A CRITICAL WIND SPEED FOR AIR-SEA BOUNDARY PROCESSES[1]

By

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INTRODUCTION

Most processes at the air-sea boundary are modified by the existing wind pattern. In general the rate of these processes varies *gradually* with wind speed. An exception seems to occur at a wind speed of about 7 m/sec; at that speed a number of apparently unrelated processes have been observed to undergo such *abrupt* changes that, in some instances, one is led to suspect the existance of discontinuities. Not only the rate, but the very nature of these processes seems to be altered at this critical wind speed.

An explanation may be found in the Kelvin-Helmholtz instability criterion; applied to the air-sea boundary this criterion gives instability for winds exceeding 6.5 m/sec. The application involves the transition from laminar to turbulent flow, a concept fundamental to fluid mechanics. Consequently oceanographic and meteorologic processes involving the air-sea boundary can be expected to change markedly at the critical wind speed.

OBSERVATIONS OF "DISCONTINUITIES" AT BEAUFORT 4

Whitecaps. Whitecaps[2] are formed when the wind speed equals or exceeds force 4 on the Beaufort scale (Beaufort 4 includes the range 5.5–7.9 m/sec, *i. e.*, 11–16 knots). The transition from a smooth sea to a sea covered by whitecaps is so striking that it serves as one of the principal indications for judging surface winds.

[1] Contribution from the Scripps Institution of Oceanography, New Series, No. 349. This work represents the results of research carried out for the Hydrographic Office, the Office of Naval Research, and the Bureau of Ships of the Navy Department, under contract with the University of California.

[2] "The white froth on the crests of waves" (Knight, 1944: 829). In England the expression "white horses" is commonly used.

Table I contains a description of the surface at various wind speeds. These speeds refer to average values; instantaneous values of the wind

TABLE I. DESCRIPTION OF SEA SURFACE AT VARIOUS WIND SPEEDS

Wind Speed Beaufort	Knots	Definition*	Description†
3	7–10	Gentle breeze	". . . produces a definite wave formation with scattered incipient whitecaps making their appearance."
4	11–16	Moderate breeze	". . . is characterized by numerous well developed whitecaps, which give the ocean a spotted appearance."
5	17–21	Fresh breeze	"The difference in the appearance of the ocean surface between forces 4 and 5 is principally one of degree. However, with force 5 spume tends to be blown from the breaking wave crests, whereas no such tendency is noticeable with force 4."

* Knight, 1944.

† U. S. Dept. of Agriculture, Weather Bureau. Instructions to marine meteorological observers, 1938.

speed may exceed the average values by as much as 50%. Therefore the lowest instantaneous wind speed sufficient to bring about whitecaps appears to lie in the range 11 to 16 knots.

Plates I, II and III show selected photographs of the sea surface at wind speeds just beneath and just above those required for whitecap formation. The optical properties of the sea surface differ at the two wind speeds. On vertical photographs, such as Plate III, foam patches were counted and their number over an area of approximately 10,000 square meters was plotted against the wind speed (Fig. 47). These counts depend somewhat upon personal judgment in selecting foam patches. There are no foam patches at wind speeds of less than 5 m/sec, but there are from 10 to 15 foam patches[3] for winds exceeding 7 m/sec.

Soaring of Birds. Woodcock (1940) has observed the soaring tactics of sea gulls over the open sea. He distinguishes between two types of free soaring: circle soaring, where "birds circle about in horizontally restricted, chimney-like up-drafts, being carried along downwind (if there is a wind) at the same time"; and linear soaring, where "the

[3] Above the critical wind speed the *number* of foam patches does not increase consistently with the wind, but their diameters do increase from 3 feet for a speed of about 7 m/sec, to 25 feet for speeds exceeding 13 m/sec.

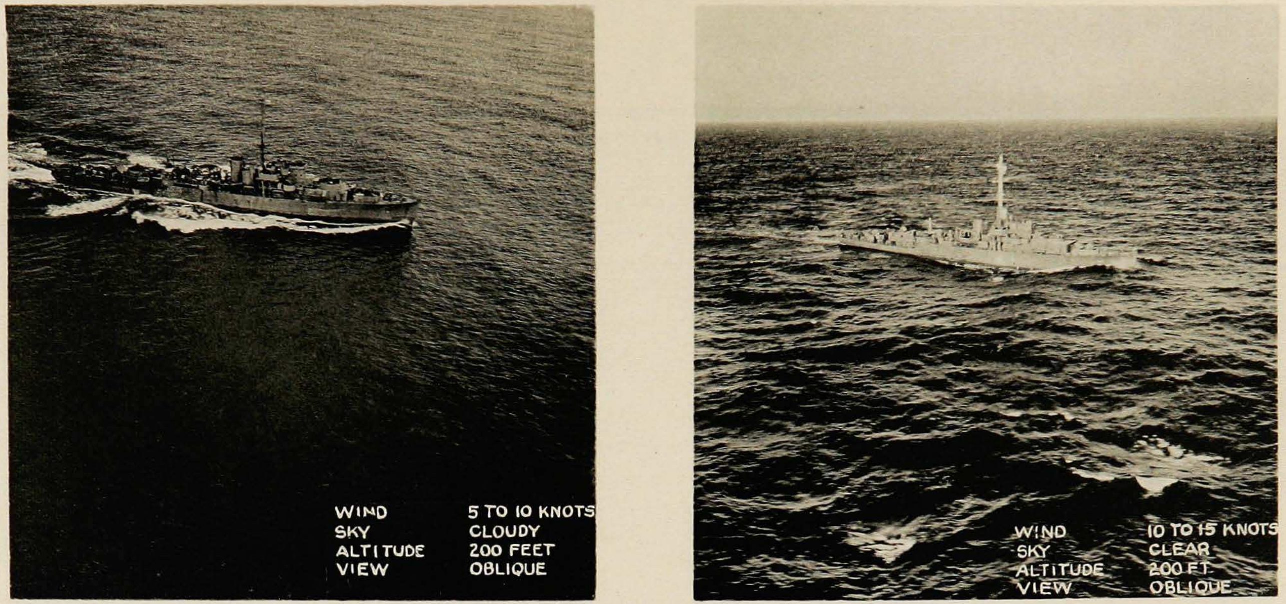
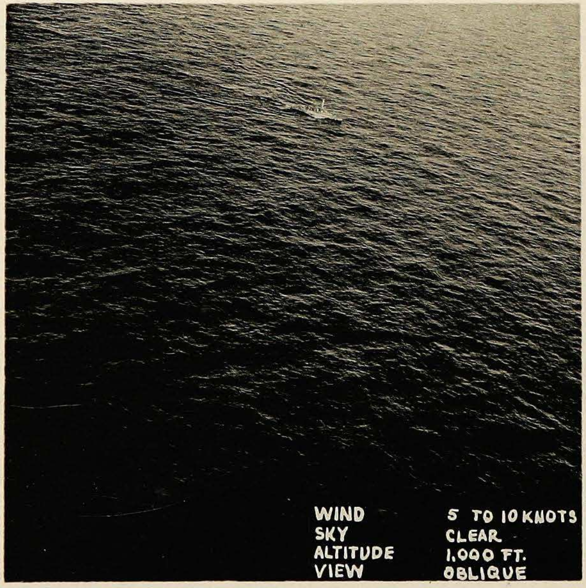


Plate I. Oblique view of the sea surface from an elevation of 200 feet. The left photograph was taken during somewhat weaker winds than the right photograph, but otherwise under identical conditions. (Official Photograph, United States Navy.)



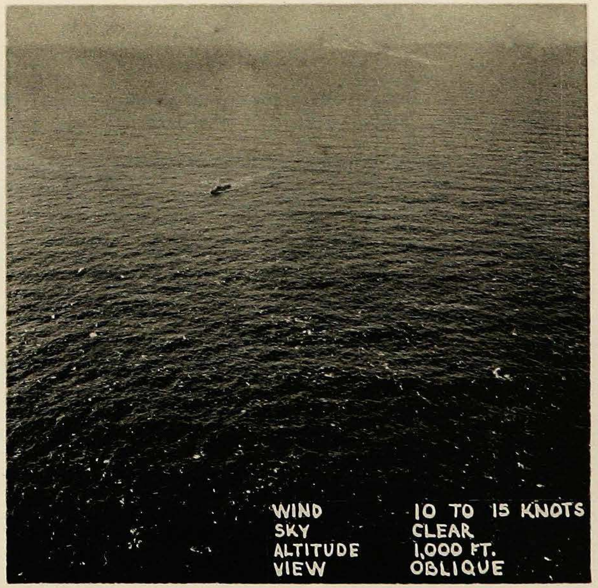


Plate II. Oblique view of the sea surface from an elevation of 1000 feet. The left photograph was taken during somewhat weaker winds than the right photograph, but otherwise under identical conditions. (Official Photograph, United States Navy.)

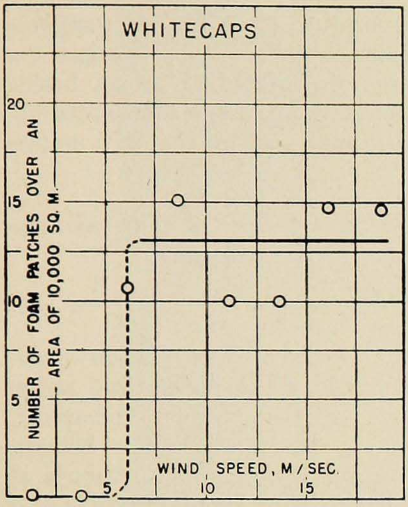


Figure 47. Foam patches are observed only for wind speeds exceding 7 m/sec.

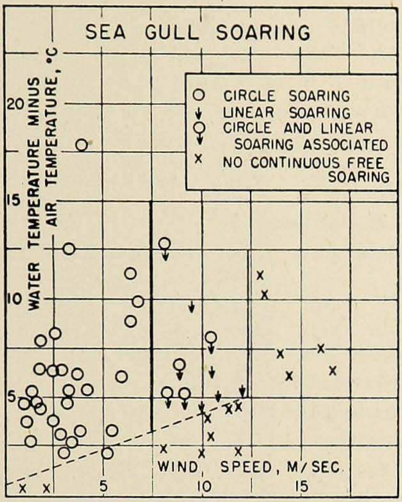


Figure 48. Observations of soaring sea gulls reveal a change in tactics at a wind speed of about 7 m/sec. This is believed to be the result of changes in the convection pattern above the sea surface. After Woodcock (1940).

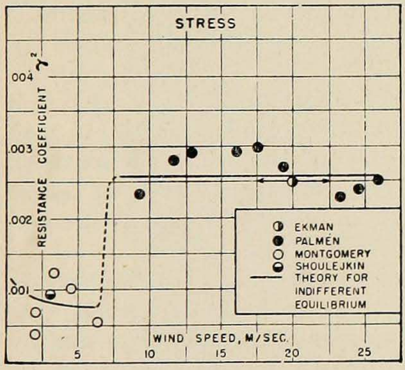


Figure 49. Observations of storm tides and of the vertical wind profile over the ocean reveal a sharp change in the resistance coefficient at a wind speed of about 7 m/sec. Arrows to both sides of Ekman's observation indicate range of wind speeds over which observation is applicable.

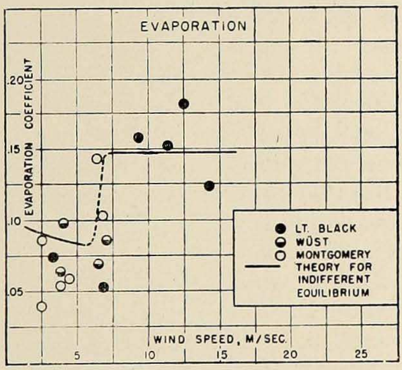


Figure 50. Observations of the humidity gradient above the sea indicate a sharp change in the evaporation coefficient at a wind speed of about 7 m/sec. After Sverdrup (1946).

birds move in a line directly against the wind flow." Observations of soaring during periods when sea temperature exceeded air temperature[4] have been plotted on Fig. 48; each of these 53 observations extended intermittently over an average time interval of six hours. For winds above 7 m/sec there is both circle-soaring and linear-soaring. According to Woodcock, "there is no linear soaring below 7 meters per second."

Resistance Coefficient. Taylor (1916) has defined a dimensionless parameter, the resistance coefficient γ^2, by the equation

$$\gamma^2 = \tau/\rho' U^2 , \tag{1}$$

where ρ' is the density of air, U the wind speed at an elevation of approximately 15 meters above sea level, and τ the mechanical stress exerted by the wind on the sea surface. Stress cannot be measured directly but may be computed from indirect evidence. Ekman (Sverdrup, *et al.*, 1942: 490) computed stress from measurements of sea level rise during a storm in the Baltic (storm tides). Palmén and Laurila (1938) made a similar study, dealing with the effect of wind on sea level in the Gulf of Bothnia, wind speeds ranging from 9 m/sec to 26 m/sec. Within this range γ^2 appears to have a constant value of about 0.0026 (Fig. 49), so that the stress equals

$$\tau = 2.6 \times 10^{-3} \rho' U^2 . \tag{2}$$

The implications of this relationship, in the light of similar "quadratic laws" for the flow of fluids over *rough* plates, have been discussed by Rossby and Montgomery (1935) and will be dealt with later. Further evidence in favour of a constant value of γ^2 for winds exceeding 6 to 8 m/sec can be found in the successful application of equation (2) to such oceanographic processes as upwelling along the coast of southern California (Sverdrup and Fleming, 1941), the generation of waves by wind action (Sverdrup and Munk, 1947), and the maintenance of equatorial currents by wind stress (Sverdrup, in press).

At low wind speeds the empirical data are much less conclusive. Rossby has found Von Kármán's equation for the flow of fluids over *smooth* plates in agreement with observations by Montgomery and by Shoulejkin of the wind profile above the sea (Rossby, 1936: fig. 1). Stress can be computed from simultaneous measurements of wind speed at two levels according to the equation

[4] In cases where the air temperature exceeds the water temperature, factors related to the stability in the lowest air layers tend to obscure the transition at the critical wind speed.

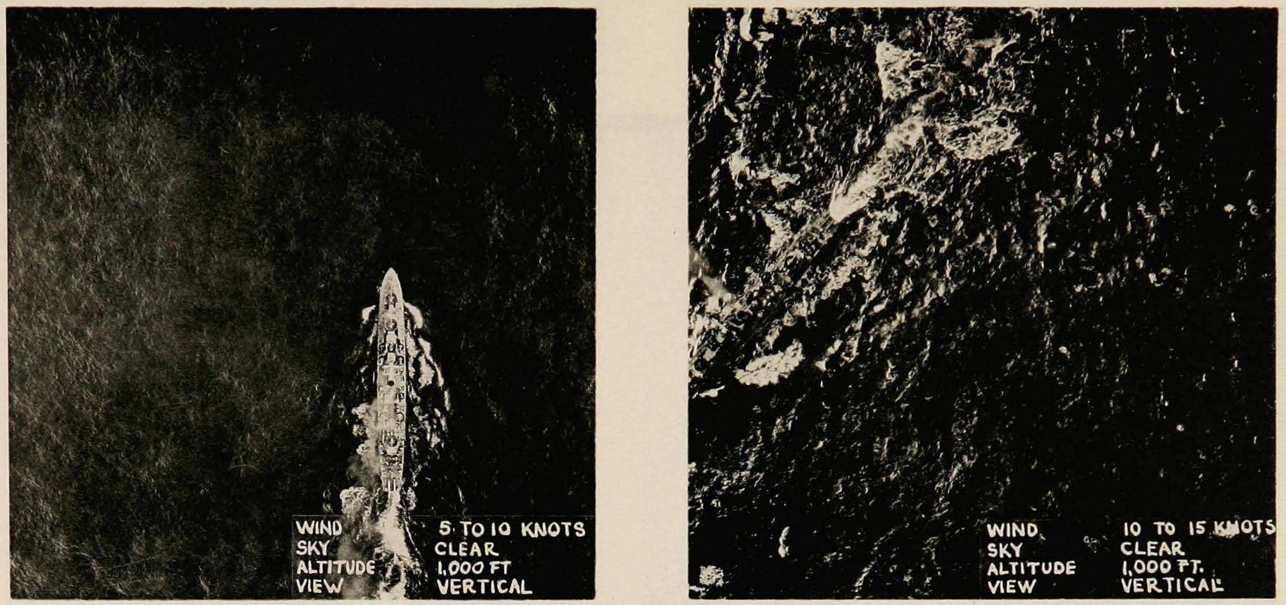


Plate III. Vertical view of the sea surface from an elevation of 1000 feet. The left photograph was taken during somewhat weaker winds than the right photograph, but otherwise under identical conditions. (Official Photograph, United States Navy.)

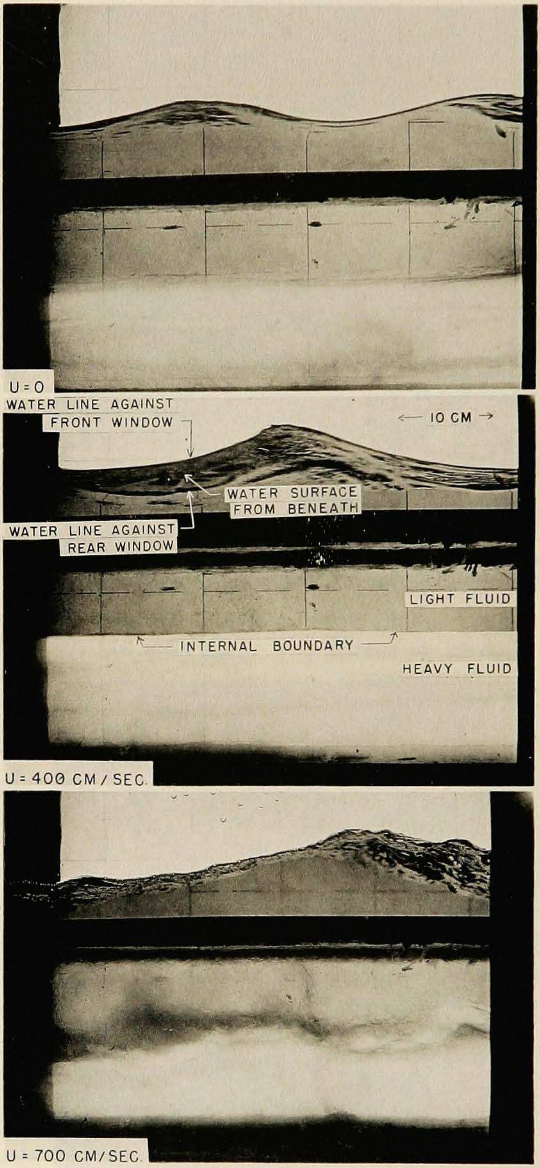


Plate IV. Effect of wind on waves generated mechanically in an experimental tank. The camera was beneath the level of the water surface outside the tank, so that the boundary is seen from below. For a wind of 700 cm/sec (bottom photograph) the water surface is rough and eddies form at the internal boundary.

$$\sqrt{\tau/\rho'} = \frac{k\,(U_2 - U_1)}{\ln\,(z_2/z_1)}, \tag{3}$$

where $k = 0.4$, U_1 and U_2 are the wind speeds at elevations z_1 and z_2, respectively. Values of the resistance coefficient for low wind speeds have been computed from these observations and plotted in Fig. 49. All wind speeds have been adjusted to an elevation of 15 meters by means of the equation

$$\frac{U_{15} - U_1}{\ln\,(z_{15} - z_1)} = \frac{U_2 - U_1}{\ln\,(z_2/z_1)}. \tag{4}$$

Montgomery's observations were made simultaneously at two levels, Shoulejkin's observations at four different levels. The solid curve segments in Fig. 49 refer to Rossby's equations for the stress over smooth and rough surfaces.

Evaporation Coefficient. Montgomery (1940) has defined another dimensionless parameter, the evaporation coefficient, by the equation

$$\Gamma = -\frac{1}{e_s - e_a}\frac{de}{d\ln z}, \tag{5}$$

where e is the vapor pressure at an elevation z above the sea surface, e_s the vapor pressure at the sea surface, and e_a the vapor pressure at a standard level above the sea surface.

Sverdrup (1946) has discussed the relationship between wind speed and the humidity gradient over the sea surface, and he finds that for indifferent equilibrium conditions the evaporation coefficient changes abruptly from a value of .08 to almost .15 at a wind speed of about 6 m/sec. His striking graph is reproduced in Fig. 50. The solid curve segments show the theoretical relationships for smooth and rough surfaces.

Laboratory Measurements. Plate IV shows three photographs taken during laboratory measurements conducted at the Institute of Hydraulic Research, University of Iowa, by Dr. H. Rouse and the author. Waves were generated by a plunger at one end of a flume and wind blown over the water surface.

The three photographs show the wave motion at gradually increasing wind speeds. By using two layers of fluid of different color and density, the water motion beneath the surface can be studied. No appreciable mixing across the internal boundary can be noted until the surface wave breaks; thereupon the laminar flow pattern is disturbed and eddies are formed at the internal boundary. Thus the effect of the critical wind speed is not confined to the sea surface and to the layers directly above and beneath the surface, but appears to make itself felt at greater depths. The laboratory measurements are summarized in Table II.

TABLE II. RESULTS OF LABORATORY MEASUREMENTS

Mean wind speed, U_{mean}, in m/sec	Above 4.5	Below 4.5
Wind speed above crests, U_{crest}, in m/sec ($U_{crest} = 1.4\ U_{mean}$)	Above 6.3	Below 6.3
Number of runs during which crests broke	10	1
Number of runs during which no breaking occurred	3	15

The wind speed U_{mean} refers to a mean water level 14 cm beneath the top of the flume. The elevation of the average breaking crest above the mean level equalled 4 cm, so that the thickness of the air layer above the crests equalled 10 cm. According to Bernoulli's principle the product of the thickness of the air layer with the air speed must remain constant and the air speed directly above the crests, U_{crest}, must equal 1.4 U_{mean}. According to Table II, breaking occurred in 77% of the runs for which the wind speed above the crests exceeded 6.3 m/sec; and in 6% of the runs for which the wind speed was less than 6.3 m/sec.

Other Observations. Langmuir's (1938) careful measurements on Lake George imply a distinct change in the convection pattern at wind speeds somewhere between 4 and 8 m/sec for indifferent equilibrium conditions. A similar change in the convection pattern is reflected in the vertical distribution of temperature and salinity in the upper layer of the ocean (Armstrong, 1947). Another indication of discontinuity is the marked change in "sea return" on radar screens at a wind speed of about 7 m/sec. Also the general noise level in the sea and the reverberation from the sea surface are affected though not apparently to an equally marked degree.

The transition from a stable to an unstable sea surface is likely to have important bearing upon the capacity of the sea surface to reflect light and other radiation; it may affect still other processes. Observations should be planned and their analyses carried out to allow for a discontinuity at Beaufort 4.

THE TURBULENCE MODEL BY ROSSBY AND MONTGOMERY

From wind observations over the ocean, Rossby and Montgomery (1935), Rossby (1936), have inferred the existence of two hydrodynamically distinct types of sea surfaces. In order to account for the profile observed at low wind speeds, Rossby assumes a 'hydrodynamically smooth' surface with a very thin (about 1 mm) laminar boundary layer directly above it. For winds exceeding a certain critical value, Rossby and Montgomery assume the sea surface as 'hydrodynamically rough' with turbulence extending to the very surface. The authors do not state the critical wind speed explicitly, but their discussion implies a value somewhere between 6 and 8 m/sec at indifferent stratification.

In this manner Rossby has been able to account for the difference in stress at low and high wind speeds; indeed the data shown in Fig. 49 contains some of the evidence used to establish the concepts of smooth and rough sea surfaces. Sverdrup (1946) has used the Rossby-Montgomery model to account theoretically for the abrupt change in evaporation at a wind speed of about 7 m/sec (Fig. 50). Woodcock (1940) follows similar reasoning to explain the change in soaring tactics of sea gulls.[5] Also, it seems likely that white caps, breaking waves in tanks, convection patterns above and beneath the sea surface, radar "sea return," and noise level in the ocean, are all related (directly or indirectly) to the hydrodynamic character of the sea surface.

The laws applied by Rossby and Montgomery to the wind profile over the sea surface are identical with laws established in laboratory experiments dealing with the flow of fluids over plates. At low wind speeds air flow over the ocean is analogous to flow of fluids over smooth plates; at high wind speeds it is analogous to flow over plates which have been artificially roughened by means of sand grains. In view of these analogies the concepts of a hydrodynamically smooth and hydrodynamically rough sea surface were introduced. However, these laboratory experiments provide no basis for assuming that the sea surface can have the characteristics of *either* a rough or a smooth surface, depending upon the wind speed. A possible basis for assuming the curious dual role of the sea surface is presented in the next section.

THE INSTABILITY CRITERION

Two fluids, of densities ρ and ρ', are separated by a sharp boundary (Fig. 51). The ratio s of the upper density to the lower density is assumed to be less than one. Let C, L, and H denote the speed,

[5] "There is some evidence that the force exerted by the wind on the sea surface cannot be expressed by a single continuous function of wind velocity and other factors, so that the critical values of these factors are also of interest to oceanographers. The soaring gulls seem to provide some clues as to the critical wind speeds at which there are marked changes in the form of convective motion, and thus in the transfer of momentum between the two mediums."

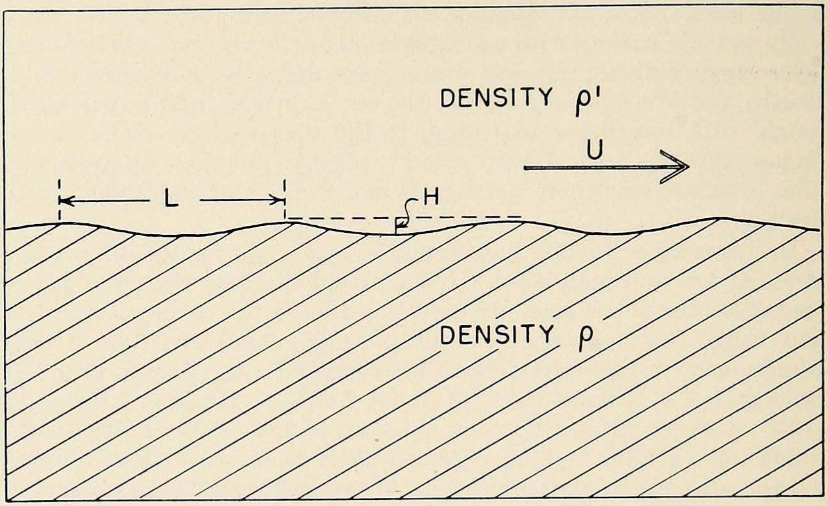


Figure 51. A fluid of density ρ' is separated by a sharp boundary from a fluid of density ρ. The upper fluid moves relative to the lower fluid with a uniform speed U. Waves at the boundary between the fluids have a height H, a length L.

length and height of waves formed at the interface in the absence of currents. If the upper fluid is moving with a speed U relative to the lower fluid, then to any particular value of the wave velocity C there corresponds a critical fluid speed, $U = U'$ (Lamb, 1932; 373–374):

$$U' = C\left[\frac{1 + s}{\sqrt{s}}\right]. \tag{6}$$

For $U < U'$ waves gradually diminish in height until they die out; for $U > U'$ waves grow according to the equation

$$H = \text{constant } e^{\alpha t}, \tag{7}$$

where

$$\alpha = \frac{2\pi}{L}\sqrt{\left[\frac{s}{(1 + s)^2}\right]U^2 - C^2}, \tag{8}$$

until the height becomes so large that certain assumptions underlying the foregoing equations are transcended.

This instability criterion was established by Helmholtz (1868), who attempted to explain cyclonic disturbances at a surface separating cold and warm air. Applied to the air-sea boundary, the criterion shows that for any value of the wind speed, no matter how small, certain waves will grow, provided their velocity is sufficiently small.

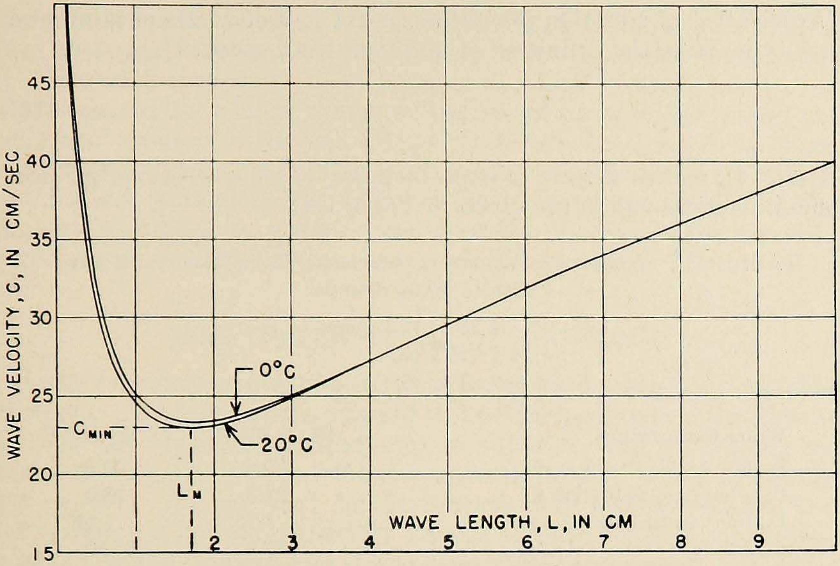


Figure 52. Wave velocity as function of wave length for two values of water temperature. The wave velocity has a minimum value, due to the opposing effects of gravity and surface tension.

Lord Kelvin (Thomson, 1871) has shown, however, that due to the effect of surface tension (σ), the wave velocity must always exceed a minimum value[6] as shown in Fig. 52:

$$C_{\min} = \left[\frac{1-s}{(1+s)^2}\frac{4g\sigma}{\rho}\right]^{1/4}. \tag{9}$$

The length of the waves of minimum velocity equals

$$L_m = \frac{2\pi}{\sqrt{1-s}}\sqrt{\frac{\sigma}{g\rho}}. \tag{10}$$

The general relationship between wave velocity and length becomes particularly simple in terms of the minimum values (Lamb, 1932: 459):

$$\frac{C}{C_{\min}} = \sqrt{\frac{1}{2}\left(\frac{L}{L_m}+\frac{L_m}{L}\right)}. \tag{11}$$

Fig. 52 shows the relationship between C and L for two extreme values of temperature.

[6] Lord Kelvin, during a boatride in the sound of Mull, which he undertook jointly with Helmholtz, was also the first to confirm the theoretically derived value for the minimum wave valocity.

Application of equation (6) to the special case of waves of minimum velocity leads to the definition of a critical wind speed, U_c,

$$U_c = C_{\min}\left[\frac{1+s}{\sqrt{s}}\right]. \tag{12}$$

For $U < U_c$ the sea surface is stable for waves of all lengths and periods. Some numerical values are given in Table III.

TABLE III. NUMERICAL VALUES OF MINIMUM WAVE VELOCITIES AND CRITICAL WIND SPEEDS

$\rho' = 1.25 \times 10^{-3}$ g/cm^3
$\rho = 1.025$ g/cm^3
$s = 1.22 \times 10^{-3}$
Salinity $S = 33°/_{oo}$

Water temperature	32° F (0° C)	68° F (20° C)
Surface tension* σ (g/sec^2)	76.4	73.5
$C_{\min}$ (cm/sec) (equation 9)	23.3	23.0
L_m (cm) (equation 10)	1.73	1.69
U_c (cm/sec) (equation 11)	668.0	658.0
U_c (knots)	13.0	12.8

* $\sigma = 75.64 - 0.144\ \vartheta$ (° C) $- 0.0221\ S$ (°/oo), (Sverdrup, *et al.*, 1942: 70).

The critical speed U_c equals about 660 cm/sec, or 13 knots, and varies only slightly with temperature. This numerical value has been quoted in a number of investigations (Thomson, 1871; Jeffreys, 1925; Thorade, 1931: 42; Lamb, 1932: 462), but no further significance has been attached to it, as all these investigations were concerned with the lowest wind capable of raising waves. According to Jeffreys (1925) ". . . the velocity of a wind just strong enough to raise waves is actually only about 110 cm/sec . . .".

The derivation of the critical speed assumes a sharp break at the boundary. Although gusts from elevations of 15–30 feet may occasionally penetrate the lowest air layers and reach the sea surface without having lost a large fraction of their momentum, the *mean* wind speed undergoes a continuous, though a rapid, change in the region directly above the sea surface. Attempts to extend the Kelvin-Helmholtz analysis to a velocity profile which does not assume a sharp break at the boundary have met with great difficulties.

The profile of the mean wind and the amount of turbulence in the air depend upon the temperature gradient in the lower air layer. During typical summer conditions the water cools the air from beneath and the air is horizontally stratified. During winter the water is likely to be warmer than the air, so that convection and turbulence stirs the lower air layers. Assuming the wind to be measured at a fixed elevation, it follows that instability of the sea surface is attained at lower wind speeds during the winter than during the summer. If whitecaps can be related to instability of the surface, this would explain the common observation that whitecaps are formed at lower wind speeds during the winter than during the summer. Compared to the effect on boundary stability of the temperature gradient in the air, any variations in surface tension or density due to the absolute variations in temperature are negligible (Table III).

DISCUSSION AND CONCLUSIONS

A clear distinction must be made between wind waves and unstable wavelets. The dimensions of wind waves are many times the dimensions of the wavelets; furthermore, wind waves can be formed by winds of only 110 cm/sec, whereas wavelets require winds exceeding 660 cm/sec. During wind speeds between 110 and 660 cm/sec the sea surface is a hydrodynamically smooth surface, but it is not a *plane* surface. Small, smooth and stable wind waves are present. In addition, occasional gusts ruffle the surface, but the small corrugations are almost immediately smoothed under the action of surface tension. Once the wind speed exceeds the critical value, these corrugations not only persist, but also grow rapidly.

Direct observation of these wavelets is difficult because of their minute size. According to Table III their length at the critical wind speed is less than 2 cm. Careful observations indicate the existence of such very small corrugations; occasionally they appear to break and leave behind a tiny trail of bubbles. The wavelets, although too small to account directly for such conspicuous features as whitecaps, may be responsible for determining the characteristic roughness of the sea surface. Table IV gives the maximum dimensions of the wavelets according to equations (6) and (11), assuming the height of the wavelets to equal between 1/3 and 1/2 their length.

TABLE IV. Dimensions of Unstable Wavelets at Various Wind Speeds According to the Kelvin-Helmholtz Theory

U' (m/sec)	L (cm)	H (cm)
6.61	1.71	0.7
10.0	7.5	3.0
15.0	17.7	7.0
20.0	31.4	12.6
30.0	71.0	28.4

According to Rossby (1936: 19), "the roughness parameter (of the sea surface) corresponding to steady moderate to strong winds seems to be in the vicinity of 0.6 cm. . . .," and within this range does not change markedly with wind speed. The average height of the unstable wavelets in the range of wind speeds between 10 and 20 m/sec is 11 cm, or roughly 20 times the roughness parameter. This factor lies intermediate between a factor of 30 suggested by Prandtl (1932) on the basis of wind tunnel experiments using sand grains for roughness and a factor of 10 determined by Sverdrup (1936) from measurements of wind over a snow field. Although the computed dimensions of the wavelets do not meet Rossby's requirement of being independent of wind speed, it may be significant that they are of the order of magnitude suggested by Rossby. In all events it is considered significant that *the Kelvin-Helmholtz theory gives instability at a wind speed which corresponds, within the limits of observational error, to the critical speed for transition from a hydrodynamically smooth to a hydrodynamically rough sea surface.*

The manner in which stability or instability bears upon turbulence and other processes at the air-sea boundary is only conjecturable, but so are any of the other causes of turbulence. Most of our knowledge is based upon the investigations by Rayleigh and Kelvin (Lamb, 1932: 670–674) into the stability of infinitely small disturbances superimposed upon flow through pipes and channels. In analogy with these investigations it may be reasoned that instability at the sea surface is the *cause* of turbulence at the boundary. From this point of view the Kelvin-Helmholtz criterion can be interpreted in terms of a nondimensional number, $U\sqrt{s}/C_{\min}(1 + s)$; a value of "one" marks the transition from laminar to turbulent flow at the boundary, just as a Reynolds number of 2000 marks the transition from laminar to turbulent flow in pipes.

The analogy may be carried further, and the pattern in the air flow over the wave crests compared to the flow of fluids around cylinders (Figs. 53, 54). As long as the flow is laminar the stream lines follow the entire wave profile, and the resultant force on the wave profile is zero (Lamb, 1932: 77). When irregular turbulent flow has set in the pressure against the wave profile is proportional to the square of the wind speed; the air flow spearates from the sea surface shortly to the lee of each crest and impinges upon the windward side of the next crest. A negative pressure or suction is likely to be found near the point of separation. An eddy is formed to the lee of each crest. The roughening of the sea surface permits the wind to grip the water and to apply greater tangential stress.

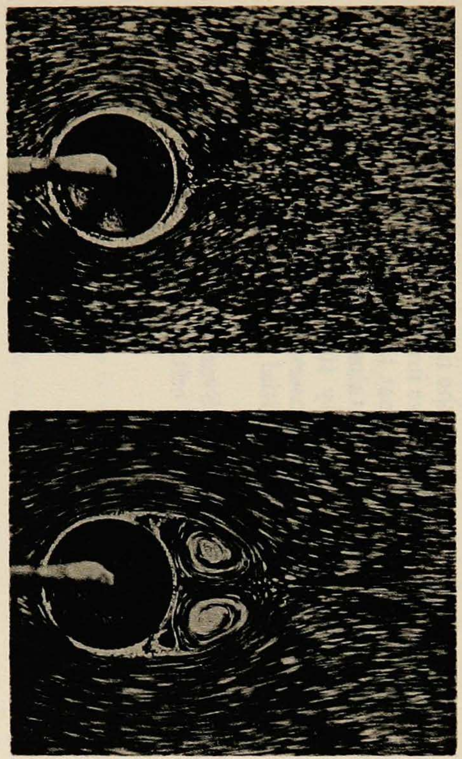


Figure 53. Observed flow of fluid past a cylindrical obstacle. Top figure: low speed; bottom figure: high speed. After Goldstein (1938: I 63).

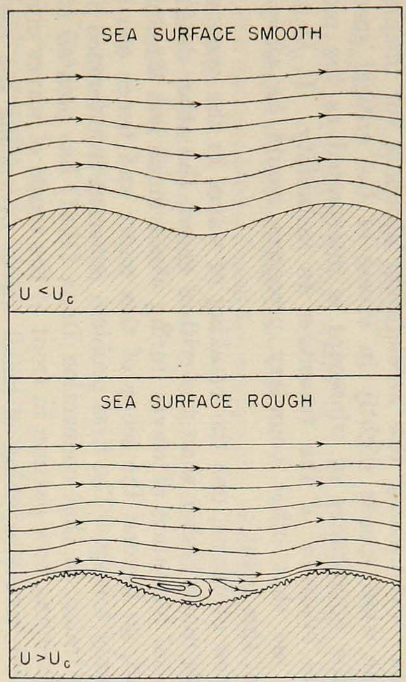


Figure 54. Assumed flow of air over waves. Top figure: less than critical speed; bottom figure: greater than critical speed.

The growth of surface waves, due to the combined action of wind pressure against the wave profile and tangential stress along the sea surface (Sverdrup and Munk, 1947: 9–12), would be accelerated by all these factors. Presumably this acceleration accounts for incipient whitecaps at wind speeds slightly in excess of the critical speed. Therefore the appearance of whitecaps is interpreted as the most direct visual evidence of the transition to instability. It is not difficult to see that other boundary processes would be affected materially at this point.

Two related problems, one the breaking of waves in the open sea during winds much higher than the critical speed, the other dealing with the initial formation of waves during winds much less than the critical speed, go beyond the scope of this paper and hence can be mentioned only briefly. The first problem concerns interference between wave trains; in that connection Unna (1941) has pointed out that the converging flow pattern in front of the crests of waves must lead to an effective steepening of small waves overriding the main wave system. The second problem has been treated by Jeffreys (1925) on the premise that for a wind speed of 110 cm/sec (the lowest wind capable of raising waves) the rate at which energy is supplied by normal wind pressure must equal the rate at which energy is dissipated by molecular viscosity. In assuming the normal pressure proportional to the square of the wind speed, Jeffreys implies fully developed turbulence at winds of 110 cm/sec, in contrast to the evidence presented in this report. Furthermore, Jeffreys obtains a value for the "sheltering coefficient" which is more than twenty times the value obtained by Sverdrup and Munk (1947: 22) from observations of the rate of growth of wind waves. The problem of initial wave formation therefore deserves further consideration.

The author is indebted to Dr. H. U. Sverdrup, Mr. R. S. Arthur and Mr. D. F. Leipper for suggestions and criticism.

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