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SLICKS, SURFACE FILMS AND INTERNAL WAVES¹

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ABSTRACT

Slicks, or calm streaks on a rippled sea, are often seen on coastal waters and lakes when the wind is light. They are formed by the ripple-damping action of a surface film of organic matter which occurs naturally on biologically productive waters. The ability of films to damp ripples varies radically with the degree of compaction of the film molecules. Such compaction may be caused by horizontally convergent flow in the water surface or by horizontal convergence of the wind stress. The characteristics of the resultant slicks vary with the agency producing compaction. One such agency, typical of summer conditions on the California Coast, is a train of long internal waves in a shallow thermocline. The periodic convergence and divergence of the horizontal component of orbital velocity associated with such waves produce alternate bands of compaction and extension in the surface film sufficient to cause visible differences in the rippling of the water by a slight breeze. The slicks are arrayed in parallel deployment over successive troughs of the internal wave and are spaced some hundreds of meters apart. Where the wave is progressive, the slicks also progress at the speed of the wave which is of the order of 25 cm/sec. The slicks are propagated as a wave of state traveling in the surface film which itself does not share the phase motion of the wave but only its local oscillations. This explanation is supported by theory, by model studies and by observations in the field. Internal wave slicks are not to be confused with the more closely spaced wind-cell slicks which predominate when the wind force is Beaufort 3 or greater.

¹ Contribution from the Scripps Institution of Oceanography, New Series No. 497.

INTRODUCTION

Description. Slicks, or calm streaks on rippled water, like those shown in Fig. 1, are commonly seen on harbor, coastal and lake waters when the wind is a slight breeze. They have been reported near California, Samoa, Australia, the Marquesas Islands, the Antarctic icepack, and occasionally in the open sea (Dietz and LaFond, 1950), in the Gulf of Panama (Forbes, 1945; Woodcock and Wyman, 1947), off the south coast of Borneo (J. D. F. Hardenberg, private communication), and in various Russian seas (Shuleikin, 1935, 1941). W. F. Royce of the U. S. Fish and Wildlife Service has photographed them over the Gulf of Maine, and B. N. Jamison on Lake Tahoe, California. At customary angles of observation, slicks appear whiter and brighter than adjacent zones of blue rippled water. From the air, looking nearly straight down into the sun's glitter on the sea, they appear as dark streaks. Viewed close at hand, they are sometimes hard to see but can be distinguished by the smooth oily appearance of wavelets and the absence of capillary crispations (Lamb, 1932; Art. 350). In the San Diego region they are observed at all seasons of the year, but in the summer months they are often strikingly distributed in parallel array, like bands of alto-cumulus "billow" clouds in the sky; the calm streaks, 10 to 50 m broad and arranged in congruent sinuous bands, are sometimes many kilometers long and are separated from one another by rippled zones about 300 m wide. In shallow water, such bands lie roughly parallel to contours of the bottom, but in deeper water they usually lie across the wind direction at various angles. They are visible only when the wind is sufficiently strong to ripple the water. On the other hand, at wind speeds greater than about 3.4 m/sec (Beaufort 3, the force required to sway small twigs and to straighten out flags), the slicks break up into a different pattern (Fig. 2) with narrower bands spaced much closer together and much shorter, their long axes always oriented nearly along the wind axis in a manner described by Langmuir (1938) and by Woodcock and Wyman (1947), who attribute the effect to wind cells. This paper is concerned only with slicks observed at wind speeds less than 3.4 m/sec, as shown in Fig. 1.

Analysis of the geometry of light reflection from the ocean, by the author and by Shuleikin (1935, 1941), has shown that the optical properties of calm streaks are due to the reduction of the average angle of reflection (attributed to flattening of the wavelets) and to the complete absence of capillary crispations as observed by Langmuir (1938).

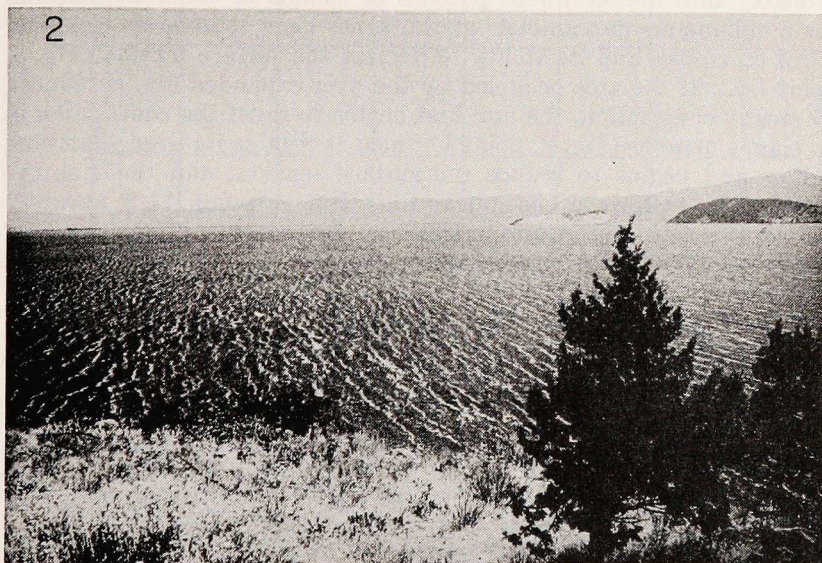
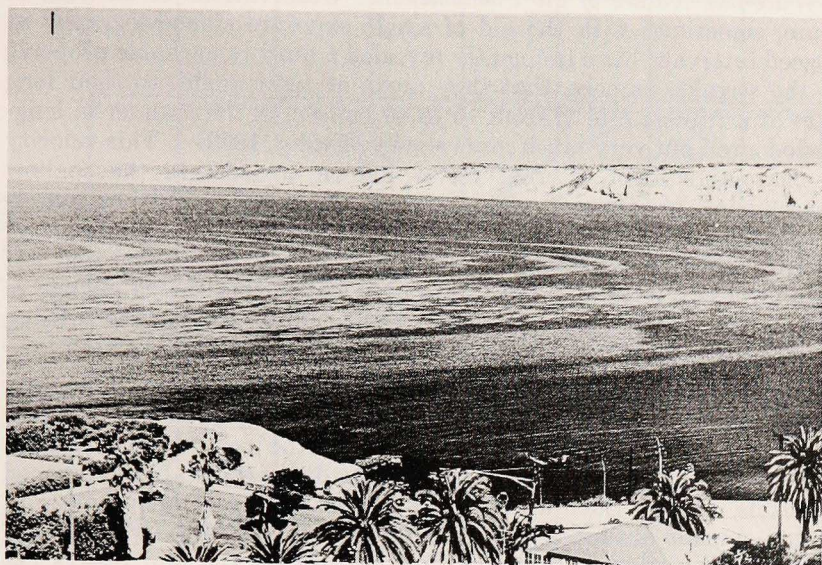


Figure 1. A series of internal wave slicks off La Jolla Cove, October 1950.

Figure 2. Wind-cell slicks, typical of wind speeds greater than 3.4 m/sec. Photo of Klamath Lake, Oregon, in July 1949 by R. Revelle.

Wave-like Nature of the Phenomenon. Observations of an hour or more, sometimes with the aid of single exposure cinematography at spaced intervals, have frequently revealed a most remarkable property of the streaks, namely, that they move at right angles to their long axes at a regular rate of from 10 to 40 cm/sec, in the manner of long-period swell but very much more slowly (Ewing, 1950). This velocity is characteristic of internal waves known to exist in the shallow thermocline of summer. The wave-like nature of this motion is revealed by the fact that dye or paper placed ahead of an advancing slick does not move along with it but is overtaken, passed and left behind nearly in its original location, thus showing that a change in the state of the sea surface which is sufficient to calm ripples is being propagated without dependence on steady flow of the water itself.

The Surface Film. The physical medium in which a change of state occurs will be shown to be a contaminating surface film of organic composition, such as is known to inhibit the formation of ripples (Aitken, 1884; Gibson, G. E., 1944) and to damp those already formed (Pockels, 1891). Such a film, since it reduces the surface tension of water, spreads until it is one molecule thick (about 6×10^{-7} cm). If spread still further by some extraneous agency, the film breaks up, probably into discontinuous patches or islands of coherent material some millimeters in diameter; at the same time it loses its damping effect on ripples and its ability to depress the surface tension (Fig. 3, point D). If the area occupied by the over-expanded film is reduced by steady contraction, the film first begins to resist the contraction in an elastic manner (Fig. 3, point C); next it regains its ripple-damping ability and begins to reduce the surface tension; and then, as the islands fuse to form a continuous film (Fig. 3, point A) it abruptly becomes nearly as incompressible as the same material in bulk (Devaux, 1914), since its molecules are then standing close-packed like sticks of chalk in a box (Langmuir, 1917).²

The Effect of Surface and Internal Waves. The discrete islands of an over-expanded film freely share the motion of water particles in the surface, and in consequence they may be crowded together to form a continuous ripple-damping film by horizontal convergence. This film may be broken up in turn if expanded by divergence. Such rhythmic convergence and divergence occur whenever a water surface is being alternately raised and lowered by a train of surface waves, and it has been shown by Hardman (1941) that, on lake water somewhat contaminated by naturally occurring organic matter, the surface tension alternately falls and rises during the passage of crests and

² For a general review of the properties of surface films, see Adam, 1941.

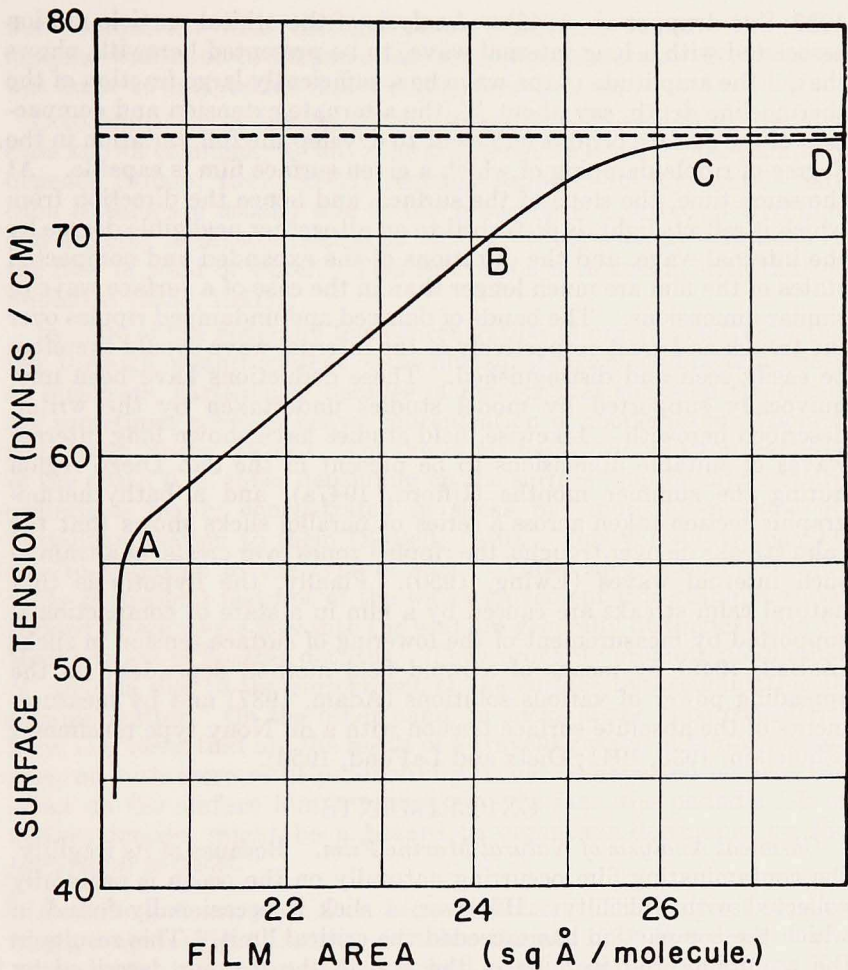


Figure 3. Variation of surface tension with degree of extension for typical film of fatty acid on water. The dashed line shows the surface tension of pure water. Redrawn from Adam, 1941.

troughs of surface gravity waves. This variation in surface tension must be accompanied by changes in the rippling, but the rapid changes of slope of the surface and the brevity of the cycle are unfavorable for the detection of such changes by visual means. On the other hand it is known that, over a long internal wave in a shallow thermocline, horizontal convergence and divergence occur throughout the upper layer over the descending and rising phases of the wave respectively (Lamb,

1932; Sverdrup, et al., 1946). Analysis of the orbital particle motion associated with a long internal wave, to be presented herewith, shows that, if the amplitude of the wave be a sufficiently large fraction of the thermocline depth, say about $\frac{1}{4}$, the alternate extension and compaction of the surface is quite sufficient to develop the full variation in the degree of ripple-damping of which a given surface film is capable. At the same time, the slope of the surface, and hence the direction from which it reflects light, is disturbed to an altogether negligible degree by the internal wave, and the durations of the expanded and compacted states of the film are much longer than in the case of a surface wave of similar dimensions. The bands of damped and undamped ripples over the trough and crest respectively of the internal wave should therefore be easily seen and distinguished. These deductions have been unequivocally supported by model studies undertaken by the writer, described herewith. Likewise, field studies have shown long internal waves of suitable dimensions to be present in the San Diego region during the summer months (Ufford, 1947a), and a bathythermographic section taken across a series of parallel slicks shows that the calm streaks lie over troughs, the rippled zones over crests, of a train of such internal waves (Ewing, 1950). Finally, the hypothesis that natural calm streaks are caused by a film in a state of compaction is supported by measurement of the lowering of surface tension in slicks. (ZoBell, 1946) by means of a rapid field method dependent on the spreading power of various solutions (Adam, 1937) and by measurements of the absolute surface tension with a du Nouy type tensimeter (Shuleikin, 1935, 1941; Dietz and LaFond, 1950).

EXPERIMENTS

Chemical Analysis of Natural Marine Film. Because of its fragility, the contaminating film occurring naturally on the ocean is ordinarily collected with difficulty. However, a slick is occasionally found in which the compaction has exceeded the critical limit. This results in the crumpling and foaming of the film in the manner described by Devaux (1914) so that a narrow band of heavy scum which has lost its elasticity is formed. Samples of such scum were collected from the Scripps Institution Pier, which projects into the open sea far from any harbor. The scum was skimmed off the water into a bucket and transferred to a separatory funnel. Even the gentlest handling destabilizes some of the suspended matter, this then permitting the escape of air bubbles and throwing down the heavier solids. The solid matter was filtered out, drawn off, dried, and weighed. E. F. Corcoran of the Biochemistry Division, Scripps Institution of Oceanography, assayed the filtered and dried solids for total carbon, using

wet combustion with chromic acid (Waksman, 1936). The solid material was found to contain about 27% organic matter. No attempt was made to analyze the liquid and gaseous phases.

Contamination of the sea surface by surface-active substances of a class known commonly to have the properties on which ripple-damping depends (Adam, 1941: 50-51) strongly supports the hypothesis that calm streaks are usually due to the presence of a surface-active film and are only indirectly related to other agencies, such as variations in the flow of wind or water. This conclusion is in accord with that of Shuleikin (1935, 1941).

In view of the generally low concentration of organic matter in the sea, the high organic content of the film substance should be of interest to biologists, particularly because of reports that slicks on the ocean are sometimes zones of heavy concentration of zooplankton (Thompson, 1862; Beebe, 1926a). On the other hand, the neuston bacteria of the ocean (i. e. those depending on the surface film for support) are sometimes highly concentrated in slicks but more often show no particular relation to them (ZoBell, 1946). This variability may be due to variation in the type of slick. Those slicks associated with fairly permanent features of the water movement (for example, those formed where rivers flow into the sea) are generally distinguishable by their content of floating detritus. They probably promote production of bacteria due to the enrichment of the surface by accumulation of organic matter. On the other hand, in slicks of the type considered here, i. e. those that are the result of a traveling wave of film-compression, no such increase of productivity would be expected because the effect on the surface film is transitory. In fact, the periodic fall of surface tension might be a hazard to organisms depending on the surface for support.

Horizontal Convergence Over Internal Waves. The simplest type of internal wave (Fig. 4) is that of Stokes (Lamb, 1932; Sverdrup, et al., 1946), where it is assumed that a fluid layer of uniform density ρ' lies over a heavier layer of uniform density ρ , their respective thicknesses being h' and h . Ufford (1947b) has shown that this simple wave is a satisfactory approximation to summer conditions in the San Diego region. As with surface waves, the primary features of internal waves depend strongly on the relation of wave length to water depth, in this case to the dimensions h' and h . We shall concern ourselves only with waves long in relation to h' , since internal waves of other dimensions have little effect at the upper surface. Compared to surface waves, the potential energy associated with a given deformation of the interface is much reduced due to the presence of the overlying layer,

while conversely the inertia of the system as a whole is increased. Consequently, compared to a surface wave of equal length, the velocity of propagation of an internal wave is reduced by a factor (depending on the relative densities of the two layers) which in the ocean is usually of the order of $10^{-3/2}$, i. e. about 1/30.

The streamlines and trajectories of the motion of individual water particles are shown in Fig. 4, both for a standing wave such as can be produced conveniently in a laboratory model and for a progressive wave such as is usually found in nature. The orbital particle motion

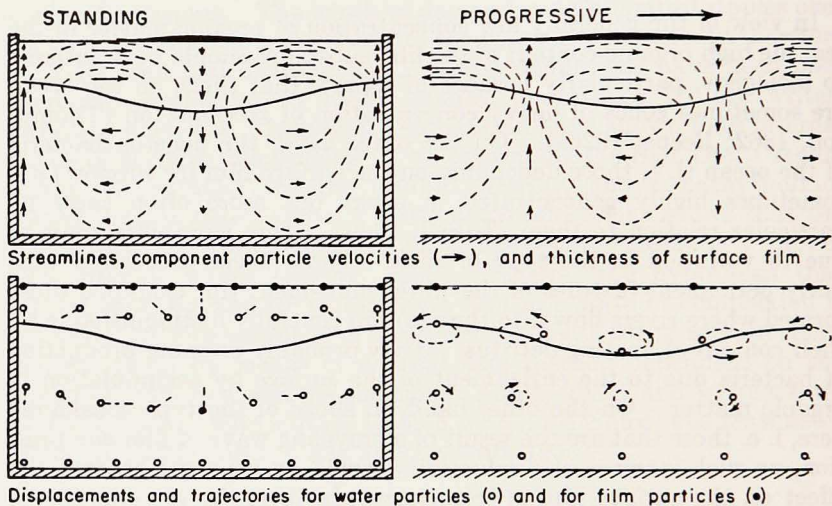


Figure 4. Long internal waves. (The extreme values of the flow and of the deformation of the interface shown together in the upper left diagram do not occur simultaneously.)

is in opposite sense in the two layers. In the upper layer the horizontal component of particle velocity, which is nearly uniform from the interface to the surface, is divergent over the rising phase of the wave and convergent over the descending phase. It can be seen by examining the distribution of surface particles in their orbits that in both types of wave the particles are most closely crowded over the trough and most dispersed over the crest of the wave. Were this superficial layer a mono-molecular film of hydrophilic substance, the condition over the trough would be one of compaction, in which ripples would be most speedily damped. If, on the other hand, the superficial film were of appreciable thickness, as in the case of a heavy coating of oil, it would be considerably thickened in this same region. As shown in Fig. 4, there is a slight vertical disturbance at the surface

which is exactly out of phase with the displacement at the interface, this displacement being required to satisfy the condition of uniform pressure along the bottom. However, the amplitude of the surface wave, which has been exaggerated in Fig. 4, is negligible in the model and in nature.

In so far as the effect on a surface film is concerned, the chief difference between standing and progressive waves is that in a standing wave the horizontal motion vanishes over the crest and trough whereas in the progressive wave it is at a maximum at these positions. Over a standing wave in a tank, a band of compaction forms alternately at the center and at the two ends of the tank. At the center, the band alternately forms by accretion from both directions during the descending phase of the wave, and then it is dispersed by scattering along each edge of the band during the rising phase. On the other hand, in the progressive wave, the zone of compaction is propagated with the same velocity as the internal wave, the zone being continuously advanced by accretion along its leading edge and scattering along its trailing edge. Over the trough, the particles are moving in the direction of wave propagation with a horizontal component of velocity

$$u' = \frac{a}{h'} c, \quad (1)$$

where c is the velocity of wave propagation, a the amplitude at the interface. A particle halfway between the crest and oncoming trough is at rest and is approached by the particles in motion over the trough. It is also accelerated so that by the time it is overtaken by the center of the zone of compaction it will have its maximum horizontal velocity, u' , which is, however, less than c . As it is passed by the wave, it loses velocity and drops out the rear of the zone of compaction, coming to rest at a point somewhat ahead of its initial one. During the remaining half-cycle over the next wave crest it will be returned to its original position by reversal of the previous motion.

Neglecting second order terms (Lamb, 1932: Art. 228), the maximum horizontal displacement of the particles over either progressive or standing waves is given by a/kh' , where k is the wave number in radians per unit of length. The amount by which the displacements from the mean positions of two neighboring particles differ is equivalent to the degree of relative extension of the surface between them, which, to the same approximation, has a/h' for its maximum value. Since the state of the surface film, and consequently its ability to damp ripples, depends on its relative compaction, a model will be similar to its prototype in this respect if the geometric similarity a/h' is preserved.

TABLE 1. KINEMATICS OF LONG INTERNAL WAVES AND EFFECT ON RIPPLE DAMPING

PARAMETER	Symbol	Unit	STANDING WAVE			PROGRESSIVE WAVE		
			FORMULA	Maximum Values in Model		FORMULA	Maximum Values Typical Ocean Wave	
				Calculated	Measured		Calculated	Measured
Potential								
Upper Layer	ϕ'		$(A \cosh ky + B \sinh ky) \cos kx e^{i\sigma t}$ $C \cosh k(y-h) \cos kx e^{i\sigma t}$			$(A \cosh ky + B \sinh ky) e^{i(kx - \sigma t)}$ $C \cosh k(y-h) e^{i(kx - \sigma t)}$		
Lower Layer	ϕ							
Density								
Upper Layer	ρ'	g/cm ³			0.998			1.0255
Lower Layer	ρ	g/cm ³			1.029			1.0244
Thickness								
Upper Layer	h'	cm			8			735
Lower Layer	h	cm			24			1000
Wavelength	Λ	cm			75			36,500
Wave Number	k		$2\pi/\Lambda$.084		$2\pi/\Lambda$.00017	
Period		sec	$2\pi/\sigma$	6.8	7.1	Λ/c	1750	1900
Frequency	σ	rad/s	(see Lamb 1932: art 231)	.941	.885	ck	.0036	.0033
Propagation Velocity	c	cm/s	σ/k	11.2	10.5	$\{g \cdot hh' / (h+h') \cdot (\rho - \rho') / \rho\}^{1/2}$	21	19
Displacement of Interface	η	cm	$a \cos kx \cos \sigma t$		2	$a \cos (kx - \sigma t)$		200
Horizontal Component of Particle Velocity	u'	cm/s	$-(a/h')c \sin kx \sin \sigma t$	2.7	4.0	$-(a/h')c \cos (kx - \sigma t)$	5.8	
Horizontal Displacement of Surface Particle	ξ	cm	$(a/kh') \sin kx \cos \sigma t$	3.0	3.1	$(a/kh') \sin (kx - \sigma t)$	1600	
Compaction of Surface Film			$-(a/h') \cos kx \cos \sigma t$.25	(see Fig. 8)	$-(a/h') \cos (kx - \sigma t)$.28	
Ripple Wavelength	λ	cm			.83	$2\pi (T/\rho g \cdot \Lambda/\lambda)^{1/2}$.3	
Modulus of Decay								
Film Compact	τ'	sec	$.322 \lambda^{1.76}$ (see Fig. 5)	.23	.23	$.322 \lambda^{1.76}$ (see Fig. 5)	.040	
Film Extended	τ	sec	$.712 \lambda^2$ (see Fig. 5)	.40		$.712 \lambda^2$ (see Fig. 5)	.064	
Ratio of Moduli of Decay	τ'/τ		$.45\lambda^{-1/4}$ (See Fig. 5)	.47		$.45 \lambda^{-1/4}$ (see Fig. 5)	.61	

The various parameters discussed above are summarized in Table I for both a standing and a progressive wave. The numerical results have to do with specific examples of a standing wave in a model and a progressive wave in the ocean which will be described presently. The analysis of Table I follows Lamb (1932: Art. 231) and Sverdrup et al. (1946: 585-589).

Ripple Damping. The precise relation between film compaction and ripple damping cannot be calculated, because there is available neither a complete theory relating damping to the state of the film nor direct empiric knowledge of changes of state with extension for a naturally occurring film.

For waves whose energy is dissipated by viscosity in the absence of interference from an elastic surface film, Stokes calculated a modulus of decay $\tau = \frac{1}{2}\nu k^2$, where ν is the kinematic viscosity and τ is defined by the relation

$$a = a_0 e^{-t/\tau}, \quad (2)$$

where a_0 is the initial amplitude and a is the amplitude at any later time t . For water, and in terms of wavelength, the numerical value of τ is given by

$$\tau = 0.712\lambda^2 \text{ seconds} \quad (3)$$

if λ , the wavelength, is expressed in centimeters (Lamb, 1932: Art 348).

Where the surface, instead of being quite free, is elastic because of contamination by a film in a critical state of extension, wave energy must be dissipated at an increased rate by the alternating tangential drag on the water as its surface is contracted and expanded by the crests and troughs of the waves (Reynolds, 1880). Lamb (1932: Art 351) attempted to "submit this explanation, to a certain extent, to the test of calculation." For wavelengths between a fraction of a millimeter and several meters, the modulus of decay, τ' , is $(8/\nu k^2 \sigma)^{1/2}$, where σ is the frequency in radians per second. The numerical value of τ' in seconds may be computed from the wavelength in centimeters by the following formulae:

$$\text{for capillary ripples } \tau' = 0.322 \lambda^{1.75} \text{ seconds}; \quad (4a)$$

$$\text{for gravity waves } \tau' = 0.375 \lambda^{1.25} \text{ seconds}. \quad (4b)$$

For longer wavelengths viscous damping is negligible and for extremely minute ripples the calculation fails because the assumptions on which it is based are violated. Curves of τ and of τ' , together with their ratio τ'/τ , are given in Fig. 5.

Reynolds' explanation (Lamb, 1895: Art 304) is based on the sup-

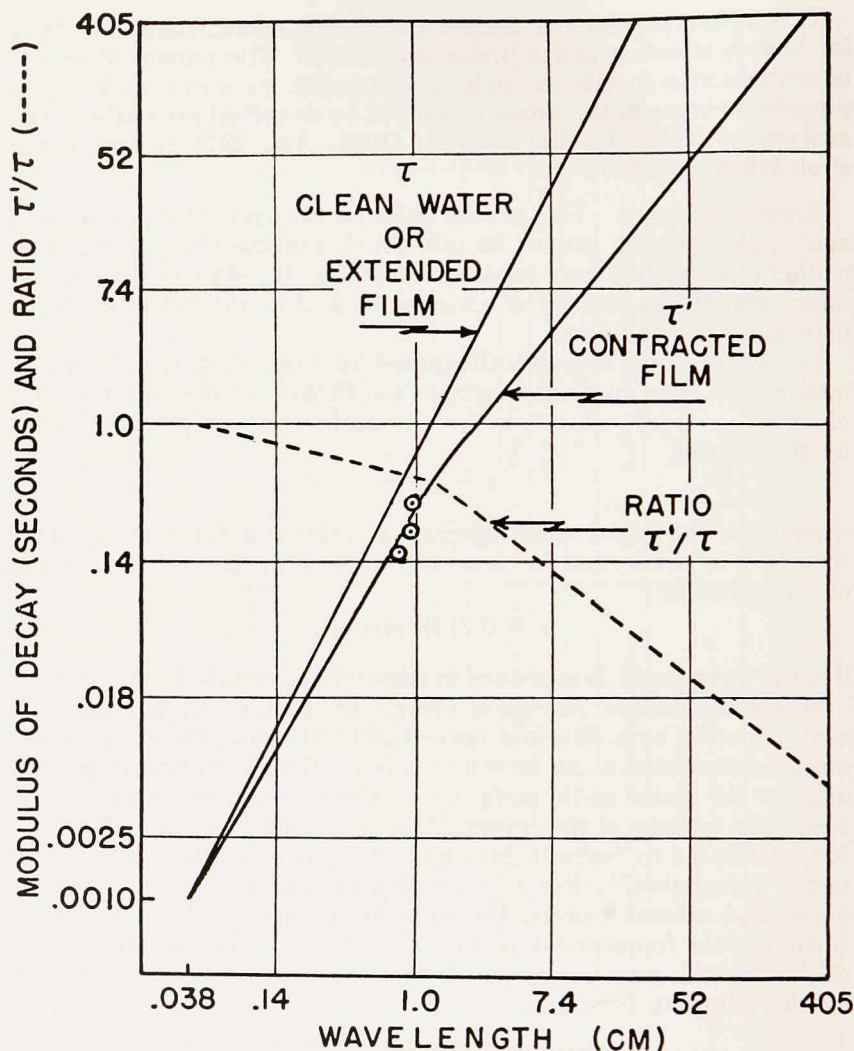


Figure 5. The modulus of decay for various wavelengths for clean water, or under an extended film (τ) and under a contracted film (τ'). The dashed line shows their ratio τ'/τ . The circled points are values observed in model experiments.

position that the surface tension varies by an amount proportional to the extension, so that

$$T = T_0 \left(1 + f \frac{d\xi}{dx} \right), \quad (5)$$

where T_0 is the tension in the undisturbed state, ξ is the horizontal displacement of a surface particle, and f is a numerical coefficient proportional to the slope of the surface tension curve (Fig. 3). But, to quote Lamb, the elasticity of the film has the effect "of practically annulling the horizontal motion at the surface. The dissipation is therefore (within limits) independent of the precise value of f ." Since the film is elastic only when the surface tension varies with the degree of compaction, it follows that τ applies to film states where f vanishes, that is, to the right of point C in Fig. 3. Likewise, τ' must apply where f is finite, as in the region to the left of point C, and must be constant where f is constant, that is to the left of B. It can be seen that the full range of damping rates between τ and τ' must be exhibited within a relatively narrow range of film extension corresponding to the shoulder of the curve between points B and C. This region is that in which the film loses its depressing effect on surface tension and hence its spreading tendency. Films in nature, if they are in equilibrium with the surface tension of water, must therefore tend to be nearly in a critical state of extension with regard to ripple damping.

The only previous experimental test of these equations appears to be that of Pockels (1891), who found a constant high damping rate, equivalent to τ' , for films compacted beyond the point B in Fig. 3 where surface tension falls rapidly and a constant low damping rate, equivalent to τ , for films compacted to any degree less than half this amount. The damping times varied by a factor of 5.7 between these extreme conditions.

For any particular film substance, the shape of the shoulder of the curve of surface tension versus extension varies with the chemical composition and temperature of the substance and with the pH and other ion-concentrations of the underlying fluid. For the film found on the ocean, the applicable curve could be determined only by delicate laboratory methods that are not readily applied in the field (Adam, 1941: 27-35). However, the Pockels results, which are for an unspecified film-substance, are typical of a wide variety of organic compounds (Adam, 1941: 50-51). Therefore, it seems reasonable to assume that, for a film initially in equilibrium with the surface tension of water, an extension and contraction of the order of $\pm 25\%$ will produce the full range of damping effect of which the film is capable, the ratio of maximum and minimum rates being, from (3) and (4a),

$$\frac{\tau'}{\tau} = 0.45\lambda^{-1/4}. \quad (6)$$

Finally, in order to examine the ability of wind of a given speed to maintain ripples in spite of viscous damping, W. H. Munk of the

Scripps Institution has made calculations (following a method due in part to Jeffreys; see Lamb, 1932: Art 348) that show the effect for various wind speeds of a contaminating film in its extended (τ) and contracted (τ') state. His main results, kindly made available as Fig. 6, are that the minimum wind speed for growth of waves is 245 cm/sec when the surface is elastic (film compacted) compared to 110 cm/sec when the surface is free (film absent or extended). In each case the first waves to achieve stability at minimum wind velocities are gravity waves. Increased wind speeds produce a spectrum of stable waves.

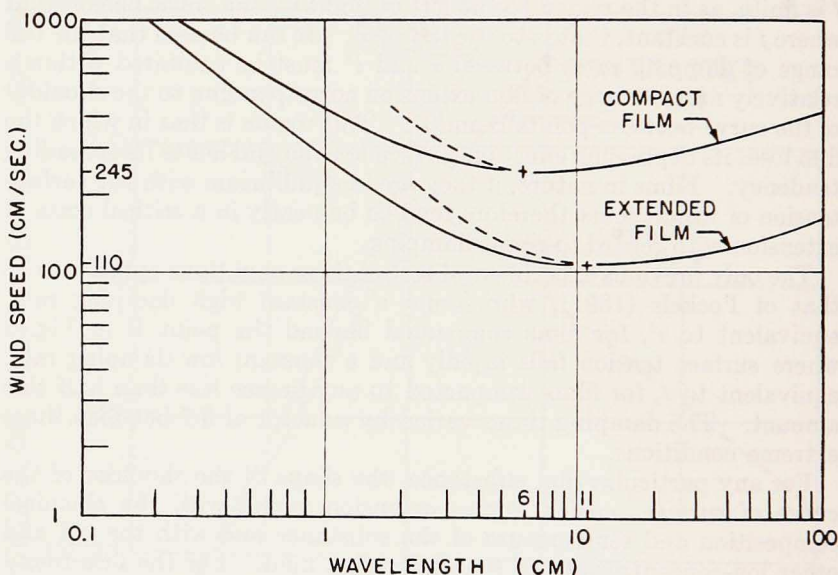


Figure 6. Limits of the spectrum of stable waves (—) and the "preferred" wave length (---) for various wind speeds.

Corresponding to each wind speed, waves of a particular "preferred" length will grow fastest. The preferred wavelength is shorter at higher wind speeds.

Summary and Hypothesis. The foregoing analysis leads to the hypothesis that whenever a long internal wave exists at the interface between two layers of water with an amplitude exceeding $\frac{1}{4}$ of the thickness of the upper layer, such a wave will produce over its troughs horizontal convergence at the upper surface sufficient to bring an organic surface film into a state of compaction where ripples will be damped at the maximum rate which the film can cause; conversely, it will produce over its crests divergence sufficient to extend the film to a

state where the rate of ripple-damping is minimal. The differences in rippling will be plainly evident, whether the ripples are produced at one point by a mechanical vibrator (as in a model experiment) or uniformly over a given area by wind blowing at a speed of 110 to 340 cm/sec. If the internal wave is a standing wave, as in a model, calm zones will appear and vanish alternately over each antinodal point of the wave, the phase being opposite at adjacent antinodes, with the calm phase coinciding at each point with the depression of the interface. If the internal wave is progressive, as in nature, calm zones will remain continuously over the troughs of the wave and will themselves move as a wave of state along the surface film, their speed being governed by the propagation velocity of the internal wave.

The foregoing hypothesis has been put to the test of experiment and observation.

Model Experiments. In order to study the effect of an internal wave on a surface film, a model was set up in the laboratory. It consisted of a standard aquarium containing a layer of clear tap water over an 8% solution of cane sugar (specific gravity 1.029) which was colored red with photographic opaque. A glass plate, slightly narrower than the tank and somewhat more than one-third its length, was laid horizontally on wire supports 6.5 cm. beneath the surface of the lower layer. A synchronous motor, connected to a suitable variable speed-reduction transmission, was used to rock the glass plate through an angle of 10° about its median horizontal axis, which was transverse to the aquarium and located $\frac{1}{4}$ the length of the aquarium from one end. This gentle motion, when the period was critically adjusted, produced a standing wave with an amplitude of 2 cm (4 cm height) and a fairly sinusoidal profile. By suitable change of the period, a half-wave or a three-halves wave was easily produced. Once established, the stratification is very stable and the layers remain distinct for several days. The upper layer can be floated onto the lower one by allowing the water to drip from a separatory funnel onto a supported watch glass, or, if care be taken to make a uniform sugar solution in advance, the lower layer can be run in below the upper layer. Lacking a suitable motor to govern the wave, the glass plate could be rocked by hand, but in this case it would be better to use a less dense sugar solution so that the period would be slower and less critical. When first examined, the free surface may be found so strongly contaminated as to be nearly inextensible, that is, in the state represented in Fig. 3, to the left of point B. (The same condition is commonly observed at the surface of newly brewed China tea.) The surface is easily cleaned by floating absorbent paper upon it and removing it with a stripping motion. A light dusting of talc revealed the horizontal motion of the surface as

well as the cycles of convergence and divergence in a spectacular way, the converging particles of talc coming together with the appearance of shock at the limiting minimum area, point A in Fig. 3. Any oil of vegetable or animal origin will restore the necessary amount of contamination to the surface should it be cleaned to an unwanted degree.

A second effect of the internal wave is the periodic variation of the thickness of the superficial layer. This was studied by placing a rather heavy film of oil on the surface, which then loses its horizontal motion but exhibits interference colors that vary through the primary visible spectrum and sometimes on into the secondary one with each cycle of the internal wave, thus showing a change of thickness of some thousand Ångstrom units.

To test the effect of film compaction on ripple damping, advantage was taken of a mechanical device designed by E. A. Williams of Santa Barbara College. It consisted of a light horizontal lucite bar 15 cm long of inverted triangular section 5 mm at the base, attached by a short vertical tube to the diaphragm of a small magnetic radio speaker. The speaker was driven by an audio-oscillator through a power amplifier at frequencies that could be varied from 10 to 60 cycles/sec. The amplitude could also be controlled. With the lucite bar just touching the water surface and with the speaker driven at 36 cycles, a regular train of 8 mm ripples could easily be seen crossing the aquarium when the surface was nearly clean. They were nearly obliterated by larger amounts of contamination. In order to photograph these minute ripples, a photoflood lamp was directed at the surface so that the shape and uniformity of its reflection could be observed. In the absence of ripples, the reflection was quite round and specular. On the other hand, when ripples were present, they broke up the reflection into corrugations within an envelope which was elongated along the axis of the ripple train. Since the ripples were subject to damping during their passage across the reflection, those to the left were steeper than those to the right, thus resulting in greater elongation to the left and hence in the egg-shaped envelopes shown in Fig. 7. The rate of damping was calculated from the asymmetry of the envelope in the compact phase and it was found to be in fair agreement with the values calculated for τ' by means of equation (4a). The experimental values for three separate cycles of the internal wave are shown as circled points in Fig. 5.

The relations between the internal wave, the extension of the surface, and the rippling are shown in Fig. 7, where the reduction of rippling over the trough of the internal wave is clearly evident. The relation between the calculated and the actual dimensions of the wave motion, film extension and ripple damping are entered in Table I.

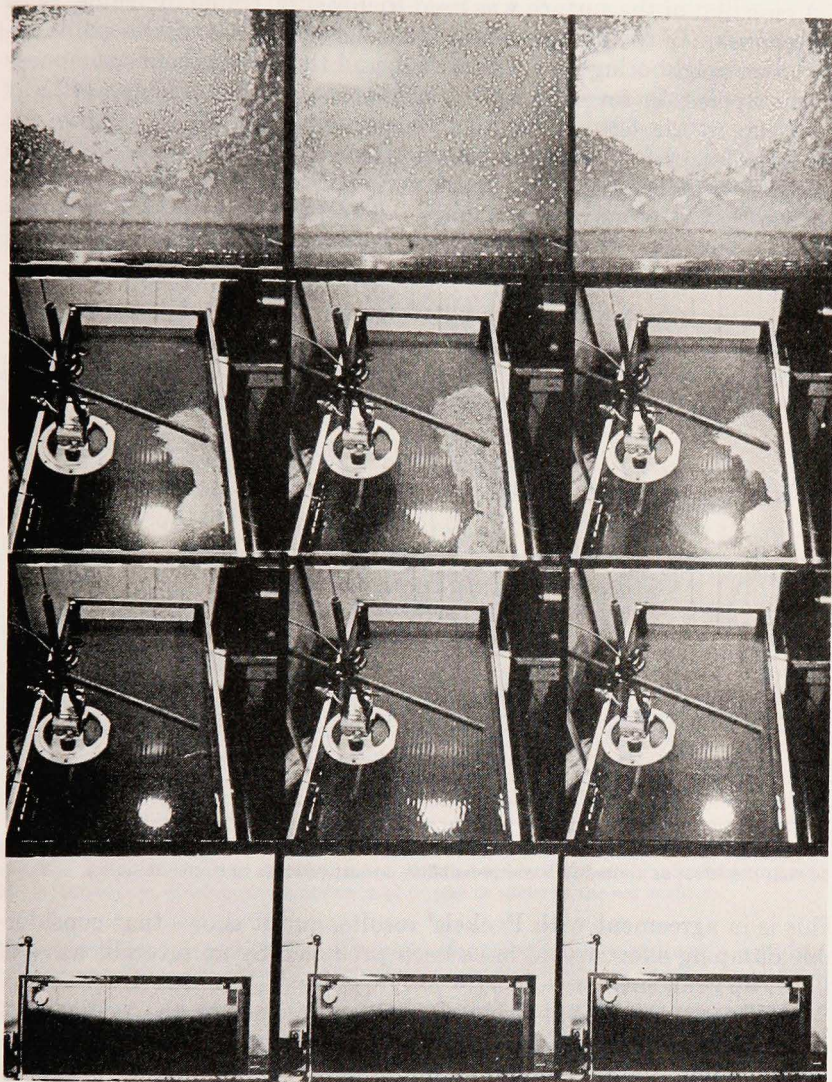


Figure 7. The model in action. Columns are matched and represent a sequence of exposures at intervals of $\frac{1}{2}$ wave period. Top row, particles of talc on the surface in alternate states of compaction and extension; second row, film compaction and ripple damping; third row, high and low rates of damping; bottom row, alternating phases of the internal wave.

A cine film of the surface was used to measure, frame by frame, both the extension of the film, as revealed by the separation of two arbitrarily chosen neighboring particles of talc, and the simultaneous steepness of the ripples, as revealed by the elongation of the envelope of light reflection to the left. The result is presented in Fig. 8, where the abrupt change of slope of the curves which show the separation of the particles indicates that there was a definite limit to the degree of compaction that the film would tolerate. The central curve, which shows the time variation of envelope extension, reveals that the ripples were fully damped at a compaction somewhat less than the maximum.

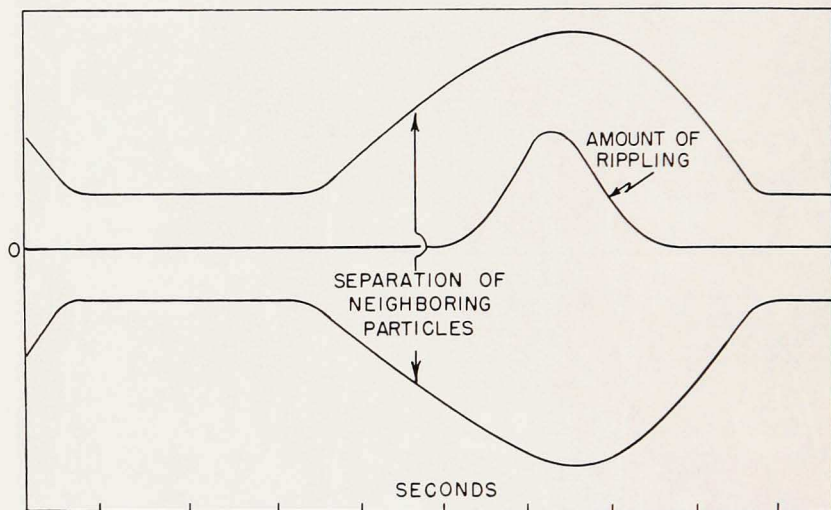


Figure 8. Extension and contraction of a surface film over a standing internal wave showing the effect of limited film compressibility and its relation to ripple damping.

This is in agreement with Pockels' results, and it shows that considerable damping effect would have been produced by an internal wave of a/h' ratio even smaller than $\frac{1}{4}$.

Finally, although it is probably implicitly clear to the reader that the transfer of the state of film compaction between adjacent antinodes of the wave was wave-like in character, yet it must be explicitly stated that such was the case. For the wave detailed in Table I, the shift of calm phase from the center to the ends of the model was over a distance of 37.5 cm, yet the maximum horizontal displacement of individual particles of the surface film was observed to be only 3.1 cm, thus proving that the ripple-damping effect was truly a change of state which was propagated independently of any but small oscilla-

tions of the medium. For reasons which will appear presently, events in nature are not favorable to such exact observation. However, it is reasonable to assume that, under similar kinematic conditions, the behavior of the sea surface is similar to that of the model surface, with the difference that, as in the ocean we expect to find progressive rather than standing internal waves so, correspondingly, we shall expect the wave of film compaction on the ocean to be propagated with a steady velocity instead of vanishing at one antinode while simultaneously appearing at the adjacent ones.

FIELD OBSERVATIONS

Long Internal Waves in the Sea. Turning now to the ocean or to large lakes to see whether internal waves have similar effects on the larger natural scale, it is easily established that the condition $a/h' \gg \frac{1}{4}$

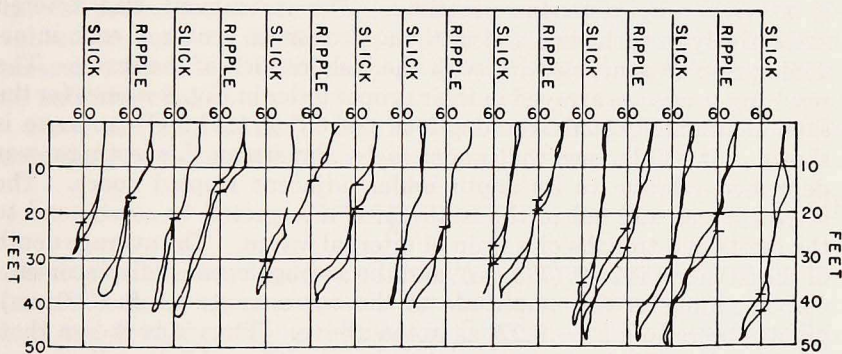


Figure 9. A series of bathythermograms taken in rapid succession from a moving ship together with simultaneous observations of slicks, showing the elevations and depressions of 60° F isotherm in relation to the presence of ripples or slicks at the sea surface.

is satisfied in the summer when the thermocline is shallow. On June 17, 1949, near San Diego, the writer used a bathythermograph (Spilhaus, 1938) to measure an internal wave with an average amplitude of 200 cm about a mean depth of 735 cm, for which the ratio a/h' is 0.27 (Fig. 9). Ufford (1947a), using a different method, measured in the same region and season (but at an unreported depth) an internal wave of 16 min period with a length of 330 m and an amplitude of 228 cm, and another of 12 min period, 130 m length and 152 cm amplitude. These were progressive waves which would produce the particle motion shown in Fig. 4. The calculated velocities associated with such a wave, together with the displacement of particles in the surface

and the resulting film compaction, are shown in Table I, where, for easy comparison, these calculations are placed alongside those for the experimental tank. It is seen that compaction, the parameter on which ripple-damping depends, is little altered in going from model to prototype. Therefore, one would expect to find parallel streaks of calm over the successive troughs of a train of such internal waves which would move along the surface at a velocity equal to the propagation velocity of the internal wave. The occurrence of such slicks was reported by the author (Ewing, 1950) and is illustrated in Fig. 1. The superficial resemblance to long-crested swell is apparent.

Bathythermograph Observations. In order to test the relation of such a series of calm streaks to internal waves, a temperature section was made, using a bathythermograph specially modified for observations shallower than 50 feet (15 m). The section was taken from a ship moving along a track normal to the long axes of the streaks and roughly at right angles to the coastline. The instrument was lowered successively in each slick and in the approximate center of each intervening rippled zone, starting with the inshore slick of the series. The resulting traces are arrayed in their proper order in Fig. 9 where, for the sake of simplicity, only the depth of the 60° F (15° C) isotherm is shown. It will be seen that under each calm streak this isotherm was depressed relative to its depth under adjacent rippled zones. The alternate rise and fall of the isotherm is interpreted to correspond to the crests and troughs of a train of internal waves. The average depth of the isotherm is 24 ft (735 cm), and the average vertical displacement, corresponding to the amplitude of the wave, is ± 6.6 ft (200 cm), giving the ratio $a/h' = 0.27$, as noted above. Thus it is shown that, where natural slicks are arranged in a series of equidistant lines over a long internal wave, they are situated over the troughs of the wave, as in the model.

The Distance between Slicks. On the proposed hypothesis, the distance between the calm streaks of a series should correspond to the wavelength of the accompanying internal wave. The interslick interval of a series (which included those previously examined by bathythermograph) was measured by sailing across the slicks in the direction described above, noting the time of passage through each successive slick on the moving tape of a sonic depth recorder. Inside the 85 m contour of water depth, the slicks were fairly evenly spaced at intervals varying from 395 to 630 m with an average interval of 486 m. These may be compared as to magnitude with the wavelengths of the two internal waves measured by Ufford in July 1945, namely 330 m and 130 m.

Along the slope of a sudden declivity beyond the 85 m contour, the slicks were crowded together at an average interval of 102 m. A photograph made by W. F. Royce shows a similar terminal aggregation of streaks in one pronounced band; Forbes (1945) has reported the same distribution in Panama Bay. These may mark regions in which short period internal waves are generated. Indeed, a mechanism whereby internal waves may be generated by a tidal current running against a submerged shelf has been proposed and experimentally demonstrated by Zeilon (1934).

Propagation and Refraction. In order to see whether serially arranged calm streaks move with the velocity of propagation of progressive internal waves, studies were made with a motion picture camera by exposing film at a rate of six frames per minute. When this film is projected at 16 frames per second, the calm streaks are seen to progress sideways to their long axes, as long crested swells do. The rate of progress was found to be 22 cm/sec on May 17, 1949 and 16 cm/sec on June 13, 1949. These velocities are comparable to those measured by Ufford (1947a) for two July internal waves which were 35 and 18.5 cm/sec.

Since the propagation of long internal waves is dependent on depth according to the equation

$$c^2 = g \frac{hh'}{h+h'} \frac{\rho-\rho'}{\rho}, \quad (7)$$

where g is the acceleration of gravity, internal waves must be refracted so as to approach parallelism to the contours of both the bottom and the thermocline (Munk and Traylor, 1947). Consequently, off the coast of San Diego, where the bottom and the thermocline gently slope away from shore, internal waves in shallow water should be oriented roughly parallel to the coast. This tendency is reflected in the orientation of slicks which parallel the shore regardless of wind direction, provided the wind is light. Where the bottom contours are sharply curved, as over a canyon, the internal waves should be convex toward shore. Correspondingly, surface streaks over La Jolla canyon have been observed to have this configuration, each one bowed congruently so that it would fit its neighbors if placed in contact.

The Reflection of Light from the Sky. The intuitive supposition that the optical effect of slicks is due to decrease in wave steepness can be rigorously examined by observing the sun's glitter path on the water. Hulburt (1934) has shown that the width of the path depends on wave steepness. Observation reveals that where a calm streak crosses the sun's path it reduces the path width to zero. Hulburt has also shown

that the light reflected by the sea, when ruffled by a 5 to 20 knot wind, is from an average altitude of 30° above the horizon. Consequently, on a clear day a rippled sea is quite blue. In slicks, however, where the wave steepness is less, the light is reflected from near the horizon, so that in contrast to the blue color of the rippled zone the calm streaks are whiter. Studies of calm streaks by Shuleikin (1935, 1941), using a somewhat different theoretical approach, yielded similar results.

Particle Motion on the Sea Surface. The hypothesis that calm streaks over internal waves are themselves propagated as waves requires that the material of the surface film shall not follow the steady phase motion of the wave but only the oscillatory horizontal particle motion. It has turned out to be difficult to observe this directly because of inability under natural conditions so to mark the film itself that its motion may be distinguished from that of water lying a few millimeters beneath and from that of objects such as bubbles having some windage profile. Close examination of the upper few millimeters of the sea when a light breeze is blowing shows surprisingly high velocities at the surface and highly developed vertical shear. Thus soluble dye put into the water becomes separated into a subsurface cloud and a superficial film. The film soon vanishes and the subsurface cloud cannot be relied on as a means of observing motion in the very surface. The best targets tried have been sheets of paper that float nearly horizontally just under the surface. These, when placed in front of an on-coming slick, were observed to be drawn into the slick, and, after some minutes, to be ejected to the rear, indicating that the propagation of slicks does not depend directly on surface flow. Nevertheless, up to the present, the conclusion that slick propagation is a wave phenomenon rests chiefly on the evidence of the model experiment.

The Effect of Wind. Only brief mention will be made of observations showing that calm streaks are oriented independently of wind direction at low wind speeds, because sufficient data are lacking to prove this independence categorically. The intuitive notion that slicks trail downwind like smoke from a chimney must be discarded as a *non sequitur* if the initial distribution of film material is uniform. Only some cross circulation of the wind or water, such as that found by Langmuir (1938) and by Woodcock and Wyman (1947), would produce this orientation, and the cases reported by them appear to involve windspeeds in excess of 4 m/sec. At lower wind speeds, downwind orientation is the least frequently observed of any, most tending to be within 30° of perpendicular to the wind. In shallow water, orienta-

tion of slicks is governed by refraction which tends toward parallelism with the contours of the bottom and of the thermocline, as stated previously. In deeper water, Forbes (1945) has shown that two trains of slicks can coexist with two different orientations, both of which obviously cannot be related to the local surface wind in the same way. Flying across the Gulf of California, the author observed slicks to be oriented about normal to the wind direction. It appears, therefore, that although slicks at low wind speeds are controlled directly by internal waves, these waves may have some preferred orientation nearly normal to the wind. In fact Zeilon (1934), by simulating the condition of a surface current excited by a steady local wind, produced experimental internal waves with crests at right angles to the wind.

DISCUSSION

Nonperiodic Slicks. Calm streaks on the sea surface are caused by a number of agencies other than internal waves. Anything that compresses the film either by mechanical means or by convergent flow will produce this result. Thus a boat sailing through the film splits it into two bands of compressed film that lie on either side of the wake, producing two ribbons of calm in which bubbles are stabilized by the compacted film (Brown, et al., 1949) and which persist much longer than the turbulence of the wake. The attendant drop in surface tension has been measured (Adam, 1937; Shuleikin, 1935, 1941).

Wind stress on the surface produces a zone of calm on the windward side of a barrier (Aitken, 1884; Adam, 1937; Hardman, 1941). The discharge of a river into the sea or into an estuary or pool also produces film accumulation through convergence and hence a calm zone (Thompson, 1862; Aitken, 1884; Adam, 1937; Schmidt, 1936).

A commonly observed slick which is not associated with internal waves forms along the axis or along both sides of tidal channels such as the outer parts of San Diego Harbor or Mission Bay. Similar slicks have been mentioned by Aitken (1884) and by Schmidt (1936) who attribute them to the wind, but this explanation is inadequate in several respects, the most important of which is that they do not usually touch the shore. Preliminary investigation by the author supports the explanation that these channel slicks result from transverse horizontal convergence associated with helicoidal flow along the channel. Such flow is known to occur in rivers below bends and in narrow straight channels with sluggish flow (Hellström, 1948; Gibson, A. H., 1909). It is independent of the wind, being caused by friction along boundaries where these are curved or of asymmetric cross section (Prandtl, 1949; Rouse, 1938: 266-268; Jeffreys, 1929).

In the open ocean, helicoidal flow occurs in the equatorial region (see Sverdrup, et al, 1946; 632-636, 708-712) and in coastal waters (Stockmann, 1946). Since overturn of water is associated with high biological productivity, it is to be expected that in these regions the surface is heavily contaminated with organic film and consequently that a slick should be found along the line of horizontal convergence over that margin of currents which is characterized by subsidence. Where a film is considerably compressed, it acts to stabilize bubbles (Brown, et al, 1949). Under this condition, a region of convergence is revealed more conspicuously by a line of foam than by a slick. Such a foam line is a persistent feature off the U. S. Pacific Coast. It has been photographed and described in detail by Isaacs (1948). A similar line of convergence with attendant slicks has been observed and photographed along the left, or "cold wall," of the Gulf Stream (Kielhorn, 1950). Along the (descending) southern margin of the South Pacific Equatorial Current, a line "3 to 4 feet wide, in the form of a long streak of bubbles and foam reaching as far as the eye could see in an east-west direction" was reported by Heard (1950). A similar narrow streak of foam over 100 miles long was photographed and described in detail by Beebe (1926a, b).

Slicks as an Oceanographic Tool. In considering the uses to which slicks and their relation to internal waves might be put, three stand out. First, slicks may be used to locate regions of horizontal convergence at the sea surface. Second, internal wave slicks represent a map of the distribution of the internal waves and so lend themselves to study of the conditions under which such waves are formed. Third, the fact that at wind speeds greater than 3.4 m/sec the film is dominated by the wind rather than by the motion of the underlying water may shed new light on quantitative aspects of the stress of wind on water.

Inhibition vs. Damping of Ripples. In closing, it is worth pointing out that the calming effect of film on water has been attributed herein solely to variation in the rate at which ripples, once generated, are damped by viscosity of the water. The possible effect of the film on the tangential stress of wind on water has been ignored on the supposition that this is relatively unimportant. It must not be overlooked, however, that careful observations have been reported (Aitken, 1884; Gibson, G. E., 1944) which indicate that the wind-induced surface current is increased by the presence of oil on water. Aitken has suggested that, by distributing the effect of stress from small eddies over areas which are large compared to the length of initially formed ripples, an elastic film discourages the irregular horizontal motion of the water surface which initiates ripple formation, favoring instead a

uniform surface drift. These matters deserve further study. Should it turn out that such variation in the action of wind on water may indeed be induced by surface film, the effects on rippling due to damping would in no way be diminished; on the contrary they would be supplemented.

CONCLUSIONS

1. The surface of coastal waters and of lakes is often coated with film of finely particulate or colloidal material, rich in organic matter.

2. Such films, when in a compact state, lower the surface tension in proportion to the degree of compaction. They therefore make the water surface elastic and increase the rate of dissipation of wave energy for wavelengths from a fraction of a millimeter to several meters. This increase more than doubles the minimum wind velocity required for the growth of waves.

3. Long internal waves in a shallow thermocline produce transient changes of state in a surface film sufficient to form bands of variable surface tension over their troughs. These bands are propagated as a wave with the same velocity as the internal wave.

4. Provided the internal wave be of sufficient amplitude in relation to its depth, light winds (less than Beaufort 3) render the bands visible as long narrow calm streaks in a rippled sea.

5. At wind forces less than 3, internal waves of short period in a shallow thermocline are usually oriented with their long axes nearly perpendicular to the wind except in shallow water where they are refracted by the thermocline or bottom to an orientation nearly parallel to the contours of one or the other of these surfaces.

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