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# Mass transport in the Stokes edge wave

by Jan Erik H. Weber<sup>1,2</sup> and Peygham Ghaffari<sup>1</sup>

## ABSTRACT

The Lagrangian mass transport in the Stokes progressive edge wave is obtained from the vertically integrated equations of momentum and mass, correct to second order in wave steepness. The cross-shore momentum balance is between the mean pressure at the sloping bottom, the radiation stress, and the pressure gradient due to the mean surface slope. In the alongshore direction, the effect of viscosity leads to a wave attenuation, and hence a radiation stress component. The frictional effect on the mean Eulerian motion is modeled through a turbulent bottom drag. The alongshore momentum balance is between the mean pressure gradient due to the surface slope, the radiation stress, and the turbulent drag on the mean Eulerian flow. It is shown that  $-\partial E/\partial y$ , where  $E$  is the total mean energy density for waves along the  $y$ -axis, is the wave-forcing term for the total mean Lagrangian momentum in the trapping region. This result is independent of the bottom slope angle. Vertically-averaged drift velocity components are obtained from the fluxes, divided by the local depth. Utilizing physical parameters relevant for field conditions, it appears the traditional Stokes drift in the Stokes edge wave is negligible compared to the mean Eulerian velocity component. The importance of this drift for the near-shore transport of effluents and suspended light sediments is discussed.

## 1. Introduction

After being regarded as a mere curiosity for many years, e.g. Lamb (1932), edge waves have fairly recently received renewed interest. This is because such waves apparently play an important role in the dynamics of coastal zone and beach erosion processes (LeBlond and Mysak, 1978). Edge waves are often considered as the major factor of the long-term evolution of the irregular coastal line, forming rhythmic crescentic bars (Bowen and Inman, 1971; Kurkin and Pelinovsky, 2003; Quevedo *et al.*, 2008). Holman and Bowen (1982) showed that the steady drift, generated by the nonlinear self-interaction of edge waves inside the bottom boundary layer, can cause a net displacement of the sediment and give rise to bottom patterns similar to those detected in the field.

We here concentrate on the Stokes edge wave (Stokes, 1846), which is the first mode in the spectrum of shelf modes that contains both discrete and continuous parts (Eckhart, 1951; Ursell, 1952; Reid, 1958). Several mechanisms for generating edge waves are possible in nature. Large-scale edge waves can be generated by direct wind stress above the

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water, by traveling air pressure, or by tsunamis (Munk *et al.*, 1956; Aida, 1967; Beardsley *et al.*, 1977; Fuller and Mysak, 1977; Golovachev *et al.*, 1992; Boss *et al.*, 1995; Kurkin and Pelinovsky, 2003; Galletta and Vittori, 2004; Monserrat *et al.*, 2006), while medium- and small-scale edge waves may occur through nonlinear interaction of wave groups or nonlinear subharmonic resonance mechanisms (Gallagher, 1971; Minzoni and Whitham, 1977; Bowen and Guza, 1978; Chapman, 1984). The occurrence of edge waves has also been demonstrated in wave tank experiments (Yeh, 1985; Mok and Yeh, 1999).

In the present study we focus on the mass transport induced by the Stokes edge wave. Earlier papers, (e.g. Kenyon (1969)) have considered the pure Stokes drift (Stokes, 1847) in inviscid edge waves applying the hydrostatic approximation, while Dore (1975) and Mok and Yeh (1999) have calculated the mass transport velocity in the viscous laminar bottom boundary layer associated with edge wave motion. But obviously, real field bottom boundary layers are turbulent. Furthermore, the frictional effect at the bottom will generate a mean interior Eulerian flow, in addition to the Stokes drift (Longuet-Higgins, 1953). It is the aim of the present paper to quantify the mean Eulerian mass transport generated by the Stokes edge wave in a turbulent ocean. When we add the Stokes flux, we obtain the total mean Lagrangian mass transport in the system. It is this transport that advects neutral tracers and bottom sediment in suspension in the region of wave trapping. In order to obtain a robust formulation, we consider the vertically integrated equations of momentum and mass, e.g. Phillips (1977), and derive the mean Lagrangian mass transport to second order in wave steepness. The vertically-averaged drift velocity is obtained by dividing the volume flux by the local depth. In this way we do not resolve the motion in the bottom boundary layer, so this method is not directed at sediment transport very close to the sea bed. However, for finer sediment that is mixed in the entire water column, and the drift of biological material, the present approach yields new and interesting results.

## 2. Mathematical formulation

We consider trapped surface gravity waves in a homogeneous incompressible fluid with a linearly sloping bottom. The motion is described in a Cartesian system, where the  $x$ -axis is situated at the undisturbed surface and directed into the semi-infinite sea, the  $y$ -axis is directed along the shore line, and the vertical  $z$ -axis is positive upwards; see the sketch in Figure 1. The corresponding velocity components are  $(u, v, w)$ . Furthermore, the pressure is  $p$  and the constant density is  $\rho$ . The bottom is given by  $z = -h = -x \tan \beta$ , where  $\beta$  ( $\leq \pi/2$ ) is the sloping angle, and the free surface by  $z = \eta$ . At the free surface the pressure is constant. In this study we disregard the effect of the earth's rotation.

We denote periodic wave variables by a tilde, and the mean flow (averaged over the wave period) by an over-bar. Mean horizontal volume fluxes ( $\bar{U}$ ,  $\bar{V}$ ) are defined by

$$\bar{U} = \overline{\int_{-h}^{\eta} u dz}, \quad \bar{V} = \overline{\int_{-h}^{\eta} v dz}. \quad (1)$$

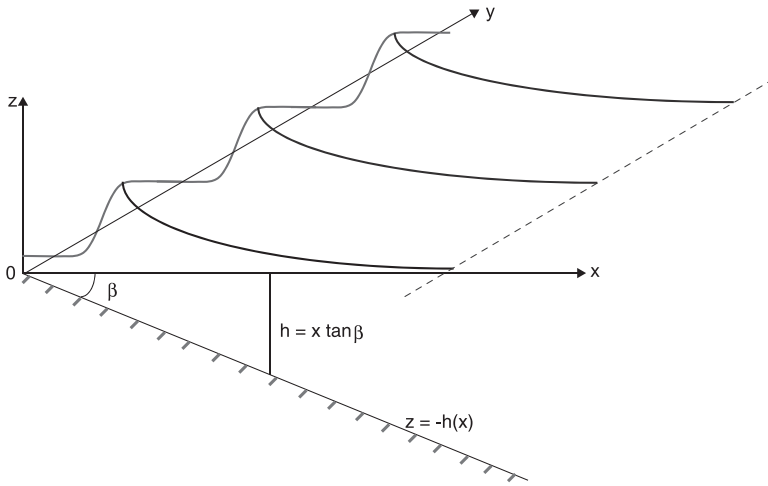


Figure 1. Sketch depicting the coordinate system, with the surface and sloping bottom included;  $y$  is the alongshore coordinate and the seawards direction is  $x \rightarrow \infty$ .

These are actually the Lagrangian fluxes, since we integrate between material boundaries (Phillips, 1977; Weber *et al.*, 2006). Integrating the governing equations in the vertical, and utilizing the full nonlinear boundary conditions at the free surface and the sloping bottom, we obtain for the mean quantities, correct to second order in wave steepness (Phillips, 1977):

$$\begin{aligned} \frac{\partial \bar{U}}{\partial t} &= -\frac{1}{\rho} \frac{\partial}{\partial x} \int_{-h}^0 \bar{p} dz - \frac{1}{2\rho} \frac{\partial}{\partial x} \overline{\bar{p}(0)\bar{\eta}} + \frac{1}{\rho} \tan \beta \bar{p}(-h) \\ &\quad - \frac{\partial}{\partial x} \int_{-h}^0 \overline{\bar{u}\bar{u}} dz - \frac{\partial}{\partial y} \int_{-h}^0 \overline{\bar{u}\bar{v}} dz - \frac{\bar{\tau}_{-h}^{(x)}}{\rho}, \\ \frac{\partial \bar{V}}{\partial t} &= -\frac{1}{\rho} \frac{\partial}{\partial y} \int_{-h}^0 \bar{p} dz - \frac{1}{2\rho} \frac{\partial}{\partial y} \overline{\bar{p}(0)\bar{\eta}} - \frac{\partial}{\partial x} \int_{-h}^0 \overline{\bar{u}\bar{v}} dz - \frac{\partial}{\partial y} \int_{-h}^0 \overline{\bar{v}\bar{v}} dz - \frac{\bar{\tau}_{-h}^{(y)}}{\rho}, \\ \frac{\partial \bar{\eta}}{\partial t} &= -\frac{\partial \bar{U}}{\partial x} - \frac{\partial \bar{V}}{\partial y}. \end{aligned} \tag{2}$$

By neglecting the effect of friction in the vertical component of the momentum equation, Phillips found for the mean pressure to this order:

$$\frac{\bar{p}}{\rho} = g(\bar{\eta} - z) - \overline{\bar{w}^2} + \frac{\partial}{\partial x} \int_z^0 \overline{\bar{u}\bar{w}} d\xi + \frac{\partial}{\partial y} \int_z^0 \overline{\bar{v}\bar{w}} d\xi, \tag{3}$$

where  $g$  is the acceleration due to gravity. As shown by Mei (1973) for the Stokes standing edge wave, the dynamic mean bottom pressure term in the  $x$ -momentum,  $\tan \beta \bar{p}(-h)/\rho$  which is missing from Phillips' derivation, must be present here. Furthermore,  $(\bar{\tau}_{-h}^{(x)}, \bar{\tau}_{-h}^{(y)})$  in (2) are the mean turbulent bottom stress components.

In this problem the oscillatory edge wave motion is influenced by viscosity. In general; for deep water waves, viscosity will affect the motion in the bulk of the fluid, while for shallow water the viscous boundary layer at the bottom will dominate. In both cases the potential part of the wave field, which is the relevant one in flux consideration, will attenuate exponentially in time. For deep water the damping coefficient will be proportional to the small viscosity coefficient, while in shallow water the damping coefficient will be larger (no slip at the bottom). In this case it is proportional to the square of the viscosity coefficient, e.g. Phillips (1977). In any case, we can obtain the potential part of the wave field by using a friction that is linear in the wave velocity, yielding the small exponential decay in time. Accordingly, we write the frictional force per unit mass on the linear wave motion as  $-r\vec{u}$ , where the constant friction coefficient  $r$  depends on the viscosity. This kind of friction does not introduce vorticity into the fluid, so we can apply the potential theory of Stokes (1846). The linearized relation between the velocity potential  $\tilde{\phi}$  and the pressure then becomes

$$\tilde{p} = -\rho \left( \frac{\partial \tilde{\phi}}{\partial t} + gz + r\tilde{\phi} \right). \quad (4)$$

In the present problem we consider waves with given frequency  $\omega$ . Then, due to friction, the wave number  $\kappa$  in the  $y$ -direction (along the coast) will be complex, i.e.  $\kappa = k + i\alpha$ , where  $k > 0$ , and  $\alpha$  is the small spatial attenuation coefficient ( $\alpha/k \ll 1$ ). In fact, for a general set of wave problems, the temporal attenuation coefficient is equal to the spatial one times the group velocity of the wave (Gaster, 1962).

For the spatially damped Stokes edge wave, we can write the complex velocity potential

$$\tilde{\phi} = -\frac{ia\omega}{k \sin \beta} \exp(-kx \cos \beta + kz \sin \beta - \alpha y + i(ky - \alpha x \cos \beta + \alpha z \sin \beta - \omega t)), \quad (5)$$

where  $a$  is the wave amplitude. The potential part of the velocity is given by  $\vec{u} = \nabla \tilde{\phi}$ , and it is easily seen that (5) satisfies the Laplace equation, and the tangential flow condition at the linearly sloping bottom. From the dynamic condition at the free surface,  $\tilde{p}(z = \hat{\eta}) = 0$ , we find to lowest order for the wave frequency and the spatial damping coefficient:

$$\omega^2 = gk \sin \beta, \quad \alpha = \frac{kr}{\omega}. \quad (6)$$

Hence waves can propagate in both directions, becoming damped as they progress along the  $y$ -axis. We here consider propagation to the right, i.e. take  $\omega > 0$ .

To determine the damping rate, we need to quantify the friction coefficient  $r$ . If we neglect the effect of a viscous boundary layer along the sloping bottom, which is permissible for large depths (i.e. large  $x$ ), we can determine the temporal wave attenuation coefficient  $\gamma$  from energy considerations (Phillips, 1977). Using real parts from potential theory, we find for the total mean energy density that

$$E = \int_0^\infty \left\{ \int_{-h}^0 \frac{\rho}{2} (\bar{u}^2 + \bar{v}^2 + \bar{w}^2) dz + \rho g \int_0^{\bar{\eta}} z dz \right\} dx = \frac{\rho g a^2}{4k \cos \beta} \exp(-2\gamma t), \quad (7)$$

while the total dissipation  $D$  in this problem is readily found to be

$$D = -\frac{\rho \nu g k a^2}{2 \cos \beta} \exp(-2\gamma t). \quad (8)$$

Assuming that  $dE/dt = D$  (Phillips, 1977) we obtain

$$\gamma = k^2 \nu, \quad (9)$$

which is exactly half the value for ordinary deep-water surface waves. Utilizing Gaster (1962), we find the spatial attenuation coefficient for this case:

$$\alpha = \frac{\gamma}{d\omega/dk} = \frac{2k^3 \nu}{\omega}. \quad (10)$$

For shallow water waves the temporal damping coefficient is related to the eddy viscosity coefficient  $\nu$  by the relation  $\gamma = \nu/(2H\delta)$ , where  $H$  is the mean depth, and  $\delta = (2\nu/\omega)^{1/2}$  is the viscous boundary-layer thickness (Phillips, 1977). A typical depth here will be that at the outer edge of the trapping region, i.e. we take  $H = \tan \beta/k$ . Then, applying Gaster's result, the spatial attenuation coefficient in this case becomes:

$$\alpha = \frac{k^2}{\tan \beta} \left( \frac{\nu}{2\omega} \right)^{1/2}. \quad (11)$$

It should be noted that (10) and (11) are the two extreme values for frictional damping in our problem. For realistic field conditions the water will be shallow. Therefore, the value given by (11) will in practice represent the magnitude of the spatial damping in our problem. In any case, the friction coefficient is obtained from (6), i.e.  $r = \omega\alpha/k$ .

Utilizing the real part of (5), it is trivial to calculate the right-hand side of (2). The  $x$ - and  $y$ -components of the Lagrangian fluxes to second order in wave steepness then becomes

$$\frac{\partial \bar{U}}{\partial t} + gh \frac{\partial \bar{\eta}}{\partial x} = \left( \frac{gka}{\omega} \right)^2 (kx \cos^2 \beta \sin \beta) \exp(-2kx \cos \beta - 2\alpha y) - \bar{\tau}_{-h}^{(x)}/\rho. \quad (12)$$

$$\frac{\partial \bar{V}}{\partial t} + gh \frac{\partial \bar{\eta}}{\partial y} = \frac{\alpha}{2k} \left( \frac{gka}{\omega} \right)^2 (kx \sin 2\beta + \sin \beta) \exp(-2kx \cos \beta - 2\alpha y) - \bar{\tau}_{-h}^{(y)}/\rho. \quad (13)$$

In this calculation we have neglected all terms proportional to  $(\alpha/k)^2$ , and higher orders. As demonstrated by Mei (1973) for standing edge waves in the absence of friction, the first term on the right-hand side of (12) is the divergence of the radiation stress component by Longuet-Higgins and Stewart (1960) plus the contribution from the dynamic bottom pressure, i.e.  $\{-\partial S_{11}/\partial x + \tan \beta \bar{p}(-h)\}/\rho$ . It is easily verified that the first term on the right-hand side of (13) is just  $\{-\partial S_{22}/\partial y\}/\rho$ , where  $S_{22}$  is given by Mei (1973) in the case when  $\alpha = 0$ .

Following Longuet-Higgins (1953), the Stokes drift  $(\bar{u}_S, \bar{v}_S)$  to second order in wave steepness for this problem is easily obtained from the linear wave solutions. By integrating in the vertical, we get the Stokes flux for this problem:

$$\bar{U}_S = \int_{-h}^0 \bar{u}_S dz = 0, \quad (14)$$

$$\bar{V}_S = \int_{-h}^0 \bar{v}_S dz = \frac{gka^2}{2\omega \sin^2 \beta} (\exp(-2kx \cos \beta) - \exp(-2kx \sec \beta)) \exp(-2\alpha y).$$

The total wave momentum in the trapped region thus becomes:

$$M = \rho \int_0^\infty \bar{V}_S dx = \frac{\rho g a^2}{4\omega \cos \beta} \exp(-2\alpha y). \quad (15)$$

For inviscid flow ( $\alpha = 0$ ), and shallow water ( $\cos \beta \approx 1$ ), this result conforms to that of Kenyon (1969). For the spatially damped Stokes edge wave, the energy density (7) becomes

$$E = \frac{\rho g a^2}{4k \cos \beta} \exp(-2\alpha y). \quad (16)$$

From (15) and (16) we note that  $E = Mc$ , where  $c = \omega/k$  is the phase speed. This is in accordance with Starr's (1959) general result for waves.

For the total momentum balance in the trapping region, we integrate the alongshore component (13) in the  $x$ -direction from the shore to infinity. Defining

$$Q = \rho \int_0^\infty \bar{V} dx, \quad B = \rho g \int_0^\infty h \bar{\eta} dx, \quad T_B = \int_0^\infty \bar{\tau}_{-h}^{(y)} dx, \quad (17)$$

we find for the rate of change of the total mean Lagrangian momentum  $Q$  that

$$\frac{\partial Q}{\partial t} = -\frac{\partial B}{\partial y} - \frac{\partial E}{\partial y} - T_B. \quad (18)$$

We note the interesting difference between the Stokes edge wave and plane surface waves in the  $y$ -direction. In the latter case the radiation stress forcing on the right-hand side would be  $-\partial(E/2)/\partial y$  for deep water waves and  $-\partial(3E/2)/\partial y$  for shallow water waves (Longuet-Higgins and Stewart, 1960). For the Stokes edge wave the water appears shallow for small  $x$ , and deep for large  $x$ . Hence, the cross-shore integration yields the in-between value  $-\partial E/\partial y$  for the forcing term, as seen from (18).

### 3. Steady mass transport

We consider the steady mass transport. The turbulent bottom stresses are modeled by the mean Eulerian velocities. In the direction normal to the coast the velocities are small. Here we can neglect the bottom friction, and the balance to lowest order in this direction is between the pressure gradient due to the mean surface slope, the radiation stress component, and the mean dynamic pressure at the sloping bottom (Mei, 1973). From (12) we then obtain for the mean surface slope

$$g\bar{\eta} = -\frac{1}{2} \left( \frac{gka \cos \beta}{\omega} \right)^2 \exp(-2kx \cos \beta - 2\alpha y). \quad (19)$$

Assuming that the alongshore mean velocity is much larger than the cross-shore one, we write the turbulent bottom stress in the  $y$ -direction:

$$\bar{\tau}_h^{(y)}/\rho = c_D |\bar{V}_E| \bar{V}_E / h^2. \quad (20)$$

where  $c_D$  is a bottom drag coefficient. From Longuet-Higgins (1953) we have for the mean alongshore Eulerian volume flux induced by friction:

$$\bar{V}_E = \bar{V}_L - \bar{V}_S. \quad (21)$$

where the Lagrangian flux  $\bar{V}_L$  is equal to  $\bar{V}$  in (1), and  $\bar{V}_S$  is given by (14). Inserting (19) and (20) into (13), we obtain for the steady mean Eulerian volume flux:

$$\bar{V}_E = \left( \frac{g\alpha a^2}{2c_D} \right)^{1/2} x \tan \beta \exp(-kx \cos \beta - \alpha y). \quad (22)$$

The present approach separates the decay of wave momentum from the frictional influence on the mean flow, which is physically sound (Jenkins, 1989; Weber and Melsom, 1993; Ardhuin and Jenkins 2006). We note from (22) that it is the divergence of the radiation stress through spatial wave decay that drives the mean Eulerian flow, while the magnitude depends on the turbulence (the roughness etc.) at the sloping bottom. A turbulent decay of wave energy ( $\alpha > 0$ ) cannot exist without a turbulent bottom drag on the mean flow, so the limit:  $\alpha$  small and finite,  $c_D \rightarrow 0$  in (22), is unphysical.

From the continuity equation we obtain for the cross-shore flux in a steady state:



$$\frac{\partial \bar{U}_E}{\partial x} = - \frac{\partial (\bar{V}_S + \bar{V}_E)}{\partial y}. \quad (23)$$

We then find, assuming that  $\bar{U}_E(x=0) = 0$ :

$$\begin{aligned} \bar{U}_E = & \frac{g\alpha a^2}{2\omega \sin^2 \beta \cos \beta} (\sin^2 \beta - \exp(-2kx \cos \beta)) \\ & + \cos^2 \beta \exp(-2kx \sec \beta) \exp(-2\alpha y) \\ & + \left( \frac{g\alpha^3 a^2}{2c_D k^4} \right)^{1/2} \frac{\sin \beta}{\cos^3 \beta} (1 - (1 + kx \cos \beta) \exp(-kx \cos \beta)) \exp(-\alpha y). \end{aligned} \quad (24)$$

We realize that from this that  $|\bar{U}_E/\bar{V}_E| = O(\alpha/k)$ , which justifies the neglect of the cross-shore velocity in the bottom drag (20).

We now define the along-shore vertically-averaged Stokes drift  $\langle v_S \rangle$ , the and Eulerian mean current  $\langle v_E \rangle$  by

$$\langle v_S \rangle = \bar{V}_S/h, \quad \langle v_E \rangle = \bar{V}_E/h, \quad (25)$$

where  $h = x \tan \beta$ . The vertically-averaged Lagrangian drift thus becomes

$$\langle v_L \rangle = \langle v_S \rangle + \langle v_E \rangle. \quad (26)$$

It easily seen that these average velocities have a maximum at the coast ( $x = 0$ ).

In order to relate our theoretical results to the natural environment, we consider shallow water, and take  $\beta = 0.1$  as a typical beach slope angle. High-frequency edge waves yield the largest drift velocities, while low-frequency waves related to the motion of atmospheric low-pressure systems have higher total mass fluxes (Kenyon, 1969). We here focus on drift velocities, and use the classic observation by Munk (1949) in the surf-beat range, giving a wave period  $T = 60$  s, and a wave amplitude  $a = 0.1$  m. For the modeling of tidal currents in the Barents Sea typical values of the eddy viscosity and bottom drag coefficients are  $\nu = 10^{-3} \text{ m}^2 \text{ s}^{-1}$  (Nøst, 1994), and  $c_D = 3 \times 10^{-3}$  (Gjevik *et al.*, 1994; Nøst, 1994), respectively. At a sloping beach, eddy viscosity estimates are higher by a factor of 10 to 50 (Apostos *et al.*, 2007), mainly due to turbulence induced by breaking waves. Reported drag coefficients seaward of the surf zone is comparable with those used for tidal current modeling. From Feddersen *et al.* (2003) we find that  $c_D \approx 2 \times 10^{-3}$  outside the surf zone. Normally, we will expect a considerable amount of turbulence within the trapping region of edge waves. Without specifying the source of turbulence, which is outside the scope of this paper, it seems reasonable to take  $\nu = 5 \times 10^{-3} \text{ m}^2 \text{ s}^{-1}$  and  $c_D = 2.5 \times 10^{-3}$  in quantifying the drift induced by the Stokes edge wave. In Figure 2 we have plotted the vertically-averaged velocities (25) and (26) as function of seaward distance for the physical parameters given here. We note that the Eulerian mean velocity is the dominating component of the Lagrangian drift velocity. In the present example we find  $\langle v_L \rangle (x=0, y=0) \approx 6 \text{ cm s}^{-1}$ . This is in fact a mean mass transport velocity which is

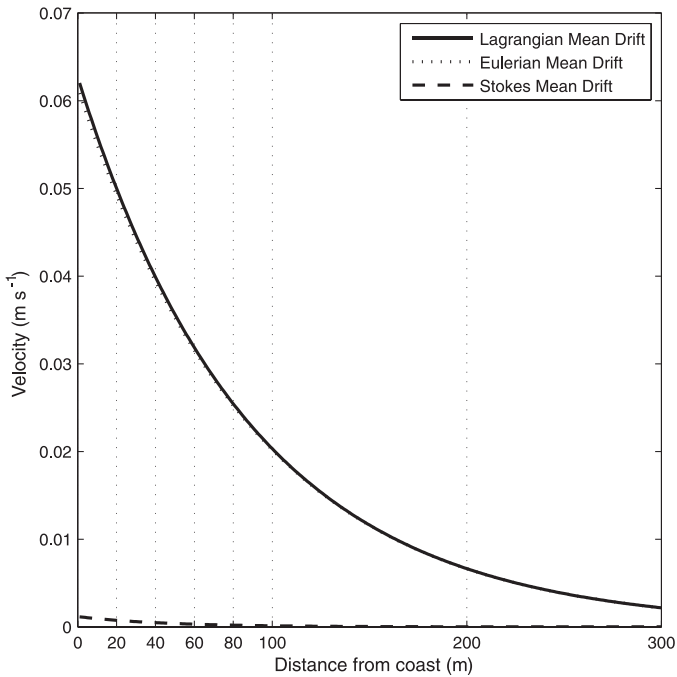


Figure 2. Mean drift velocity components (25) and (26) at  $y = 0$ , vs. distance from the coast  $x$ . Dashed line depicts  $\langle v_S \rangle$ , dotted and solid lines depict  $\langle v_E \rangle$ , and  $\langle v_L \rangle$ , respectively. The physical parameters are:  $a = 0.1$  m,  $T = 60$  s,  $\nu = 5 \times 10^{-3} \text{ m}^2 \text{ s}^{-1}$ , and  $c_D = 2.5 \times 10^{-3}$ .

comparable to traditional wind surge velocities. The drift velocity components decay exponentially in the seaward direction. The alongshore drift becomes negligible outside the wave trapping zone  $x > L$ , where  $L = \pi/k \approx 280$  m in this example.

#### 4. Discussion and concluding remarks

In this paper we have shown for the Stokes edge wave that the time rate of change of the total Lagrangian mean momentum is forced by the divergence of the total energy density  $\partial E/\partial y$ , independent of the bottom slope. This is exactly mid-way between the deep and shallow water values for ordinary surface waves (Longuet-Higgins and Stewart, 1960). This appears to be a novel result.

Furthermore, we have derived an analytical expression for the vertically-averaged Lagrangian drift velocity induced by the Stokes edge wave. This drift is composed of a Stokes drift component plus an Eulerian mean velocity, where the latter arises from the effect of bottom friction. Examples from moderately sloping beaches show that the mean Eulerian part of the velocity dominates, and are by far the largest contribution to the Lagrangian drift velocity, as seen from Figure 2. This has not been reported in the literature before.

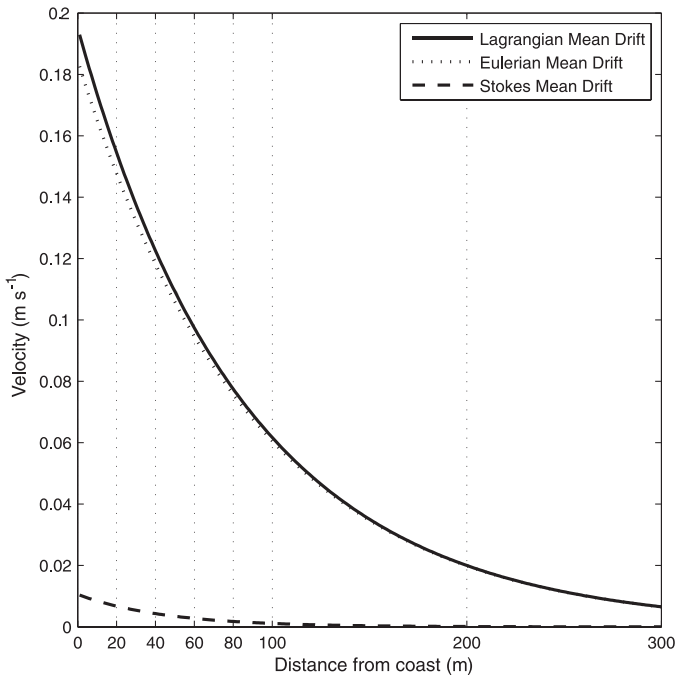


Figure 3. Same as in Figure 2, but with wave amplitude  $a = 0.3$  m.

For given wave amplitude, the drift velocities increase with decreasing slope angle. In the example given by Kenyon (1969), the slope angle was 0.02 radians, yielding a Stokes drift at the shore of  $15 \text{ cm s}^{-1}$ . This is rather extreme. For a more reasonable slope like 0.1 radians, as in the present example, the Stokes drift at the shore becomes  $0.1 \text{ cm s}^{-1}$ , which is negligible, compared with the Eulerian mean component of about  $6 \text{ cm s}^{-1}$ , as seen from Figure 2. The contribution from the Stokes drift increases with increasing amplitude. This is obvious from Figure 3, where we have plotted the drift velocity components when the amplitude is 0.3 m, which is probably on the larger side for high-frequency edge waves. Still we must conclude that the Stokes drift contribution is negligibly small. The Stokes drift contribution also increases with decreasing value of the viscosity coefficient. However, even with a molecular viscosity coefficient ( $\nu = 1.2 \times 10^{-6} \text{ m}^2 \text{ s}^{-1}$ ) in this problem, which is highly unrealistic, the Stokes drift contribution would be smaller than the mean Eulerian flow.

The present approach does not resolve the viscous bottom boundary layer, so the results should not be used to assess the drift of heavy bottom sediments. However, it yields the vertically-averaged drift in the water column as function of the seaward distance. In practice, the maximum of this drift current occurs near the shore. We think that this drift may be of importance for the transport along the shore of biological material, pollutants, as well as light sediments in suspension. The Stokes edge wave and the associated drift

current can have either direction along a straight coast. This is important to keep in mind when we try to estimate the whereabouts of effluents released in the near-shore zone.

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