total volume, however, it would obviously be present at twice the concentration.

3. *Longitudinal Horizontal Mixing.* If the mixing is not complete lengthwise between the sections defining the limits of a volume segment, the calculated exchange ratio will be too large and consequently the accumulation will be too small. If, for example, the mixing were complete over half, instead of over the total distance traversed by a particle of water during a flood tide, one should apply the same exchange ratio to two adjacent segments instead of to the single segment in order to compute the actual accumulation. The volume segment would thus contain double the accumulation calculated by the unmodified theory. It is possibly this error which results in the low calculated values at the outer ends of both Raritan Bay and Alberni Inlet. Unfortunately, there is no independent criterion which can be used to correct the exchange ratio for this condition. It can be recognized only by comparison of calculated with observed distributions.

The subdivision of the estuary is an integral and essential part of the theory. Sanitary engineers have considered the entire tidal prism

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[5] This description applies to the northern hemisphere and would be reversed in the southern hemisphere.

at any cross section as though it were available for the dilution of introduced pollution. This inherently assumes that the mixing is complete longitudinally throughout the length of the estuary. Such a use of the tidal prism is an extreme example of incomplete longitudinal mixing. The above discussion shows that such an assumption will give too small an accumulation of river water, since a single exchange ratio is applied to the estuary as a unit. This leads to an exaggerated estimate of the rate of flushing. The application of the theory described in this paper to sanitary engineering problems is discussed by Ketchum (1951).

Two criteria are available to determine whether the tidal exchange theory will give valid estimates of the distribution of fresh and salt water within the estuary. Where current data are available, comparison of the average excursion per tide with measured tidal currents provides a valuable check on the definition of the volume segments. It should be remembered, however, that sufficient current data are rarely available to obtain an accurate average flow across the complete cross section. Approximations must be used, such as those described by Keulegan and Hall (1950), as was done in this paper for Raritan Bay. Absolute agreement cannot be expected.

The salinity data can be used to determine whether mixing is vertically complete or not, and if it is not complete the correction for vertical mixing can be applied. The comparison of average salinity at any cross section in the estuary with a plot of the calculated longitudinal distribution of salinity, such as that shown in Fig. 3, gives a check on the horizontal mixing anomaly for the part of the estuary where salinity data are available.

The value of the theory lies in the fact that it describes the exchanges throughout the estuary which are necessary to maintain the steady state distribution. This implies that the physical processes controlling the distribution are incorporated in the theory, though the individual effect of each process cannot yet be evaluated. The major practical advantage of the theory is that it permits calculation of the distribution of fresh water and salinity to be expected in any given estuary with a variety of river flows. The calculations can be made without undertaking field work, since existing data found in tide tables, water supply journals, and navigation charts are used. The validity of the calculated distribution can be checked by comparison with salinities at high tide at various points within the estuary.

The theory has been developed with river water as an indicator substance. It should be pointed out that the same exchange ratio will apply to anything suspended or dissolved in the water. Thus, for example, if one were interested in the distribution of plankton, fish eggs, or pollution, the exchange ratio multiplied by the observed concentration would give a measure of the movements of the material to be expected during a tidal cycle. If, however, there is not a continuous and constant rate of supply of the material, the steady state conditions would not apply. If the rate of supply is known, however, the exchanges can be calculated and the distribution deduced as the tidal exchange distributes the material throughout the estuary.

SUMMARY

1. An empirical theory is presented which describes the exchanges between various parts of an estuary as a result of tidal oscillations and which permits the calculation of the average distribution of fresh and salt water within the estuary.

2. The characteristics of the estuary used in the calculation are the mean range of tides, the river flow, and the topography, all readily available for most estuaries.

3. The fundamental assumptions are: a) That a steady state distribution of fresh and salt water will exist throughout the estuary if the river flow is constant; b) that complete mixing will occur within volume segments defined horizontally by the width of the estuary and by the average excursion of a particle of water on the flood tide and vertically by the depth of the mixed layer. Discrepancies produced by incomplete mixing are discussed.

4. The calculated distributions of river water are compared with observed distributions for three very different estuaries, Raritan River and Bay, Alberni Inlet, and Great Pond. The tidal prism for these three estuaries includes more than a thousandfold range of volume, and the river flow varies by a factor of 240. The theory produces results which correspond closely with observed distributions of salinity and fresh water in all three estuaries.

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