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## The Effects of Prior Calculus Classes on Success in Organic Chemistry

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The Effects of Prior Calculus Classes on Success in Organic Chemistry

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Honors Thesis

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**Abstract**

Within University of Nebraska – Omaha (UNO) chemistry-based degree programs, organic chemistry along with at least two semesters of calculus are required courses. There is, however, no requirement for the order in which these courses should be taken. This is mainly due to the notion that for chemistry courses, organic chemistry is detached from the typical quantitative concepts studied in other chemistry-based courses. This is untrue, however, as many organic chemistry concepts rely on three-dimensional spatial visualizations and manipulations. Spatial awareness skills are strengthened within higher-level calculus classes. A survey was administered to organic chemistry students enrolled in 2022-2023. The students were surveyed and interviewed to determine correlations between taking calculus courses and grades received in the first semester of organic chemistry as well as their confidence levels toward concepts related to three-dimensional spatial visualizations. The results of this survey will be discussed to show the effects of taking calculus prior to taking organic chemistry and general student success with organic chemistry concepts.

Keywords: *Survey, Correlation, Chemical Education, Organic Chemistry, Calculus, Polarity*

## Introduction

Within the chemistry major degree program, it is well known that math is a skill used in almost every chemistry course available. Analytical practices are prevalent in all general chemistry and instrumental based chemistry courses and there is a clear correlation between the need for mathematical skills and being successful with chemical concepts. Algebra and statistics are regularly used when students are learning how to do mole conversions and generating calibration curves for chemical analysis. There is an exception to this kind of analytical thinking within the chemistry-based classes required for students. Organic chemistry is often seen as unique in its separation from analytical concepts. Commonly, organic chemistry concepts focus on the qualitative reactions and chemical processes with little focus on actual quantitative analysis. The concepts focused on rely more heavily on processes, patterns, mechanisms, and memorization of items to be applied to chemistry concepts.

Organic chemistry is seen as one of the most difficult classes that many chemistry, biology, and pre-med students will ever have to take. Currently, the fail rate for organic chemistry across the nation is over 50%, with the course being described as the ultimate weed-out course.<sup>1</sup> It has a notorious reputation, and many professors have difficulty achieving high success rates in their classes. A few different skills are needed to succeed in organic chemistry, including a great work ethic and study method, as well as skills previously unused in lower-level general chemistry courses. These skills range from the ability to memorize concepts and sequences as well as being able to think in a more complex chemical way.

A major aspect to success in organic chemistry courses is that students must develop a level of spatial awareness.<sup>2</sup> A student may be asked to visualize qualitative aspects of a molecule like geometry and polarity instead of giving a numerical value. This requires students to be able

to create models of molecules in their minds and oftentimes rotate them and manipulate bonds within the mental image as well. Even though these are seen as qualitative manipulations, students use mental spatial modeling methods to identify and categorize molecules which allows for reaction mechanisms to be better understood and behaviors of molecules to be predicted. There is a claim that students who lack the ability to be spatially aware of molecules are shown to perform weaker than those with a more defined skill of mental visualization.<sup>3</sup> Students who cannot understand how a molecule works in a three-dimensional manner when looking at a two-dimensional drawing could struggle with stereochemistry, polarity, electron density, and how reagents work mechanistically when stereochemistry is a concern. When identifying a molecule's polarity, for example, students must be able to visualize the three-dimensional positioning of electronegativity vectors which add and subtract to determine how polar or non-polar that specific molecule is.<sup>4</sup>

Within studies on students' spatial awareness skills, those who have a better ability to visualize and manipulate three-dimensional objects in the mind also tend to be more successful in mathematics.<sup>5</sup> This may blur the distinction between organic chemistry being a strictly qualitative study. The thought that the concepts learned within organic chemistry are strictly unrelated to a mathematical approach is not true when considering the use of spatial awareness as a mathematical concept in terms of three-dimensional graphing and vectors. The study of vectors in a three-dimensional space relates directly to the course curriculum of calculus 3 at UNO. This is not a required course for those who are in chemistry-based courses, however there are a selection of students who will continue to take calculus 3 in their time as a student. As it currently stands, all chemistry majors are required to take two semesters of calculus, however there is no requirement or suggestions to have taken it before taking organic chemistry.

Through the survey of current UNO organic chemistry students, three research questions were sought to be answered. First, does having two semesters of calculus completed before taking organic chemistry increase students' success in organic chemistry courses? Second, is there a difference in the confidence levels of students in three-dimensional visualization with and without a calculus background? Third, does an analytical or creative mindset of a student have an impact on the understanding of mental visualization concepts in organic chemistry? Answering these questions would give insight into the potential importance of calculus courses for organic chemistry students' success with the intent to increase student grades and decrease the fail rate for both organic chemistry 1 and 2.

## **Methods**

To be able to gain insight into organic chemistry students' experiences and skills and to see any early correlations, a preliminary pilot survey was administered to organic chemistry 1 students in the fall of 2022. Developed using Qualtrics survey software, the pilot survey contained four demographic questions along with four Likert questions and five free-response questions. The pilot survey was distributed to 83 organic chemistry 1 students with a 26% response rate. This pilot survey data was reported in a poster in the spring of 2023 and used to refine the final, primary survey. The primary survey was distributed to 104 students with a response rate of 22.12%. Developed using the same Qualtrics software, the primary survey contained six demographic questions, four Likert questions, and six free-response questions. The primary survey was open for 30 days and students were given the opportunity to identify if they were willing to participate in one-on-one interviews.

Students were asked what the highest level of calculus course they had completed along with the grade received, and they were sorted accordingly. The two groups divided were those who had completed no calculus or just calculus 1 and those who had completed calculus 2 or 3. These two groups were used to compare average grades within the respective courses they have taken or are currently taking. The groups are broken down into four individual groups for Likert and free responses as those without calculus experience, those with only calculus 1 experience, those with calculus experience through calculus 2, and those with calculus experience through calculus 3. Those with no calculus made up 34.7% of the population, followed by 26.1% of students in each group having only taken calculus 1 or calculus 2. Finally, 13.1% of the students completed calculus 3.

A series of interview questions were asked to three students within organic chemistry 2 courses who also completed the survey. They were asked to explain their thoughts more thoroughly about three-dimensional visualization, potential benefits of calculus courses, and their take on analytical versus creative mindsets in the lens of an organic chemistry student. These students remain anonymous and are only referred to as student 1, 2, or 3.

## **Results**

Within the survey responses, it was found that in the grade distributions for the different student groups, there were some differences in average grades. The four groups were broken down into two groups. The first group observed are those who have not taken any calculus or who have finished or are currently enrolled in calculus 1. The second group are those who have



completed or are currently enrolled in calculus 2 or 3. The data shows that 83.3% of those who have taken calculus 1 received an A within the course (Figure 1).

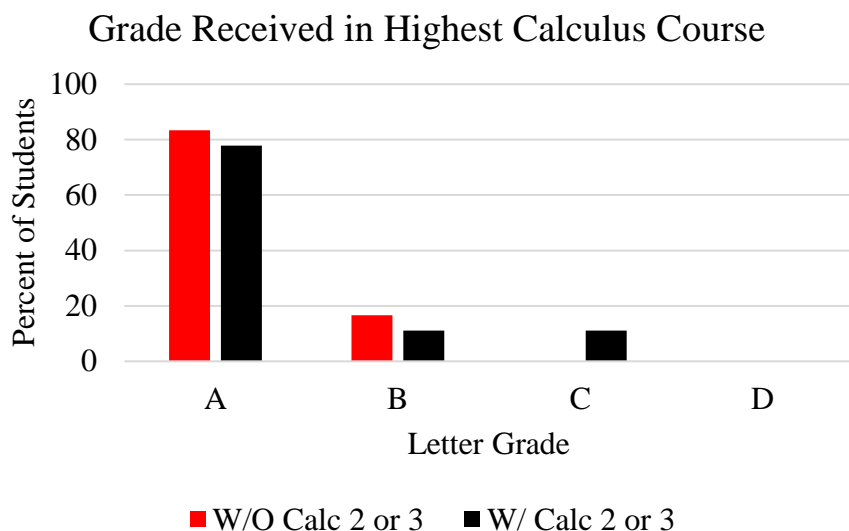


Figure 1: A distribution of the grade achieved in a student's highest level calculus course. The red bars indicate those who have either not taken any calculus courses or who have taken only calculus 1. The black bars indicate those who have completed calculus 2 or 3.

Of those who completed calculus 1, 16.7% received a B. For those who continued to further calculus classes, 77.8% have achieved an A in their respective highest calculus course. This is followed by 11.1% receiving a B and the remaining 11.1% receiving a C. For organic chemistry 1 students without higher calculus experience, 42.9% of students received an A and 42.9% of students received a C (Figure 2).

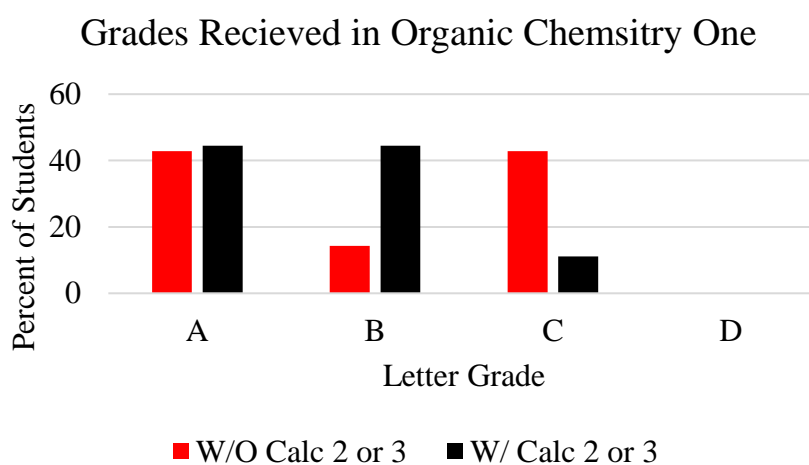


Figure 2: A distribution of the grade achieved in organic chemistry 1. The red bars indicate those who have either not taken any calculus courses or who have taken only calculus 1. The black bars indicate those who have completed calculus 2 or 3.

The remaining 14.3% received a B for the first semester of organic chemistry. For the second group, students that have more experience with calculus concepts, 44.4% of students received an A and 44.4% of students received a B. The remaining 11.1% received a C in the course. For organic chemistry 2, 50.0% of those without calculus received an A for the course (Figure 3).

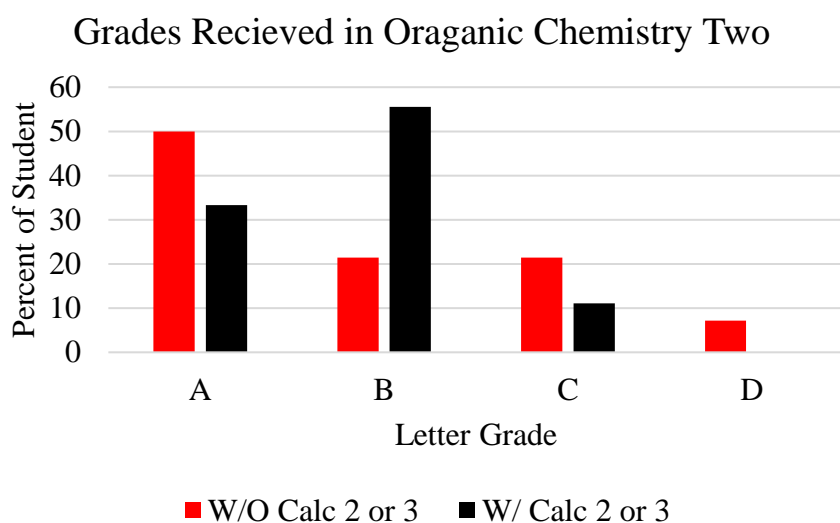


Figure 3: A distribution of the grade achieved in organic chemistry two. The red bars indicate those who have either not taken any calculus courses or who have taken only calculus 1. The black bars indicate those who have completed calculus 2 or 3.

Additionally, 21.42% of students received a B, and 21.42% of students received a C.

Finally, the remaining 7.1% received a D for the course. For those with calculus 2 or 3, 33.3% of students received an A for the course, and 55.6% of students received a B. The last 11.1% received a C for the second semester of organic chemistry.

The Likert scale questions asked were broken down into four separate groups. When asked how confident the student was in understanding three-dimensional geometries of molecules, those who had no calculus or only calculus 1 gave an average response of neither confident nor not confident (Table 1). Those who had calculus 2 and 3 experience, however, had an average response of being somewhat confident. All four groups had a most common response of somewhat confident. (Table 2). When asked about confidence levels with regard to visualizing polarity of three-dimensional molecules, all groups responded with an average of being somewhat confident, with that also being the most common response across the board. When

asked about three-dimensional visualization, those who completed calculus 1 were neither confident nor not confident on average. All of the other groups had an average response of somewhat confident. For the most common responses, those who had no calculus, calculus 1, and calculus 2 responded somewhat confident most frequently. Those who have taken calculus 3, however, had the most common response of very confident. Finally, when asked how confident they are in determining the polarity of a molecule without the use of a physical model, all groups on average responded with somewhat confident.

Within the free response questions, there was some insight into individual ideas when relating organic chemistry concepts to analytical and creative processes. Those who had taken at least one semester of calculus courses were asked if visualizing in three-dimensional ways has gotten easier since taking any calculus courses. Those who have only taken calculus 1 were very sure that it had not changed at all. Those with calculus 2 did not see a connection until completing calculus 2. They had found that their ability to mentally picture three-dimensional objects had increased in some cases. Those with calculus 3 were overall inconclusive as some students were unsure, some said no, and some said yes. When asked if being able to visualize molecules in three-dimensions is vital for organic chemistry success, students had more mixed responses. Those without any calculus responded evenly with four yes and four no responses. Those who had taken calculus 3 were also unsure and replied with one yes, one no, and one neither response. Responses from those who had taken only calculus 1 were very definitive with yes being the answer across the board. Calculus 2 students said yes that visualization could be helpful when considering organic chemistry courses and concepts. There were a few comments, however, that stated that it may not be entirely required, as many other skills are applied to organic chemistry courses.

When all students were asked to describe themselves as either analytically or creatively minded, all students from all four groups unanimously said analytical. Most mentioned using logical approaches to problems using pattern recognition and algorithmic problem-solving. Some mentioned the struggle with generating new concepts and ideas, however it was easy to memorize and apply new concepts taught explicitly. The thought process of thinking with numbers instead of shapes was brought to light in this question. When asked how having an analytical mind could help in organic chemistry courses, all four groups answered similarly. By having an analytical approach to organic chemistry concepts, pattern recognition and application can be used to apply reagents learned in the classroom to extended examples. There is mention of being able to critically analyze synthesis problems in organic chemistry to help break down each individual step. The synthesis of a molecule involves the use of reagents and mechanisms to generate a final chemical product, usually in a sequence of a few reactions. This goes hand in hand with the idea that understanding each individual chemical process on a molecular and bonding level can help students fundamentally understand the synthesis processes used in organic chemistry. An analytical mind is also mentioned to be helpful for those needing to memorize the many reagents used in organic chemistry. The students were then asked how they think having a creative mind could help in organic chemistry. In all categories, the students replied that having the three-dimensional spatial awareness could be beneficial for an organic chemistry course. Several students relate generating a synthesis to a creative process as it takes a base knowledge of reagents, however eventually you do have to use trial and error and problem-solving to find a solution. The three-dimensional visualization of molecules with a creative mind is said to be beneficial when trying to understand how different configurations of molecules are attacked by different reagents on a molecular level.

When asked a question about how they felt about organic chemistry 2, the students who had less calculus when compared to those who had higher-level calculus experience actually showed a difference in response. Those who had less calculus speak about how difficult organic chemistry 1 was in comparison to organic chemistry 2. They mentioned that organic chemistry 2 is a difficult class, but nowhere as difficult as the first semester of the course. Those with calculus 2 and 3 experience, however, talk about how much more difficult calculus 2 is compared to calculus 1.

For student interviews, three students were individually interviewed with two having calculus experience and one not having any. Students 1, 2, and 3 were all asked the same series of questions. For the two who took calculus courses, students 1 and 2, both had taken courses at least through calculus 2. When asked if they had ever had any modeling experience, student 1 had mentioned some use of ball and stick modeling for molecules. Student 1 had finished up through calculus 2 and received a B letter grade for the course. When asked how calculus could have been potentially helpful in organic chemistry concepts, student 1 stated that they had initially not seen any connections between the two courses. There was mention, however, that calculus courses had introduced some terminology like vectors and the addition of vectors that could be applied to organic chemistry concepts, specifically polarity. When asked further about polarity, student 1 stated that the most difficult part of understanding the concept of polarity was applying it to larger molecules. It was stated as well that polarity was only something that was difficult if the molecule was unable to be visualized or seen in three dimensions, whether that be in the mind or with physical models. When asked directly how well student 1 was able to visualize in three-dimensional ways, they replied that the ability to do so was learned through practice over time. They stated that new molecules are more difficult as they become more

intricate, but with experience and practice with mental spatial manipulations, it is made simpler. Student 1 determined themselves to be an analytical thinker but gave credit to needing creative problem-solving skills to be successful.

Student 2 was asked the same series of questions and identified themselves to have completed calculus 3 with an A letter grade. This student had no prior experience with three-dimensional modeling in the past and never found polarity to be a difficult concept to understand and apply. When asked if their calculus experience was beneficial within organic chemistry, they stated that the concept of vectors taught in calculus allowed them to quickly catch on to polarity concepts with ease. Student 2, when asked if they believed taking calculus before organic chemistry was helpful, stated that it was very beneficial. They stated that working through calculus courses helped build a new style of study method. There was a comparison of how memorizing specific equations to apply in calculus felt similar to learning specific reagents for organic reactions. Student 2 stated that they have very little problem with three-dimensional visualization in their mind in general. Student 2 also believed themselves to be an analytical thinker but credited that a creative mind should be able to do the same in terms of being spatially aware and able to manipulate shapes in the mind.

For student 3, there was no calculus experience before taking organic chemistry 1 or 2. Student 3, however, had extensive experience with three-dimensional rendering software and had practiced using three-dimensional sketches for this program. They were asked how that experience could have helped in terms of understanding the organic chemistry course content. Student 3 mentioned that they were able to apply their skills practiced by using three-dimensional modeling software to help with mental visualization of molecules. This helped with determining specific concepts like stereochemistry, the configurations of parts of a molecule,

along with identifying polarity of larger molecules. Student 3 declared themselves to be analytical when approaching organic chemistry. When asked, however, if being creative would be helpful in succeeding in organic chemistry courses, they mentioned that it may be easier for those who could creatively think and understand mental three-dimensional images and manipulations.

### **Discussion**

When looking at the grade distributions for those who had taken calculus 1, there is a higher percentage of A letter grades received, with a few more Bs as well. Those who took higher-level calculus courses tended to obtain an A with a 77.8% majority. The only Cs received were those who took higher calculus courses, which is possibly expected as the difficulty and intensity of the calculus courses increased. Overall, however, it can be determined that on average, all students surveyed have the potential to succeed in their respective courses.

When looking at the grade distributions for organic chemistry 1, those who had taken calculus 2 or 3, generally scored higher in the class than those with little to no calculus experience. When looking at the free responses, it was common that those with higher calculus experience also found the first semester of organic chemistry to be more enjoyable and seemingly easier than the second semester of organic chemistry. For organic chemistry 2, however, there are more of those with no calculus experience with an A letter grade than those with higher calculus experiences. Those with calculus 2 or 3, have most of their letter grades in the B letter grade category. This coincides with the general opinion of those with less calculus experience finding organic chemistry 2 easier than the first semester. In organic chemistry 1, students must answer more questions directly related to molecular geometry. Those with higher-level calculus experience may excel in those topics due to their spatial awareness and lead to

better grades on average. In organic chemistry 2, topics like synthesis that are more advanced do not require as much molecular geometry consideration. This could be why students without advanced calculus courses are still able to succeed and have a better average grade for the second semester course. This correlation is something to explore further with student opinion and grades for each semester once both semesters of organic chemistry are completed.

In general, all students averaged a response of being somewhat confident across the board for the Likert questions. Any noticeable differences are very minimal, however may allow for some insight into individual groups. For those without higher calculus courses, there was more uncertainty or even lack of confidence in average responses when asked if they understood three-dimensional geometries of molecules. Those with only calculus 1 experience were also uncertain in their confidence on average about three-dimensional visualization. Another difference to be highlighted in the most common responses reflects on those who have completed calculus 3. Most commonly, students with the most calculus experience were very confident in their abilities to visualize molecules in a three-dimensional way.

Within the free responses of the students, every student identified themselves as being generally analytical with their thought processes. When asked how an analytical approach can be beneficial in organic chemistry courses, many offered the response of problem-solving. This is an interesting correlation that most students within the organic chemistry courses related a problem-solving process to being a strictly analytical approach. When individual students were interviewed, however, problem-solving skills were mentioned when asked about creative abilities used in organic chemistry courses. This may indicate a need to have students define what they mean by problem-solving. This could define problem-solving as an analytical or creative process exclusively or see if it is a process that requires both analytical and creative



thinking. This could be further explored in later surveys and interview processes. The interviewees tended to see the value of taking a calculus course along with organic chemistry, however none of them believed there was a strict correlation between their personal calculus experiences and their success level in organic chemistry 1 or 2.

When looking at the data, there may be a slight correlation between those who took calculus courses and their level of success in specifically organic chemistry 1. Those who have had higher-level calculus experience with calculus 2 or 3 found that organic chemistry 1 was more enjoyable and scored higher, indicating there may be a correlation between content of calculus courses and the first semester of organic chemistry. A further analysis of the concepts learned in calculus 2 and 3 and those learned in organic chemistry 1 could be done to see if more specific correlations arise. For those who have not had calculus experience, organic chemistry 2 seems to be where their success and enjoyment lie. There could be further studies done to see if there are specific concepts being taught in organic chemistry 2 where those who have not experienced calculus courses succeed more specifically than those with a calculus background.

### **Conclusion**

With surveys and interviews with organic chemistry 2 students, the research questions proposed were able to be explored and some conclusions were able to be drawn from the data. When seeing if having two semesters of calculus completed before taking organic chemistry increases the students' success in organic chemistry courses, a possible correlation can be drawn between those with less calculus experience and those with higher calculus experience. Those without calculus courses seemed to have struggled more in the first semester of organic chemistry on average with those who have taken calculus 2 or 3 doing better. In the second semester of organic chemistry, however, those with more calculus courses taken had lower letter

grades and described the second semester of organic chemistry as being difficult while those with little calculus found the second semester easier. This could be studied further with a larger population of organic chemistry students who have both taken and not taken various levels of calculus courses.

The second research question observed is to see if there is a difference in confidence levels in three-dimensional visualization with and without a calculus background. Reviewing the Likert question responses, it seems that there may be a relatively similar level of confidence across the board for students of all levels of calculus experience. When taking into consideration the interviewee responses, those with calculus did seem very confident in their ability to visualize and manipulate objects and molecules in their mind. This is also seen in the most common Likert response for those with calculus 3 being very confident. Better data could possibly be found by a larger sample of students. This would give an opinion-based response on a student's personal thoughts on their own abilities. A way to expound on this may be to have a set of problems or questions where students have to apply these spatial awareness skills and compare confidence levels before and after having to apply the skills. This could also give a more definitive answer on what groups may be more skilled at three-dimensional manipulations.

Finally, the third research question addressed was if an analytical or creative mindset of a student has an impact on the understanding of mental visualization concepts in organic chemistry. This question is the one that needs much more data collection to give a conclusion on as every student surveyed described themselves as having an analytical mind. The interesting conflict of the classification of problem-solving skills being an analytical process or creative process could be addressed. Once problem-solving is more defined, it could be determined if a student is self-proclaimed as analytical, but actually follows a more creative process while

thinking. This could also be partnered with a simple personality test to help assign a better description of those who are analytically or creatively minded, or if there is a spectrum of both.

Future work on this concept could be done through a series of more data collection over the span of a few years. This could give a view of students from many different backgrounds and skill levels to find better correlations between skills learned in calculus and skills applied in organic chemistry. If there were a way to gain the funds, an in-depth critical review of individual concepts taught in calculus and concepts taught in organic chemistry could be done to see where students bridge that knowledge across the two different disciplines. There could be an implementation of a concept exam to see how students specifically perform on those individual learning objectives in the two courses and see if there is a correlation between concepts students commonly do well on in the courses.

When looking into possible ways to increase organic chemistry student grades and decrease the drop and fail rate, one could consider how other classes could be made prerequisites to encourage success. Almost every student who will take organic chemistry at UNO is also required to take calculus 1 and 2 to complete their degree program. There is no requirement at this time for the calculus courses to be taken in any order when compared to most chemistry courses. If at least calculus 1 was a prerequisite for organic chemistry 1, it would encourage students to take their semester of calculus 2 during the same semester of organic chemistry 1. This could potentially give students the proper skills in spatial awareness that are possibly not vital to success but do seem to help students succeed in their organic chemistry courses. By preparing students as much as possible for the critical thinking, study skills, and three-dimensional awareness needed in organic chemistry, there is a chance of decreasing the drops, fails, and withdrawals that are so commonly seen in organic chemistry courses.

**Appendix**

Table 1: Average Likert responses to survey questions from organic chemistry students with various levels of calculus experience.
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Likert Question	Average Responses			
	No Calculus	With Calc 1	With Calc 2	With Calc 3
How confident are you in the understanding of 3-dimensional geometries of molecules?	Neither	Neither	Somewhat Confident	Somewhat Confident
How confident are you in your understanding of polarity on 3-dimensional molecules?	Somewhat Confident	Somewhat Confident	Somewhat Confident	Somewhat Confident
How confident are you in being able to have 3-dimensional visualization in your mind?	Somewhat Confident	Neither	Somewhat Confident	Somewhat Confident
How confident are you in determining the polarity and geometry of a molecule without a physical model set to use?	Somewhat Confident	Somewhat Confident	Somewhat Confident	Somewhat Confident

Table 2: Most common Likert responses to survey questions from organic chemistry students with various levels of calculus experience.

Likert Question	Most Common Responses			
	No Calculus	With Calc 1	With Calc 2	With Calc 3
How confident are you in the understanding of 3-dimensional geometries of molecules?	Somewhat Confident	Somewhat Confident	Somewhat Confident	Somewhat Confident
How confident are you in your understanding of polarity on 3-dimensional molecules?	Somewhat Confident	Somewhat Confident	Somewhat Confident	Somewhat Confident
How confident are you in being able to have 3-dimensional visualization in your mind?	Somewhat Confident	Somewhat Confident	Somewhat Confident	Very Confident
How confident are you in determining the polarity and geometry of a molecule without a physical model set to use?	Somewhat Confident	Somewhat Confident	Somewhat Confident	Somewhat Confident

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