NOTES ON A THEORY OF THE THERMOCLINE[1]

By

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INTRODUCTION

The very large number of bathythermograms which have been taken during the last few years have established the essential features of the temperature structure in the upper layers of the ocean. When these layers are heated during a period of calm, the bathythermograms show a marked temperature gradient all the way to the surface, as shown in Fig. 1. If subsequently a strong wind starts to blow, the upper layers are stirred until an almost homogeneous layer is formed, bounded beneath by a region of marked temperature gradient, the thermocline. The transition from the homogeneous layer to the thermocline is usually sudden. If the wind increases in intensity the thermocline moves downward, but the characteristic *shape* of the temperature-depth curve remains essentially unchanged.

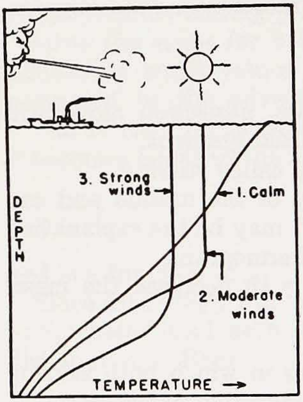

Figure 1. Schematic presentation of the development of a wind stirred layer and thermocline.

It has been customary to interpret the wind-stirring effect in terms of Ekman's famous theory for wind driven currents on a rotating globe (Ekman, 1905; Sverdrup, *et al.*, 1942: 492–494). Ekman assumes *homogeneous* water and a constant value[3] for the eddy

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[3] Ekman (1905) also gives a solution for the case where the eddy viscosity is proportional to the shear. This does not modify the essential meaning of our remarks.

viscosity, and obtains for a solution current vectors whose azimuths vary linearly and whose magnitudes decrease exponentially with depth from the very surface downward. Therefore, the Ekman spiral does not contain the concept of a homogeneous upper layer with a well defined lower boundary. Nevertheless, this concept has often been brought in by means of the artifice of identifying the thickness of the mixed layer with a depth where the current equals $e^{-\pi}$ times the surface current and is directed opposite to it.

The difficulty can be resolved, at least qualitatively, by taking into account the effect of the density stratification on the scale of turbulence. Ekman (1905) stated:

> It is obvious that [the eddy coefficient] cannot generally be regarded as a constant when the density of the water is not uniform within the region considered. For [the eddy coefficient] will be greater within the layers of uniform density and comparatively small within the transition-layers where the formation of vortices must be much reduced owing to the differences of density.

The reduction in the vertical eddy conductivity in the thermocline leads to high resistance to heat flux from above, with ". . . the paradoxical result that the water-masses below . . . are the more 'protected' against heating the stronger the heating . . ." (Helland-Hansen, 1930: 44). Surface heating therefore tends toward sharpening the existing temperature gradient in the thermocline and further reducing the turbulence. At the same time, the reduction in turbulence permits relatively easy slippage of water layers with respect to one another, and the resulting shear in the thermocline represents a source for producing turbulent energy. The ultimate outcome must be a steady-state distribution which represents a balance between the stabilizing influence of temperature gradients and the unstabilizing influence of current gradients. It is this steady-state solution which is being sought.

The foregoing discussion indicates how intimately the problems of current and of temperature distributions are related. Mathematically they are linked by the coefficients of eddy viscosity and conductivity, both of which are functions of the gradients of temperature *and* current. For the eddy viscosity such a function has been introduced by Rossby and Montgomery (1935); for the eddy conductivity a new relationship will be derived. These functions will be substituted into the differential equations governing the distributions of temperature and current, and the differential equations solved *simultaneously*.

Ekman was well aware of the desirability of incorporating these thermodynamic considerations into his dynamic treatment of ocean currents. He wrote that ". . . this is however a complication which cannot be taken into account in the general theory . . ." We have been fortunate to have at our disposal the new Differential Analyzer of the University of California. With this computer, numerical solutions have been found for 21 examples; 14 examples correspond to actual conditions for which complete observational material was available, and which were in agreement with the assumptions underlying the theory. The most restrictive of these assumptions is that the net heat flux be directed downward and that there be no convective stirring such as might be induced by surface cooling due to evaporation.

EQUATIONS OF TURBULENT FLUX UNDER STABLE CONDITIONS

The downward flux, per unit area, of momentum, of heat and of salt, due to turbulent processes, equal respectively

$$F_M = - A_V V', \quad F_T = - c_p A_T T', \quad F_S = - A_S S', \qquad (1 \text{ a, b, c})$$

where c_p is the specific heat at constant pressure, A_V, A_T, and A_S the dynamic eddy coefficients (dimensions mass length^{-1} time^{-1}) of viscosity, conductivity and diffusivity, respectively. The accents (′) denote differentiations with respect to depth:

$$V' = \partial V/\partial z, \qquad T' = \partial T/\partial z, \qquad S' = \partial S/\partial z, \qquad (2)$$

positive z being directed downward. It is customary to refer to V' as shear and to

$$\tau = - F_M \qquad (3)$$

as stress.

The coefficients A_V, A_T and A_S are not physical constants of the fluid but depend upon the state of its motion and may vary with time and position. Two factors tend to reduce the values of the turbulence coefficients: boundaries and stable stratification. Since the effect of the boundary is limited to the upper few meters of the ocean, we shall be concerned only with stability.

Let the subscript "*o*" denote the case of *neutral stability*. The ratio A/A_o for any one of the three coefficients in equation (1) is assumed to depend explicitly on stability, shear and gravity:

$$A/A_o = f\,(E, V', g), \qquad (4)$$

where the stability E depends upon the temperature and salinity distributions

$$E = E\,(T', S', T, S), \qquad (5)$$

and these distributions vary with depth and time. The theory of the wind-stirred layer is contained in the solution $T(z)$ of equations (4), (5) and the three equations (1).

Stability, Temperature and Salinity Gradients. It is convenient, whenever possible, to express the stability as a function of temperature gradient or salinity gradient only:

$$E = a\,T' \qquad \text{or} \qquad E = b\,S'. \tag{6}$$

To examine the implications of equation (6) and to determine the coefficients a and b, we make use of equation XII, 12–14, in Sverdrup, *et al.* (1942), according to which

$$E = \frac{1}{\rho}\frac{\partial \rho}{\partial z} \doteq 10^{-3}\frac{\partial \sigma_t}{\partial z} \tag{7}$$

except at great depth. Expanding by partial differentiation one finds

$$E = 10^{-3}\left(\frac{\partial \sigma_t}{\partial T}T' + \frac{\partial \sigma_t}{\partial S}S'\right) = 10^{-3}\left(\frac{\partial \sigma_t}{\partial T} + \phi\frac{\partial \sigma_t}{\partial S}\right)T', \tag{8}$$

or

$$E = 10^{-3}\left(\frac{1}{\phi}\frac{\partial \sigma_t}{\partial T} + \frac{\partial \sigma_t}{\partial S}\right)S', \tag{9}$$

where, provided S and T are functions of z only,

$$\phi = \frac{S'}{T'} = \frac{dS}{dT} \tag{10}$$

is the slope of the line denoting the T-S relationship of the water mass. Comparison between equations (6) and (8) or (9) gives

$$a = 10^{-3}\left(\frac{\partial \sigma_t}{\partial T} + \phi\frac{\partial \sigma_t}{\partial S}\right),\quad b = 10^{-3}\left(\frac{1}{\phi}\frac{\partial \sigma_t}{\partial T} + \frac{\partial \sigma_t}{\partial S}\right). \tag{11}$$

To simplify computations, the parameters a or b will be considered independent of depth in subsequent integrations. This involves the assumptions that over the range of integrations the T-S relationship is linear, with variations in temperature and salinity small. The errors introduced by these assumptions are generally small compared to those inherent in Ekman's assumption of a constant eddy viscosity.

TABLE I. VALUES OF $\partial\sigma_t/\partial T$

T \ S	33°/oo	35°/oo	37°/oo
0 °C	−.045	−.055	−.060
10 °C	−.170	−.172	−.175
20 °C	−.260	−.262	−.265

TABLE II. VALUES OF $\partial\sigma_t/\partial S$

T \ S	33°/oo	35°/oo	37°/oo
0 °C	.806	.806	.807
10 °C	.779	.780	.780
20 °C	.760	.761	.763

Effect of Stability and Shear on Eddy Coefficients. According to the Π-theorem in dimensional analysis, the quantities E, V', and g, on which the ratio of eddy coefficients was assumed to depend (equation 4), can be combined into only one type of nondimensional number, the Richardson number r (Brunt, 1939: 237):

$$r = g\,E/(V')^2. \tag{12}$$

In selecting a particular functional relationship we are guided by the limiting conditions

$$A \rightarrow A_o \text{ for } r \rightarrow 0, \tag{13}$$

$$A \rightarrow 0 \text{ for } r \rightarrow \infty. \tag{14}$$

Equation (14) states that for very high stability the flux of momentum heat and salt takes place by molecular processes only.

A simple set of equations which satisfy (13) and (14) is

$$A_V = A_o\,(1 + \beta_V\, r)^{-n_V}, \tag{15}$$

$$A_T = A_o\,(1 + \beta_T\, r)^{-n_T} = A_S, \tag{16}$$

where the eddy coefficients for *neutral* stability

$$A_{V,\,o} = A_{T,\,o} = A_{S,\,o} = A_o \tag{17}$$

have been assumed equal,[4] and where β_V, n_V, β_T, n_T are positive constants, not yet determined. The coefficients pertaining to heat flux and salinity diffusion are assumed to equal one another but to differ from the coefficient pertaining to momentum flux (see below).

Equation (15) is a relationship derived by Rossby and Montgomery (1935). It will be shown that the constants in (16) can be evaluated from known relationships, and that the resulting equations are in agreement with observations.

DETERMINATION OF CONSTANTS

Application of Investigations by Rossby and Montgomery and by Sverdrup. From energy considerations, Rossby and Montgomery (1935) derived an equation identical in form to equation (15), with $n_V = \frac{1}{2}$. Sverdrup (1936), on the basis of his measurements over a snow field at Spitzbergen, estimates β_V at 10 to 13, and probably nearer the lower limit. We shall adopt the value $\beta_V = 10$.

[4] "In recent years a great deal of work on the relationship between A_s and A_V has been done in the field of Aeronautics. The assumption of Reynold and Prandtl that $A_s = A_V$ has been confirmed in all cases when the fluid is homogeneous" (Taylor, 1931, notation ours).

Jacobsen's Investigation. From his extensive analysis of salinity and current observations in Randers Fjord and Schultz Ground, Jacobsen (1913) postulates that under stable conditions the turbulent elements give off their momentum rapidly to their surroundings, whereas other properties, such as temperature and salinity, are exchanged slowly. The elements are therefore moved to new surroundings by gravitational forces before equalization can take place. As a consequence, ". . . the effect of stability on turbulence is two-fold. In the first place, the turbulence is reduced, leading to smaller values of the eddy viscosity, and, in the second place, the type of turbulence is altered in such a manner that the accompanying eddy diffusivity becomes smaller than the eddy viscosity" (Sverdrup, *et al.*, 1942: 477).

The first conditions, $A_V \leqslant A_o$, is already satisfied by equation (15); the implication of the other condition,

$$X_j \equiv A_S/A_V \leqslant 1, \tag{18}$$

remains to be investigated. Substituting from equations (15) and (16) gives

$$X_j = (1 + \beta_V\, r)^{n_V}\, (1 + m\,\beta_V\, r)^{-n_T}, \tag{19}$$

where

$$m = \beta_T/\beta_V. \tag{20}$$

For $r = 0$ one finds

$$X_j = 1,\ d\,X_j\,/d\,r = -\,\beta_V\, n_T\,(m - n_V/n_T), \tag{21}$$

so that

$$m > n_V/n_T, \tag{22a}$$

in order for Jacobsen's condition (18) to be satisfied (Fig. 2). The special case

$$m = n_V/n_T, \tag{22b}$$

for which X_j approaches 1 asymptotically as r approaches zero, seems more likely from a physical point of view than the general case for which $d\,X_j\,/d\,r$ has a finite value at $r = 0$.

For very large values of r,

$$X_j \doteq m^{-n_T}\,(\beta_V\, r)^{n_V - n_T}, \tag{23}$$

and

$$n_T > n_V, \tag{24}$$

in order for (18) to be satisfied. Equations (22) and (24) impose certain restrictions upon the numerical values of n_T and β_T.

Taylor's Investigation. Further restrictions are imposed by an entirely different condition proposed by Taylor (1931):

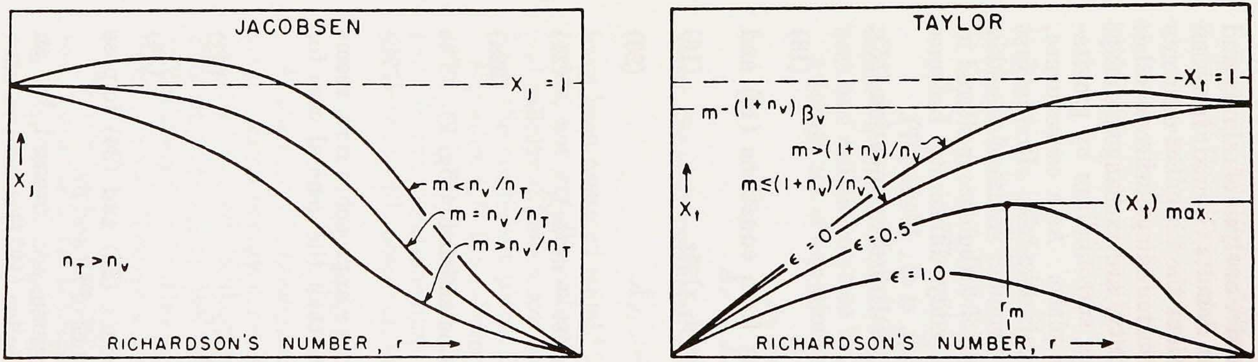

Figure 2. Limitations imposed by Jacobsen's and by Taylor's conditions upon the constants in equations (15) and (16). See text.

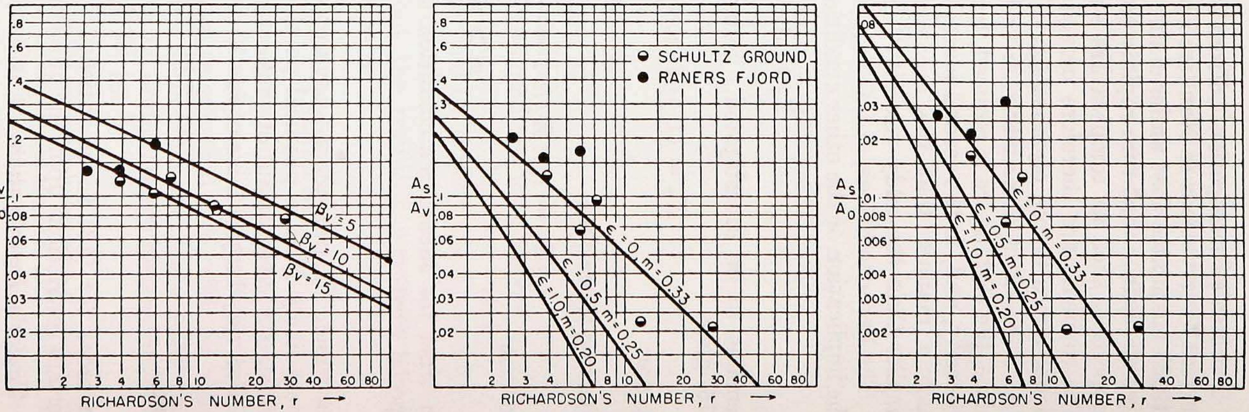

Figure 3. Comparison of theoretical relationships between eddy coefficients and Richardson's number with that derived from Jacobsen's observations of current and salinity. The left part of the figure confirms the value of β_V obtained by Sverdrup from meteorological measurements over a snow field. The central and right parts are consistent with values of ϵ and m derived from physical considerations.

In the case of a stratified fluid with density increasing downward any disturbance to steady laminar flow necessarily involves an increase in gravitational potential energy. It is this increase which causes infinitely small disturbances of a fluid at rest to be stable. In cases of turbulent flow the rate at which energy flows into a layer of the fluid owing to the action of the fluid above and below it must be greater than the rate at which gravitational energy increases in the same portion of the fluid.

Taylor shows that the fraction of available energy which is used to increase gravitational potential energy equals $r\,A_T/A_V$, and his argument requires therefore that

$$X_t \equiv r\,A_T/A_V \leqslant 1. \tag{25}$$

Again returning to (15) and (16) one finds

$$X_t = r\,(1 + \beta_V\,r)^{n_V}\,(1 + m\,\beta_V\,r)^{-n_T}. \tag{26}$$

For very small values of r, equation (26) reduces to $X_t \doteq r$, independent of β_T and n_T (Fig. 2). For large values of r,

$$X_t \doteq m^{-n_T}\,\beta_V{}^{n_V - n_T}\,r^{1+n_V-n_T}, \tag{27}$$

so that $n_T \geqslant 1 + n_V$, or

$$n_T = 1 + n_V + \varepsilon, \quad \varepsilon \geqslant 0, \tag{28}$$

in order for Taylor's condition (25) to be satisfied.

In general X_t reaches a maximum, to be designated by $(X_t)_{\max}$, for $r = r_m$ (Fig. 2), where r_m is a root of the equation

$$m\,\varepsilon\,\beta_V{}^2\,r_m{}^2 + \beta_V\,[m\,(n_V + \varepsilon) - (1 + n_V)]\,r_m - 1 = 0. \tag{29}$$

As long as Taylor's condition holds for $(X_t)_{\max}$ it certainly must hold for any other values of X_t, and we may express equation (25) in the more definite form

$$(X_t)_{\max} \leqslant 1. \tag{30}$$

For the special case

$$\varepsilon = 0, \text{ and } m \leqslant (1 + n_V)/n_V, \tag{31}$$

a true maximum does not exist, but X_t remains bounded (Fig. 2). The term $(X_t)_{\max}$ in equation (30) is then interpreted as $m^{-(1+n_V)}\,\beta_V{}^{-1}$, the value of X_t at $r = \infty$. The special conditions stated in equation (31) appear more reasonable than any of the other possible combinations, as these other combinations require that, with increasing stability, the fraction X_t of the available energy going into potential energy should first increase and then decrease.

Equations (28), (29), and (30) define a minimum value for m for any fixed value of ε. Limitations imposed by Jacobsen's and by Taylor's conditions on the constants n_T and β_T are summarized in Table III.

Taylor's condition imposes the more severe restriction on n_T, whereas Jacobsen's condition imposes the more severe restriction on m, or β_T.

TABLE III. LIMITS TO CONSTANTS IMPOSED BY JACOBSEN'S AND BY TAYLOR'S CONDITIONS

Constant	n_V	n_T		ε	n_T	m	
Source	Rossby	Jacobsen	Taylor	Assumed	—	Jacobsen	Taylor
Equation	—	(24)	(28)	—	(28)	(22a)	(28) (29) (30)
Values	0.5	$\geqslant 0.5$	$\geqslant 1.5$	0.0*	1.5	$\geqslant 0.33$†	$\geqslant 0.22$
—	—	—	—	0.5	2.0	$\geqslant 0.25$	$\geqslant 0.10$
—	—	—	—	1.0	2.5	$\geqslant 0.20$	$\geqslant 0.07$

* Preferable for reasons stated following equation (31).

† $m = 0.33$ preferable to $m > 0.33$ for reasons stated following equation (22b).

For reasons referred to in the footnotes to Table III, the combination

$$\varepsilon = 0, n_T = 1.5, m = 0.33 \qquad (32)$$

appears to be the most reasonable. Substituting from (15) and (16) gives

$$A_V = A_o (1 + \beta_V r)^{-1/2}, \beta_V = 10$$
$$A_S = A_T = A_o (1 + \beta_T r)^{-3/2}, \beta_T = \beta_V/3 = 3.33 . \qquad (34)$$

Some Numerical Values. Taylor (1931) has used Jacobsen's observations to test the validity of his criterion, and he finds condition (25) satisfied. The same observations have been used in Fig. 3, eliminating, however, the observation at 15 meters, where the shear shows a sudden change from neighboring depths which is inconsistent with the general trend.

The left part of Fig. 3 indicates the validity of equation (15), using $\beta_V = 10$. It has already been stated that this value was derived from meteorological observations; its applicability to oceanographic data may indicate that β_V represents a universal constant. The curves in the middle and right parts of Fig. 3 were computed for three values of ε and the corresponding *minimum* values of m (second column from right in Table III.) Increasing m shifts the curves toward the left away from the observations. The empirical data are consistent with the choice of values proposed in equation (32) on the basis of physical reasoning.

THE DISTRIBUTIONS OF VELOCITY AND TEMPERATURE

General Solution. The problem consists of solving simultaneously the five equations

$$\tau = A_V V', \qquad F_T = -c_p A_T T', \tag{1a, b}$$

$$A_V = A_o (1 + \beta_V r)^{-1/2}, \quad A_T = A_o (1 + \beta_T r)^{-3/2} \tag{33, 34}$$

$$r = g\, a\, T' (V')^{-2} \tag{35}$$

in the five unknowns V, T, A_V, A_T and r. Solving for r gives

$$\frac{g\,(-a)\, F_T\, A_o}{c_p\, \tau^2} = r\,(1 + \beta_V r)\,(1 + \beta_T r)^{-3/2}, \tag{36}$$

where, in general, a, F_T, A_o, and τ are all functions of z, and these functions must be specified in order that solutions $r = r\,(z)$ can be found. It will be convenient to introduce functions K and R so that equation (36) becomes

$$K\,(z) = R\,(r). \tag{37}$$

The distributions of velocity and temperature are best expressed in terms of r;

$$\frac{V'}{V_o'} = \frac{(1 + \beta_T r)^{3/4}}{r^{1/2}}, \quad V_o' = \frac{g\,(-a)\, F_T}{c_p\, A_o}, \tag{38}$$

$$\frac{T'}{T_o'} = (1 + \beta_T r)^{3/2}, \quad T_o' = -\frac{F_T}{c_p\, A_o}. \tag{39}$$

The above relationships are plotted in Fig. 4.

Depth and Sharpness of Thermocline. Before taking up the special case of the Ekman spiral, a few quite general remarks concerning the character of the thermocline can be made. These remarks are applicable to transient as well as steady-state conditions.

Two features which are usually prominent on bathythermograms are: (a) the maximum temperature gradient, $T' = T_{max}'$ at depth $z = z_g$; (b) the "knee," $T'' = T_{max}''$ at depth $z = z_c$. As a simple case, assume T_o' independent of z. It can be seen from Fig. 4 that in order for the temperature gradient to have a maximum value, r and hence $K\,(z)$ must reach a maximum. Other factors remaining equal, *the stress must be at a minimum where the temperature gradient is at a maximum. The maximum temperature gradient will be the larger, the smaller the value of the minimum stress.*

At the depth z_c of maximum curvature $T''' = 0$, or, according to (39),

$$r'' + (\tfrac{1}{2})\, \beta_T\, (1 + \beta_T r)^{-1}\, r'^2 = 0. \tag{40}$$

Combining this with the equations

$$K' = \frac{dR}{dr} r' , \tag{41}$$

$$K'' = \frac{d^2R}{dr^2} r'^2 + \frac{dR}{dr} r'' , \tag{42}$$

and substituting $\beta_T = \beta_V/3$, gives, after much manipulation,

$$K'' + QK'^2 = 0, \tag{43}$$

where

$$Q = \frac{2\,\beta_V\,(1 + \beta_T\,r)^{3/2}\,(\beta_V^2\,r^2 + 16\,\beta_V\,r - 15)}{(\beta_V^2\,r^2 + 11\,\beta_V\,r + 6)^2} \tag{44}$$

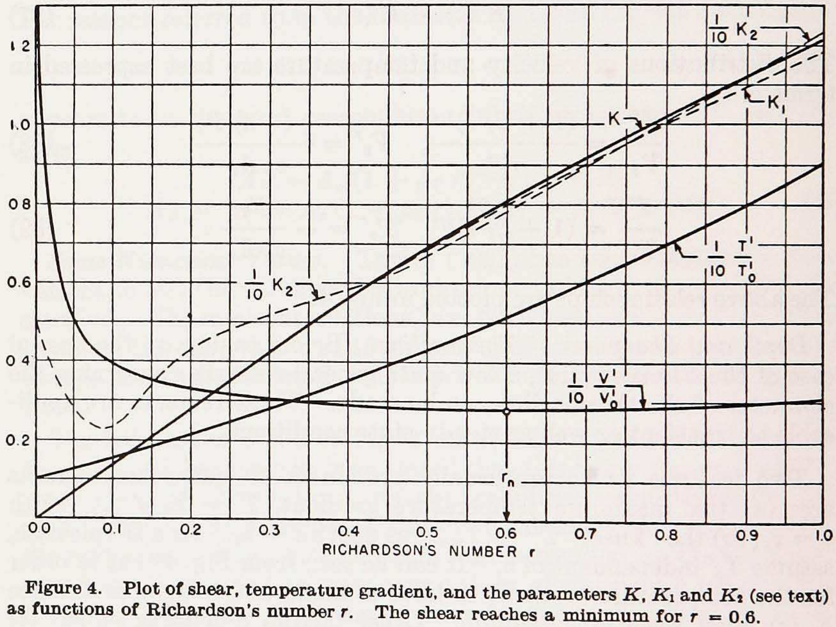

Figure 4. Plot of shear, temperature gradient, and the parameters K, K_1 and K_2 (see text) as functions of Richardson's number r. The shear reaches a minimum for $r = 0.6$.

is a function of r, or in view of (37), a function of K. The solution to (43) is

$$K_1 = C_1\,z + C_2, \tag{45}$$

where

$$K_1 = \int e^{-\int Q dK} dK$$

has been plotted in Fig. 4. Except for large values of r, $K_1 \doteq K$. *Therefore the depth* z_c *of maximum curvature in the* T(z) *curve equals approximately the depth of zero curvature in the* K(z) *curve.*

The maximum curvature can be computed from

$$T_{\max}'' = 3/2\ \beta_T\ (1 + \beta_T\ r)^{1/2}\ r'.$$

According to (40) and (45)

$$r' = \frac{K'}{R'} = \frac{C_1\, e^{\int Q dK}}{R'}$$

and

$$T_{\max}'' = K_1'\ (z_c)\ K_2\ (z_c), \tag{47}$$

where

$$K_2 = \frac{(3/2)\,\beta_T\,(1 + \beta_T\, r)^{1/2}\, e^{\int Q dK}}{R'} \tag{48}$$

is plotted in Fig. 4. Thus if $K(z)$ is known, the depth of the thermocline and the sharpness of the "knee" on the temperature-depth curve can readily be computed.

Minimum Shear Layer. The shear reaches a minimum value

$$V'_{\min} = \left[\frac{3\sqrt{3}}{2}\,\frac{g\,(-a)\,F_T\,\beta_T}{c_p\,A_o}\right]^{1/2} \tag{49}$$

when

$$r = r_n = 2/\beta_T = 0.6. \tag{50}$$

The corresponding values of A_V/A_o, A_T/A_o and K are 0.38, 0.19, and 0.81 respectively. The region of minimum shear occurs somewhat above the thermocline (Fig. 6). At greater depths the shear is larger because of the high stability which permits relatively easy slippage of water layers with respect to one another; at lesser depths the shear is larger because of the driving effect of the wind. Measurements to determine the possible existence of a minimum shear layer could serve as a critical test of this theory, especially as no corresponding feature is contained in Ekman's solutions.

The Modified Ekman Spiral. For steady-state conditions, with the frictional force balanced by the Coriolis force, the equations of motion take the form

$$\begin{aligned} \tau_x' &= \frac{d}{dz}\,(A_V\, V_x') = -\,C\, V_y \\ \tau_y' &= \frac{d}{dz}\,(A_V\, V_y') = C\, V_x\,, \end{aligned} \tag{51}$$

where

$$C = 2\omega \sin \varphi,$$

ω being the earth's rotational velocity, and φ the latitude, and where

$$\tau = \tau_x^2 + \tau_y^2, \qquad V' = V_x'^2 + V_y'^2, \qquad V = V_x^2 + V_y^2 \quad (52)$$

are the magnitudes of the stress, shear and velocity vectors, respectively. Equations (51) are identical in form with those solved by Ekman, the difference being that here A_V is a function of temperature gradient and shear, whereas Ekman treated A_V as constant, or a function of shear only.

Solutions were obtained by solving equations (1), (33), (34), (35) and (51) simultaneously on the differential analyzer. In order to have steady-state temperature distribution, the heat flux F_T was assumed independent of depth.

At the surface we set, with Ekman,

$$\tau_x = 0, \tau_y = -\tau_a \qquad \text{for } z = 0, \quad (53)$$

where τ_a is the stress of the wind on the sea surface, assuming the wind to blow along the positive y-axis. At great depths the only reasonable boundary condition is the one prescribed by Ekman, namely that the velocity vanishes. This condition cannot be reconciled with the existence of a minimum value for the velocity gradient unless the heat flux vanishes at great depth (equation 49). It appears that the distribution of temperature and current cannot both be stationary at the same time.

Without the bottom boundary conditions the solutions to the equations are no longer uniquely determined. To make the problem definite, we shall assume the components of surface current to be given by Ekman's equations

$$V_x = V_y = \frac{\tau_a}{\sqrt{2 \rho A_o C}} \text{ for } z = 0. \quad (54)$$

Values of A_o and τ_a as functions of the wind speed were read off Fig. 5.

The former are based on a comparison of Ekman's theory with observations (Sverdrup, *et al.*, 1942: 494), the latter on recent calculations[5] taking into account the roughness of the sea surface and the gustiness of the wind.

Fig. 6 shows the results of a sample calculation (run 3b, Table IV). Minimum shear is found at a depth of 15 meters, the maximum

[5] A discussion by Mr. T. Saur is in preparation.

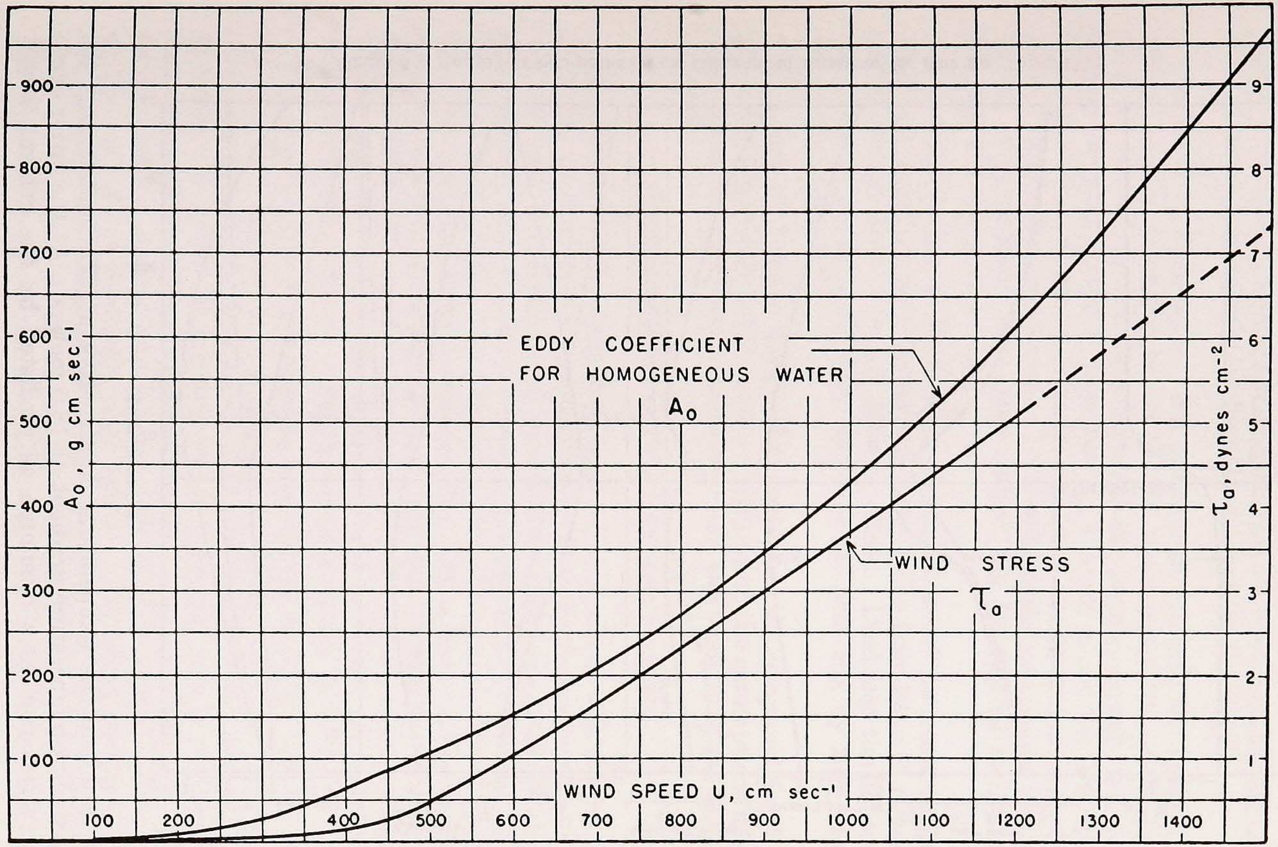

Figure 5. Relationships between wind speed at standard anemometer elevations, the wind stress per unit area of sea surface, and the eddy coefficient for homogeneous water.

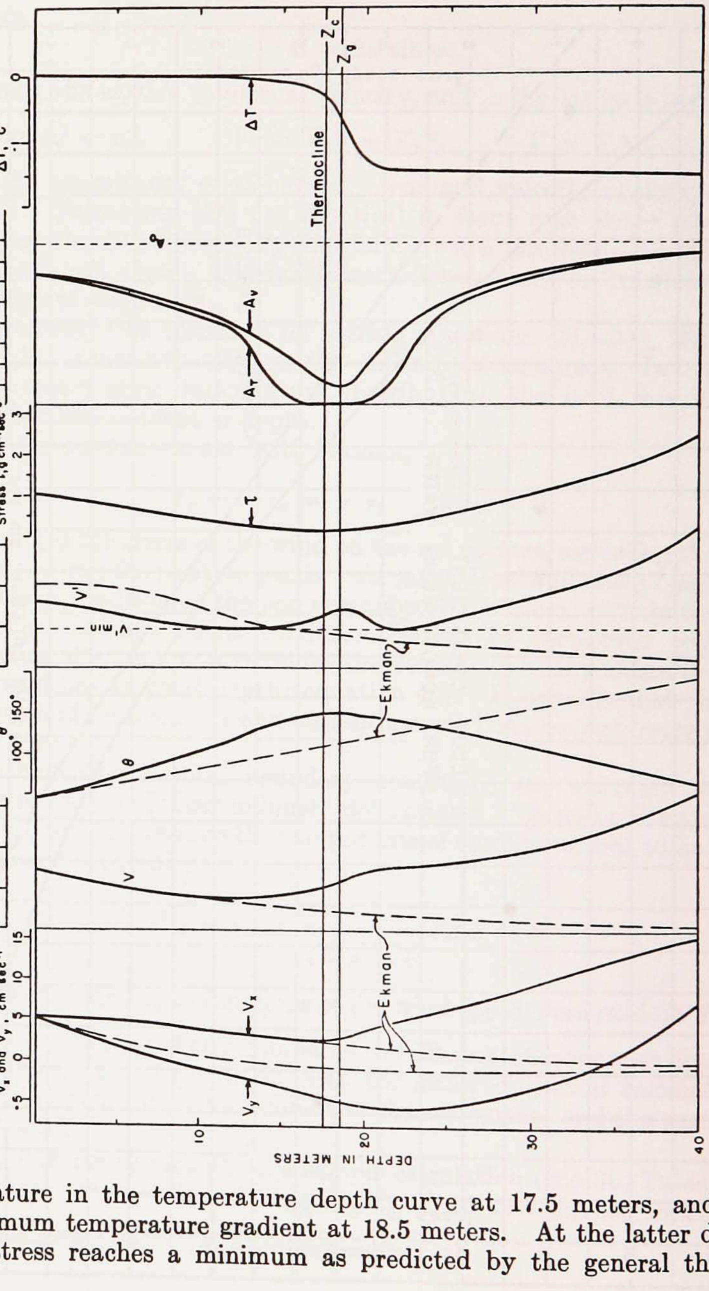

Figure 6. Results of computation on Differential Analyzer for Run 3b, Table IV.

curvature in the temperature depth curve at 17.5 meters, and the maximum temperature gradient at 18.5 meters. At the latter depth the stress reaches a minimum as predicted by the general theory.

Because of the simplifying assumption that F_T is a constant, the solution becomes increasingly inadequate the greater the depth.

Variation in Parameters. All parameters entering the differential equations and, according to our theory, the distributions of temperature and current in the upper layers of the oceans, depend upon four factors: the wind speed U, heat flux F_T, latitude φ, and the T-S correlation factor a. Some of these factors enter the equations more than once; U enters in three places. To determine the role played by each factor, computations have been carried out for typical conditions, varying one factor at a time.

The first five lines in Table IV show the results of the computations. Doubling the wind speed lowers the thermocline from 22 to 61 meters! Other factors have a much smaller effect. Doubling the latitude raises the thermocline by 40%, doubling the heat flux by 20%, doubling a by only 15%.

The next two lines in Table IV indicate that the computed depth of the thermocline does not depend critically upon the assumptions made in equation (54) as to the relative direction and velocity of the surface current.

COMPARISON WITH OBSERVATIONS

In selecting specific cases to be used for comparison with theory the following considerations were involved: (1) bathythermograph observations must be adequate; (2) evaporation relatively small; (3) heat flux positive; and (4) advection small, *i. e.*, no strong currents. The depth of thermocline was computed for: (1) a relatively clear-cut situation observed from a weather ship; (2) a broad strip in mid-Pacific, for which the computed and observed changes of thermocline depth with *latitude* could be compared; and (3) a lake at various times of the year, for which the computed and observed changes with *season* could be compared. Results are summarized in Table IV.

The weather ship was located at Lat. 49° 50′ N, Long. 145° 30′ W; the time of observation was September 28–29, 1943. Five bathythermographs taken during the 20-hour period were averaged to minimize the effect of internal waves. During this time interval there was condensation on the sea surface. The heat flux was computed from the general heat budget, following the method described on pp. 100–124 of Sverdrup, *et al.* (1942). The constant a was obtained from a hydrographic station occupied by the USS Bushnell on August 19, 1934, at Lat. 49° 29′ N, Long. 174° 31′ W. The observed depth of maximum curvature of the bathythermogram

TABLE IV. SUMMARY OF COMPUTATIONS ON DIFFERENTIAL ANALYZER

Run Number	*Wind Speed*	*Latitude*	*Heat Flux*	T–S *Correlation*	*Surface Speed*	*Wind Angle*	*Wind Stress*	*Eddy Viscosity*	*Depth z in Meters*		
	U (cm sec^{-1})	φ	$10^3 F_T$ (cal cm^{-1} sec^{-1})	$10^4 a$ (°C)$^{-1}$	$(v)_{z=0}$ (cm sec^{-1})	θ	τ (dynes cm^{-2})	A_0 (g cm^{-1} sec^{-1})	*Minimum Shear*	*Thermocline* ($T''' = 0$) *Computed*	*Observed*
	Variation of Parameters										
1h	600	30° N.	2.00	2.00	9.60	45.0°	1.02	155.0	19.6	22.0	—
2a	1200	30	2.00	2.00	24.30	45.0	5.15	619.0	58.9	61.0	—
3a	600	60	2.00	2.00	9.60	45.0	1.02	155.0	—	14.0	—
4	600	30	1.00	2.00	9.60	45.0	1.02	155.0	22.8	26.5	—
5	600	30	2.00	4.00	9.60	45.0	1.02	155.0	14.7	19.0	—
1j	600	30	2.00	2.00	9.60	48.4	1.02	155.0	18.1	21.5	—
3b	600	60	2.00	2.00	7.30	45.0	1.02	155.0	13.5	17.5	—
	Weather Ship (September, 1943)										
6	630	49	3.89	2.18	8.75	45.0	1.20	171.0	13.8	16.2	38.0
	Mid-Pacific NW–SE Strip (Average Summer)										
7	617	15	0.49	2.85	14.25	45.0	1.12	164.0	38.6	44.0	76.0
17	505	21	0.76	2.85	6.58	45.0	0.50	110.0	15.5	20.0	43.0
13	530	24	0.97	2.80	7.45	45.0	0.63	121.0	15.0	19.0	48.0
18	462	28	1.32	2.65	3.91	45.0	0.31	91.8	6.5	11.0	33.0
8	365	32	1.73	2.72	2.59	45.0	0.16	49.6	—	5.6	19.0
15	440	38	2.04	2.80	2.78	45.0	0.24	83.2	—	6.3	14.0
19	440	43	2.10	2.60	2.64	45.0	0.24	83.2	0.5	6.0	17.0
9	542	48	2.14	2.43	5.90	45.0	0.69	126.0	10.0	13.0	27.0
	Sweetwater Lake (April, May, June, July, August)										
10	350	32	1.18	1.89	2.58	45.0	0.15	43.7	3.5	7.2	6.5
14	330	32	2.28	2.07	2.63	45.0	0.14	36.6	—	4.5	6.5
11	365	32	2.00	1.33	2.59	45.0	0.16	49.6	2.3	7.5	5.5
16	325	32	2.20	2.23	2.50	45.0	0.13	35.0	—	3.0	4.0
12	300	32	1.18	1.83	2.39	45.0	0.11	27.5	2.3	4.5	4.0

trace was at 38 meters, the computed depth at 16 meters, giving a ratio of 0.43.

Computation for a mid-Pacific NW-SE strip was made for the summer season, during which evaporation is relatively small. The region is shown in the inset of Fig. 7. Values of *a* were computed from

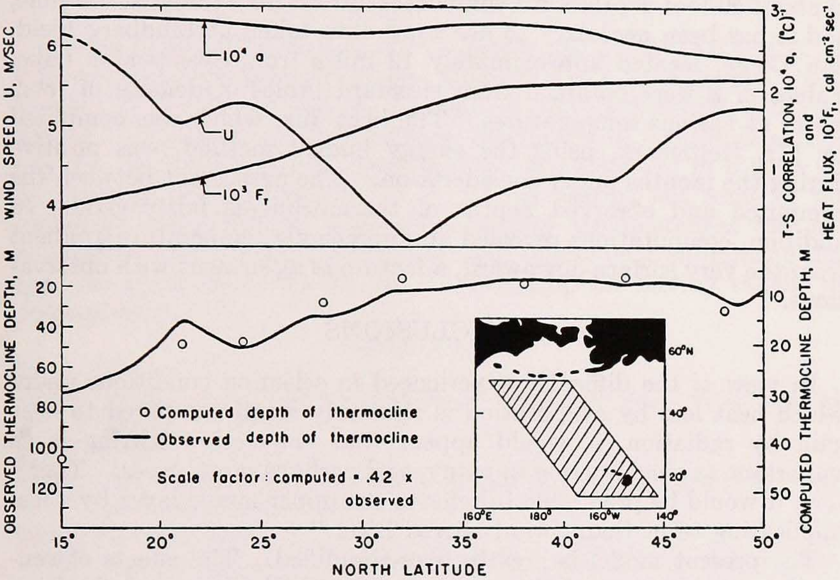

Figure 7. Comparison between computed and observed depths of thermocline as functions of latitude for the broad strip in the mid-Pacific shown in inset. The upper part gives the variations of the basic parameters entering the calculations.

selected USS Bushnell hydrographic stations. Wind speeds were obtained from pilot charts of the North Pacific. Values of heat flux for each five-degree square were kindly put at our disposal by Dr. W. C. Jacobs. These were computed on the basis of energy budget considerations, and they may be considerably in error because of uncertainties involved in some of the data. The observed depths of the thermocline were taken from a compilation by Mr. E. C. LaFond, using all data available prior to January 1945. For the extreme northern and southern portions of the area, the averages were adjusted for recent observations. Fig. 7 shows the variation with latitude of the basic parameters and of the observed and computed depths of thermocline. The computed depth is much too shallow, bearing roughly the same ratio to the observed depth as was found for the weather ship. Except for the southernmost station, the percentage *variations* in the depth of thermocline are similar for the computed and observed values.

Observations for Sweetwater Lake, California, taken during 1944 and 1945, have kindly been placed at our disposal by Mr. B. E. Holtsmark. Temperature observations were taken from a barge in water of about 50-foot depth. No wind observations were taken at the time, and it has been necessary to use wind data taken at Lindberg Field, San Diego, located approximately 12 miles from Sweetwater Lake. Values of a were obtained from standard tables of density of fresh water at various temperatures. The heat flux, which was computed by Mr. Holtsmark, using the energy budget method, was positive during the months under consideration. The agreement between the computed and observed depths of thermocline is fairly good. In addition, computations revealed an appreciable temperature gradient from the very surface downward, a feature in agreement with observations.

CONCLUSIONS

In view of the difficulty experienced in selecting conditions under which heat loss by evaporation is relatively small compared to heat gain by radiation, it would appear that convective stirring is as important as wind stirring in many, and perhaps most, cases. Therefore, it would be preferable to refer to the upper mixed layer by a less implicating term than "wind stirred layer."

The present model is greatly over-simplified. The effects of convective stirring, and of the convergence and divergence of the horizontal current field, must exert predominant influences in certain regions. If the heat flux had been assumed to decrease quasi-exponentially with depth, to take into account the absorption of radiation, more consistent results would have been obtained. In view of the close inter-relation between the temperature and current distributions, the turbidity of the water might be an important factor in the dynamics of wind currents.

Except in the case of Sweetwater Lake, where the winds were quite low, the computed depths of the thermocline are somewhat less than half the recorded depths. Perhaps this discrepancy is due to the assumed relationship $A_o(U)$ (Fig. 5) for the eddy coefficient under *indifferent* equilibrium which is based largely on measurements made when the density stratification was stable.

It has been demonstrated: that the sharp transition between the mixed layer and the thermocline can be accounted for theoretically by letting the eddy coefficients be functions of the stability and shear;

that the theory gives a thermocline depth which, although too shallow, is of the right order of magnitude; that this depth depends on wind speed, latitude, heat flux, and the *T-S* correlation, in the order stated, and that the variation with each of these factors occurs in a reasonable direction.

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