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ON THE ORIGIN OF INTERNAL TIDE WAVES IN THE OPEN SEA^{1,2}

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ABSTRACT

Several mechanisms are examined which might explain the observed preponderance of internal waves of tidal period. It is concluded that the effect of the earth's rotation on the velocity of internal waves is an important factor in favoring the generation of such internal waves.

The phenomenon of internal waves with periods of the tide generating forces, designated briefly as internal tides, occurs not only in shallow seas and straits but in the open ocean as well. The amplitude of these internal tides and the water transport connected with them are much larger than those of the normal surface tides. These internal tides exert a much greater influence on the thermohaline structure of the oceans than do normal tides. Fr. Nansen and Bj. Helland-Hansen were the first to notice the existence of such internal waves, but Otto Pettersson was the first to associate these large periodic displacements of water masses with the tide generating forces. Such internal tides have been observed during all subsequent oceanographic expeditions where anchor stations were made in deep water. The anchor stations of the METEOR in the North and South Atlantic oceans have been discussed in the METEOR publications, and those of the SNELLIUS Expedition have been elaborated by Dr. Lek. The later cruises of the METEOR (1937, 1938) in the North Atlantic and in the area between the Cape Verde and Canary islands, the observations of the ALTAIR at anchor stations in the mean current of the Gulf Stream north of the Azores, the many cruises of the ATLANTIS in the western North Atlantic, the observations of the DANA Expedition, etc., all have shown that internal tides are as common as tides at the surface of the ocean.

Recent observations, taken on the Marine Life Research cruises of the Scripps Institution of Oceanography, have shown again very definitely that the influence of internal tides is great in this area also. This influence can be very disconcerting when attempting to obtain

¹ Contribution from the Scripps Institution of Oceanography, New Series, No. 471.

² I should like to draw attention to a paper by Haurwitz (1950) which appeared while this paper was in press. The conclusions reached in Haurwitz' paper are similar to the ones presented here.

the mean distribution of oceanographic elements, and therefore knowledge of the behavior of internal tides is required in order to eliminate their influence (Defant, 1950). However, aside from a practical application, the rising and sinking of immense water masses is in itself one of the interesting phenomena of dynamic oceanography; in fact, it is every bit as interesting as normal tides at the surface.

By theoretical and experimental research we are well informed concerning the physical nature of internal waves. Early theoretical models treated two-layered bodies of water with a physical discontinuity between them. Later the problem of internal waves of tidal periods in water masses of continuous density variations with depth were studied, especially by Fjeldstad (1933) and Groen (1948).

Although we know what internal waves with tidal periods are, there still are many difficulties on how such internal waves with large amplitudes develop and why they are so frequent, i. e., why they occur everywhere and at all times. Although observations at anchor stations, up to the present time, do not give accurate data about the velocity of internal tides in the open ocean, they do show clearly and distinctly that internal tides are related to tides at the surface. But the form of this coupling is not the same everywhere, though it seems to remain fixed with time at a fixed locality. This coupling leads to the assumption that internal tides are *forced* waves. If that is so, then they travel with a velocity about 100 times that of the theoretical velocity of *free* internal waves.

Free internal waves with tide periods, once they are generated, will soon lose every connection with the tide at the surface, and the coupling between the two wave systems would vary with time and distance. Since the generation of internal waves requires but little energy, the internal tides generated at different points would spread freely in all directions, the result being a chaos of internal tides in the ocean and the impossibility of finding a suitable method of eliminating their influence. But this is not the case, for internal tides are very regular and are strictly coupled to normal tides at the surface. With the latter they form a well determined wave system. This has been shown clearly by the observations of the Marine Life Research program

In work with the METEOR data I have already dealt with the question of the generation of internal tides, but at that time it was not possible to give a satisfactory answer. Now, in connection with the Marine Life Research observations, I have been occupied again with this problem, and I would like to submit a few points of view which might make this question clearer. In the following discussion I will work mostly with a model of two superposed water bodies.

In the case of tidal waves in deep water, the entire water masses

will be moved with the same periodic tidal velocity; there will be no change of current with depth. If the lower water mass lies on a rough bottom, friction will decrease the intensity of tidal currents and will change the phase of these currents. This influence makes itself felt up to a certain height above the bottom, this height depending on the turbulence of the tidal current. If the lower water mass is not too deep, as on an ocean shelf, then the friction will influence the total underlayer to some extent. On the boundary, shearing stresses will occur which can force a dislocation of the boundary layer. These will produce internal waves or, if they are already present, they will affect amplitude.

The theoretical solution, given previously in the METEOR work (Defant, 1932), is

$$\frac{\zeta_2}{\zeta_1} = 1 - \frac{1}{(h_2/h_1)M + 1}, \quad (1)$$

where h_1 designates the height of the upper homogeneous water mass and ρ_1 its density; the corresponding values for the bottom layer are h_2 and ρ_2 ; $\Delta\rho$ is $(\rho_2 - \rho_1)/\rho$; ζ_1 and ζ_2 are the displacements of the surface and of the boundary; at most they are the amplitudes of the corresponding waves. The function M is given by

$$M = 1 - \frac{tgh(1+i)\beta_2 h_2}{(1+i)\beta_2 h_2},$$

where $\beta_2 = \sqrt{\sigma/2\nu_2}$, and ν_2 is the eddy viscosity of the lower layer. Since M is always slightly less than 1, ζ_1 is larger than ζ_2 . Therefore this process is incapable of increasing the amplitude of internal waves of tidal character.

A second point of view may be illustrated by the following problem. Let the upper water mass move with constant velocity U_1 , the lower with the constant velocity U_2 , both parallel to the undisturbed discontinuity layer. A tide generating force,

$$X = f e^{i(\sigma t - \kappa x)}, \quad (2)$$

acting in a horizontal direction on the whole system, will produce tide waves at the surface and on the discontinuity layer. The frequency is denoted by σ , the wave number by κ ; the velocity of propagation $c = \sigma/\kappa$ is determined by the force. Under which conditions then will real internal waves, whose amplitudes are much larger than the amplitudes at the surface, be generated?

The solution is not difficult to find, since the vertical motion in both layers can be neglected, according to the first approximation in tide

theory. In the equations for the amplitudes of the surface and of the boundary layer, the expression

$$c(c - U_1)[(c - U_2)^2 \omega_1 \omega_2 + (c - U_1)^2 \rho_1 / \rho_2] - [(c - U_2)^2 \omega_1 + (c - U_1)^2 \omega_2 \rho_1 / \rho_2 + c(c - U_1) \omega_1 \Delta \rho] \cdot g/k + \Delta \rho g^2 / k^2 = D_1 \quad (3)$$

always appears as denominator, wherein $\omega_1 = ctgh \kappa h_1$ and $\omega_2 = ctgh \kappa h_2$. But this expression, when set equal to zero, is the equation which determines the velocity of the *free* waves of the system. For $U_1 = U_2 = 0$ this equation becomes the well known equation of the velocity of free waves with the wave number κ :

$$c^4(\omega_1 \omega_2 + \varphi_1 / \rho_2) - c^2(\omega_1 + \omega_2)g/\kappa + \Delta \rho g^2 / \kappa^2 = D_2 = 0. \quad (4)$$

The two solutions of this equation for long waves are

$$c_1 = \sqrt{g(h_1 + h_2)} \quad \text{and} \quad c_2 = \sqrt{\Delta \rho g h_1 h_2 / (h_1 + h_2)}. \quad (5)$$

The velocity c_1 is valid for waves at the surface, which are associated with displacements of the boundary layer of smaller amplitudes than those at the surface. c_2 is the velocity of free internal waves.

In the case of *forced* waves the case of *resonance* corresponds to $D_2 = 0$. Theoretically the amplitude of forced waves will be infinitely large. Note that the internal waves will be infinitely large if the condition $c = c_2$ is exactly fulfilled. But this never occurs, since the magnitude of c for tidal waves in the oceans is about 200 m/sec and that of c_2 only 2 m/sec. The same is true for the more general equation (3) which also contains the influence of steady currents. For values of c , U_1 and U_2 that occur in nature, D_1 is a very large number, so that the generated internal waves will remain small—much smaller than those at the surface. Consequently they are not internal waves in a strict sense.

The same conclusions follow from the computation of the ratio of the amplitudes of the two generated tide waves. One finds with good approximation:

$$\frac{\zeta_1}{\zeta_2} = 1 - \frac{(c - U_2)^2 h_1 / h_2}{gh_1 \Delta \rho - c(c - U_1) \rho_1 / \rho_2}. \quad (6)$$

Since internal waves have larger amplitudes on the boundary layer:

$$\Delta \rho g h_1 - c(c - U_1) \rho_1 / \rho_2 \geq (c - U_2)^2 h_1 / h_2.$$

Since $c \gg U_1$ and U_2 ,

$$c^2 \leq \Delta \rho g h_1 h_2 / (h_1 + h_2);$$

in other words, c must be smaller than the velocity of *free* internal

waves. However, this is never the case. We see that this factor does not contribute to the generation of large internal waves. This factor has been considered in greater detail by Haurwitz (1948), who arrives at essentially the same conclusion.

The condition $D_1 = 0$ has another significance which needs a few words of explanation. The solution (3) for c leads to a few difficulties. Since we want internal waves for which amplitudes of contemporary waves at the surface are extremely small, we may take, with good approximation, this amplitude equal to zero. Then we replace the surface by a rigid horizontal plane, in which the vertical motions are zero. The solution for c is then:

$$c = \frac{\rho_1 U_1 \omega_1 + \rho_2 U_2 \omega_2}{\rho_1 \omega_1 + \rho_2 \omega_2} \pm \sqrt{\frac{g}{\kappa} \frac{\rho_2 - \rho_1}{\rho_2 \omega_2 + \rho_1 \omega_1} - \rho_1 \rho_2 \omega_1 \omega_2 \left(\frac{\rho_2 - \rho_1}{\omega_2 \rho_2 + \omega_1 \rho_1} \right)^2}. \quad (7)$$

The first term on the right side is the convection velocity which corresponds approximately to a mean velocity of two currents. The first term below the root is the velocity of free internal waves; the second term with the negative sign, which always decreases the stability of the internal waves, gives the velocity of inertial waves. Dynamic instability will occur if this term is larger than the first, and therefore the condition for dynamically unstable internal waves will be:

$$(U_2 - U_1)^2 > \frac{g}{\kappa} (\rho_2 - \rho_1) [1/\rho_1 \omega_1 + 1/\rho_2 \omega_2]. \quad (8)$$

This criterion of stability is discussed in all its details by Bjercknes, et al. (1933) from the point of view of the generation of cyclones in the atmosphere. Here we are interested with the question as to when dynamic instability occurs in the ocean. The condition for this is:

$$(U_2 - U_1) > \sqrt{\frac{\rho_2 - \rho_1}{\rho_1 \rho_2} (\rho_2 h_1 + \rho_1 h_2) g}. \quad (9)$$

If h_2 is large, which is the case in the open sea, the inequality is reduced to $(U_2 - U_1) > \sqrt{\Delta\rho g h_2}$. The quantity on the right-hand side is always large, since h_2 is large, even with small $\Delta\rho$. Such steady currents will not occur in nature, and the inequality (9) is never fulfilled. Hence internal tide waves are always stable in the open ocean. But on shelf water or in shallow straits internal waves may easily become unstable. If $h_1 = h_2$, we have

$$U_2 - U_1 > \sqrt{g h_1 \frac{\rho_2^2 - \rho_1^2}{\rho_1 \rho_2}}, \quad (10)$$

with $h_1 = 100 - 150$ m, $U_2 - U_1 > \frac{1}{2}$ to 1 m/sec. This velocity difference is easily reached by steady currents. In fact, observations at anchor stations in straits have shown that normal stable internal waves can become unstable (Defant, 1948). The observations at anchor stations by the DANA Expedition in the Strait of Gibraltar during the spring of the year show distinctly that large internal waves will be unstable, thus leading to internal surf. Also, internal waves on the shelf off the West African Coast tend towards instability and formation of internal surf. This surf may destroy the thermocline, and only in this case will the phenomenon of upwelling of cold water develop off the coast. The instability of internal waves is still more pronounced in the Strait of Messina, where it produces the famous eddies of Scylla and Charybdis (Mazarelli, 1938; Defant, 1940). When the boundary layer between the converging tidal currents in the two water masses is brought to the surface on one side of the Strait by Coriolis' force, the instability of the internal waves will generate a large eddy. Undoubtedly these eddies are nothing but the effect of dynamic instability of internal waves in this shallow Strait.

The two points of view just discussed lead to negative results. I will now discuss a third point of view, which may account in part at least for the generation of internal tide waves. Until now we have neglected the Coriolis' force. Perhaps this force can be neglected in the first approximation in small regions or in narrow straits, but in the open ocean this is surely not permissible. Since we have already shown that the steady current in the two superposed water masses does not lead to large internal waves, we can disregard those currents. By working with long waves we can neglect the vertical accelerations; the equations of motion of the system are:

$$\begin{aligned} \frac{\partial u_1}{\partial t} &= \lambda v_1 - g \frac{\partial \zeta_1}{\partial x} + X, & \frac{\partial u_2}{\partial t} &= \lambda v_2 - g \frac{\rho_1}{\rho_2} \frac{\partial \zeta_1}{\partial x} - g \Delta \rho \frac{\partial \zeta_2}{\partial x} + X, \\ \frac{\partial v_1}{\partial t} &= -\lambda u_1 - g \frac{\partial \zeta_1}{\partial y}, & \frac{\partial v_2}{\partial t} &= -\lambda u_2 - g \frac{\rho_1}{\rho_2} \frac{\partial \zeta_1}{\partial y} - g \Delta \rho \frac{\partial \zeta_2}{\partial y}, \end{aligned} \quad (11)$$

where X is given by (2) and $\lambda = 2\Omega \sin \varphi$ is the Coriolis' parameter; the influence of friction is neglected. To these dynamic equations must be added the equation of continuity for both layers:

$$\begin{aligned} \frac{\partial \zeta_1}{\partial t} + h_2 \left[\frac{\partial u_2}{\partial x} + \frac{\partial v_2}{\partial y} \right] + h_1 \left[\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} \right] &= 0 \\ \frac{\partial \zeta_2}{\partial t} + h_2 \left[\frac{\partial u_2}{\partial x} + \frac{\partial v_2}{\partial y} \right] &= 0. \end{aligned} \quad (12)$$

If we take

$$\frac{u_1}{v_1} = \frac{U_1}{V_1} e^{i(\sigma t - \kappa x)}, \quad \frac{u_2}{v_2} = \frac{U_2}{V_2} e^{i(\sigma t - \kappa x)} \quad \text{and} \quad \frac{\zeta_1}{\zeta_2} = \frac{Z_1}{Z_2} e^{i(\sigma t - \kappa x)},$$

then the previous system of equations gives six equations for computing the six unknowns U, V, Z . With

$$\frac{\sigma^2 - \lambda^2}{\kappa^2} = \epsilon^2, \quad (13)$$

we have as denominator of all these quantities the expression:

$$\epsilon^4 - (h_1 + h_2)g\epsilon^2 + \Delta\rho g^2 h_1 h_2 = D_3. \quad (14)$$

It has the same form as the relation (4), transformed for tidal waves. But instead of being the velocity of forced waves, c enters the influence of the earth's rotation through the quantity ϵ . Whereas c has the magnitude of 200 m/sec and the quantity D_2 is a large number, as shown before, under certain conditions ϵ can be small or nearly equal to the velocity of free internal waves. With these values, however, D_3 will be zero. If T_i is the period of inertial waves (equal to 12 pendulum hours), then

$$\epsilon^2 = c^2[1 - (T/T_i)^2]. \quad (15)$$

For the diurnal tide wave, $T = 24$ hours and ϵ will be zero at 30° latitude, but for the semidiurnal lunar wave, $T = 12.43$ hours and ϵ will be zero at 74° latitude.

For small ϵ equation (14) has, in the first approximation, the form:

$$D_4 = \Delta\rho g^2 h_1 h_2 - g(h_1 + h_2)\epsilon^2.$$

D_4 will be zero for

$$\epsilon^2 = \Delta\rho g h_1 h_2 / (h_1 + h_2). \quad (16)$$

When $\epsilon = c_2 =$ the velocity of free internal waves, then *resonance* exists for forced internal waves. In other words, the amplitudes of internal tide waves will be very large at these latitudes. This is not the case for the normal tides at the surface, for which

$$D_5 = \epsilon^2[\epsilon^2 - g(h_1 + h_2)]. \quad (17)$$

D_5 remains a large number when $\epsilon = c_2$.

Therefore, the rotation of the earth provides a factor which is able to increase the amplitudes of internal waves, and internal waves will thus be a larger wave motion than the tide at the surface.

If we add steady currents to the two superposed layers, the denominator assumes, for forced internal waves, the form

$$(c - U_2)^2 \left[1 - \frac{\lambda^2}{(\sigma - \kappa U_2)^2} \right] h_1 + \frac{\rho_1}{\rho_2} (c - U_1)^2 \left[1 - \frac{\lambda^2}{(\sigma - \kappa U_1)^2} \right] h_2 - \Delta \rho g h_1 h_2 = D_6. \quad (18)$$

The term with Coriolis' parameter will be changed only insofar as we have $(\sigma - \kappa U)^2$ instead of σ^2 . The influence of this change goes in the desired direction of increasing the resonance, but the effect is very small.

In the foregoing results there is one difficulty, which I wish to discuss. The width of the resonance area is very small, at most 2 to 3 degrees of latitude. Outside this small strip, D_3 is always large and the amplitudes of internal waves will not be influenced greatly by the rotation of the earth. This surprising result, however, is the consequence of the way the problem has been formulated. The difficulties are the same as those in the theory of surface tides, and they can be removed if the boundary conditions are taken into account.

Finally, the equations of the problem show directly that the rotation of the earth must have a great influence upon internal wave motion. We will suppose in this case that the density increases in an arbitrary manner with depth. Eliminating pressure from the equation of motion,

$$\begin{aligned} \frac{\partial \rho}{\partial x} &= \frac{\lambda}{g} \frac{\partial \rho v}{\partial z} - \frac{1}{g} \frac{\partial \rho \dot{u}}{\partial z} \\ \frac{\partial \rho}{\partial y} &= -\frac{\lambda}{g} \frac{\partial \rho u}{\partial z} - \frac{g}{g} \frac{\partial \rho \dot{v}}{\partial z}, \end{aligned} \quad (19)$$

where the z -axis is positive downwards. If $u = u_0 \sin \sigma t$ and $v = v_0 \cos \sigma t$ designate tidal currents, then (19) gives

$$\begin{aligned} \frac{\partial \rho}{\partial x} &= \frac{\sigma}{g} \left[s \frac{\partial \rho v_0}{\partial z} - \frac{\partial \rho u_0}{\partial z} \right] \cos \sigma t, \\ \frac{\partial \rho}{\partial y} &= \frac{\sigma}{g} \left[\frac{\partial \rho v_0}{\partial z} - s \frac{\partial \rho u_0}{\partial z} \right] \sin \sigma t, \end{aligned} \quad (20)$$

where $s = \lambda/\sigma$. To these equations we must add the equation of continuity, and, as shown by Fjeldstad in his theory of internal waves, we obtain equations for u and v in which the denominator is again the quantity ε . With the same values of T , resonance will occur in the same latitudes. But the equations (20) already show clearly that internal waves are also associated with a variation of the density gradient with depth. If internal waves exist only in the y -direction, then $\partial \rho / \partial x = 0$, and we have $u_0 = s v_0$; in other words, the tidal current u in the x -direction will then be

$$\frac{\partial \rho}{\partial y} = \frac{\sigma}{g} \epsilon^2 v_0 \frac{\partial \rho}{\partial z} \sin \sigma t, \quad (21)$$

provided the tidal current v is independent of z , as in the case of simple tidal waves. This means that every change in density with depth is followed by a periodical sinking and rising of isopycnals across the tidal current. As a numerical example, take

$v_0 = 1$ m/sec, $\sigma_s = 1.45 \times 10^{-4}$ sec $^{-1}$, and $\lambda = 0.7 \times 10^{-4}$ sec $^{-1}$ and $\Delta\rho/\Delta z = 4 \times 10^{-3}$ g cm $^{-3}$ per 30 m = 1.33×10^{-6} g cm $^{-4}$. This gives $\Delta\rho/\Delta y = 1.52 \times 10^{-11}$ g cm $^{-4}$; that is, the isopycnals show an elevation of 5.7 m over a distance of 500 km. The range of internal waves is 11.4 m. Therefore, strong tidal current must always be connected with large internal waves, and it is not surprising that the oceanographic observations at anchor stations in the open ocean always show large internal waves. If we wish to eliminate these waves from the observations of oceanographic surveys, we must know the law to which they are subjected. But we are still far from this aim.

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