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REFLECTION OF TSUNAMIS[1]

BY

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INTRODUCTION

Waves are reflected whenever they move into a region where their speed of propagation changes. It has generally been found that for appreciable reflection a large change in speed of propagation must occur over a distance which is small compared to a wave length. Since tsunamis are shallow water waves, they are propagated at a speed c which depends on the depth h according to

$$c = \sqrt{gh} .$$

A typical tsunami, one having a period of 20 minutes for example, has a wave length of 90 miles in water of 5,000-ft. depth. At the continental slopes and walls of deep oceanic trenches, an appreciable depth change takes place over a distance which is small compared with this typical wave length, and reflection is to be expected. The object of this paper is to examine the observational evidence for tsunami reflection and to make a comparison with results which are derived theoretically.

OBSERVATIONAL EVIDENCE OF REFLECTION

Occasionally a tide gauge record of a tsunami shows a marked amplitude increase long after the arrival of the first waves. Such a feature has been considered as possible evidence of reflection if its time of occurrence coincides with the expected arrival time of waves traveling

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[1] Contributions from the Scripps Institution of Oceanography, New Series, No. 377. This work represents results of research carried out for the Office of Naval Research and the Hydrographic Office of the Department of the Navy under contract with the University of California.

[2] We would like to take this occasion to express our great appreciation for the privilege of working and studying under the direction of Dr. H. U. Sverdrup. The training and guidance which he has provided have been and will continue to be invaluable to us.

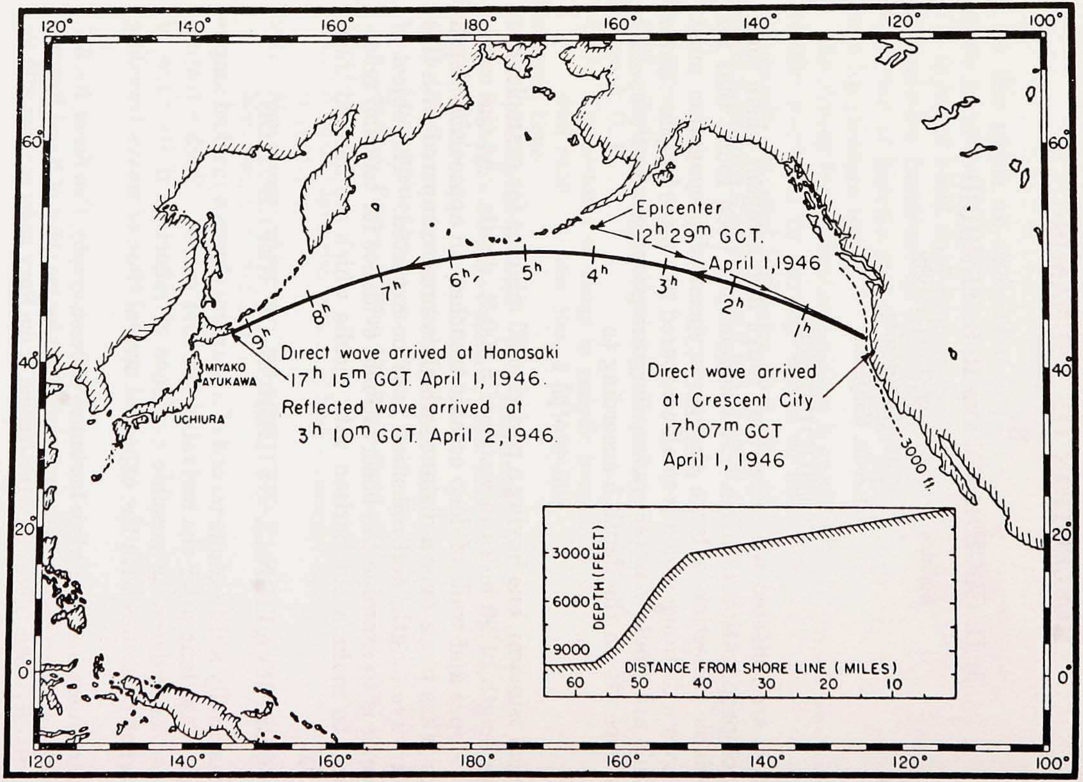

Figure 1. Arrival times at Hanasaki of direct and reflected waves for tsunami of April 1, 1946. Heavy solid line indicates probable path of reflected waves reaching Japanese Coast. Inset shows a typical bottom profile along a line from shore near Crescent City.

over a path involving at least one reflection. Amplitude variations appear frequently in records of tsunamis. The evidence for reflection is most conclusive, therefore, when the amplitude increases greatly during a short interval of time and then remains relatively large for a period of several hours.

Two records discussed by Hart (1931: 192–198) are ideal in this respect, since the tsunamis were associated with earthquakes east of New Zealand, and the records show no indication of tsunami-type waves at Sydney until the arrival of waves supposedly reflected. Of about 25 additional records examined by the present writers, those from four Japanese stations (location in Fig. 1) for the tsunami of April 1, 1946 contain indication of reflection[3] (Figs. 2–5), and of these only the evidence from Hanasaki is conclusive.

To bring out the general trend of wave height, individual wave heights were averaged in overlapping two-hour intervals centered at each of the first 20 hours of the tsunami. The arrival of a train of reflected waves may be expected to show up as a relatively steep slope in the curve of average height.

The first few waves were small in all four cases, hardly exceeding 10 cm. at most. After seven to ten hours had passed, the average heights had increased two- to seven-fold (Figs. 2–5). The most sudden increase occurred at Hanasaki, where the average height grew from 6 cm. to 24 cm. in the time interval 0200 to 0500 G. C. T. The slope of the curve of average height is steepest at 0310 G. C. T., 9 hr. and 55 min. after the arrival of the first wave. A rough refraction diagram indicates that waves could reach Hanasaki by way of a reflection at or near the coast of southern Oregon (Fig. 1). The expected time interval from the arrival of the first wave to the arrival of such a reflected wave is 9 hr. and 24 min., which is close to the interval actually observed.

THEORY

The reflecting mechanisms which are considered in this paper are the rather abrupt changes in depth embodied in the continental slopes and walls of deep oceanic trenches. As a first approximation, the size of reflected waves is calculated for a simple model defined as follows:

1) The depth changes abruptly along a straight line (Fig. 6, showing the co-ordinate system which is used).

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[3] Evidence from records and a refraction diagram received from the Japanese Central Meteorological Observatory through the Hydrographic Office, Department of the Navy. In the refraction diagram prepared by the observatory, wave trains reflected near various sections of the North American coast are shown.

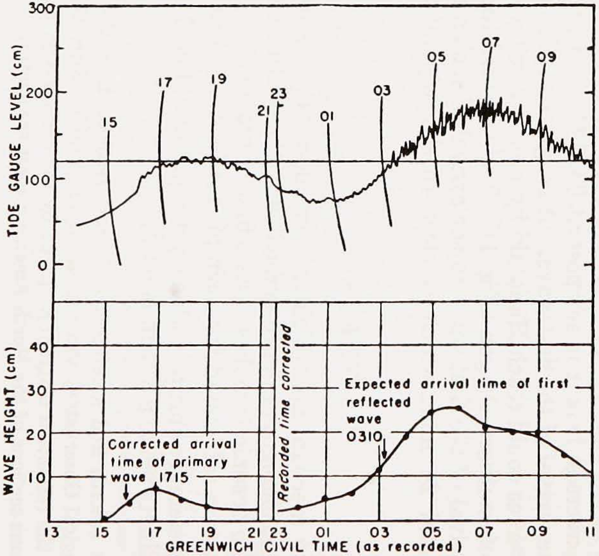

FIGURE 2. Hanasaki tide gauge record for tsunami of April 1, 1946, and graph of average wave height.

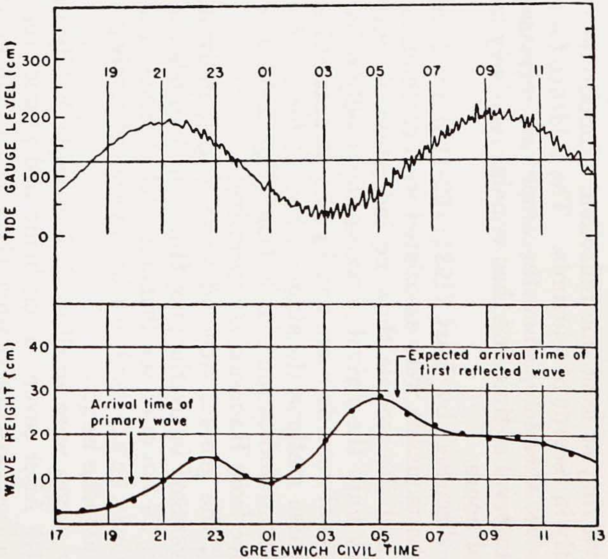

FIGURE 3. Miyako tide gauge record for tsunami of April 1, 1946, and graph of average wave height.

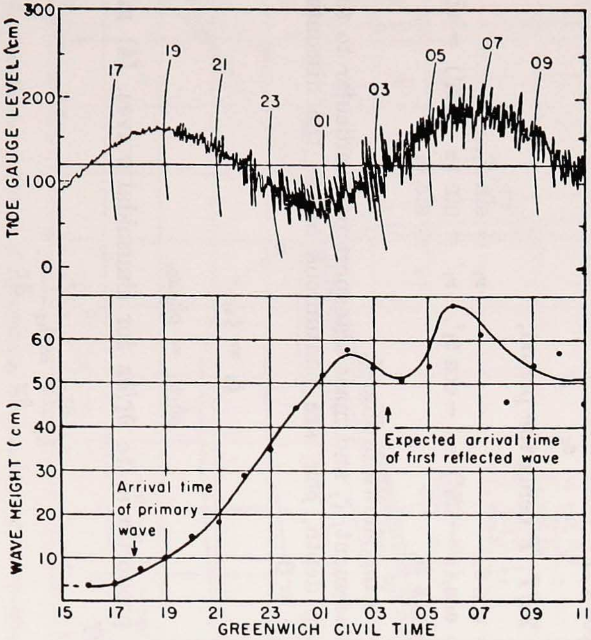

Figure 4. Ayukawa tide gauge record for tsunami of April 1, 1946, and graph of average wave height.

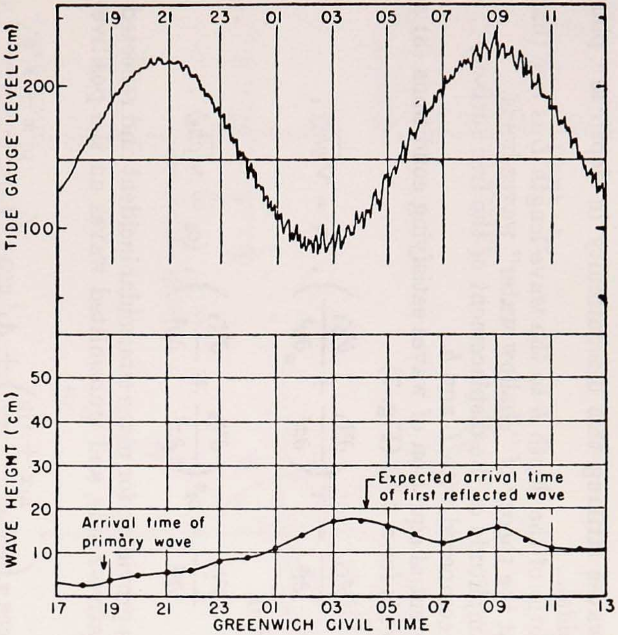

Figure 5. Uchiura tide gauge record for tsunami of April 1, 1946, and graph of average wave height.

2) The waves striking this discontinuity in depth are plane and sinusoidal.
3) The ratio of the depth h to the wave length L is less than 0.05, so that the theory of "shallow water" waves holds.
4) The amplitude of the displacement of the free surface, ζ, is very small compared with L and h.

The differential equation of waves satisfying conditions (3) and (4) of the model is for $x > 0$ (Fig. 7)

$$\frac{\partial^2 \zeta_1}{\partial t^2} = c_1^2 \left(\frac{\partial^2 \zeta_1}{\partial x^2} + \frac{\partial^2 \zeta_1}{\partial y^2} \right), \ (c_1 = \sqrt{gh_1}) \ ,$$

and for $x < 0$,

$$\frac{\partial^2 \zeta_2}{\partial t^2} = c_2^2 \left(\frac{\partial^2 \zeta_2}{\partial x^2} + \frac{\partial^2 \zeta_2}{\partial y^2} \right), \ (c_2 = \sqrt{gh_2}) \ .$$

Solutions providing for plane sinusoidal incident and reflected waves on the negative-x side, and transmitted waves on the positive-x side, are:

$$\zeta_1 = A_1 \cos \sigma \left(t + \frac{\mu_1 x + \nu_1 y}{c_1} \right) + A_1' \cos \sigma \left(t + \frac{\mu_1' x + \nu_1' y}{c_1} \right), \qquad (1)$$

$$\zeta_2 = A_2 \cos \sigma \ \left(t + \frac{\mu_2 x + \nu_2 y}{c_2} \right), \qquad (2)$$

where $\sigma = 2\pi/T$, T being the period,

and $\mu_1 = \cos \theta_1,$ $\nu_1 = \sin \theta_1,$
$\mu_1' = \cos (\pi - \theta_1') = -\cos \theta_1',$ $\nu_1' = \sin (\pi - \theta_1') = \sin \theta_1',$
$\mu_2 = \cos \theta_2,$ $\nu_2 = \sin \theta_2;$

these angles being shown in Fig. 7.

The displacement, ζ, and mass transport perpendicular to the discontinuity in depth, $\rho h u$, are continuous across the discontinuity. That is, at $x = 0$

$$\zeta_1 = \zeta_2, \qquad (3)$$

and

$$\rho h_1 u_1 = \rho h_2 u_2. \qquad (4)$$

Since u is proportional to $\partial \zeta / \partial x$ for sinusoidal waves, (4) may be expressed by

$$h_1 \frac{\partial \zeta_1}{\partial x} = h_2 \frac{\partial \zeta_2}{\partial x} \, . \qquad (4')$$

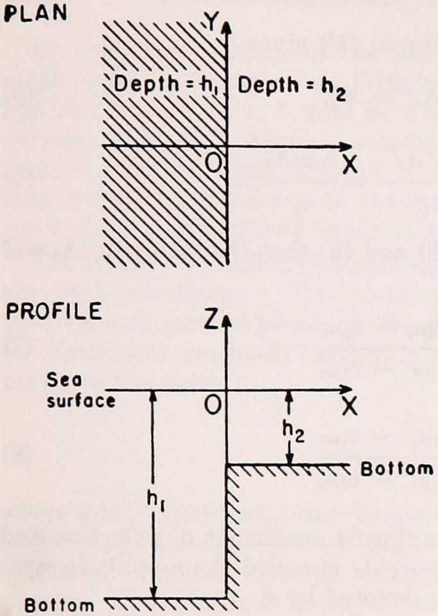

FIGURE 6. Schematic diagram showing abrupt depth change assumed in theory.

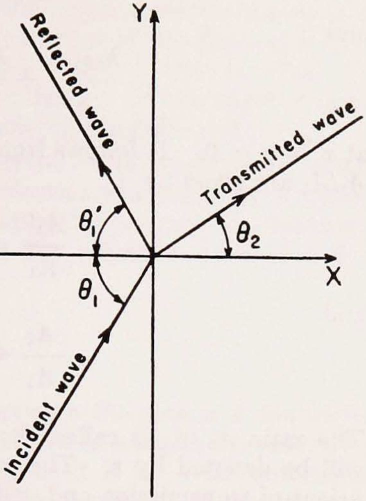

FIGURE 7. Direction of travel of incident wave, reflected wave, and transmitted wave.

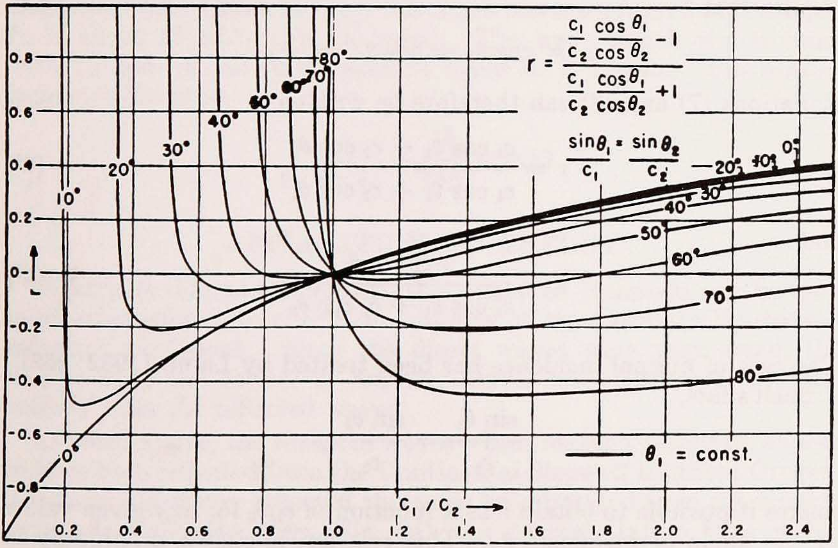

FIGURE 8. Amplitude coefficient of reflection, r, as a function of the velocity ratio, c_1/c_2, for various values of the angle of incidence θ_1.

Substituting (1) and (2) into (3) and (4′) gives

$$A_1 + A_1' = A_2, \tag{5}$$

and

$$\frac{h_1\mu_1A_1}{c_1} + \frac{h_1\mu_1'A_1'}{c_1} = \frac{h_2\mu_2A_2}{c_2}, \tag{6}$$

at $x = y = 0$. It follows from (5) and (6) that the ratios A_1'/A_1 and A_2/A_1 are given by

$$\frac{A_1'}{A_1} = \frac{c_2\mu_2 - c_1\mu_1}{c_1\mu_1' - c_2\mu_2}, \tag{7}$$

and

$$\frac{A_2}{A_1} = \frac{c_1\mu_1' - c_1\mu_1}{c_1\mu_1' - c_2\mu_2}. \tag{8}$$

The ratio A_1'/A_1 is called the amplitude coefficient of reflection and will be denoted by r. The ratio A_2/A_1 is called the amplitude coefficient of transmission and will be denoted by s.

The law of reflection,

$$\theta_1 = \theta_1',$$

implies that

$$\mu_1 = \cos\theta_1 = \cos\theta_1' = -\mu_1'.$$

Equations (7) and (8) can therefore be written

$$r = \frac{c_1\cos\theta_1 - c_2\cos\theta_2}{c_1\cos\theta_1 + c_2\cos\theta_2}, \tag{7′}$$

and

$$s = \frac{2\,c_1\cos\theta_1}{c_1\cos\theta_1 + c_2\cos\theta_2}. \tag{8′}$$

The case of normal incidence has been treated by Lamb (1932: 262).

Snell's law,

$$\frac{\sin\theta_1}{c_1} = \frac{\sin\theta_2}{c_2},$$

makes it possible to obtain r as a function of c_1/c_2 for any given values of the angle of incidence θ_1. This relation is shown graphically in Fig. 8.

An important feature of the relation which can be seen from the graph is that r changes very little between an angle of incidence of 0° and 20°, when $c_1/c_2 > 1$, that is, when the incident waves approach a decrease in depth. Another noteworthy feature of the relation is that there is a critical angle of incidence when $c_1/c_2 < 1$, that is, when the incident wave approaches an increase in depth. If the angle of incidence exceeds this critical value, total reflection is indicated ($r = 1$).

The reflection coefficient has been derived on the assumption of an abrupt depth change. The theory of reflection of sound from a region between two parallel planes where the speed of propagation increases (or decreases) gradually indicates that the reflection coefficient depends on the ratio

$$\frac{d}{L/\sin\theta},$$

where d is the perpendicular distance between the planes, L the wave length, and θ the angle of incidence. If the ratio is small, the reflection coefficient is nearly as large numerically as it is for a discontinuous change. If the ratio is large, the reflection coefficient is nearly zero. Assuming the same criterion to apply to tsunamis, the reflection coefficient calculated above (Eq. 7′) is a fairly good approximation at steep slopes. For example, at the Continental Slope off northern California (near Crescent City) the depth increases from 3,000 to 9,900 ft. in about 15 mi.[4] (Fig. 1, inset). The wave length of a tsunami with 15 min. period in a depth of 9,900 ft. is 116 mi. For normal incidence the ratio

$$\frac{d}{L/\sin\theta} = \frac{15}{116} = 0.13\ .$$

APPLICATION OF THEORY

Of various tide gauge records, that taken at Hanasaki can be used most satisfactorily to check the validity of the theoretically derived reflection coefficient. Since the direct waves were very small, the heights after the arrival of reflected waves are considered to result entirely from the reflected waves.

As noted above, the reflected waves which reached Hanasaki appear to have been reflected from the Continental Slope off southern Oregon. The height of waves reflected there can be estimated from the heights recorded at Cresent City (Fig. 9a), the nearest station from which

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[4] Miles refers to statute miles throughout this paper.

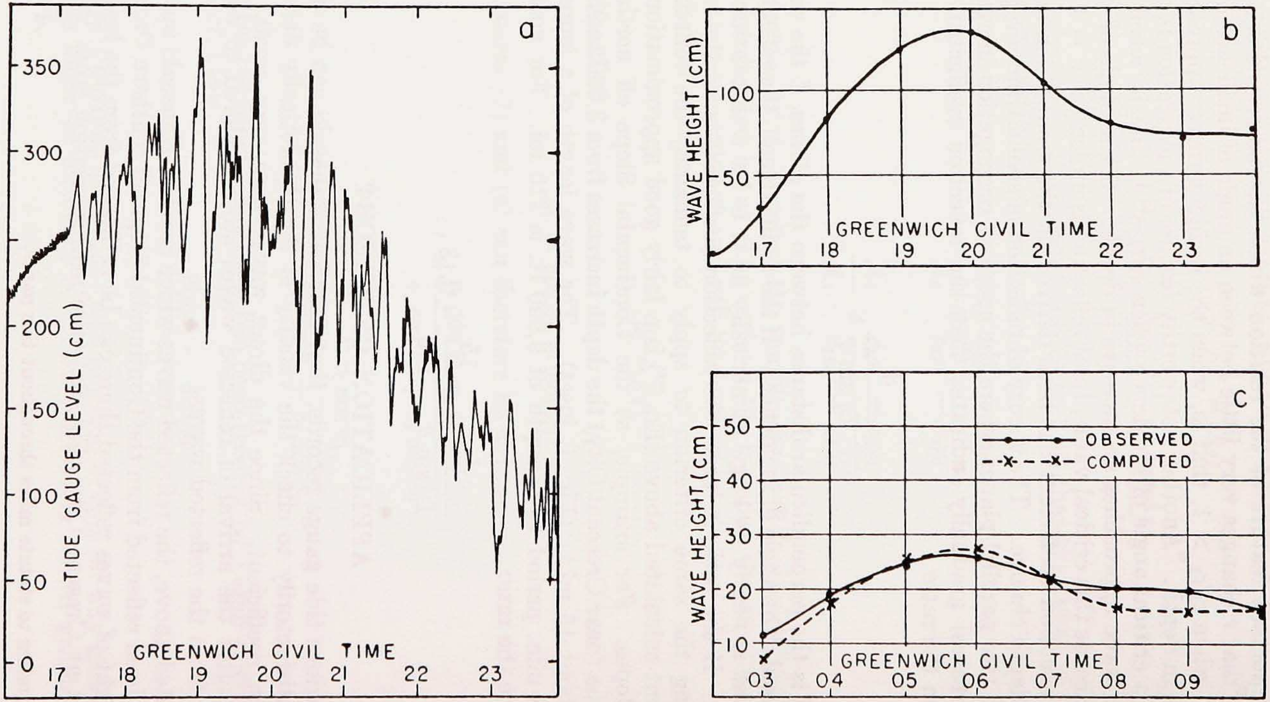

Figure 9. a. Crescent City tide gauge record for tsunami of April 1, 1946. b. Graph of average wave height for Crescent City. c. Comparison between observed and computed average wave heights at Hanasaki.

records are available. The heights averaged over two hour intervals, H_C, have been computed for each of the first 10 hours (Fig. 9b) in the same manner as they were for Hanasaki.

A typical bottom profile along a line seaward from near Crescent City is shown in the inset of Fig. 1. To facilitate computation, the bottom contours have been assumed to be straight lines parallel to the coastline. The actual contours do not differ widely from this picture.

Since the waves approach the coast almost at right angles, they diverge very little, and so Green's law (Lamb, 1932: 275),

$$\frac{H_1}{H_2} = \left(\frac{h_2}{h_1}\right)^{1/4},$$

can be used to obtain an estimate of the wave heights at 3,000 ft. from the recorded wave heights at Crescent City. The depth of the water at the tide well is assumed to be 20 ft. Thus the wave height at 3,000 ft., H_{3000}, is given by

$$H_{3000} = H_C\left(\frac{20}{3000}\right)^{1/4}. \qquad \text{(A)}$$

The height of waves reflected from the slope at 9,900 ft. is obtained by applying the theory developed above. The reflection and transmission coefficients for incidence at 10° are nearly the same as those for normal incidence, namely

$$r = 0.27,$$

$$s = 1 + r = 1.27,$$

for $c_1/c_2 = \sqrt{9900/3000}$. The height of waves reflected at 9,900 ft., H_R, is then given by

$$H_R = \frac{r}{s} H_{3000} = \frac{0.27}{1.27} H_{3000}. \qquad \text{(B)}$$

Reflection along the path (Fig. 1) of the reflected wave from the slope off southern Oregon to the Japanese Trench can be neglected, since there are no abrupt changes in depth.

The effect of distance D on the waves is obtained by using the relation

$$\frac{H_1}{H_2} = \left(\frac{D_2}{D_1}\right)^{1/3},$$

which applies to the first few waves of an impulsively generated wave train (Jeffreys, *et al.*, 1946: 485). In the present calculation D_1 is approximately 1,900 mi., and D_2 approximately 6,300 mi. The height off Hanasaki, at 9,900 ft., is given by

$$H_J = \left(\frac{1900}{6300}\right)^{1/3} H_R . \tag{C}$$

The height at the Hanasaki tide well, H_H, is obtained from H_J by Green's law. The depth of water at the tide well is again assumed to be 20 ft. Thus

$$H_H = \left(\frac{9900}{30}\right)^{1/4} H_J . \tag{D}$$

The average wave height at Crescent City during the first two hours of the tsunami is 85 cm. The calculation of the corresponding reflected wave height at Hanasaki is given in the table below:

(A) Wave height at 3,000 ft.,

$$H_{3000} = 85\left(\frac{20}{3000}\right)^{1/4} = 25 \text{ cm.}$$

(B) Height of wave reflected at 9,900 ft.,

$$H_R = 25\left(\frac{0.27}{1.27}\right) = 5.3 \text{ cm.}$$

(C) Wave height at 9,900 ft. off Hanasaki,

$$H_J = 5.3\left(\frac{1900}{6300}\right)^{1/3} = 3.6 \text{ cm.}$$

(D) Wave height at 20 ft. at Hanasaki,

$$H_H = 3.6\left(\frac{9900}{20}\right)^{1/4} = 17 \text{ cm.}$$

The travel time of the reflected wave from the Oregon Coast to Hanasaki is about 10 hours. The heights calculated in the above manner compare favorably with the average of the heights recorded at Hanasaki between 9 and 11 hours later (Fig. 9c). This close agreement is probably in part fortuitous, since many factors such as convergence and resonance have not been taken into account. The order of magnitude of the theoretical reflection coefficient, upon which the calculated height is critically dependent, seems to lead to agreement with this single set of observations.

The ratio between the average heights during the first 10 hours at Crescent City and the corresponding heights at Hanasaki remains nearly constant and serves to identify the waves at the two stations.

REFLECTION FROM CONTINENTAL SLOPES

Shepard (in press) gives 5° 20′ for the average slope and 10–20 mi. for the average range in width of the continental slopes bordering the Pacific Ocean. The depth at which the slope begins may vary from 600 to 3,000 ft. The present theory implies that the reflection coefficient at such average slopes is between 0.2 and 0.4 for normal incidence and between 0.15 and 0.3 for incidence at 45°. Thus, according to the theory, the reflected waves are of secondary but not negligible magnitude. At a given station, however, the reflected waves may be favored by convergence and be of primary magnitude. This might explain why the reflected waves are prominent only in relatively few cases.

REFLECTION FROM TRENCH WALLS

As pointed out above, the theory implies that a wave reaching a trench wall from shallower water is reflected totally if the angle of incidence exceeds a critical value. The arrival of the highest and most damaging waves at Napoopoo and Hokeena on the west coast of Hawaii about 18 hours after the April 1 earthquake (Shepard, *et al.*) can be explained by assuming that nearly total reflection occurred along the eastern edge of the Japan Trench and again at the Bonin Trench.

CONCLUSIONS

The data examined give good evidence that reflections do occur. One case has been subjected to quantitative treatment, and the observed wave heights were found to be in good agreement with those calculated on the basis of theory. The theory indicates that the reflected waves can be expected to be considerably smaller than the direct waves, except under very special circumstances. This conclusion is in agreement with observations.

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