TRAJECTORIES OF SMALL BODIES SINKING SLOWLY THROUGH CONVECTION CELLS[1]

By

HENRY STOMMEL[2]

Woods Hole Oceanographic Institution

INTRODUCTION

Mr. J. C. Neess of the University of Wisconsin has indicated to the writer in a personal communication that he has often observed a greater variability in plankton tows taken up or down wind in a lake than when made across wind. He suggested that this might be due to convective or wind-induced cellular motion in the water, which in some manner should act to concentrate plankton in linear arrays in the direction of the wind, thus leaving relatively clear lanes between. This fact suggested that it might be of interest to set up a simple cell analytically and to see what kinds of paths or trajectories the particles of various settling velocities would describe.

The existence of long strip wind-induced cellular motion in lakes and oceans has been demonstrated (Langmuir, 1938; Woodcock, 1941). The nature of the trajectories should not depend necessarily on the nature of the forces producing the cellular motion, so that the results should apply equally well to thermally-induced cellular motion. The layer of wind-induced or thermally-induced strip cells may be thought of in the following way. The layer is divided into a number of strips (or cells) lying side by side with their lengths parallel to the wind. The water in each cell is rotating around the central axis of each cell, the sense of rotation in any given cell being opposite to that of its immediate neighbors, much like the rotation of a long train of gears. The boundaries of these cells are regions of sinking and rising water alternately. In addition to the rotating movement there is a movement in the direction of the wind so that a given particle of water describes a helix with axis horizontal and in the direction of the wind.

Fig. 1 shows a cross section view of two neighboring cells. The arrows indicate the direction of water movement in the plane perpendicular to the wind direction; the curved lines are streamlines. The motion of the water in the direction of the wind clearly has no effect

[1] Contribution No. 469 from the Woods Hole Oceanographic Institution.

[2] The writer wishes to thank Dr. C.-G. Rossby for reading this paper and suggesting several important improvements.

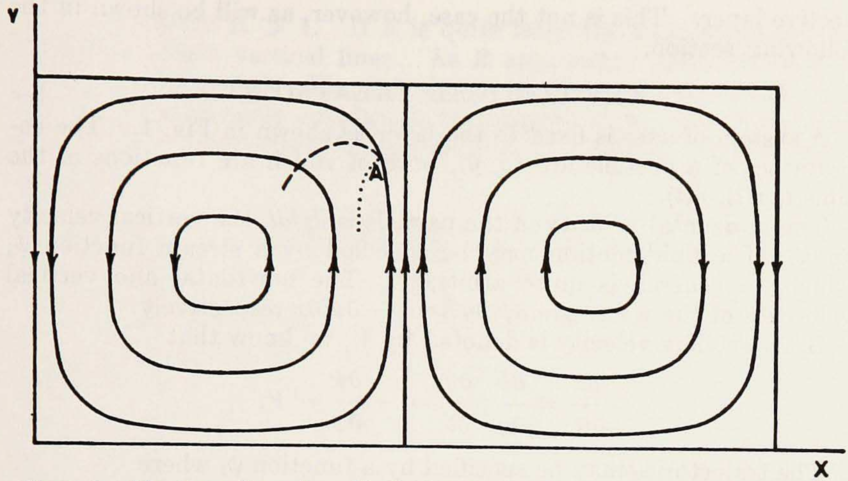

Figure 1. The streamlines of an idealized vertical section across a pair of convection cells. The arrows show the direction of the water flow.

upon the trajectory of a particle immersed in the cell except to move it uniformly in the direction of the wind. We may consider the problem as essentially two dimensional.

A small body whose density is the same as that of the water will have no tendency to settle. Therefore its trajectory will lie along the streamlines of Fig. 1—in other words it will simply move along with the water. Particles whose densities are greater than that of the water will tend to sink with a constant settling velocity, and if placed in still water they will tend to settle vertically downward in straight line trajectories. In case of convective motion, as shown in Fig. 1, such a heavy particle introduced at point *A* cannot rise along the streamline as would a particle with zero settling velocity; rather, it will start to fall immediately (for example, along the dotted line), and, if heavy enough, will fall through the bottom of the convective layer. A particle with a smaller settling velocity, if placed initially at point *A*, will begin to rise at first because the rising current is strong there, but eventually it will be swept horizontally to a region where the upward velocity is less and hence it will fall away (for example, along the dashed line).

One naturally wonders if this particle will be caught up again into the ascending current and so execute an endless number of circuits, or whether it is bound eventually to fall out of the layer. As a matter of fact, one begins to wonder whether all particles with a settling velocity—no matter how small—will eventuallly fall out of the convective layer. This is not the case, however, as will be shown in the following section.

THEORETICAL TREATMENT

A system of axes is fixed to the layer as shown in Fig. 1. The coordinates of a particle are (x, y), both of which are functions of the time t; $x(t)$, $y(t)$.

The horizontal velocity of the particle is dx/dt, the vertical velocity dy/dt. The fluid motion may be specified by a stream function ψ, which, in general, is quite arbitrary. The horizontal and vertical velocities of the water are $\partial\psi/\partial y$ and $-\partial\psi/\partial x$ respectively.

If the settling velocity is denoted by V, we know that

$$\frac{dx}{dt} = \frac{\partial\psi}{\partial y}; \frac{\partial y}{\partial t} = -\frac{\partial\psi}{\partial x} - V.$$

The trajectories may be specified by a function ψ_1 where

$$\frac{dx}{dt} = \frac{\partial\psi_1}{\partial y}; \frac{dy}{dt} = -\frac{\partial\psi_1}{\partial x}.$$

The function ψ_1 is therefore given by the following expression

$$\psi_1 = \psi + Vx.$$

The trajectory of a particle is a curve along which ψ_1 is constant. The result is quite general. In the case of convection cells the simplest stream function to consider is

$$\psi = \psi_0 \sin x \sin y.$$

The streamlines of Fig. 1 were constructed from this function.

In this case the trajectories are given by

$$\frac{\psi_1}{\psi_0} = \sin x \sin y + Rx,$$

where the quantity R is defined by

$$R = V/\psi_0.$$

The trajectories shown in Figs. 2, 3, and 4 were constructed from this function.

DISCUSSION OF RESULTS

It is convenient to discuss the trajectories for different settling velocities in terms of the quantity R: the ratio of the settling velocity to the maximum upward water velocity.

The case where R $\gg$ 1. If R is quite large the trajectories are essentially straight vertical lines. As R approaches unity the trajectories become curved. Fig. 2 is drawn for the limiting case $R = 1$. The particles all settle out, but as they approach the region of maximum upward motion they diverge and fall more slowly, as if to avoid that region.

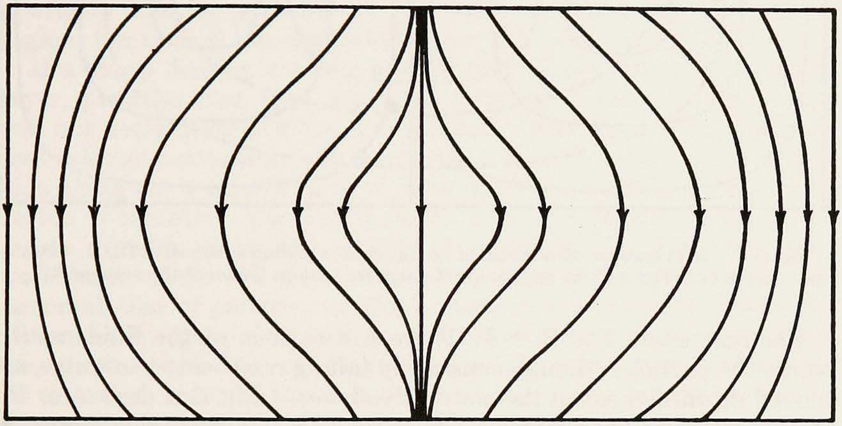

Figure 2. The trajectories of small bodies whose sinking velocity is just sufficient to insure that they all settle out.

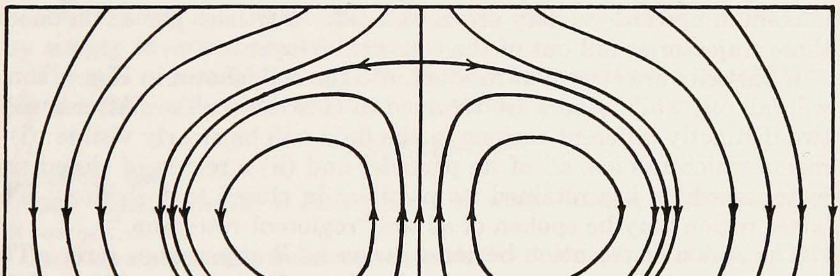

Figure 3. The trajectories of small bodies whose sinking velocity is just one-half of those in Fig. 2. The closed trajectories lie within a region conveniently called the region of retention.

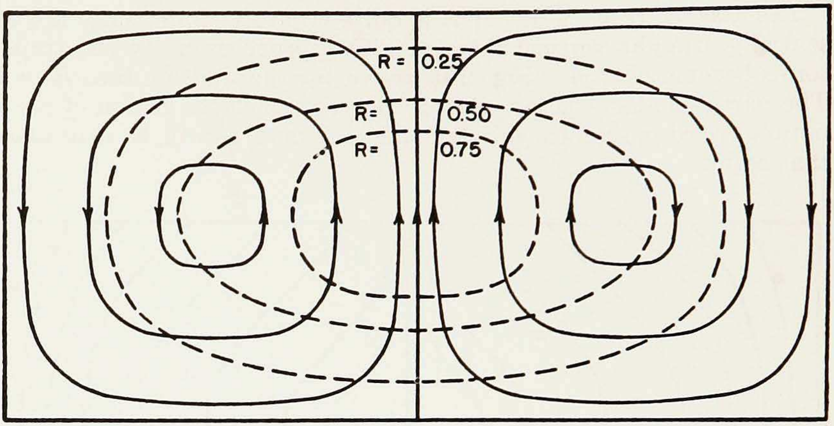

Figure 4. Solid lines are streamlines of the convective motion as shown in Fig. 1. Dashed lines are the boundaries of the regions of retention for various values of the number R.

The case where 1> R >0: There are regions of the fluid motion where the particles, though constantly falling relative to the water, are moved upward through the convective layer. Fig. 3 is drawn for the case $R = 0.5$. The interesting feature of this figure is that there are closed trajectories in the upward current. A particle placed on one of these trajectories makes an endless number of circuits and does not fall out of the convective layer despite the fact that it is always falling relative to the fluid. The trajectories more remote from the region of maximum upward velocity are not closed. Particles placed on one of these trajectories fall out of the convective layer.

If particles are strewn at random into the cell, shown in Fig. 4, some will fall out while others are retained in closed circuits. After awhile two distinctly different regions in the fluid will be clearly visible: (i) a region which has lost all of its particles and (ii) a region of closed trajectories which has retained its particles in closed trajectories. The latter region may be spoken of as the "region of retention."

The region of retention becomes larger as R approaches zero. The boundaries of the regions of retention for values of R, equal to 0.25, 0.50, and 0.75, are shown in Fig. 4.

If particles of different settling velocities are introduced simultaneously, a sorting process ensues, in which heavier ones are concentrated near to the upward water currents and lighter ones extend to greater distances from the upward currents.

The population density of particles of a given R will be zero outside the region of retention, but it will not be uniform within that region.

Near the upward currents the space velocity of the particles is smaller than in other portions of the retention region, and therefore, from the principle of continuity, the density of population will be greater in the center of the regions of retention.

In actual convection cells there is always some degree of turbulence present, so that there will always be some turbulent exchange across trajectories of particles. In such a case a region of retention must gradually lose its particles by turbulent exchange with the outside region, the rate of loss depending upon the degree of turbulence.

If a sharp density increase occurs directly beneath the convective layer, particles that have a settling velocity in the convective layer will not necessarily sink through the underlying water. In nonturbulent cellular convection this would mean that there was an accumulation along the x-axis (Fig. 1) of those particles which were not in the region of retention, the accumulation being particularly dense on the x-axis directly under the regions of retention. In the presence of turbulence there will be a slight turbulent exchange between this accumulation of particles on the bottom of the convective layer and those in the region of retention. On the whole, an equilibrium would be established with a certain fraction of the population in the region of retention, most of the remainder on the x-axis directly under the region of retention, and a very small fraction moving en route between these two centers of population through the "clear" region.

Particles less dense than water have an upward velocity relative to the water (e. g., pelagic sargassum, providing settling is independent of depth). They cannot escape through the free surface. It would be evident that the preceding discussion may be applied to this case by simply inverting the picture.

There is a possibility that the method here advanced might be applicable to problems in convective clouds.

REFERENCES

LANGMUIR, IRVING

1938. Surface motion of water induced by wind. Science, *87* (2550): 119–123.

WOODCOCK, A. H.

1941. Surface cooling and streaming in shallow fresh and salt waters. J. Mar. Res., *4* (2): 153–161.