



Smith, S., Nguyen, D. H., Knowles, J., Lowenberg, M. H., & Biggs, S. (2023). *An initial bifurcation analysis of an EV pickup truck*. Abstract from NODYCON 2023, Rome, Italy.

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NODYCON 2023

Third International Nonlinear Dynamics Conference

Rome, June 18-22, 2023

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An Initial Bifurcation Analysis of an EV Pickup Truck

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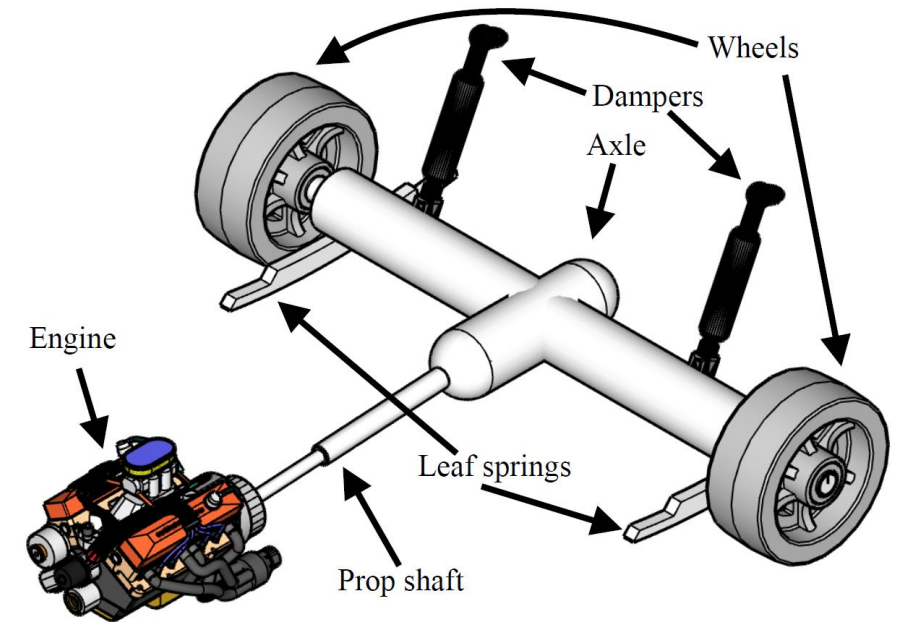
Jaguar Land Rover³

NODYCON 2023, ROME, JUNE 18-22, 2023



Introduction

- Vehicles with a beam axle setup can undergo a problem known as “axle tramp”.

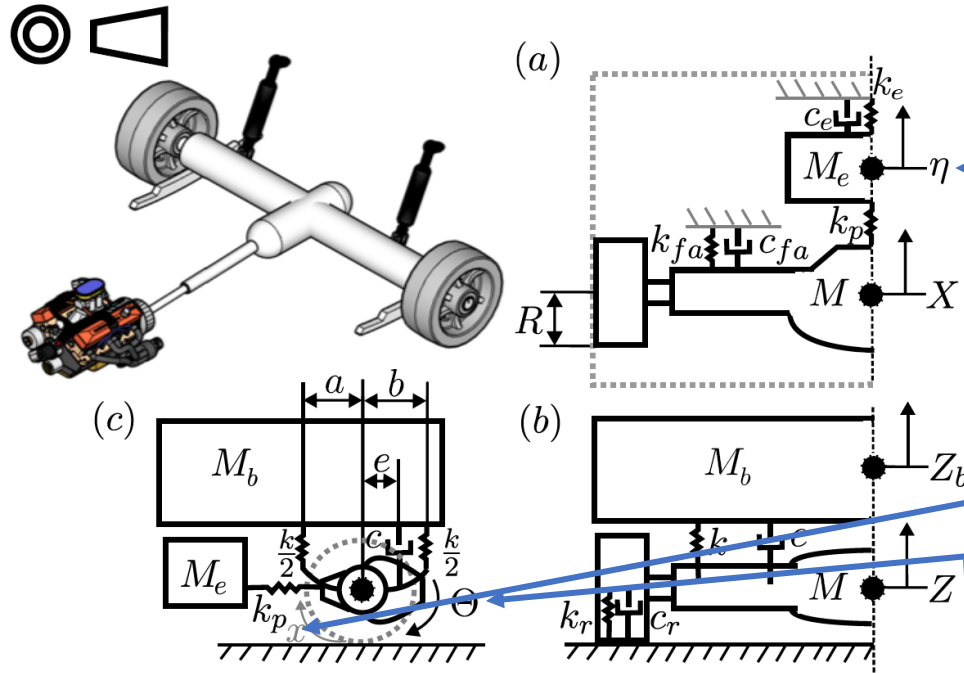


- Electrification requires changes to the powertrain which can alter the problem.
- Study compares bifurcations in a car and electric truck model.



Beam Axle Model

- Equations of motion for a 6DOF beam axle;



Longitudinal;

- Axle & wheel longitudinal displacement, X
- Engine longitudinal displacement, η

Vertical;

- Axle & wheel vertical displacement, Z
- Body vertical displacement, Z_B

Angular;

- Wheel angular displacement, δx
- Axle pitch, θ

$$\ddot{X} = (-c_{fa}\dot{X} - k_{fa}X + F + (k_{fa}n + kph)\theta + k_p h \eta) / (m + M)$$

$$\ddot{\eta} = (-C_e \dot{\eta} - k_p \eta - k_e \eta + k_p X - kph\theta) / (M_e)$$

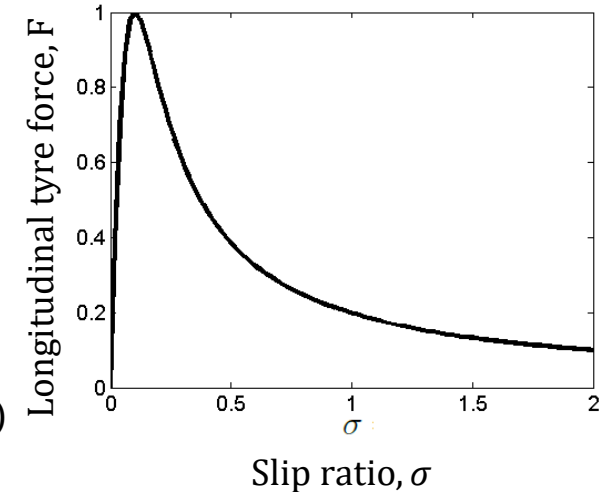
$$\ddot{Z} = (C(\dot{Z} - \dot{Z}_b) + k(Z_b - (Z - R)) - k_r(Z - R) - c_r \dot{Z} - mg - Ce\dot{\theta} - k(b - a)\theta) / (m + M)$$

$$\ddot{Z}_b = (-C(\dot{Z}_b - \dot{Z}) - k(Z_b - (Z - R)) - (M_b)g) / (M_b)$$

$$\ddot{\delta x} = (-c_1 \dot{\delta x} - k_1 \delta x - F) / i_y$$

$$\ddot{\theta} = (-Ce^2 \dot{\theta} - (k(a^2 + b^2) + k_{fa}n^2 + kph^2)\theta - Ce\dot{Z} - k(b - a)(Z - R) + (kn + kph)X - k_p h \eta)$$

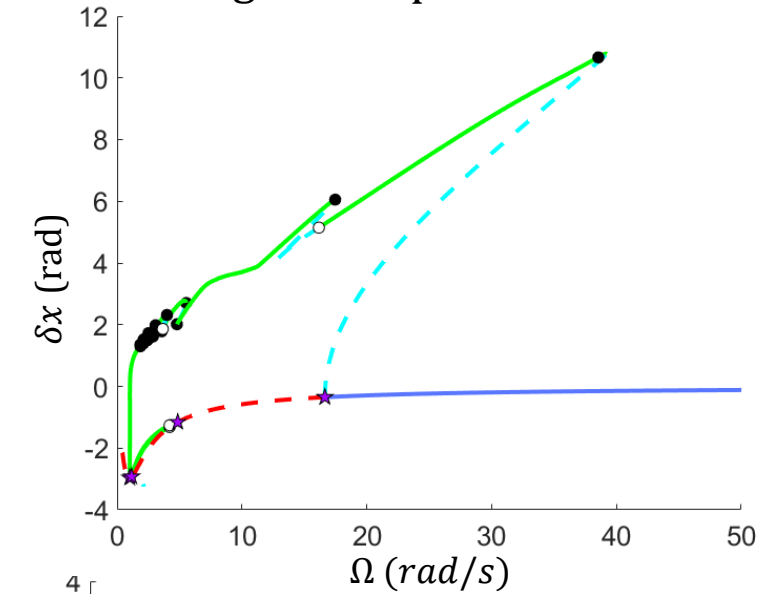
k=stiffness, c=damping, m=masses, F=tyre force, i=inertia, g=gravity, other=dimensions



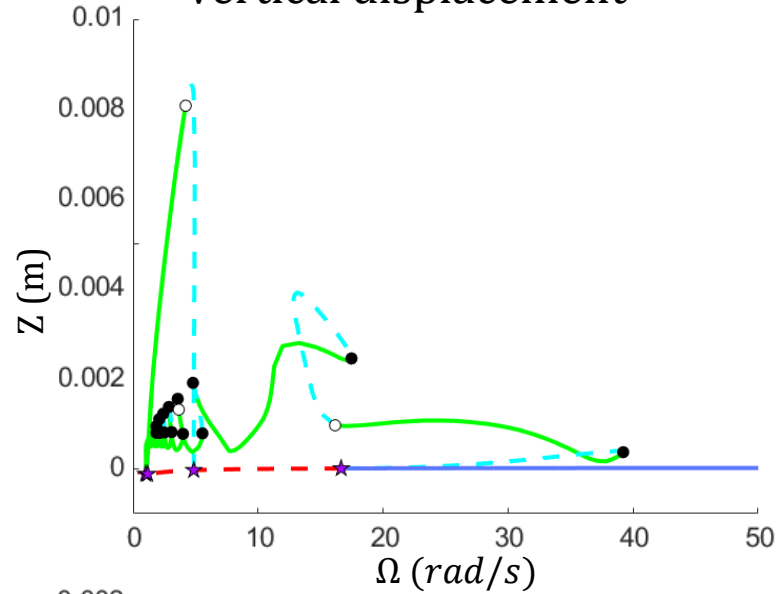


Car Results

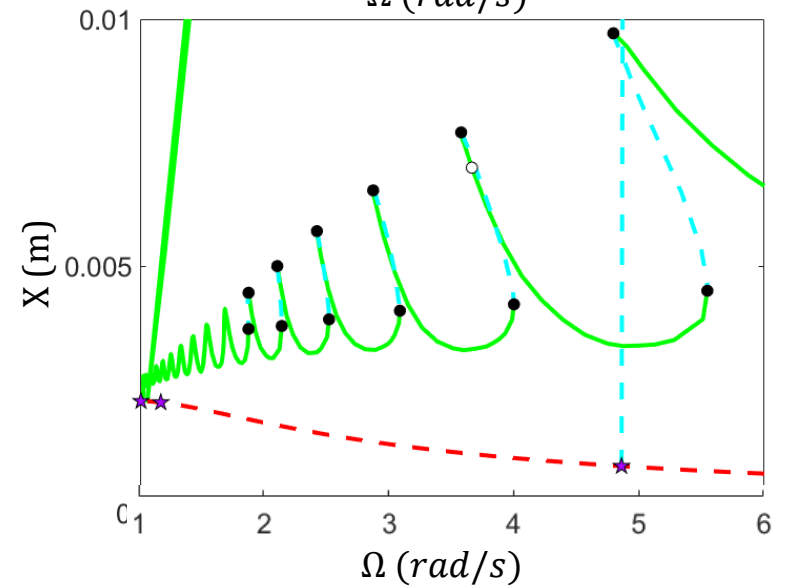
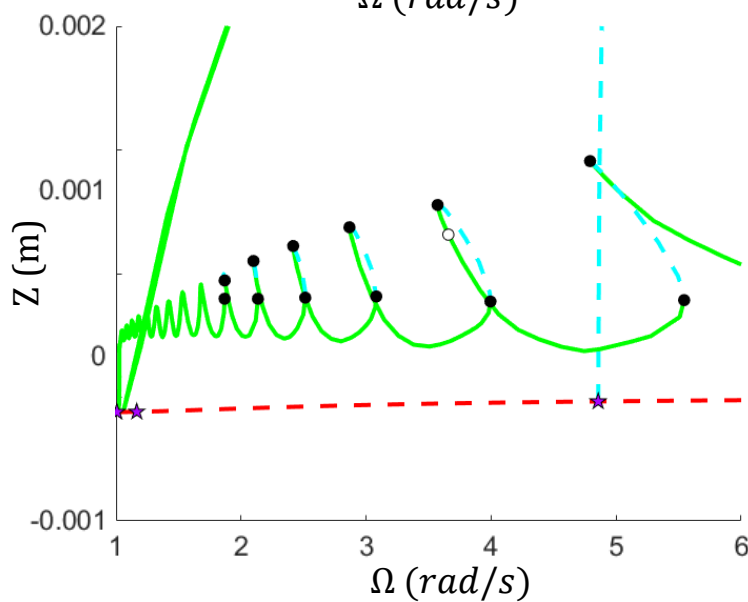
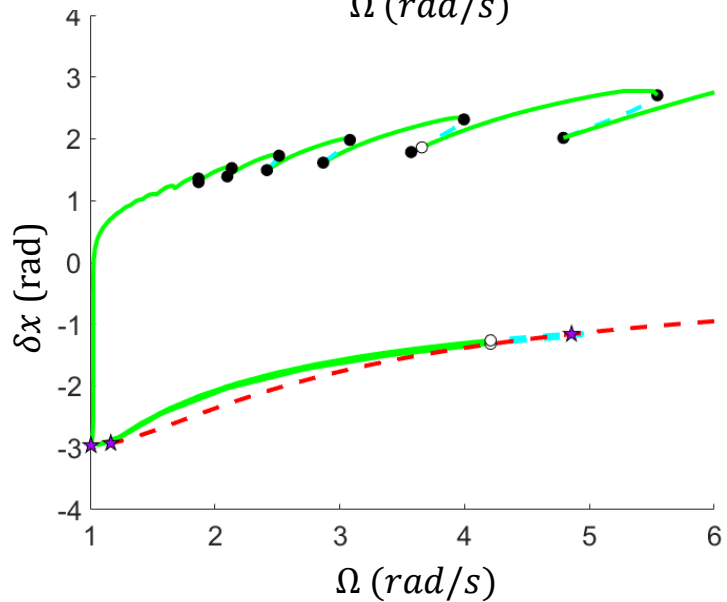
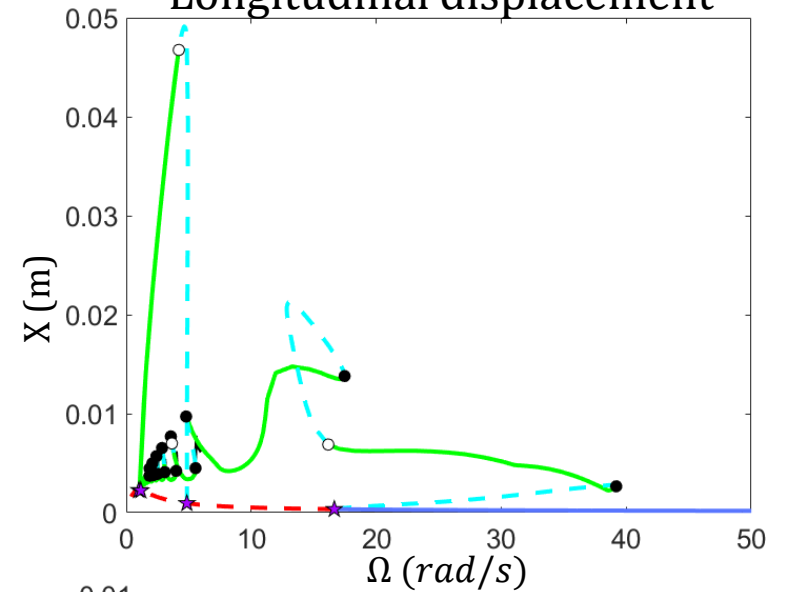
Angular displacement



Vertical displacement



Longitudinal displacement





From Petrol Car to Electric Truck

- Internal Combustion Engine Car



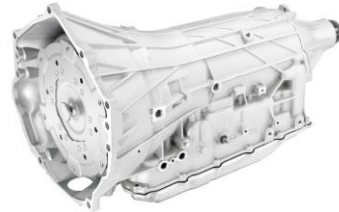
Crank



Dual Mass Flywheel



Clutch



Transmission



Halfshaft



Wheel

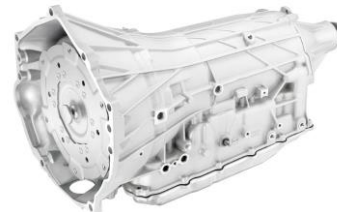


Tyre

- Electric Truck



Electric Motor



Transmission



Halfshaft



Wheel



Tyre



From Petrol Car to Electric Truck

- Key Parameter Changes From Car Model

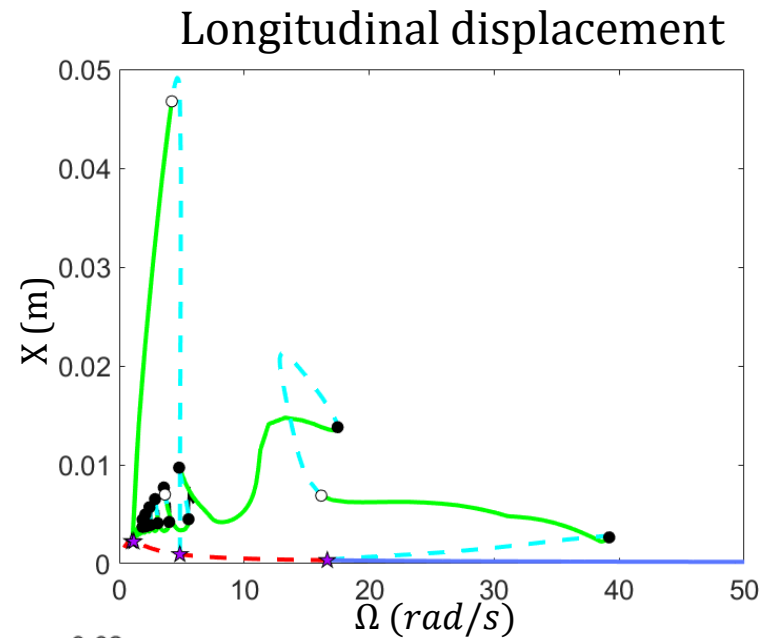
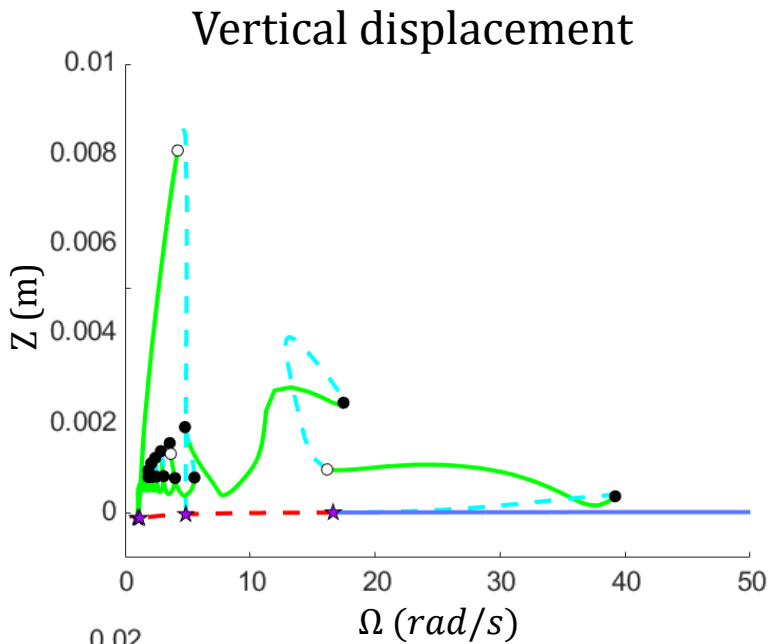
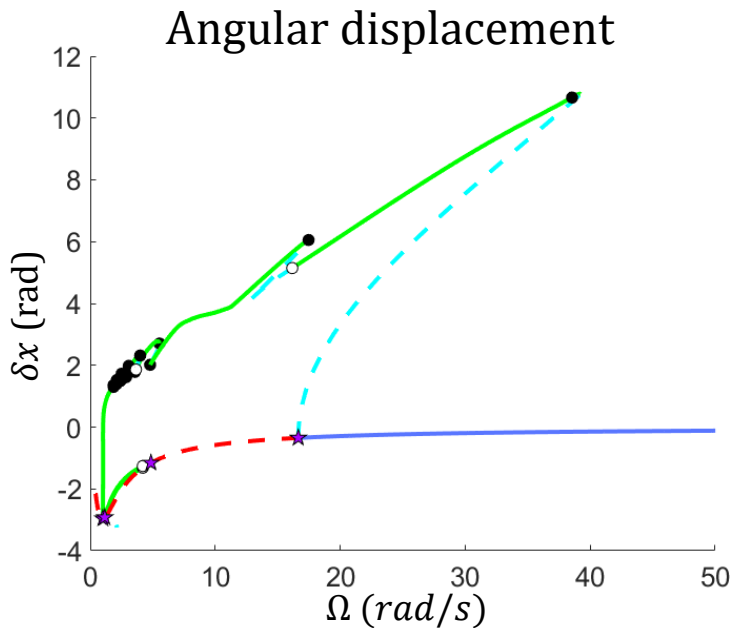
Parameter	Car	EV Truck	Unit	Approx change
Wheel radius	0.28	0.45	m	1.5x
Wheel mass	18	54	Kg	3x
Mass of axle assembly	31	92	Kg	3x
Mass of body	120	360	Kg	3x
Axle pitch inertia	0.27	0.81	Kgm ²	3x
Suspension stiffness	11000	33000	N/m	3x
Longitudinal engine stiffness	500000	1530000	N/m	3x
Engine/Motor mass	160	40	Kg	0.25x
Torsional wheel stiffness (referred)	165	3850	N/m	20x
Torsional wheel inertia (referred)	1.2	2	Kgm ²	1.5x



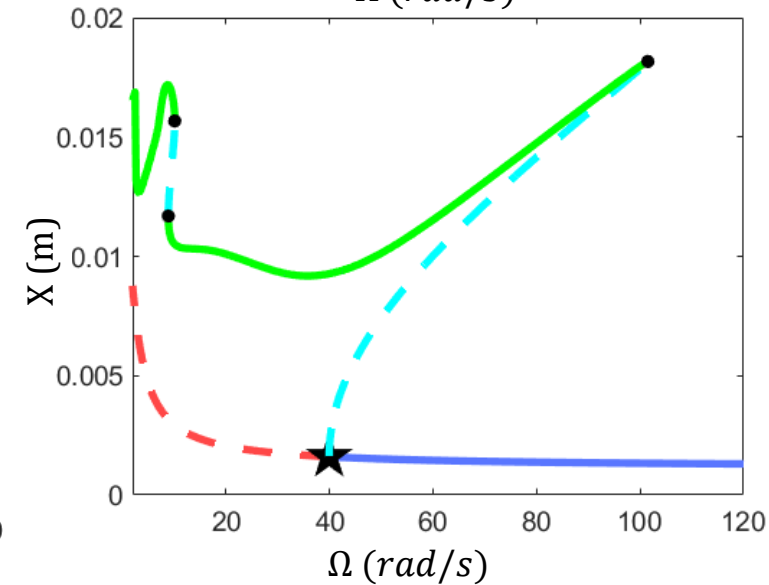
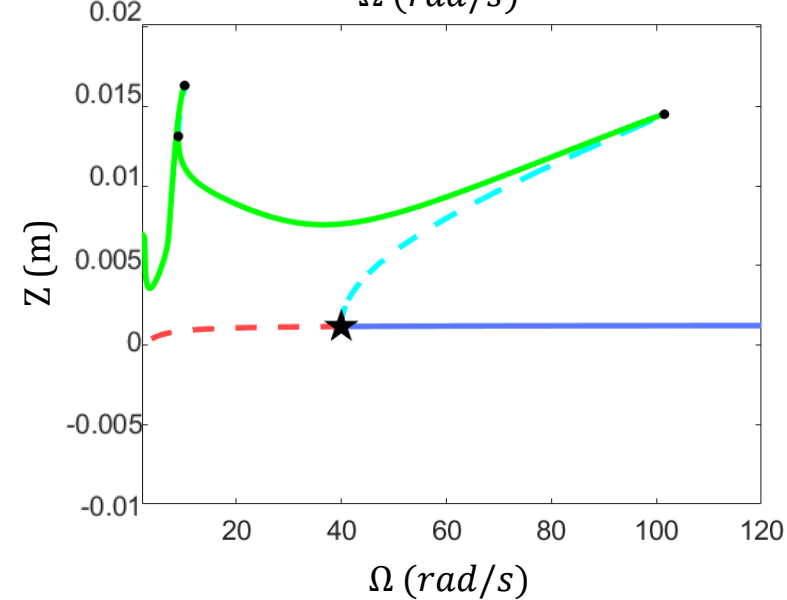
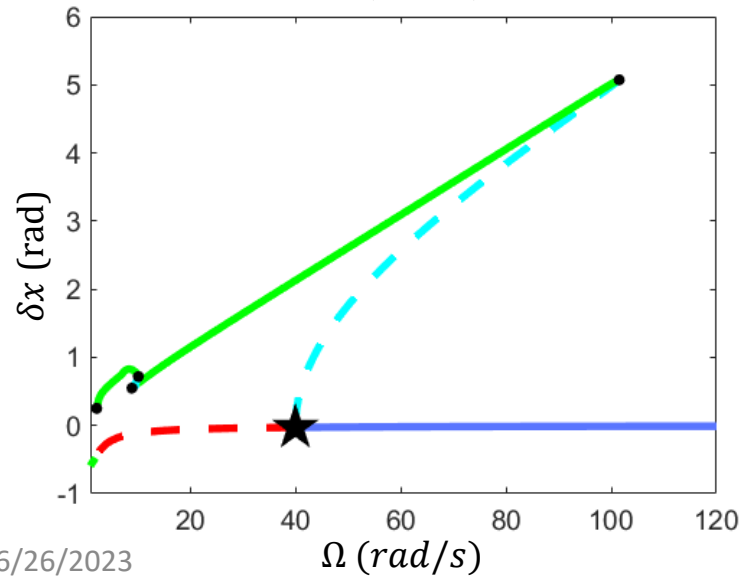


Results

ICE Car

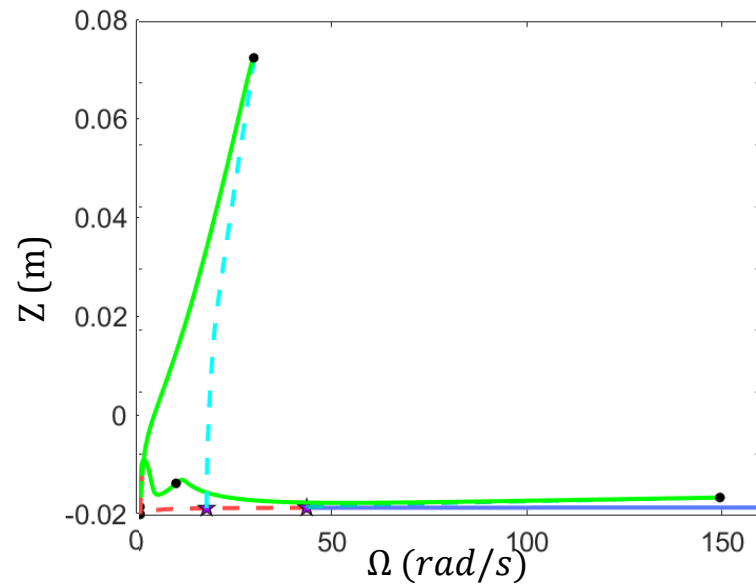
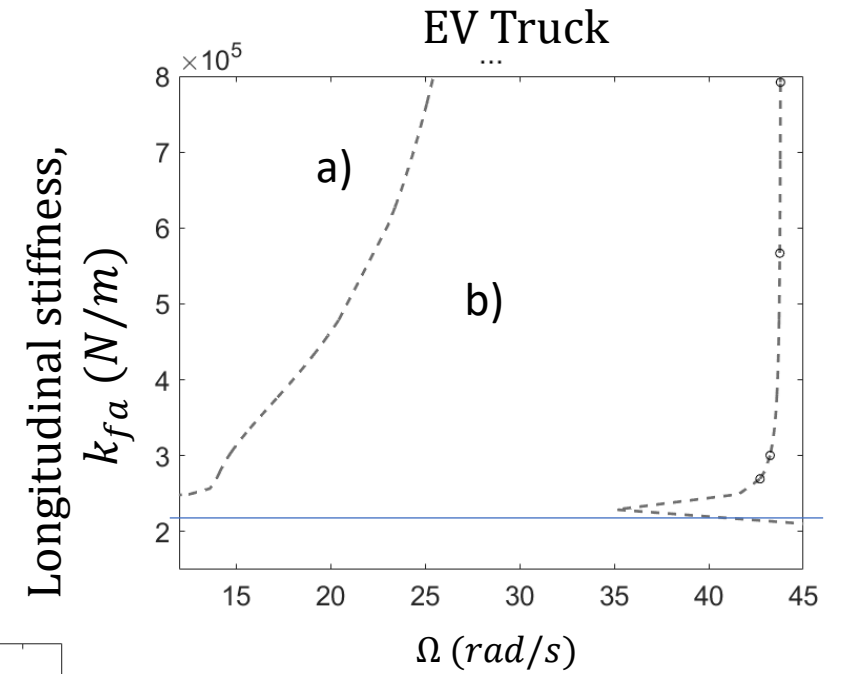
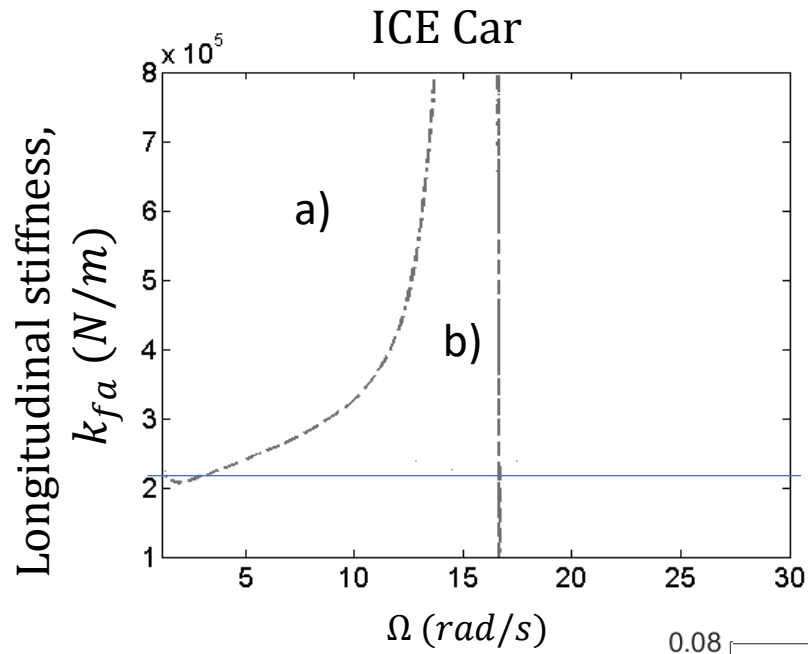


EV Truck





Results





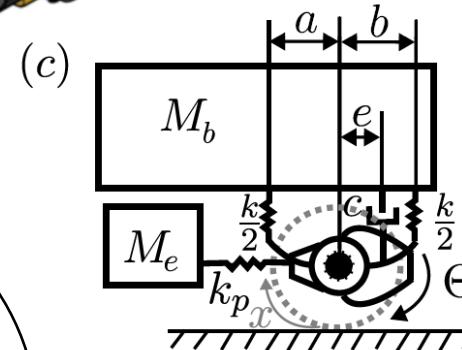
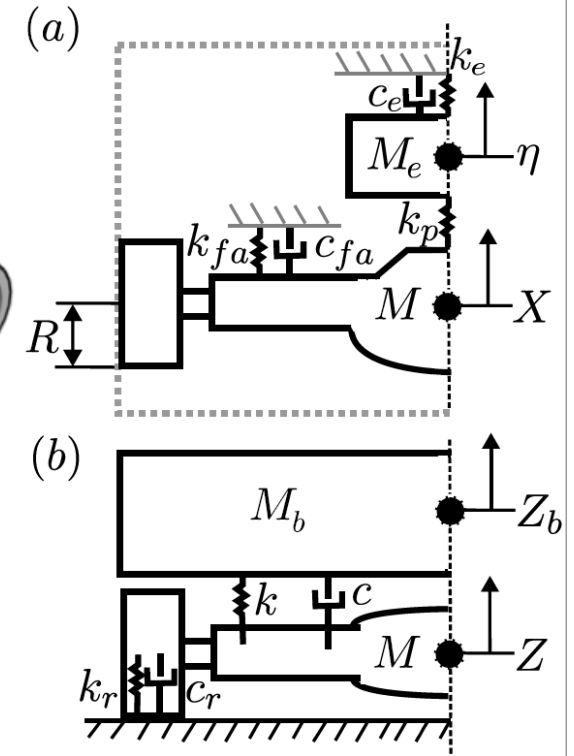
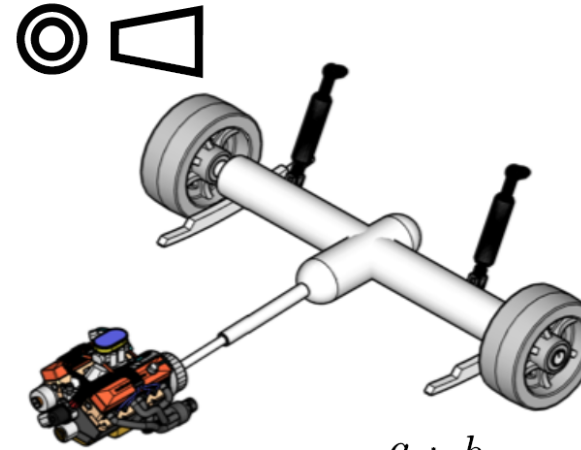
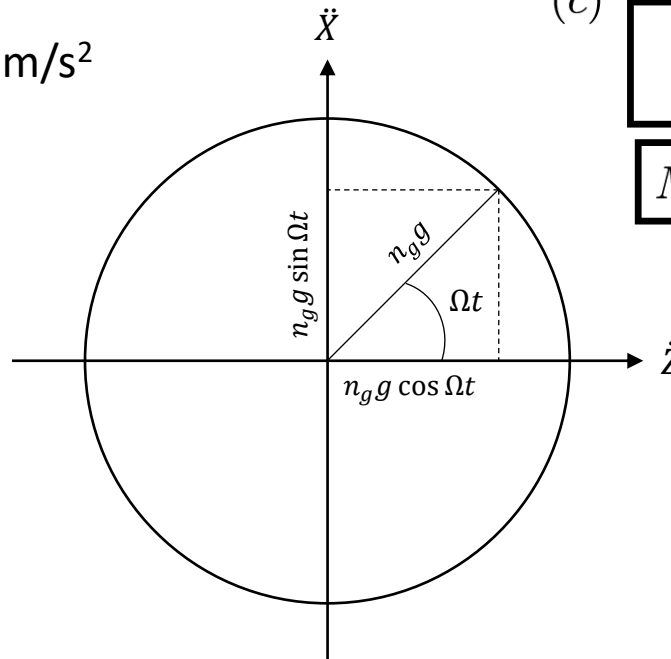
Harmonic Forcing

Additional vibration due to harmonic forcing from powertrain:

$$\ddot{Z} = \dots + n_g g \cos \Omega t$$

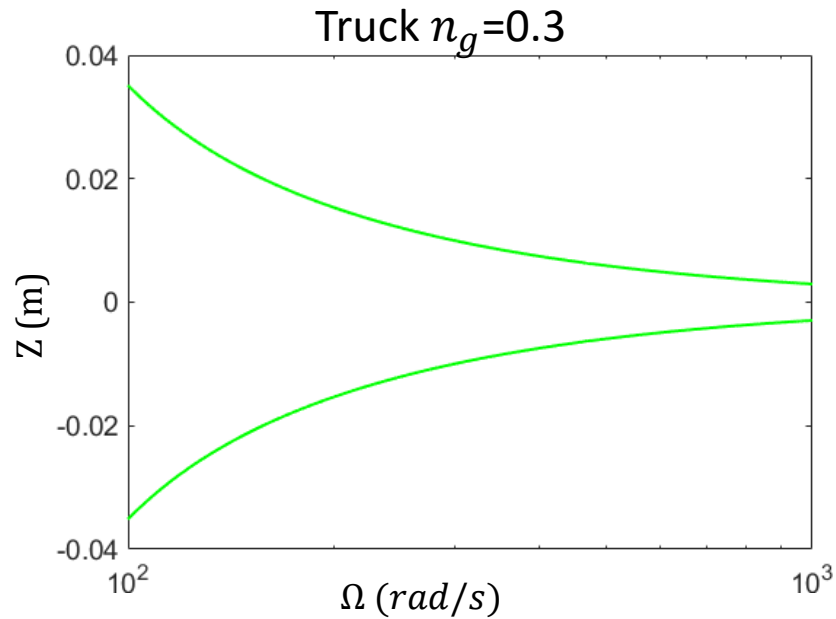
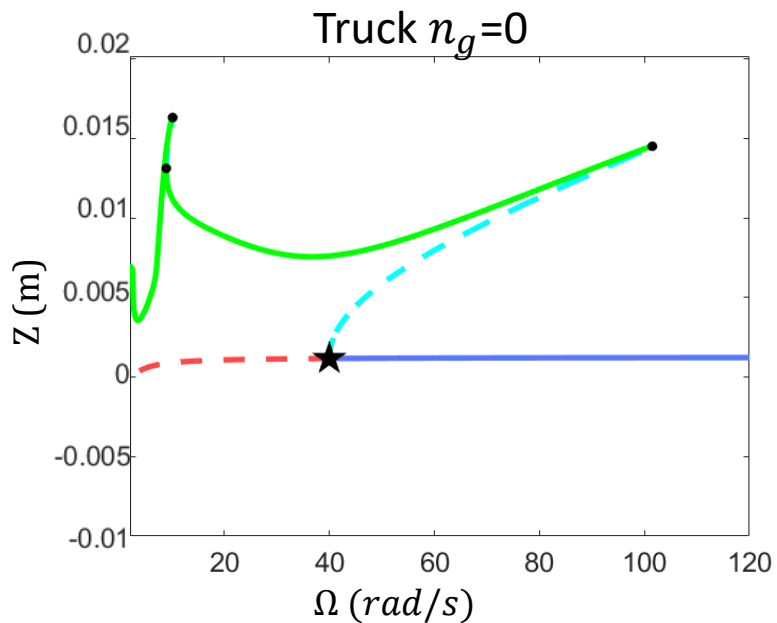
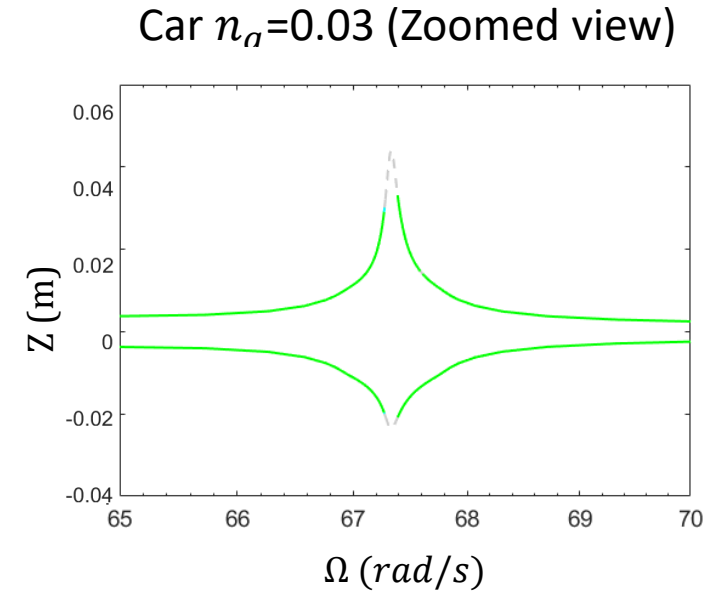
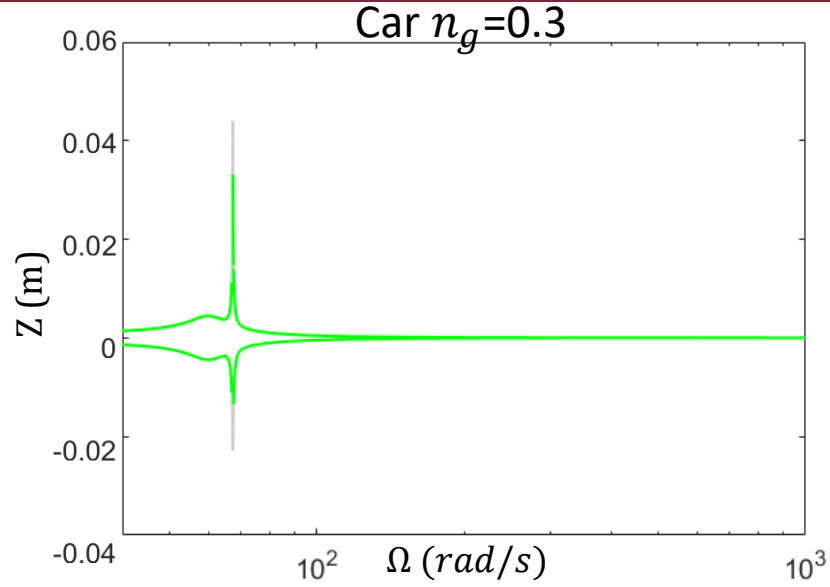
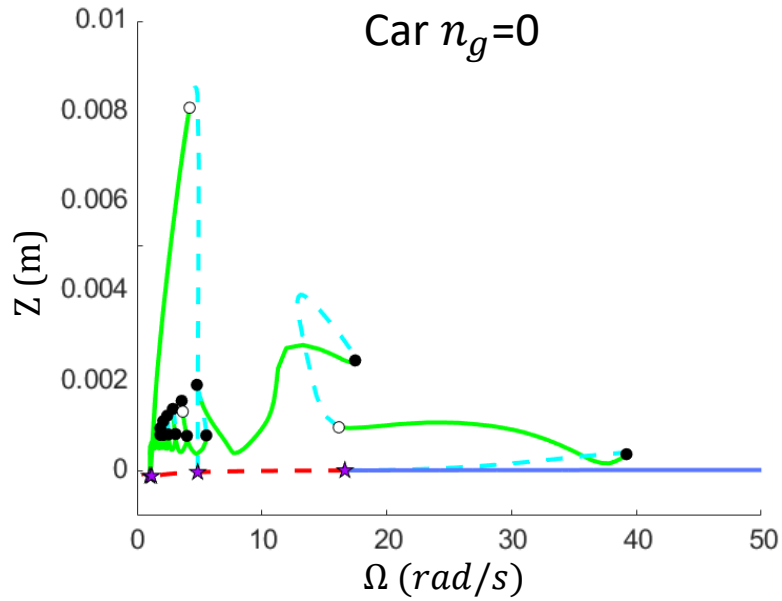
$$\ddot{X} = \dots + n_g g \sin \Omega t$$

$n_g g$: acceleration due to vibration in m/s^2
 Ω : vibration frequency (rad/s)





Harmonic Forcing





- Work presents a preliminary bifurcation study into axle tramp of an EV Truck
- Electric system appears to change the axle tramp problem
 - **System still undergoes tramp**
 - Only one limit cycle in EV parametrisation
 - Amplitudes larger in the vertical component, similar size in longitudinal displacement, smaller in angular displacement
 - **More instabilities are observed at lower speed ranges (less stable equilibria) meaning oscillations occur for a larger speed range**
- System forced with harmonic term
 - Truck does not exhibit unstable behaviour as found in the car
- Future work
 - Explore mechanisms by which bifurcations evolve



- [1] R. Sharp, “Nature and Prevention of Axle Tramp,” *Proc. Inst. Mech. Eng. Automob. Div.*, vol. 184, no. 3, pp. 41–54, 1969, doi: 10.1243/pime_auto_1969_184_011_02.
- [2] A. Zargartalebi and K. H. Shirazi, “Dynamic modelling of axle tramp in a sport type car,” *Shock Vib.*, vol. 20, no. 4, pp. 711–723, 2013, doi: 10.3233/SAV-130779.
- [3] S. Smith, J. Knowles, B. Mason, and S. Biggs, “International Journal of Bifurcation and Chaos A bifurcation analysis and sensitivity study of brake creep groan,” *Int. J. Bifurc. & Chaos*.
- [4] D. H. Nguyen, M. H. Lowenberg, and S. A. Neild, “Frequency-domain bifurcation analysis of a nonlinear flight dynamics model,” *J. Guid. Control. Dyn.*, vol. 44, no. 1, pp. 138–150, 2021, doi: 10.2514/1.G005197.