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NOTE ON THE USE OF THE *T-S* CORRELATION FOR DYNAMIC HEIGHT ANOMALY COMPUTATIONS[1]

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1. The convenience of the bathythermograph for making rapid detailed surveys of the thermal structure in vertical sections of the ocean is leading to an accumulation of data on ocean temperatures far exceeding that on the corresponding salinities. Since dynamic height anomaly computations require a knowledge of both the temperature and the salinity as functions of depth, it is the purpose of this note to determine, in the case where only temperature data are available, whether or not it is possible to use the known *T-S* correlation for a given water mass to supply information on salinity which is sufficiently accurate to be used in dynamic height and current computations.

2. The functional relation between T and S, as shown by the *T-S* diagram for any particular water mass, is such that for a given temperature T the value of salinity S is given only within a certain "range of uncertainty," which is denoted by R_s. The quantity R_s varies for different water masses and also for different temperatures within the same water mass.

From a knowledge of the approximate dependence of R_s upon the pressure (depth) a rough estimate of the range of uncertainty R_D in the dynamic height may be made. For these purposes the *T-S* curve is divided into pressure intervals Δp of approximately constant R_s.

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Calculation of the mean range of uncertainty in the specific volume anomaly (indicated by R_δ) is then possible for each Δp. The products $R_\delta \cdot \Delta p$ are formed and summed up from the surface to the depth of the pressure reference level. The final quantity $\Sigma R_\delta \cdot \Delta p$ may be written as R_D. Because this represents the entire range of uncertainty in the determination of the dynamic height, the probable error in dynamic height may be taken as about $\pm \frac{1}{4} R_D$.

To facilitate this operation—which need merely be approximate—Table I has been constructed. It is prepared directly from the usual tables for δ_s (1, 4).

TABLE I—Brief Table for Approximate Calculation of the Uncertainty in Specific Volume Anomaly from the Uncertainty in Salinity

Salinity Uncertainty R_s(°/oo)	*Specific Volume Anomaly Uncertainty* R_δ	*Salinity Uncertainty* R_s(°/oo)	*Specific Volume Anomaly Uncertainty* R_δ
0.00	0.00000	0.30	0.00023
.02	2	.40	30
.04	3	.50	38
.06	5	.60	46
.08	6	.70	53
.10	8	.80	61
.15	11	.90	69
.20	15	1.00	76
.25	19	2.00	152

As an example of the use of Table I, some stations made at the Grand Banks in June 1940 by the International Ice Patrol (3) are considered. Rather than include a great number of stations the discussion will be limited to the small area covered by stations 3275, 3276, 3277, and 3278. Upon plotting these data on a *T-S* plane it is immediately apparent that the curve may be broken up into five depth intervals

TABLE II—Computation Scheme for Dynamic Height Anomaly Uncertainty for Grand Banks Data

p (decibars)	Δp	R_s	R_δ	$R_\delta \cdot \Delta p$
0				
25	25	0.20	0.00015	0.00375
200	175	1.00	76	13300
300	100	0.20	15	1500
400	100	0.10	8	1000
1000	600	0.05	4	3000
				$R_D = 0.19175$

with an approximately constant R_s. The computation may then be made according to the scheme in Table II. From the results given in this table it is concluded that the probable error introduced into the dynamic height by trying to compute it with the use of the *T-S* correlation for the Grand Bank area would be excessive, about $\pm$ 0.05 meters.

An example showing the other extreme is found by determining, for purposes of illustration, the dynamic height of the 300 decibar surface above the 1000 decibar surface in the Sargasso Sea where the *T-S* correlation (2) is an extremely good one. The computation is given in Table III. The total range of uncertainty in dynamic height

TABLE III—COMPUTATION SCHEME FOR DYNAMIC HEIGHT ANOMALY

p (decibars)	Δp	R_s	R_δ	$R_\delta \cdot \Delta p$
300				
400	100	0.12	0.00010	0.01000
1000	600	0.01	6	3600
				R_D = 0.04600

is of the order of 0.046 meters, and the probable error would then be $\pm$ 0.012 meters, which is good.

3. In those cases where the use of the *T-S* diagram for dynamic computations seems feasible (after a preliminary examination using the method in Section 2) a considerable simplification of the actual calculations is possible by constructing special tables for each water mass. These new tables would give the specific volume anomaly δ as a sum of only two terms, so that

$$\delta = [\tau] + [\tau, p] .$$

An example of such tables appears in Section 5. The term $[\tau]$ includes the usual terms δ_s, δ_T and $\delta_{s,\,T}$, whereas the term $[\tau, p]$ includes $\delta_{s,\,P}$ and $\delta_{T,\,P}$. It is also possible to include a column, in the table for $[\tau]$, which would give the probable error in δ due to the uncertainty involved in using the *T-S* curve. The accumulation of the errors may then be carried through by integration over depth, just as δ is integrated, and a more accurate estimate of the error will result than can be obtained through the use of Table I.

4. The range of uncertainty R_s in the salinity for a given temperature may arise from the following causes:

a. R_s will be very large wherever the slope of the *T-S* curve is small, as is the case in Baffin Bay water.

b. R_s will be large whenever seasonal variations and mixing occur. These conditions predominate, for example, near the surface.

c. Measurements of T and S at any station are made only for certain discrete values of the depth. Even if the precision of individual discrete measures were indefinitely high, an error would still be introduced due to the assumption that the specific volume varies linearly or "smoothly" between the points where the measures are made. Indeed, most recent experience with the bathythermograph indicates a wide prevalence of very irregular variations.

d. The accuracy of individual salinity measurements used in drawing up the T-S curve is limited, which fact implies an interesting point about the use of the T-S curve, since there may be an advantage in using the T-S curve for bodies like the Sargasso Sea instead of using individual measures. The value of R_s in the Sargasso Sea water below 300 meters is so small that it is possible that it arises from the inaccuracies of the salinity determinations themselves rather than from actual fluctuations in the composition of the water mass. In such a case the use of the T-S diagram would be more accurate than the use of individual measures for dynamic current computations.

5. In order to see how these ideas work out in practice, the following example has been prepared from "Atlantis" Stations 1637–1642 across the Gulf Stream at Onslow Bay. The T-S curve for this section shows that it is an intermediate case; it is not as scattered as in the case of the Grand Banks, but on the other hand, it is not as ideal as the one for the deep Sargasso Sea.

First, the anomaly of the dynamic height was computed by using the standard tables of Bjerknes and Sandstrom (1), from the formula

$$\delta = \delta_s + \delta_T + \delta_{s,T} + \delta_{s,P} + \delta_{T,P},$$

and by summing over the first 1000 meters of depth. The summations for stations 1637–8 were carried only for the entire depth to the ocean bottom, since in each of these instances the ocean depth is less than 1000 meters. This value of the dynamic height anomaly for each station is indicated in Table IV.

A T-S diagram for these six stations was then constructed, and a mean curve drawn. Both S and R_s were scaled off as functions of T; the results appear in Table V. Using this table and the Bjerknes-Sandstrom tables (1), special tables (as described above in Section 3) were constructed for this water mass, and these are included here as Tables VI and VII. Specific volume anomalies were then computed from Tables VI and VII and summed over the first 1000 meters. The resulting dynamic height anomalies appear in Table IV.

TABLE IV—COMPARISON OF DYNAMIC HEIGHT ANOMALIES OF "ATLANTIS" ONSLOW BAY STATIONS AS COMPUTED BY THE STANDARD TABLES OF BJERKNES AND SANDSTROM AND THOSE BY THE SPECIAL TABLES VI AND VII

(1) "Atlantis" Station Number	1639	1640	1641	1642	1637	1638
(2) Reference Level (meters)	1000	1000	1000	1000	189	461
(3) Dynamic height anomaly (meters) as computed by standard tables	1.690	1.756	1.711	1.472	0.473	1.027
(4) Dynamic height anomaly (meters) computed from Tables VI and VII	1.713	1.834	1.746	1.527	0.459	1.035
(5) Differences: (4) − (3) (meters)	0.023	0.078	0.035	0.055	−0.014	0.008

TABLE V—THE *T-S* RELATION FOR ONSLOW BAY, INCLUDING THE RANGE OF UNCERTAINTY OF SALINITY

Temperature T	*Salinity* S	*Uncertainty of Salinity* R_s
27.5	36.00	0.20
25.0	36.15	.20
22.5	36.35	.40
20.0	36.45	.30
17.5	36.40	.05
15.0	35.95	.05
12.5	35.62	.05
10.0	35.20	.04
7.5	35.08	.04
5.0	35.06	.04
2.5	35.04	.04

A comparison of the dynamic height anomalies, as computed by the standard technique and those computed by Tables VI and VII, shows (Table IV) that a considerable deviation exists between the results computed by the two different methods. This must be due to a variation of the water composition within the mass that was chosen. However, if the stations are divided into two groups—stations 1637–8, and stations 1639–42, the mean difference of the first group is negative, and that of the second group is + 0.047 meters. This suggests that there is a real change of water mass from one group to the other. As

TABLE VI—Special Onslow Bay Table of the Correction [τ]

T	[τ]	T	[τ]
27	0.00438	14	0.00122
26	404	13	112
25	370	12	105
24	336	11	99
23	302	10	94
22	270	9	81
21	241	8	71
20	212	7	60
19	189	6	48
18	166	5	37
17	150	4	27
16	142	3	19
15	132	2	9

TABLE VII—Special Onslow Bay Table of the Correction [τ, p]

p \ T	2½	5	7½	10	12½	15	17½	20	22½	25	27½
decibars	0	0	0	0	0	0	0	0	0	0	0
100	1	1	2	2	3	3	3	3	4	4	4
200	1	2	3	4	5	6	7	8	7	8	8
300	2	4	5	7	8	8	11	11	12	12	12
400	2	5	6	9	11	13	14	15	16	17	17
500	3	6	8	11	13	16	17	18	19	20	21
600	4	7	10	13	16	18	20	22	23	24	25
700	4	8	12	15	18	21	23	25	26	28	29
800	5	10	13	17	21	24	26	29	30	32	33
900	6	11	15	19	23	27	30	33	34	36	37
1000	6	12	17	21	26	30	33	36	38	39	41

a matter of fact, this may be seen from an examination of the *T-S* curve, where station 1637 differs most widely from the mean distribution. Had 1637 and 1638 been omitted from the construction of the *T-S* curve, the agreement of the two methods for the remaining stations would have been much better. The probable error of dynamic height anomaly in using the short tables instead of the standard tables for stations 1639–42 (omitting 1637–8 from the correlation diagram) would be about ± 0.02 meters. It is interesting to compare this probable error with that arising from the salinity measurements themselves. The error in the salinity measurements from these stations is estimated by Iselin (2) to be about ± 0.02 °/₀₀ as a maximum. To this salinity error there corresponds a maximum error in dynamic height anomaly of about ± 0.02 meters.

As a result it appears that in certain restricted regions the *T-S* diagram may be used for rough dynamic computations. For more detailed survey work where great accuracy of the result is desired the method using the *T-S* diagram is clearly unsuitable.

6. It is certain that if more soundings were available for each of the stations there would have been more accurate values of dynamic height anomaly. However, there are only a certain number, and therefore there is little *a priori* reason to assume any particular kind of interpolation between these points. Some idea can be obtained of the inaccuracies introduced through a linear interpolation by omitting alternate soundings. For this purpose the data for each station were split into two sets of alternate soundings, and the dynamic height anomalies were computed for these as though they were the only data available in each case. The results, using the standard tables (1), are given in Table VIII, along with the differences between these values

TABLE VIII. Error Due to Using a Linear Interpolation between Soundings as Indicated by Splitting the Soundings for Each Station into Two Alternate Sets

(1) "Atlantis" Station	1639	1640	1641	1642
(2) Dynamic height anomaly from complete data (meters)	1.690	1.756	1.711	1.472
(3) Dynamic height anomaly from first half of soundings taken alternately	1.711	1.803	1.754	1.453
(4) Dynamic height anomaly from second half of soundings taken alternately	1.707	1.765	1.698	1.429
(5) Difference: (3) − (2)	0.021	0.047	0.043	−0.019
(6) Difference: (4) − (2)	0.017	0.009	−0.013	−0.043

and the values computed from the complete data. In all cases it is seen that the dynamic height anomalies as computed by the two methods are considerably different—of the order of accuracy in which we are interested. It is essential to recognize, therefore, that the values in which all the data are used are subject to errors nearly of the same (though somewhat smaller) magnitude. This source of error—the interpolation error—appears in addition to the other errors. However, it is possible to eliminate the interpolation error almost completely from the dynamic height computations if use is made of a bathythermogram in conjunction with the more exact bottle data.

It would be unwise to claim too much for the techniques proposed in this note. However, it may be possible (for certain very homogeneous water masses such as the Sargasso Sea), by judicious use of the *T-S* correlation curve, the bathythermogram, the usual bottle samples and temperatures, that errors due to both individual measurements of salinity and interpolation may be largely suppressed, with the result that the dynamic heights so obtained would be certain to an additional half decimal place or better.

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There is very little which is original with the writer in the present note, since it is the result of the thoughts of a number of people. The possibility of using the *T-S* diagram for dynamic computations was first mentioned to the writer by Dr. C.-G. Rossby. From subsequent correspondence it appears that Mr. C. O'D. Iselin and Mr. Floyd Soule had considered the idea years ago. An estimate of the desired accuracy in work of this kind was obtained from some unpublished notes of Dr. R. B. Montgomery. In addition, the writer has had the opportunity to discuss these and related matters with Messrs. Frederick Fuglister, V. P. Starr and A. C. Vine.

REFERENCES

1. Bjerknes, V. and J. W. Sandstrom
   1910. Dynamic meteorology and hydrography. Pt. I. Statics. Publ. Carneg. Instn., No. 88: 1–68.
2. Iselin, C. O'D.
   1936. A study of the circulation of the western North Atlantic. Pap. phys Oceanogr. Meteorol., *4* (4): 6.
3. Soule, F. M.
   1940. International ice observation and ice patrol service in the North Atlantic Ocean. Bull. U. S. Cst. Guard, No. 30: 36–40.
4. Sverdrup, H. U.
   1933. Vereinfachtes Verfahren zur Berechnung der Druck- und Massenverteilung im Meere. Geofys. Publ., *10* (1): 1–9.