NOTE ON THE CORRECTION OF REVERSING THERMOMETERS[1]

BY

H. U. SVERDRUP

Scripps Institution of Oceanography, University of California
La Jolla, California

Corrections have to be applied to the readings of protected and unprotected reversing thermometers because they are read at a temperature that differs from that at the time of reversal. In deriving these corrections it is generally assumed that Feruglio's (1912) equation [(5) below] is exact. It is shown here that Feruglio's equation is not quite exact, but a close approximation. From this equation practical formulae, equations (6) and (8) below, can be derived very simply without further sacrifice of accuracy.

PROTECTED THERMOMETERS

Notations.

T_w	: true water temperature at reversal.
T	: reading of reversing thermometer.
I	: index correction of thermometer.
$T' = T + I$	: reading of reversing thermometer corrected for index error.
$\Delta T = T_w - T'$	: correction to be applied with proper sign to reading in order to obtain true water temperature ($T_w = T' + \Delta T$).
v_0	: volume of mercury below scale division "0 degree," expressed in °C.
$v_w = v_0 + T_w$	: volume of mercury at true water temperature.
$v_t = v_0 + T'$	: volume of mercury at reading.
t	: temperature of auxiliary thermometer, that is, temperature of main thermometer at reading.
$1/K$	: coefficient of thermal expansion of system glass-mercury expressed in unites °C^{-1}. For Jena glass 59^{III}, $K = 6100$, and for Jena glass 16^{III}, $K = 6300$.

[1] Contribution from the Scripps Institution of Oceanography, New Series, No. 304. This work represents results of research carried out for the Hydrographic Office, the Office of Naval Research, and the Bureau of Ships of the Navy Department under contract with the University of California.

The change in volume of the torn-off column of mercury is

$$dv = \frac{1}{K} vdt. \qquad (1)$$

Therefore, assuming K to be constant in the temperature range T_w to t:

$$\int_{T_w}^{t} \frac{dv}{v} = \frac{1}{K} \int_{T_w}^{t} dt, \qquad (2)$$

or

$$ln \frac{v_t}{v_w} = \frac{1}{K} (t - T_w) = - (T_w - t) \frac{1}{K}. \qquad (3)$$

Since

$$v_t = v_0 + T' = v_0 + T_w - \Delta T,$$

$$v_w = v_0 + T_w,$$

it follows

$$ln \frac{v_t}{v_w} = ln \left(1 - \frac{\Delta T}{v_0 + T_w}\right) = - \frac{T_w - t}{K}. \qquad (4)$$

Expanding the logarithm in series and omitting terms, which, if retained, would result in a maximum error in ΔT of 0.002°, one obtains Feruglio's (1912) formula:

$$\Delta T = \frac{(T_w - t (v_0 + T_w)}{K}. \qquad (5)$$

Introducing $T_w = T' + \Delta T$ and omitting a term containing $(\Delta T)^2$:

$$\Delta T = \frac{(T' - t)(v_0 + T')}{K - (T' - t) - (v_0 + T')}. \qquad (6)$$

This formula is nearly identical with the formulae by Schumacher (1923) and Hidaka (1932), but it is slightly more accurate. The advantage of the present derivation is that formula (4) is exact if K is a constant and that any desired degree of accuracy can be achieved by taking more terms into account when expanding the logarithm in (4) in series. This is in effect done by Hansen (1934).

In the above, the index correction has been applied before computing ΔT, but in practice this refinement is not necessary if the index correction is less than 0.1°. In (6) T' can be replaced by T, the reading of the reversing thermometer, and the index correction can be combined with ΔT to give the total correction. Furthermore, since the numerical value of the sum $(T' - t) + (v_0 + T')$ generally is about 100, the denominator in (6) can be taken as 6000 or 6200 for Jena glass 59^{III} and 16^{III} respectively.

UNPROTECTED THERMOMETERS

Notations. As above, adding subscript u to reading and corrected values of unprotected thermometer. Thus $T_{w,u}$ represents the true value recorded by the unprotected thermometer at reversal and $\Delta T_u = T_{w,u} - T_u'$.

Equations (1) to (3) remain unaltered but (4) becomes

$$ln \frac{v_t}{v_w} = ln \left(1 - \frac{\Delta T_u}{v_0 + T_{w,u}}\right) = - \frac{T_w - t}{K}, \quad (7)$$

which is exact if K is constant. Expanding the logarithm and introducing $T_{w,u} = T_u' + \Delta T_u$ leads to the close approximation:

$$\Delta T_u = \frac{(T_w - t)\,(v_0 + T_u')}{K - (T_w - t)}. \quad (8)$$

Equation (8) is nearly identical with Schumacher's formula (1923).

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