THE MOTION OF ATMOSPHERIC DISTURBANCES ON THE SPHERICAL EARTH

By

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When the perturbations of the pressure distribution due to rapidly moving cyclones are eliminated by plotting charts of the mean distribution of the pressure for a number of days, centres of larger dimensions and of a more permanent character appear, as has been shown by the work carried out by Rossby and his collaborators (*7*). On winter maps of the northern hemisphere, as a rule at least five of these perturbations are found: the Icelandic and the Aleutian Lows, the Azores, Asiatic and Pacific Highs. They can hardly be considered as mere perturbations of the general circulation, but must rather be regarded as important parts of it, as is evidenced by their semi-permanent character. Nevertheless, they will in the following sometimes be referred to as perturbations or disturbances, since they are treated mathematically as perturbations of an undisturbed current.

Rossby (l. c.) has given a simple theory relating their dimensions and velocities with the zonal component of the general atmospheric circulation. He assumes that the lateral extent of these centres, that is their width in meridional direction, is infinite, and furthermore that the earth may be regarded as flat. The effect of the first simplification has been considered previously (*5*). The difference between the velocity of the centre and the undisturbed zonal velocity of the general circulation is smaller the smaller the lateral extent of the centre. Similarly, a centre of finite lateral extent becomes stationary at a smaller undisturbed zonal velocity than a centre of infinite lateral extent.

These centres extend over a considerable fraction of the earth's circumference. It appears necessary, therefore, to take the spherical shape of the earth into account. When the earth is regarded as a sphere it is also possible to allow for the change of the variation of the Coriolis parameter $2\omega \sin \varphi$ with the latitude. Hitherto the latitudinal variation of this parameter has been regarded as constant.

It would be possible to base a rigorous calculation on a previous investigation in which the wave motions on the rotating earth were studied (*4*). Instead, a much more simplified treatment will be

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given here. The formulae obtained are, of course, not strictly accurate. But the error is very small, especially when the number of waves around the earth's circumference is greater than four.

The atmosphere will be considered as incompressible for the sake of simplicity, although the results hold for any autobarotropic fluid or gas. Let ϑ be the colatitude, increasing towards S, λ the longitude, increasing towards E, a the earth's radius. The velocity of the zonal circulation of the atmosphere may be denoted by V. If α is the angular velocity of the air motion relative to the earth

$$V = \alpha\, a \sin\vartheta. \tag{1}$$

α will be assumed as constant for the sake of simplicity. u and v are the components of the perturbation velocity towards S and E. They may be small compared to V. The vorticity relative to the earth ζ is given by

$$\zeta = \frac{1}{a\sin\vartheta}\left\{\frac{\partial}{\partial\vartheta}[\sin\vartheta\,(v + V)] - \frac{\partial u}{\partial\lambda}\right\}. \tag{2}$$

The absolute vorticity, that is, the vorticity relative to the earth plus the vorticity due to the earth's rotation $2\omega\cos\vartheta$, must be constant

$$\zeta + 2\omega\cos\vartheta = \text{const.} \tag{3}$$

Differentiating with respect to time, it follows that

$$\left(\frac{\partial}{\partial t} + \alpha\frac{\partial}{\partial\lambda}\right)\left\{\frac{1}{a\sin\vartheta}\left[\frac{\partial}{\partial\vartheta}(\sin\vartheta v) - \frac{\partial u}{\partial\lambda}\right]\right\} + \\ u\frac{\partial}{a\partial\vartheta}\left[\frac{1}{a\sin\vartheta}\frac{\partial}{\partial\vartheta}\left(\sin\vartheta V\right)\right] = \frac{2\omega\sin\vartheta}{a}u. \tag{4}$$

The equation of continuity may be written in the form

$$\frac{1}{a\sin\vartheta}\left[\frac{\partial(\sin\vartheta u)}{\partial\vartheta} + \frac{\partial v}{\partial\lambda}\right] = 0 \tag{5}$$

This equation is not strictly correct since the existence of an undisturbed zonal velocity V implies the presence of a pressure gradient, and therefore a variation of the depth of the fluid, in meridional direction. But a more rigorous analysis based on the paper mentioned previously showed that this effect may be neglected without serious error.

The form of (5) suggests the introduction of a stream function χ by the equations

$$(6) \qquad u = -\frac{\partial \chi}{a \sin \vartheta \, \partial \lambda}, \qquad v = \frac{\partial \chi}{a \partial \vartheta}.$$

Then (4) becomes

$$(7) \quad \left(\frac{\partial}{\partial t} + \alpha \frac{\partial}{\partial \lambda}\right) \frac{1}{\sin \vartheta} \left\{ \frac{\partial}{\partial \vartheta} \left(\sin \vartheta \frac{\partial \chi}{\partial \vartheta} \right) + \frac{1}{\sin \vartheta} \frac{\partial^2 \chi}{\partial \lambda^2} \right\} + 2(\omega + \alpha) \frac{\partial \chi}{\partial \lambda} = 0.$$

Since perturbations progressing in easterly or westerly direction are to be studied, it may be assumed that

$$(8) \qquad \chi(t, \lambda, \vartheta) = \cos(\beta t + m\lambda) f(\vartheta).$$

β is the frequency, m the wave number. The period

$$(9) \qquad \tau = \frac{2\pi}{\beta},$$

and the wave length

$$(10) \qquad L = \frac{2\pi a \sin \vartheta}{m}.$$

If β is positive, the waves travel westward; if it is negative, they travel eastwards. Substituting the expression (8) for χ in the differential equation (7), it is found that $f(\vartheta)$ must satisfy the equation

$$(11) \qquad \frac{1}{\sin \vartheta} \frac{d}{d\vartheta} \left(\sin \vartheta \frac{df}{d\vartheta} \right) + \left(\frac{2\omega m}{\beta + \alpha m} - \frac{m^2}{\sin^2 \vartheta} \right) f = 0.$$

At the poles f must vanish, since the value of χ must here be the same for every value of λ. This is only possible if f is an associated Legendre polynomial,

$$(8a) \qquad f = C \sin^m \vartheta \frac{d^m P_n(\cos \vartheta)}{d(\cos \vartheta)^m} = C\, P_n{}^m (\cos \vartheta),$$

C being an arbitrary constant. Equation (11) is the differential equation of the associated Legendre polynomials if

$$(12) \qquad \frac{2(\omega + \alpha)\, m}{\beta + \alpha m} = n(n+1); \qquad n = 1, 2, 3, \ldots$$

Thus, the length of the period $\frac{2\pi}{\beta}$ is given by (12). $d^m P_n(\cos \vartheta)/d(\cos \vartheta)^m$ is a polynomial in $\cos \vartheta$ of the degree $n - m$. It vanishes on $n - m$ parallels between the poles. If $n - m$ is odd, the equator is one of these nodal parallels; if it is even, $P_n{}^m$ is different from zero at the equator. In either case, the nodal parallels are symmetrical to the equator.

When $n = m$ the nodal parallels vanish and the sphere is divided into sectors by the nodal meridians. The functions P_n^m are therefore called sectorial harmonics. It will be noted that always $m \leqq n$. If $0 < m < n$, the sphere is divided into quadrangles as shown in Figure 61. The functions P_n^m when $0 < m < n$ are the tesseral spherical harmonics. If $m = 0$, the nodal meridians disappear, and the functions P_n^0 are the ordinary Legendre polynomials or zonal spherical harmonics P_n.

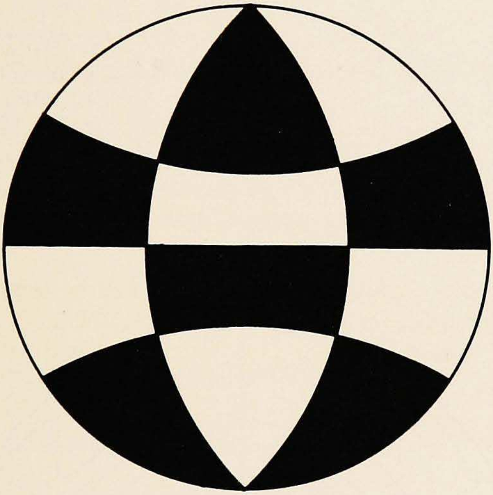

Figure 61. Tesseral spherical Harmonic; schematic.

Tables of the associated Legendre polynomials up to $n = 8$, $m = 8$, have been published by Tallquist (*9*) and by A. Schmidt (*8*). The numerical values of the functions published by both authors differ by a certain factor. Schmidt's form is preferable for numerical work because it is chosen so that the order of magnitude of the functions for different values of n and m remains the same. The position of the nodal parallels as taken from these tables is given in Table I. The tesseral functions P_n^n which have no nodal parallels are omitted.

The stream function for the perturbation motion is given by

$$\chi = C \cos(\beta t + m\lambda)\, P_n^m(\cos\vartheta) \tag{13}$$

and the stream function for the total, disturbed and undisturbed velocity

$$\chi + X = C \cos(\beta t + m\lambda)\, P_n^m(\cos\vartheta) - \alpha a \cos\vartheta. \tag{13a}$$

TABLE I

LATITUDE OF THE NODAL PARALLELS OF P_n^m

(The Functions P_n^n which have no Nodal Parallels are Omitted).

$n = 2$	$m = 1$	0°
$n = 3$	$m = 1$	±26.5°
	$m = 2$	0°
$n = 4$	$m = 1$	0°; ±40°
	$m = 2$	±22.2°
	$m = 3$	0°
$n = 5$	$m = 1$	±16.25°; ±50°
	$m = 2$	0°; ±35.3°
	$m = 3$	±19.5°
	$m = 4$	0°
$n = 6$	$m = 1$	0°; ±28°; ±56°
	$m = 2$	±14.5°; ±44°
	$m = 3$	0°; ±31.5°
	$m = 4$	±17.5°
	$m = 5$	0°
$n = 7$	$m = 1$	±12°; ±36.3°; ±60.5°
	$m = 2$	0°; ±35°; ±60°
	$m = 3$	±13°; ±39.8°
	$m = 4$	0°; ±38.7°
	$m = 5$	±16°
	$m = 6$	0°
$n = 8$	$m = 1$	0°; ±21.3°; ±42.5°; ±64°
	$m = 2$	±10.9°; ±32.7°; ±55°
	$m = 3$	0°; ±22.7°; ±46°
	$m = 4$	±12°; ±36.7°
	$m = 5$	0°; ±26.5°
	$m = 6$	±15°
	$m = 7$	0°

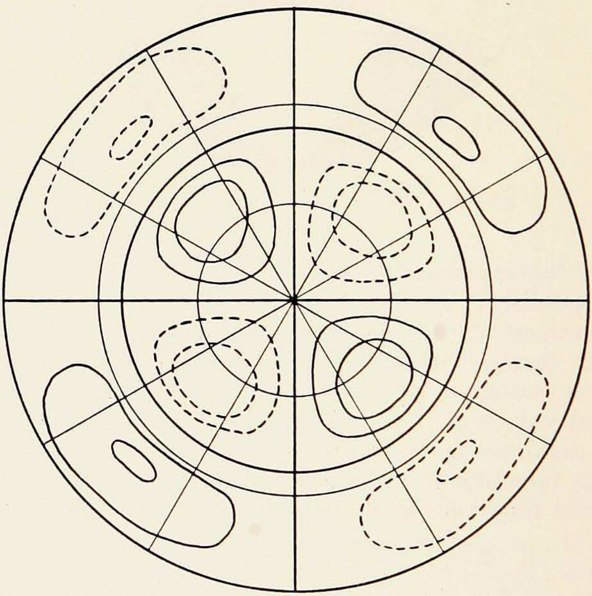

Figure 62. Perturbation stream lines for $n = 5$, $m = 2$. The centre of the figure is the pole, the outer circle the equator. Regions of different directions of motion are distinguished by full and broken curves.

The perturbation streamlines for $n = 5$, $m = 2$, are shown in Figure 62 as an example. The nodal meridians are 90° apart, the nodal parallels are at the equator and at 35.3° latitude. Thus the hemisphere is divided into eight cells. In each two adjoining cells the direction of the perturbation motion is opposite. The streamlines are drawn in arbitrary units. The distance between two streamlines is inversely proportional to the velocity of the motion.

In order to obtain the total stream function given by (13a), the term due to the undisturbed motion should be added to the function represented by Figure 62. If this is done, the streamlines surround the pole, and the effects of the perturbation appear as north-southerly undulations of the stream lines. The amplitude of this undulation depends on the constant C, which has to be regarded as arbitrary at present. (See also p. 264.)

When the stream function is known, the components of the perturbation velocity can be determined with the aid of the equations (6). The perturbation pressure is given by the equations of motion

$$(14)^2 \quad \begin{aligned} \frac{\partial v}{\partial t} + \alpha \frac{\partial v}{\partial \lambda} + 2(\alpha + \omega)\cos\vartheta u &= -\frac{1}{\rho}\frac{\partial p}{a \sin\vartheta\, \partial\lambda} \\ \frac{\partial u}{\partial t} + \alpha \frac{\partial u}{\partial \lambda} - 2(\alpha + \omega)\cos\vartheta v &= -\frac{1}{\rho}\frac{\partial p}{a\partial\vartheta} \end{aligned}$$

With some simple transformations

$$(15) \quad p = C\, 2(\alpha + \omega)\cos(\beta t + m\lambda)\left[\frac{n+1}{n}\cos\vartheta\, P_n{}^m - \frac{n-m+1}{n(n+1)} P_{n+1}{}^m\right].$$

From the geostrophic wind relation it follows that the undisturbed pressure

$$(16) \quad P = P_0 + \rho a^2 \alpha\omega \sin^2\vartheta.$$

This value of P has to be added to p to obtain the total pressure.

From the equations (9) and (10) for the period τ and the wave length L it follows that the velocity of the perturbation

$$(17) \quad c = \frac{L}{\tau} = \frac{a\sin\vartheta}{m}\beta.$$

[2] These equations could have been used to derive (4) instead of the theorem that the vorticity must remain constant.

With the value of β given by (12)

$$c = \omega a \sin \vartheta \left[\frac{2}{n(n+1)} - \frac{\alpha}{\omega} \frac{n(n+1) - 2}{n(n+1)} \right] \tag{18}$$

The bracket represents the ratio of the velocity c of the disturbance to the velocity of a point fixed on the earth due to the earth's rotation. Table II gives values of the velocity for different n and α/ω

TABLE II

VELOCITY OF THE CELLS IN M/SEC AT 45° LATITUDE

n	2	3	4	5	6	7	8	9	10	11	12
α/ω = 1/10	87.4	27.4	+3.3	−8.7	−15.6	−19.6	−22.8	−24.8	−26.2	−27.3	−28.2
α/ω = 1/20	98.5	41.0	18.1	6.6	−0.03	− 4.0	− 6.8	− 8.7	−10.8	−11.2	−12.0
α/ω = 1/30	102.0	45.5	23.0	12.0	5.2	1.2	− 1.5	− 3.4	− 4.8	− 5.8	− 6.6

at 45° latitude. In considering this table, it should be kept in mind that for smaller wave numbers m and smaller n the approximation is not as good as for larger ones. However, this restriction does not seriously affect the present considerations. Positive figures indicate motion towards west, negative figures towards E, as can be seen from the form of the expression for χ (eq. (13)). If the zonal undisturbed current is zero, the motion of the disturbances is always westwards. This shows that these disturbances are identical with the oscillations studied by Margules (*6*) which are referred to as oscillations of the second class. The length of the period of these oscillations in sidereal days is given with sufficient approximation by

$$\tau = \frac{\beta}{\omega} = \frac{2m}{n(n+1)} \tag{4}$$

for the higher modes of oscillations. The same formula follows from eq. (12) if $\alpha = 0$. This type of oscillation is only possible on a rotating surface since the period becomes infinite when $\omega = 0$.

For smaller n, that is for disturbances of large dimensions, the motion is directed towards W, for smaller disturbances towards E. In the first case the effect of the latitudinal variation of the Coriolis force is preponderant, in the second case the effect of the undisturbed zonal velocity. For each disturbance n there is a certain velocity α_s of the zonal current at which the velocity of propagation vanishes. Then the disturbance remains stationary. It follows from (18) that

(19) $$\frac{\alpha_s}{\omega} = \frac{2}{n(n+1) - 2}.$$

The values of α_s/ω and of the metric velocities at different latitudes are shown in Table III for some values of n. The last line contains the wave length of the disturbance—that is the extent of a high and the following low when $n - m = 0$; in other words, when the perturbations have their greatest possible lateral extent, from pole to pole.

The stationary velocity is smaller, the smaller the dimensions of the disturbance. The same rule holds for stationary disturbances of infinite extent on the flat earth according to the following formula given by Rossby (1939, eq. 22).

$$V_s = \frac{2\omega a}{m^2} \cos^3 \varphi.$$

The letters here have the same meaning as in the preceding formulae. The geographic latitude $\varphi = 90° - \vartheta$. This formula may be compared with (19) for the case of a spherical earth. To obtain the closest possible agreement of the assumptions involved in the derivation of both formulae, let the disturbance on the spherical earth extend from pole to pole so that $m = n$. From (19)

$$m = -\tfrac{1}{2} + \sqrt{\frac{2\omega}{\alpha_s}} \sqrt{1 + \frac{9}{8}\frac{\alpha_s}{\omega}}$$

since the negative root must be omitted. The ratio α_s/ω is small according to Table III. Therefore approximately,

$$m = \sqrt{\frac{2\omega}{\alpha_s}}$$

or if the expression (1) for the metric wind velocity is substituted

(20) $$V_s = \frac{2\omega a \cos \varphi}{m^2}.$$

This expression agrees with the one derived by Rossby for a flat earth at the equator (and at the pole since here $V_s = 0$). It is to be expected that the formula for a flat earth gives the best approximation near the equator where the convergence of the meridians and the effect of the spherical shape of the earth are smallest. Moreover, the latitudinal variation of the Coriolis force—which has to be regarded as constant when the earth is assumed flat—changes least near

TABLE III

STATIONARY WIND VELOCITIES

n	2	3	4	5	6	7	8	9	10	11	12
a_s/ω	0.50	0.20	0.111	0.0715	0.050	0.037	0.0286	0.0227	0.0185	0.0154	0.0130
V_s at 30°	200.5	80.0	43.2	28.7	20.0	14.8	11.5	9.1	7.4	6.17	5.20 m/sec
V_s at 45°	164	65.5	36.4	23.5	16.4	12.1	9.4	7.45	6.06	5.05	4.26 m/sec
V_s at 60°	116.0	46.4	30.6	20.3	14.1	10.5	7.4	6.4	5.2	4.4	3.7 m/sec
L at 45° if $n = m$	14200	9450	7090	5660	4710	4050	3540	3140	2840	2580	2360 km

the equator. The approximation involved in the derivation of eq. (20) is better the smaller a_s/ω. a_s/ω decreases as n increases. Thus the effect of the spherical form of the earth decreases with decreasing dimensions of the perturbations, as is to be expected. In comparing the two formulae it should be remembered that Rossby's formula is based on the assumption that the zonal metric velocity is constant, while in this paper the angular velocity is considered as constant.

The mean pressure maps for February show two stationary highs and two lows in temperate latitudes on the northern hemisphere; the number m of disturbances around the earth's circumference is equal to two. According to Bjerknes (*2*) the angular velocity of the zonal current is nearly constant in temperate latitudes, and its maximum value in the troposphere is approximately 0.5×10^{-5} sec^{-1}, so that $a/\omega = 0.07$. It will be seen from Table III that this figure is very close to the approximate value for the stationary case, 0.0715 when $n = 5$. The perturbation streamlines would then be those shown in Figure 62, over which the streamlines of the undisturbed zonal motion have to be superimposed in order to obtain the total motion. The streamlines shown in Figure 62 are, of course, much more symmetrical than those which are observed in the atmosphere. Nevertheless, the close agreement of the actual angular velocity of the air with the theoretical stationary one for $n = 5$, $m = 2$, seems significant.

If the earth had a homogeneous surface, any of the disturbances given by (13) or a linear combination of these disturbances could exist. The heterogeneities of the earth's surface, in particular the unequal distribution of water and land, tend to set up perturbations whose location is determined by these geographical factors. The forces which generate the centres considered here are active especially along the coast lines where the atmospheric solenoids are concentrated, as was discussed in a previous paper (*5.*).

If the effect of these solenoidal forces on the motion of the disturbances is to be taken into account, the left-hand side of (7) is not equal to zero, but to the external forces. Their intensity is mainly a function of the geographical distribution; only to a much lesser extent does it depend on the time. It can therefore be assumed that the force is represented by a function $F(\lambda, \varphi)$ of λ and φ only. Equation (7) becomes then an inhomogeneous equation whose solution is composed of two parts. The first is the solution of the homogeneous equation which is a linear combination of the expressions of the form (8). It represents the possible free oscillations of the fluid system. They are subjected to the damping influence of friction which eventually must bring the motion to a standstill. The second part consists of a particular solution of the inhomogeneous equation which can be found by well-known methods. It gives the perturbations generated by the external forces. They correspond to the forced oscillations of vibrating systems and will be referred to as forced disturbances. Since the generating force is independent of the time the forced disturbances are also independent of the time. It has been shown previously[3] that the stationary forced perturbations have the same wave lengths as the free stationary perturbations. The amplitude of a forced perturbation increases as the difference between the wave length of the free perturbation and of the generating force decreases. When the difference vanishes, the amplitude reaches a maximum; resonance occurs. This is analogous to the phenomenon of resonance in the case of harmonic oscillations, except that there the resonance is between the periods of the generating force and of the free oscillation. In the present case where the generating force is a function not of time, but of space, its wave length takes over the rôle of the period.

In the light of these considerations, the close agreement between the observed zonal velocity on the northern hemisphere during February and the theoretical stationary zonal velocity for the perturbations of the form $\cos 2\lambda\, P_5^2(\cos \vartheta)$ may be interpreted in the following manner. The distribution of the force over the earth can be represented by a series of spherical surface harmonics. The part of the solution which represents the effect of the generating force then appears as a series of terms each giving the effect of one of the members in the series development for the external force. The amplitudes of the terms depend on the amplitudes of the terms in the series for the

[3] B. Haurwitz: l. c.

generating force and on the degree of resonance. It may therefore be concluded that the agreement between the observed and the theoretical zonal velocity in the case mentioned previously is such a resonance phenomenon. Neither the generating force nor the resulting perturbation is exactly of the form $\cos 2\lambda\, P_5^2(\cos \vartheta)$. It is not even necessary that this term in the series development of the external force should have a very high amplitude. Nevertheless, due to resonance, the corresponding term in the series for the resulting perturbations reaches a very high amplitude. The other harmonic components of the force affect the solutions too, although not as much. But owing to their existence the stream function does not very closely resemble the symmetrical picture shown in Figure 62.

For a complete study it would be necessary to analyze the external force, similar to the methods of the tidal theory, and to see which components have wave lengths close to those of the free perturbations. These will be predominant and their amplitudes largest. Such an analysis may be rather difficult since the mathematical formulation of the effect of the generating solenoidal force is not quite obvious.

In the meantime it may be useful to determine the dominant terms by a more empirical method of trial and error, as has been done here in a particular case. A few more general ideas can, however, be formulated as a result of the theoretical considerations.

Equation (19) and Table III show that n is larger and thus the dimensions of the perturbation patterns smaller the smaller the zonal velocity α_s. The zonal velocity α_s may be taken as an index for the general circulation. It depends on the intensity of the meridional pressure gradient. Thus, with a strong circulation index (α_s large) there will be a few large centres. When the circulation is weak the perturbation pattern appears broken up into numerous smaller cells. These ideas agree with those developed by Rossby[4] for the plane earth, and with the results obtained by Allen (*1*) from a study of the five-day-mean-pressure charts. Allen's charts show a few strongly developed centres of high and low pressure when the zonal circulation is strong; in other words, when the meridional pressure gradient is intense. On the other hand, when the zonal circulation is weak, a great number of smaller pressure centres exist; the various "centres of action," like the Aleutian low and the Pacific high, appear broken up into two or more weaker centres.

A knowledge of the distribution of these centres of action and their development is essential if the weather development is to be forecast for a longer period. Thus, the zonal circulation intensity which ap-

[4] C. G. Rossby: l. c.

pears to regulate this pressure distribution can serve as a valuable help in long range forecasting.

The next step towards a solution of this problem is to derive a method by which the changes in the zonal circulation intensity can be determined in advance. The zonal circulation intensity depends, through the geostrophic wind relation, on the meridional pressure gradient which has been called the circulation index by Rossby. No satisfactory theory has been given yet to account for the variations of this quantity. If the view is accepted that the meridional pressure gradient is a result of the temperature differences between lower and higher latitudes, it would seem that its variations depend on the intensity with which heat is transported from lower to higher latitudes. Defant (*3*) has discussed how such a transport of heat can be effected by the cyclones of temperate latitudes. Using his concept of a horizontal mixing of colder northerly and warmer southerly air whereby the cyclones act as turbulent eddies, it may be assumed that at first when a strong zonal circulation exists intense centres of high and low pressure are present. Due to the horizontal mixing between high and low latitudes the meridional differences of temperature are reduced, the zonal current becomes smaller and the perturbation patterns break up into smaller, more numerous centres according to the results of the present investigation. With the breaking-up the horizontal mixing becomes less effective, so that now the temperature differences between north and south increase again. In this manner the variations between strong and weak zonal circulations with their effect on the large scale weather situation appear as a quasi-periodic process. The zonal circulation can only attain certain lower and upper limits. When these limits are reached it will be intensified or reduced again. Obviously, not very much regularity of the fluctuation can be expected in view of the many factors coming into play. But it should be possible by the development of the theory and an analysis of the weather situations to determine these factors and their importance, to that it becomes feasible to forecast, at least, say, at the peak of the circulation intensity, when the next minimum is to be expected, and vice versa.

SUMMARY

To eliminate the fast moving disturbances of smaller extent such as the cyclones of temperate latitudes, charts of the mean pressure distribution for five or more days may be plotted. These charts show the semi-permanent centres of action, for instance, the Icelandic and Aleutian Low and the Pacific and Azores High. The slow changes of position and intensity of these centres determine the large scale

variations of the weather. The spherical shape of the earth and the variation of the deflecting force of the earth's rotation with the latitude determine the horizontal dimensions of the possible cells of high and low pressure and their velocity with respect to the general zonal motion of the atmosphere. For each possible distribution of cells there is a zonal velocity of the atmosphere at which the system remains stationary. If the position of the semi-permanent highs and lows is fixed and determined by the distribution of water and land, and if the zonal velocity of the atmosphere is of the right magnitude, their intensity will be strengthened, provided that the "forced" distribution of the cells coincides with the possible "free" distribution on a homogeneous earth. This is similar to the strengthening of forced oscillations which are in resonance with free oscillations.

When the zonal circulation is strong, the intensity of the semi-permanent centres of high and low pressure is great, and the number of centres is small. When the zonal circulation is weak, these centres break up into smaller and less intensive centres. A mechanism is suggested which may account for the observed changes between times of strong and weak zonal circulation.

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