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Relation between variations in the intensity of the zonal circulation of the atmosphere and the displacements of the semi-permanent centers of action[1]

by C.-G. Rossby and Collaborators[2]

This paper attempts to interpret, from a single point of view, several at first sight independent phenomena brought into focus through the synoptic investigations carried on at the Massachusetts Institute of Technology during the last few years. Since this interpretation is very largely based on a consideration of the changes in vorticity which must occur in vertical air columns which are displaced from one latitude to another and since such vorticity changes play a fundamental role also in Ekman's general ocean current theory (Ekman 1932), the results would appear to be of enough interest to physical oceanographers to warrant their publication in this journal. The particular phenomena brought out in the course of our studies are listed below.

1. Mean monthly isentropic charts for the month of August from the last few years show that the atmosphere over the United States at that time of the year is characterized by a well-marked anticyclonic cellular structure, each cell or eddy consisting of a dry current from the north and a moist current from the south, the total diameter of each cell being from 1500 to 2500 miles (Namias and Wexler 1938). Eddies of this type also represent the most outstanding feature of the *daily* isentropic charts for the summer season; the fact that the mean monthly charts show the same structure proves that the individual eddies are very nearly stationary and that they frequently regenerate in certain preferred locations. A comparison of the mean charts from different years shows that in spite of marked differences from year to year, one

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tongue of moist air always enters the United States from the southeast around or to the west of El Paso in New Mexico. Another moist tongue from the south frequently appears over western Florida. This recurrence suggests that the *position* of the eddies very largely must depend upon geographic factors. Nothing is known, however, about the dynamic factors which presumably control the *size* of the eddies.

2. During the last three years we have constructed daily weather maps for the Northern Hemisphere for use in an effort to extend the time range of our synoptic forecasts. In the course of these studies seven-day, later five-day mean pressure charts were constructed weekly. Underlying the construction of these mean charts is the assumption that the averaging process, at least partially, removes perturbations in the pressure distribution associated with the rapidly moving wave cyclones. The remaining perturbations are of larger dimensions and appear to determine the path of the wave disturbances. On the winter maps there are normally at least five such perturbations to be seen, the Icelandic and the Aleutian Lows, the Azores and the Asiatic Highs and finally the Pacific High, but one or several of these centers frequently breaks up into two parts. It is well known that these perturbations at higher levels no longer appear as closed isobaric systems but merely as undulations in the prevailing zonal pressure distribution. The sea level perturbations sometimes move westward several weeks in succession, finally splitting up into several parts. No explanation of these displacements and of the ultimate breaking up of the perturbations has been offered.

3. Five-day mean isentropic charts and five-day mean pressure charts for the three kilometer level have been constructed weekly. These upper level mean pressure charts often indicate the existence, during the winter season, of a trough of low pressure over the United States. The trough may remain stationary for several weeks in succession, but hitherto no adequate explanation of its position and displacement has been offered. The significance of the trough is best seen from the isentropic charts which show that even a fairly feeble low pressure trough over the Mississippi Valley with west-southwesterly gradient wind over the eastern part of the country in isentropic representation appears as a strong moist current extending from the Gulf northeastward towards the middle Atlantic coast.

4. In an attempt to obtain simple indices to the intensity of the general zonal circulation of the Northern Hemisphere, five-day means of the mean pressure on each latitude circle were computed and plotted weekly in the form of pressure profiles from equatorial to polar regions. Such curves were first used systematically by H. H. Clayton (1923). These curves of mean pressure against latitude have on several occasions shown marked trends towards increasing or decreasing zonal circulation, persisting through several weeks.

In the attempt to understand the dynamics of the upper level trough over the United States, the author found great help in a remarkable paper by J. Bjerknes (1937) which offers a simple explanation for the displacement of perturbations superimposed upon the zonal pressure
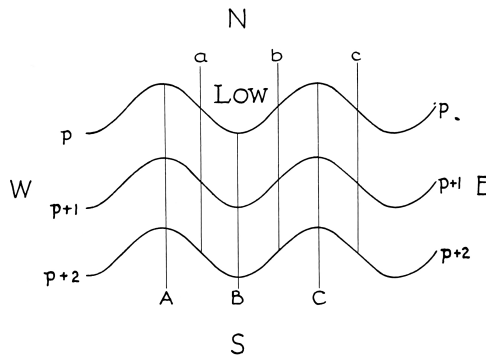
Figure 11. Sinusoidal Perturbation on Zonal Motion.

distribution which normally prevails in the upper part of the troposphere. Bjerknes studied the amount of air transported across a section perpendicular to two consecutive isobars and found that this transport depends upon the curvature of the isobars and upon the latitude of the section. Thus, if the pressure distribution is not strictly zonal but characterized by a certain perturbation, regions of convergence and divergence and hence of rising and falling pressure are created which lead to a displacement of the perturbation. Bjerknes' reasoning may be seen from Figure 11. Disregarding at first variations in latitude, it is evident that more air is transported through the sections A and C where the curvature is anticyclonic than through section B, where the curvature of the isobars is cyclonic. Thus horizontal divergence is created between B and C and a drop in pressure occurs to the east of the trough at B, while horizontal convergence and a rise in pressure will occur between A and B. As a result the trough will move eastward.

Since the variations in the transport and thus in the magnitude of the pressure changes increase with decreasing radius of curvature of the isobars, it follows that for the same amplitude short waves must move more rapidly eastward than long waves.

Considering now solely the effect of variation in latitude, it may be seen from the gradient wind relationship that more air is transported between consecutive isobars in low latitudes than in high. Thus the transport across B is greater than the transport across A or C and hence there must be convergence and pressure rise between B and C, divergence and pressure fall between A and B. The variations in transport are independent of the wave length of the perturbation, but increase with its amplitude. In the absence of other factors, the latitude effect here described would lead to a westward displacement of the perturbations.

For perturbations of the dimensions actually observed in the free atmosphere the two effects are approximately of the same magnitude. It follows that long waves must travel westward and shorter waves eastward. An intermediate wave length must exist such that the corresponding perturbations remain stationary.

The stream line pattern to which the above analysis by Bjerknes applies must satisfy not only the equation of continuity but also the equations of motion. If one considers the very simplest case of an ideal (non-friction) homogeneous, incompressible atmosphere in *purely horizontal motion*, the two equations of motion may, through the elimination of pressure, be compressed into a single equation expressing the conservation of absolute vorticity. Thus, if f represents the Coriolis' parameter ($f = 2\Omega \sin \varphi$) and consequently also the vertical component of the vorticity due to the rotation of the earth, and if ζ is the vertical component of the vorticity of the motion of the air relative to the earth's surface, then each individual vertical column must satisfy the condition

$$f + \zeta = \text{ constant.} \tag{1}$$

In this equation

$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}. \tag{2}$$

It is assumed that the x-axis points eastward, the y-axis northward. Vorticity is counted positive for cyclonic rotation, negative for anticyclonic rotation. Thus equation (1) implies that an air column which is displaced towards higher latitudes, where the cyclonic vertical component of the earth's rotation is stronger, will experience a decreasing cyclonic, or increasing anticyclonic, rotation. A column displaced towards lower latitudes will experience an increasing cyclonic, or decreasing anticyclonic, rotation.

It follows from (1) that the relative vorticity ζ, must remain constant in the absence of variations in latitude. The stream lines in Figure 11 show cyclonic vorticity between a and b, anticyclonic vorticity between b and c and hence such a pattern cannot be established through purely horizontal motion except possibly as the result of latitude variations. It is the purpose of the simple analysis below to show that the latitude variations are sufficient to bring about the required vorticity variations.

It follows from (1) that the individual variation of vorticity with time must obey the law

$$\frac{d\zeta}{dt} = -\frac{df}{dt}. \tag{3}$$

Since the Coriolis' parameter does not depend on longitude or time, it follows that

$$\frac{df}{dt} = v \frac{\partial f}{\partial y} = \beta v \left(\beta = \frac{\partial f}{\partial y} \right) \tag{4}$$

and hence

$$\frac{d\zeta}{dt} = -\beta v. \tag{5}$$

Table I. Variation of β with latitude

φ	$10^{13} \cdot \beta = \frac{2\Omega \cos \varphi}{R} \cdot 10^{13}$
90°	0.0 cm^{-1} sec^{-1}
75°	0.593
60°	1.145
45°	1.619
30°	1.983
15°	2.212
0°	2.290

In (5) β represents the rate at which the Coriolis' parameter increases northward. It is easily seen that β may be computed from the equation

$$\beta = \frac{\partial f}{\partial y} = \frac{\partial f}{R \partial \varphi} = \frac{2\Omega \cos \varphi}{R}, \tag{6}$$

R being the mean radius of the earth. Table I gives the results of such a computation. In the subsequent analysis it is necessary to treat β as a constant and hence independent of y. This assumption is reasonably justified near the equator where β has its maximum and its rate of variation in per cent is very small, but represents a rather severe restriction in higher latitudes. Thus β decreases by about 29% from 45° N. to 60° N.

We shall now apply (5) to the case of a zonal current of uniform constant velocity U upon which is superimposed a perturbation with the velocity components u' and v'. Thus the total velocities are

$$u = u' + U, v = v'. \tag{7}$$

Since the zonal current U is independent of x and y it follows that

$$\zeta = \zeta' = \frac{\partial v'}{\partial x} - \frac{\partial u'}{\partial y}. \tag{8}$$

The vorticity equation (5) may be expanded and gives then

$$\frac{\partial \zeta}{\partial t} + u \frac{\partial \zeta}{\partial x} + v \frac{\partial \zeta}{\partial y} = -\beta v \tag{9}$$

or

$$\frac{\partial \zeta}{\partial t} + U \frac{\partial \zeta}{\partial x} + \left[u' \frac{\partial \zeta}{\partial x} + v' \frac{\partial \zeta}{\partial y} \right] = -\beta v \tag{10}$$

For small perturbations the terms within the bracket may be neglected as small terms of second order (ζ, u' and v' are all small terms of the first order). Thus the equation reduces to

$$\frac{\partial \zeta}{\partial t} + U \frac{\partial \zeta}{\partial x} = - \beta v'. \tag{11}$$

Now let us look for perturbations traveling without change in shape: If c is the eastward speed of such a wave, it follows that

$$\frac{\partial \zeta}{\partial t} = - c \frac{\partial \zeta}{\partial x} \tag{12}$$

and consequently (11) transforms into

$$(U - c) \frac{\partial \zeta}{\partial x} = - \beta v'. \tag{13}$$

In the case of perturbations independent of y it follows from (8) that

$$\zeta = \frac{\partial v'}{\partial x} \tag{14}$$

and thus

$$(U - c) \frac{\partial^2 v'}{\partial x^2} = - \beta v'. \tag{15}$$

In the case of a simple sinusoidal disturbance of the wave length L the perturbation must be given by an expression of the form

$$v' \eqsim \sin \frac{2\pi}{L} (x - ct). \tag{16}$$

Substitution of (16) in (15) shows that

$$c = U - \frac{\beta L^2}{4\pi^2}, \tag{17}$$

This expression gives the wave velocity in terms of the gradient wind velocity and the wave length L.

It appears that the waves become stationary when

$$c = U - \frac{\beta L_s^2}{4\pi^2} = 0, \quad L_s = 2\pi \sqrt{\frac{U}{\beta}}. \tag{18}$$

Waves of greater length than L_s travel westward (c negative), shorter waves travel eastward. A combination of (17) and (18) gives

$$c = U \left(1 - \frac{L^2}{L_s^2} \right), \tag{19}$$

Table II. Stationary Wave Length in km as Function of Zonal Velocity (U) and Latitude (φ)

φ \ U	4 m/sec	8 m/sec	12 m/sec	16 m/sec	20 m/sec
30°	2822 km	3990 km	4888 km	5644 km	6310 km
45°	3120	4412	5405	6241	6978
60°	3713	5252	6432	7428	8304

Table III. Velocity Deficit ($U - c$) as Function of Number of Perturbations (n) and Latitude (φ)

φ \ n	2	3	4	5	6	7
30°	150.7 m/sec	67.0	37.7	24.1	16.7	12.8
45°	82.0	36.5	20.5	13.1	9.1	6.7
60°	29.0	12.9	7.3	4.6	3.2	2.4

from which the relation between wave length and speed is most readily seen. The stationary wave length is given in Table II as a function of the latitude and of the zonal circulation U.

The total number of waves (n) around the circumference of the earth at the latitude φ, is given by

$$nL = 2\pi R \cos \varphi \tag{20}$$

Substitution of (20) in (17) gives

$$U - c = \frac{2\Omega R}{n^2} \cos^3 \varphi \tag{21}$$

or, for stationary waves,

$$U = \frac{2\Omega R}{n_s^2} \cos^3 \varphi. \tag{22}$$

The velocity deficit $U - c$ obtained from (21) is given in Table III as a function of the number of perturbations and of the latitude. It is evident that if the zonal velocity distribution is known equation (22) enables one to determine the number of stationary perturbations possible in each latitude.

The preceding analysis applies to simple harmonic waves. The case of an arbitrary solitary perturbation observed at the time $t = 0$ is somewhat more complicated but may be treated with the aid of Fourier integrals. Thus, in the case of a symmetric solitary pressure ridge or trough centered around $x = o$, one finds

$$v' = \int_0^\infty F(\mu) \sin \mu \left[x - Ut + \frac{\beta}{\mu^2} t \right] d\mu, \tag{23}$$

Table IV. Maximum Zonal Velocity in February According to Bjerknes

Lat.	70°	60°	53°	45°	30°	28°
U_{max} (mps)	10	[16]	20	[23]	[29]	30

provided

$$v_o' = \int_0^\infty F\ (\mu)\ \sin\ \mu x \cdot d\mu.$$ (24)

represents the velocity distribution in the initial perturbation (v_o' is antisymmetric). It follows that the harmonics corresponding to large wave lengths $\left(\mu = \frac{2\pi}{L} < \frac{2\pi}{L_s}\right)$ must travel westward, those of shorter wave length ($L < L_s$) eastward, and thus a large initial perturbation may actually split in two centers, traveling in opposite directions.

V. Bjerknes and collaborators have computed the zonal velocity distribution from the zonal temperature distribution and from the zonal pressure distribution at the ground (V. Bjerknes and coll., 1933). The computation was made separately for the months of February and August. The aerological data used in this computation were, unfortunately, not entirely satisfactory for the purpose, since soundings from stations in widely different longitudes had to be combined (Pavia, Agra, Batavia). Bjerknes' diagram shows that the maximum velocity normally occurs just below the tropopause. The maximum velocities for the month of February have been taken from this diagram and are listed below in Table IV. Intermediate values have been determined by interpolation and are enclosed within brackets. A study of the tabulated values indicates that this portion of the atmosphere rotates with a practically constant relative angular velocity. A comparison with Table III shows that the number of stationary perturbations must be between two and three in 60° N. and slightly above four in latitude 30° N. This result is in fair agreement with the observed dimensions both of the permanent lows and of the subtropical high pressure cells.

In applying the preceding analysis to the semi-permanent centers of action, it must of course be emphasized that these centers are maintained in their normal position by permanent solenoidal fields acting alone or in cooperation with topographic factors. Thus there is no obvious reason why the character of these thermally or topographically produced "forced" perturbations should agree in detail with the stationary "free" perturbations analyzed above. To the extent that these forced perturbations are the result of solenoidal circulation they are controlled primarily by the distribution of solenoids in a vertical surface following a latitude circle around the earth. In such a surface the solenoids will be found to have their maximum concentration wherever the latitude circle crosses the boundary between continent and ocean. (Bjerknes and coll. 1933, pp. 686–693). During the warm season and in lower latitudes the concentration of solenoids is particularly marked along the west coasts of North America and of Europe Africa, during the cold season and in lower latitudes along the Pacific coast of Asia and over the eastern coast of North America. During

Table V. Component from the West (in m.p.s.) of Vectorial Mean Wind

Station	Lat. N.	6 km.	8 km.	10 km.	12 km.	Va Winter 14 km.
Ellendale	46.0	11.2	10.3	10.7	—	—
Omaha	41.2	10.7	8.8	8.6	—	—
Broken Arrow	36.0	14.3	12.6	13.7	20.3	—
Groesbeck	31.5	13.9	12.8	14.2	17.8	—
						Vb Summer
Ellendale	46.0	10.2	11.9	11.2	11.1	—
Omaha	41.2	6.3	7.4	6.1	9.6	11.5
Broken Arrow	36.0	2.8	3.1	3.3	4.0	3.8
Groesbeck	31.5	0	−0.5	−0.5	0.9	0.5

either season the two principal regions of solenoidal activity are much further apart than the "free" stationary wave length determined above. Each one of these solenoidal zones will create a practically stationary perturbation in the zonal circulation and in the trail of this perturbation a series of stationary waves must develop in somewhat the same fashion as the standing waves which are sometimes observed in the clouds on the lee side of a mountain ridge. Mathematically the problem may be expressed by saying that if, through solenoidal or topographic influences, a perturbation of the zonal circulation is maintained at the line $x = o$, such that the perturbation velocity and vorticity there have arbitrarily prescribed values,

$$v'_{x=o} = v_o', \quad \zeta'_{x=o} = \left(\frac{\partial v'}{\partial x}\right)_{x=0} = \zeta_o', \tag{25}$$

then it follows that from (15) that

$$v' = v_o' \cos \frac{2\pi x}{L_s} + \frac{\zeta_o' L_s}{2\pi} \sin \frac{2\pi x}{L_s}, \tag{26}$$

for all values of $x > 0$. This solution corresponds to a series of stationary perturbations of the wave length L_s. It is the author's opinion that this reasoning may help to explain the fact that the Pacific high pressure belt during the winter season generally splits into two separate cells. It is probable that upper level troughs formed in this fashion readily develop into frontogenetic zones near sea level.

Bjerknes' summer section is, unfortunately, very much disturbed by the Indian monsoon, and must therefore be used with a great deal of caution. It shows a velocity maximum of about 30 mps. in latitude 40° N. and would thus correspond to about four stationary high pressure cells in this latitude. However, the observed zonal velocity distribution over the United States differs rather markedly from the one computed by Bjerknes. In Tables Va and Vb are given the components from the west of the vectorial mean wind velocity at

different heights above four stations in the vicinity of the meridian of Omaha, Nebraska, for winter and summer. They were determined graphically from vectorial mean winds published by the U. S. Weather Bureau (Stevens 1937). These velocities are considerably less than those computed from the zonal temperature distribution. The tables show that during the winter season the west wind velocity is around 12 mps. up to about 12 km., whence it increases rapidly upward. In summer time the wind distribution is characterized by a strong anticyclonic shear in the north and by weak velocities over the greater part of the country. Assigning a value of 16 mps. to the zonal circulation during the winter season one finds, from Table III, that the number of permanent perturbations would be about 6 at 30° N., and between 4 and 5 at 45° N. Unfortunately there are no pilot balloon data available concerning the wind distribution at higher latitudes in the same meridian. If Bjerknes' computed value of the zonal circulation at latitude 60° N. is combined with the observed wind data at lower latitudes, the resulting zonal circulation is compatible with the number of perturbations observed in the polar regions. The slightly larger number, i.e. slightly smaller dimensions, of the subtropical cells is not incompatible with the actual dimensions of the anticyclones of lower latitudes.

Table Vb indicates that the summer season is characterized by a very weak zonal circulation over the United States with strong anticyclonic shear zone near the Canadian border. If a value of 4 mps. is assigned to the zonal wind, one obtains from Table II a stationary wave length of about 3000 km., in good agreement with the dimensions of the anticyclonic cells observed on our mean isentropic charts for the summer season.

The barotropic perturbations analyzed above are not associated with any available energy supply and are therefore stable and incapable of changing into real vortices. However, it is well known that a shear zone such as the one observed in the northern part of the country during the summer season is dynamically unstable and must break up into vortices which convert the energy of the zonal current into vortical kinetic energy. In this case, the equation for the conservation of vorticity should be modified to include the vorticity of the zonal current. Thus if the vorticity of the zonal current is designated by Z it follows that equation (13) changes into

$$(U - c)\frac{\partial \zeta'}{\partial x} + v'\frac{\partial Z}{\partial y} = - \beta v' \tag{27}$$

or, since

$$Z = - \frac{\partial U}{\partial y}, \tag{28}$$

$$(U - c)\frac{\partial \zeta'}{\partial x} = - \left(\beta - \frac{\partial^2 U}{\partial y^2}\right)v'. \tag{29}$$

An inspection of Table Vb shows that $\frac{\partial^2 U}{\partial y^2}$ at the most is about 25% of β.

Since $\frac{\partial^2 U}{\partial y^2}$ is positive, the stationary wave length is thus increased by a maximum of about 13%. It should be remembered, however, that the original solution no longer strictly holds

since the zonal velocity (U) is a function of y and the perturbation thus also must depend on that coordinate.

The solenoidal field along the western coast of North America in combination with the steep mountain ranges creates a permanent perturbation (trough) near the coast and the permanent perturbation thus maintained will set up a series of standing perturbations in the zonal pressure distribution further down stream. The shearing zone along the northern border of the United States must necessarily break up into eddies which take care of the dissipation of the kinetic energy generated further to the north. It seems likely that the maximum size to which the frictionally driven eddies can grow is determined by the grid provided by these semi-permanent perturbations.

This grid would seem to explain the appearance of a moist tongue over western Florida in the isentropic mean charts for August and suggests an explanation for the occasional occurrence of a third moist tongue in the vicinity of Bermuda.

The large semi-permanent Bermuda high pressure cell observed on the seasonal mean sea level pressure charts, on the north side of which this grid of smaller perturbations is superimposed, is not explained by the preceding analysis but appears to require a permanent and widespread field of solenoids for its maintenance. It is also possible that the larger dimensions of this system are associated with the higher zonal wind velocities prevailing near the base of the stratosphere.

The simple analysis carried out above does not include the effect of changes in depth of the moving fluid column. A single layer atmosphere, moving with a constant zonal velocity U must have a depth D_o which decreases northward. This slope gives rise to a pressure gradient which balances the Coriolis' force associated with the zonal motion. Thus

$$U = -\frac{g}{f}\frac{\partial D_o}{\partial y}.\tag{30}$$

It follows that fluid columns moving north or south are going to change their depth and as a result their vorticity will change. In addition, the traveling perturbations themselves cause deformations of the free surface which in turn produce changes in vorticity. To analyze the problem completely we must return to the general equations of motion. For a homogeneous incompressible atmosphere of thickness D they may be compressed into the following system:

$$\frac{d\zeta}{dt} = -\frac{df}{dt} - (f+\zeta)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\tag{31}$$

$$\tfrac{1}{2}\frac{dq^2}{dt} = -g\left(\frac{dD}{dt} - \frac{\partial D}{\partial t}\right)\tag{32}$$

$$\frac{1}{D}\frac{dD}{dt} = -\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right).\tag{33}$$

The first of these equations expresses the conservation of vortices, the second the conservation of energy, and the third the conservation of mass. In these equations q^2 is the kinetic energy per unit mass and g the acceleration of gravity.

In our particular case

$$u = U + u', v = v', q^2 = U^2 + 2u'U + (u'^2 + v'^2) \tag{34}$$

and

$$D = D_0 + \delta \tag{35}$$

D_o being the depth (decreasing northward) of the undisturbed atmosphere in zonal motion and δ the deformation due to the travelling perturbations. Neglecting squares and products of u' and v' one finds

$$\tfrac{1}{2}\frac{dq^2}{dt} = U\frac{du'}{dt} = U\left(\frac{\partial u'}{\partial t} + U\frac{\partial u'}{\partial x}\right) \tag{36}$$

and

$$\frac{dD}{dt} = \frac{\partial \delta}{\partial t} + U\frac{\partial \delta}{\partial y} + v'\frac{\partial D_o}{\partial y} \tag{37}$$

or, because of (30),

$$\frac{dD}{dt} = \frac{\partial \delta}{\partial t} + U\frac{\partial \delta}{\partial x} - \frac{fU}{g}v'. \tag{38}$$

If we replace $f + \zeta$ on the right hand side of (31) with a constant f_o[3] and introduce a constant mean value D_∞ for the undifferentiated D in equation (33), equations (31) to (33) are readily solved. For perturbations which are independent of the y-coordinate, one obtains finally an equation for the wave speed which may be written

$$(U - c)\frac{4\pi^2}{L^2} = \beta + \frac{cf_o^2}{gD_\infty - (U - c)^2}. \tag{39}$$

It is interesting to note that for $U = 0, \beta = 0$, this equation gives the wave velocity obtained by Sverdrup (1926) in his study of tidal waves on the North Siberian shelf. For $U = 0, \beta = 0, f_o = 0$ it reduces to the well known formula for long gravitational waves

$$c = \pm\sqrt{gD_{oo}}. \tag{40}$$

For standing perturbations $(c = 0)$ it reduces to the relation obtained earlier in this paper by elementary methods,

$$L_s = 2\pi\sqrt{\frac{U}{\beta}}. \tag{41}$$

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3. This substitution is justified in view of the small contribution of the divergence term in (31) to the individual change in vorticity in a single-layer barotropic atmosphere.

For a homogeneous atmosphere the product gD_{00} has a value of about $8 \cdot 10^8 \cdot \mathrm{cm}^2 \cdot \mathrm{sec}^{-2}$. For the perturbations in which we are interested $(U - c)^2$ is of the order of magnitude $10^7 \cdot \mathrm{cm}^2 \cdot \mathrm{sec}^{-2}$ or less. Thus it is a permissible approximation to write

$$(U - c) \frac{4\pi^2}{L^2} = \beta + \frac{c f_o^2}{g D_{oo}}, \tag{42}$$

or, after solving for c,

$$c = \frac{U - \dfrac{\beta L^2}{4\pi^2}}{1 + \dfrac{L^2}{4\pi^2 \lambda^2}}, \tag{43}$$

in which expression the length λ, defined by

$$\lambda = \frac{1}{f_o} \sqrt{g D_{oo}}, \tag{44}$$

has a value of about 2800 km. for a homogeneous atmosphere. Even for wave lengths double the value of λ the wave velocity determined by (43) is only 10% less than the one obtained from the approximate formula (17). This formula has an important forecasting implication. If a series of permanent perturbations exists at the time $t = 0$ and a study of consecutive pressure profiles suggests that the zonal circulation is slowing down, it follows that the perturbations will be displaced westward, but the rate of this displacement must be less than the reduction in the zonal circulation velocity.[4] It is likewise evident that an acceleration of the zonal circulation speed will bring about an eastward displacement of the perturbations, but again at a slower rate than the change in the zonal velocity.

It was stated earlier than an arbitrary perturbation, once created, will be displaced in such a fashion that harmonics of greater wave length will move westward, those of shorter wave length eastward, and that this process may lead to a splitting up of the perturbation. It is now evident that such displacements may be produced also by changes in the zonal circulation. It is evident that once the centers of action have been displaced from their normal positions, the fairly stationary solenoidal fields responsible for their generation and maintenance may lead to a regeneration of the centers in their normal position. In this way double centers will again be produced. An inspection of our northern hemisphere charts appears to indicate that this is particularly likely to happen when the Asiatic High, as a result of a slowing down of the general zonal circulation, has been displaced far to the west (towards Europe) of its normal position.

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4. A preliminary analysis of perturbations in the upper, moving portion of a double layer atmosphere with a bottom layer at rest indicates that the denominator in (43) then may reach values of 4 or 5; thus the rate of displacement of non-stationary perturbations is greatly reduced. However, the final answer to this question cannot be given until a satisfactory theoretical analysis has been carried out. The same preliminary result applies to a double-layer atmosphere in which the motion occurs in the lower layer.
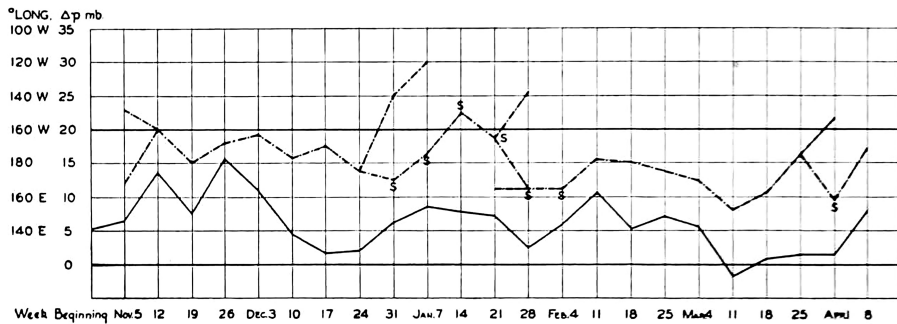
Figure 12. Relation between position of Aleutian Low and Zonal Circulation Intensity. Broken Line
   Indicates Longitude of Aleutian Low, as Determined from our Weekly Five Day Mean Pressure
   Charts and Full Line gives Zonal Circulation Intensity as Indicated by Mean Pressure Difference
   on the same Charts between 35° N. and 55° N.

Two practical examples to illustrate the preceding theoretical analysis are presented
below. Figure 12 contains a comparison for the winter 1938–1939 between the longitude
of the Aleutian Low, as determined by my colleague Dr. H. C. Willett from the five day
mean pressure chart prepared weekly under his direction at the Massachusetts Institute of
Technology, and the intensity of the zonal circulation as measured by the pressure differ-
ence between latitudes 35° N. and 55° N. obtained by averaging this pressure difference all
around the globe on the same charts. There are a few cases of splitting of the center and in
these cases double points have been entered. If in each case the stronger of the two centers
(marked *s*) is selected, it is readily seen that a good correlation is obtained between the two
curves. There is also some evidence of a few days lag of the longitude curve relative to the
circulation index. The correlation coefficient between the two curves is 0.665. A later and
more accurate determination of the zonal circulation index resulted in a correlation coeffi-
cient of 0.695. Finally, to prove that the variations in the intensity of the zonal circulation
index are not merely the result of the longitude variations of the Aleutian Low, a revised
zonal circulation index was computed which excluded a section of 120 degrees of longitude
over the Pacific. Even then a correlation of 0.542 with the longitude of the Aleutian Low
was obtained.

It may at first sight appear unreasonable to compare the displacement of the Aleutian
Low with the zonal circulation intensity in a much lower latitude. It must be remembered,
however, that the positions and displacements of the semi-permanent centers of action
depend upon the intensity of the zonal motion in the free atmosphere well above the seat of
the advective phenomena. The author has tried to demonstrate, in a previous paper (Rossby
1938), that at least during the summer season, the westerlies south of the polar front are to a
large extent maintained by isentropic stresses which transmit the motion of the polar vortex
to the surface layers in somewhat lower latitudes. On the basis of this analysis, it appears
reasonable to measure the intensity of the polar vortex in the free atmosphere by the sea
level pressure gradient to the south of the polar front.
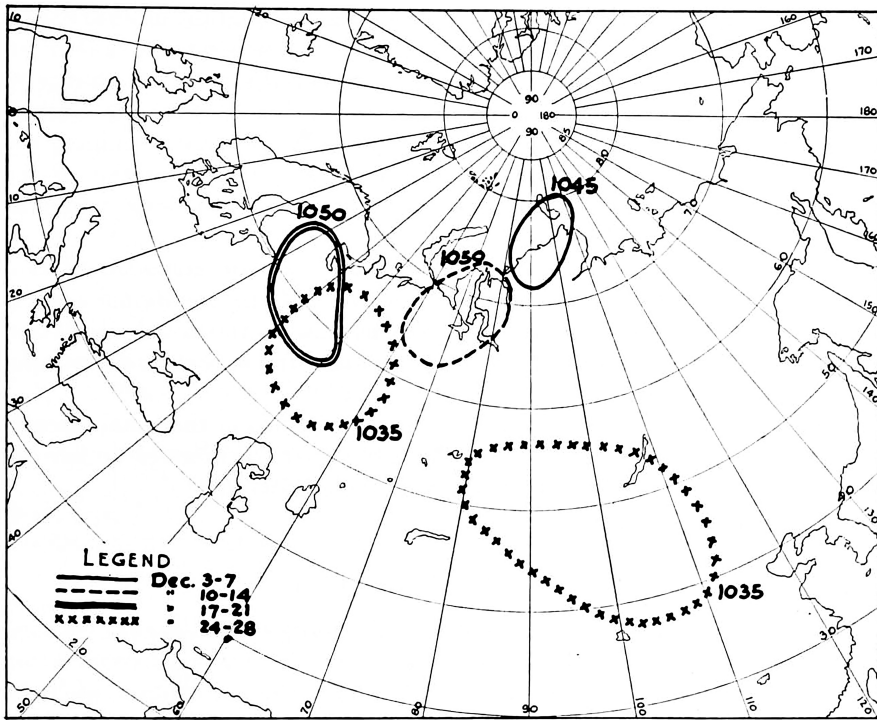
Figure 13. Successive positions of last closed isobar in Asiatic High during December 1938. These
  isobars were obtained from our weekly five day mean pressure charts.

Figure 13 shows the successive positions of the innermost closed isobar of the Siberian
High during a few weeks in December, 1938, as determined from the weekly five day
mean pressure charts for this period. An inspection of the circulation index in Figure 12
shows that this period was characterized by a continuous decrease in the zonal circulation
intensity, and Figure 13 shows that the Siberian High during this period advanced towards
northwestern Europe, where its arrival caused a complete change in weather type. This
example is particularly significant, since the High is located in entirely different latitudes
from the ones used in determining the circulation intensity. It is further more located over the
interior of the continent where the normal zonal pressure distribution is no longer discernible.

It is perhaps not superfluous to emphasize that the zonal circulation intensity varies with
the intensity of the solenoids in meridional planes, while the maintenance of permanent
perturbations on this zonal circulation primarily must be the result of solenoids contained
in vertical planes parallel to the latitude circles.

Further examples of the relation between variations in the zonal circulation intensity and
displacements of the atmospheric centers of action are discussed by Dr. Willett in another
place (Willett 1939).

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