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Euclid's Elements for High School Classrooms

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Euclid's Elements for High School Classrooms

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by

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Abstract: Euclid's Elements for High School Classrooms

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What is our goal when teaching students geometry? Is it to obtain a passing grade, be able to construct geometric figures, know all of the necessary terms? I would like to propose that the purpose is to cultivate a love for the logic, art, and argument of geometry. The original geometry book written by Euclid was used as a guide for two thousand years and led many of our historical geometers to discoveries. Instead of relying on memorizing confusing acronyms for congruent figures, let's give the students an opportunity to see the work behind the properties. By using the methods outlined in this curriculum, students will experience geometry, study how to accurately justify their work, learn how to discuss, disagree, and defend respectfully, and know the math behind the theorems. To evaluate the curriculum, a small pilot study was conducted on the impact on student learning. Moreover, the impact and value of this curriculum was evaluated by two experts who specialize in teaching mathematics at the high school level.

Acknowledgments

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1. Introduction

"By and large, Euclidean geometry is presented from the perspective of the 20th century philosophy of mathematics as a set of sentences that follow from axioms…As a result, students implicitly adopt a view that mathematics is but an endless chain of theorems governed by logical consequence with no beginning and no actual aims." [20]

Yet in accepting the thought above, we are encouraged to overlook the beauty that flows from the formation of mathematics, especially Geometry. The students in our modern schools see math as a means to an end, whether that be high school graduation, a career in engineering or medicine, or just a good enough grade in their current class. This mindset flows directly from the desire to make education more progressive. William Whewell (1794-1866), a professor of moral philosophy at the University of Cambridge was one of the fighters to remove Euclid from our schools with the argument that Geometry's focus on reason made it a "permanent study" while Algebra and Calculus are "progressive studies" and needed to be mastered first and foremost [21]. Whewell was soon joined by mathematician James Joseph Sylvester (1814-1897) who said, "The early study of Euclid made me a hater of geometry" [21]. Other mathematical giants joined this progressive revolution and have, over the last 150 years, shifted our Geometry education to use strictly modern textbooks.

While there is room and space for modern geometric teaching in our classrooms, we must not rely solely on theorems, congruences, and proofs and forget the simplicity in which they were formulated. "To study Euclid's Elements in terms of ancient Greek

mathematics, one has to put away the modern techniques relying on the facts that line segments have lengths represented by real numbers, triangles have areas calculated by formula, etc…" [20]. As bold as the presented opinions may seem, I am not the first to desire a return to the classics. A few previously published textbooks on this matter include, *Euclid's Elements with Exercises* by Kathryn Goulding and *Geometry - A Classical Approach: A Revision of Euclid's Elements Books I - VI* by Mr. Martin I Ohara M. Ed. In the world of Classical Education the use of Euclid is expected, yet this project provides a student based, hands on approach that is not fully adopted by other texts. The use of limited drawing tools, the specific choice of propositions, and the classroom layout set this apart as a new model of embracing Geometry.

As students are coached in understanding readings laid out by logic and using reasoning to form their arguments, the dissection of Euclid's Elements comes to life! Students are involved in the math presented and in working only with the tools of definitions, postulates, common notions, or propositions already proven. A challenge is presented for students to create math without modern measuring tools, extended explanations, or even numbers! And this is where the beauty lies: in creating an atmosphere where discovery is supported, debate is encouraged, discussions are expected, and learning grows from the basics.

This project includes the Student Workbook, Answer Key, and Teacher Guide for selected propositions from the first three books of Euclid's Elements. Students in this course should have finished high school Algebra and it would be beneficial if they have had

training in rhetoric or debate. The material is meant to be taught using the classical

model which operates on a discussion based collaborative classroom. Each student is

expected to participate in all components of the class and to be an active part of in class

discussions.

Each lesson is laid out with the students investigating what was intended by Euclid and

other mathematicians of the time. The students are then expected to present their

material to the class, question one another, and engage in discussion based on the

methods, logic, and beauty behind each proposition. The material in this project is set to

cover approximately 109 school days. Teachers are provided with an outline of the

learning for each day but are also encouraged to take the freedom and adjust the activities

based on their class makeup.

To evaluate the effectiveness of the curriculum, I conducted a small pilot study by

teaching a portion of the material to the class I currently teach, an 8th grade Algebra class

at Lifegate Christian School. Students were given a 21-item initial assessment and final

assessment covering angle relations when two parallel lines are cut by a transversal and I

will use an F-test to test for equality of variances and then I will use an appropriate paired

t-test to test if we can reject the null hypothesis that the mean scores would be the same

on the initial assessment and final assessment. This analysis is reported in the Data

section.

Moreover, the curriculum will undergo content expert assessment. Two experienced high

school math teachers will look over the curriculum, give an overall assessment on its

usefulness and potential impact and suggest changes. This assessment is reported in the

Assessment/Review of the Material Section.

2. Material

2.1 Teacher Guide

GEOMETRY

AS TOLD BY
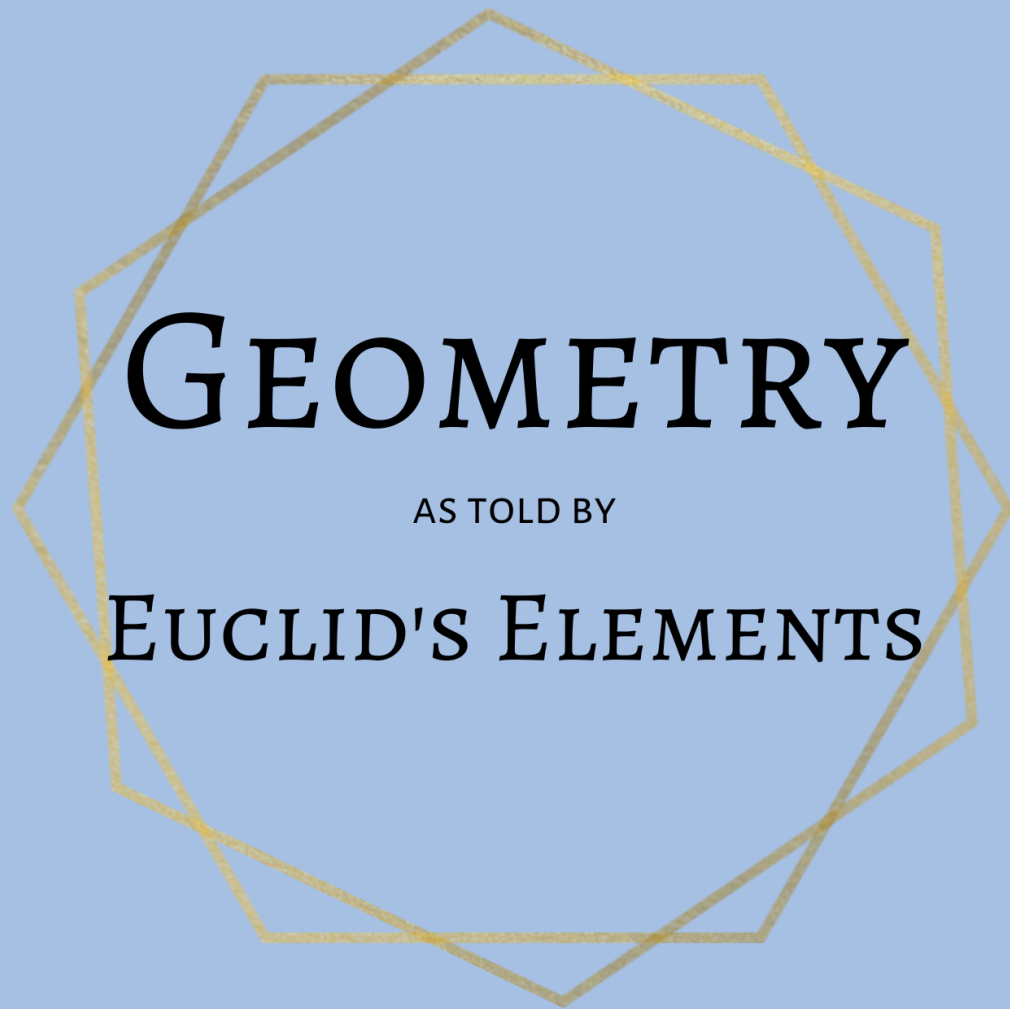
EUCLID'S ELEMENTS

Teacher Guide

Welcome!

You are currently looking at the Teacher Guide for Geometry as told by Euclid's Elements. By using this, you are provided with:

I. Lesson Outlines

   A. Lesson

      1. The chapter and section that you are in, for example 3.5 is chapter 3 section 5

   B. Days

      1. Approximately how many days each lesson will take

   C. Topic

      1. A generalized idea of the topics being covered in that lesson

   D. What to Do

      1. A day by day outline of how the lesson could be formatted

   E. Homework

      1. What homework could be given each day based on the lesson

This class is set up to be based in socratic discussion, thus the teacher is not directly lecturing but guiding students in their learning and discussions.

A Few Notes:

Socratic Discussion

You will notice that quite a few sections ask you to "open the floor for socratic discussion as a class" this means

  1) the students have already reviewed the questions
  2) have your chairs in a circle so that the students can all see one another
  3) students will raise their hand to talk
  4) no interrupting is allowed
  5) this is designed to be 90% student led, teachers should only join if something needs to be corrected or the class needs to be redirected

Euclid's Elements

We will be using the original Geometry textbook that was developed by Euclid as the basis for this course. The textbook from which our material comes is published by Green Lion Press. An online version of Euclid's Elements can also be found at http://aleph0.clarku.edu/~djoyce/elements/elements.html.

Homework

You will notice that quite a few lessons will have no homework. Homework will be used in this curriculum to make sure students are prepared for the next lesson of learning. If there is quality discussion in class, it is likely that the lesson will not require extra work to be done at home.

When the students are assigned Exercises for the section, there may only be a few exercises because each one asks for a defense of their answer, thus students who are taking their time to give a quality response will not be expected to do excessive work.

Quizzes

Yes, it may seem like there are a lot of quizzes over vocabulary each chapter, there are! It is integral that your students know each term, postulate, common notion, and proposition by memory in order to use them when needed

<u>Supplies</u>

1. Each student will be supplied with
    a) Euclid's Elements Book - Green Lion Publishing
    b) Pencil bag containing:
        i. compass
        ii. protractor
        iii. pencils
        iv. eraser
        v. highlighter
        vi. red/black pen
        vii. notecards on a binder ring
2. Notebook
3. Folder
4. A Student Version of Geometry as Told by Euclid's Elements

<u>Workbook Layout</u>

Each proposition is stated and then there is room below for the students to have space to draw the proposition and take notes.

If there are following discussion questions, they are listed below the proposition.

Lesson	Days	Topic	What to do	Homework
1.1	4	The Importance of Logic	**Day 1**: Introduce **logos** Have students independently read about Protagoras and Socrates and make notes on their thoughts/questions. Read Protagoras as a class, and open the floor for socratic discussion as a class - repeat with Socrates **Day 2:** Repeat the process from Day 1 with Plato and Diodorus Cronus. **Day 3**: Repeat the process with Aristotle, Eubulides, and Chrysippus. **Day 4**: 1.1 Discussion questions - have students take time in class to independently write down their answers to each question Open the floor for a discussion on each question - respectful disagreeing is okay	**Day 1**:None **Day 2**: None **Day 3**: None **Day 4**: None
1.2	2	Reasoning	**Day 1:** Open with defining reason and using the quote about Plato and Aristotle (our friends from lesson 1.1) to lead into the class discussion questions - have the students consider and write notes on each question individually and then discuss them as a class Notes: Statement definition and examples **Day 2**: Notes: Inference definition and examples Notes: Syllogism definition and examples Notes: Argument definition	**Day 1**: None **Day 2**: 1.2 Exercises

Lesson	Days	Topic	What to do	Homework
1.3	1	Invalidity and Counterexamples	**Day 1:** Notes: Counterexample definition and example Notes: Fallacy definition and fallacy examples	**Day 1**: 1.3 Exercises
1.4	1	Composing and Defending an Argument	**Day 1**: Notes: Goals on composing an argument 1.4 Discussion questions - have students take time in class to independently write down their answers to each question - then discuss them as a class	**Day 1:** None
2.1	1	Euclid's History	**Day 1:** Have each student independently read *About the Author: Euclid* on page xix of your Elements and answer the Discussion questions, then go through the questions as a class For question 2 - there are notes about what was happening in Greece, before giving these, see what the students can come up with from their knowledge of world history *Depending on how much conversation you get into over the discussion questions, you may have time and space to start lesson 2.2 today as well	**Day 1:** None
2.2	1	Euclidean Terminology	**Day 1:** Have each student independently read *Some Euclidean Terminology* on page xxiii of their Elements and fill out the chart in their notes As a class discuss the terms that were introduced as they will be unfamiliar but we be used often	**Day 1:** None

Lesson	Days	Topic	What to do	Homework
3.1	3	Book 1 Definitions	**Day 1**: Have the students write down the Definitions and draw a picture demonstrating each one Have each student create flashcards with the term on the front and a picture and definition on the back **Day 2**: Discussion - dive into what Euclid was saying with his definitions - What does it mean for a point to be that which has no part? Does that make sense? What was he trying to convey? **Day 3**: Memorization - game	**Day 1:** Create flashcards for the terms **Day 2:** None **Day 3**: Study your terms, you will need to know what they mean in order to use them - quiz coming up
3.2	4	Postulates and Common Notions	**Day 1**: Quiz over definitions 1-8 Have the students write down the Postulates and draw a picture demonstrating each one Have the students write down the Common Notions Have the students make flashcards of the Postulates and the Common Notions - add these to the definition cards 3.2 Discussion questions - have students take time in class to independently write down their answers to each question - then discuss them as a class **Day 2**: Quiz over definitions 1-16 Use the rest of class time to work on memorizing the rest of the definitions for quiz tomorrow	**Day 1**: 3.2 questions - if not finished in class **Day 2:** Study your terms, you will need to know what they mean in order to use them - quiz coming up

Lesson	Days	Topic	What to do	Homework
			Day 3: Quiz over definitions 1-20, Use the rest of class time to work on memorizing the postulates and common notions for quiz tomorrow **Day 4:** Quiz over postulates, and common notions	**Day 3**: Continue studying definitions, postulates, and common notions - they should become second nature to you **Day 4**: Quiz over postulates and common notions - when this is finished, start proposition 1 as a class Book 1 Quizzes
3.3	3	1.1 1.2 1.3	**Day 1**: Read through Book 1 - Proposition 1 as a class as you work/write it out on the board *Make sure to note definition 15 and Common Notion 1 which give you some of the freedom you need to complete certain steps for the first proposition *While your students will be creating this by hand, here is a link to Geogebra where the teacher can work it out electronically before the lesson for further understanding: Book 1 - Proposition 1 You will be doing propositions 1-3 as a class to set an example as to what your students will be presenting moving forward.	**Day 1:** None

Lesson	Days	Topic	What to do	Homework
			Day 2: Flex day - extra time may be needed for the first three propositions If you do not need an extra day, have students study for their quiz tomorrow **Day 3**: Quiz over definitions, postulates, and common notions	**Day 2**: None **Day 3**: Quiz over definitions, postulates, and common notions
3.4	8	1.4 1.5 1.6 1.8	**Day 1**: Set the students into partners to read through Book 1 - Proposition 4 Select student groups will be presenting their understanding of this proposition for the class tomorrow *While your students will be creating this by hand, here is a link to Geogebra where the teacher can work it out electronically before the lesson for further understanding: Book 1 - Proposition 4 **Day 2**: Select two different groups to get up and present their method and understanding of the proposition as a class Socratic discussion over the discussion questions *If you have room, let both groups presenting leave their drawings on the whiteboard as they may be helpful to reference during the discussion time	**Day 1**: Proposition 4 discussion questions Be prepared to present Proposition 4 tomorrow **Day 2**: None

Lesson	Days	Topic	What to do	Homework
			Day 3: Set the students into partners to read through Book 1 - Proposition 5 - until you find consistent partners, set the student with different partners each day so that they learn to work well with anyone in the class Select student groups will be presenting their understanding of this proposition for the class tomorrow *While your students will be creating this by hand, here is a link to Geogebra where the teacher can work it out electronically before the lesson for further understanding: Book 1 - Proposition 5. After your students present, it would be interesting to go through this Geogebra presentation with them as it is laid out very cleanly. **Day 4**: Select two different groups to get up and present their method and understanding of the proposition as a class Socratic discussion over the discussion questions *If you have room, let both groups presenting leave their drawings on the whiteboard as they may be helpful to reference during the discussion time As a class, go through Proposition 5 and create a color coded guide for each step - use a different color the highlight the shape and write the step that is taking place	**Day 3**: Proposition 5 and discussion questions Be prepared to present Proposition 5 tomorrow **Day 4**: None

Lesson	Days	Topic	What to do	Homework
			AF = AG AB = AC And so on. **Day 5**: Set the students into partners to read through Book 1 - Proposition 6- until you find consistent partners, set the student with different partners each day so that they learn to work well with anyone in the class Select student groups will be presenting their understanding of this proposition for the class tomorrow *While your students will be creating this by hand, here is a link to Geogebra where the teacher can work it out electronically before the lesson for further understanding: Book 1 - Proposition 6. **Day 6**: Select two different groups to get up and present their method and understanding of the proposition as a class Socratic discussion over the discussion questions *If you have room, let both groups presenting leave their drawings on the whiteboard as they may be helpful to reference during the discussion time Create a chart where you will record a short summary of each proposition so they are easy to memorize and reference.	**Day 5**: Proposition 6 and discussion questions Be prepared to present Proposition 6 tomorrow **Day 6**: None

Lesson	Days	Topic	What to do	Homework
				Day 7: Proposition 8 and discussion questions Be prepared to present Proposition 8 tomorrow

<table>
<tr><td>1.1</td><td>Construct an equilateral triangle on a given straight line.</td></tr>
<tr><td>1.2</td><td>Given a point and a line</td></tr>
<tr><td>1.3</td><td>Draw 2 lines of the same length with a compass</td></tr>
<tr><td>1.4</td><td>Side Angle Side Congruence theorem</td></tr>
<tr><td>1.5</td><td>Base Angles Theorem</td></tr>
<tr><td>1.6</td><td>Converse of the Base Angles Theorem</td></tr>
</table>

Day 7: Set the students into partners to read through Book 1 - Proposition 8 - until you find consistent partners, set the student with different partners each day so that they learn to work well with anyone in the class

Select student groups will be presenting their understanding of this proposition for the class tomorrow

*While your students will be creating this by hand, here is a link to a Youtube tutorial where the teacher can work through it electronically before the lesson for further understanding: Book 1 - Proposition 8.

Lesson	Days	Topic	What to do	Homework
			Day 8: Select two different groups to get up and present their method and understanding of the proposition as a class Socratic discussion over the discussion questions *If you have room, let both groups presenting leave their drawings on the whiteboard as they may be helpful to reference during the discussion time	**Day 8**: None
3.5	6	1.9 1.10 1.11 1.12	By this point, your students should be able to handle more than one proposition per class period, so instead of allowing two days for each proposition, we will condense it to a day and a half for each. **Day 1:** Set the students into partners to read through Book 1 - Proposition 9 - until you find consistent partners, set the student with different partners each day so that they learn to work well with anyone in the class Select student groups will be presenting their understanding of this proposition for the class tomorrow *While your students will be creating this by hand, here is a link to a Geogebra tutorial where the teacher can work through it electronically before the lesson for further understanding: Book 1 - Proposition 9 and Book 1 - Proposition 10.	**Day 1:** Proposition 9 and discussion questions Be prepared to present Proposition 9 tomorrow *If you have finished Proposition 9 with good quality, start on Proposition 10.

Lesson	Days	Topic	What to do	Homework
			Day 2: Select two different groups to get up and present their method and understanding of the proposition as a class	**Day 2:** Proposition 10 and discussion questions
			Day 3: Select two different groups to get up and present their method and understanding of the proposition as a class	**Day 3:** None **Day 4:** Proposition 11 and discussion questions
			Day 4: Set the students into partners to read through Book 1 - Proposition 11 *While your students will be creating this by hand, here is a link to a Youtube tutorial where the teacher can work through it electronically before the lesson for further understanding: Book 1 - Proposition 11 and Book 1 - Proposition 12	*If you have finished Proposition 11 with good quality, start on Proposition 12. **Day 5:** Proposition 12 and discussion questions **Day 6:** None
			Day 5: Set the students into partners to read through Book 1 - Proposition 12 Select student groups will be presenting their understanding of propositions 11 and 12 for the class tomorrow	
			Day 6: Select two different groups to get up and present their method and understanding of the proposition as a class	
3.6	4	1.13 1.14 1.15	**Day 1:** Set the students into partners to read through Book 1 - Proposition 13 and 14 **Day 2:** Present on 13 and 14	**Day 1:** Presentation on 13 and 14 along with discussion questions **Day 2:** None

Lesson	Days	Topic	What to do	Homework
			Day 3: Set the students into partners to read through Book 1 - Proposition 15 **Day 4:** Work through Proposition 15 Fill out chart with next propositions you have covered 	**Day 3:** Presentations on 15 along with discussion questions **Day 4:** Finish chart if you didn't in class

1.1	Construct an equilateral triangle on a given straight line.
1.2	Given a point and a line
1.3	Draw 2 lines of the same length with a compass
1.4	Side Angle Side Congruence Theorem
1.5	Base Angles Theorem
1.6	Converse of the Base Angles Theorem
1.8	Side Side Side Congruence Theorem
1.9	To bisect an angle
1.10	To bisect a line
1.11	To draw a perpendicular line
1.12	To draw a perpendicular line through a point
1.13	Supplementary angles
1.14	Every straight line is 180 degrees
1.15	Vertical Angles

Lesson	Days	Topic	What to do	Homework
			*At the end of this guide, there is a completed version of the Propositions Summary Chart Teacher Resources: [Book 1 - Proposition 13](), [Book 1 - Proposition 14]() and [Book 1 - Proposition 15]()	
3.7	10	1.16 1.17 1.18 1.19 1.20 1.22	**Day 1:** Today the students will work through 16 and 17 individually. **Day 2:** For presentations, have one student get up and complete only one step, then another student picks up where they left off and only completes one more step, and so on until the proposition is finished. This may not turn out as perfectly as we would like, but it is a lesson in adaptability and seeing how well the students studied the proof to fully understand it. **Day 3:** Finish going over propositions 16 and 17, go through discussion questions as a group (socratic). **Day 4**: Set the students into partners to read through Book 1 - Proposition 18 and 19 **Day 5**: Select two different groups to get up and present their method and understanding of the proposition as a class Socratic discussion over the discussion questions **Day 6:** Flex day - use this if needed - we don't want to rush their understanding	**Day 1:** Finish work on 16 and 17 and discussion questions **Day 2:** Discussion questions **Day 3:** None **Day 4**: Propositions 18 - 19 and discussion questions **Day 5**: None **Day 6:** None

Lesson	Days	Topic	What to do	Homework
			Day 7: Set the students into partners to read through Book 1 - Proposition 20 and work through the discussion questions **Day 8:** Select two different groups to get up and present their method and understanding of the proposition as a class Socratic discussion over the discussion questions After Prop 20 - Research drawings done by Escher. Are we back at our question of drawing versus a demonstration in words? In this proposition, what is obvious in the drawing (at least if you wanted the shortest distance to lunch) is here proved to be true. But can some drawings seem obvious but turn out to be misleading or outright false? Have you seen such drawings by Escher? [11] **Day 9:** Set the students into partners to read through Book 1 - Propositions 21 and 22 and work through the discussion questions **Day 10:** Present propositions 21 and 22 *If there is extra time, feel free to continue the Escher discussion Teacher Resources: Book 1 Proposition 16, Book 1 Proposition 17, Book 1 Proposition 18, Book 1 Proposition 19, Book 1 Proposition 20, Book 1 Proposition 21, Book 1 Proposition 22	**Day 7:** Proposition 20 and discussion questions **Day 8:** Escher research and questions - if not completed in class **Day 9:** Propositions 21 and 22 **Day 10:** None

Lesson	Days	Topic	What to do	Homework
3.8	1	1.23	**Day 1:** Go through constructing and discussing proposition 23 as a class. This may be teacher led since the others have been student led Teacher Resource: [Book 1 Proposition 23](#)	**Day 1:** None
3.9	3	1.24 1.25 1.26	**Day 1:** Set the students into partners to read through Book 1 - Propositions 24 and 25 **Day 2**: Select two different groups to get up and present their method and understanding of the proposition as a class Socratic discussion over the discussion questions **Day 3**: Go through constructing and discussing proposition 23 as a class. This may be teacher led since the others have been student led Teacher Resource: [Book 1 Proposition 24](#), [Book 1 Proposition 25](#), [Book 1 Proposition 26](#)	**Day 1:** Proposition 24 and discussion questions *If you have finished Proposition 24 with good quality, start on Proposition 25 **Day 2:** Proposition 25 and discussion questions **Day 3**: None
3.10	6	1.27 1.28 1.29 1.30 1.31	**Day 1:** Set the students into partners to read through Book 1 - Propositions 27 and 28 **Day 2:** Select two different groups to get up and present their method and understanding of the proposition as a class Socratic discussion over the discussion questions **Day 3:** Set the students into partners to read through Book 1 - Propositions 29 and 30	**Day 1:** Propositions 27 and 28 and discussion questions **Day 2:** None **Day 3:** Propositions 29 and 30 and discussion questions

Lesson	Days	Topic	What to do	Homework
			Day 4: Select two different groups to get up and present their method and understanding of the proposition as a class Socratic discussion over the discussion questions **Day 5:** Set the students into partners to read through Book 1 - Proposition 31 **Day 6:** Select two different groups to get up and present their method and understanding of the proposition as a class Socratic discussion over the discussion questions Teacher Resources: [Book 1 Proposition 27](), [Book 1 Proposition 28](), [Book 1 Proposition 29](), [Book 1 Proposition 30](), [Book 1 Proposition 31]()	**Day 4:** None **Day 5:** Proposition 31 and discussion questions **Day 6:** None
3.11	1	1.32	**Day 1:** Go through constructing and discussing proposition 23 as a class. This may be teacher led since the others have been student led Teacher Resource: [Book 1 Proposition 32]()	**Day 1:** None
3.12	4	1.33 1.34 1.46	**Day 1:** Today the students will work through 33 and 34 individually. **Day 2:** For presentations, have one student get up and complete only one step, then another student picks up where they left off and only completes one more step, and so on until the proposition is finished. This may not turn out as perfectly as we would like, but it is a lesson in adaptability and seeing how well the students studied the proof to fully understand it.	**Day 1:** Propositions 33 and 34 **Day 2:** Discussion questions for 33 and 34

Lesson	Days	Topic	What to do	Homework
			Day 3: Finish going over propositions 33 and 34, go through discussion questions as a group (socratic). **Day 4:** Go through constructing and discussing Proposition 46 as a class. This may be teacher led since the others have been student led Teacher Resources: [Book 1 Proposition 33], [Book 1 Proposition 34], [Book 1 Proposition 46]	**Day 3:** None **Day 4:** None
3.13	5	1.47 1.48	**Day 1:** For the next 3 days, students will be working through either 47 or 48 (both if they have time!) and creating a video that works through the proposition *Whether you would like this to be in partners or individual is up to you - due to having only three days to work on it, partner work may be more efficient *Student may be creative with this, they may record themselves talking about it, voice over a powerpoint, use a screen recorder of them working through it using an online graphing calculator, just to name a few ideas **Day 2:** Work time on proofs **Day 3:** Work time on proofs **Day 4:** Presentations	**Day 1:** Video over Proposition 47 or 48 **Day 2:** Video over Proposition 47 or 48 **Day 3:** Video over Proposition 47 or 48 **Day 4:** None

Lesson	Days	Topic	What to do	Homework
			Day 5: Presentations *Finish your chart of the Proposition summaries for book #1 - work on this in class if there is time Teacher Resources: [Book 1 Proposition 47](#), [Book 1 Proposition 48](#)	**Day 5:** Make sure all of Book 1 Propositions are added to your summary chart
4.1	2	Book 2 Definitions	**Day 1**: Have the students write down the Definitions and draw a picture demonstrating each one and a counter example Have each student create flashcards with the term on the front and a picture and definition on the back **Day 2**: Memorization - game	**Day 1:** Create flashcards for the terms **Day 2**: Study your terms, you will need to know what they mean in order to use them - quiz coming up
4.2	3	2.1 2.2 2.3	The method of how we will go about showing proofs in books 2 and 3 is different from Book 1 because Book 1 was focused on providing a base for all of our future work. Since we are adjusting our methods, you will work through the first three proportions of Book 2 as a class **Day 1:** Proposition 1 construction and steps/justification layout - teacher led **Day 2:** Proposition 2 construction and steps/justification layout - teacher led	**Day 1:** None **Day 2:** None

Lesson	Days	Topic	What to do	Homework
			Day 3: Proposition 3 construction and steps/justification layout - teacher led Teacher Resources: [Book 2 Proposition 1](#), [Book 2 Proposition 2](#), [Book 2 Proposition 3](#)	**Day 3:** None
4.3	4	2.4 2.5 2.6	**Day 1:** Set students into groups to work through Propositions 4 and 5 **Day 2:** Select two different groups to get up and present their method and understanding of the proposition as a class Socratic discussion over the discussion questions **Day 3:** Proposition 6 with partner **Day 4:** Select two different groups to get up and present their method and understanding of the proposition as a class Socratic discussion over the discussion questions Teacher Resources: [Book 2 Proposition 4](#), [Book 2 Proposition 5](#), [Book 2 Proposition 6](#)	**Day 1:** Finish Propositions 4 and 5 for presentations tomorrow and discussion questions **Day 2:** None **Day 3:** Proposition 6 discussion questions **Day 4:** None
4.4	3	2.7 2.8	**Day 1:** Set students into groups to work through Propositions 7 and 8	**Day 1:** Finish Propositions 7 and 8 for presentations tomorrow and discussion questions

Lesson	Days	Topic	What to do	Homework
			Day 2: Select two different groups to get up and present their method and understanding of the proposition as a class Socratic discussion over the discussion questions **Day 3:** Add all of Book 2 to your Proposition Summary Chart Teacher Resources: [Book 2 Proposition 7], [Book 2 Proposition 8]	**Day 2:** None **Day 3:** None
5.1	1	Book 3 Definitions	**Day 1**: Have the students write down the Definitions and draw a picture demonstrating each one and a counter example Have each student create flashcards with the term on the front and a picture and definition on the back Memorization - game	**Day 1:** Create flashcards for the terms Study your terms, you will need to know what they mean in order to use them - quiz coming up
5.2	6	3.1 3.2 3.3 3.4 3.5	**Day 1:** Today the students will work through 1 and 2 individually. **Day 2:** For presentations, have one student get up and complete only one step, then another student picks up where they left off and only completes one more step, and so on until the proposition is finished. This may not turn out as perfectly as we would like, but it is a lesson in adaptability and seeing how well the students studied the proof to fully understand it.	**Day 1:** Proposition 1 and 2 constructions, step/justification chart, and full understanding **Day 2:** None

Lesson	Days	Topic	What to do	Homework
			Day 3: Finish going over Propositions 1 and 2 Start work on Propositions 3, 4, and 5 - no rush - take your time to fully understand what Euclid is doing and what it means in the math world! **Day 4:** Work time on Propositions 3, 4, and 5 **Day 5:** Present 3, 4, and 5 in the same manner as we did 1 and 2. **Day 6:** Present 3, 4, and 5 in the same manner as we did 1 and 2. Teacher Resources: [Book 3 Proposition 1](), [Book 3 Proposition 2](), [Book 3 Proposition 3](), [Book 3 Proposition 4](), [Book 3 Proposition 5]()	**Day 3:** None **Day 4:** Finish 3, 4, and 5 **Day 5:** None **Day 6:** None
5.3	6	3.6 3.9 3.10 3.11 3.12 3.13	**Day 1:** Set students into groups to work through Propositions 6 and 9 **Day 2:** Select two different groups to get up and present their method and understanding of the propositions as a class **Day 3:** Set students into groups to work through Propositions 10 and 11 **Day 4:** Select two different groups to get up and present their method and understanding of the proposition as a class **Day 5:** Set students into groups to work through Propositions 12 and 13 **Day 6:** Select two different groups to get up and present their method and understanding of the proposition as a class	**Day 1:** Have propositions 6 and 9 completed and ready to present **Day 2:** None **Day 3:** Have propositions 10 and 11 completed and ready to present **Day 4:** None **Day 5:** Have propositions 12 and 13 completed and ready to present

Lesson	Days	Topic	What to do	Homework
			Teacher Resources: [Book 3 Proposition 6](#), [Book 3 Proposition 9](#), [Book 3 Proposition 10](#), [Book 3 Proposition 11](#), [Book 3 Proposition 12](#), [Book 3 Proposition 13](#)	**Day 6:** None
5.4	6	3.14 3.15 3.17 3.19 3.20 3.21	**Day 1:** Add the first group of Book 3 Propositions to your Proposition Summary Chart Set students into groups to work through Propositions 14 and 15 **Day 2:** Select two different groups to get up and present their method and understanding of the propositions as a class **Day 3:** Set students into groups to work through Propositions 17 and 19 **Day 4:** Select two different groups to get up and present their method and understanding of the propositions as a class **Day 5:** Set students into groups to work through Propositions 20 and 21 **Day 6:** Select two different groups to get up and present their method and understanding of the propositions as a class Teacher Resources: [Book 3 Proposition 14](#), [Book 3 Proposition 15](#), [Book 3 Proposition 17](#), [Book 3 Proposition 19](#), [Book 3 Proposition 20](#), [Book 3 Proposition 21](#)	**Day 1:** Have propositions 14 and 15 completed and ready to present **Day 2:** None **Day 3:** Have propositions17 and 19 completed and ready to present **Day 4:** None **Day 5:** Have propositions 20 and 21 completed and ready to present **Day 6:** None
5.5	5	3.22 3.25 3.26 3.27	**Day 1:** Today the students will work through 22 and 25 individually.	**Day 1:** Propositions 22 and 25 constructions, step/justification chart, and full understanding

Lesson	Days	Topic	What to do	Homework
			Day 2: For presentations, have one student get up and complete only one step, then another student picks up where they left off and only completes one more step, and so on until the proposition is finished. This may not turn out as perfectly as we would like, but it is a lesson in adaptability and seeing how well the students studied the proof to fully understand it. **Day 3:** Finish going over Propositions 22 and 25 Take your time to fully understand what Euclid is doing and what it means in the math world! Start on Propositions 26 and 27 **Day 4:** Let students work on 26 and 27 individually - if everyone is ready, start presenting like we did on day 2 **Day 5:** Flex day Finish up presentations if needed Teacher Resources: Book 3 Proposition 22, Book 3 Proposition 25, Book 3 Proposition 26, Book 3 Proposition 27	**Day 2:** None **Day 3:** Propositions 26 and 27 **Day 4:** None **Day 5:** None
5.6	5	5.28 5.30 5.32 5.34	**Day 1:**Set students into groups to work through Propositions 28 and 30 **Day 2:** Select two different groups to get up and present their method and understanding of the propositions as a class	**Day 1:** Have propositions 28 and 30 completed and ready to present **Day 2:** None

Lesson	Days	Topic	What to do	Homework
			Day 3: Set students into groups to work through Propositions 32 and 34 **Day 4:** Select two different groups to get up and present their method and understanding of the propositions as a class **Day 5:** Flex day Make sure all of your Book 3 Propositions are added to your Proposition Summary Chart Teacher Resources: Book 3 Proposition 28, Book 3 Proposition 30, Book 3 Proposition 32, Book 3 Proposition 34	**Day 3:** Have propositions 32 and 34 completed and ready to present **Day 4:** None **Day 5:** None - if chart is complete

Proposition Summary Chart

1.1	Construct an equilateral triangle on a given straight line.
1.2	Given a point and a line
1.3	Draw 2 lines of the same length with a compass
1.4	Side Angle Side Congruence Theorem
1.5	Base Angles Theorem
1.6	Converse of the Base Angles Theorem
1.8	Side Side Side Congruence Theorem
1.9	To bisect an angle
1.10	To bisect a line
1.11	To draw a perpendicular line
1.12	To draw a perpendicular line through a point
1.13	Supplementary angles
1.14	Every straight line is 180 degrees
1.15	Vertical Angles
1.16	In a triangle, exterior angles are greater than either of the interior or opposite angles
1.17	Sum of two angles in a triangle is always less than 180 degrees
1.18	The longest side of a triangle is always opposite the largest angle
1.19	Converse of Proposition 18
1.20	In a triangle, the sum of two sides is always greater than the third side
1.22	Use three given lines and construct a triangle
1.23	Given an angle, construct a congruent angle on a given line at a given point
1.24	In two isosceles triangles, if the top angle of one is larger, the base segment of that triangle will be larger than the other triangle
1.25	Converse of Proposition 24
1.26	Angle Side Angle

| 1.27	Converse of the Alternate Interior Angles Theorem
1.28	If corresponding angles are congruent, then the lines are parallel
1.29	If two lines are cut by a transversal, then the corresponding angles are congruent and the interior angles on the same sides add up to 180 degrees
1.30	Transversal Property of Parallel Lines
1.31	Through a given point, to draw a straight line parallel to a given straight line
1.32	Sum of the two opposite interior angles is equal to the outside angle - The sum of all the interior angles is equal to 180 degrees
1.33	Forming a parallelogram
1.34	A parallelogram cut in half from one vertex divides the area of the shape in half
1.46	On a given straight line to construct a square
1.47	The Pythagorean Theorem
1.48	The Converse of the Pythagorean Theorem
2.1	If you cut a rectangle into pieces, the sum of the pieces is the original rectangle
2.2	If you cut a square into pieces, the sum of the pieces is the original rectangle
2.3	If you cut a rectangle into a square and a rectangle, then the rectangle is equal to the sum of the new square and rectangle
2.4	If you cut a straight line, the square on the whole is equal to the sum of squares on the parts
2.5	If you cut a straight line into equal and unequal segments, the whole is equal to the square and the extra unequal parts
2.6	If you bisect a straight line and add a segment to it, you can recreate the original using your new parts.
2.7	If you cut a straight line, then the square on the whole and one of the segments together are equal to twice the rectangle contained by the whole, the segment, and the square on the remaining segment
2.8	If you cut a straight line four times, the whole rectangle with one of the segments is equal to the square and the original segment
3.1	To find the center of a given circle
3.2	If two points on the circumference are taken, the line between these points will fall

	within the circle
3.3	If a diameter is bisected by a line not through the center, then they meet at right angles, and if they meet at right angles, then the line bisects the diameter
3.4	If two straight lines cut one another but are not through the center, they do not bisect one another
3.5	If two circles cut one another, they do not have the same center
3.6	If two circles touch one another, they do not have the same center
3.9	If a point be taken in a circle and more than one line falls on that point, then the point is in the center
3.10	A circle does not cut a circle at more points than two
3.11	If two circles touch internally, the straight line produced from both of their centers will fall on the point where the circle intersect
3.12	If two circles touch externally, the straight line joining their centers will pass through the point of contact
3.13	A circle cannot touch a circle at more than one point, internally or externally.
3.14	In a circle, equal straight lines are equally distant from the center and those that are equally distant from the center are equal to one another
3.15	Of straight lines in a circle, the diameter is the greatest and those nearer to the center are always greater than the more remote
3.17	From a given point to draw a straight line touching a given circle
3.19	If a straight line be extended from the tangent of the circle it will touch the center
3.20	In a circle, the circumference angle is double the center angle when they share a base
3.21	In a circle, the angles in the same segment are equal to one another
3.22	The opposite angles of quadrilaterals in circles are equal to 180 degrees
3.25	Given a segment of a circle, to describe the complete circle of which it is a segment
3.26	In equal circles equal angles stand on equal circumferences, where they stand at the centers or at the circumferences
3.27	In equal circles equal straight lines cut off equal circumferences
3.28	In equal circles equal circumferences are subtended by equal straight lines

3.30	To bisect a given circumference
3.32	If a straight line through a circle, and from the point of contact there be drawn across, in the circle, a straight line cutting the circle, the angles which it makes with the tangent will be equal to the angles in the alternate segments of the circle
3.34	From a given circle to but off a segment admitting an angle equal to a given rectilinear angle

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2.2 Student Workbook
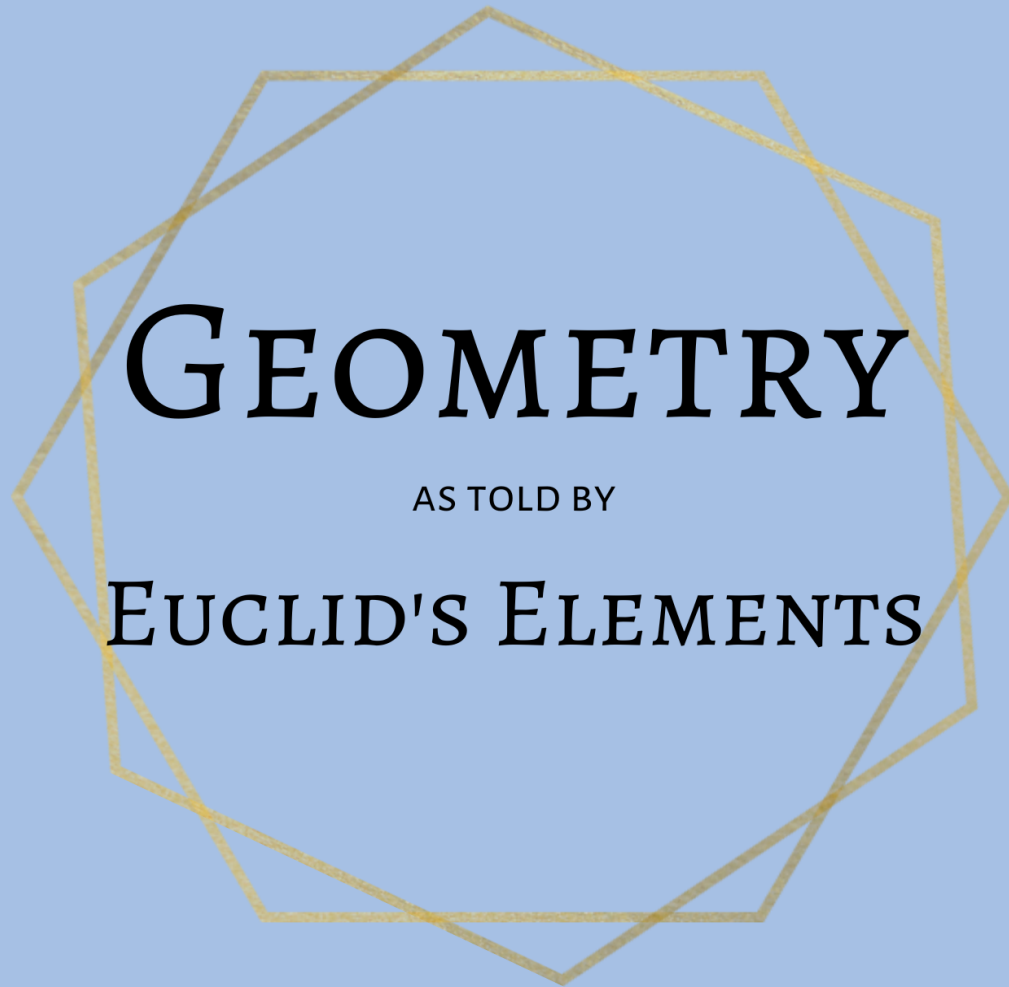
GEOMETRY

AS TOLD BY

EUCLID'S ELEMENTS

Student Workbook

<u>Welcome!</u>

You are currently looking at the Student Version of Geometry as told by Euclid's Elements. This text goes along with Euclid's Elements (Green Lion Press). By using this you are provided with:

I. Discussion questions

    A. These questions are meant to challenge your thinking and spark discussion

    B. Review the questions and formulate your own answers before discussing with the class

II. Space for building each proposition

    A. For each proposition you will be given space to draw out Euclid's steps and evaluate what is happening

    B. Consider what previous material we have to work with (Definitions, Postulates, Common Notions, previously proven propositions) and evaluate Euclid's methods

    C. Summarize each proposition in your own words

III. Usability - what is the best format for engaging in this material

    A. The Socratic Classroom

        1. Allow for open, respectful discussion

        2. Be methodical with your thoughts and how you form arguments

        3. Challenge one another

        4. Think outside the box

    B. Student Led

        1. The teacher will guide, but the work is led by the students

        2. Propositions will be led by student presentations

        3. Students will lead discussions following the propositions

    C. Engage

        1. Take yourself back to Greece in 300 BC and watch the material in Euclid's Elements come alive as you engage with it!

Table of Contents

Logic

1.1 - The Importance of Logic

logic (n.)

mid-14c., *logike*, "branch of philosophy that treats forms of thinking; the science of distinction of true from false reasoning," from Old French *logique* (13c.)

From Latin *(ars) logica* "logic,"

From Greek *(he) logike (techne)* "(the) reasoning (art)," from fem. of *logikos* "pertaining to speaking or reasoning" (also "of or pertaining to speech"), from *logos* "reason, idea, word".

Formerly also *logick*. Sometimes formerly plural, as in *ethics*, but this is not usual. Meaning "logical argumentation" is from c. 1600. Contemptuous *logic-chopper* "sophist, person who uses subtle distinctions in argument" is from 1846. [10]

Logic in the words of the Ancient Philosophers

Protagoras (~470 BCE)
Athens, Greece

Lawyer who understood that every argument has two sides

Protagoras is known primarily for three claims [1]
   (1) that man is the measure of all things (which is often interpreted as a sort of radical relativism)
   (2) that he could make the "worse (or weaker) argument appear the better (or stronger)" and
   (3) that one could not tell if the gods existed or not.

Socrates (~450 BCE)
Athens, Greece

Socrates is well known for these thoughts:

(1) "If a man knew anything, he could "give an account (*logos*)" of it [to others], and in his hands that maid-of-all-work among Greek words takes on the meaning "definition" or something closely approaching it." [2]
(2) Knowledge was the ultimate good and ignorance the ultimate evil.

Plato(~410 BCE) :
Athens, Greece
Author of The Republic and Symposium, a well known philosopher who worked at a school called Academy

Plato is well known for saying:
(1) "Do not train a child to learn by force or harshness; but direct them to it by what amuses their minds, so that you may be better able to discover with accuracy the peculiar bent of the genius of each."
(2) "The heaviest penalty for declining to rule is to be ruled by someone inferior to yourself."
(3) "If women are expected to do the same work as men, we must teach them the same things."
(4) "The object of education is to teach us to love what is beautiful."
(5) Nothing beautiful without struggle."
(6)"Opinion is the medium between knowledge and ignorance" [3]

Diodorus Cronus (~400 BCE)
Alexandria, Egypt

Diodorus and his pupil Philo worked out the beginnings of a logic that took propositions, rather than terms, as its basic elements. They, along with Eubulides were part of the Dialectic School - a Greek school focused on logic. [4]

Aristotle (~370 BCE)
Stagira, Greece

Aristotle developed the syllogism. In logic, a *syllogism* is a form of deductive reasoning consisting of a major premise, a minor premise, and a conclusion. Adjective: *syllogistic*. Also known as a *categorical argument* or a *standard categorical syllogism*. The term syllogism is from the Greek, "to infer, count, reckon"

Here is an example of a valid syllogism:

Major premise: All mammals are warm-blooded.

Minor premise: All black dogs are mammals.

Conclusion: Therefore, all black dogs are warm-blooded. [11]

Aristotle is well known for saying:
(1) "It is the mark of an educated mind to be able to entertain a thought without accepting it."
(2) "Excellence is never an accident. It is always the result of high intention, sincere effort, and intelligent execution; it represents the wise choice of many alternatives – choice, not chance, determines your destiny."
(3) "The high-minded man must care more for the truth than for what people think."
(4) "He who cannot be a good follower cannot be a good leader."
(5) "All men by nature desire to know."
(6) "The Law is Reason free from Passion." [5]

Eubulides of Miletus (~350 BCE)
Athens, Greece

Known for inventing logical paradoxes, his four most famous are:
  (1) The Liar: "Does a man who says that he is now lying, speak truly?"
  (2) The Horns: if you have not lost something, you still have it. You have not lost horns. So you still have horns.
  (3) The Hooded Man: a man enters the room, wearing a hood. You do not know who the hooded man is. But the hooded man is your brother. So you do not know who your brother is.
  (4) The Heap: if you have one grain of sand it does not make a heap. If you add one grain of sand to grains that do not make a heap, they still do not make a heap. So, starting with one grain, and adding grain after grain, you will never have a heap, even when the sand is piled up to the height of a mountain. [6]

Chrysippus (~250 BCE)
Athens, Greece

Chrysippus was a major contributor to **Stoicism** - an ancient Greek school of philosophy founded at Athens by Zeno of Citium. The school taught that virtue, the highest good, is based on knowledge; the wise live in harmony with the divine Reason that governs nature, and are indifferent to the change of fortune and to pleasure and pain.

Discussion
1. What is logic and why do we need it?

2. What are the results when someone argues without the use of logic? Think of an example.

3. Why are we only studying people who lived, taught, and learned over 2,000 years ago?

4. Complete the map of where the ancient thinkers lived.

5. Which thinker do you agree with the most and why?

6. Is there a philosopher that you disagree with? Why?

1.2 - Reasoning

reasoning (n.)

late 14c., *resouning*, "exercise of the power of reason; act or process of thinking logically;" also an instance of this, a presentation of reasons or arguments. [10]

Thinking enlightened by logic.

Thus to Plato reason was the messenger of the gods. To Augustine reason was the eye of the soul, given to man by God that he may comprehend God. [7]

Discussion
1. Why do we reason?

2. What are the differences between reasoning and arguing?

3. What problems could arise as we reason with others?

Statement: declarative sentence which must be either true or false but not both

When we are wanting to make an argument we avoid vague terms or terms that have multiple meanings.

Examples that are not statements.

Mike is tall.

Jessica is so loud!

Examples that are statements.

The goldenrod is the Nebraska state flower.

The American flag is red, white, and blue.

Euclid uses the word **Proposition** for Statement. He will state a proposition and then use logical arguments to construct or prove it.

Inference - the process of reasoning from what we think is true to what else is true - does not have to be logical

Logical - a valid argument
Illogical - an invalid argument

Examples of logical inferences

Statement - The forecast is for a sunny day.
Inference - The sun will be out at 3pm today.

Statement - You are dropped off at school at 7:50 am and the parking lot is full.
Inference - School starts at 8:00 am.

Examples of illogical inferences
   Statement - No one has less expensive sleeping bags than Camping Express - each bag is $25.99.
   Inference - Camping Express has the least expensive sleeping bags.

   Statement - Brian, Jenna, and Lauren went to the beach to hang out.
   Inference - Jenna and Lauren rode in the same car to get to the beach.

Syllogism - comes from the Greek word *syllogismos* which means conclusion or inference
         - kind of logical argument that uses deductive reasoning to arrive at a conclusion from two given propositions that are assumed to be true

Most famous syllogism by Aristotle:
   Statement 1: All men are mortal.
   Statement 2: Socrates is a man.
   Conclusion: Therefore, Socrates is mortal.

Examples of Syllogisms
   Statement 1: Two lines form a cross when they intersect perpendicularly.
   Statement 2: These lines are intersecting perpendicularly.
   Conclusion: These lines form a cross.

   Statement 1: If it is sunny, then we went to the park.
   Statement 2: It is sunny.
   Conclusion: We went to the park.

Examples that are not Syllogisms
   Statement 1: If it is a Sunday in June, then we will get pizza for dinner.
   Statement 2: It is a Sunday.
   Conclusion: We got pizza for dinner.

Statement 1: If it is cold out then everyone wears their blue hats.
Statement 2: It is 45 degrees outside.
Conclusion: Everyone is wearing their blue hats.

Statement 1: If it is Monday, then we will go to the movies.
Statement 2: We went to the movies.
Conclusion: It is Monday.

Argument - reasons that support a conclusion; made up of premises, something that has already been proved or is assumed to be true

1.2 Exercises

1. Determine whether the sentence is a statement, defend your reasoning.
    a. Chocolate chip cookies are the best.

    b. Mark got first place because he was the fastest runner at the track meet.

    c. John Adams was the second president of the United States.

    d. All Americans over the age of 16 own a Tesla.

2. Determine whether the inference is logical or illogical, defend your reasoning.
    a. Statement: The menu only has hot dogs or cheeseburgers.
       Inference: You will order either a hot dog or a cheeseburger.

    b. Statement: The Christmas lights turn on after it gets dark outside.
       Inference: The lights will be on by 10pm.

    c. Statement: Anyone over the age of 16 can get their driver's license.
       Inference: Anna is 17 so she has her driver's license.

3. Determine whether the statements form a syllogism or not, defend your reasoning.
    a. Statement 1: If the book is at least 150 pages long, then it will be put in the library.
       Statement 2: The book is 139 pages long.
       Conclusion: The book is not in the library.

    b. Statement 1: If Caroline runs to the tree, then she will climb the tree.
       Statement 2: Caroline climbed the tree.
       Conclusion: Caroline ran to the tree.

c. Statement 1: If Frederich makes a sandwich, then the family will sit down to eat lunch.
   Statement 2: Frederich made a grilled cheese.
   Conclusion: The family sat down to each lunch.

1.3 - Invalidity and Counterexamples

Once something is assumed or proven to be true, the only way to change that is to provide a **counterexample** that proves it invalid.

Here is an example that is invalid:

Statement 1: All birds have feet.	All A have B
Statement 2: All octopus ink is black.	All C is D
Conclusion: Therefore all birds are black.	Therefore all A are D

Counterexample: A counterexample to the example above would be a bluebird.

Here is an example that is invalid:

Statement 1: All of the students run track.	All A do B
Statement 2: Some of the students play baseball.	Some A do C
Conclusion: Therefore all of the track runners play baseball.	Therefore all B do C

Counterexample: A counterexample would be a student who runs track but does not play baseball.

Fallacies - flaws in logic which lead to an invalid argument

Examples of Fallacies
If we ban Hummers because they are bad for the environment, eventually the government will ban all cars, so we should not ban Hummers.

Even though it's only the first day, I can tell this is going to be a boring course.

I drank bottled water and now I am sick, so the water must have made me sick.

People who don't support the proposed state minimum wage increase hate the poor.

If you were a true American you would support the rights of people to choose whatever vehicle they want. [8]

1.3 Exercises

Explain what the fallacy is in each statement and why it is a flaw in logic.
1. A well trained dog knows how to sit and speak, if your animal does not know these, then it is not a dog.

2. I have had one meal at the new restaurant and from that I know that I will not like their whole menu.

3. I went for a walk and now I have a headache, the walk must have made my head hurt.

4. Write two examples of invalid arguments and then identify a counterexample for each.
*Show the logic (All A have B…) as well.

1.4 - Composing and Defending an Argument

Steps in composing an argument:

1. Identify your goal
   a. In order to reach a conclusion, you must know what you want to achieve
   b. Euclid will state the goal right after the title of his Proposition
2. Identify your premises
   a. What truths do you have that you can work with
   b. Think about what has already been defined, assumed, and proven
3. Reason to a conclusion
   a. Use your premises along with logic to reason to your conclusion
4. Conclude
   a. Restate your goal (theorem, proposition) and that you have proven it

Discussion
What is our goal when we compose an argument?

What is our goal when we defend an argument?

Why do we need to defend/prove things that were done by mathematicians 2000 years ago?

What can you use to defend your argument?

Will I have to have everything memorized?

2: History and Purpose

2.1 - Euclid's History

Read *About the Author: Euclid* on page xix of your Elements.

Discussion

1. Approximately when did Euclid live?

2. What else was happening in Greece at the time? In the world?

3. Who wrote an Elements book before Euclid? Why do we use Euclid's book now?

4. Fill in the information about Euclid's works:

Work	Information/Importance
Data	
On divisions of figures	
Porisms	
Treatise on conic sections	
Optics and Catoptrics	

2.2 - Euclidean Terminology

Read *Some Euclidean Terminology* on page xxiii of your Elements.

Divisions of a Proposition	Every proposition contains: 1. 2. 3. 4. 5. 6. **Necessary in every proposition**
Therefore, etc.	
Q.E.D.	
Q.E.F.	

Porism (Corollary)	
Lemma	
Reductio ad absurdum	
Figure and boundary	

3: Book 1

3.1 - Definitions

Point:

Line:

Extremities:

Straight Line:

Surface:

Lines:

Plane Surface:

Plane Angle:

Rectilinear:

Right Angle:

Perpendicular:

Obtuse Angle:

Acute Angle:

Boundary:

Figure:

Circle:

Center of a Circle:

Diameter:

Semicircle:

Rectilinear Angles:

Trilateral Figures:

Quadrilateral Figure:

Multilateral Figure:

Equilateral Triangle:

Isosceles Triangle:

Scalene Triangle:

Right-Angled Triangle:

Obtuse-Angled Triangle:

Acute-Angled Triangle:

Square:

Oblong:

Rhombus:

Rhomboid:

Trapezia:

Parallel:

3.2 - Postulates and Common Notions

Postulates

Let the following be postulated:

1.

2.

3.

4.

5.

Common Notions

1.

2.

3.

4.

5.

3.2 Discussion

1. Why did Euclid have to state these things?

2. Is there a reason for only having 5 for each?

3. Do you think there should be more? Why or why not?

3.3 - Propositions 1-3

Book 1 - Proposition 1
On a given finite straight line to construct an equilateral triangle.

Discussion

1. How do we know that the circles will intersect?

2. How to we know that $\overline{AC} = \overline{BC} \Rightarrow \overline{AB} = \overline{AC} = \overline{BC}$?

Book 1 - Proposition 2
To place at a given point [as an extremity] a straight line equal to a given straight line.

Discussion
1. Write out the order or equalities that were established throughout this proposition.

Book 1 - Proposition 3
*Given two unequal straight lines, to cut off from the greater a straight line equal to the
less*

Discussion
1. Describe the syllogism used in this proposition.

2. Postulate 3 said that we could describe a circle with any center and distance. So why couldn't
we just use length C as the given distance and one end of the line AB? Why did we need Prop.
2? Is it because in Postulate 3 we have the distance given, and in Prop. 3 we just have a line?
[11]

3.4 - Proposition 4-6, 8

Book 1 - Proposition 4
If two triangles have the two sides equal to two sides respectively, and have the angles contained by the equal straight lines equal, they will also have the base equal to the base, the triangle will be equal to the triangle, and the remaining angles will be equal to the remaining angles respectively, namely those which the equal slides subtended.

Discussion

1. We consider this a "congruence proof". Define congruent.

2. How does Proposition 4 feel different than the first three?

3. Can you think of any method to prove that the areas, the remaining angles, and the base are equal?

4. In this proof, it seems that Euclid is saying it is okay for us to pick up items and move them to show congruence. Why have we not been able to do this with any other propositions? What are the pros and cons of using this method? Does it take away from the legitimacy of the argument?

Book 1 - Proposition 5

In isosceles triangles the angles at the base are equal to one another, and, if the equal straight lines be produced further, the angles under the base will be equal to one another.

Discussion

1. Restate what this proposition has proven in your own words.

2. Is there a more simple way to complete this proof?

3. How many new triangles did Euclid create? Draw and label all the congruent triangles in the space below.

Book 1 - Proposition 6

If in a triangle two angles be equal to one another, the sides which subtend the equal angles will also be equal to one another.

Discussion

1. This proposition claims that triangle *BDC* is less than triangle *BAC* but never explicitly proves it, how do we know it is an accurate claim to make?

*Proof by Contradiction: this proposition introduces a new method called *reductio ad absurdum*. If we can't find a way to prove something directly, we assume the contrary and see whether that leads to a contradiction.

2. Could this be proven directly?

3. Is the contradiction proof more persuasive than a direct proof?

4. Is there a possible flaw or fallacy in assuming a contrary? What would that be? What would we have to be careful about? [11]

5. What Common Notion does this heavily rely on and why?

Book 1 - Proposition 8
If two triangles have the two sides equal to two sides respectively, and have also the base equal to the base, they will also have the angles equal which are contained by the equal straight lines.

Discussion

1. This proposition does not end with an explicit statement that the two triangles are equal. What previous proposition could we use along with this in order to show the two triangles as equal.

2. Are you keeping track of the congruence proofs? Congruent triangles are the ones that have all corresponding sides and angles equal (from which it follows that they are equal in area). Congruent triangles will be essential building blocks for future proofs, but as we continue to develop the structure, it might start to seem that *any* equal sides and angles will make triangles congruent. What kinds of congruence proofs do we have so far?

3.5 - Propositions 9-12

Book 1 - Proposition 9
To bisect a given rectilineal angle.

Discussion

1. What does bisect mean?

2. Why was it important for Euclid to be able to bisect an angle? Does this even matter if we don't have the tools to measure the angle in degrees?

3. Since this seems like a basic proposition, why was this not one of our first propositions?

Book 1 - Proposition 10
To bisect a given finite straight line.

Discussion

1. How did we define a line?

2. Of what is a "breadthless length" made up?

3. Is there always going to be an even way to divide this in half? What if there isn't?

Book 1 - Proposition 11

To draw a straight line at right angles to a given straight line from a given point on it.

Discussion

1. This follows from proposition 10 where we almost did this exact construction with point D. Why do you think Euclid separated the two ideas?

Book 1 - Proposition 12
To a given infinite straight line, from a given point which is not on it, to draw a perpendicular straight line.

Discussion

1. How would you define infinite?

2. Why is it important for the line to be infinite in this proof?

3.6 - Propositions 13-15

Book 1 - Proposition 13

If a straight line set up on a straight line makes angles, it will make either two right angles or angles equal to two right angles.

Discussion

1. Write out using geometry symbols what this proposition is showing.

2. Why is Euclid using the phrase "two right angles"? What term might we use instead?

Book 1 - Proposition 14

If with any straight line, and at a point on it, two straight lines not lying on the same side make the adjacent angles equal to two right angles, the two straight lines will be in a straight line with one another.

Discussion

1. Restate what this proposition has proven in your own words.

Book 1 - Proposition 15

If two straight lines cut one another, they make the vertical angles equal to one another.

Discussion

1. Write out the thought process that this proposition followed using Geometric symbols.

3.7 - Propositions 16-20, 22

Book 1 - Proposition 16

In any triangle, if one of the sides be produced, the exterior angle is greater than either of the interior and opposite angles.

Discussion

1. Is there a more succinct way Euclid could have proved this?

2. Restate what this proposition has proven in your own words.

Book 1 - Proposition 17

In any triangle two angles taken together in any manner are less than two right angles.

Book 1 - Proposition 18
In any triangle the greater side subtends the greater angle.

Book 1 - Proposition 19
In any triangle the greater angle is subtended by the greater side.

Discussion

1. Define converse.

2. This is a converse of the previous proposition. If we already know that a proposition is true, do we need to prove it's converse?

3. Could the proposition be true and the converse be false?

Book 1 - Proposition 20
In any triangle two sides taken together in any manner are greater than the remaining one.

Discussion

1. Write out the inequalities that Euclid walks us through in this proposition.

2. Proculus, a 5th Century commentator writing in Alexandria, says that the Epicureans liked to ridicule this proposition by saying that any donkey knows its truth, because no donkey will approach its food along two sides of a triangle rather than in a straight line. Indeed, it seems intuitively obvious just looking at or imagining a triangle. But is that a proof? Proculus says that perceiving the truth of something is not enough: one must know *why* it is true for it to be knowledge. Do you agree? [11]

Book 1 - Proposition 22

Out of three straight lines, which are equal to three given straight lines, to construct a triangle: thus it is necessary that two of the straight lines taken together in any manner should be greater than the remaining one.

Discussion

1. Why must any two of the straight lines be greater than the third? Of course, if that were not true we could show from Prop. 20 that we couldn't construct a triangle. But is there more? How do we know what the two circles will intersect such that the triangle we need will result?

2. Each of the propositions we have investigated focuses on different aspects of geometry (lines, points, triangles, etc.). Make a chart to categorize the propositions that we have worked through based on their geometric focus.

3.8 - Propositions 23

Book 1 - Proposition 23

On a given straight line and at a point on it to construct a rectilineal angle equal to a given rectilineal angle.

Discussion

1. Since in 1.4 we were allowed to pick up the triangle and apply it, why can't we do that here? (We noticed that this possible shortcut could have applied to earlier propositions as well.) Are we seeing a reluctance to use this technique? Do we share Euclid's discomfort? [11]

3.9 - Propositions 24-26

Book 1 - Proposition 24

If two triangles have the two sides equal to two sides respectively, but have the one of the angles contained by the equal straight lines greater than the other, they will also have the base greater than the base.

Book 1 - Proposition 25

If two triangles have the two sides equal to two sides respectively, but have the base greater than the bse, they will also have the one of the angles continued by the equal straight lines greater than the other.

Discussion

1. Write out the thought process that this proposition followed using Geometric symbols.

Book 1 - Proposition 26

If two triangles have the two angles equal to two angles respectively, and one side equal to one side, namely, either the side adjoining the equal angles, or that subtending one of the equal angles, they will also have the remaining sides equal to the remaining sides and the remaining angle equal to the remaining angle.

Discussion

1. List out the triangle congruence proofs we have covered.

3.10 - Propositions 27-31

Book 1 - Proposition 27

If a straight line falling on two straight lines make the alternate angles equal to one another, the straight lines will be parallel to one another.

*This is called "parallel lines cut by a transversal".

Discussion

1. We have now started discussing parallel lines. Did you expect to see a triangle in this proof?

2. Euclid's 5th Postulate, "the parallel postulate" has been considered controversial. Do you think it is an assumption that should be freely accepted?

3. Can we picture that strange hypothetical triangle well enough to operate on it with a proof? How careful do we have to be with a drawing representing the impossible if we are to make sure that we are not misled? Or does it have nothing to do with the drawing and we just have to make sure the logical steps in the words are airtight? [11]

Book 1 - Proposition 28
If a straight line falling on two straight lines make the exterior angle equal to the interior and opposite angle on the same side, or the interior angles on the same side equal to two right angles, the straight lines will be parallel to one another.

Discussion
1. You may have noticed by now that we are doing and creating math, but Euclid has yet to use any numbers. What are your thoughts on this? Is it still math? Do we ever need to use numbers?

2. What would you say is the importance of parallel lines a) in math and b) in the world?

Book 1 - Proposition 29
A straight line falling on parallel straight lines make the alternate angles equal to one another, the exterior angle to the interior and opposite angle, and the interior angles on the same side equal to two right angles.

Discussion
1. This proposition is proving a converse. Why do we need to prove converses?

2. When Postulate 5 was first presented, it may have seemed wordy and clumsy. Are you starting to appreciate its power now? [11]

*Many have tried to improve on Postulate 5, or to actually prove it, but it seems that their proofs all in some way covertly rely on the postulate. You might be tempted to start trying out some proofs of your own for this postulate now that you have the tools of the first twenty-nine propositions. It's tricky terrain on which even the great have stumbled, but the effort can be entertaining and instructive even if, or especially if, it ends in failure. [11]

Book 1 - Proposition 30
Straight lines parallel to the same straight line are also parallel to one another.

Discussion
1. This feels a lot like a syllogism. Define syllogism and determine whether this qualifies as one.

Book 1 - Proposition 31
Through a given point to draw a straight line parallel to a given straight line.

Discussion
1. Restate this proposition in your own words.

3.11 - Propositions 32

Book 1 - Proposition 32

In any triangle, if one of the sides be produced, the exterior angle is equal to the two interior and opposite angles, and the three interior angles of the triangle are equal to two right angles.

3.12 - Propositions 33-34, 46

Book 1 - Proposition 33

The straight lines joining equal and parallel straight lines [at the extremities which are] in the same directions [respectively] are themselves also equal and parallel.

Discussion

1. Define parallelogram

2. Does this make you appreciate the math, logic, and work that Euclid has put into the theorems prior to this one?

Book 1 - Proposition 34
In parallelogramic areas the opposite sides and angles are equal to one another, and the diameter bisects the areas.

Discussion
1. "Diameter" is a term that we have seen used for circles, but here Euclid uses it for a quadrilateral. How would you define diameter so that it works for this figure as well?

2. Euclid talks about a "parallelogramic area" in this proposition; it is the only place he uses the explicit term area. The word area does not appear any other place (except in words quoting this proposition). Area is not defined. (Def. 5 is of a surface: "A surface has length and breadth only.") But here in the second part of Prop. 34, where the diameter is said to bisect the area, it seems to speak of the size of a figure, or about a relationship between areas of figures. So what is area? Is it about size and comparison of size? Or is it more about occupying space? [11]

3. You may have noticed that Eeclid has yet to define or prove anything using numbers (except the vague use of "the measurement is that of two right angles"). Without using numbers, is the measurement of area useful?

Book 1 - Proposition 46
On a given straight line to describe a square.

Discussion
1. Is it frustrating that we have to prove everything we do?

2. Is this a reminder that an act of thought is only complete when we *know* that what we did is what we intended? We need to know *what* we've built. Is Euclid saying, "You have not done what I asked until you have apprehended what you did"? [11]

3.31 - Propositions 47-48

Book 1 - Proposition 47

In right-angled triangles the square on the side subtending the right angle is equal to the squares on the sides containing the right angle.

Discussion

1. Work your way through this proposition again and write out your thought process.

2. This very famous proposition, attributed to Pythagoras, provides a very satisfying application of congruent triangles and the propositions exacting the relationships of triangles and quadrilaterals between parallel lines. Do you have a feeling of a culmination of the long careful construction that has been Book1?   We have one more loose end to tie up here, but this magnificent proposition is a worthy finale to the foundation of the *Elements*.

Book 1 - Proposition 48

If in a triangle the square on one of the sides be equal to the squares on the remaining two sides of the triangle, the angle contained by the remaining two sides of the triangle is right.

Discussion

1. Did the converse of 1.47 need to be proved? Do some converses need to be proved but not others? Or is it just that some converses are false and others true, so without proving any given converse you don't know for sure? Are geometric converses and logical converses different sorts of things?

4: Book 2

4.1 - Definitions

Rectangular Parallelogram:

Gnomon:

Book 2 Format

As we saw in Book 1, the focus was mainly on laying the geometry foundations so that we would have the freedom to use basic skills without needing to prove them each time. Now in Book 2, we are going to focus on the specific steps and justification, the bones of proof writing, to show the logic of each proposition. Thus instead of specific discussion questions at the end of each proposition, we will use a Step/Justification t-chart while we work the proposition.

It will look like:

Step	Justification
Line from A to B	Postulate 2
Bisect AB at C	1.9

For each step that the proposition uses, it must be backed up by a justification from something Eulcid has, assumed, defined, or proven already.

4.2 - Propositions 1-3

Book 2 - Proposition 1

If there be two straight lines, and one of them be cut into any number of segments whatever, the rectangle contained by the two straight lines is equal to the two rectangles contained by the uncut straight line and each of the segments.

Book 2 - Proposition 2

If a straight line be cut at random, the rectangles contained by the whole and both of the segments are equal to the square on the whole.

Book 2 - Proposition 3

If a straight line be cut at random, the rectangle contained by the whole and one of the segments is equal to the rectangle contained by the segments and the square in the aforesaid segment.

4.3 - Propositions 4-6

Book 2 - Proposition 4

If a straight line be cut at random, the square on the whole is equal to the squares on the segments and twice the rectangle contained by the segments.

Book 2 - Proposition 5

If a straight line be cut into equal and unequal segments, the rectangle contained by the unequal segments of the whole together with the square on the straight line between the points of section is equal to the square on the half.

Before you begin this proposition - take note that 1.43, a proposition that we did not cover in book 1, says that complements always equal each other.

You will also need Book 1 Proposition 36: Parallelograms which are on equal bases and within the same parallels equal one another

Complements

Book 2 - Proposition 6

If a straight line be bisected and a straight line be added to it in a straight line, the rectangle contained by the whole with the added straight line and the added straight line together with the square on the half is equal to the square on the straight line made up of the half and the added straight line.

4.4 - Propositions 7-9

Book 2 - Proposition 7

If a straight line be cut at random, the square on the whole and that on one of the segments both together are equal to twice the rectangle contained by the whole and the said segment and the square on the remaining segment.

Book 2 - Proposition 8

If a straight line be cut at random, four times the rectangle contained by the whole and one of the segments together with the square on the remaining segment is equal to the square described on the whole and the aforesaid segment as on one straight line.

5: Book 3

5.1 - Definitions
Equal Circles:

Touch a Circle:

Touch One Another:

Equally Distant from the Center:

Greater Distant:

Segment of a Circle:

Angle of a Segment:

Angle in a Segment:

Stand Upon:

Sector of a Circle:

Similar Segments of Circles:

5.2 - Propositions 1-5

Book 3 - Proposition 1
To find the center of a given circle.

Book 3 - Proposition 2

If on the circumference of a circle two points be taken at random, the straight line joining the points will fall within the circle.

Book 3 - Proposition 3

If in a circle a straight line through the center bisect a straight line not through the center, it also cuts it at right angles; and if it cut it at right angles, it also bisects it.

Book 3 - Proposition 4

If in a circle two straight lines cut one another which are not through the center, they do not bisect one another.

Book 3 - Proposition 5
If two circles cut one another, they will not have the same center.

5.3 - Propositions 6, 9-13

Book 3 - Proposition 6
If two circles touch one another, they will not have the same center.

Book 3 - Proposition 9

If a point be taken within a circle, and more than two equal straight lines fall from the point on the circle, the point taken is the center of the circle.

Book 3 - Proposition 10
A circle does not cut a circle at more points than two.

Book 3 - Proposition 11

If two circle touch one another internally, and their centers be taken, the straight line joining their centers, if it be also produced, will fall on the point of contact of the circles.

Book 3 - Proposition 12

If two circles touch one another externally, the straight line joining their centers will pass through the point of contact.

Book 3 - Proposition 13
A circle does not touch a circle at more points than one, whether it touch it internally or externally.

5.4 - Propositions 14, 15, 17, 19 - 21

Book 3 - Proposition 14

In a circle equal straight lines are equally distant from the center, and those which are equally distant from the center are equal to one another.

Book 3 - Proposition 15
Of straight lines in a circle the diameter is greatest, and of the rest the nearer to the center is always greater than the more remote.

Book 3 - Proposition 17
From a given point to draw a straight line touching a given circle.

Book 3 - Proposition 19
If a straight line touch a circle, and from the point of contact a straight line be drawn at right angles to the tangent, the center of the circle will be on the straight line so drawn.

Tangent: 1590s, "meeting at a point without intersecting," from Latin *tangentem* (nominative *tangens*), present participle of *tangere* "to touch," First used by Danish mathematician Thomas Fincke in *"Geomietria Rotundi"* (1583). [11]

Book 3 - Proposition 20

In a circle the angle at the center is double the angle at the circumference when the angles have the same circumference as base.

Book 3 - Proposition 21

In a circle the angles in the same segment are equal to one another.

5.5 - Propositions 22, 25-27

Book 3 - Proposition 22

The opposite angles of quadrilaterals in circles are equal to two right angles.

Book 3 - Proposition 25

Given a segment of a circle, to describe the complete circle of which it is a segment.

Book 3 - Proposition 26

In equal circles equal angles stand on equal circumferences, whether they stand at the centers or at the circumferences.

Book 3 - Proposition 27

In equal circles angles standing on equal circumferences are equal to one another, whether they stand at the centers or at the circumferences.

5.6 - Propositions 28, 30, 32, 34

Book 3 - Proposition 28
In equal circles equal straight lines cut off equal circumferences, the greater equal to the greater and the less to the less.

Book 3 - Proposition 30
To bisect a given circumference.

Book 3 - Proposition 32

If a straight line touch a circle, and from the point of contact there be drawn across, in the circle, a straight line cutting the circle, the angles which it makes with the tangent will be equal to the angles in the alternate segments of the circle.

Book 3 - Proposition 34

From a given circle to cut off a segment admitting an angle equal to a given rectilineal angle.

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2.3 Answer Key

GEOMETRY

AS TOLD BY

EUCLID'S ELEMENTS
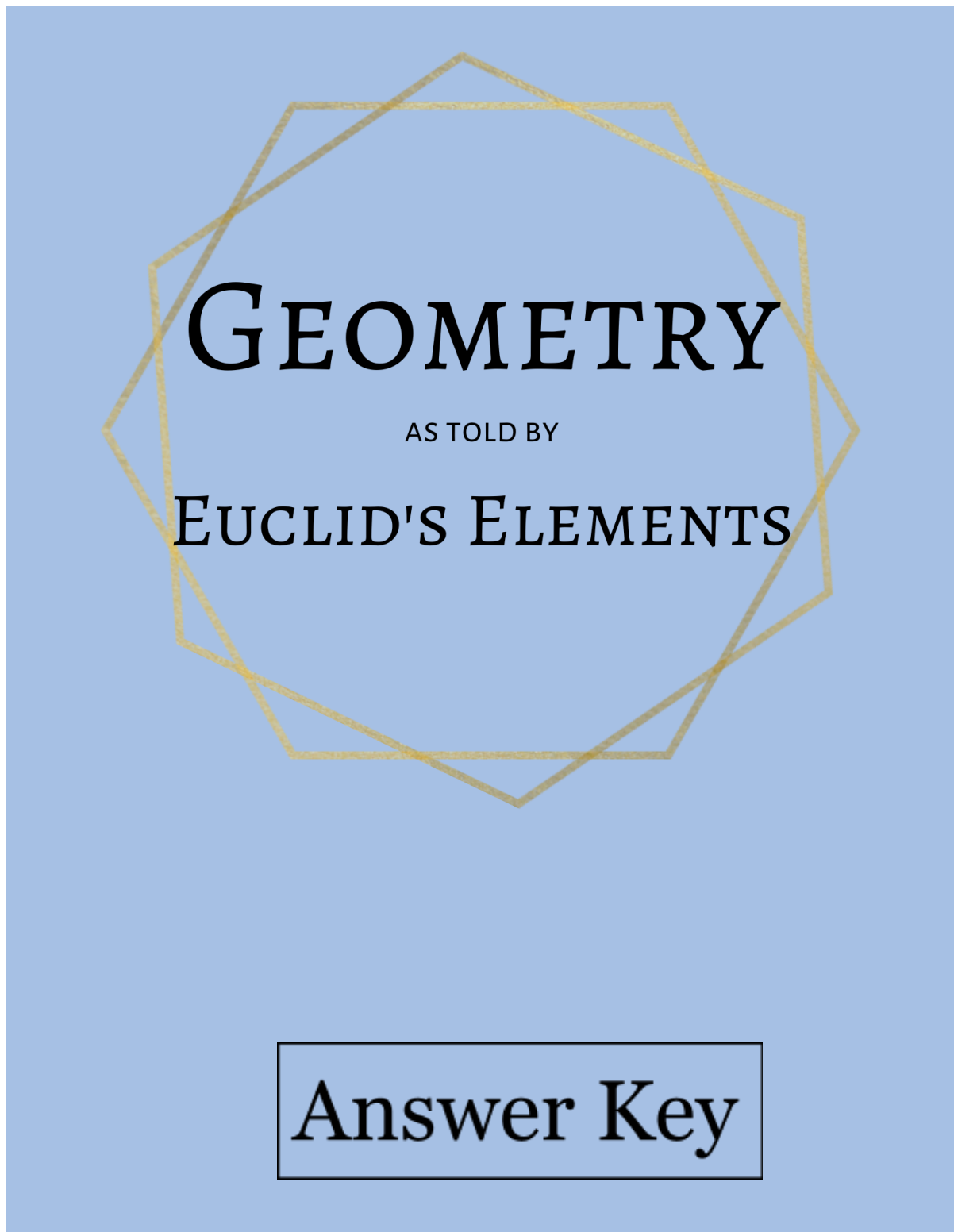
Answer Key

<u>Welcome!</u>

You are currently looking at the Answer Key for Geometry as told by Euclid's Elements. This answer key is synonymous with the Student Version. By using this, you are provided with:

I. Direct answers to questions or problems

II. Possible student answers

    A. There are questions where multiple answers are possible and possible responses have been provided

    B. There will be times when the students come up with another way to prove a proposition and the class will need to logically reason through legitimacy of their result

III. Areas for open discussion

    A. Some questions are posed simply to start discussion and do not have direct answers

IV. Drawings of each proposition

    A. While students are expected to draw each proposition, the drawings have been provided for you here

This class is set up to be based in socratic discussion, thus the teacher is not directly lecturing but guiding students in their learning and discussions.

Table of Contents

1: Logic

1.1 - The Importance of Logic

logic (n.)

mid-14c., *logike*, "branch of philosophy that treats forms of thinking; the science of distinction of true from false reasoning," from Old French *logique* (13c.)

From Latin *(ars) logica* "logic,"

From Greek *(he) logike (techne)* "(the) reasoning (art)," from fem. of *logikos* "pertaining to speaking or reasoning" (also "of or pertaining to speech"), from *logos* "reason, idea, word".

Formerly also *logick*. Sometimes formerly plural, as in *ethics*, but this is not usual. Meaning "logical argumentation" is from c. 1600. Contemptuous *logic-chopper* "sophist, person who uses subtle distinctions in argument" is from 1846. [10]

Logic in the words of the Ancient Philosophers

Protagoras (~470 BCE)
Athens, Greece

Lawyer who understood that every argument has two sides

Protagoras is known primarily for three claims [1]
  (1) that man is the measure of all things (which is often interpreted as a sort of radical relativism)
  (2) that he could make the "worse (or weaker) argument appear the better (or stronger)" and
  (3) that one could not tell if the gods existed or not.

Socrates (~450 BCE)
Athens, Greece

Socrates is well known for these thoughts:

(1) "If a man knew anything, he could "give an account (*logos*)" of it [to others], and in his hands that maid-of-all-work among Greek words takes on the meaning "definition" or something closely approaching it." [2]
(2) Knowledge was the ultimate good and ignorance the ultimate evil.

Plato(~410 BCE) :
Athens, Greece
Author of The Republic and Symposium, a well known philosopher who worked at a school called Academy

Plato is well known for saying:
(1) "Do not train a child to learn by force or harshness; but direct them to it by what amuses their minds, so that you may be better able to discover with accuracy the peculiar bent of the genius of each."
(2) "The heaviest penalty for declining to rule is to be ruled by someone inferior to yourself."
(3) "If women are expected to do the same work as men, we must teach them the same things."
(4) "The object of education is to teach us to love what is beautiful."
(5) Nothing beautiful without struggle."
(6)"Opinion is the medium between knowledge and ignorance" [3]

Diodorus Cronus (~400 BCE)
Alexandria, Egypt

Diodorus and his pupil Philo worked out the beginnings of a logic that took propositions, rather than terms, as its basic elements. They, along with Eubulides were part of the Dialectic School - a Greek school focused on logic. [4]

Aristotle (~370 BCE)
Stagira, Greece

Aristotle developed the syllogism. In logic, a *syllogism* is a form of deductive reasoning consisting of a major premise, a minor premise, and a conclusion. Adjective: *syllogistic*. Also known as a *categorical argument* or a *standard categorical syllogism*. The term syllogism is from the Greek, "to infer, count, reckon"

Here is an example of a valid categorical syllogism:

Major premise: All mammals are warm-blooded.

Minor premise: All black dogs are mammals.

Conclusion: Therefore, all black dogs are warm-blooded. [11]

Aristotle is well known for saying:
(1) "It is the mark of an educated mind to be able to entertain a thought without accepting it."
(2) "Excellence is never an accident. It is always the result of high intention, sincere effort, and intelligent execution; it represents the wise choice of many alternatives – choice, not chance, determines your destiny."
(3) "The high-minded man must care more for the truth than for what people think."
(4) "He who cannot be a good follower cannot be a good leader."
(5) "All men by nature desire to know."
(6) "The Law is Reason free from Passion." [5]

Eubulides of Miletus (~350 BCE)
Athens, Greece

Known for inventing logical paradoxes, his four most famous are:
   (1) The Liar: "Does a man who says that he is now lying, speak truly?"
   (2) The Horns: if you have not lost something, you still have it. You have not lost horns. So you still have horns.
   (3) The Hooded Man: a man enters the room, wearing a hood. You do not know who the hooded man is. But the hooded man is your brother. So you do not know who your brother is.
   (4) The Heap: if you have one grain of sand it does not make a heap. If you add one grain of sand to grains that do not make a heap, they still do not make a heap. So, starting with one grain, and adding grain after grain, you will never have a heap, even when the sand is piled up to the height of a mountain. [6]

Chrysippus (~250 BCE)
Athens, Greece

Chrysippus was a major contributor to **Stoicism** - an ancient Greek school of philosophy founded at Athens by Zeno of Citium. The school taught that virtue, the highest good, is based on knowledge; the wise live in harmony with the divine Reason that governs nature, and are indifferent to the change of fortune and to pleasure and pain.

Discussion
1. What is logic and why do we need it?
Logic is the thought, value, and understanding of knowledge. We need it in order to think deeper, it helps us reason.

2. What are the results when someone argues without the use of logic? Think of an example.

Arguing without logic is like going on a road trip without a map - the intended destination is not usually achieved.

Examples:

   A: Dogs are the best animal because they are fast.

   B: What do you use your fast dog to do?

   A: Nothing, it just makes him look cool.

   B: So does being cool make him the best?

   B: How are you defining cool?

   A: How I want to and I'm right.

   And so on, in this example, person A is not using logic and ends up making a pointless argument.

   A: The two triangles are congruent because they look like they are the same size.

   B: Are all of their corresponding angles the same measure?

   A: Yeah, it looks like it!

   B: Even if the angle measures are off by one degree, the triangles are no longer congruent.

   A: Good thing I know they are because they look the same!

   In this example, person A is not using basic math facts to back up the argument which makes him seem to be lacking sense.

3. Why are we only studying people who lived, taught, and learned over 2,000 years ago?

We are studying the original logicians - these men struggled through the discovery of this way of thinking and set out formal methods for arguing logically. There have been more recent logical discoveries, but they all lie on the work of the originals.

4. Complete the map of where the ancient thinkers lived.

5. Which thinker do you agree with the most and why?
Open to individual student responses.

6. Is there a philosopher that you disagree with? Why?
Open to individual student responses.

1.2 - Reasoning

reasoning (n.)

late 14c., *resouning*, "exercise of the power of reason; act or process of thinking logically;" also an instance of this, a presentation of reasons or arguments. [10]

Thinking enlightened by logic.

Thus to Plato reason was the messenger of the gods. To Augustine reason was the eye of the soul, given to man by God that he may comprehend God. [7]

Discussion

1. Why do we reason?

   We reason in order to come to a logical conclusion based on the information we know/are given.

2. What are the differences between reasoning and arguing?

   Arguing usually comes with emotion based on feelings while reasoning comes with passion based on the truth.

3. What problems could arise as we reason with others?

   False information

   Letting emotion control the discussion

   Not defining your terms the same

Statement: declarative sentence which must be either true or false but not both

When we are wanting to make an argument we avoid vague terms or terms that have multiple meanings.

Examples that are not statements

Mike is tall.

Who is defining tall?

Jessica is so loud!

How is loud defined?

Examples that are statements.

The goldenrod is the Nebraska state flower.

Statement, this is a fact.

The American flag is red, white, and blue.

Statement, this is a fact.

Euclid uses the word **Proposition** for Statement. He will state a proposition and then use logical arguments to construct or prove it.

Inference - the process of reasoning from what we think is true to what else is true - does not have to be logical

Logical - a valid argument
Illogical - an invalid argument

Examples of logical inferences

Statement - The forecast is for a sunny day.
Inference - The sun will be out at 3pm today.
This inference is logical because it uses the forecast of a sunny day, something we know is true, to infer that the sun will shine.

Statement - You are dropped off at school at 7:50 am and the parking lot is full.
Inference - School starts at 8:00 am.
This inference is logical because it uses the statement of time and the facts of you being dropped off to a full parking lot to infer that school starts soon.

Examples of illogical inferences
    Statement - No one has less expensive sleeping bags than Camping Express - each bag is $25.99.
    Inference - Camping Express has the least expensive sleeping bags.
    This inference is illogical because other retailers could sell sleeping bags for the same price as Camping Express which would not make it the least expensive.

    Statement - Jack, Jenna, and Lauren went to the beach to hang out.
    Inference - Jenna and Lauren rode in the same car to get to the beach.
    While this is possible, this inference is illogical because it does not directly follow from the statement we were given.

Syllogism - comes from the Greek word *syllogismos* which means conclusion or inference
    - kind of logical argument that uses deductive reasoning to arrive at a conclusion from two given propositions that are assumed to be true

Most famous syllogism by Aristotle:
    Statement 1: All men are mortal.
    Statement 2: Socrates is a man.
    Conclusion: Therefore, Socrates is mortal.

Examples of Syllogisms
    Statement 1: Two lines form a cross when they intersect perpendicularly.
    Statement 2: These lines are intersecting perpendicularly.
    Conclusion: These lines form a cross.

    Statement 1: If it is sunny, then we went to the park.
    Statement 2: It is sunny.
    Conclusion: We went to the park.

Examples that are not Syllogisms
    Statement 1: If it is a Sunday in June, then we will get pizza for dinner.
    Statement 2: It is a Sunday.
    Conclusion: We got pizza for dinner.
    While this may be true, since we do not know whether it is June or not, we cannot guarantee the conclusion.

Statement 1: If it is cold out then everyone wears their blue hats.
Statement 2: It is 45 degrees outside.
Conclusion: Everyone is wearing their blue hats.
This conclusion is not valid because the term "cold" is not well defined.

Statement 1: If it is Monday, then we will go to the movies.
Statement 2: We went to the movies.
Conclusion: It is Monday.
While this conclusion may be true, the syllogism does not hold because we must prove the accuracy of the premise (If it is Monday) before we can assume the conclusion (then we will go to the movies).

Argument - reasons that support a conclusion; made up of premises, something that has already been proved or is assumed to be true

1.2 Exercises

1. Determine whether the sentence is a statement, defend your reasoning.
   a. Chocolate chip cookies are the best.
   Not a statement because "best" is not defined.

   b. Mark got first place because he was the fastest runner at the track meet.
   Statement, this explains exactly why Mark got 1st place.

   c. John Adams was the second president of the United States.
   Statement, this is a fact.

d. All Americans over the age of 16 own a Tesla.
Not a statement, this is false.

2. Determine whether the inference is logical or illogical, defend your reasoning.
a. Statement: The menu only has hot dogs or cheeseburgers.
Inference: You will order either a hot dog or a cheeseburger.
This inference is logical, if you only have two options to eat, you will choose one of them to eat.

b. Statement: The Christmas lights turn on after it gets dark outside.
Inference: The lights will be on by 10pm.
This inference is logical because it is dark by 10pm in the winter months.

c. Statement: Anyone over the age of 16 can get their driver's license.
Inference: Anna is 17 so she has her driver's license.
This inference is illogical. Just because Anna is old enough to have her driver's license, that does not infer that she chose to get it.

3. Determine whether the statements form a syllogism or not, defend your reasoning.
a. Statement 1: If the book is at least 150 pages long, then it will be put in the library.
Statement 2: The book is 139 pages long.
Conclusion: The book is not in the library.
These statements form a syllogism. The first statement gives the options, the second statement chooses an option, and the third statement gives a logical conclusion.

b. Statement 1: If Caroline runs to the tree, then she will climb the tree.
Statement 2: Caroline climbed the tree.
Conclusion: Caroline ran to the tree.
These statements do not form a syllogism. The second statement does not accurately follow from the first statement, thus the conclusion cannot follow from the second statement.

c. Statement 1: If Frederich makes a sandwich, then the family will sit down to eat lunch.
   Statement 2: Frederich made a grilled cheese.
   Conclusion: The family sat down to each lunch.
   These statements form a syllogism. Since a grilled cheese is a type of sandwich,
   statement 2 follows from statement 1, thus the conclusion is logical.

1.3 - Invalidity and Counterexamples

Once something is assumed or proven to be true, the only way to change that is to provide a **counterexample** that proves it invalid.

Here is an example that is invalid:
Statement 1: All birds have feet.   All A have B
Statement 2: All octopus ink is black.   All C is D
Conclusion: Therefore all birds are black.   Therefore all A are D

Counterexample: A counterexample to the example above would be a bluebird.
While the premises are true, the conclusion does not logically follow.

Here is an example that is invalid:
Statement 1: All of the students run track.   All A do B
Statement 2: Some of the students play baseball.   Some A do C
Conclusion: Therefore all of the track runners play baseball.   Therefore all B do C

Counterexample: A counterexample would be a student who runs track but does not play baseball.
There may be track runners who play baseball, but by using the universal qualifier "all" this conclusion is not necessarily true.

Fallacies - flaws in logic which lead to an invalid argument

Examples of Fallacies
If we ban Hummers because they are bad for the environment, eventually the government will ban all cars, so we should not ban Hummers.
This is a fallacy because the speaker is comparing all cars to Hummers which is a flaw.

Even though it's only the first day, I can tell this is going to be a boring course.
This is a fallacy because it is not based on knowledge or experiment, it is an opinion taken with little experience.

I drank bottled water and now I am sick, so the water must have made me sick.
This is a fallacy because it is working in a closed environment and assuming that nothing else is happening internally or externally that could cause the sickness.

People who don't support the proposed state minimum wage increase hate the poor.
This is a fallacy because it is too simple. There is more to the lack of support, but this argument is trimming the entire decision down to one opinion.

If you were a true American you would support the rights of people to choose whatever vehicle they want. [8]
This is a fallacy because it uses the word "true" to engage your emotions of being a "loyal American" and wanting to support what that means.

1.3 Exercises

Explain what the fallacy is in each statement and why it is a flaw in logic.
1. A well trained dog knows how to sit and speak, if your animal does not know these, then it is not a dog.
The term "well trained dog" is not equivalent to "dog", thus you can have a dog without it being well trained, that is - knowing how to sit and speak.

2. I have had one meal at the new restaurant and from that I know that I will not like their whole menu.
This is a fallacy because it is too simple. Unless the restaurant only serves one item, it is an oversimplification to assume that by not liking one item you will dislike the entire menu.

3. I went for a walk and now I have a headache, the walk must have made my head hurt.
This is a fallacy because it ignores everything else that you have done during the day that could have caused the headache along with what is going on inside your body that could have caused it.

4. Write two examples of invalid arguments and then identify a counterexample for each.
*Show the logic (All A have B…) as well.
Open to individual student responses.

1.4 - Composing and Defending an Argument

Steps in composing an argument:

1. Identify your goal
   a. In order to reach a conclusion, you must know what you want to achieve
   b. Euclid will state the goal right after the title of his Proposition
2. Identify your premises
   a. What truths do you have that you can work with
   B. Think about what has already been defined, assumed, and proven
3. Reason to a conclusion
   a. Use your premises along with logic to reason to your conclusion
4. Conclude
   a. Restate your goal (theorem, proposition) and that you have proven it

Discussion

What is our goal when we compose an argument?
Our goal is to use what we know in order to logically reach a conclusion.
This may include
   1) Referencing material that has already been defined or proven.
   2) Drawing a picture to clearly represent an idea
   3) Putting your information into order so that it flows logically

What is our goal when we defend an argument?
Do we just want to "win"? No, our goal is to prove that we can do something that we couldn't before based only on components that we know are true.

We do not want to let emotion into our argument, only the truth.

Why do we need to defend/prove things that were done by mathematicians 2000 years ago?
The goal in this method is to see the beauty and creativity behind the thinking. We will study methods and try to think of our own.

We will sit in the frustration of only being able to use a few premises in order to come to a conclusion.

We will have to work with only what we have been given and are unable to use the phrase, "because I know it" or "because ____ says so" - our goal is to learn how to truly know why and how mathematical connections are taking place.

What can you use to defend your argument?

You are allowed to use Euclid's definitions, his postulates, his common notions, and any proposition that has already been proven or shown.

Will I have to have everything memorized?

You will need to have a full understanding of each explanation you use, whether that be a definition, a common notion, a postulate, or a proposition.

2: History and Purpose

2.1 - Euclid's History

Read *About the Author: Euclid* on page xix of your Elements.

Discussion

1. Approximately when did Euclid live?
347 BCE

2. What else was happening in Greece at the time? In the world?
Thales (625 - 545)
594: Solon replaces the Draconian law in Athens and lays the foundation for Democracy. He introduced to Athens the first coinage and a system of weights and measures.
Pythagoras (569 - 475)
510: Alcmaeonid family and Spartans free Athens from tyranny. Introduction of Democracy in Athens.

WARS

403: Democracy restored in Athens
399: Trial and execution of Socrates
380: Plato establishes the Athens Academy
347: Plato dies [9]

Open to individual student responses.

3. Who wrote an Elements book before Euclid? Why do we use Euclid's book now?
Hippocrates, Leo, and Theudius of Magnesia had all written Elements books prior to Euclid, but Euclid's superseded them so completely that they are now known only from Eudemus's references as preserved by Proclus.

4. Fill in the information about Euclid's works:

Work	Information/Importance
Data	An aid in problem solving by analysis
On divisions of figures	Dividing figures into other, unlike, figures
Porisms	Propositions belonging to the modern theory of transversals and to projective geometry
Treatise on conic sections	Four books, superseded by Apollonius's *Conics*
Optics and Catoptrics	Geometrical optics

2.2 - Euclidean Terminology

Read *Some Euclidean Terminology* on page xxiii of your Elements.

Divisions of a Proposition	Every proposition contains: 1. **Enunciation** - states what is given and what is that which is sought 2. Setting out - marks off what is given, by itself, and adapts it beforehand for use in the investigation 3. Definition or speculation - states separately and makes clear what the particular thing is which is sought 4. Construction or machinery - add what is wanting to the datum for the purpose of finding what is sought 5. **Proof** - draws the required inference by reasoning scientifically from acknowledged facts 6. **Conclusion** - reverts again to the enunciation, conforming what has been demonstrated **Necessary in every proposition**
Therefore, etc.	An exact restatement of the enunciation It is necessary to repeat what you set out to prove at the conclusion.
Q.E.D.	Quod erat demonstrandum - "that which was to have been demonstrated" or "precisely what was required to be proved" Usually represents a theorem
Q.E.F.	Quod erat faciendum - "that which was to have been done" or "precisely what was required to be done" Usually represents a construction

Porism (Corollary)	From what has been demonstrated some other theorem is revealed at the same time without propounding it A theorem that follows from another theorem
Lemma	Any proposition which is assumed for the construction of something else - an assumption made in order to prove something A minor proposition that is proved in order to prove something greater
Reductio ad absurdum	Proofs that reach an impossible conclusion - proof by contradiction - assume your proposition is false and work with it until you reach a contradiction which makes your original proposition true
Figure and boundary	Figure - that which has a boundary or boundaries

3: Book 1

3.1 - Definitions

Point: A point is that which has no part

Line: A line is breadthless length

Extremities: The extremities of a line are points

Straight Line: A straight line is a line which lies evenly with the points on itself

Surface: A surface is that which has length and breadth only

Lines: The extremities of surfaces are lines

Plane Surface: A plane surface is a surface which lies evenly with the straight lines on itself

Plane Angle: A plane angle is the inclination to one another of two lines in a plane which meet one another and do not lie in a straight line

Rectilinear: And when the lines containing the angle are straight, the angle is called rectilinear

Right Angle: When a straight line set up on a straight line makes the adjacent angles equal to one another, each of the equal angles is right

Perpendicular: When a straight line set up on another straight line makes four right angles, the straight line standing on the other is called a perpendicular to that on which it stands

Obtuse Angle: An obtuse angle is an angle greater than a right angle

Acute Angle: An acute angle is an angle less than a right angle

Boundary: A boundary is that which is an extremity of anything

Figure: A figure is that which is contained by any boundary or boundaries

Circle: A circle is a plane figure contained by one line such that all the straight lines falling upon it from one point among those lying within the figure are equal to one another

Center of a Circle: The point where all equal lines inside a circle intersect (taken from circle definition)

Diameter: A diameter of the circle is any straight line drawn through the center and terminated in both directions by the circumference of the circle, and such a straight line also bisects the circle

Semicircle: A semicircle is the figure contained by the diameter and the circumference cut off by it. And the center of the semicircle is the same as that of the circle.

Rectilinear Angles: Rectilinear figures are those which are contained by straight lines

Trilateral Figures: Trilateral figures are those contained by three straight lines

Quadrilateral Figure: Quadrilateral figures are those contained by four straight lines

Multilateral Figure: Multilateral figures are those contained by more than four straight lines

Equilateral Triangle: An equilateral triangle is that which has its three sides equal

Isosceles Triangle: An isosceles triangle is that which has two of its sides alone equal

Scalene Triangle: A scalene triangle is that which has its three sides unequal

Right-Angled Triangle: A right-angled triangle is that which has a right angle

Obtuse-Angled Triangle: An obtuse-angled triangle is that which has an obtuse angle

Acute-Angled Triangle: An acute-angled triangle is that which has three acute angles

Square: A square is that which is both equilateral and right-angled

Oblong: An oblong is that which is right-angled but not equilateral

Rhombus: A rhombus is that which is equilateral but not right-angled

Rhomboid: A rhomboid is that which has its opposite sides and angles equal to one another but is neither equilateral nor right-angled

Trapezia: All other quadrilaterals

Parallel: Parallel straight lines are straight lines which, being in the same plane and being produced indefinitely in both directions, do not meet one another in either direction

3.2 - Postulates and Common Notions

Postulates

Let the following be postulated:

1. To draw a straight line from any point to any point.

2. To produce a finite straight line continuously in a straight line.

3. To describe a circle with any center and distance.

4. That all right angles are equal to one another.

5. That, if a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.

Common Notions

1. Things which are equal to the same thing are also equal to one another.

2. If equals be added to equals, the wholes are equal.

3. If equals be subtracted from equals, the remainders are equal.

4. Things which coincide with one another are equal to one another.

5. The whole is greater than the part.

3.2 Discussion

1. Why did Euclid have to state these things?

In order to be able to use these as truth, Euclid had to state them. It was not acceptable to assume that everyone knew them or agreed they were true.

2. Is there a reason for only having 5 for each?

The number 5 is not specific, but it is important to note that there were very few of them. Euclid did his best to bring us back to the ground level and build geometry from there.

3. Do you think there should there be more?

While it seems convenient to have more, it would take away from the beauty of geometrically being able to construct so much when given so little.

3.3 - Propositions 1-3

Book 1 - Proposition 1
On a given finite straight line to construct an equilateral triangle.
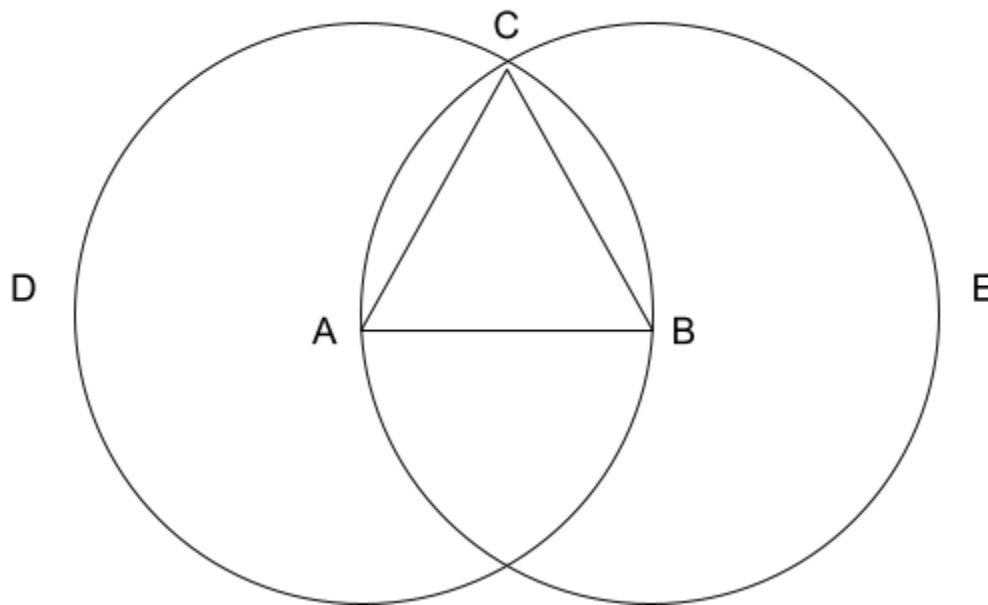
Discussion

1. "and from the point C in which the circles cut one another": How do we know that the circles will intersect?

We know that the circles will intersect because they have A and B as their centers respectively. By Postulate 3 we are allowed to describe a circle with any center and distance, thus we use that and see the following. The circle ACE has B as a center and a radius AB, thus A will be on its circumference just as the circle BCD has A as a center and a radius AB, thus B will be on its circumference. Since A is a part of BCD and B is a part of ACE, the circles will intersect at two points, one of them being C.

2. How to we know that $\overline{AC} = \overline{BC} \Rightarrow \overline{AB} = \overline{AC} = \overline{BC}$?

All of these lines are radius of circles that are congruent. And by Common Notion1, we know that things which are equal to the same thing are also equal to one another.

Book 1 - Proposition 2

To place at a given point [as an extremity] a straight line equal to a given straight line.
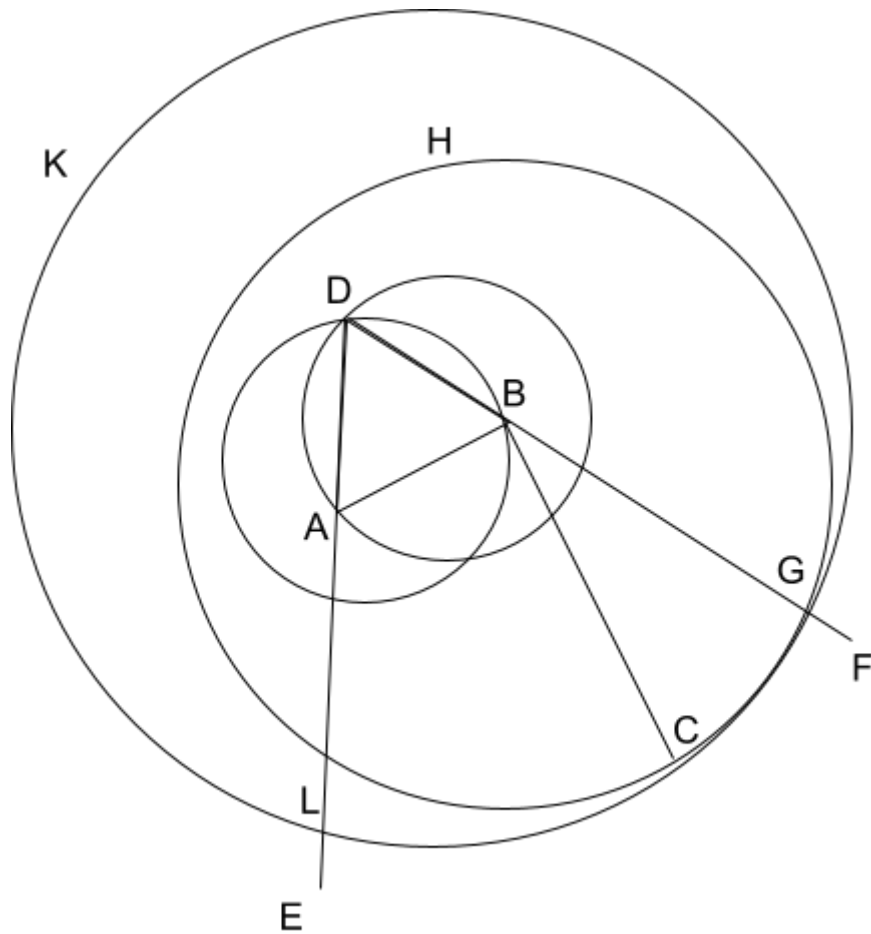
Discussion

1. Write out the order of equalities that were established throughout this proposition.

BC ≅ BG
DL ≅ DG
DA ≅ DB
AL ≅ BG
Al ≅ BC

Book 1 - Proposition 3

Given two unequal straight lines, to cut off from the greater a straight line equal to the less

C
_____

D

A   E   B
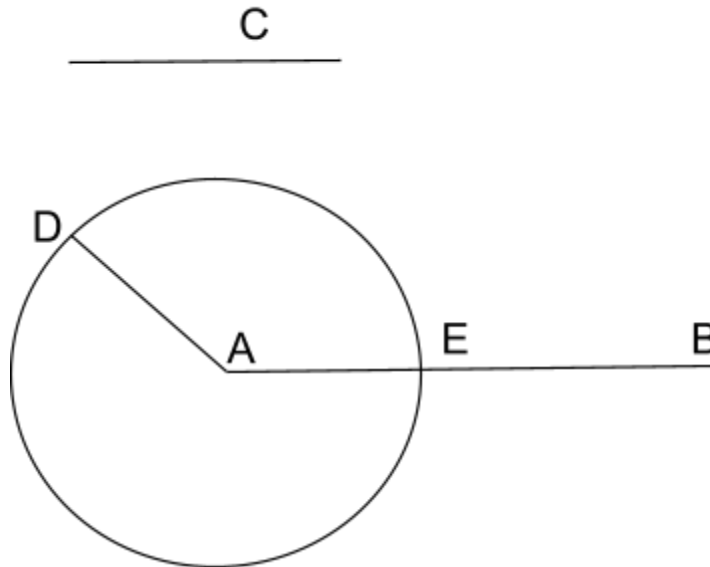
Discussion

1. Describe the syllogism used in this proposition.
   Premise 1: C = AD
   Premise 2: AD = AE
   Conclusion: Therefore AE = C

2. Postulate 3 said that we could describe a circle with any center and distance. So why couldn't we just use length C as the given distance and one end of the line AB? Why did we need Prop. 2? Is it because in Postulate 3 we have the distance given, and in Prop. 3 we just have a line? [11]

> Yes, postulate 3 lets us describe a circle of any given size, but we needed to use common notion 1 to show their equality. We needed to use proposition 2 in order to be able to draw the line *AD* which then leads us to using that as the radius for circle *F,* which then lets us cut *AB* off at *E* and we can use the fact that *AD* and *AE* are both radius to show their equality.

3.4 - Proposition 4-6, 8

Book 1 - Proposition 4

If two triangles have the two sides equal to two sides respectively, and have the angles contained by the equal straight lines equal, they will also have the base equal to the base, the triangle will be equal to the triangle, and the remaining angles will be equal to the remaining angles respectively, namely those which the equal slides subtended.
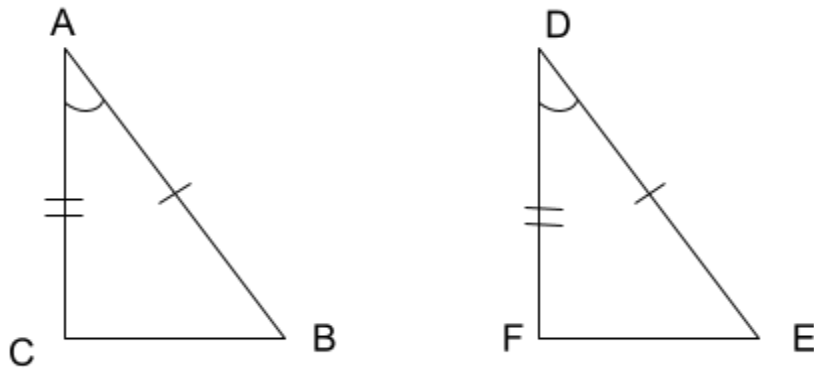
Discussion

1. We consider this a "congruence proof". Define congruent.
   Congruent: exactly the same in size and shape

2. How does Proposition 4 feel different than the first three?
   The first three propositions are constructing basic geometric figures while this proposition asks us to prove congruence between two triangles.

3. Can you think of any method to prove that the areas, the remaining angles, and the base are equal?

**We have yet to define a way to find the area of a triangle, so in order to show that the areas are congruent, we would need to cut out one shape and place it on top of the other, which is unrealistic and not precise.

We also do not have access to a protractor, but maybe we could use our compass and measure the distance between the sides.

We could also use our compass, set to a predetermined (random) length and create three similar circles that have each vertex of the triangle as their center and then compare the shape on the first triangle to that on the second triangle. In doing so, if the triangles are not congruent, we will see the three circles intersect at different points on each triangle.

*There is not necessarily a right or wrong answer here, this question is meant to start discussion, not find an answer.

4. In this proof, it seems that Euclid is saying it is okay for us to pick up items and move them to show congruence. Why have we not been able to do this with any other propositions? What are the pros and cons of using this method? Does it take away from the legitimacy of the argument?

Open to individual student responses.

Possible thoughts:

Pros	Cons
It is convenient	There is no firm argument to make that involves the components: side lengths or angle measures
Less writing, less defending	Argument/proof/defense could be torn apart more easily
	How do we know that the shape kept its original size and shape when it was moved?
	It makes the previous proposition work less meaningful

Book 1 - Proposition 5
In isosceles triangles the angles at the base are equal to one another, and, if the equal straight lines be produced further, the angles under the base will be equal to one another.
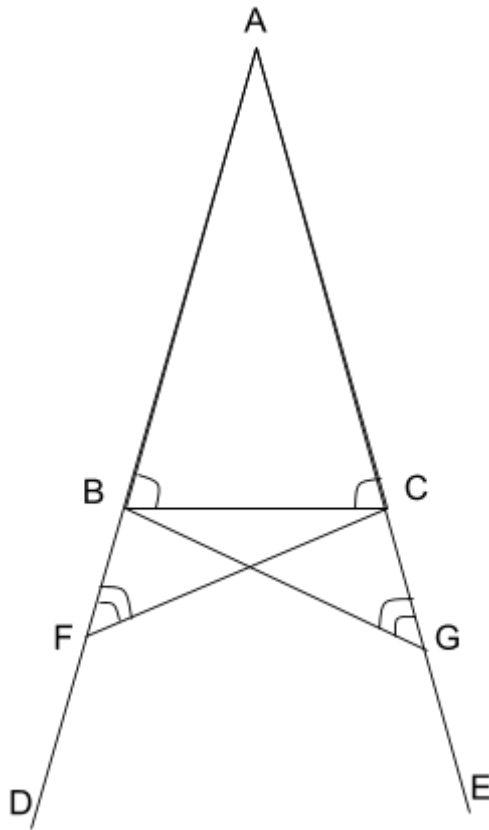
Discussion
1. Restate what this proposition has proven in your own words.
   This proposition is the Base Angles Theorem.
   When the base angles of two triangles are congruent, then triangles are congruent.

2. Is there a more simple way to complete this proof?
   Open to individual student responses.

3. How many new triangles did Euclid create? Draw all the congruent triangles in their own space?
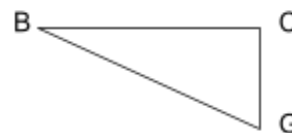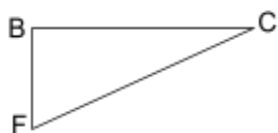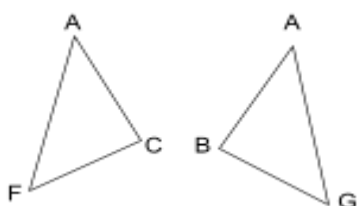
Book 1 - Proposition 6
If in a triangle two angles be equal to one another, the sides which subtend the equal angles will also be equal to one another.
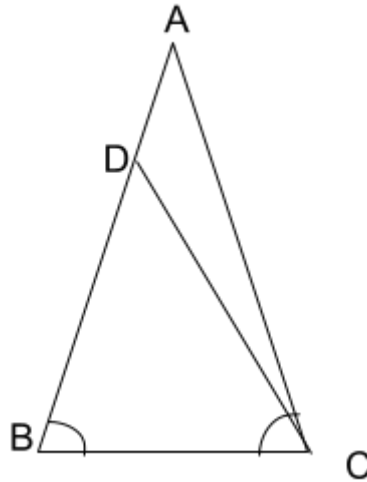
Discussion
1. This proposition claims that triangle *BDC* is less than triangle *BAC* but never explicitly proves it, how do we know it is an accurate claim to make?

   The side DC is smaller than the side AC and the side BD is smaller than the side BA while side BC is shared, thus the triangle BDC is going to be smaller than triangle BAC.

   *We know that BD is shorter than BA because D is cut off before A

*Proof by Contradiction: this proposition introduces a new method called *reductio ad absurdum*. If we can't find a way to prove something directly, we assume the contrary and see whether that leads to a contradiction.

2. Could this be proven directly?

   This proposition would be hard to prove directly without a protractor (Euclid did not have one) but it is possible.

3. Is the contradiction proof more persuasive than a direct proof?

It can be. A contradiction gets you to believe one way and then proves the absurdity. It can make the actual proof seem stronger when it is the counterexample of the contradiction.

4. Is it possible to find fallacy when you assume a contradiction? What should we be cautious of?

Yes!

It can be that the contradiction turns out to be true and we have to go back and find our misstep.

It can be that we have missed steps in hopes of reaching a certain conclusion.

5. What Common Notion does this heavily rely on and why?

This proposition relies on Common Notion 5, "the whole is greater than the part". This comes into play with the side lengths and the triangles.

Book 1 - Proposition 8

If two triangles have the two sides equal to two sides respectively, and have also the base equal to the base, they will also have the angles equal which are contained by the equal straight lines.
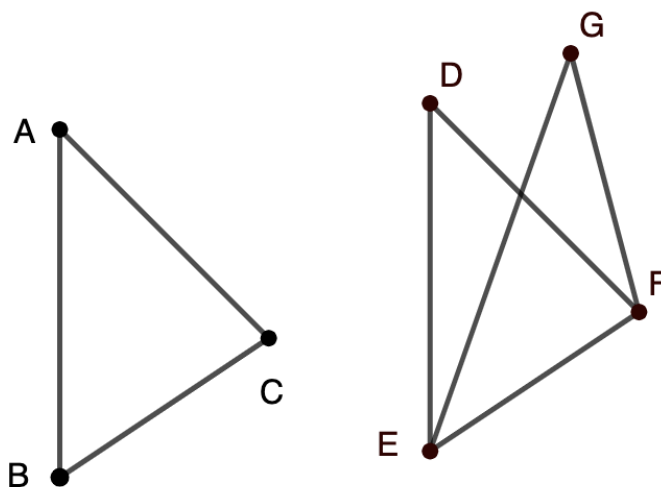
Discussion

1. This proposition does not end with an explicit statement that the two triangles are equal. What previous proposition could we use along with this in order to show the two triangles as equal.

In order to show the triangles are equal, you would need to apply 1.4 to both the sides and the angles in 1.8.

2. Are you keeping track of the congruence proofs? Congruent triangles are the ones that have all corresponding sides and angles equal (from which it follows that they are equal in area). Congruent triangles will be essential building blocks for future proofs, but as we continue to develop the structure, it might start to seem that *any* equal sides and angles will make triangles congruent. What kinds of congruence proofs do we have so far?

SAS - Side Angle Side - 1.4
SSS - Side Side Side - 1.8
Base Angles Theorem - 1.5

3.7 - Propositions 9-12

Book 1 - Proposition 9
To bisect a given rectilineal angle.
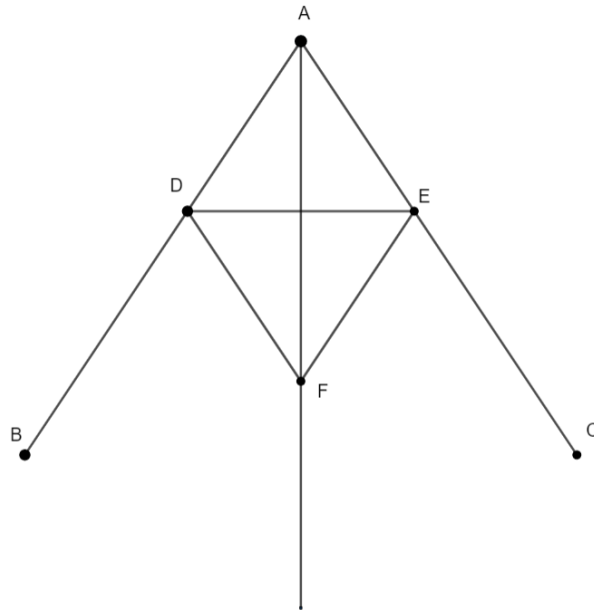
Discussion

1. What does bisect mean?

Bisect means to cut in half.
Bi - Latin: *bi* - twice, double
Sect - Latin: *secare* - to cut

2. Why was it important for Euclid to be able to bisect an angle? Does this even matter if we don't have the tools to measure the angle in degrees?

Being able to cut something in half accurately adds to our toolbox of skills that we can now use. This is one that will be used for many propositions in the future!

3. Since this seems like a basic proposition, why was this not one of our first propositions?

It does seem like it could have been proven earlier, but in order to prove this, we need 1.3 and 1.8.

Book 1 - Proposition 10
To bisect a given finite straight line.
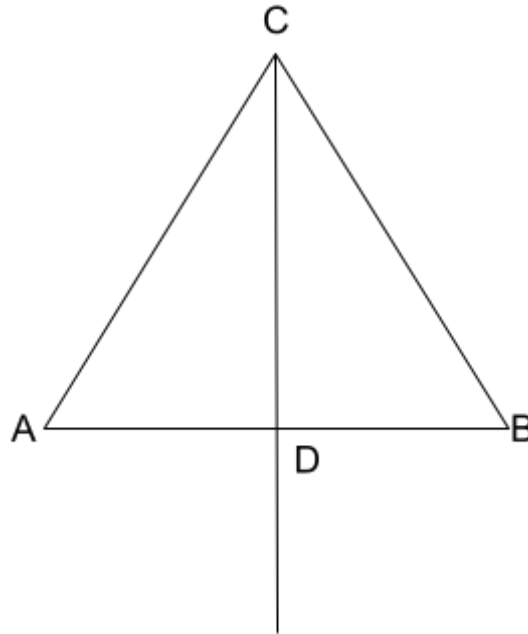
Discussion
1. How did we define a line?
    Line: a breadthless length

2. Of what is a "breadthless length" made up?
   What is a line actually made up of? If it is infinitely many points, then could we possibly split infinity? If we have an odd number, can we split the middle of something that we don't fully know what it is?

3. Is there always going to be an even way to divide this in half? What if there isn't?
   Having the ability to bisect a line gives us the chance to "measure" things without having a labeled ruler or measuring device. Parts of Geometry are based on the precision of cutting shapes to form them into what they need to be, thus being able to accurately find the halfway point of a line is very beneficial.

Book 1 - Proposition 11

To draw a straight line at right angles to a given straight line from a given point on it.
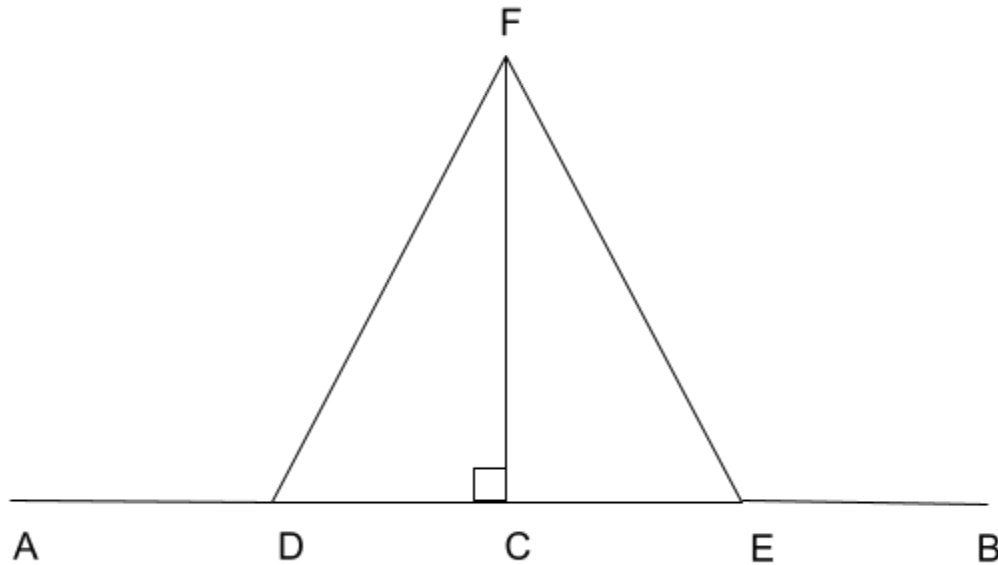
Discussion

1. This follows from proposition 10 where we almost did this exact construction with point D. Why do you think Euclid separated the two ideas?

Proposition 10 shows us how to bisect a line where proposition 11 is how to make two lines perpendicular. While the work required in these two propositions is incredibly similar, the results are different and perhaps Euclid wanted to make a point in proving two different thoughts that both deserve their own attention.

Book 1 - Proposition 12

To a given infinite straight line, from a given point which is not on it, to draw a perpendicular straight line.
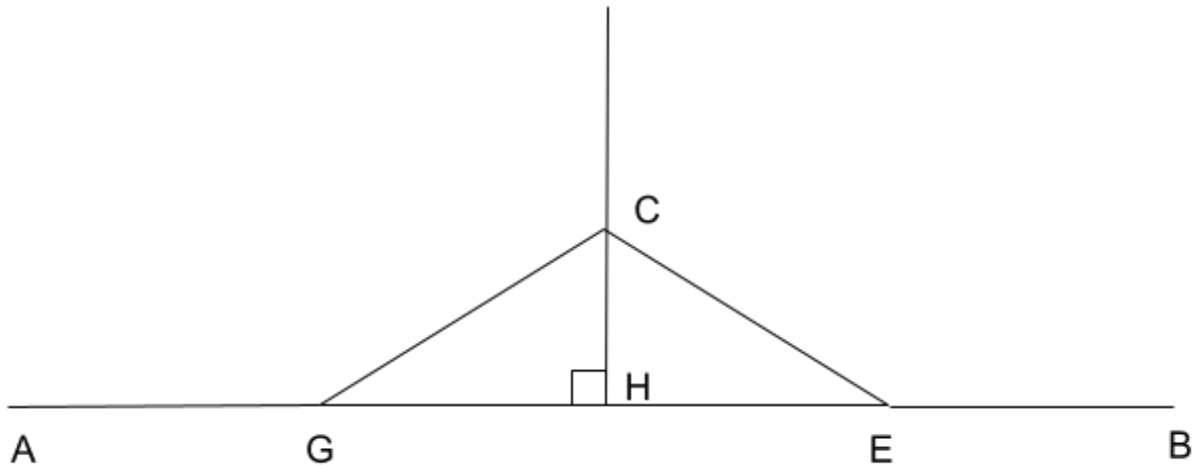
Discussion

1. How would you define infinite?

Latin *infinitus* "unbounded, unlimited, countless, numberless" [10]

2. Why is it important for the line to be infinite in this proof?

This line must be infinite because there were no restrictions on the placement of C.

3.9 - Propositions 13-15

Book 1 - Proposition 13

If a straight line set up on a straight line makes angles, it will make either two right angles or angles equal to two right angles.
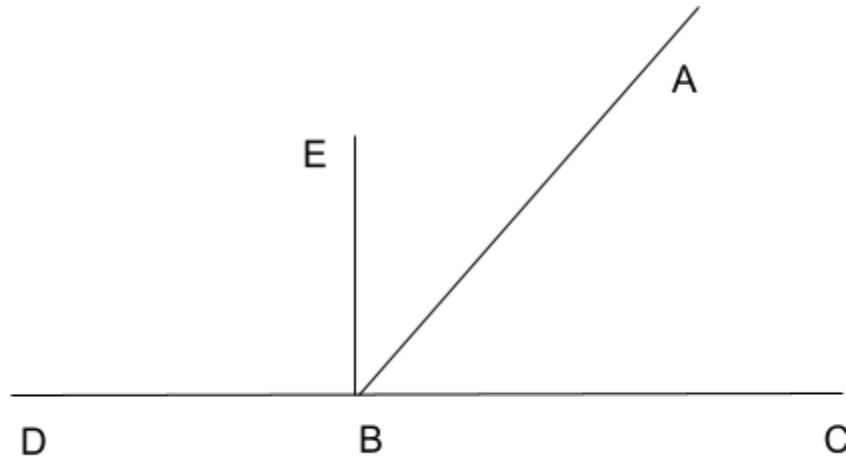
Discussion

1. Write out using geometry symbols what this proposition is showing.

   Either $\angle ABC = 90$ and $\angle ABD = 90$ or $\angle ABC + \angle ABD = 180$.

2. Why is Euclid using the phrase "two right angles"? What term might we use instead?

   Euclid uses this term because he does not actually use any numbers in his geometry. All of his calculations are done with the figures that we are given or can draw. His use of "two right angles" is referencing a 180 degree angle.

Book 1 - Proposition 14
If with any straight line, and at a point on it, two straight lines not lying on the same side make the adjacent angles equal to two right angles, the two straight lines will be in a straight line with one another.
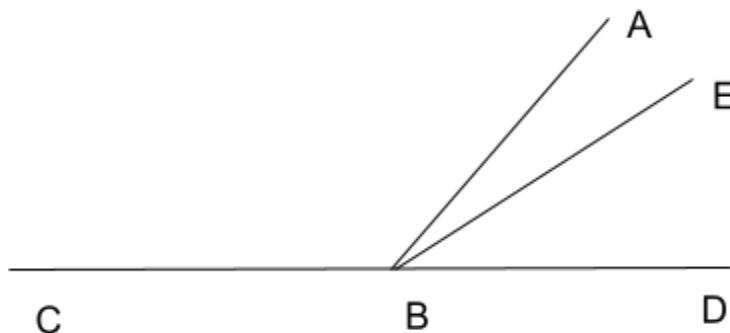
Discussion
1. Restate what this proposition has proven in your own words.
   Open to individual student responses.
   $\angle ABC + \angle ABD = 180$

Book 1 - Proposition 15

If two straight lines cut one another, they make the vertical angles equal to one another.
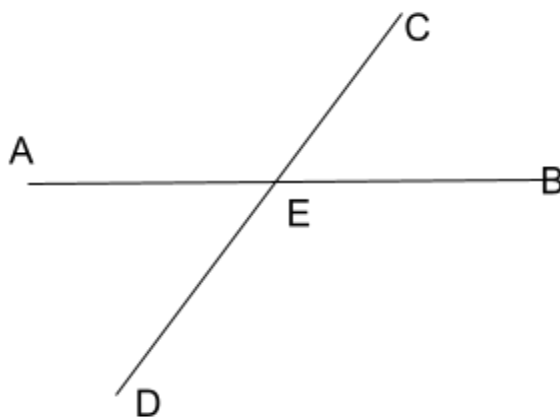
Discussion

1. Write out the thought process that this proposition followed using Geometric symbols.

∠CEA + ∠AED = 180

∠AED + ∠DEB = 180

∠CEA + ∠AED = ∠AED + ∠DEB

Therefore ∠CEA = ∠DEB

3.10 - Propositions 16-20, 22

Book 1 - Proposition 16

In any triangle, if one of the sides be produced, the exterior angle is greater than either of the interior and opposite angles.
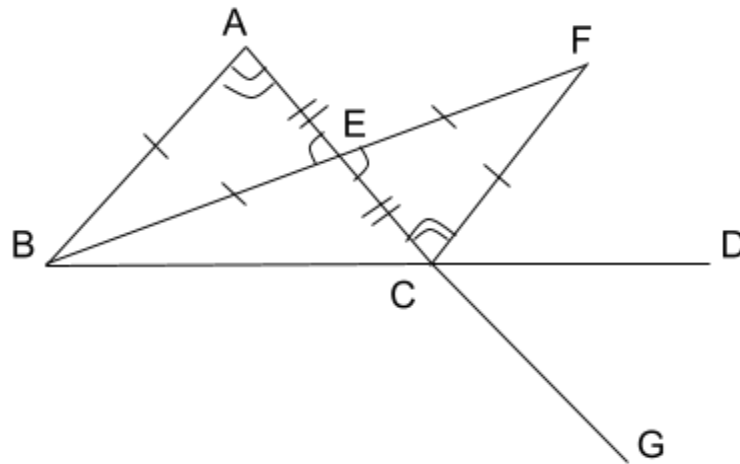
Discussion

1. Is there a more succinct way Euclid could have proven this?

Open to individual student responses.

Euclid has been succinct so far, so based on what we have been given, I am sure he tried to prove this as efficiently as he could.

2. Restate what this proposition has proven in your own words.

The exterior angle of a triangle is greater than the sum of the two opposite interior angles.

Book 1 - Proposition 17
In any triangle two angles taken together in any manner are less than two right angles.
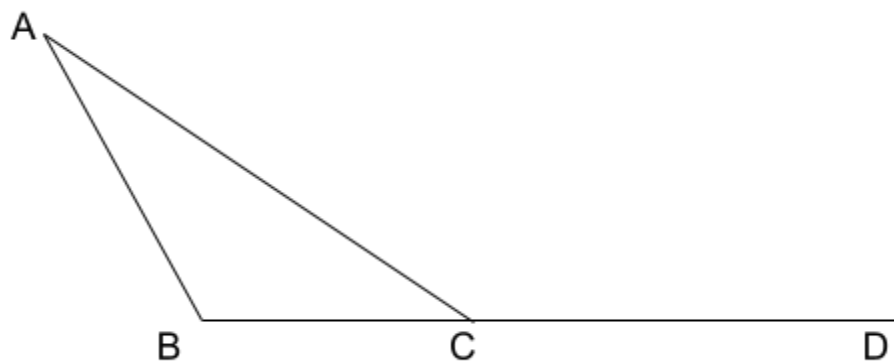
Book 1 - Proposition 18
In any triangle the greater side subtends the greater angle.
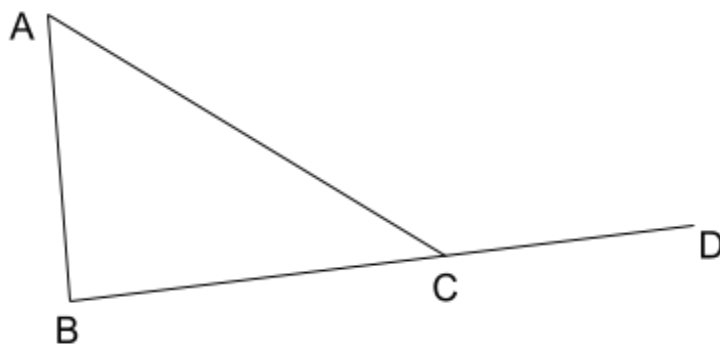
Book 1 - Proposition 19
In any triangle the greater angle is subtended by the greater side.
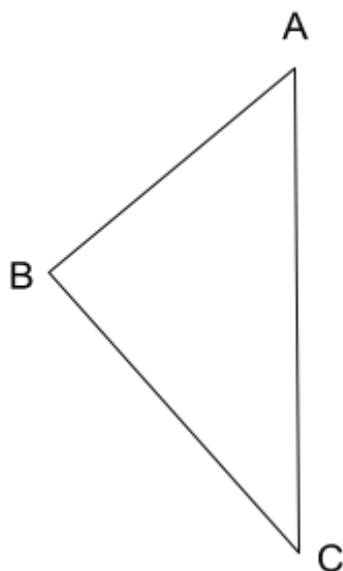
Discussion

1. Define converse.

1550s, originally in mathematics as "thing or action that is the exact opposite of another." [10]

Geometrically we write:

Proposition: P→Q, "If P, then Q"

Converse: Q→P, "If Q, then P"

2. This is a converse of the previous proposition. If we already know that a proposition is true, do we need to prove it's converse?

Yes, the converse needs to be proven. In math, we do not trust material that has not been proven true.

*Remember Euclid only started with 5 common notions and 5 postulates that he was assuming were true and he built his geometry empire upon those. Thus now we are able to take his assumptions along with definitions and previously proven propositions to accomplish so much!

3. Could the proposition be true and the converse be false?

The converse of a proposition does not have to be true. Even if it is true, it may be very difficult to prove.

Book 1 - Proposition 20

In any triangle two sides taken together in any manner are greater than the remaining one.
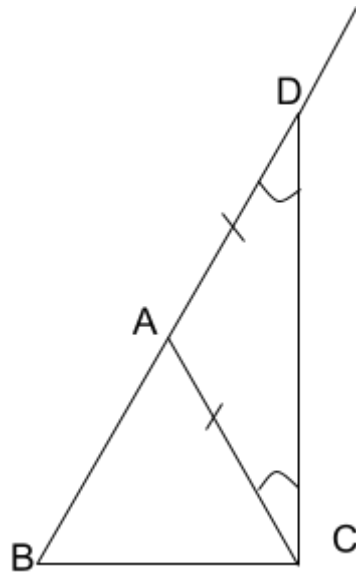
Discussion

1. Write out the inequalities that Euclid walks us through in this proposition.

AB + AC > BC

AB + BC > AC

BC + CA > AB

2. Proculus, a 5th Century commentator writing in Alexandria, says that the Epicureans liked to ridicule this proposition by saying that any donkey knows its truth, because no donkey will approach its food along two sides of a triangle rather than in a straight line. Indeed, it seems intuitively obvious just looking at or imagining a triangle. But is that a proof? Proculus says that perceiving the truth of something is not enough: one must know *why* it is true for it to be knowledge. Do you agree? [11]

Open to individual student responses.

Book 1 - Proposition 22
Out of three straight lines, which are equal to three given straight lines, to construct a triangle: thus it is necessary that two of the straight lines taken together in any manner should be greater than the remaining one.

Discussion
1. Why must any two of the straight lines be greater than the third? Of course, if that were not true we could show from Prop. 20 that we couldn't construct a triangle. But is there more? How do we know what the two circles will intersect such that the triangle we need will result?

2. Each of the propositions we have investigated focuses on different aspects of geometry (lines, points, triangles, etc.). Make a chart to categorize the propositions that we have worked through based on their geometric focus.

3.13 - Propositions 23

Book 1 - Proposition 23

On a given straight line and at a point on it to construct a rectilineal angle equal to a given rectilineal angle.
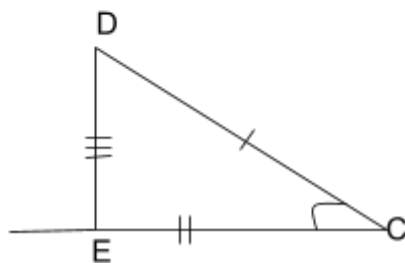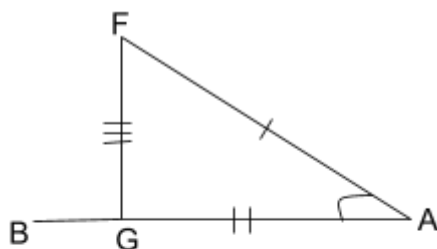
Discussion

1. Since in 1.4 we were allowed to pick up the triangle and apply it, why can't we do that here? (We noticed that this possible shortcut could have applied to earlier propositions as well.) Are we seeing a reluctance to use this technique? Do we share Euclid's discomfort? [11]

Open to individual student responses.

3.14 - Propositions 24-26

Book 1 - Proposition 24

If two triangles have the two sides equal to two sides respectively, but have the one of the angles contained by the equal straight lines greater than the other, they will also have the base greater than the base.
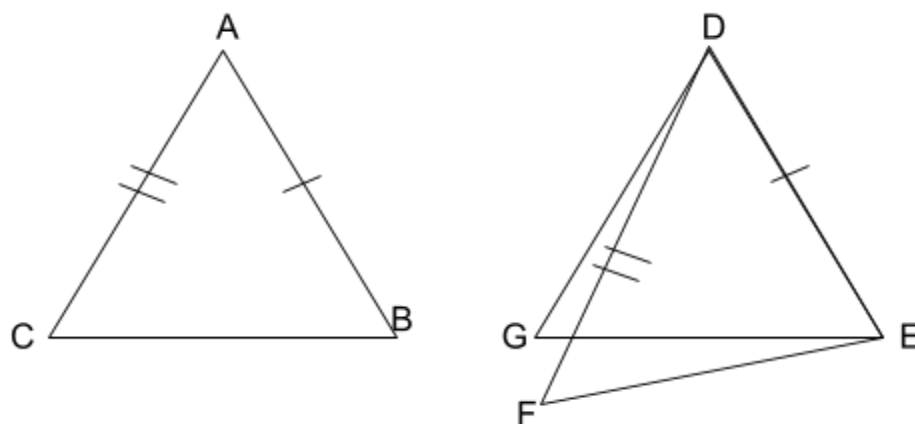
Book 1 - Proposition 25

If two triangles have the two sides equal to two sides respectively, but have the base greater than the bse, they will also have the one of the angles continued by the equal straight lines greater than the other.

Discussion

1. Write out the thought process that this proposition followed using Geometric symbols.

If AB ≅ DE and AC ≅ DF but ∠A > ∠D, then BC > EF

Book 1 - Proposition 26

If two triangles have the two angles equal to two angles respectively, and one side equal to one side, namely, either the side adjoining the equal angles, or that subtending one of the equal angles, they will also have the remaining sides equal to the remaining sides and the remaining angle equal to the remaining angle.
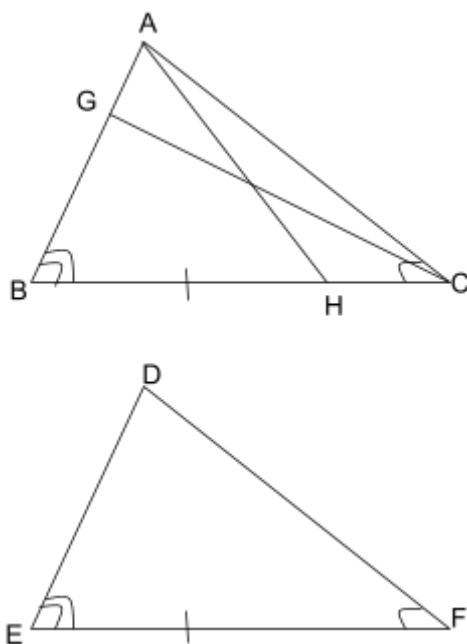
Discussion

1. List out the triangle congruence proofs we have covered.

    1.5 - Base Angles Theorem

    1.6 - Converse of the Base Angles Theorem

    1.8 - Side Side Side

    1.26 - Angle Side Angle

3.16 - Propositions 27-31

Book 1 - Proposition 27

If a straight line falling on two straight lines make the alternate angles equal to one another, the straight lines will be parallel to one another.
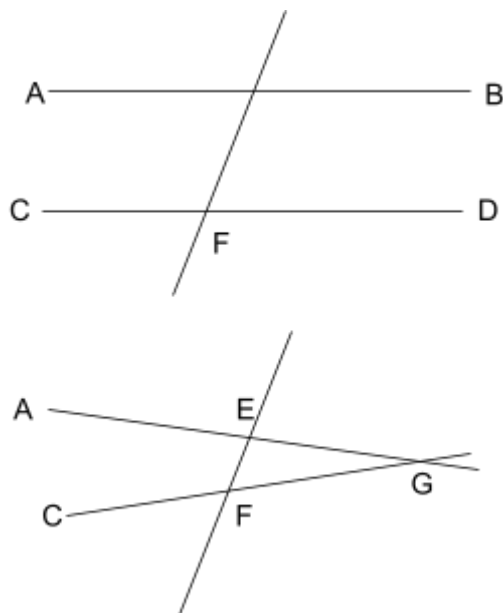
*This is called "parallel lines cut by a transversal".

Discussion

1. We have now started discussing parallel lines. Did you expect to see a triangle in this proof?

Open to individual student responses.

2. Euclid's 5th Postulate, "the parallel postulate" has been considered controversial. Do you think it is an assumption that should be freely accepted?

Postulate 5: That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.

Open to individual student responses.

3. Can we picture that strange hypothetical triangle well enough to operate on it with a proof? How careful do we have to be with a drawing representing the impossible if we are to make sure that we are not misled? Or does it have nothing to do with the drawing and we just have to make sure the logical steps in the words are airtight? [11]

Open to student discussion.

Are any of these proofs reliant on the drawing or are they all based on the logic backing it up?

Talk about the difference between QEF (that which was to be done) and QED (that which was demonstrated).

Book 1 - Proposition 28

If a straight line falling on two straight lines make the exterior angle equal to the interior and opposite angle on the same side, or the interior angles on the same side equal to two right angles, the straight lines will be parallel to one another.
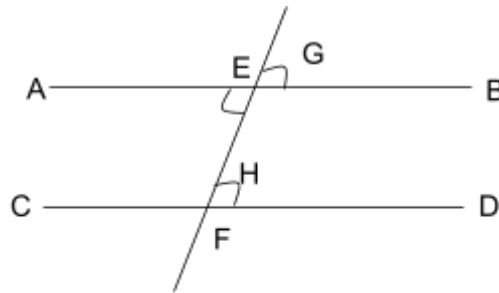
Discussion

1. You may have noticed by now that we are doing and creating math, but Euclid has yet to use any numbers. What are your thoughts on this? Is it still math? Do we ever need to use numbers?

Euclid is showing us a beautiful representation of math that can be proven based on a short list of assumptions. While he did not use specific numbers, he was able to have students all draw the same figures and see the same relationships between lines, angles, and geometric figures. What he has done is incredible!

2. What would you say is the importance of parallel lines a) in math and b) in the world?
   Open to individual student responses, but some answers may include:

   a) Show us the relation between lines, determining slopes on a graph, classifying figures such as parallelograms and trapezoids.

   b) Parallel roads, walls in houses that will stay up and hold up other floors or the ceiling, railroad tracks, bridges.

Book 1 - Proposition 29
A straight line falling on parallel straight lines make the alternate angles equal to one another, the exterior angle to the interior and opposite angle, and the interior angles on the same side equal to two right angles.
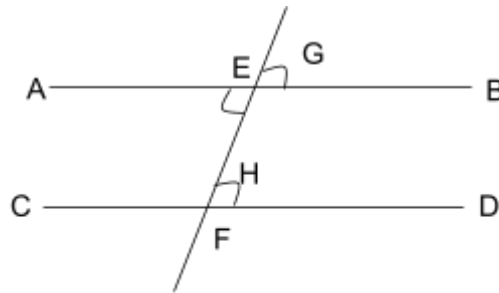
Discussion

1. This proposition is proving a converse. Why do we need to prove converses?

There are times when we don't want the original proof, but we want to use the converse based on the situation we are given and it is easier to have it already proven than to prove it again each time we want to use it.

2. When Postulate 5 was first presented, it may have seemed wordy and clumsy. Are you starting to appreciate its power now? [11]

Open to individual student responses.

*Many have tried to improve on Postulate 5, or to actually prove it, but it seems that their proofs all in some way covertly rely on the postulate. You might be tempted to start trying out some proofs of your own for this postulate now that you have the tools of the first twenty-nine propositions. It's tricky terrain on which even the great have stumbled, but the effort can be entertaining and instructive even if, or especially if, it ends in failure. [11]

Book 1 - Proposition 30

Straight lines parallel to the same straight line are also parallel to one another.

Discussion

1. This feels a lot like a syllogism. Define syllogism and determine whether this qualifies as one.

Syllogism: A form of deductive reasoning consisting of a major premise, a minor premise, and a conclusion.

Major premise: $\angle AGK = \angle GHF$
Minor premise: $\angle GHF = \angle GKD$
Conclusion: $\angle AGK = GKD$

Book 1 - Proposition 31

Through a given point to draw a straight line parallel to a given straight line.
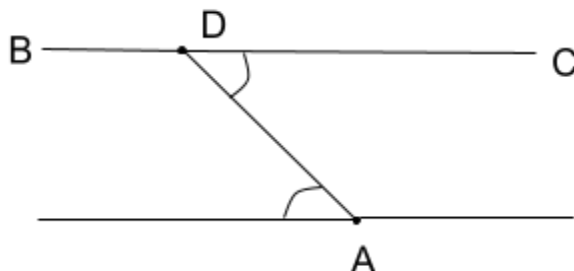
Discussion

1. Restate this proposition in your own words.

Through any point one one line exists that is parallel to a given line.

3.17 - Propositions 32

Book 1 - Proposition 32

In any triangle, if one of the sides be produced, the exterior angle is equal to the two interior and opposite angles, and the three interior angles of the triangle are equal to two right angles.
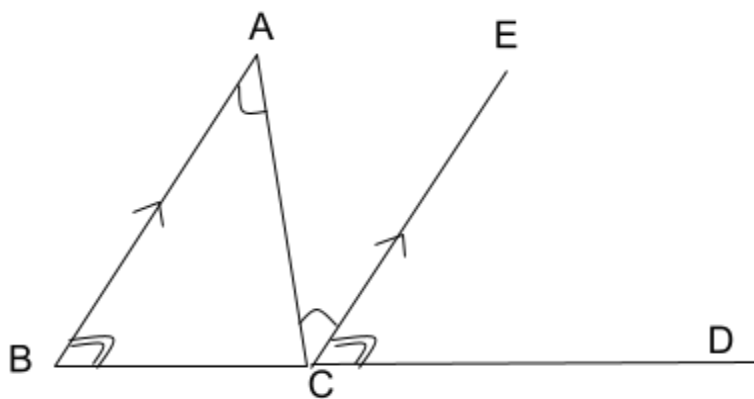
3.18 - Propositions 33-34, 46

Book 1 - Proposition 33

The straight lines joining equal and parallel straight lines [at the extremities which are] in the same directions [respectively] are themselves also equal and parallel.
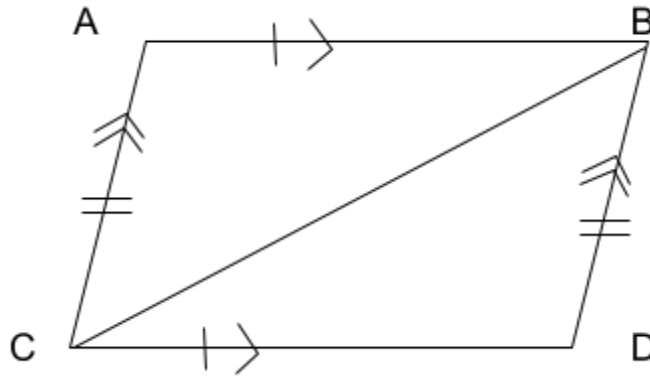
Discussion

1. Define parallelogram

Parallelogram: "quadrilateral whose opposite sides are parallel," 1560s, from French *parallélogramme* (1550s) and directly from Latin *parallelogrammum*, from Greek *parallelogrammon* noun use of a neuter adjective meaning "bounded by parallel lines," [10]

2. Does this make you appreciate the math, logic, and work that Euclid has put into the theorems prior to this one?

Rhetorical question - open to individual student responses.

Book 1 - Proposition 34
In parallelogramic areas the opposite sides and angles are equal to one another, and the diameter bisects the areas.
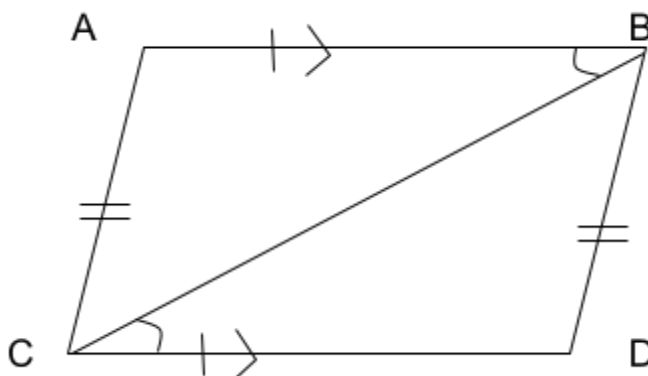
Discussion

1. "Diameter" is a term that we have seen used for circles, but here Euclid uses it for a quadrilateral. How would you define diameter so that it works for this figure as well?

Diameter: dia - across

metra - measure

The measurement across a shape.

2. Euclid talks about a "parallelogramic area" in this proposition; it is the only place he uses the explicit term area. The word area does not appear any other place (except in words quoting this proposition). Area is not defined. (Def. 5 is of a surface: "A surface has length and breadth only.") But here in the second part of Prop. 34, where the diameter is said to bisect the area, it seems to speak of the size of a figure, or about a relationship between areas of figures. So what is area? Is it about size and comparison of size? Or is it more about occupying space? [11]

Open to individual student responses.

Area is the two-dimensional measure of a figure.

3. You may have noticed that Eeclid has yet to define or prove anything using numbers (except the vague use of "the measurement is that of two right angles"). Without using numbers, is the measurement of area useful?

The definition of area is useful because we are laying out a foundation and can then apply numbers to problems once we know from where the rules, ideas, math comes from.

Book 1 - Proposition 46
On a given straight line to describe a square.
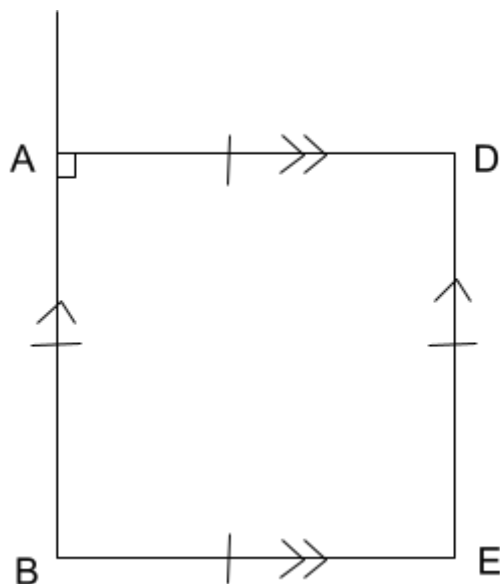
Discussion

1. Is it frustrating that we have to prove everything we do?

   In math, unless we can prove it, we cannot use it. And each proof can be destroyed by a simple counterexample.

2. Is this a reminder that an act of thought is only complete when we *know* that what we did is what we intended? We need to know *what* we've built. Is Euclid saying, "You have not done what I asked until you have apprehended what you did"? [11]

Is any of the math we are doing "real" or is it all just concepts?

Unless math can be applied to a real world concept, does it hold any value?

3.19 - Propositions 47-48

Book 1 - Proposition 47

In right-angled triangles the square on the side subtending the right angle is equal to the squares on the sides containing the right angle.
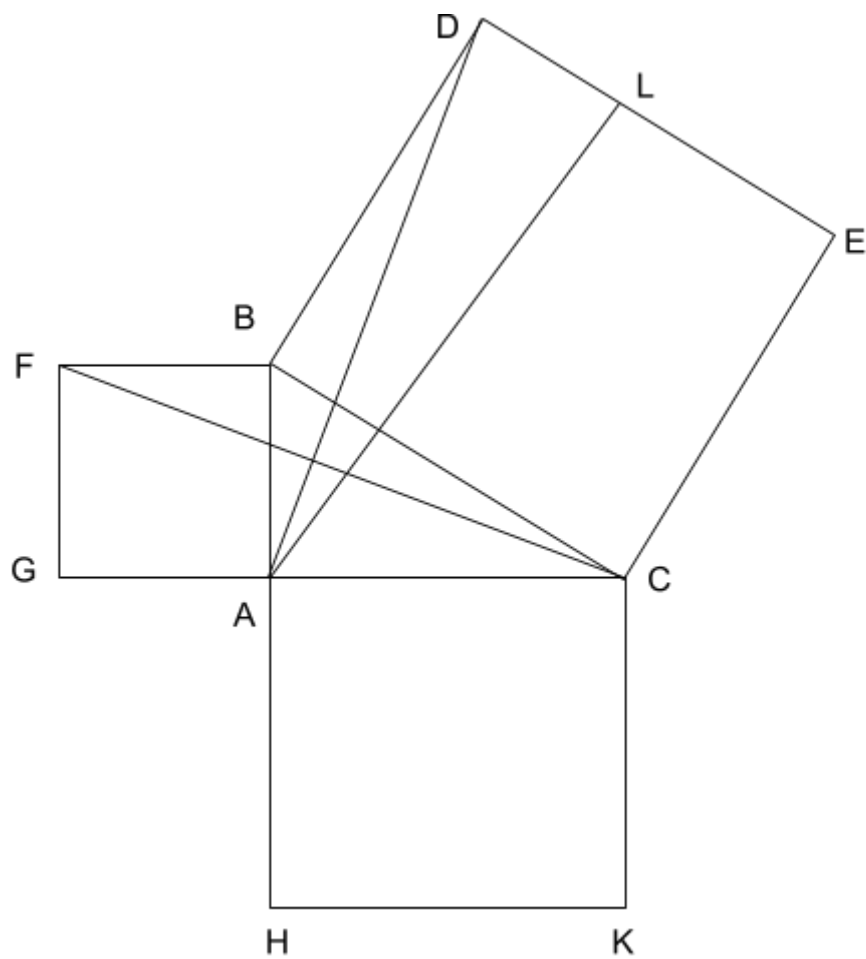
Discussion

1. Work your way through this proposition again and write out your thought process.

Possible response:

$\angle DCB = \angle FBA$

$\angle DCB + \angle ABC = \angle FBA + \angle ABC$

$\angle DBA = \angle FBC$

$AD = FC$

$\triangle ABD = \triangle CBF$

$\square BL = 2\triangle ABD$

$\square GB = 2\triangle CBF$

$\square BL = \square GB$

2. This very famous proposition, attributed to Pythagoras, provides a very satisfying application of congruent triangles and the propositions exacting the relationships of triangles and quadrilaterals between parallel lines. Do you have a feeling of a culmination of the long careful construction that has been Book1? We have one more loose end to tie up here, but this magnificent proposition is a worthy finale to the foundation of the *Elements*.

Since book 1 is all about laying the foundations for what we are going to do, it is satisfying to wrap it up with a proposition and its converse that require us to use almost everything we have learned thus far.

Book 1 - Proposition 48
If in a triangle the square on one of the sides be equal to the squares on the remaining two sides of the triangle, the angle contained by the remaining two sides of the triangle is right.
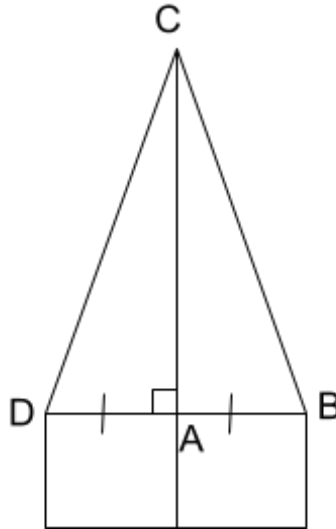
Discussion
1. Did the converse of 1.47 need to be proved? Do some converses need to be proved but not others? Or is it just that some converses are false and others true, so without proving any given converse you don't know for sure? [11]

The converse of a proposition is not always true, thus it must be proven in order to be accepted and in order to be used.

4: Book 2

4.1 - Definitions

Rectangular Parallelogram: contained a rectangular parallelogram is said to be contained by the two straight lines containing the right angle

Gnomon: in any parallelogrammic area let any one whatever of the parallelograms about its diameter with the two complements be called a gnomon

Gnomon

Book 2 Format

As we saw in Book 1, the focus was mainly on laying the geometry foundations so that we would have the freedom to use basic skills without needing to prove them each time. Now in Book 2, we are going to focus on the specific steps and justification, the bones of proof writing, to show the logic of each proposition. Thus instead of specific discussion questions at the end of each proposition, we will use a Step/Justification t-chart while we work the proposition.

It will look like:

Step	Justification
Line from A to B	Postulate 2
Bisect AB at C	1.9

For each step that the proposition uses, it must be backed up by a justification from something Eulcid has, assumed, defined, or proven already.

4.2 - Propositions 1-3

Book 2 - Proposition 1

*If there be two straight lines, and one of them be cut into any number of segments whatever,
the rectangle contained by the two straight lines is equal to the two rectangles contained by
the uncut straight line and each of the segments.*

Step	Justification
Draw BF at a right angle to BC	1.11 - Draw a perpendicular line
Let BG = A	1.3 - Draw two lines the same length
Let GH be drawn parallel to BC	1.31 - Draw a line parallel to a given line
BH = BK + DL + EH	The whole is a sum of its parts
BH = A x BC BK = A x BD DL = A x DE EH = A x EC	1.34
Therefore A x BC = A x BD + A x DE + A x EC	
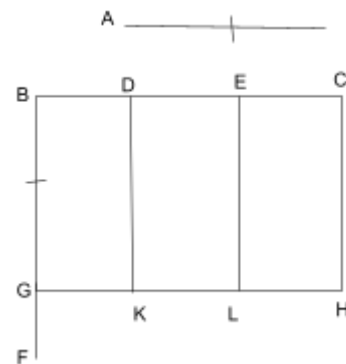
* Algebraically this proposition is the distributive property, notice
what is being proven:

a(b + c + d) = ab + ac + ad
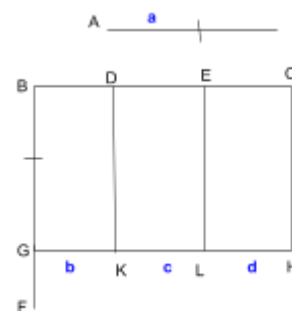
Book 2 - Proposition 2

If a straight line be cut at random, the rectangles contained by the whole and both of the segments are equal to the square on the whole.

Step	Justification
Let ▢ADEB be described on AB	1.46 - On a given straight line to construct a square
Let CF be drawn parallel to AD and BE	1.31 - Draw a line parallel to a given line
AE = AF + CE	The whole is a sum of its parts
BA = BE	Definition of square
Therefore BA x AC + AB x BC = ▢AB x AB	
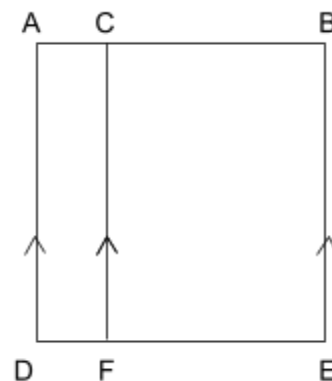
*Algebraically this proposition is the square of a binomial, notice what is being proven:

$(a + b)^2 = (a + b) \cdot b + (a + b) \cdot a$
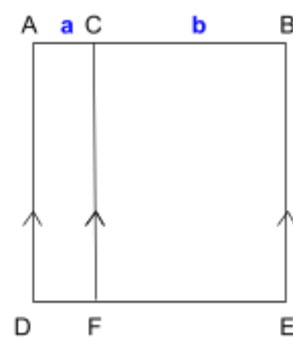
Book 2 - Proposition 3
If a straight line be cut at random, the rectangle contained by the whole and one of the segments is equal to the rectangle contained by the segments and the square in the aforesaid segment.

Step	Justification
Let ☐CDEB be described on CB	1.46 - On a given straight line to construct a square
Let ED be drawn through to point F	Postulate 1
Let AF be drawn parallel to CD and BE	1.31 - Draw a line parallel to a given line
AE = AD + CE	The whole is a sum of its parts
BE = BC	Definition of square
AE = AB x BC	The whole is a sum of its parts
DC = CB	Definition of square
AD = AC x CB	The whole is a sum of its parts
Therefore AB x BC = (AC + CB)xBC	
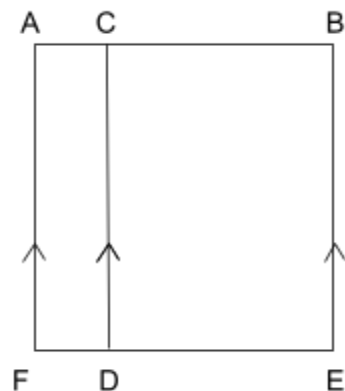
*Algebraically this proposition can be written as:
AB x BC = (AC + CB)xBC
$(a + b) \bullet b = (a + b) \bullet b$
$ab + b^2 = ab + b^2$
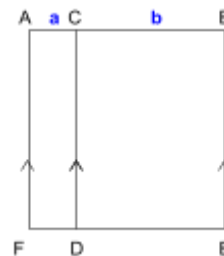
4.2 - Propositions 4-6

Book 2 - Proposition 4

If a straight line be cut at random, the square on the whole is equal to the squares on the segments and twice the rectangle contained by the segments.

| Step	Justification
Let ☐ADEB be described on AB	1.46 - On a given straight line to construct a square
Let BE be joined	Postulate 1
Let CF be drawn parallel to AD and EB	1.31 - Draw a line parallel to a given line
Let HK be drawn parallel to AB and DE	1.31 - Draw a line parallel to a given line
∠CGB = ∠ABD	1.29 - If you have two parallel lines cut by a transversal, the corresponding angles are congruent
∠ADB = ∠ABD since BA = AD	1.5 - Base angles theorem
∠CGB = ∠GBC and BC = CG	1.6 - Converse of the base angles theorem
Cb = GK and CG = KB therefore GK = KB and CGKB is a square	Definition of square
By the same argument HF is a square	Definition of square
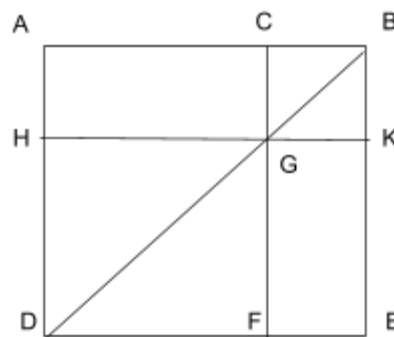
AG = GE and AG = AC x CB	
GC = CB	Definition of square
GE = AC x CB	
AC x GE = 2(AC + CB)	
HF + CK + AG + GE = ADEB	The whole is a sum of its parts
Therefore AB x AB = AC x AC + BC x BC + 2(AC + BC)	

*Algebraically this proposition can be written as:
AB x AB = AC x AC + BC x BC + 2(AC x BC)
$(a + b)^2 = a^2 + b^2 + 2ab$
$a^2 + 2ab + b^2 = a^2 + 2ab + b^2$

Book 2 - Proposition 5

If a straight line be cut into equal and unequal segments, the rectangle contained by the unequal segments of the whole together with the square on the straight line between the points of section is equal to the square on the half.

Before you begin this proposition - take note that 1.43, a proposition that we did not cover in book 1, says that complements always equal each other.

You will also need Book 1 Proposition 36: Parallelograms which are on equal bases and within the same parallels equal one another

Step	Justification
Let ▭ CEFB be described on CB	1.46 - On a given straight line to construct a square
Let BE be joined	Postulate 1
Let DG be drawn parallel to CE and BF	1.31 - Draw a line parallel to a given line
Let KM be drawn parallel to AB and EF	1.31 - Draw a line parallel to a given line
The complement CH = complement HF	1.43 - Definition of complement
CF + DM = HF + DM	Common Notion 2
CM = DF	
CM = AL since AC = CB, therefore AL = DF	1.36 - Parallelograms which are on equal bases and within the same parallels equal one another

AKHD = CH + DM + HF	The whole is a sum of its parts
AH = gnomon NOP	Definition of gnomon
AH + LG = gnomon NOP + LP	Common Notion 2
AD x DB + CD x CD = CB x CB	

*Algebraically this proposition is called Difference of Squares:

$(AD \times DB) + CD \times CD = CB^2$

$(a + b)(a - b) + b^2 = a^2 \Rightarrow (a + b)(a - b) = a^2 - b^2$
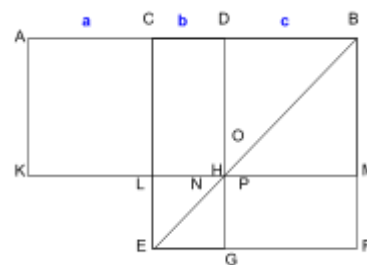
Book 2 - Proposition 6

If a straight line be bisected and a straight line be added to it in a straight line, the rectangle contained by the whole with the added straight line and the added straight line together with the square on the half is equal to the square on the straight line made up of the half and the added straight line.
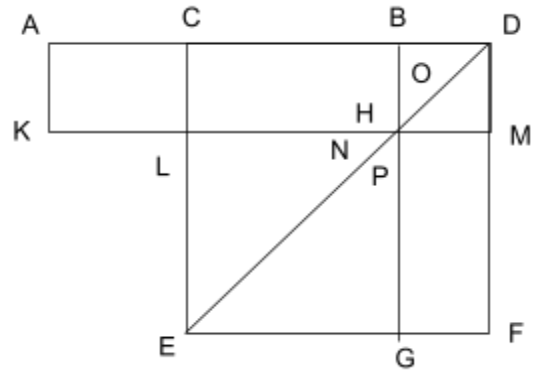
Step	Justification
AB is bisected at C	1.10 - to bisect a line
Add AB be drawn through to D	Postulate 1
Let ☐CEFD be described on CD	1.46 - On a given straight line to construct a square
Let DE be joined	Postulate 1
Let BG be drawn parallel to EF and DF	1.31 - Draw a line parallel to a given line
Let KM be drawn parallel to AB and EF	1.31 - Draw a line parallel to a given line
Let AK be drawn parallel to CL and DM	1.31 - Draw a line parallel to a given line

AC = CB, then AL = CH	1.36 - Parallelograms which are on equal bases and within the same parallels equal one another
CH = HF, thus AL = HF	1.43 - Definition of complement
AL + CM = HF + CM	Common Notion 2
AM = gnomon NOP	Definition of gnomon
DM = DB	
AM = AD x DB = gnomon NOP	Common Notion 1
LG = BC x BC, then AD x DB + CB x CB = gnomon NOP + LG	
But gnomon NOP + LG =☐AD x BD +☐BC ,therefore☐AD x BD +☐BC =☐CD	

*Algebraically this proposition is called square of a binomial:

☐AD x BD +☐BC =☐CD

$(2a + b)b + a^2 = (a + b)^2$

$a^2 + 2ab + b^2 = a^2 + 2ab + b^2$
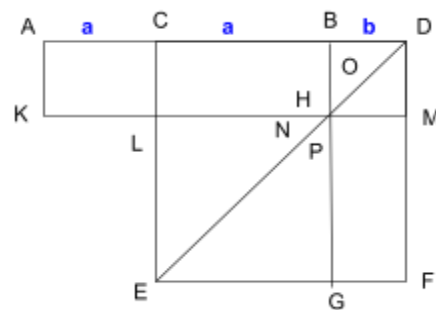
4.2 - Propositions 7-8

Book 2 - Proposition 7

If a straight line be cut at random, the square on the whole and that on one of the segments both together are equal to twice the rectangle contained by the whole and the said segment and the square on the remaining segment.

Step	Justification
Let AB be cut at C	
Let ☐ADEB be described on AB	1.46 - On a given straight line to construct a square
AG = GE	1.43 - Definition of complement
Let CF be added thus AG + CF = GE + CF and AF = CE	Common Notion 2
AF + CE = 2AF	Common Notion 2
AF + CE = gnomon KLM + CF	Definition of gnomon
2AF = gnomon KLM + CF	
2(AB x BC) = 2AF 2(AB x BC) = gnomon KLM + CF 2(AB x BC) + DG = gnomon KLM + CF + DG	Common Notion 2
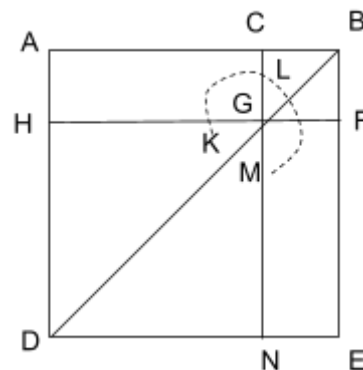
2AF + HN = □AB + □BC	
HN = □AC and AF = AB x BC	
Therefore 2(AB x BC) + □AC = □AB + □BC	

*Algebraically this proposition can be written as:

□AB + □BC = 2 x AC x BC + □AC

$(a + b)^2 + b^2 = 2(a + b)b + a^2$

$a^2 + 2ab + b^2 = a^2 + 2ab + b^2$

Book 2 - Proposition 8

If a straight line be cut at random, four times the rectangle contained by the whole and one of the segments together with the square on the remaining segment is equal to the square described on the whole and the aforesaid segment as on one straight line.

Step
CK = KN
QR = RP
☐CK = ☐BN
☐GR = ☐KP
☐CK = ☐KP
CG = GQ
☐AG = ☐MQ
☐QL = ☐RF
☐MQ = ☐QL
Gnomon STU = sum of all 8 areas = ☐4(AG + CK) = ☐4AK = 4AB•BD
Gnomon STU + ☐OH = AD•AD = 4•AD•BD + AC•AC
(AB + BC)•(AB + BC) = 4AB•BC + AC•AC

*Algebraically this proposition can be written as:

$4AB \cdot BC + AC^2 = (AB + BC) \cdot (AB + BC)$

$4(a + b) \cdot b + a^2 = (a + b + b)(a + b + b)$

$4ab + b^2 + a^2 = a^2 + ab + ab + ab + b^2 + b^2 + ab + b^2 + b^2$

$a^2 + 4b^2 + 4ab = a^2 + 4b^2 + 4ab$

5: Book 3

5.1 - Definitions

Equal Circles: those the diameter of which are equal, or the radii of which are equal

Touch a Circle: a straight line is said to touch a circle when it meets the circle, is produced, and does not cut the circle

Touch One Another: circles are said to touch one another when they meet but do not cut one another

Equally Distant from the Center: in a circle, straight lines are equally distant from the center when the perpendicular drawn to them from the center are equal

Greater Distant: the straight line is said to be greater distant on which the greater perpendicular falls

Segment of a Circle: the figure contained by a straight line and a circumference of a circle

Angle of a Segment: that contained by a straight line and a circumference of a circle

Angle in a Segment: the angle, which, when a point is taken on the circumference of the segment and straight lines are joined from it to the extremities of the straight line which is the base of the segment, is contained by the straight lines so joined

Stand Upon: when the straight lines containing the angle cut off by a circumference, the angle is said to stand upon that circumference

Sector of a Circle: the figure which, when an angle is constructed at the center of the circle, is contained by the straight ines containing the angle and the circumference cut off by them

Similar Segments of Circles: those which admit equal angles, or in which the angles are equal to one another

5.2 - Propositions 1-5

Book 3 - Proposition 1
To find the center of a given circle.

Step	Justification
Construct ⊙ABC	Postulate 3
Draw line AB	Postulate 1
Bisect AB at D	1.10
Connect DC perpendicular to AB	1.11
∠ADG = ∠GDB	1.8
∠GDB is right	1. Def 10
∠FDB is right	1. Def 10

∠GDB = ∠FDB	CN 1

Book 3 - Proposition 2
If on the circumference of a circle two points be taken at random, the straight line joining the points will fall within the circle.
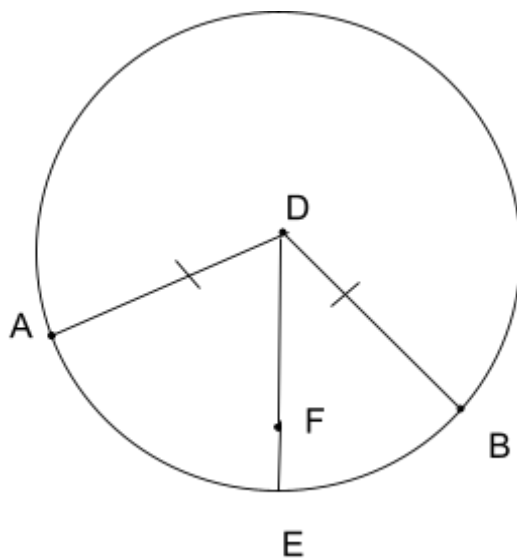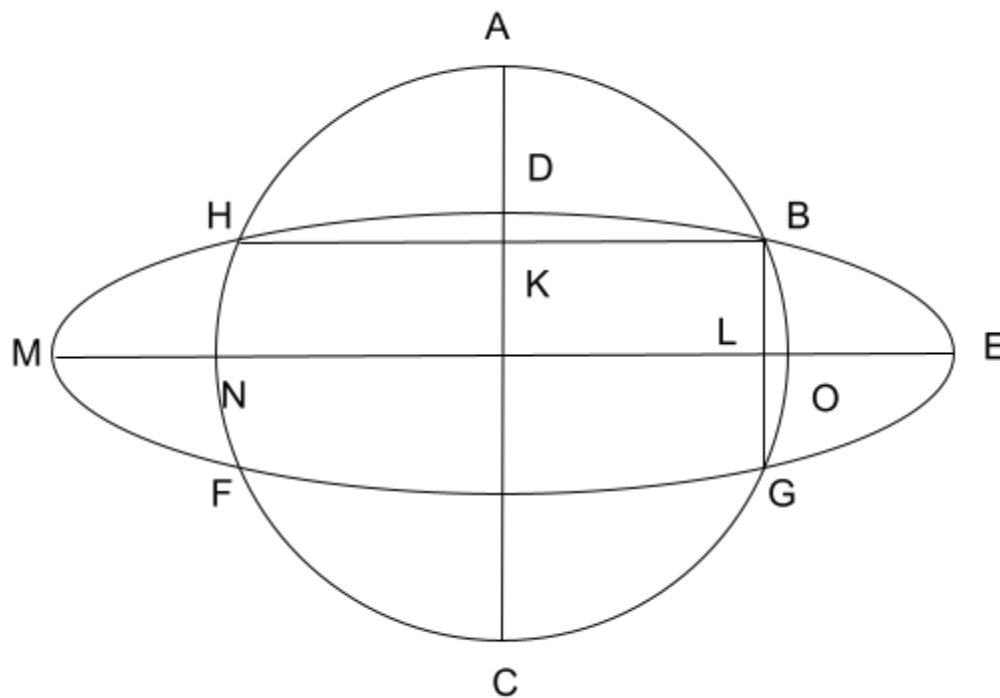

Step	Justification
Construct ◯ABC	Postulate 3
Let D be the center of ABC	3.1
Construct DA	Postulate 1
Construct DB	Postulate 1
Construct DE	Postulate 1
Construct DF	Postulate 1
Since DA = DB, ∠DAE = ∠DBE	1.5

But $\angle$DAE = $\angle$DBE, therefore DB = DE	1.19
But DB = DF and DE is longer	
Therefore a straight line from A to B will not fall outside the circle.	

Book 3 - Proposition 3

If in a circle a straight line through the center bisect a straight line not through the center, it also cuts it at right angles; and if it cut it at right angles, it also bisects it.
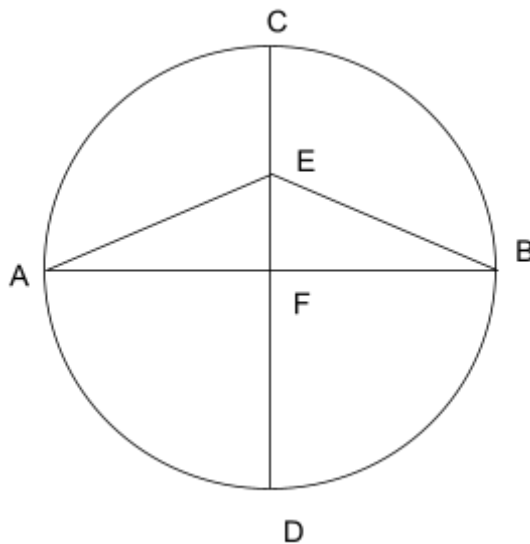
Step	Justification
Construct ◯ABC	Postulate 3
Draw AB not through the center	Postulate 1
Let CD through the center of the circle bisect AB	1.10
E is the center of ◯ABC	3.1
Construct EA	Postulate 1
Construct EB	Postulate 1
AF = FB and FE is common	

∠EA = ∠EB therefore ∠AFE = ∠BFE	1.8
∠AFE is right and ∠BFE is right	1. Def 10
∠EAF = ∠EBF	1.5
△AFE = △BFE	1.26
Therefore AF = FB	

Book 3 - Proposition 4

If in a circle two straight lines cut one another which are not through the center, they do not bisect one another.
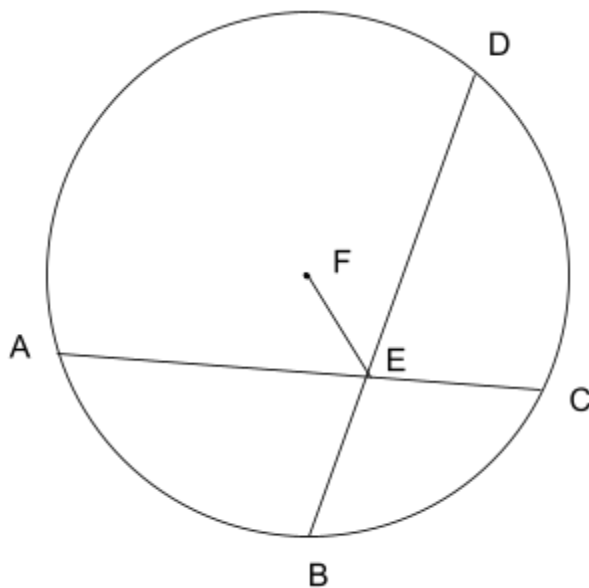
Step	**Justification**
Construct ◯ABCD	Postulate 3
Draw AC not through the center	Postulate 1
Draw BD not through the center	Postulate 1
Let AC bisect BD at E	1.10
Let F be the center of ◯ABCD	3.1
Draw FE	Postulate 1

Since FE bisects AC it cuts it at right angles, therefore ∠FEA is right	3.3
Since FE bisects BD it cuts it at right angles, therefore ∠FEB is right	3.3
Then ∠FEA = ∠FEB which is impossible!	
Therefore AC and BD do not bisect one another	

Book 3 - Proposition 5

If two circles cut one another, they will not have the same center.
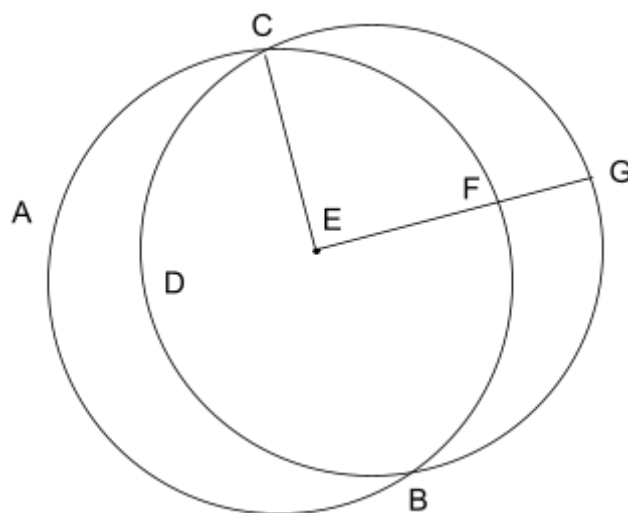
Step	Justification
Construct ◯ABC	Postulate 3
Construct ◯CDG	Postulate 3
Let ◯ABC cut ◯CDG at B and C	
Let E be the center of both ABC and CDG	3.1
Since E is the center of ABC, EC = EF	1. Def 15
Since E is the center of CDG, EC = EG	1. Def 15
But EC = EF, thus EF = EG	

The less to the greater which is impossible!	
Therefore E is not the center of ◯ABC and ◯CDG	

5.3 - Propositions 6, 9-13

Book 3 - Proposition 6
If two circles touch one another, they will not have the same center.
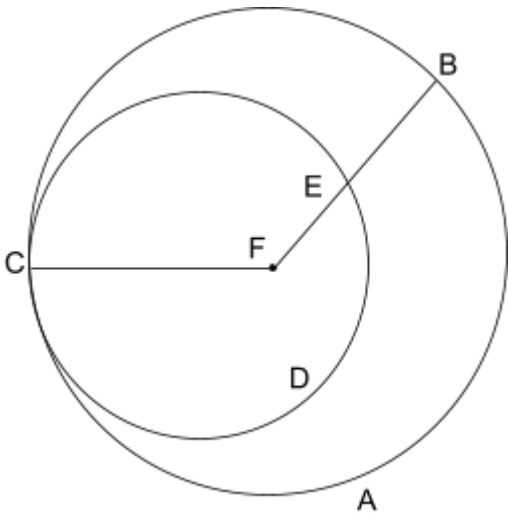
Step	Justification
Construct ◯ABC	Postulate 3
Construct ◯CDE	Postulate 3
Let ABC touch CDE at C	3. Def 2
Let F be the center of ◯ABC and ◯CDE	1. Def 15
Construct FC	Postulate 1
Draw FEB at random	Postulate 1
FC = FB	1. Def 15

FC = FE	1. Def 15
Therefore FB = FE	
The less to the greater which is impossible!	
Therefore F is not the center of ◯ABC and ◯CDE	

Book 3 - Proposition 9

If a point be taken within a circle, and more than two equal straight lines fall from the point on the circle, the point taken is the center of the circle.
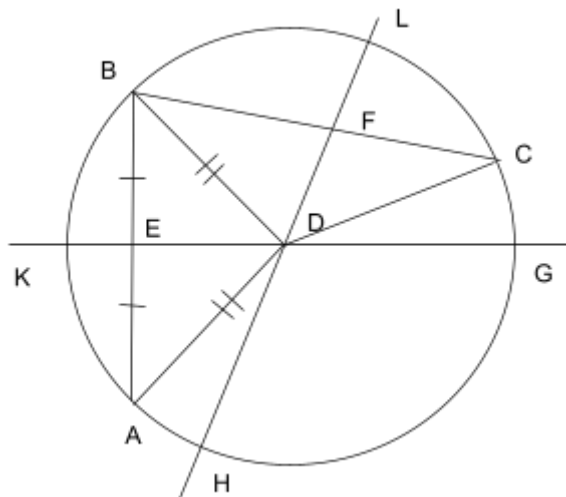
Step	Justification
Construct ◯ABC	Postulate 3
Let D be the center of ◯ABC	3.1
Construct AC, DB, DC	Postulate 1
Join AB and let it be bisected at E	1.10
Join BC and let if be bisected at F	1.10
∠AED = ∠BED	1.8
∠AED is right and ∠BED is right	1. Def 10
Center of ◯ABC is on GK	3.1 porism

Center of ◯ABC is on HL	3.1 porism
GK and HL only intersect at D	
Therefore D is the center of ◯ABC	

Book 3 - Proposition 10
A circle does not cut a circle at more points than two.

Step	Justification
Construct ○ABC	Postulate 3
Construct ○DEF	Postulate 3
Let ○ABC cut ○DEF at BGFH	
Let BH be bisected at K	1.10
Let BG be bisected at L	1.10
The center of ○ABC is on AC	3.1 porism

The center of ◯ABC is on NO	3.1 porism
AC and No only meet at P, thus that is the center of ABC	
But P is also the center of DEF but two circles which cut one another cannot have the same center	3.5

Book 3 - Proposition 11

If two circles touch one another internally, and their centers be taken, the straight line
joining their centers, if it be also produced, will fall on the point of contact of the circles.
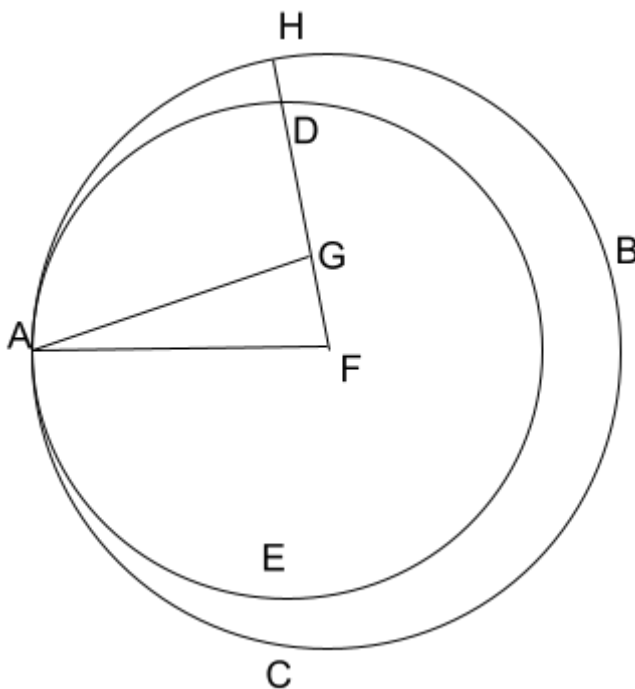
Step	Justification
Construct ○ABC	Postulate 3
Construct ○ADE	Postulate 3
Let ○ABC and ○ADE touch each other internally at point A	3. Def 3
Let F be the center of ○ABC	3.1
Let G be the center of ○ADE	3.1

Draw FGH, AF, AG	Postulate 1

AG + GF > FA = FH
   - FG   -FG
    AG > GH	

AG = GD therefore GD > GH	
The less than the greater which is impossible!	
Therefore FG will fall on A.	

Book 3 - Proposition 12

If two circles touch one another externally, the straight line joining their centers will pass through the point of contact.
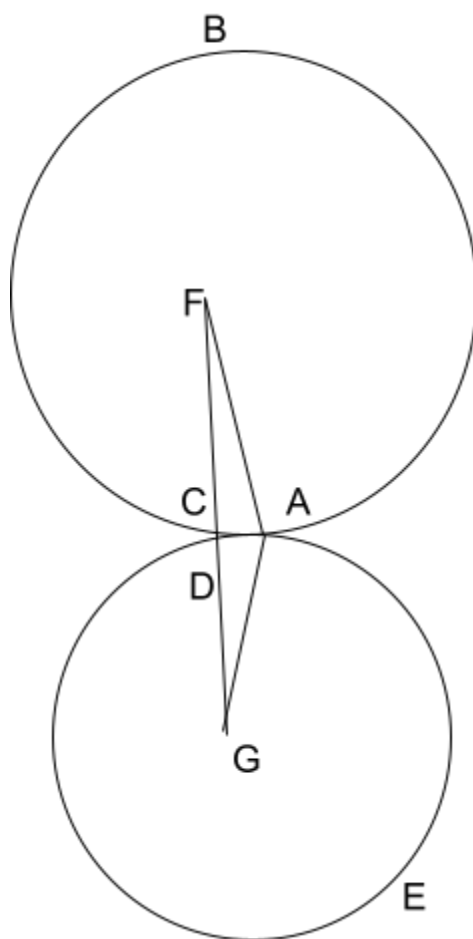
Step	Justification
Let ◯ABC and ◯ADE touch each other externally at point A	3. Def 3
Let F be the center of ◯ABC	3.1
Let G be the center of ◯ADE	3.1

Let FCDG be joined not through A	
Let FA and FG be joined	
FA = FC since F is the center	1. Def 15
GA = GD since G is the center	1. Def 15
FA + AG = FC + GD FG > FA + AG But it is also less which is impossible!	1.20
Therefore a straight line joined from F to G will not fail to pass through the point of contact A	

Book 3 - Proposition 13

A circle does not touch a circle at more points than one, whether it touch it internally or externally.
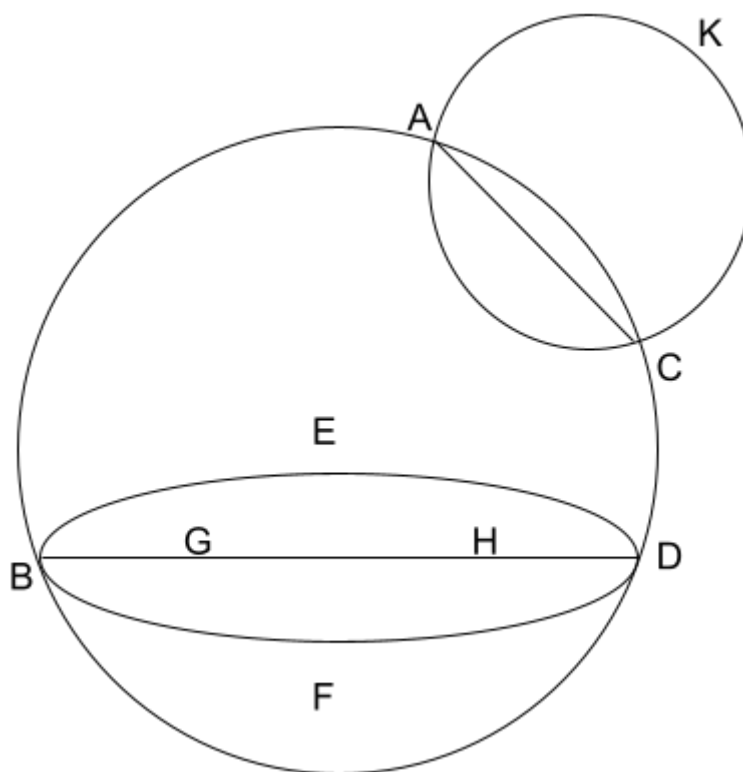
Step	Justification
Let ◯ABCD touch ◯EBFD tat D and B	
Let G be the center of ◯ABCD	3.1
Let H be the center of ◯EBFD	3.1
Therefore GH will fall on BD	3.11
Since G is the center of ◯ABCD, BG = GD, BG>HD, and BH>HD	

Since H is the center of ◯EBFD, BH = HD	
But it was proved to be greater, which is impossible!	

Therefore a circle does not touch a circle internally at more than two points.

*Nor does it externally

5.4 - Propositions 14, 15, 17, 19 - 21

Book 3 - Proposition 14

In a circle equal straight lines are equally distant from the center, and those which are equally distant from the center are equal to one another.
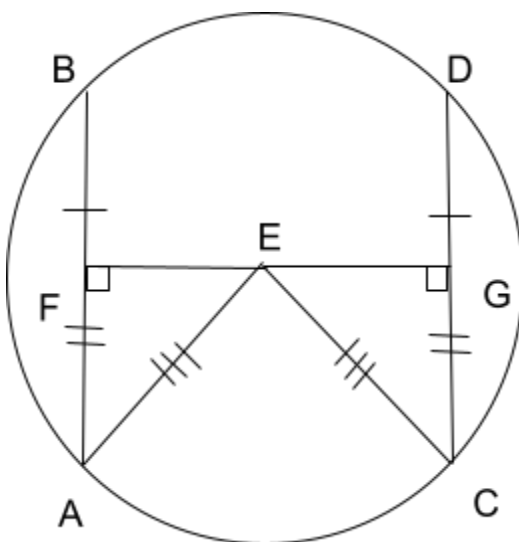
Step	Justification
Construct ◯ABDC and join AB and CD	
Let E be the center	3.1
Construct EF perpendicular to AB	1.11
Construct EG perpendicular to CD	1.11
Let AE and EC be joined	Postulate 1
EF bisects AB	3.1

$AF = FB$ $AB = 2AF$ $CD = 2CG$	
$AE = EC$ therefore $AE^2 = EC^2$	
$AF^2 + FE^2 = AE^2$ $AF^2 + FE^2 = CG^2 + GE^2$ $AF^2 = CG^2$ $FE^2 + GE^2$ $FE = GE$	
AB and CD are equally distant from the center	3. Def 4
$EF = EG$ and $AB = CD$	
$EF^2 + AF^2 = AE^2$ $EG^2 + GC^2 = CE^2$ $EF^2 + FA^2 = EG^2 + GC^2$ $EF^2 = EG^2$ $EF = EG$	
$AF^2 = CG^2$ $AF = CG$	
$AB = 2AF$ $CD = 2CG$	
Therefore $AB = CD$	

Book 3 - Proposition 15

Of straight lines in a circle the diameter is greatest, and of the rest the nearer to the center is always greater than the more remote.
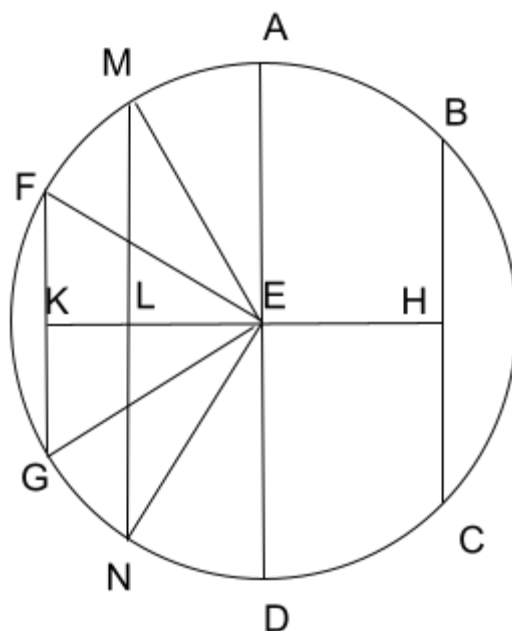
Step	Justification
Construct ◯ABCD with diameter AD and center E	1. Def 17 3.1
Let BC be nearer to AD and FG be more remote	
Let EH be constructed perpendicular to BC	1.11
Let EK be constructed perpendicular to FG	1.11

EK > EH	3. Def 5
EL = EH	
Let LM be constructed perpendicular to EK	1.11
Construct ME, EN, FE, and EG	Postulate 1
Since EH = EL, BC = MN	3.14
Since AE = EM and ED = EN, AD = ME + EN	
But ME + EN > MN	
MN = BC and AD > BC	
ME + EN = FE + EG	
∠MEN > ∠FEG and MN > FG	1.24
But MN = BC	
Therefore AD is greatest and BC > FG	

Book 3 - Proposition 17
From a given point to draw a straight line touching a given circle.
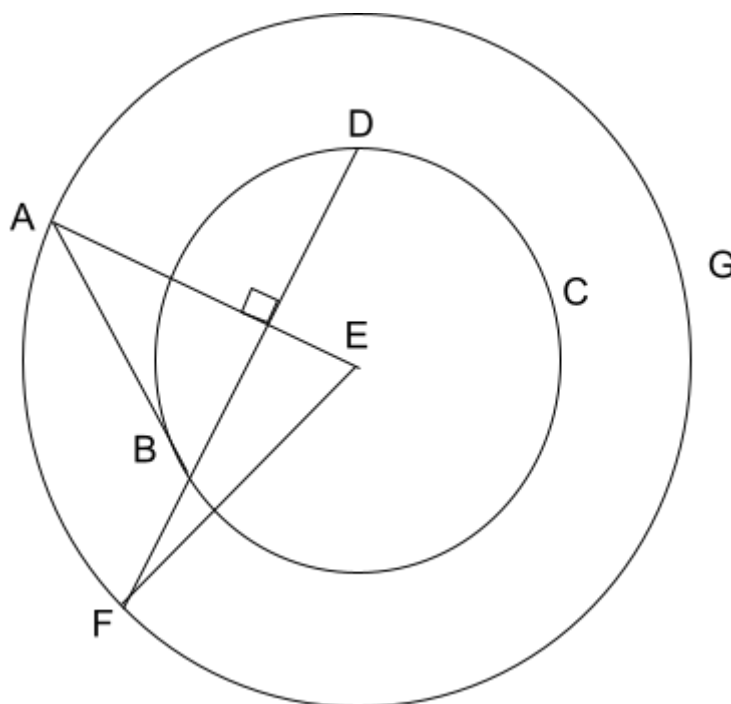
Step	Justification
Construct ◯BCD with E as it's center	3.1
Let AE b joined	Postulate 1
Construct ◯AFG with distance EA	Postulate 3
Let DF be constructed perpendicular to EA	1.11
Construct EF and AB	Postulate 1

EA = EF and ED = EB EA + EB = EF + EB DF = AB	
△DEF = △BEA	1.4
But ∠EDF is right therefore ∠EBA is right	
AB touches ◯BCD	3.16 porism
Therefore from the given point A the straight line AB has been drawn touching ◯BCD	

Book 3 - Proposition 19

If a straight line touch a circle, and from the point of contact a straight line be drawn at right angles to the tangent, the center of the circle will be on the straight line so drawn.

Tangent: 1590s, "meeting at a point without intersecting," from Latin *tangentem* **(nominative** *tangens***), present participle of** *tangere* **"to touch," First used by Danish mathematician Thomas Fincke in** *"Geomietria Rotundi"* **(1583). [11]**
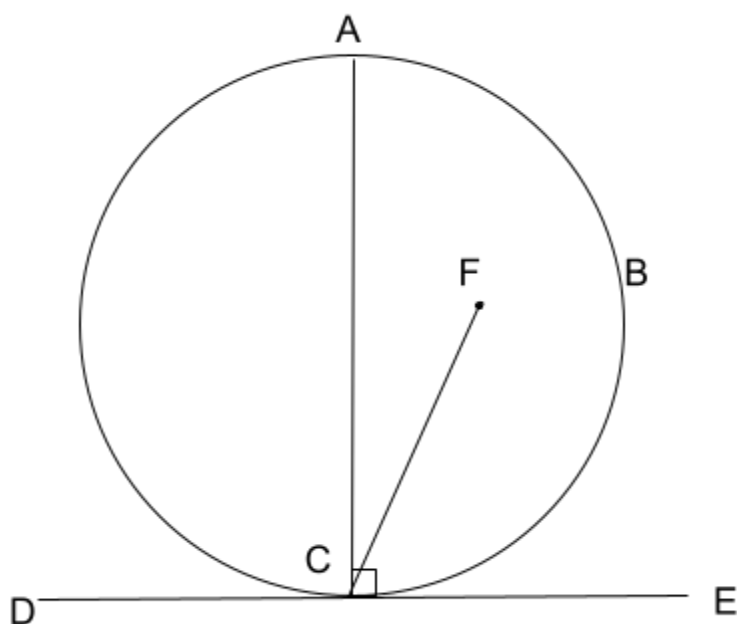
Step	Justification
Construct ◯ABC and let DE touch it at C	
Let CA be drawn perpendicular to DE	1.11
Let F be the center of ◯ABC and F not on CA	
Construct FC	Postulate 1

FC is perpendicular to DE therefore ∠FCE is right	3.18
But ∠ACE is also right therefore ∠FCE = ∠ACE	
Which is impossible!	
Therefore F is not the center of ◯ABC	

Book 3 - Proposition 20

In a circle the angle at the center is double the angle at the circumference when the angles have the same circumference as base.
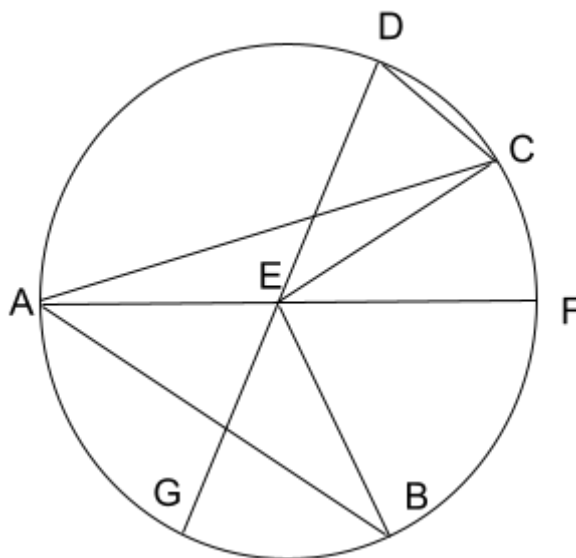
Step	Justification
Construct ◯ABC with ∠BEC at its center	Postulate 3
∠BAC at the circumference	
Let ∠BEC and ∠BAC share base BC	
Let AE be joined to F	Postulate 1
EA = EB	
∠EAB = ∠EBA ∠EAB + ∠EBA = 2∠EAB	1.5
∠BEF = ∠EAB + ∠EBA ∠BEF = 2∠EAB	1.32

For the same reason $\angle FEC = 2\angle BAC$	
DE be produced to G, but the same argument $\angle BEC = 2\angle BDC$	

Book 3 - Proposition 21

In a circle the angles in the same segment are equal to one another.
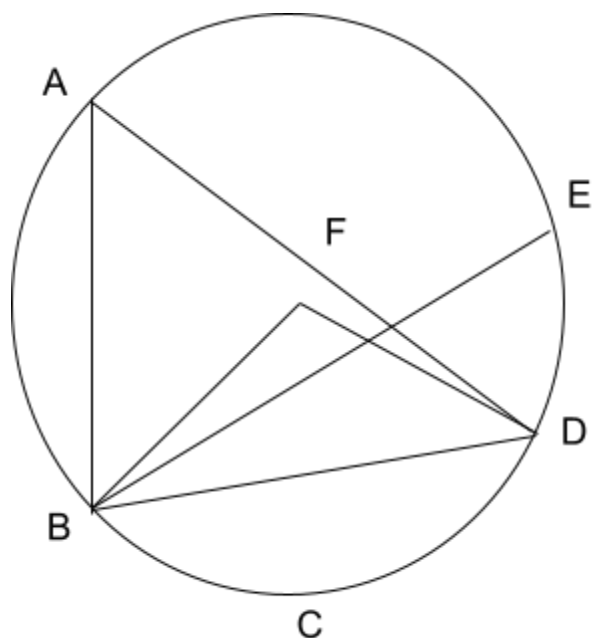
Step	Justification
Construct ◯ABCD	
Let ∠BAD and ∠BED be constructed so they share base BD	
Let F be the center of ◯ABCD	
Construct BF and FD	Postulate 1
Since ∠BFD is at the center ∠BFD = 2∠BAD	3.20
For the same reason ∠BFD = 2∠BED	

Therefore $\angle BAD = \angle BED$

5.5 - Propositions 22, 25-27

Book 3 - Proposition 22

The opposite angles of quadrilaterals in circles are equal to two right angles.
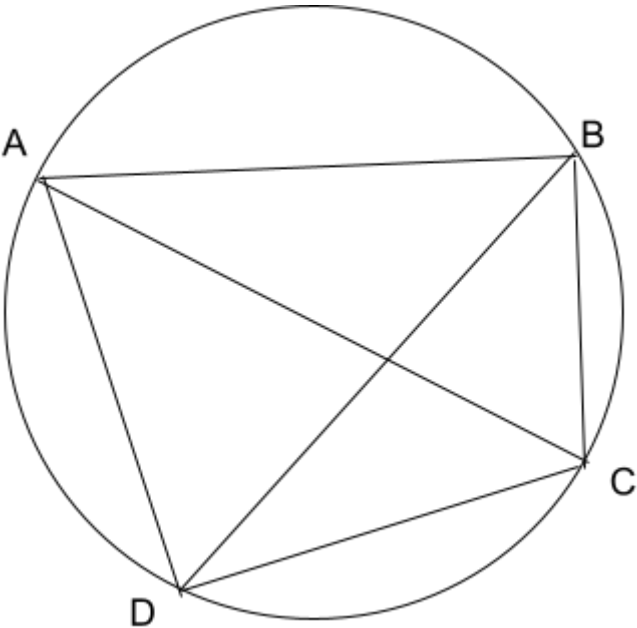
Step	Justification
Construct ○ABCD and let ABCD be constructed inside the circle	
∠CAB + ∠ABC + ∠BCA = 180	1.32 *180 = two right angles
∠CAB = ∠BDC	3.21
∠ACB = ∠ADB	
∠ADC = ∠BAC + ∠ACB	

$\angle ADC + \angle ABC = \angle BAC + \angle ACB + \angle ABC$	
$\angle ADc + \angle ABC = 180$	
For the same reason $\angle BAD + \angle DCB = 180$	

Book 3 - Proposition 25

Given a segment of a circle, to describe the complete circle of which it is a segment.
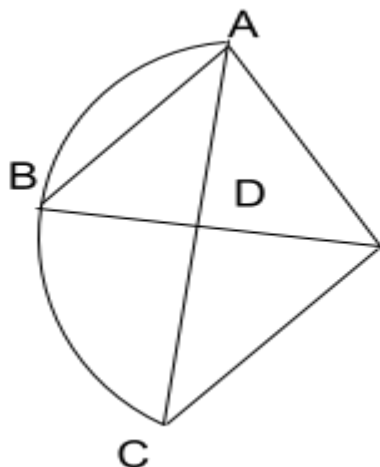
Step	Justification
Let ABC be a segment	3. Def 6
Let AC be bisected at D	
Let DB be constructed perpendicular to AC	1.11
Construct AB	Postulate 1

Book 3 - Proposition 26

In equal circles equal angles stand on equal circumferences, whether they stand at the centers or at the circumferences.
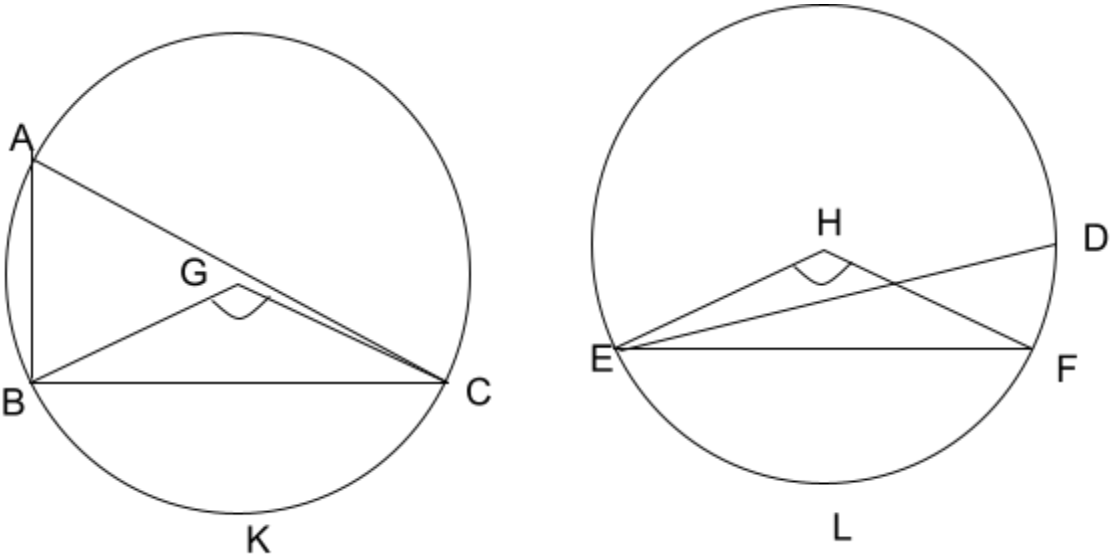
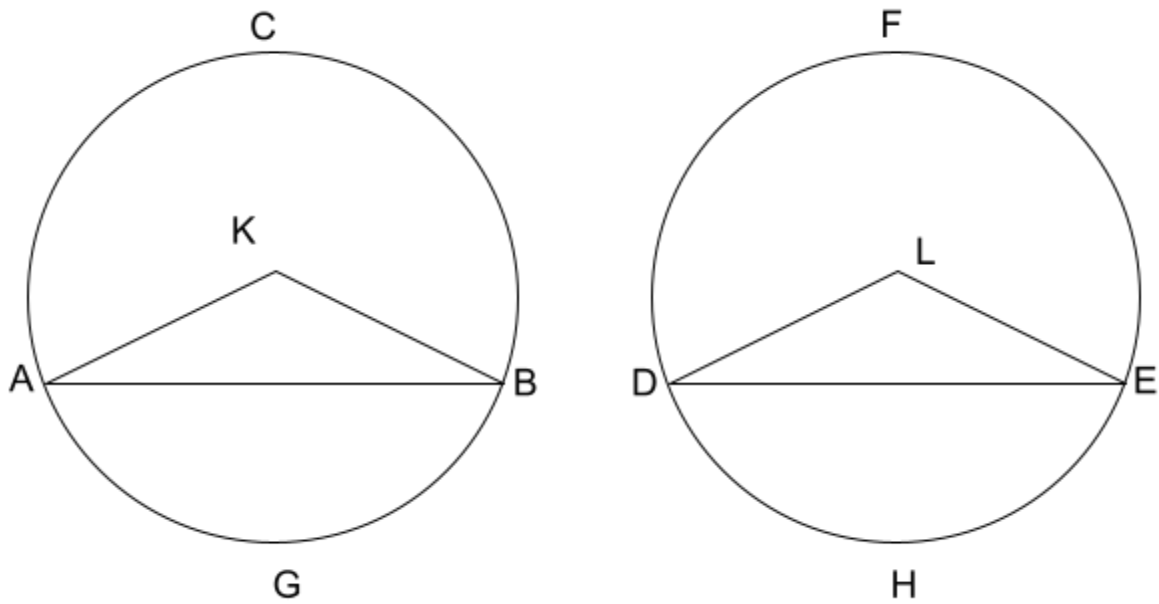| Step	Justification
Construct ◯ABC = ◯DEF	Postulate 3
∠BGC = ∠EHF	
Since ◯ABC = ◯DEF, the radii are equal	3. Def 1
BG = GC	1.4
∠A = ∠D	
Segment BAC = Segment EDF	3. Def 11
Therefore circumference BKC = circumference ELF	

Book 3 - Proposition 27

In equal circles angles standing on equal circumferences are equal to one another, whether they stand at the centers or at the circumferences.
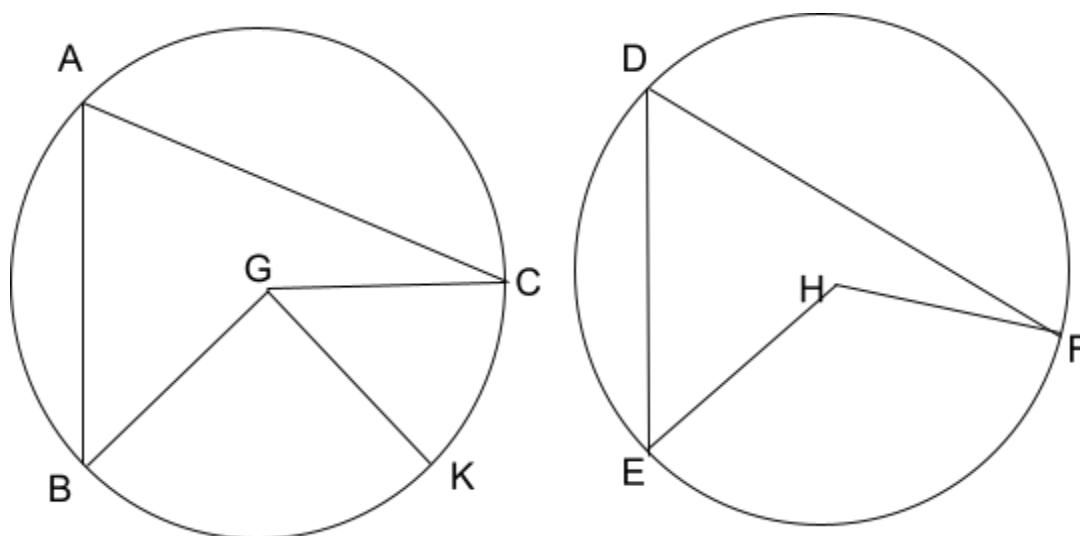
Step	Justification
Construct ◯ABC with center G and ◯DEF with center H - let ◯ABC = ◯DEF	Postulate 3
∠BGC > ∠EHF ∠BGK = ∠EHF	
Circumference BK = Circumference EF	
But EF = BC therefore BK = BC	
The less to the greater which is impossible!	
∠BGC = ∠EHF ∠A = ½ ∠BGC ∠D = ½ ∠EHF	

Therefore the angle at A is also equal to the angle at D

5.6 - Propositions 28, 30, 32, 34

Book 3 - Proposition 28

In equal circles equal straight lines cut off equal circumferences, the greater equal to the greater and the less to the less.

Step	Justification
Construct ◯ABC = ◯DEF	Postulate 3
Let circumference ACD > circumference AGB and let circumference DFE > circumference DHE	
Let K be the center of ◯ABC	
Let L be the center of ◯DEF	

Construct AK, KB, DL, LE	
AK + KB = DL + LE AB = DE	
∠AKB = ∠DLE	1.8
Circumference AGB = circumference DHE	3.26
Therefore circumference ACB is equal to circumference DFE	

Book 3 - Proposition 30
*To bisect a given circumference.*
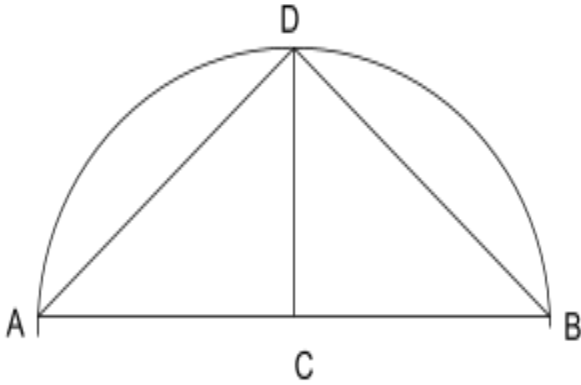
Step	Justification
Construct AB and let it be bisected at C	1.10
Construct CD perpendicular to AB	1.11
Construct DA and DB	
Since AC = CB and CD is common, AC + CD = BC + CD	
∠ACD = ∠BCD because they are both right	
AD = DB	1.4
Circumference AD + circumference DB < a semicircle	3.28
Circumference AD = circumference DB	

Therefore the given circumference has been bisected a point D

Book 3 - Proposition 32

If a straight line touch a circle, and from the point of contact there be drawn across, in the circle, a straight line cutting the circle, the angles which it makes with the tangent will be equal to the angles in the alternate segments of the circle.
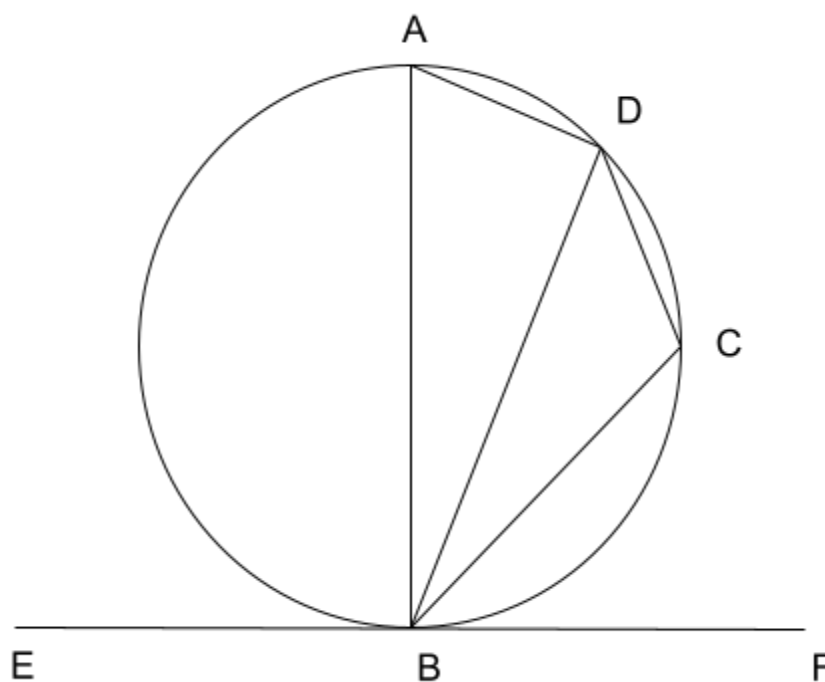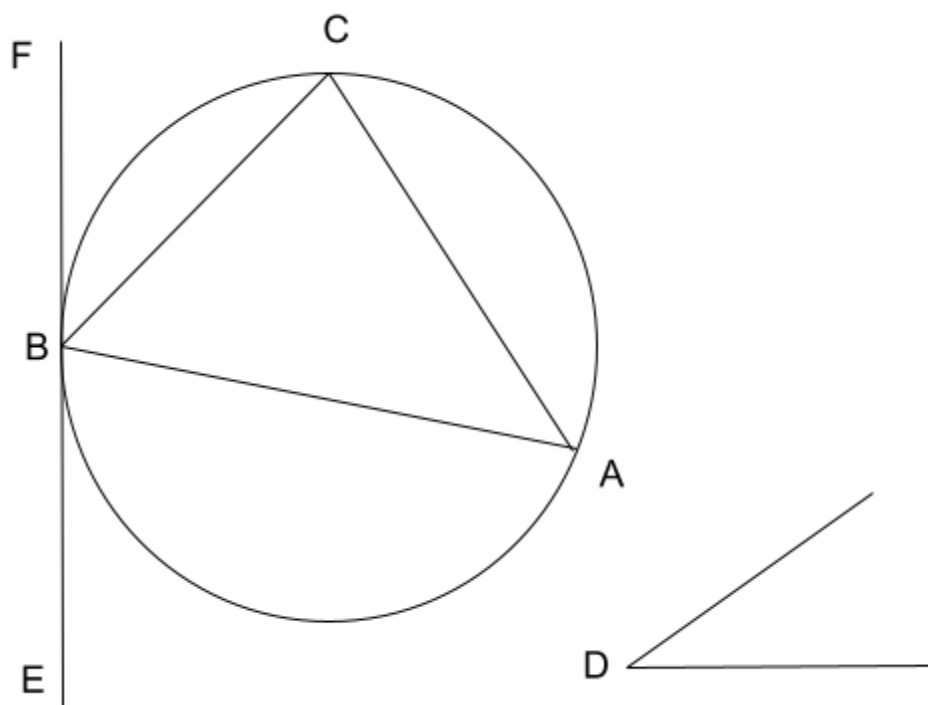

Step	Justification
Construct ○ABCD and let EF touch it at point B	
Construct BD across ○ABCD	Postulate 1
Construct BA perpendicular to EF	
Construct AD, DC, CB	Postulate 1
The center of ○ABCD is on AB	3.19

BA is a diameter and ∠ADB is right	3.31
∠BAD + ∠ABD = one right angle ∠ABF is also right	
∠ABF = ∠BAD + ∠ABD -∠ABD -∠ABD ∠DBF = ∠BAD	
Since ABCD is a quadrilateral, the opposite angles equal 2 right angles	3.22
∠DBF + ∠DBE = 2 right angles	
∠DBF + ∠DBE = ∠BAD + ∠BCD	
∠BHD = ∠DBF	
∠DBE = ∠DCB	

Book 3 - Proposition 34

From a given circle to cut off a segment admitting an angle equal to a given rectilineal angle.

Step	Justification
Construct ◯ABC with EF touching ◯ABC at B	Postulate 3
Let ∠FBC be constructed equal to ∠D	1.23
∠FBC = ∠BAC	3.32
Since ∠FBC = ∠D, then ∠BAC = ∠D	

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2.4 Quizzes

Euclid's Elements Book 1 - Quiz 1

Name:

Class:

Date:

---

1. Point:

2. Line:

3. Extremities:

4. Straight Line:

5. Surface:

6. Lines:

7. Plane Surface:

8. Plane Angle:

Euclid's Elements Book 1 - Quiz 2

Name:
Class:
Date:

1. Point:

2. Line:

3. Extremities:

4. Straight Line:

5. Surface:

6. Lines:

7. Plane Surface:

8. Plane Angle:

9. Rectilinear:

10. Right angles:

   Perpendicular:

11. Obtuse Angle:

12. Acute Angle:

13. Boundary:

14. Figure:

15. Circle:

16. Center:

Euclid's Elements Book 1 - Quiz 3

Name:
Class:
Date:

1. Point:

2. Line:

3. Extremities:

4. Straight Line:

5. Surface:

6. Lines:

7. Plane Surface:

8. Plane Angle:

9. Rectilinear:

10. Right angles:

Perpendicular:

11. Obtuse Angle:

12. Acute Angle:

13. Boundary:

14. Figure:

15. Circle:

16. Center:

17. Diameter:

18. Semicircle:

19. Rectilinear:

Trilateral:

Equilateral:

Isosceles:

Scalene:

Right-angled:

Obtuse-angled:

Acute-angled:

Quadrilateral:

Square:

Oblong:

Rhombus:

Rhomboid:

Trapezia:

Multilateral:

20. Parallel:

Euclid's Elements Book 1 - Quiz 4

Name:

Class:

Date:

Postulates

1.

2.

3.

4.

5.

Common Notions

1.

2.

3.

4.

5.

Euclid's Elements Book 2 - Quiz 1

Name:
Class:
Date:

1. Contained:

2. Gnomon:

Book 1 review:
1. Point:

2. Line:

3. Extremities:

4. Straight Line:

5. Surface:

6. Lines:

7. Plane Surface:

8. Plane Angle:

9. Rectilinear:

10. Right angles:

Perpendicular:

11. Obtuse Angle:

12. Acute Angle:

13. Boundary:

14. Figure:

15. Circle:

16. Center:

17. Diameter:

18. Semicircle:

19. Rectilinear:

Trilateral:

Equilateral:

Isosceles:

Scalene:

Right-angled:

Obtuse-angled:

Acute-angled:

Quadrilateral:

Square:

Oblong:

Rhombus:

Rhomboid:

Trapezia:

Multilateral:

20. Parallel:

Postulates

1.

2.

3.

4.

5.

Common Notions

1.

2.

3.

4.

5.

Euclid's Elements Book 3 - Quiz 1

Name:
Class:
Date:

1. Equal circles:

2. Touch a circle:

3. Circles are said to touch one another:

4. Equally distant from the center:

5. Greater distance:

6. Segments of a circle:

Euclid's Elements Book 3 - Quiz 2

Name:
Class:
Date:

7. Angle of a segment:

8. Angle in a segment:

9. Base of the segment:

10. Stand upon:

11. Sector of a circle:

12. Similar segments of circles:

Euclid's Elements Book 3 - Quiz 3

Name:
Class:
Date:

1. Equal circles:

2. Touch a circle:

3. Circles are said to touch one another:

4. Equally distant from the center:

5. Greater distance:

6. Segments of a circle:

7. Angle of a segment:

8. Angle in a segment:

9. Base of the segment:

10. Stand upon:

11. Sector of a circle:

12. Similar segments of circles:

Book 2 review:

1. Contained:

2. Gnomon:

Book 1 review:

1. Point:

2. Line:

3. Extremities:

4. Straight Line:

5. Surface:

6. Lines:

7. Plane Surface:

8. Plane Angle:

9. Rectilinear:

10. Right angles:

    Perpendicular:

11. Obtuse Angle:

12. Acute Angle:

13. Boundary:

14. Figure:

15. Circle:

16. Center:

17. Diameter:

18. Semicircle:

19. Rectilinear:

Trilateral:

Equilateral:

Isosceles:

Scalene:

Right-angled:

Obtuse-angled:

Acute-angled:

Quadrilateral:

Square:

Oblong:

Rhombus:

Rhomboid:

Trapezia:

Multilateral:

20. Parallel:

Postulates

1.

2.

3.

4.

5.

Common Notions

1.

2.

3.

4.

5.

3. Data

While I have used the method of socratic discussions through Euclid's Elements to teach Geometry in two previous high school classes, my current teaching assignment is in middle school. Due to the material taught in middle school the data I could collect with these methods was limited, but I was able to give an initial assessment, lessons, and a final assessment over angle relations when two parallel lines are cut by a transversal, see Appendix.

The class was made up of twelve eighth grade students who are studying Algebra. The assessments I used are included in the Appendix and the class results are given below. After taking the initial assessment, students were given were given the workbook pages for propositions 1.15, 1.29, and 1.30 which cover *Vertical Angles*, *If two lines are cut by a transversal, then the corresponding angles are congruent and the interior angles on the same sides add up to 180 degrees*, and *Transversal Property of Parallel Lines* respectively. The class worked through these propositions collaboratively along with the names and identities that are commonly used in geometry for angle relations. This learning process took 8 class days. At the end, students were given the final assessment.

The results show that my students were coming into this unit with very little or no knowledge of angle relations between parallel lines. I had anticipated for this to last 6 class days, but as we dissected the propositions slowly and then discussed their meanings in the realm of geometry, the unit was extended to 8 days. The results show that this unit was successful for most of the students, but there were still components that were lost in the process. Overall, this was a valuable unit for this class and I anticipate teaching it similarly for other classes.

	Initial Assessment Scores 21 points total	%	Final Assessment Scores 21 points total	%
Student 1	2	9.5%	13	62%
Student 2	4	19%	21	100%
Student 3	1	4.8%	20	95%
Students 4	0	0%	21	100%
Student 5	2	9.5%	21	100%
Student 6	5	25%	17	81%
Student 7	3	14%	15	71%
Student 8	4	19%	20	95%
Student 9	7	33%	14	67%
Student 10	2	9.5%	18	86%
Students 11	5	25%	17	81%
Student 12	5	25%	20	95%
Class Average	40/12 = 3.33	16%	217/12=18.1	86%

Using a 21-item test that measured angle relations when two parallel lines are cut by a transversal, students took an initial assessment (M=3.33, SD=2.02) prior to the lesson and a final assessment (M=18.08, SD=2.87) after the lesson. A paired t-test was used to see if we can reject the null hypothesis that the mean scores would be the same on the pre-test and post-test. The increase in scores was highly significant, $t(11)$=12.6224, p<0.0001 meaning we reject the null hypothesis and conclude that the material was highly effective in helping the students learn the content.

4. Assessment/Review of the Material

The text *Geometry as told by Euclid's Elements* provides a fresh approach to using Euclid in the Geometry classroom. Before students are immersed in the propositions, they are asked to explore the world of logic and reasoning. These skills lead into a student led, teacher guided, understanding of the Elements.

This project is made up of three main components, the Student Workbook, the Answer Key, and the Teacher Guide. The Student Workbook provides space for propositions to be drawn, thoughts to be annotated, and discussions to be formed. This format gives the students a consistent foundation for learning. In the socratic model provided, students are leading the class time and will therefore be reliant on their work and personal notes, thus this space is vital to the course. The Answer Key is synonymous with the Student Workbook but provides answers or suggested answers for the questions, discussion, and propositions. While teachers should be masters of their content, this key provides a guide for the conversations that could aid in whole class learning. The last component, the Teacher Guide provides a day by day outline for how to structure the class. It is consistent over all of the material provided and gives structure to the course while still allowing for freedom in the time and space of classroom discussions or presentations.

In comparison to other classical Geometry texts and modern Geometry textbooks, this work shows a stronger desire for the student to develop their Geometry basis while shifting from solely being the learner to also being the creator, the instructor, and the master of their work.

The material was reviewed by Elizabeth Ball, Instructor at the University of Nebraska Omaha and math teacher at Trinity Classical Academy, who said, "This student workbook and teacher guide is well organized and easy to follow. It offers a hands-on approach to Euclid's Elements, expecting the student to participate in discussion and thought. It maintains Euclid's approach of geometry, not with numbers and measurement but in comparison and proof. I appreciate the discussions of logical thinking and proof, not just presented as a formal mathematical application but also in daily life as this is one of the main goals of students studying Geometry. The commitment to stay true to Euclid's Elements is fantastic. The addition of the introduction of other mathematicians applicable to the topics really helps to bring the humanity back to the geometry classroom."

Matthew Mueller, math teacher at Concordia Omaha High School, also reviewed the material and found the pacing of the teacher guide to be superb and found an appreciation for studying reasoning and logical arguments. Mueller does not actively use the classical model daily, but said, "I see the Socratic discussion being a valuable tool to help students develop arguments, reason, think critically and build public speaking skills all while learning geometry. These are all valuable skills, and these skills are not being taught in most classrooms."

The review process of this material also brought to light certain aspects that were overlooked in the original writing of this text.

1) When used in a classroom where students have been studying the classical model, the material flows well, but in the situation where new students are new to this way of learning, the wording and methods used can be confusing.

2) The different teaching and learning methods used are very limited.

3) Any alignment to state or national mathematics standards is not included.

All of these critiques are valid and are addressed in the following section.

When *Geometry as told by Euclid's Elements* is fully examined, the reader will find a text that is based in the desire for students to truly experience Geometry and grow in the logical beauty laid out by Euclid. Students and teachers alike will mature mathematically from their time spent in this material!

4.1 Adjustments to the Material

This material has been reviewed by Elizabeth Ball, Matthew Mueller, Dr. Roslanowski, Dr. Rech, Dr. Grangenett, and Dr. Matthews. After conferencing with these given individuals, the following suggestions were made for adjustments to the material: 1) Questions more understandable for non English speakers or non classical learners, 2) Application practice of the propositions, 3) Material aligned with state standards, and 4) Summary of each proposition.

I have returned to the material and added more depth to some of the questions in order to avoid language or learning methodology barriers. Since this material does not flow like a typical high school textbook, there will still be areas that need clarification from the instructor.

I agree with Elizabeth Ball that there needs to be other forms of application for the propositions. Even the most academic student may not desire to sit and work out proofs for each class. In the Teacher Guide, I have provided a few different learning techniques where students will create a video or presentation to explain what they have done. As teachers, we know that each class brings a variety of students who come with different learning styles that can help engage and challenge other learners in the room. I am very interested in adding in different learning strategies that will add variety while sticking to the core of the Elements.

When looking at lining the material up with the state or national standards for a high school Geometry class, I felt that would be a necessary task for a later date when more of the final textbook is finished. That is not a part of this project but I appreciate and value the feedback.

It is not explicitly stated in any of the provided resources that students should be summarizing each proposition in their own words, but as discussion flows, it is expected for this to happen. While there is the Proposition Summary Chart that is made as a part of Book 1, I have now included "Summarize each proposition in your own words" in the Introduction to the Student Version. As teachers become more familiar with the material and flow of the class, I would expect the students summarizing the propositions in their own words to be an expectation.

5. Conclusion/Next Steps

The opportunity to create, test, review, and adjust the material for *Geometry as told by Euclid's Elements* has been pivotal in my teaching career. Each step of this process has challenged me to think differently about the process of learning and teaching. I anticipate the day when I am blessed with the opportunity to actively use this with students.

The material in this project is only the first half of a completed year long course on Geometry using Euclid's Elements. I plan to complete the rest of the material by Summer of 2024, have the material formally reviewed in order to be published, and present at conferences for the classical, Christian, and homeschool communities.

The final material will include selected propositions from books 6, 7, 8, 9, 11, 12, 13. While Book 1 is the foundations of points, lines, angles, and triangles, Book 2 uses what was developed in Book 1 to introduce geometry, and Book 3 is focused on circles, the other books that will have material taken from them focus on similar figures, number theory, solids, and geometric measurements. We are unable to indulge in all four hundred and sixty five propositions, but this course will allow students to immerse themselves in a condensed version of the Elements where they discover the beauty that Euclid has laid out and called Geometry.

When the material is finished and ready to be presented, I would like to start by introducing this material at the Nebraska Christian Home Education Association conference. Following that I plan to present at the Consortium for Classical Lutheran Education conference. As

discussed earlier, I currently do not have the material aligned with standards which may be a problem, but ultimately I would like to present this at the National Council of Teachers in Mathematics conference. In presenting, my goal is to introduce this type of learning to the math education world as well as the beauty of Geometry that is presented in Euclid's Elements.

As each component of this project was developed, new challenges were presented. And yet, as the students will come to know through this course, we grow in adversity. I believe that this material has the power to change students' hearts and minds towards mathematics. The methods used provide space for curiosity, joy, anger, and excitement all in a safe classroom environment. As I continue the work, I anxiously anticipate the use of this in classrooms all over the world!

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7. Appendix

Congruence and Similarity Initial Assessment   Name:
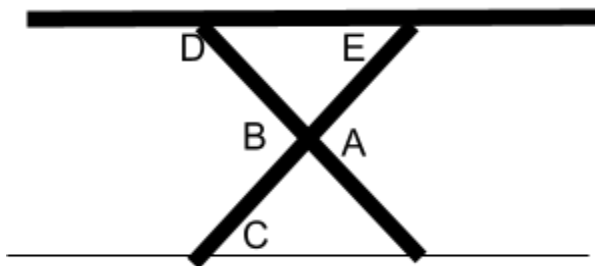   Class:
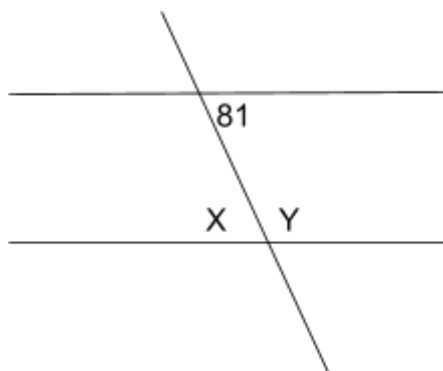   Date:

1. Define congruent:

2. Define supplementary:

3. You are building a table and you want the top to be parallel to the ground. If $m\angle A = 95$ and $m\angle C = 42.5$ find $m\angle D$, $m\angle E$, and $m\angle B$.
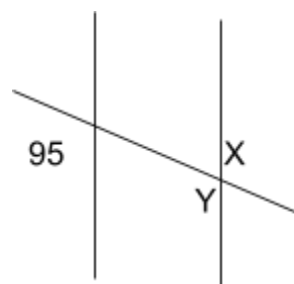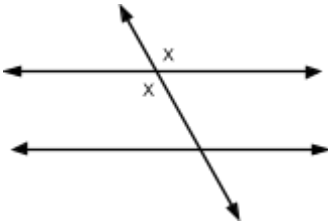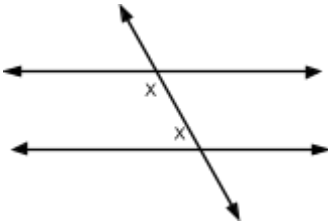

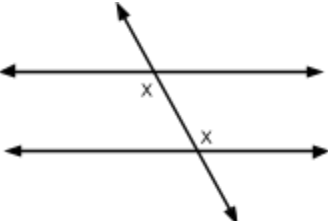4. Find the missing angle values.
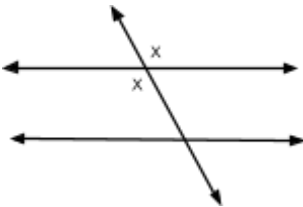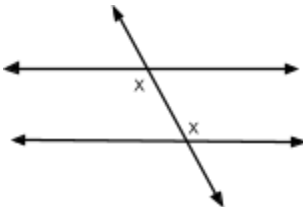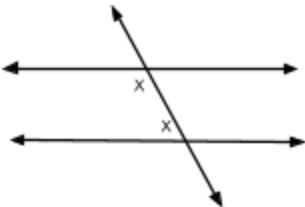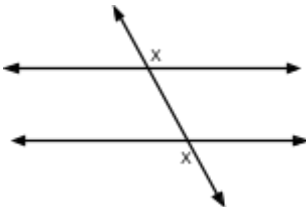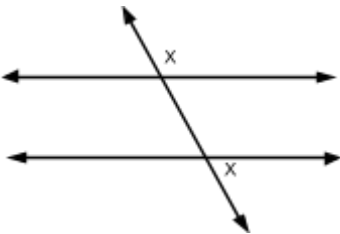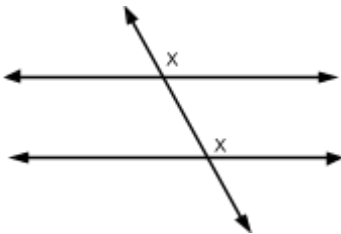
a)

b)

5. Give the name of each angle relation and then tell whether the angles are congruent or supplementary.

_____ These angles are _____	_____ These angles are _____
_____ These angles are _____	_____ These angles are _____
_____ These angles are _____	_____ These angles are _____

Congruence and Similarity Final Assessment

Name:
Class:
Date:

1. Define congruent:

2. Define supplementary:

3. You are building a table and you want the top to be parallel to the ground. If $m\angle A = 92$ and $m\angle C = 56$ find $m\angle D$, $m\angle E$, and $m\angle B$.
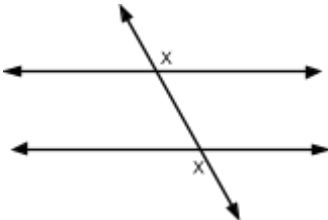

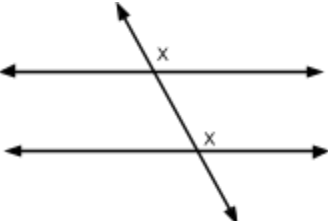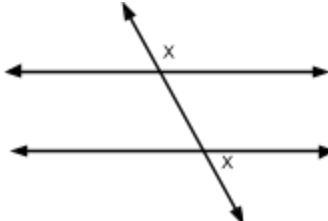4. Find the missing angle values.

a)

b)

5. Give the name of each angle relation and then tell whether the angles are congruent or supplementary.

_____ These angles are _____	_____ These angles are _____
_____ These angles are _____	_____ These angles are _____
_____ These angles are _____	_____ These angles are _____