NOTE ON THE FLOW OF STREAMS IN A ROTATING SYSTEM

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Certain similarity of the observed flow in the ocean and in the atmosphere to the behavior of a wake stream, for which a complete, experimentally verified theory is available (*1*), led Rossby (*2*) to initiate the extension of the wake stream idea first to a rotating system and secondly, to a stratified medium. Several interesting predictions were made by Rossby as a result of his theoretical investigation; and it was the purpose of the experiments carried out by the author to investigate to what extent these predictions could be verified in the laboratory. The experiments were of an exploratory nature and the conclusions can in no way be regarded as final.

Preliminary tests were made in a small cylindrical tank subject to cyclonic rotation and with a jet so arranged that the influx of water took place radially from the periphery. With this system the path described by water injected through a point jet at the surface was demonstrated for various radii of the inertia circle and these flow patterns were compared with those obtained in the same fashion but with the jet extending the whole depth of the layer of water being investigated. In the latter case the flow patterns differed radically from the inertia circle, the anticyclones being very much larger for the deep jet than for the surface, point jet. This indicated that certain pressure gradients were established which served to balance to some extent the Coriolis' forces. That such pressure gradients will be established has been demonstrated mathematically by Taylor (*3*).

It was evident, however, that with the jet issuing from the side of the tank, the wall effect contributed to the balancing pressure gradients. To minimize this wall effect, therefore, a large tank 6 feet in diameter was constructed and the jet inflow arranged to take place radially outwards from the center. The water was brought to the tank from a stationary head cistern by means of a gland around the hollow axis of the tank and issued through a vertical slit in this hollow axis. In this system it was possible to vary the following parameters at will:—

(I) the depth, from 5 cm. to 40 cms.
(II) the head at the jet, from 10 cms. to 100 cms.
(III) the rotational speed, from $1\frac{3}{4}$ r.p.m. to 10 r.p.m.

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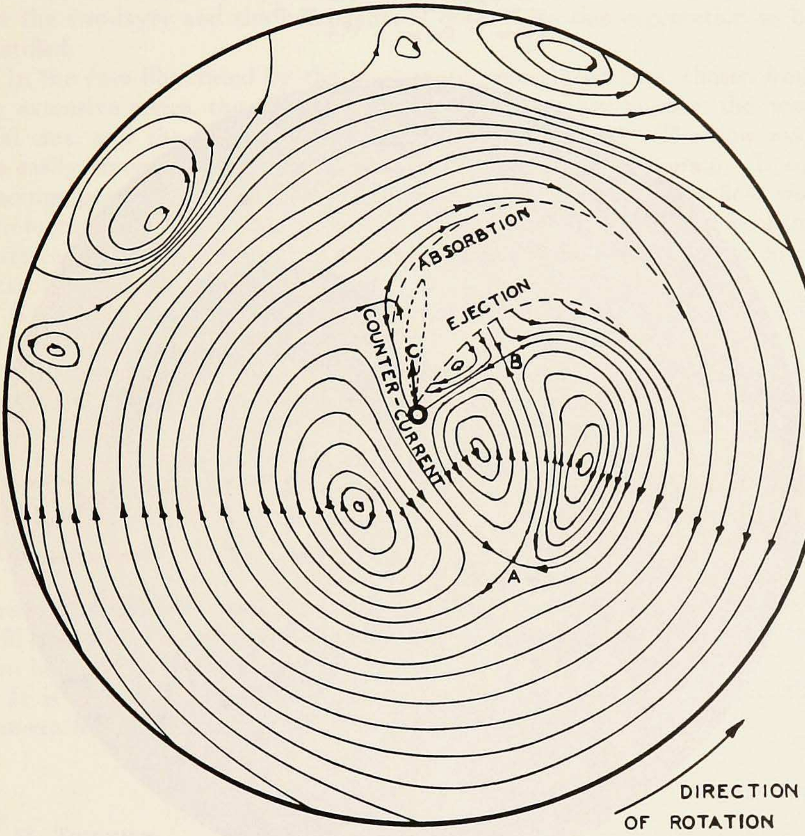
From Taylor's theory there is reason to expect, as indicated above, that the Coriolis or deflecting forces due to rotation would be completely balanced by pressure gradients resulting from the motion. Therefore, in a deep homogeneous medium with two dimensional motion, the flow pattern should be the same regardless of rotation and in the cylindrical tank described would take the form of a cyclone on the left side of the jet looking downstream and a symmetrically placed anticyclone on the right. Although Taylor's conditions were in no sense satisfied in the experimental arrangement described, they are approached more closely as the depth of the water in the tank increases, and therefore the flow patterns should show greater symmetry in the deeper cases. This was found to be the case; though, again, even in the deepest test made, the pressure gradients were far from completely balancing the deflecting force.

Rossby's extension of the wake stream theory to a rotating system calls for a counter-current on the left hand side of the jet, due to absorption there and ejection on the right side. This counter-current is brought about, in a homogeneous fluid, by deformations of the free surface and should be expected to be most marked for shallow depths. Thus it is seen that, whereas the counter-current does not appear to be compatible with the conception of completely balanced Coriolis forces, the two results are actually complementary cases at the extremes of the depth scale, viz.—in shallow layers the deformation of the free surface is percentually great compared to the depth and therefore the counter-current can be established, but in deep layers, where the surface deformations are percentually negligible, Taylor's reasoning applies.

This was well demonstrated in the six foot tank. With a head of 40 cms. and a rotation of 5 revolutions per minute, a clear counter-current accompanying a completely asymetrical flow pattern was observed for a depth of 6 cms., while for the same conditions of head and rotation, *i. e., the same radius of the inertia circle,* no counter-current and a markedly more symmetrical flow pattern was found when the depth was increased to 24 cms.

It has been pointed out that the effect of a given deformation of the free surface increases with diminishing depth, but decrease of depth also has the undesirable characteristic of accentuating the influence of bottom friction. The surface deformation effect can, however, be intensified, without decreasing the depth to the extent that bottom friction would play a noticeable role, by utilizing a stratified medium. Moreover, in nature, stratification often exists in the atmosphere and always is present in the ocean so, that, while Taylor's conclusions are true for a homogeneous deep fluid, Rossby's speculations as to the motion in shallow depths and stratified media are the more significant in Meteorology and Oceanography by virtue of their closer relationship to the natural state.

In order to demonstrate this effect of stratification a rotating tank was built having a four foot diameter and with 12 cms. depth; apart from its size this tank was similar in all respects to the larger one described above. Into the four foot tank was poured first a layer 6 cm. deep of a mixture, specific gravity 1.2, of Carbon Tetrachloride and Kerosene, and then above



KEY DIAGRAM

Figure 10. Surface flow in four foot rotating tank with stratification.

this the jet injected the wake stream into an overlying layer of water of equal depth. The water was allowed to overflow all around the edge of the tank, and when it is pointed out that the jet was only 12 thousandths of an inch wide it becomes evident that the distributed overflow would not introduce any serious effect on the flow pattern. In the ideal case with this stratification, if the lower liquid were to rotate with the tank as a solid, deformations of the free water surface would be magnified at the internal boundary by a factor

$$\frac{\rho_{lower}}{\rho_{lower} - \rho_{upper}} = \frac{1.2}{0.2} = 6,$$

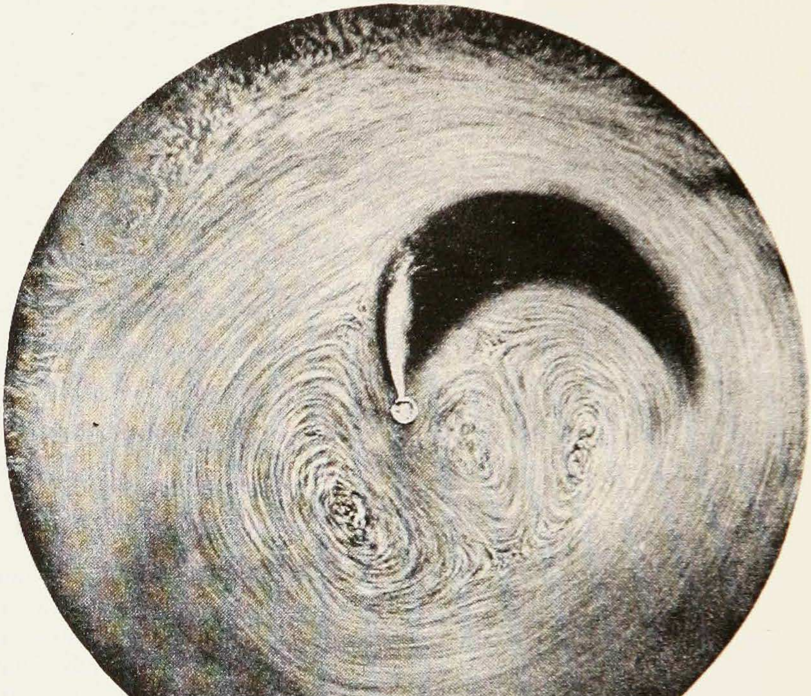


Figure 11. Surface flow in four foot rotating tank with stratification, ten revolutions per minute cyclonic rotation and 100 cms. head at jet

and therefore the Rossby effects are magnified while the Taylor balance is diminished. In the practical experiment it is impossible to prevent the circulation of the water above from imparting a similar motion to the underlying denser liquid, and, hereby, a tendency to barotropy, which would nullify the effectiveness of the stratification, occurs. Complete barotropy is prevented by bottom friction acting on the lower liquid and by the fact that the jet only plays in the overlying water and therefore a more intense counter-current and a less symmetrical flow is expected in the stratified than in the corresponding homogeneous case. Comparison of flow patterns for the two-layer and the homogeneous case shows this expectation to be justified.

In the case illustrated by the photograph reproduced here, chosen from an extensive series, the rotation was 10 revolutions per minute, the head 100 cms. and the two-layer stratification was employed. The flow may be easily traced by reference to the accompanying Key Diagram. Lycopodium powder was used as the indicating substance and the flow was photographed with the camera rotating with the tank. Examination of the pattern reveals that several of the features predicted are ratified to some extent at least in the experiment.

(I) A fully developed counter-current is established on the left hand side of the stream near the jet.

(II) Further downstream absorption on the left side is noticeable and such absorption implies the existence of a return current there also, but this counter-current is masked by the water sweeping around in the general anticyclonic motion and overlapping the wakestream.

(III) Ejection downstream on the right hand side is evident while on this side the only absorption noticeable occurs in the immediate proximity of the jet.

While these features to some extent are in accordance with Rossby's predictions it is also evident that important modifications to his analysis will have to made before the behavior of a wake stream in a rotating system can be completely explained.

It is interesting to note the flow at A and B in the photograph; the camera has caught two hyperbolic points in a classical pose.

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