LATERAL MIXING IN THE DEEP WATER OF THE SOUTH ATLANTIC OCEAN*

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The deep-sea circulation of the Atlantic has recently been carefully analyzed by Wüst (1935) who, on the basis of the tongue-like distribution of temperature and salinity, has drawn conclusions as to the direction of flow at different depths. He considers that an exchange of deep water between the North Atlantic Ocean and the Antarctic Ocean is maintained by a flow to the north along the bottom of Antarctic water and a flow to the south at lesser depths of North Atlantic deep water and Mediterranean water.

The temperature and salinity sections from the Atlantic Ocean clearly show that the Antarctic bottom water loses its identity as it flows north since temperature and salinity both increase in the direction of flow and, similarly, that the North Atlantic deep water loses its identity as it flows south since temperature and salinity both decrease in the direction of flow. Defant (1936) has accounted for these changes by considering processes of mixing in a vertical direction. He has shown that the distribution of temperature and salinity may depend on the ratio between the velocity of the flow and the virtual coefficient of diffusion, and, in the case of the South Atlantic Ocean, he has computed the numerical values of this ratio which correspond to the observed distributions.

It has been assumed that the lateral coefficients of diffusion are of the same order of magnitude as the vertical and that the effect of lateral mixing is insignificant because of the small horizontal gradients, but in view of recent results it appears desirable to examine the correctness of these assumptions. Rossby and his collaborators have shown that in the atmosphere a large-scale quasi-horizontal isentropic mixing takes place and Rossby (1936) has suggested that such lateral mixing may take place in the ocean as well, where it should follow σ_t-surfaces. Parr (1938) has demonstrated the importance of such lateral mixing to the distribution of temperature and salinity off the Grand Banks of Newfoundland, and by means of as yet unpublished data off the coast of California R. H. Fleming and the author (Sverdrup and Fleming, in press) have found that there the lateral mixing is

* Contributions from the Scripps Institution of Oceanography. New Series, No. 75.

of dominating importance between depths of 200 and 400 meters, the coefficient having a numerical value of about $2 \times 10^6 \text{cm}^2\text{sec}^{-1}$. Seiwell (1938) has found that in the Caribbean Sea distribution of temperature and salinity will not be influenced by lateral mixing unless the coefficient is greater than 10^6, and Montgomery (1939) has shown that in the Equatorial Counter Current of the Atlantic the salinity distribution can be accounted for by introducing a coefficient of lateral mixing of $4 \times 10^7 \text{cm}^2\text{sec}^{-1}$ and neglecting the effect of vertical mixing.

It is evident that a lateral mixing of such intensity as that found off the coast of southern California or that suggested by Montgomery in the region of the Atlantic Equatorial Counter Current may be of importance to the general distribution of temperature and salinity, and our first question is, therefore, whether factors are active which can maintain an intensive lateral mixing at great depths. So far, evidence for the existence of lateral mixing has been obtained from coastal areas where the development of quasi-horizontal eddies may be ascribed to the effect of the solid boundaries, or from an area characterized by swift currents in opposite directions where eddies may develop along the boundaries of the currents. Rossby (1938) has, however, recently shown that changes of the winds over the ocean may produce currents reaching from the surface to the greatest depths. All such motion will be directed at random and Rossby's conclusion, therefore, can be interpreted as showing that changing wind conditions may produce variable currents in the deep water, but no flow in a definite direction. If this is the case, a large-scale lateral mixing may take place. In view of these conclusions, it appears worth while to examine to what extent processes of lateral mixing contribute towards maintaining the observed distribution of temperature and salinity in the deep sea.

In a two-dimensional system the general condition for a stationary distribution of a conservative property, s, is:

$$(1) \qquad \frac{\partial s}{\partial t} = \frac{\partial}{\partial l}\left(A_l \frac{\partial s}{\partial l}\right) + \frac{\partial}{\partial n}\left(A_n \frac{\partial s}{\partial n}\right) - u\frac{\partial s}{\partial x} - w\frac{\partial s}{\partial z} = 0$$

where u and w are the velocity components, and where A_l and A_n are the coefficients of lateral and vertical mixing expressed as coefficients of diffusion.

Equation (1) is far too complicated to form a basis for any study, but from this equation three others can be derived, introducing different assumptions as to the variables. Assuming $w = 0$, A_n and A_l to be constants, and the effect of lateral mixing to be negligible, one obtains

$$(2) \qquad A_n \frac{\partial^2 s}{\partial n^2} - u\frac{\partial s}{\partial x} = 0$$

Assuming $w = 0$, A_n and A_l to be constants and the effect of vertical mixing to be negligible, one obtains:

$$(3) \qquad A_l \frac{\partial^2 s}{\partial l^2} - u \frac{\partial s}{\partial x} = 0$$

Finally, assuming $w = 0$, $u = 0$, and A_n and A_l to be constants, one obtains

$$(4) \qquad A_l \frac{\partial^2 s}{\partial l^2} + A_n \frac{\partial^2 s}{\partial n^2} = 0$$

Defant (1936) has used equation (2) in a modified form which takes into account the fact that the deep-sea currents need not be strictly horizontal and has computed values of A/u in the deep and bottom water of the Atlantic Ocean and in the Antarctic intermediate water. In his study of the Equatorial Counter Current of the Atlantic Ocean, Montgomery (1939) has used the equivalents of equation (2) and (3) which apply to a three-dimensional system, and in their discussion of conditions off the coast of southern California Sverdrup and Fleming (in press) used the same equations. Equation (4) has so far not been made use of.

It is of interest, in the first place, to observe that equations (2) and (4) have some common integrals. Thus, one of the tongue-like distributions discussed by Defant (1929) satisfies (2) and is represented by

$$s = s_0 + \delta e^{-\alpha x} \cos \pi \frac{z}{2l}$$

where

$$\alpha = \frac{\pi^2}{4l^2} \frac{A_z}{u}$$

The same distribution also satisfies (3) if

$$\alpha = \frac{\pi}{2l} \sqrt{\frac{A_x}{A_z}}$$

The two solutions are identical if

$$\frac{A_x}{A_z} = \left(\frac{u}{A_z}\right)^2 \frac{4l^2}{\pi^2}$$

In a numerical example Defant introduces $l = 300$ m $= 3 \times 10^4$cm and $A/u = 20$. With these values one obtains

$$A_x = 0.9 \times 10^6 A_z$$

Defant's numerical values were selected in order to represent a tongue-like distribution of salinity of dimensions corresponding to those encountered in the ocean. It is of interest to observe that the same distribution can be explained as a result of processes of mixing alone provided that the ratio between the vertical and lateral coefficients of diffusion is about 10^6 which is the ratio found off the coast of California. Thus, tongue-like distributions may result from horizontal flow and vertical mixing or from processes of mixing only.

Another solution of equation (3) can be written

$$s = s_0 + \delta \frac{\cos h\,[\beta(h - z)]}{\cos h\,[\beta\, h]} \sin \pi \frac{x}{2l}$$

provided that

$$\frac{A_x}{A_z} = \frac{4l^2}{\pi^2} \beta^2$$

Placing the z-axis vertically downwards and considering the distribution of the property within the interval $0 < z < h$ and $0 < x < 2l$ one obtains curves which have a slight similarity to isotherms in a vertical north-south section through an ocean and the author (Sverdrup, 1939) therefore suggested that the distribution of deep-sea temperature might result from processes of mixing only.

A closer examination of the distribution of temperature and salinity in the South Atlantic Ocean has now been completed, but this has clearly shown that the above suggestion is not correct as far as conditions in that ocean are concerned. The results do indicate, however, that in general the processes of lateral mixing are of greater importance than those of vertical mixing.

The examination was based on the horizontal charts of temperature and salinity by Wüst (Wüst and Defant, 1936). The selected area extends between latitudes 60° and 30° S. and from the South Atlantic Ridge to the South American Continent and the South Antillean Arc. The eastern boundary passes through the points 30° S., 15° W., 40° S., 15° W. and 60° S., 10° E. In this area average values of temperature and salinity were computed along every 5° of latitude at the depths 2000 m., 2500 m., 3000 m., 3500 m., and 4000 m. The average values were used for constructing vertical and horizontal curves by means of which the smooth isotherms and isohalines in Figures 64 and 65 were plotted. The following discussion will deal only with conditions between 2500 m. and 4000 m. within which interval of depth the corresponding average values of σ_t vary only between 27.80 and 27.87. The σ_t curves 27.82 and 27.84 have been plotted in Figure 64.

A further study of these vertical sections which represent average conditions in the western part of the South Atlantic Ocean may be criticized on the basis that by the process of averaging, the characteristic details are removed. It may particularly be pointed out that the tongue of cold water

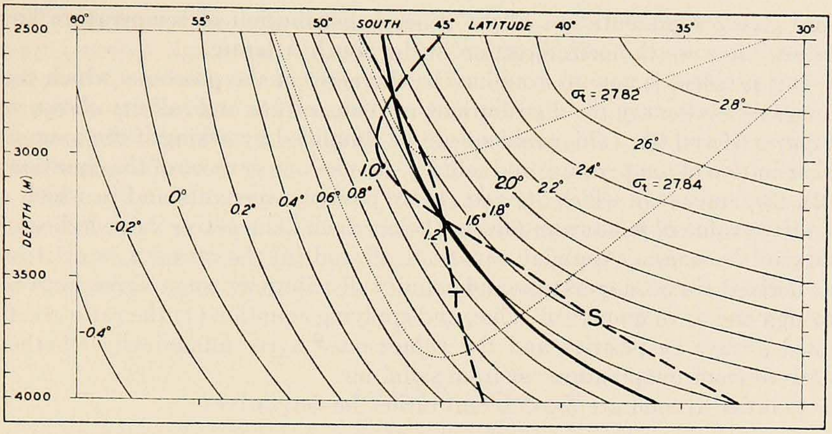

Figure 64. Average distribution of temperature in the South Atlantic Ocean between latitudes 60° and 30° S. and between depths of 2500 and 4000 meters, based on Wüst's horizontal charts. Lines marked *T* and *S* represent the boundaries between flow to the north and flow to the south as derived from temperature and salinity, respectively.

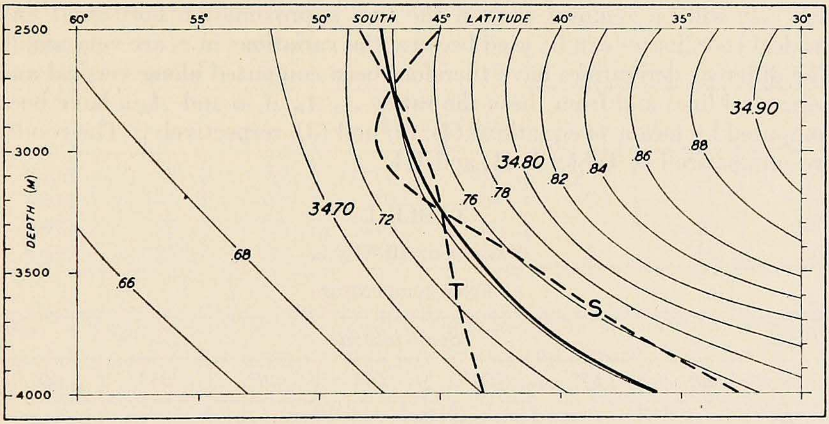

Figure 65. Average distribution of salinity in the South Atlantic Ocean between latitudes 60° and 30° S. and between depths of 2500 and 4000 meters, based on Wüst's horizontal charts. Lines marked *T* and *S* represent the boundaries between flow to the north and flow to the south as derived from temperature and salinity, respectively.

of low salinity along the coast of South America ought to be considered as a separate phenomenon. It can be emphasized on the other hand that a comparison between our sections and those prepared by Wüst (1935) on the basis of individual stations all show similar features and our sections therefore give a representation of the general distribution of temperature and salinity in a south-north direction in the South Atlantic.

Our problem is now to examine the character of the processes which can maintain stationary the distributions of temperature and salinity shown in Figures 64 and 65. This problem can be simplified by asking if the average distribution of temperature and salinity satisfies one or more of the equations (2), (3), and (4) in which A_n and A_l are positive constants and in which a positive value of u indicates flow to the north and a negative value indicates flow to the south. Applying equations (2) and (3) the ratios A_n/u or A_l/u, as derived from temperature and salinity distribution, must agree both as to sign and as to numerical value, and applying equation (4) the ratio A_l/A_u must always be positive and the values must agree numerically whether derived from temperatures or from salinities.

In order to compute the different ratios the derivatives

$$\frac{\partial s}{\partial l}, \frac{\partial^2 s}{\partial l^2} \text{ and } \frac{\partial^2 s}{\partial n^2}$$

must be found. For this purpose it should be known whether the flow is horizontal and whether the lateral mixing takes place in a horizontal direction. It will be assumed that in the first approximation horizontal and vertical coordinates can be used because the variations in σ_t are very small. The different derivatives have therefore been computed along vertical and horizontal lines and from these the ratios A_x/A_z, A_z/u and A_z/u have been computed by means of equations (4), (2) and (3), respectively. The results are summarized in Tables I, II, and III.

TABLE I

VALUES OF $10^{-5}A_x/Az$

a. From temperature

Depth	*South latitude*						
m	*60°*	*55°*	*50°*	*45°*	*40°*	*35°*	*30°*
2500	?	−7	−6	−7	−8	−30	−71
3000	?	?	−8	3	−11	−25	−230
3500	?	6	11	−60	7	1	−89
4000	?	6	31	?	7	?	−15

b. From salinity

Depth m	*60°*	*55°*	*50°*	*45°*	*40°*	*35°*	*30°*
2500	−41	−8	−16	12	−48	−36	−42
3000	95	12	10	−195	−66	−67	−47
3500	−78	−22	−8	−25	?	−45	−27
4000	−40	−50	17	−100	−74	−230	200

Table I contains the values of $10^{-5}A_x/A_z$. If this ratio shall have any physical significance it must, as already stated, be positive because A_x and A_z are both positive constants. It is seen, however, that the ratio is mainly *negative* from which it follows that processes of mixing alone can not account for the observed distribution of temperature and salinity in the area under consideration.

Table II contains the values of the ratio A_z/u. The coordinates are selected in such manner that a positive sign corresponds to flow towards the north and a negative sign corresponds to flow towards the south. In many instances it was not possible to determine this ratio, particularly because

TABLE II

VALUES OF A_z/u

a. From temperature

Depth	*South Latitude*						
m	*60°*	*55°*	*50°*	*45°*	*40°*	*35°*	*30°*
2500	?	150	420	−100	−76	−10	−8
3000	?	?	80	450	−34	−16	−2
3500	?	−220	−180	340	120	?	−24
4000	?	−170	−70	?	110	?	−120

b. From salinity

Depth m	*60°*	*55°*	*50°*	*45°*	*40°*	*35°*	*30°*
2500	6	75	75	−330	−50	−60	−35
3000	−6	−32	−70	−34	−39	−31	−26
3500	23	50	120	39	?	−25	−27
4000	84	65	−160	29	38	14	15

the vertical second derivative of temperature was too small. From the table it is seen that both the magnitude and the sign of the ratio vary irregularly and that no consistency exists between results derived from the temperature and from the salinity distribution except between latitudes 40° and 30° S. This area overlaps a small portion of the area which was examined by Defant (1936), and the distribution of the sign of the ratio corresponds to that found by Defant, but our values are three or four times larger than his. His examination was, however, based on different data and a complete agreement can therefore not be expected. The main point is that in the larger part of the section under consideration the distribution of temperature and salinity can not be accounted for as resulting from vertical mixing and horizontal flow, but in the northern portion of the section these processes may be of importance.

Table III contains the values of $10^{-7}A_z/u$ as derived from temperature and salinity, and the bottom portion of the table gives the averages. In this case there is a remarkable regularity in the variation of the sign and in the numerical values. On the whole, there is a good agreement between results derived from temperature distribution and from salinity distribution. Positive values indicating flow to the north are found in the southern part

TABLE III

VALUES OF $10^{-7}A_z/u$

a. From temperature

Depth m	*South latitude* 60°	55°	50°	45°	40°	35°	30°
2500	12	5	?	−7	−6	−3	−6
3000	8	13	7	−13	−4	−4	−6
3500	10	15	20	?	−8	−9	−19
4000	4	11	22	34	?	−10	−18

b. From salinity

Depth m	60°	55°	50°	45°	40°	35°	30°
2500	2	6	12	?	−24	−21	−15
3000	6	4	7	?	−26	−20	−12
3500	18	11	9	10	?	−11	−7
4000	34	32	26	29	29	?	−29

c. Averages

Depth m	60°	55°	50°	45°	40°	35°	30°
2500	6	6	?	?	−15	−12	−10
3000	7	8	7	?	−15	−12	−9
3500	14	13	14	?	?	−10	−13
4000	19	22	24	32	?	?	−24

of the section and the horizontal distance to which this flow reaches increases with increasing depth. The numerical values obtained from temperature and salinity are in fair agreement. In the northern part of the section all values are negative, indicating flow to the south, and in this case the extent of the flow to the south decreases with increasing depth. Thus, the distribution of temperature and salinity can be accounted for if horizontal mixing dominates and if in the southern portion of the section the flow is directed to the north and in the northern portion of the section to the south, the two currents being separated by a boundary surface which slopes downward from south to north. In Figures 64 and 65 are entered the boundaries between the flow to the north and the flow to the south as derived from temperature and salinity data. The lines do not quite coincide, for which reason an average line has been plotted.

The good agreement within the flow to the north of the values of A_x/u derived from temperatures and from salinities indicates perhaps that there the flow is nearly horizontal. Within the flow to the south the ratios derived from the temperature distribution are, however, much smaller than those derived from the salinity distribution, which may indicate that there the flow is not horizontal. An examination of the sections in Figures 64 and 65 shows that the ratios can be brought into better agreement by assuming descending motion. Within this flow significant differences in density are found as seen from the σ_t curves which are entered in Figure 65. If it is assumed that both the current and the lateral mixing are directed along σ_t surfaces the values of A_x/u derived independently from the temperature and salinity distributions are actually brought into agreement and are both about 8×10^7, that means somewhat smaller than the averages in the bottom part of Table III.

The above results demonstrate that in the section under discussion the distribution of temperature and salinity can be completely accounted for by nearly horizontal flow which is directed to the north in the southern part, and to the south in the northern part, and by processes of lateral mixing which in the southern part, where uniform density distribution is encountered, is horizontal, but in the northern part follows σ_t surfaces. If the velocity of the currents is about 1 cm/sec the coefficient of lateral mixing will be about 10×10^7 or twice as large as the maximum value found by Montgomery in the Equatorial region. Within both currents the velocity decreases with depth if the coefficient of lateral mixing can be considered constant.

It was pointed out that north of latitude 40° S. the vertical mixing may be of importance because the ratio A_z/u was consistent. This result is perhaps significant. If the lateral mixing is maintained by the effect of variable winds one must expect the lateral mixing to be most intense in the regions of variable wind systems, but less important over areas of constant wind conditions. In the southern part of the South Atlantic Ocean strong and variable winds dominate but in the central part lower wind velocities and less variable conditions are encountered, such that for these reasons a decrease of the intensity of the lateral mixing from south to north may occur.

It is not surprising that the coefficient of lateral mixing appears to be about 100 times greater than that off southern California. There a small area was examined within which the eddies associated with the lateral mixing were of dimensions 10 to 20 km. Here a large part of the open ocean is considered and the dimensions of the eddies are probably several hundred kilometers. The lateral mixing dealt with here probably corresponds to the macro-turbulence of the atmosphere.

The boundary surface between the deep currents to the north and to the south represents a region of convergence. The south-flowing Atlantic deep water must rise towards the surface in this region and the same is true as far as the greater amount of the north-flowing Antarctic water is concerned. This water can not sink under the south-flowing water because this would lead to bottom currents of unreasonable velocities. One is therefore forced to conclude that both the Antarctic water and the Atlantic deep water rise along the boundary surface separating two currents and that the water

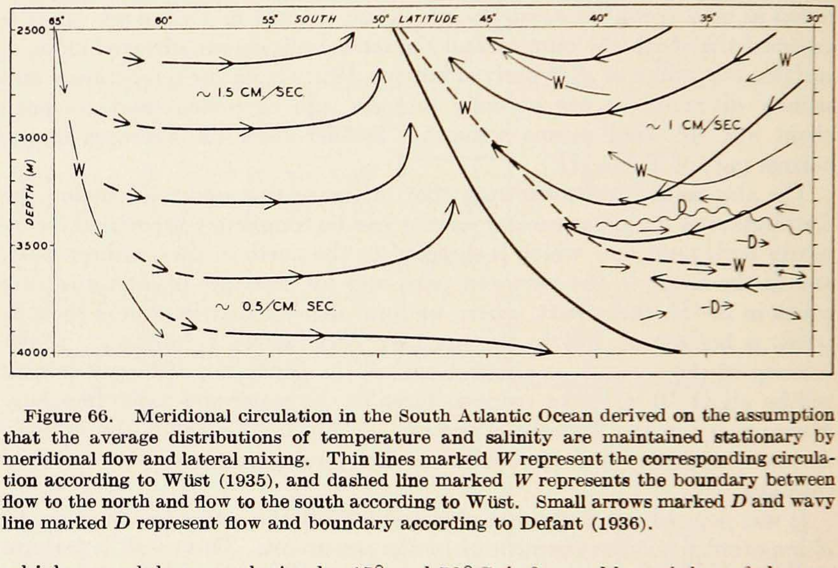

Figure 66. Meridional circulation in the South Atlantic Ocean derived on the assumption that the average distributions of temperature and salinity are maintained stationary by meridional flow and lateral mixing. Thin lines marked *W* represent the corresponding circulation according to Wüst (1935), and dashed line marked *W* represents the boundary between flow to the north and flow to the south according to Wüst. Small arrows marked *D* and wavy line marked *D* represent flow and boundary according to Defant (1936).

which ascends between latitudes 45° and 50° S. is formed by mixing of the two types. This conclusion is in agreement with the fact that the ascending water actually is intermediate in character between that of the two opposing types. This analysis therefore leads to an amplification of the picture of the deep-sea currents of the South Atlantic which has been prepared by Wüst (1935). Figure 66 gives a schematic representation of the deep-water flow based upon the present analysis, and in the same figure are entered the arrows which are shown in the same area by Wüst and the boundary surface which Wüst has determined between the Atlantic deep water and the Antarctic bottom water. The agreement is good except in the northern part where Wüst places the boundary surface higher than according to our results. In the figure is also shown Defant's boundary surface between the Atlantic deep water and the bottom water, and small arrows indicating his computed directions of flow. Defant's boundary surface also falls somewhat above the one determined here. Our analysis shows, in addition to these features, that the Antarctic deep water flows north between latitudes 60° and 50° S. and contributes to the ascending motion between latitudes 50° and 45° S.

The order of magnitude of the ascending motion is about 10^{-2} cm/sec or 10 m. per day, if it is assumed that the ascending motion takes place over a 500-km. wide area and that the velocities of meeting deep currents are about 1 cm/sec. An ascending motion of 10 m. per day appears large and it is therefore possible that the current velocities are less than 1 cm/sec. Wattenberg (1938) has pointed out, however, that south of latitude 50° S. one must assume a considerable ascending motion of the deep water in order to account for the high phosphate content of the surface waters during the period of the year when an enormous phytoplankton production takes place. The present picture is, therefore, in fair agreement with all of the available information.

In conclusion it should be emphasized that the above analysis should be considered as a first attempt to show that processes of lateral mixing must be taken into account when drawing conclusions as to currents in the deep-sea based on the distribution of properties. The results give added weight to the concept that large-scale lateral mixing takes place, but it should be admitted that many more data are needed for an examination which can lead to a final picture of the processes which are operating.

SUMMARY

1. The importance of lateral mixing in maintaining the distribution of temperature and salinity is discussed.

2. In the case of a two-dimensional system the general equation of stationary conditions is broken down into three separate equations, one by means of which the ratio A_z/u (A_z, coefficient of vertical mixing, u, horizontal velocity) can be computed if the distribution of properties is maintained by horizontal flow and vertical mixing, one by means of which the ratio A_x/u (A_x, coefficient of lateral mixing) can be computed if the distribution is maintained by horizontal flow and lateral mixing, and one by means of which the ratio A_x/A_z can be computed if the distribution is maintained by processes of mixing only.

3. Sections of average temperature and salinity distribution in the western South Atlantic Ocean between 60° and 30° S. were derived from the horizontal charts by Wüst and Defant, and the three ratios A_z/u, A_x/u, and A_x/A_z were computed.

4. It was found that only the ratio A_x/u showed consistent values over the entire section. It was concluded that between 2500 and 4000 m. Antarc-

tic deep water flows north, the flow reaching to about 50° S. at 2500 m. and to about 40° S. at 4000 m. The Atlantic deep water, on the other hand, flows south, the flow reaching to about 40° S. at 2500 m. and to about 35° S. at 4000 m. Between the two currents is an area of convergence, which slopes downward from south to north.

5. The distribution of temperature and salinity can be accounted for as a result of these currents and processes of lateral mixing which in the uniform Antarctic deep water are horizontal but in the Atlantic deep water are directed along σ_t surfaces. If the average current velocities are about 1 cm/sec the coefficient of lateral mixing is about $10^8 cm^2 sec^{-1}$. The ascending velocity in the area of convergence is about 10 m. per day and the ascending water represents a mixture of Antarctic deep water and Atlantic deep water. In the northern part of the area vertical mixing may be of importance as shown by Defant. The results amplify those by Defant and Wüst and the conclusions are generally in agreement with their findings.

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