A Retro Fit Kit to the Exhaust of a JetCat P100 for Electrical Power Generation

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Abstract

This paper presents the design and analysis of a power generator retro fit kit that bolts on a JetCat P100 in place of the stock nozzle. The motivation for this study is for small high-speed aircraft that would benefit from a power supply to their payload. The approach to this study was to first create a cycle analysis to determine the restraints and characteristics of the flow through the stock JetCat P100. From this analysis it was determined that the optimal place to extract power is from the exhaust, since the exhaust has the highest mass flow rate potential to be harvested from. Finally, detailed drawings of each component were made and analyzed. The final design is a power turbine in the nozzle that is connected to a generator through a gear box. When the exhaust flow spools up the turbine, the generator will produce an AC current. This current will go through a rectifier converting it into DC current that can be stored in a battery. This set up will produce an excess of 500 watts of power when the engine is running at full throttle. Observation from this study can inform potential aircraft and engine designers for small high-speed aircrafts.

Nomenclature

A = Area
AC = Alternating Current
c_p = Specific Heat Capacity
DC = Direct Current
g_c = Gravity Constant
M = Mach Number
m_{dot} = Mass Flow Rate
MFP = Mass Flow Parameter
P = Pressure
PTO = Power Take Off
R = Specific Gas Constant
SFC = Thrust Specific Fuel Consumption
T_t = Total Temperature
T/W = Thrust to Weight Ratio
V = Velocity
W = Work
γ = Specific Heat

I. Introduction

The goal of this project is to design an attachment to the stock JetCat P100RX that can efficiently extract power from the air flow and convert it into electrical power while maximizing the thrust to weight ratio (T/W). By maximizing the T/W ratio the design can be attached to any existing high-speed aircraft powered by a JetCat P100RX and expect similar performance with the added benefit of electrical power. The electrical power generated can be used for many different purposes when installed on small high-speed aircraft. For example, providing power to the payload abroad the aircraft. An image of a stock JetCat P100RX with dimensions and its specifications are in Figure 1 and Table 1.

Table 1. OEM Engine Specifications

Engine Specifications	
Max Thrust	22.5 lbf (100 N)
Weight	2.38 lbf
T/W	9.45
EGT_{max}	1328 ^{o}F
EGV_{max}	1427.17 ft/s
Mass Flow Rate	0.64 lbm/s
SFC	1.834 (lbm/hr)/lbf

Figure 1. JetCat P100RX Dimensions

Some limitations were placed on this project. The first being that the internal components of the engine cannot be modified. Second is that the fuel flow rate cannot be changed. Finally, the design has to have a similar outer diameter to the engine so that it can fit in an aircraft's stock engine bay. These limitations are to make the product more marketable to small high speed aircraft users, like the Department of Defense, and small engine manufacturers that produce engines that do not offer power take off.

The requirements and limitations of this senior design capstone project are provided by Air Force Research Laboratory (AFRL). The AFRL is one of the major stakeholders of this competition, as they receive monthly updates from the team and have the right to obtain our final design and the data with it. The AFRL hosts the competition as well in Dayton, Ohio where the final design is tested in their laboratory. Oklahoma State University is the other major stakeholder in this senior design capstone project. The university provides the team with a lead professor, a graduate assistant, and workspace for the project. If the AFRL does not choose to obtain a patten for the final design, Oklahoma State can also obtain the rights to the design.

II. Background and Research

A. Concepts

At the start of the design period, the team came up with three possible ways that electrical power could be pulled off of the engine; from the inlet air flow, the exhaust air flow, and from the shaft. All three of these have their pros and cons, but ultimately the team decided to move forward with extracting the power from the exhaust flow. The exhaust air flow has the highest mass flowrate and air speed velocity. Therefore, power can be extracted from it with the least effect on the engine's performance. Also, extracting energy from the shaft could overload the turbine since it is designed to only power the compressor. Additionally, the amount of airflow into the inlet changes depending on the flight mission making the device that would extract electricity less efficiently when not operating on design.

To extract power from the flow, a turbine will be placed in the nozzle. Allowing the exhaust airflow to spool the turbine. The outside of the turbine will have gear teeth so that it can be connected to an electrical generator through a gear box. Figure 2 shows the first design concept without the electrical system.

Figure 2. Preliminary CAD of Concept

B. Turbine

To start designing the turbine, a preliminary cycle analysis (PCA) was conducted in Mathcad. The equations utilized in the PCA can be referenced to the PCA of Real Engines chapter of Mattingly's Elements of Propulsion textbook[1]. A PCA is based on the power balance equation, Eq. (1), and total temperature equation, Eq. (2). The mass flowrate and level of technology for each engine component had to be assumed. A mass flowrate of .507 lbm/s was assumed based on the data from the engine spec sheet. Then each component's level of technology (LOT) was determined by changing their values until the result mirrored the corrected physical tests results. When these LOTs were run through the Mathcad program it found that the output thrust was 23 lbf, the same amount of thrust that was calculated during physical tests on the engine.

$$W_{compressor} = W_{Turbine} => m_{dot,air} * Cp_c * (T_{t3} - T_{t2}) = (m_{dot,air} + m_{dot,fuel}) * Cp_t * (T_{t4} - T_{t3}) \quad \textbf{(1)}$$

$$T_t = T + \frac{V^2}{2 * g_c * c_{pt}} \quad \textbf{(2)}$$

Then, the PCA was redone in AEDsys to identify the effects of taking power out of the flow, appendix A shows both PCA for Mathcad and AEDsys. Using the same LOTs at M_0=0 and an altitude of 900 ft. for Stillwater, OK, resulted in a thrust of 23 lbf. Then the PCA was tested with different amounts of power take-off (PTO) showing the effects of taking power out of the flow. It was found that taking the minimum amount of power out of the flow, like 500 Watts, decreased the thrust to 22 lbf. However, additional testing found that 2 kW of power could be extracted from the flow while maintaining the thrust at 22 lbf. Since more power could be taken off of the flow without affecting the thrust, it was decided that 2kW would be extracted from the flow. The AEDsys software utilized correlates with Mattingly's Aircraft Engine Design textbook[2].

Next the total temperature change across the power turbine was found using Eq (3). The work is 2 kW, the Cp is .2974 BTU/lbm/R, the total mass flow rate is .513 lbm/s and a T_{t5} of 1788 R. Then the equation is solved to find a $T_{t5.5}$ of 1775 R.

$$W = Cp * m_{dot}(T_{t5.5} - T_{t5}) \quad \textbf{(3)}$$

This data was then put into AEDsys's Turbine Preliminary Analysis Program (TURBN) to design the turbine shape. In TURBN the rotor angular velocity was assumed to be 2600 rad/s to meet up with the chosen electrical motor. Also, the degree of the flow coming out of the stock turbine was assumed to be 5 degrees to account for the swirling. The data from the TURBN program can be seen in appendix B, and the vane and blade design are in Figure 3. Applying the results, a CAD model of the turbine was made, Figure 4.

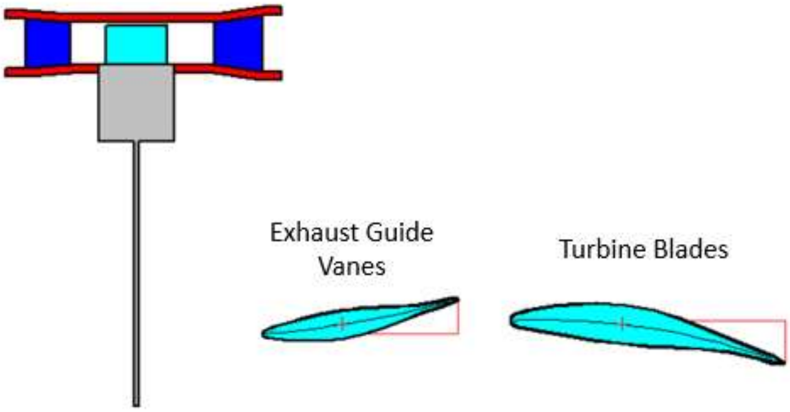


Figure 3. TURBN Results for Turbine Design

Figure 4. CAD Model of Turbine Design

C. Nozzle

The pressures and temperatures calculated in the PCA are now able to be used in the nozzle sizing. This then allowed for Mach at the exit of the nozzle to be calculated from the pressure at the nozzle. Then using the exit area of the exhaust gas vanes given from TURBN the mass flow parameter (MFP) equating, Eq. (4), and Eq. (5), can be used to size the nozzle outlet area, the results can be seen in Appendix C.

$$\mathrm{MFP}_9 := \mathrm{M}_9 \cdot \sqrt{\gamma_c \cdot \frac{g_c}{R_n}} \cdot \left[1 + \left(\frac{\gamma_c - 1}{2}\right) \cdot \mathrm{M}_9^{\,2}\right]^{\left[\frac{\gamma_c + 1}{2 \cdot (1 - \gamma_c)}\right]} \tag{4}$$

$$A_9 = \frac{mdot_{total} * \sqrt{T_{t9}}}{MFP_9 * P_{t9}} \tag{5}$$

The nozzle was designed in a similar way to past APOP teams that were required to increase the thrust to weight of the JetCat P100. The past team had great results with their nonlinear aerospike design with a great amount of increase in their thrust to weight ratio. Therefore, a nonlinear aerospike was chosen for the exhaust because it will prevent flow separation and unsmooth pressure gradients. It also allows for a more centralized pressure from the exhaust, resulting in a higher thrust. Improving the design concept from the past APOP team, the tip of the spike was cut to prevent pressure build up in case exhaust gas seep into the middle section. The final nozzle design can be seen in Figure 5.

Figure 5. Final Nozzle Design

D. Electrical System

To convert the mechanical energy from the turbine into electrical power, an electrical generator was used. Since the turbine is spinning at 22,000 RPM a motor that is rated for 50,000 rpm was chosen. The motor will be connected to the turbine through a gear that has half as many teeth as the turbine so the gear will spin at twice the speed. The motor that was used for this project is seen in Figure 6 and has a kV rating of 2,150 and a max power of 1,800 Watts.

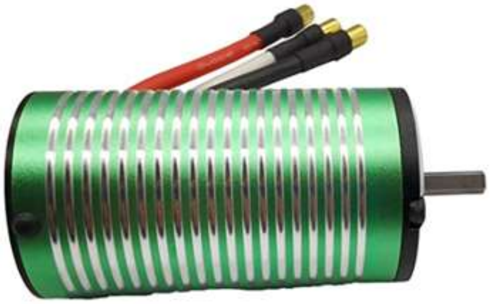


Figure 6. Electric Motor

The electric motor will be connected to a rectifier to convert the current from AC to DC. Then, the DC current will travel though a power meter that is attached to five water heating element resistors connected in parallel. A schematic of the set up can be seen in Figure 7.

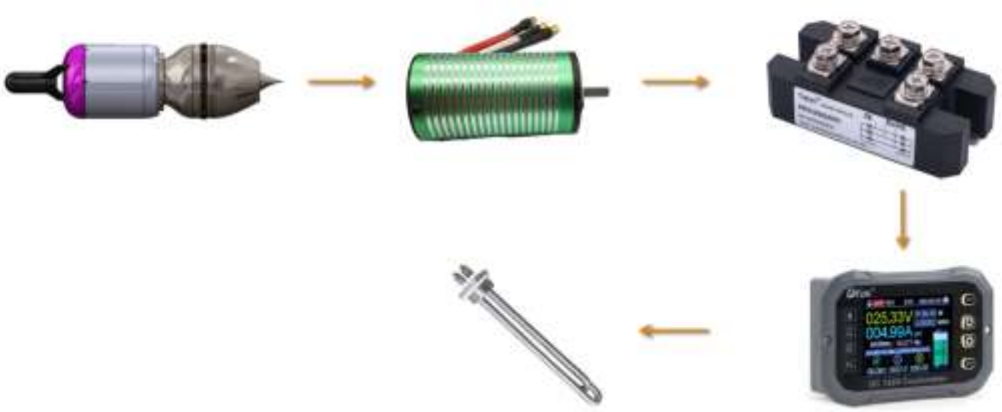


Figure 7. Electric Generation Flow Path

Connected to the brushless motor was a twin 30 mm brushless motor fan that cooled the motor during operation. The motor cooling fan was powered utilizing the generated power from the exhaust flow. Ideally, the cooling fan would be powered using a battery, but that would have added additional weight to the system. The cooling fan can be seen in Figure 8.

Figure 8. Engine Cooling Fan

E. Final Design

To be able to withstand the operating temperature of 1788 R and the centrifugal forces from spinning at 22,000 RPM, a Titanium alloy was chosen for the turbine to be additive manufactured out of. Then to save on weight the titanium alloy was used for the other components. The nozzle inlet, turbine, and nozzle will be held together with a bolt and a locking nut. The head of the bolt can be tightened by an allen wrench inserted through the hole in the tip of the nozzle. The turbine will be connected to the shaft through a high heat, high speed bearing so it will be able to spin freely and withstand the forces. There is a lip on the front of the turbine to hold it in place against the top run of the bearing in case the turbine expands and weakens the press fit between the turbine and the bearing. The bottom run of the bearing will be held in place by the inlet exhaust guide vane and the nozzle. Then the electric motor will be held in place using the same bolts that attach the nozzle to the engine and bolts through the lip on the inlet exhaust guide vane. Finally, the electric motor has a gear attached to it that will interact with the gear teeth on the outside of the turbine. The gear ratio between the two gears is 2:1, making the electric motor shaft will spin twice as fast as the turbine. Figure 9 shows the complete set up, Figure 10 shows how the bearing sets, and Figure 11 shows how the parts will fit together.

Figure 9. Side View of the Complete Set Up

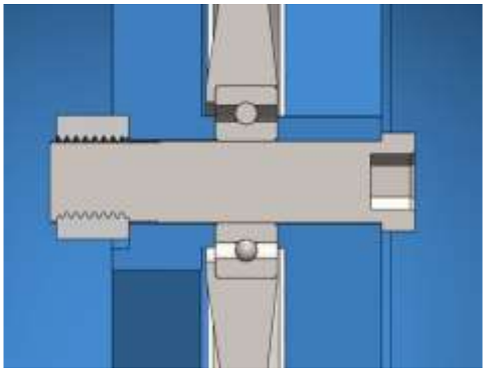


Figure 10. Cross Section View of the Bearing Compartment

Figure 11. Exploded View of the Complete Set Up

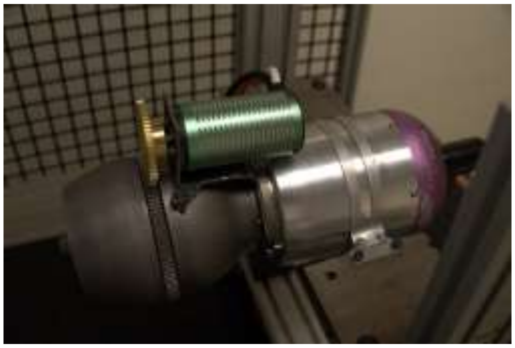


Figure 12. Fabricated Assembly

By correcting the size of the nozzle, the efficiency of the nozzle will be increased from .95 to .98. The PCA was conducted again in AEDsys to find the final theoretical amount of thrust the engine will produce. With the new nozzle design the engine will produce 25 lbf of thrust. Using the Solidworks weight tool it was found that the total weight increases from adding the new nozzle design is 2.01 lbf. With this weight and thrust increase the T/W ratio will decrease to 6.38. For the electrical side, using the rated efficiency of the rectifier and the battery controller with the KV rating the system will be able to produce 2,000 watts.

Predicted Engine Specifications	
Max Thrust	25 lbf
Weight	4.39 lbf
T/W	6.38
Electric Generation	2,000 Watts

Table 2. Predicted Engine Specifications

III. Results

Solidworks was used to conduct a finite element analysis (FEA) on all of the parts at max operating parameters, 27,000 RPM and 1788 R. The parts were tested at a higher RPM than expected to allow for more safety. As seen in the results in Figure 13 and Figure 14, all of the strains and displacements that the parts will experience will only have a minimal effect on them. The stresses on the turbine are well below the max stress level and the stresses on the complete set up are only exceeded on the bolt holes, but these holes will have more support from the bracket holding the electric motor in place.

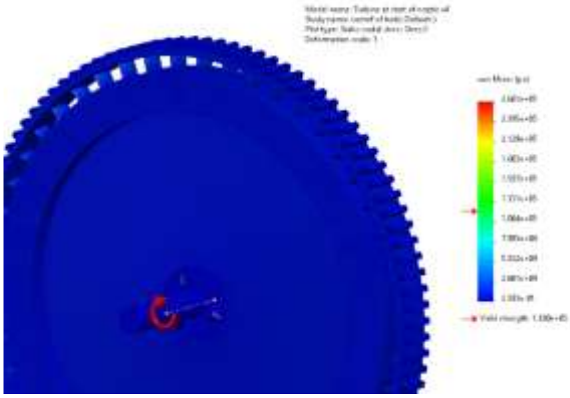


Figure 13. Stress Graph on Just the Turbine

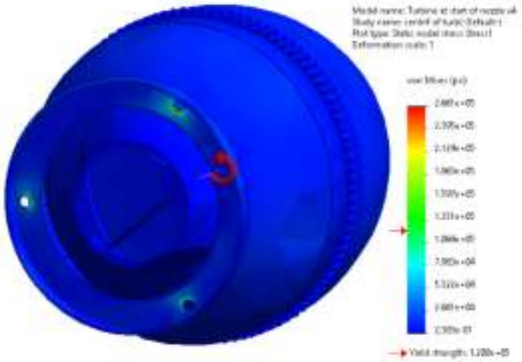


Figure 14. Stress Graph on Complete Set Up

F. Test Setup and Procedure

First, the team 3d printed a bracket to mount two brushless motors. These two brushless were then connected by a coupling fitting and were attached to a test stand. Next, a battery was connected to one brushless motor, while a three-phase rectifier and a power meter were placed on the other. These devices were placed on a separate table next to the test stand. Next, the brushless motor connected to the battery was powered up by the battery. This brushless motor then operated at various RPM ranges, which in return powered the other brushless motor at various RPM ranges through the coupling fitting. The electrical power, current, and voltage could then all be recorded at various RPM ranges from the power meter connected to the brushless motor. The resultant graph of voltage vs motor rpm is seen in Figure 15. From this test the team found that at the design point for the electric motor the electrical system will produce around 600 watts.

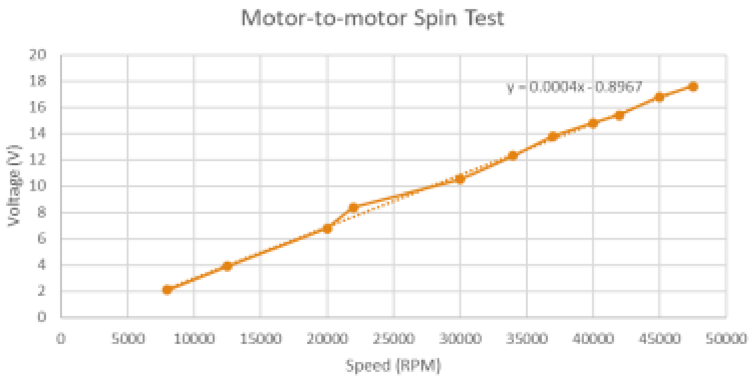


Figure 15. Voltage Vs Speed for Our Electrical System

Figure 16. Brushless Motor Mount

Figure 17. Electrical System

After testing the electrical system the team moved on to testing the retrofit kit on the Jetcat P100 engine. This was done by atttaching the sytem to the engine by the nozzle bolts. Once the system is then attached to the engine, the engine was securely attached to a test stand, as shown in Figure 18 below, that is connected to a cart. Once the mechanical systems was secured to the test stand then the electrical system was connected. The electrical system is composed of a load, generator, and wireless v/a meter, shown in both Figure 18 and Figure 20. A visual inspection of the area around the turbine engine was done to make sure that there are no hazards that could compromise the test. One team member started and controlled the engine with a servo tester. Then another teammate collected electrical data with the wireless v/a meter. At the beginning of the testing the atmospheric conditions was collected. Then the team member started the engine, brought it to idle and checked for issues, while another member starts the collection of thrust data. Once each of these tasks have been done the team started increasing the rpm of the engine to 40% then 60% throttle and turned off. Then the nozzle was taken apart to make sure the components had not failed before 100% throttle. During each of these increments, watts of electrical power was measured by the v/a meter.

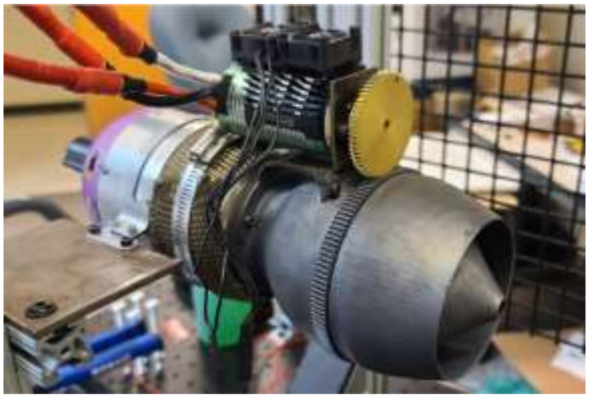


Figure 18. Engine on Test Stand

Figure 19. Moment Arm Thrust Measurement Stand

Figure 20. Electrical System on Cage

During our test in Dayton, Ohio, we achieved significantly less thrust, and watts produced. This could have been due to a variety of factors. One of the main reasons for this decrease in thrust and watts was that once the exhaust gases reached our bearing, the bearing started to carbonize creating significantly more frictional resistance on the turbine until the bearing finally seized up. We believe this caused the turbine to spin significantly slower than our design rpm which caused a significant loss in watts as well as thrust. Our test data from the test in Dayton is shown below as well as the thrust and voltage vs time graphs. It can be seen on the voltage vs time graph that the turbine seized around 70% throttle while the test ended at 80% throttle. The voltage can be seen to decrease at each throttle percentage when held at constant throttle which we believe is due to the bearing starting to seize and creating extra frictional resistance.

Tested Engine Specifications	
80% Throttle	6.87 lbf
Weight	5.14 lbf
T/W	1.34
Electric Generation	53 Watts

Table 3. Data Collected from Testing in Dayton

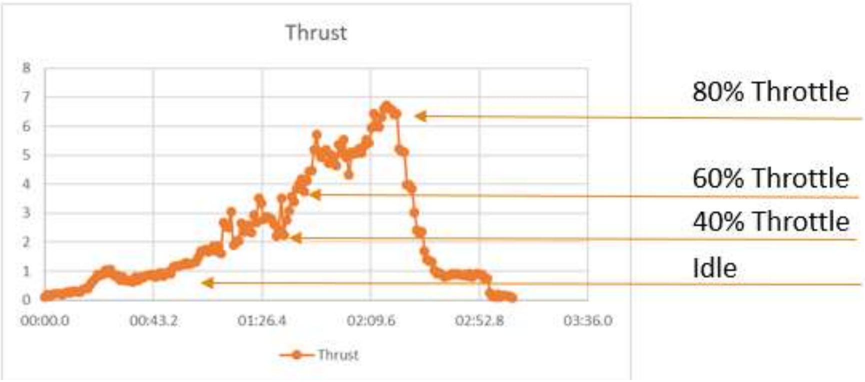


Figure 21. Dayton Testing Thrust vs Time

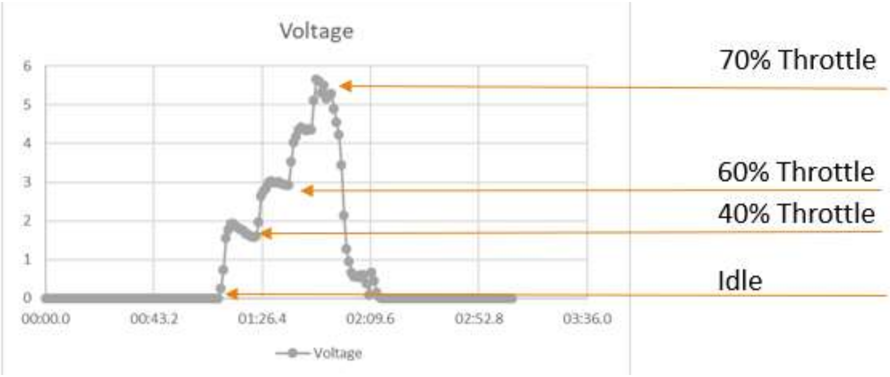


Figure 22. Dayton Testing Voltage vs Time

IV. Conclusion

The observations from the study show that the T/W will decrease by 86%, but the engine will have the additional benefit of having 53 watts of electrical power that can be used as needed. The amount of thrust and electrical power produced are expected to increase drastically when tested again in the future with a more robust bearing that is better suited for the hot environment. During testing, the design showed no signs of wearing or had any catastrophic failures to the design or the engine. This leads the team to assume that the design is solid and can withstand longer run times.

Moving forward with the design there are some changes that the team would recommend. The first change is to increase the RPM of the turbine. This will help the turbine create more torque to overcome added resistance from the bearing and electric motor. Also, increasing the amount of power take off will increase the height of the turbine blades and decrease the flow separation. Finally, having a bolt on front lip to the turbine instead of it being built into the turbine will make it easier to change the internal diameter of the turbine for a better fit on to the shaft.

V. References

[1] Mattingly, Jack D., and Keith M. Boyer. *Elements of Propulsion: Gas Turbines and Rockets*. AIAA, 2016.

[2] Mattingly, Jack D., et al. *Aircraft Engine Design*. American Institute of Aeronautics and Astronautics, Inc, 2018.

VI. Appendix

A. PCA

Mathcad

JetCat P100 PCA:

Given/ Measured Assumed Results

Flight Conditions:

Alititude:= 0ft $M_0 := 0$

$P_0 := 14.2\text{psi}$ $T_0 := 557.91\text{R}$ $a_0 := 1125.33\,\frac{\text{ft}}{\text{s}}$ $V_0 := M_0 \cdot a_0 = 0$

Level of Technology:

$\pi_d := 0.97$ $\pi_b := .93$ $\pi_n := 0.95$ $\eta_b := 0.88$ $\eta_{mech} := 0.97$ $\pi_c := 2.9$

$e_c := 0.84$ $e_t := 0.85$

LOT:

pi_d = 2

pi_b = 1

pi_n = 2

η_mech = 2

e_c = 2

e_t = 2

Other Values:

$c_{pc} := 0.24\,\frac{\text{BTU}}{\text{lbmR}}$ $c_{pt} := 0.27\,\frac{\text{BTU}}{\text{lbm}\cdot\text{R}}$ $g_c := 32.2\,\frac{\text{lbm}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2}$ $h_{PR} := 18700\,\frac{\text{BTU}}{\text{lbm}}$

$\gamma_c := 1.4$ $\gamma_t := 1.33$ $\text{P0\_P9} := 1$

Jetcat website/ manual:

$\text{m\_dot}_{air} := 0.23\,\frac{\text{kg}}{\text{s}} = 0.507 \cdot \frac{\text{lbm}}{\text{s}}$

$\text{m\_dot}_{fuel} := .0131\,\frac{\text{lbm}}{\text{s}}$

$\text{m\_dot}_{total} := \text{m\_dot}_{air} + \text{m\_dot}_{fuel} = 0.52 \cdot \frac{\text{lbm}}{\text{s}}$

PCA:

$$\text{Tt0\_T0} := 1 + \frac{\gamma_c - 1}{2} \cdot M_0^{\,2} = 1 \qquad \tau_r := \text{Tt0\_T0} = 1$$

$$\text{Pt0\_P0} := \text{Tt0\_T0}^{\frac{\gamma_c}{\gamma_c - 1}} = 1 \qquad \pi_r := \text{Pt0\_P0} = 1$$

$$f := \frac{\text{m\_dot}_{fuel}}{\text{m\_dot}_{air}} = 0.026$$

Temperatures:

$$T_{t0} := \text{Tt0\_T0} \cdot T_0 = 557.91 \cdot \text{R}$$

$$T_{t2} := T_{t0} = 557.91 \cdot \text{R}$$

$$T_{t3} := T_{t2} \cdot \pi_c^{\frac{\gamma_c - 1}{\gamma_c \cdot e_c}} = 801.387 \cdot \text{R}$$

$$T_{t9} := 1787.67\text{R}$$

$$T_{t5} := T_{t9} = 1.788 \times 10^3 \cdot \text{R}$$

$$T_{t4} := T_{t5} + \frac{\left[c_{pc} \cdot \left(T_{t3} - T_{t2}\right)\right]}{\left[(1 + f) \cdot \eta_{mech} \cdot c_{pt}\right]} = 2.005 \times 10^3 \cdot \text{R}$$

$$\pi_t := \left(\frac{T_{t5}}{T_{t4}}\right)^{\frac{\gamma_t}{(\gamma_t - 1) \cdot e_t}} = 0.58$$

$$\mathrm{Pt9\_P9} := \pi_r \pi_d \pi_c \pi_b \pi_t \pi_n \cdot \mathrm{P0\_P9} = 1.442$$

$$\mathrm{Tt9\_T9} := \mathrm{Pt9\_P9}^{\frac{\gamma_t - 1}{\gamma_t}} = 1.095 \qquad T_9 := \frac{T_{t9}}{\mathrm{Tt9\_T9}} = 1.632 \times 10^3 \cdot \mathrm{R}$$

Pressures:

$$P_{t0} := \mathrm{Pt0\_P0} \cdot P_0 = 2.045 \times 10^3 \cdot \mathrm{psf}$$

$$P_{t2} := P_{t0} \cdot \pi_d = 1.983 \times 10^3 \cdot \mathrm{psf}$$

$$P_{t3} := P_{t2} \cdot \pi_c = 5.752 \times 10^3 \cdot \mathrm{psf}$$

$$P_{t4} := P_{t3} \cdot \pi_b = 5.349 \times 10^3 \cdot \mathrm{psf}$$

$$P_{t5} := P_{t4} \cdot \pi_t = 3.104 \times 10^3 \cdot \mathrm{psf}$$

$$P_{t9} := P_{t5} \cdot \pi_n = 2.948 \times 10^3 \cdot \mathrm{psf}$$

Figures of Merit:

$$V_9 := \left[\left(T_{t9} - T_9 \right) \cdot 2 \cdot g_c \cdot c_{pt} \right]^{\frac{1}{2}} = 1.449 \times 10^3 \cdot \frac{\mathrm{ft}}{\mathrm{s}}$$

$$F := \mathrm{m\_dot}_{total} \cdot V_9 = 23.427 \cdot \mathrm{lbf}$$

$$\mathrm{F\_m}_{dot} := \frac{(1+f) \cdot V_9 - V_0}{g_c} = 46.164 \cdot \frac{\mathrm{lbf}}{\frac{\mathrm{lbm}}{\mathrm{s}}}$$

$$\mathrm{SFC} := \frac{f}{\mathrm{F\_m}_{dot}} = 2.015 \cdot \frac{\left(\frac{\mathrm{lbm}}{\mathrm{hr}} \right)}{\mathrm{lbf}}$$

$$\eta_p := \frac{2 \cdot g_c \cdot V_0 \cdot \mathrm{F\_m}_{dot}}{(1+f) \cdot V_9^2 - V_0^2} = 0$$

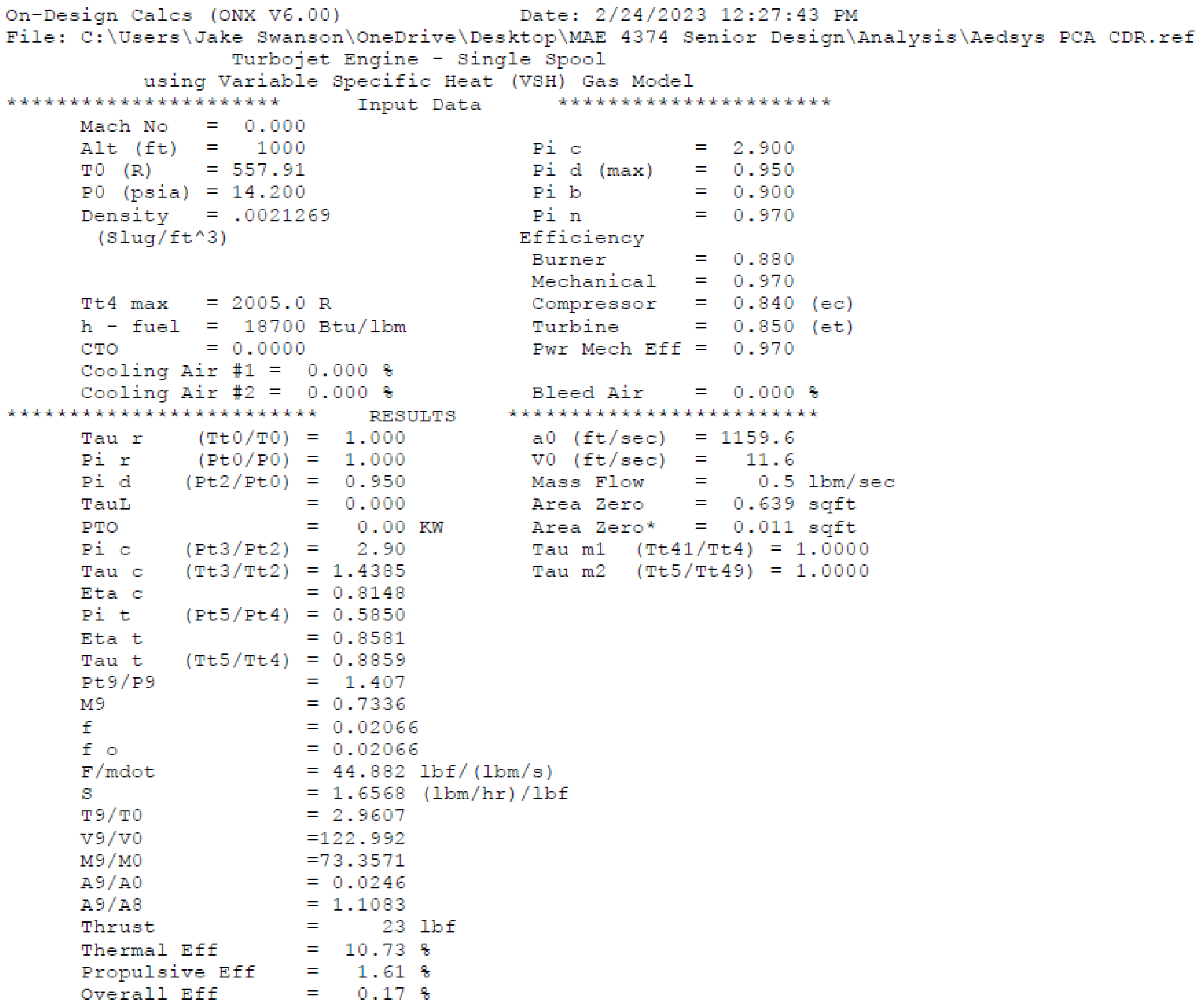
$$\eta_{th} := \frac{(1+f) \cdot V_9^2 - V_0^2}{2 \cdot g_c \cdot f \cdot h_{PR}} = 0.089$$

$$\eta_o := \eta_{th} \cdot \eta_p = 0$$

PCA Results Comarison:

Mathcad:	**AEDSys:**	**Testing:**
F = 23.427lbf	F = 23lbf	F = 23.57lbf
Specific Thrust = 46.164 lbf/lbm/s	Specific Thrust = 44.882 lbf/lbm/s	Specific Thrust = 45.33 lbf/lbm/s
SFC = 2.015 lbm/hr/lbf	SFC = 1.6568 lbm/hr/lbf	SFC = 2.052 lbm/hr/lbf
Propulsive Efficiency = 0%	Propulsive Efficiency = 1.61%	
Thermal Efficiency = 8.9%	Thermal Efficiency = 10.73%	
Overall Efficiency = 0%	Overall Efficiency = 0.17%	

AEDsys



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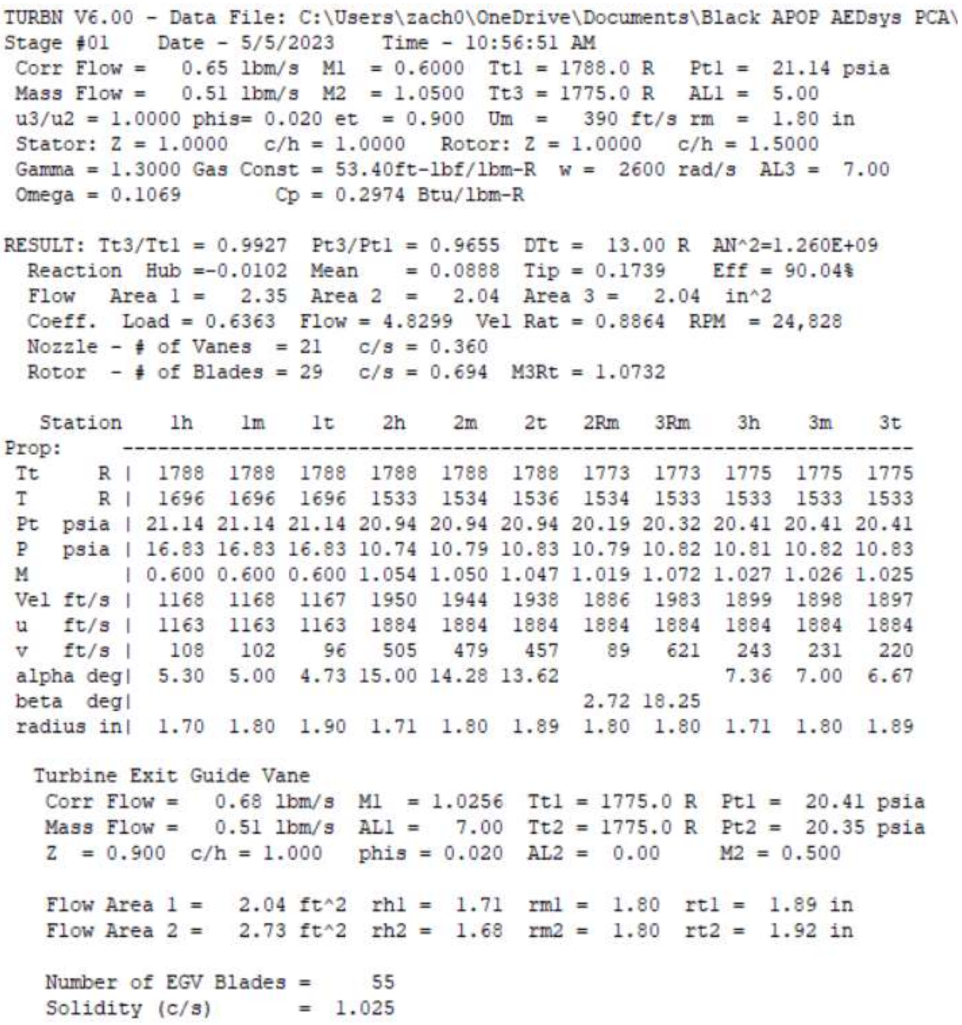
On-Design Calcs (ONX V6.00)              Date: 2/24/2023 12:27:43 PM
File: C:\Users\Jake Swanson\OneDrive\Desktop\MAE 4374 Senior Design\Analysis\Aedsys PCA CDR.ref
                  Turbojet Engine - Single Spool
           using Variable Specific Heat (VSH) Gas Model
**********************      Input Data      **********************
      Mach No    =  0.000
      Alt (ft)   =   1000                       Pi c          =  2.900
      T0 (R)     = 557.91                       Pi d (max)    =  0.950
      P0 (psia)  = 14.200                       Pi b          =  0.900
      Density    = .0021269                     Pi n          =  0.970
       (Slug/ft^3)                             Efficiency
                                                Burner        =  0.880
                                                Mechanical    =  0.970
      Tt4 max    = 2005.0 R                     Compressor    =  0.840 (ec)
      h - fuel   =  18700 Btu/lbm               Turbine       =  0.850 (et)
      CTO        = 0.0000                       Pwr Mech Eff  =  0.970
      Cooling Air #1 =  0.000 %
      Cooling Air #2 =  0.000 %                 Bleed Air     =  0.000 %
*************************    RESULTS    *************************
      Tau r     (Tt0/T0)  =  1.000              a0 (ft/sec)   = 1159.6
      Pi r      (Pt0/P0)  =  1.000              V0 (ft/sec)   =   11.6
      Pi d     (Pt2/Pt0)  =  0.950              Mass Flow     =    0.5 lbm/sec
      TauL                =  0.000              Area Zero     =  0.639 sqft
      PTO                 =   0.00 KW           Area Zero*    =  0.011 sqft
      Pi c     (Pt3/Pt2)  =   2.90              Tau m1  (Tt41/Tt4)  = 1.0000
      Tau c    (Tt3/Tt2)  = 1.4385              Tau m2  (Tt5/Tt49)  = 1.0000
      Eta c               = 0.8148
      Pi t     (Pt5/Pt4)  = 0.5850
      Eta t               = 0.8581
      Tau t    (Tt5/Tt4)  = 0.8859
      Pt9/P9              =  1.407
      M9                  = 0.7336
      f                   = 0.02066
      f o                 = 0.02066
      F/mdot              = 44.882 lbf/(lbm/s)
      S                   = 1.6568 (lbm/hr)/lbf
      T9/T0               = 2.9607
      V9/V0               =122.992
      M9/M0               =73.3571
      A9/A0               = 0.0246
      A9/A8               = 1.1083
      Thrust              =     23 lbf
      Thermal Eff         =  10.73 %
      Propulsive Eff      =   1.61 %
      Overall Eff         =   0.17 %
```

B. AEDsys TURBN



```
TURBN V6.00 - Data File: C:\Users\zach0\OneDrive\Documents\Black APOP AEDsys PCA\
Stage #01    Date - 5/5/2023    Time - 10:56:51 AM
 Corr Flow =   0.65 lbm/s  M1  = 0.6000  Tt1 = 1788.0 R   Pt1 =  21.14 psia
 Mass Flow =   0.51 lbm/s  M2  = 1.0500  Tt3 = 1775.0 R   AL1 =  5.00
 u3/u2 = 1.0000 phis= 0.020 et  = 0.900  Um  =   390 ft/s rm  =  1.80 in
 Stator: Z = 1.0000   c/h = 1.0000   Rotor: Z = 1.0000   c/h = 1.5000
 Gamma = 1.3000 Gas Const = 53.40ft-lbf/lbm-R  w =  2600 rad/s  AL3 =  7.00
 Omega = 0.1069        Cp = 0.2974 Btu/lbm-R

RESULT: Tt3/Tt1 = 0.9927  Pt3/Pt1 = 0.9655  DTt =  13.00 R  AN^2=1.260E+09
  Reaction  Hub =-0.0102  Mean     = 0.0888  Tip = 0.1739    Eff = 90.04%
  Flow   Area 1 =   2.35  Area 2  =   2.04  Area 3 =   2.04  in^2
  Coeff.  Load = 0.6363  Flow = 4.8299  Vel Rat = 0.8864  RPM  = 24,828
  Nozzle - # of Vanes  = 21   c/s = 0.360
  Rotor  - # of Blades = 29   c/s = 0.694  M3Rt = 1.0732

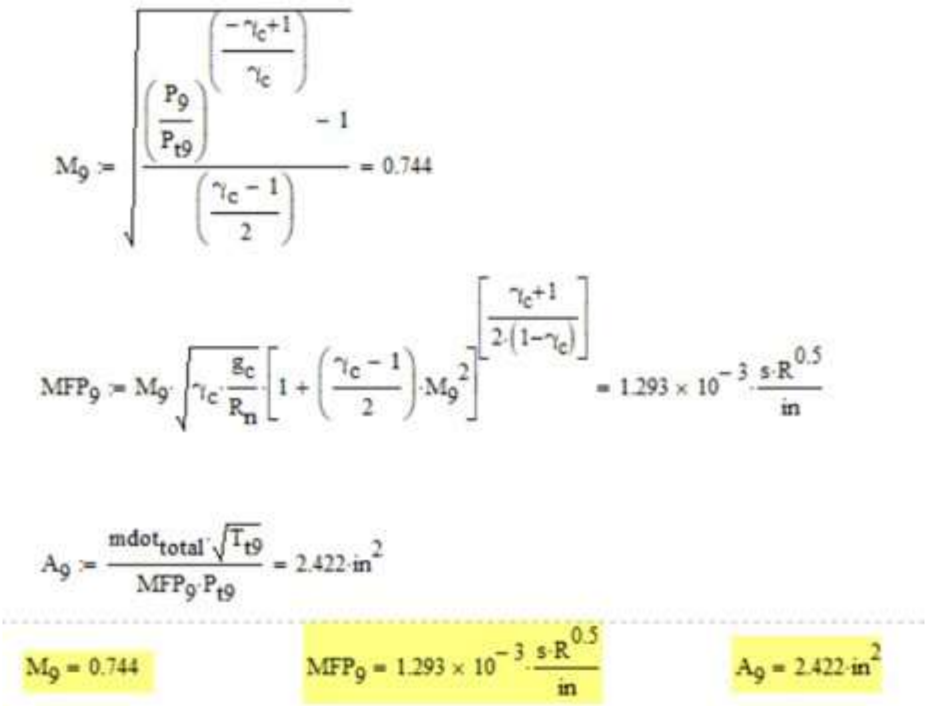
   Station    1h    1m    1t    2h    2m    2t   2Rm   3Rm    3h    3m    3t
Prop:      ------------------------------------------------------------------
 Tt     R |  1788  1788  1788  1788  1788  1788  1773  1773  1775  1775  1775
 T      R |  1696  1696  1696  1533  1534  1536  1534  1533  1533  1533  1533
 Pt  psia | 21.14 21.14 21.14 20.94 20.94 20.94 20.19 20.32 20.41 20.41 20.41
 P   psia | 16.83 16.83 16.83 10.74 10.79 10.83 10.79 10.82 10.81 10.82 10.83
 M        | 0.600 0.600 0.600 1.054 1.050 1.047 1.019 1.072 1.027 1.026 1.025
 Vel ft/s |  1168  1168  1167  1950  1944  1938  1886  1983  1899  1898  1897
 u   ft/s |  1163  1163  1163  1884  1884  1884  1884  1884  1884  1884  1884
 v   ft/s |   108   102    96   505   479   457    89   621   243   231   220
 alpha deg|  5.30  5.00  4.73 15.00 14.28 13.62              7.36  7.00  6.67
 beta  deg|                                      2.72 18.25
 radius in|  1.70  1.80  1.90  1.71  1.80  1.89  1.80  1.80  1.71  1.80  1.89

  Turbine Exit Guide Vane
   Corr Flow =   0.68 lbm/s  M1  = 1.0256  Tt1 = 1775.0 R  Pt1 =  20.41 psia
   Mass Flow =   0.51 lbm/s  AL1 =   7.00  Tt2 = 1775.0 R  Pt2 =  20.35 psia
   Z  = 0.900  c/h = 1.000   phis = 0.020  AL2 =  0.00     M2 = 0.500

   Flow Area 1 =   2.04 ft^2  rh1 =  1.71  rm1 =  1.80  rt1 =  1.89 in
   Flow Area 2 =   2.73 ft^2  rh2 =  1.68  rm2 =  1.80  rt2 =  1.92 in

   Number of EGV Blades =     55
   Solidity (c/s)       =  1.025
```

C. MFP



$$M_9 := \sqrt{\frac{\left(\frac{P_9}{P_{t9}}\right)^{\left(\frac{-\gamma_c+1}{\gamma_c}\right)} - 1}{\left(\frac{\gamma_c - 1}{2}\right)}} = 0.744$$

$$MFP_9 := M_9 \cdot \sqrt{\gamma_c \cdot \frac{g_c}{R_n}} \cdot \left[1 + \left(\frac{\gamma_c - 1}{2}\right) \cdot M_9^{\,2}\right]^{\left[\frac{\gamma_c+1}{2\cdot(1-\gamma_c)}\right]} = 1.293 \times 10^{-3} \cdot \frac{s \cdot R^{0.5}}{in}$$

$$A_9 := \frac{mdot_{total} \cdot \sqrt{T_{t9}}}{MFP_9 \cdot P_{t9}} = 2.422 \cdot in^2$$

$M_9 = 0.744$ $MFP_9 = 1.293 \times 10^{-3} \cdot \frac{s \cdot R^{0.5}}{in}$ $A_9 = 2.422 \cdot in^2$