

YALE PEABODY MUSEUM

P.O. BOX 208118 | NEW HAVEN CT 06520-8118 USA | PEABODY.YALE. EDU

JOURNAL OF MARINE RESEARCH

The *Journal of Marine Research*, one of the oldest journals in American marine science, published important peer-reviewed original research on a broad array of topics in physical, biological, and chemical oceanography vital to the academic oceanographic community in the long and rich tradition of the Sears Foundation for Marine Research at Yale University.

An archive of all issues from 1937 to 2021 (Volume 1–79) are available through EliScholar, a digital platform for scholarly publishing provided by Yale University Library at <https://elischolar.library.yale.edu/>.

Requests for permission to clear rights for use of this content should be directed to the authors, their estates, or other representatives. The *Journal of Marine Research* has no contact information beyond the affiliations listed in the published articles. We ask that you provide attribution to the *Journal of Marine Research*.

Yale University provides access to these materials for educational and research purposes only. Copyright or other proprietary rights to content contained in this document may be held by individuals or entities other than, or in addition to, Yale University. You are solely responsible for determining the ownership of the copyright, and for obtaining permission for your intended use. Yale University makes no warranty that your distribution, reproduction, or other use of these materials will not infringe the rights of third parties.



This work is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 4.0 International License.
<https://creativecommons.org/licenses/by-nc-sa/4.0/>



ATMOSPHERIC TIDES¹

By

JACOB BJERKNES

University of California, Los Angeles

The subject of atmospheric tides has been and still is a great challenge to the theoretical meteorologist. Despite the many brilliant contributions to atmospheric tidal theory from the time of Laplace up to the present, one of the essential tidal features—the surprisingly great amplitudes of the semidiurnal tide produced by the sun—still calls for a really convincing explanation. This article does not bring any refinements in existing tidal theories, but it does provide the suggestion that the model to which the theories have hitherto been applied is not sufficiently realistic. With a more true-to-nature model of the semidiurnal atmospheric tide, there seems to be a possibility for a simple explanation of the great tidal amplitude.

Fig. 1 summarizes the empirical facts about the solar harmonic components W_2^2 , Z_2 and W_4^3 as observed in the sea-level atmospheric pressure. Note the dominating character of the sectorial progressive 12h wave, W_2^2 , with its maximum pressure oscillation of ± 1.3 mb at the equator. The standing 12h oscillation, Z_2 and the tesseral progressive 8h oscillation W_4^3 are, in comparison, quite weak. A global 24h oscillation also exists, but it is so mixed up with the thermal sea- and land-breeze and with valley-mountain circulations of 24h period that it proves difficult to isolate. The evidence from pinpoint islands in the tropical Pacific without local 24h wind systems is that the global diurnal tide has an amplitude of the order one-fifth of that of the semidiurnal tide.

All lunar tidal components in the atmosphere are much weaker than the corresponding solar ones, and it is only the sectorial progressive wave, W_2^2 , that has been found empirically (Chapman, 1918). The observed lunar tide pressure oscillation amounts to ± 0.09 mb at the equator, with a decrease towards higher latitudes in the same fashion as shown for the solar W_2^2 in Fig. 1.

The contrast to the ocean tides is striking. In the oceans the moon produces a stronger tidal component than the sun because of the greater tidal force exerted by the moon. In the atmosphere that relation is reversed, although the ratio between lunar and solar tidal forces must be the same. The classical explanation of this phenomenon is based on the assumption of resonance. The atmosphere is

¹ U. C. L. A. Department of Meteorology, Papers in Meteorology, No. 11.

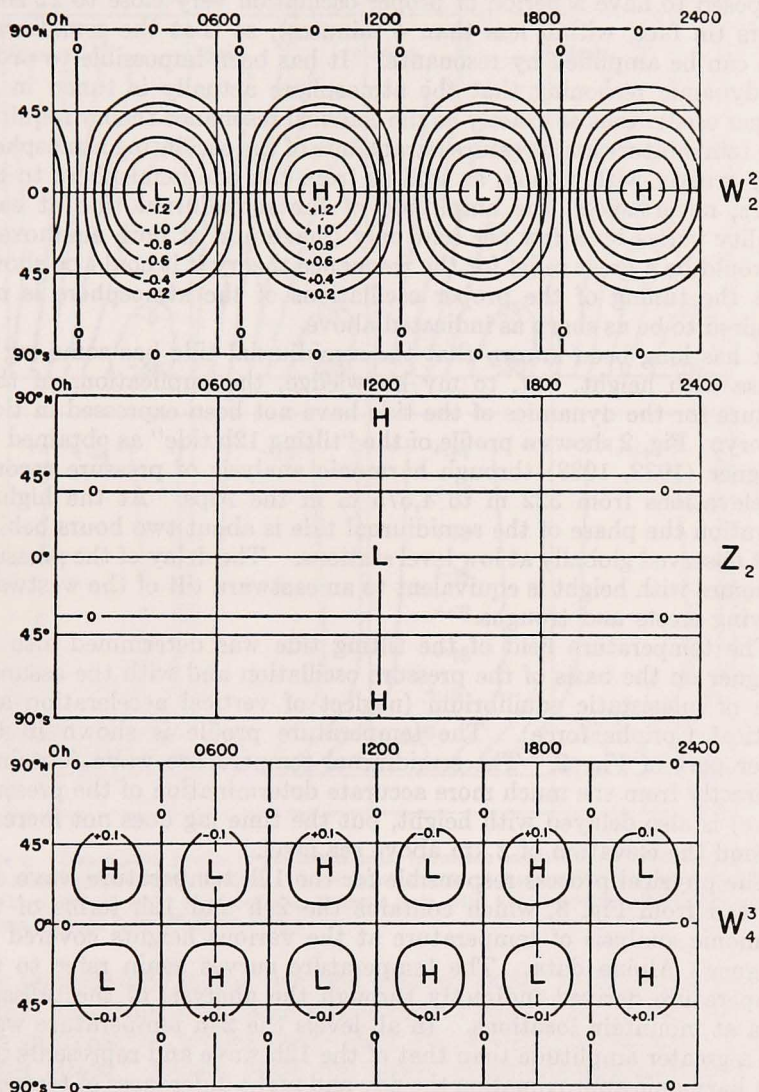


Figure 1. The planetary harmonic components of atmospheric tides in solar time as observed at sea level: W_2^2 , the progressive sectorial 12h oscillation (pressure interval 0.2 mb); Z_2 , the standing 12h oscillation, depicted at 11h or 23h Greenwich time (pressure amplitude about 0.06 mb at the equator and 0.12 mb at the poles); W_4^3 , antisymmetric part of the progressive 8h oscillation averaged from November to February (pressure interval 0.1 mb). From Bartels, 1939.

supposed to have a period of proper oscillation very close to 12 solar hours (in fact, within less than 4 minutes), so that the semidiurnal tide can be amplified by resonance. It has been impossible to prove by dynamic reasoning that the atmosphere actually is tuned in its proper oscillations as exactly as the classical resonance theory requires. The temperature and pressure parameters of the oscillating atmosphere vary greatly with season, as well as from the one hemisphere to the other; nevertheless, the amplitude of the semidiurnal tide at each locality varies by a few per cent only (maximum at both equinoxes). It would be a great relief for the resonance theory if it could be shown that the tuning of the proper oscillations of the atmosphere is not required to be as sharp as indicated above.

It has long been known that the semidiurnal tide has some lag in phase with height, but, to my knowledge, the implications of that feature for the dynamics of the tide have not been expressed in tidal theory. Fig. 2 shows a profile of the "tilting 12h tide" as obtained by Wagner (1932, 1938) through harmonic analysis of pressure records at elevations from 522 m to 4,575 m in the Alps. At the highest elevation the phase of the semidiurnal tide is about two hours behind that observed globally at low level stations. The delay of the pressure extremes with height is equivalent to an eastward tilt of the westward moving crests and troughs.

The temperature field of the tilting tide was determined also by Wagner on the basis of the pressure oscillation and with the assumption of quasistatic equilibrium (neglect of vertical acceleration and vertical Coriolis force). The temperature profile is shown in the lower part of Fig. 2. The semidiurnal temperature wave (obtained indirectly from the much more accurate determination of the pressure wave) is also delayed with height, but the time lag does not increase beyond the elevation of 2 km above sea level.

The physical process responsible for the 12h temperature wave can be read from Fig. 3, which contains the 24h and 12h terms of the harmonic analysis of temperature at the various heights covered by Wagner's Alpine data. The temperature curves again refer to the temperature derived indirectly through the analysis of the pressure data at mountain locations. In all levels the 24h temperature wave has a greater amplitude than that of the 12h wave and represents in a first harmonic approximation the day and night differences of temperature. The 12h wave of temperature is of such a phase that it changes the 24h sine function into one with a steep temperature increase in the morning hours and a more gentle decrease of temperature in the afternoon. The physical process involved, both in the 24h and in the 12h temperature wave, is obviously that of radiation, which calls for a

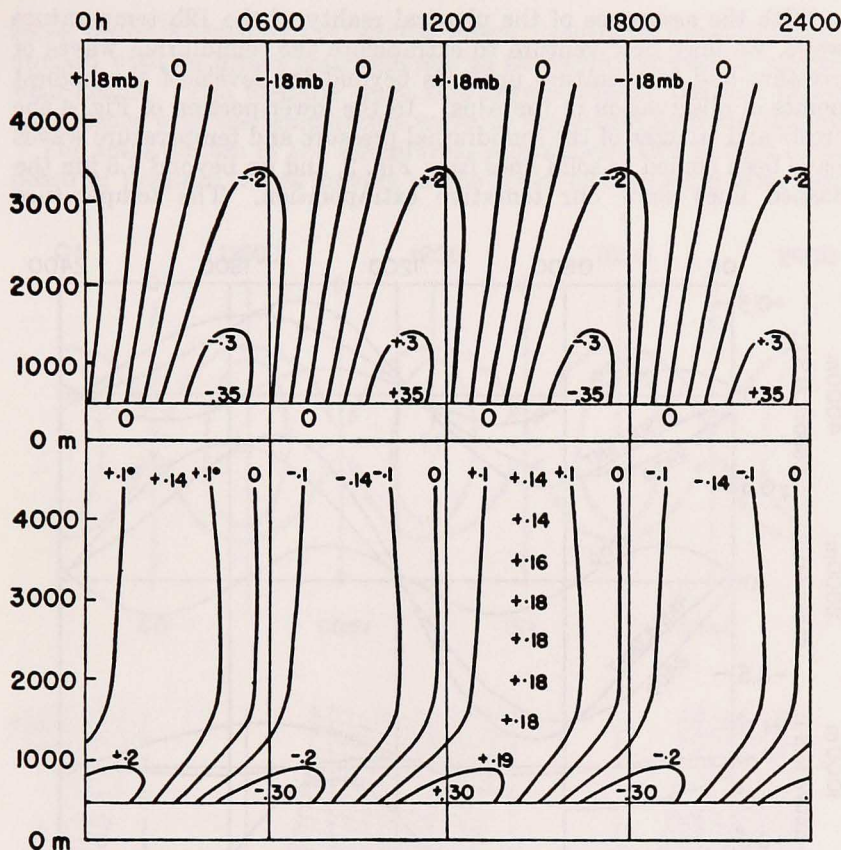


Figure 2. *Upper part:* Profile of the semidiurnal harmonic component of pressure, based on data from Alpine summit stations. *Lower part:* Profile of the semidiurnal component of temperature derived from corresponding pressure data with the assumption of quasistatic conditions. (Data from Wagner, 1932.)

higher temperature in the daytime than at night; more specifically, radiation also calls for a rapid rise in temperature after sunrise and a slower decrease of temperature after the diurnal maximum. The "sunrise effect," known from the shape of thermograms near the ground, is revealed also by the temperature curves at higher levels. In fact, the ratio of the amplitudes of the 12h and 24h temperature waves increases with height, indicating that the sunrise effect on the shape of the diurnal curve of free-air temperature becomes more accentuated with height.

With the assurance of the physical reality of the 12h temperature wave, we may now venture to extrapolate the semidiurnal waves of pressure and temperature upwards beyond the levels of the highest points of observation in the Alps. In the lower portion of Fig. 4 the crests and troughs of the semidiurnal pressure and temperature waves have been copied as solid lines from Fig. 2, and up beyond 4.5 km the dashed lines show our tentative extrapolation. The temperature

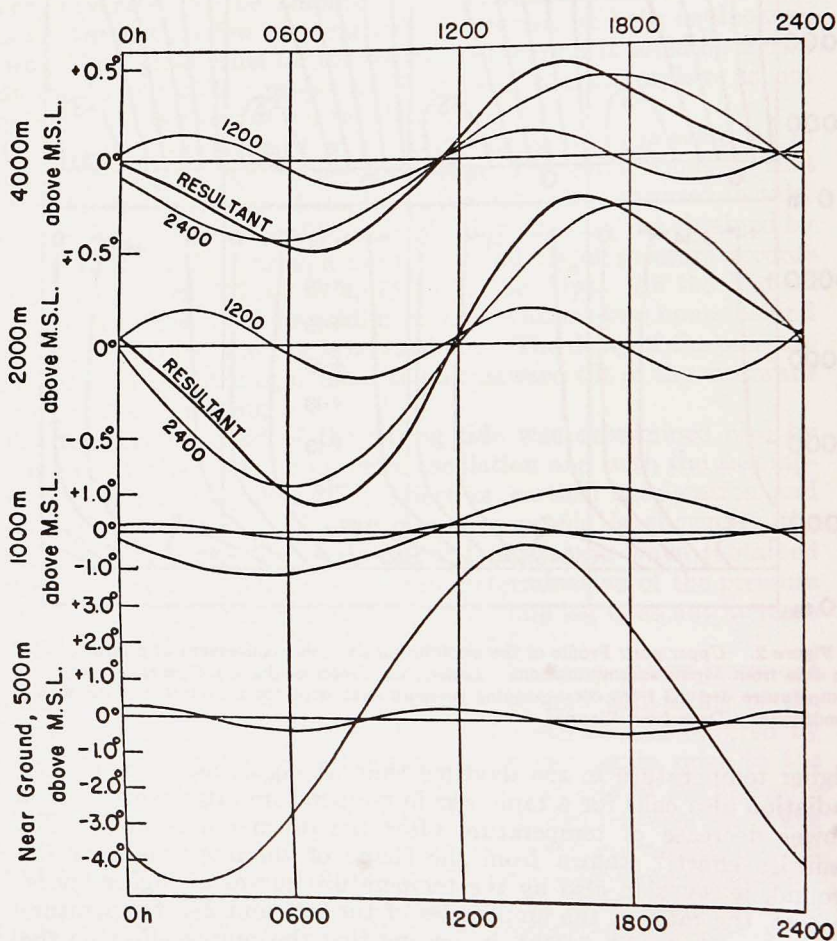


Figure 3. Diurnal and semidiurnal harmonic components of temperature, derived from harmonic analysis of pressure records at Alpine summit stations. (Data from Wagner, 1932.)

wave, which is timed by the "sunrise effect," continues quasi-vertically upward with an insignificant tilt due to the earlier arrival of sunrise with height. The pressure wave tilts asymptotically towards phase coincidence with the temperature wave, as prescribed by hydrostatics.

The total picture of the semidiurnal atmospheric tide is thus composed of pressure waves moving at the speed of local time from east

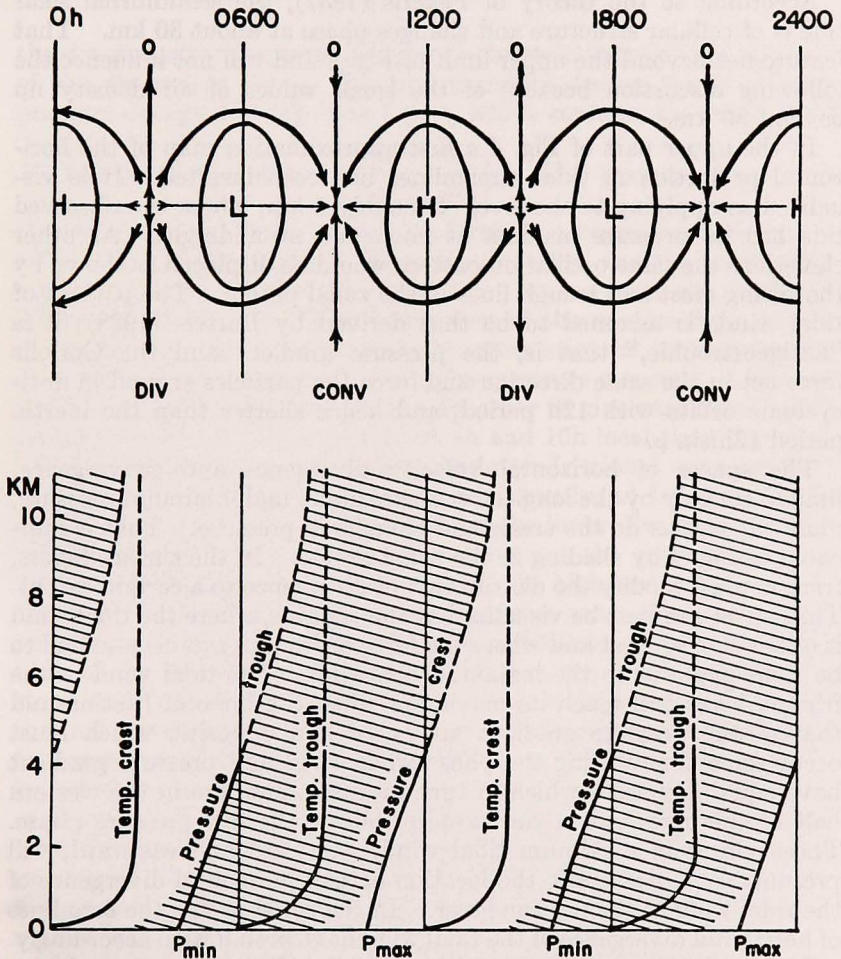


Figure 4. Upper part: Oscillation pattern of the semidiurnal tide with phase times as observed at roughly 4.5 km. Lower part: Profile of the tilting semidiurnal tide. Shaded space indicates convergence of the horizontal velocity field; unshaded space, divergence.

to west and tilting backwards with height all the way up to the stratosphere. At 4.5 km, about halfway through the atmosphere by weight, the phase times of the pressure waves are those of a simple semi-diurnal gravitational tide produced by the sun. Below 4.5 km the pressure waves have earlier phase, and above 4.5 km later phase, than that of a simple gravitational tide because of the tilt produced by thermal asymmetry.

According to the theory of Pekeris (1937), the semidiurnal solar tide is of cellular structure and changes phase at about 30 km. That feature lies beyond the upper limit of Fig. 4 and will not influence the following discussion because of the small values of air density up beyond 30 km.

In the upper part of Fig. 4 a first approximation map of the horizontal projection of tidal streamlines has been inserted. It is visualized as applying to the level of roughly 5 km, where the observed tide has its pressure maxima at noon and at midnight. At other elevations the same oscillation pattern would be displaced as shown by the tilting crest and trough lines in the zonal profile. The pattern of tidal winds is assumed to be that derived by Bartels (1928); it is "antigeostrophic," that is, the pressure gradient and the Coriolis force act in the same direction and force the particles around in anticyclonic orbits with 12h period, and hence shorter than the inertia period $12h/\sin \phi$.

The spaces of horizontal velocity divergence and convergence, limited roughly by the longitudes of maximum and minimum pressure, must tilt just as do the crests and troughs of pressure. That is indicated in Fig. 4 by shading in the zonal profile. In the surface layers, friction must modify the distribution of divergence to a certain extent. That effect can best be visualized at the equator, where the tidal wind is either east or west and where the force of friction can be assumed to be right opposite to the instantaneous wind. The tidal wind in the friction layer will reach its maximum when the force of friction and that of the pressure gradient are equal and opposite, which must occur some time during the phase when wind and pressure gradient have equal direction, which in turn means somewhere in the western half of the high and in the western half of the low pressure phase. Those places of maximum tidal wind, westward and eastward, will presumably indicate also the location of zero horizontal divergence of the tidal wind in the friction layer. In the zonal profile the zero lines of horizontal divergence of the tidal wind have been drawn accordingly.

The tendency equation applied to the semidiurnal pressure change at the ground (p = pressure, t = time, g = acceleration of gravity, ρ = air density, v_x and v_y = horizontal wind components),

$$(1) \left(\frac{\partial p}{\partial t} \right)_0 = - \int_0^{\infty} g\rho \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} \right) dz - \int_0^{\infty} g \left(v_x \frac{\partial \rho}{\partial x} + v_y \frac{\partial \rho}{\partial y} \right) dz,$$

reduces to

$$(1') \quad \left(\frac{\partial p}{\partial t} \right)_0 = - \int_0^{\infty} g\rho \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} \right) dz,$$

the density advection term being only about 10^{-3} times the magnitude of the divergence term. We may therefore discuss the semidiurnal pressure change directly from Fig. 4, which contains the zonal profile

of horizontal divergence, $\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} \right)$. The pressure rise at the

ground, centered around 7h and 19h local time, comes from the surplus of horizontal convergence in the vertical column, and the pressure fall, centered around 1h and 13h local time, comes from the surplus of horizontal divergence. The mechanism of the westward progression of the pressure oscillation lies in the alternation of convergence and divergence in the proper phase to give pressure rise west of the high and pressure fall west of the low. In order to give zero value for the pressure tendency at the ground at 4h and 16h local time, the upper divergence must just compensate the lower frictional convergence in the right hand integral in (1'), and analogously, at 10h and 22h the upper convergence must compensate for the lower divergence. Without the frictional divergence and convergence the pressure tendency would have been positive at the crest of the high and negative at the trough of the low, which would have meant a progressively increasing amplitude of the semidiurnal pressure oscillation. The great amplitude of the semidiurnal pressure oscillation can therefore be understood as an effect of its tilt in space, which in turn is the result of its thermal asymmetry. The semidiurnal pressure oscillation will maintain an adjusted intensity at which the frictional effect near the ground is just strong enough to check the growth of the wave motion.

An exact theory, built on the rough outline above, will probably show that a less rigorous approach of the period of proper oscillations to 12 solar hours is required for tilting pressure waves than for non-tilting ones. The big semidiurnal pressure amplitudes would then cease to be a theoretical enigma.

The 24h solar tide, which was also analyzed by Wagner, does not show any appreciable tilt of pressure crests and troughs and will therefore not be subject to the intensification described for the 12h

solar tide. As for the lunar tide there is no reason to expect any thermal asymmetry with inherent tilt and intensification, and hence it is understandable that the lunar tide remains much weaker than the solar tide.

The tilt of the semidiurnal atmospheric tide has of course no analogy in the ocean, because the ocean has no semidiurnal temperature oscillation in depth. The theory of ocean tides, which in a general way has set the pattern for the theory of atmospheric tides, will call for a generalization to fit the tilting semidiurnal atmospheric tide.

REFERENCES

BARTELS, JULIUS

1928. Gezeitenschwingungen der Atmosphäre. Wien-Harms Handbuch der Experimentalphysik. Geophysik. Teil 1: 161-210, diagr. p. 206. Akademische Verlagsgesellschaft, Leipzig. 699 pp.
1939. Sonnen- und mondentägige Luftdruckschwankungen. Hann-Süring Lehrbuch der Meteorologie, pp. 276-306, diagr. p. 302. Willibald Keller, Leipzig. 480 pp.

CHAPMAN, S.

1918. The lunar atmospheric tide at Greenwich, 1854-1917. *Quart. J. R. met. Soc.*, 44: 271-280.

PEKERIS, C. L.

1937. Atmospheric oscillations. *Proc. roy. Soc. London, (A)* 158: 650-671.

WAGNER, A.

1932. Der tägliche Luftdruck- und Temperaturgang in der freien Atmosphäre und in Gebirgstälern. (Gerland's) *Beitr. Geophys.*, 37: 315-344.
1938. Theorie und Beobachtung der periodischen Gebirgswinde. (Gerland's) *Beitr. Geophys.*, 52: 408-449.