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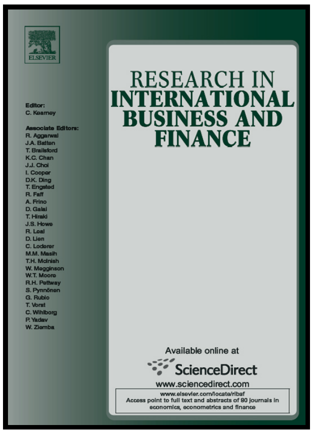

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EUROPEAN SYSTEMIC CREDIT RISK TRANSMISSION USING DYNAMIC BAYESIAN NETWORKS

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Abstract

The analysis of systemic credit risk is one of the most important concerns within the financial system. Its complexity lies in adequately measuring how the transmission of systemic default spreads through assets or financial markets. The transmission structure of systemic credit risk across several European sectoral CDS is studied by dynamic Bayesian networks. The new approach allows for a more advanced analysis of systemic risk transmission, including long-term and more complex relationships. The modelling reveals as relevant only relationships between the original series and one- and two-lagged series. Network structure learning displays a robust and stationary underlying risk transmission structure, pointing to a consolidated transmission mechanism of systemic credit risk between CDSs. Between 5% and 40% of sectoral CDS series variances are explained by the network relationships. The modelling allows us to ascertain which relationships between the CDS series show positive (amplifier) and negative (reducer) effects of systemic risk transmission.

Keywords

CDS, systemic credit risk, dynamic Bayesian networks, vector autoregressive moving average models, Markov Chain Monte Carlo

JEL classification: C11, C45, G15, G32.

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1. Introduction

Systemic risk is inherent to the financial system and consists of the transmission of shocks that can affect financial markets or institutions, which are nowadays highly interconnected due to international financial globalisation. Shocks occurring in the credit market are especially important, as the default of one company or sector could spill over to the entire financial system. This is particularly relevant and accentuated in periods of financial or economic crisis. The study of the transmission of systemic credit risk is increasingly on the radar of regulators, policymakers and the academic community in order to prevent and attempt to mitigate the devastating effects that the spread of this risk could have on the financial system. The transmission of systemic credit risk is a complex problem, difficult to study given the large number of factors involved and the diversity of possible dynamics. Because of the non-triviality of the challenge, the goal of analysing this transmission is of particular interest.

In a Bayesian statistics context, we propose the use of Dynamic Bayesian Networks (DBNs) to address this problem. This opens up a new approach in the study of systemic credit risk interconnections. These networks, DBNs, are defined by two fundamental elements: Network Structure Learning (NSL) and Parameter Learning (PL), which make up the entire statistical learning process, from model construction and selection to model estimation and diagnosis. In our case, the structure of the DBN is built, after inducing stationarity, using ARMA (Autoregressive Moving Average) Models, which determine the dynamic relationships present in the network. NSL is performed using unconditional independence tests, conditional independence tests and network scores, combining exhaustive and heuristic exploration methods. PL is carried out through simulation methods based on Markov Chain Monte Carlo (MCMC), using both maximum likelihood and penalised regression approaches. The proposed modelling allows for a more advanced analysis of the transmission of systemic credit risk.

This study is the result of combining the financial literature that analyses the transmission of systemic credit risk and the statistical literature on Bayesian networks with dynamic models. These lines of research overlap in the area related to the analysis of interconnections computed via networks. Within the line of research that analyses connections in credit risk, we focus on credit derivative contracts, specifically Credit

Default Swaps (CDS). The most widely used measures of systemic risk are based on information on CDS spreads, which are forward-looking and reflect the market's perception of the credit risk of a particular issuer (Chamizo and Novales, 2020).

The first studies focused on systemic risk using the CDS market emerged in the wake of the global financial crisis. These papers deal with the analysis of the decomposition of systemic risk using US and European sovereign CDSs (Bhansali *et al*., 2008; Ang and Longstaff, 2013), sectoral CDS (Novales and Chamizo, 2019) or using data from financial institutions (Suh *et al*., 2013). Another line of credit risk research includes the study of the transmission of credit risk using networks based on the methodology of Diebold and Yilmaz (2015). However, few recent studies refer to the analysis of systemic credit risk based on the transmission between various nodes in a network. In this regard, Kanno (2020), Brownlees *et al*. (2021) and Naifar and Shahzad (2021) focus on network analysis using different techniques for the CDSs of international financial institutions. The literature on Bayesian networks is less extensive, though increasingly frequent in recent years (Scutari and Denis, 2021). Following this line, the literature has begun to be interested in the study of financial series (Fortunato *et al*., 2017). However, to the best of our knowledge, there has been no application of Dynamic Bayesian Networks to the analysis of credit risk and, more specifically, to the systemic credit risk component.

The literature related to the application of networks to analyse the transmission of systemic credit risk in the context of Bayesian statistics is still to be explored, and this study is presented as the beginning of a fresh line of research. The limitations still present in the study of credit risk transmission mean that the adoption of a new approach, such as that offered by DBNs, should be welcomed in view of the potential innovations it can provide. Thus, the novel contribution of this paper is to analyse the transmission of credit risk through networks in the context of Bayesian statistics.

The aim of this research is to unravel the structure of systemic credit risk transmission across European sectoral CDSs during the COVID-19 crisis. The recent pandemic has not only had devastating consequences for public health, but also an unprecedented economic impact, destabilising development and the global economy.

We organise the research into several objectives. Our first goal is to understand the structure of risk transmission underlying European sectoral CDSs. The second is to determine the overall dynamics of systemic credit risk transmission and to understand how credit risk is transmitted across sectors. Our third aim is to determine the proportion of new systemic credit risk that results from risk transmission between sectors, as well as from new market innovations. In addition, we would like to provide an alternative to the classical transmission analyses in which available expert information on the transmission of systemic risk could be introduced. It should be made clear that the focus of the paper is on the results relating to the procedure for obtaining the transmission structure and not on the interpretation of the results of the credit risk transmission itself, which will be the subject of future research.

Our initial approach to modelling the problem explores a DBN in which the relationships between the contemporaneous sectoral CDS series and the delayed series consider lags between one and five. This analysis leads to three important results. First, of all the relationships considered for our Bayesian network, only a few, namely a part of those between the original series and the series delayed with one or two lags, are found to be relevant. Second, there is an underlying systemic credit risk transmission structure among European CDSs and this is robust and stationary over time. By performing network structure learning following the various strategies proposed, we have been able to show how systemic credit risk spreads across the different European CDS, thereby obtaining the risk transmission structure corresponding to the DBN. This structure has been verified by checking that the relationships are robust and consistent with the complete data series. Third, between 5% and 40% of the new systemic credit risk is explained by the transmission among the different CDS series. This is a high but reasonable proportion that quantifies the importance of risk transmission. By analysing the posterior distributions of all the parameters, we have come to understand how risk is transmitted along the DBN, as well as learning what proportion of the new risk is a consequence of the transmission of risk from earlier time points. The effects of lagged series on sectoral CDS are either positive, as a direct propagation of systemic risk, or negative, as a correction of the propagated risk. The proposed modelling allows for a more advanced analysis of the transmission of systemic risk among European sectoral CDS series.

The rest of the paper is organised as follows. Section 2 includes the literature review that summarises the lines of research followed and explains where exactly this research fits in. Section 3 presents the European sectoral CDS data, including a brief descriptive analysis. Section 4 explains the methodology used. Section 5 provides the results for network structure learning (NSL) and parameter learning (PL). Finally, section 6 concludes and summarises the most significant results and suggests some lines of future research.

2. Review of the Literature

This research merges two important strands of literature, one analysing the transmission of systemic credit risk through the CDS market and the other dealing with Bayesian statistics. The intersection between these lines of financial and mathematics research lies in network modelling.

The study of systemic risk using the CDS market arose in the aftermath of the global financial crisis. The large amount of credit exposure that occurred during 2007 and 2008 raised concerns about how systemic credit risk is decomposed and transmitted. Bhansali *et al.* (2008) use a linearized three-jump model for the US and employ European indices CDS data to conclude that systemic risk has become the most important source of total credit risk, with levels of idiosyncratic and sectoral credit risk remaining relatively constant. Using US and Eurozone sovereign CDS data, Ang and Longstaff (2013) estimate a multifactor credit model in order to decompose the systemic credit risk. Their results show that in both areas it is highly correlated, with eurozone countries accounting for a larger fraction of total credit risk compared to the US. They also find systemic risk to be closely linked to financial market variables, especially equity returns. Novales and Chamizo (2019) use an international sample of sectoral CDSs to split the credit risk of individual firms into its components. Systemic and sectorial components explain around 65% of credit risk in European industrial and financial sectors and 50% in North American sectors, while 35% and 50% of risk, respectively, is of an idiosyncratic nature.

Another line of research has looked exclusively at the financial sector. Focusing on CDSs of large US financial institutions, Suh *et al.* (2013) propose a structural credit risk model to measure systemic risk. Contributions to systemic risk arising from the default risk of an individual institution vary over time, increase during the crisis period compared to the pre-crisis period, and are related to the risk inherent in equity returns during the crisis period. The components of systemic risk for European sovereign and bank CDSs are addressed by Farina *et al.* (2019) following a probability of default approach. In peripheral countries the risk is concentrated in a joint country-banking shock at various levels, while in the Euro-core countries the risk is characterised by a systemic European shock at the country level, with an idiosyncratic component at the banking system level and at the individual bank level.

Within the analysis of systemic credit risk transmission, the network approach has become popular. The first studies using CDS data are based on the Diebold and Yilmaz (2015) methodology and where focused on computing the network structure via the Generalised VAR model. These researchers address the transmission of credit risk in the CDS market, but not specifically the study of systemic risk. Greenwood-Nimmo *et al.* (2019) and Chen *et al*. (2020) analyse credit risk transmission between European sovereign CDSs. Bostanci and Yilmaz (2020) extend the sample to an international CDS sovereign sample and Sun *et al.* (2020) analyse the transmission between the CDS market and other markets. None of these studies approach the topic using sectoral CDS data.

Strictly speaking, the network approach, which analyses how credit risk is transmitted between the different nodes of a network, has become popular. The network approach has important advantages over alternative analyses due to the good performance of these models and their interpretability. In this regard, the literature has centred on the CDS financial sector. Focusing on CDSs of US financial institutions, Kanno (2020) studies systemic risk using different measures of network centrality. The findings suggest that three of six main banks played a key role in the network in the past. In addition, the theoretical analysis of contagion defaults reveals that one contagious default was triggered by many stand-alone defaults during the global financial crisis. Brownlees *et al.* (2021) apply a new approach to estimating interconnectedness for large eurozone financial institutions by extending the standard reduced form credit risk model. The results show that the network captures systematic factors and other interdependence relationships. The larger the financial entity, the more interconnected it is. Moreover, time evolution analysis reveals that stressed financial institutions become hubs in the credit risk network during the crisis. Naifar and Shahzad (2021) construct a CDS financial network consisting of sovereign credit risk spillover effects across the most COVID-19 affected countries through a tail-event driven network risk technique. Their findings show that the connectedness between CDS spreads was higher during the COVID-19 crisis and changed over time. China, Russia and Brazil are the countries that most transmitted-received credit risk during the pandemic period.

The Bayesian approach, based on Bayes theorem, combines the construction of complex models with the inclusion of known prior information about the parameters in the models (Gelman *et al.*, 2014). Bayesian statistics allows for a new approach to frequentist statistics, both in conceptual and practical terms. In the Bayesian context, the parameters can be considered as random variables, which provides a more realistic approach in many cases where the parameter is of interest in itself (Lindley and Smith, 1972). In addition, the Bayesian approach to analysis not only uses the data, but the results are also based on a priori sources of information known prior to data sampling. Modern Bayesian inference is based on simulation methods, which allow one to calculate the a posteriori distribution of the parameters when direct sampling is not possible (Gilks *et al.*, 1995). The literature on Bayesian statistics is very extensive due to its myriad fields of application and some advantages that the Bayesian approach provides compared to frequentist statistics (Bauwens *et al*., 2000). Within the diverse literature on Bayesian statistics, in this study we focus on the literature associated with dynamic models, due to the nature of the problem studied (West and Harrison, 2006).

Bayesian networks are a class of probabilistic models under the umbrella of Bayesian statistics. Bayesian belief networks are very useful and powerful models that combine probabilistic reasoning and graphical modelling (Spiegelhalter *et al.*, 1993) and can successfully manage the different elements of uncertainty and causality in complex problems (Cowell *et al*., 2007; Jensen and Nielsen, 2007). These models are composed of a combination of random variables and the relationships between them, defined from a directed acyclic graph (Quigley *et al.*, 2013). The nature of these models is clearly dynamic, due to the possibility of introducing series as the nodes of the network itself. The literature on Bayesian networks is less extensive, though increasingly frequent in recent years (Scutari and Denis, 2021). The fields of application of Bayesian networks are also very diverse, as they are compatible with most contexts in which the goal is to analyse the relationship between time series (Nagarajan *et al.*, 2013; Heckerman, 2008). Bayesian networks are compatible with multiple models that define the structure, such as Vector Autoregressive Moving Average Models (Neapolitan, 2004). In the case of introducing an autoregressive structure into Bayesian networks, there are dynamic Bayesian networks (DBNs), which are of special interest for studying the relationships among financial time series (Neil *et al.*, 2005; Fortunato *et al.*, 2017). DBNs take into account the temporal nature of the data and consider the possible relationships between the series over time, which is essential for analysing the transmission of systemic risk. Thus, the novel contribution of this paper is that it analyses the transmission of systematic credit risk through networks in the context of Bayesian statistics.

Combining the network approach to perform the analysis of systemic credit risk transmission on CDS data with the possibility of working with DBNs, we arrive at our proposed line of research, which, despite the limited literature, arises naturally from the two topics raised. Thus, this study makes important contributions in the different areas that comprise it. On the one hand, our data refer to European sectoral CDSs, which is a novel contribution since the literature on sectoral CDS is very scarce and the little that does exist does not employ a network approach. On the other hand, the DBN approach introduces a new methodology, not previously considered in this literature and with great potential. Finally, this work analyses data associated with the current COVID-19 crisis, a period for which few risk transmission studies have been conducted. For all these reasons, this study represents a novel contribution to the literature.

3. Data and exploratory analysis

We analyse the transmission structure of systemic credit risk across ten European industrial sectors using CDS spreads[1] obtained from Refinitiv Eikon. We focus on the movements of the data themselves to understand the underlying risk, as CDS spreads are themselves a measure of credit risk. We also consider CDS contracts maturing in five years, as they are the most liquid and traded credit risk derivatives (Norden and Weber, 2009; Ballester *et al*., 2016). Daily frequencies are used in order to analyse how systemic credit risk is transmitted across different sectors in Europe.

Specifically, we have taken the daily data from December 2007 to April 2021 (3,690 observations) and have focused on the period from December 2019 to April 2021 (370 observations), a span that includes the recent COVID-19 crisis. We are interested in the estimation of the model itself, so we have used the entire data set to assess the model

[1] We define a CDS as a financial derivative contract that exchanges the credit risk of a product for a premium. CDSs are financial swap agreements in which the seller of the CDS will compensate the buyer in the event of a debt default. In this way, a CDS allows the buyer to insure himself in the event of default of the reference asset. CDS are mainly used for two purposes: to hedge risk and to speculate. CDSs are defined by the credit risk being exchanged and by the term over which it is exchanged. CDS spreads are quoted on the market in basis points.

fit.[2] The sectoral CDSs analysed include the following sectors: banking, consumer goods, electricity, energy, manufacturing, other financial firms, services, sovereigns, telephony, and transport.

Figure 1 displays the time evolution of CDS spreads for each of the ten sectors considered and Table A1 in the supplementary material shows their main summary statistics. Initially, horizontal movements are observed in the CDS spreads of the different sectors, from December 2019 to the beginning of March 2020, when COVID-19 expanded globally and became pandemic. During the month of March, it can be seen how the CDS spread increased considerably, hitting levels much higher than those observed in the previous months, reaching the maximums of the entire sample. The highest default risk observed in the period under analysis corresponds to Services (303.03 bp) and Transport (447.1 bp), the sectors most affected by the COVID-19 crisis. The most stable sectors, with the lower CDS spreads (on average), are Sovereign and Consumer goods. Subsequently, at the beginning of June 2020, there was a sharp decline in sectoral CDSs. This coincides with the timing of a number of institutional announcements in Europe about the possibility of summer foreign travel. These notices were very well received by the market, which ensured that the risk of default in the various sectors would be significantly lower. Lastly, a final marked decline is observed during November 2020, at the time when news of the discovery of the first effective vaccines against COVID-19 began. In short, the three events described above, strongly associated with the COVID-19 crisis, are the ones that have driven the largest movements during the sample period.

In order to analyse how the movements between the different CDSs are related and to ensure the stationarity of the data, we calculate the log returns of the CDS series:

$$X_i(t) = log\left(\frac{Z_i(t)}{Z_i(t-1)}\right)$$

where $Z_i(t)$ denotes the CDS spread of sector i at time t and $X_i(t)$ refers to the log returns of such sectoral CDSs. The results show that the returns follow a non-Gaussian

[2] There is an important risk associated with selecting the whole set as a training sample, namely overfitting. This potential problem will not be a risk for our model in practice, as the construction of the model penalises overfitting significantly. In short, it makes sense to use the entire data set for model fitting, so we select all 3,690 observations for the model construction assessment.

distribution, with a higher kurtosis.[3] However, this will not contradict the Gaussian distribution assumptions made in section 4, as we will assume a Gaussian distribution for the model residuals, and not for the CDS returns.

Figure 2 shows the relationships between the different sectors measured by the contemporaneous correlations between the different series. Note that this figure shows only part of the structure of relationships that can be captured by our network, as we fit a model in which for each series the covariates are the log returns of spreads of the rest of the CDS series, as well as all the autoregressive terms (of the other series and of the series itself). That is, we will consider both partial and conditional correlations. All correlations between any pair of sectoral CDSs are positive, which is actually intuitive, a consequence of the fact that all industrial sectors are part of the same economy and spreads are measures of risk. Obviously, not all correlations are equal and some CDS sectors are more related than others. The sovereign sector stands out, showing the lowest correlations with the rest of the sectors. This analysis is extended later, when after modelling we perform a formal analysis of the correlations, as well as an analysis of the correlations between lagged time points, seeing that a multivariate autoregressive structure should be introduced.

4. Methods

This section presents the methods we have used to analyse the transmission of systemic risk across industrial sectors in Europe after considering each sector as a node within a network composed of all sectors. Our objective is to analyse how systemic risk spreads from one node to the rest. We frame our analysis within the Bayesian statistics approach and use dynamic Bayesian networks (DBNs) to take into account the temporal nature of the data. Within this framework, we use autoregressive and moving average models to capture temporal relationships and perform the so-called network structure learning (NSL) and parameter learning (PL) (Koski and Noble, 2011) to analyse the structure of the network and understand the relationships between the nodes. The approach offers the possibility of introducing expert information to improve the analysis.

4.1. The Bayesian approach

[3] Histograms of European sectoral CDS spread returns and their main shape statistics (standard deviation, skewness and kurtosis) are available in the supplementary material, in Figure A1 and Table A2, respectively.

Bayesian inference is a process of fitting a probabilistic model to a data set and of summarising the results using probability distributions over model parameters and unobserved data, such as predictions for new observations (Gelman *et al.*, 2014). Inference from the known values of a variable y is performed using a property on the conditional probability, known as Bayes Rule:

$$p(\theta|y) = \frac{p(\theta, y)}{p(y)} = \frac{p(\theta)p(y|\theta)}{p(y)}$$

where $p(y) = \sum_{\theta} p(\theta)p(y|\theta)$ if the set for θ is discrete or $p(y) = \int p(\theta)p(y|\theta)d\theta$ if it is continuous. From this formula we could summarise that the a posterior information, $p(\theta|y)$, is obtained after updating the prior information, $p(\theta)$, with the sample information, $p(y|\theta)$.

The prior distributions can be considered according to two interpretations. On the one hand, they can be interpreted as representing a population of possible values of the parameter vector θ. On the other hand, in the more subjective sense, they can be seen as the place to express our knowledge and uncertainty about the parameter vector. The process of Bayesian inference involves moving from a prior distribution, $p(\theta)$, to a posterior distribution, $p(\theta|y)$.

Many statistical applications involve multiple parameters that are related to each other in some way through the structure of the problem, with the joint probability model reflecting their structure of dependencies. We denote the parameters θ_j as a random sample with a given distribution. A key element is to see that the observed data can be used to estimate the distribution of the θ_j parameters, even though the parameter values are themselves unobserved. The idea of positing a hierarchical model arises naturally; with the observed data we will conditionally model some parameters, which in turn are given by a certain probability function in terms of other parameters, known as hyperparameters.

Working with complex Bayesian hierarchical models, it can be complicated to obtain the posterior distribution analytically. To obtain the posterior distribution, one of the most commonly used possibilities is simulation. We rely on simulation methods based on Markov Chain Monte Carlo (MCMC). Different simulation techniques have been developed to carry out this process, Gibbs Sampling being one of the most widely used.

In this study we have used the implementation of this method through the program "WinBUGS" (Lunn *et al*., 2009), called directly from R (R Core Team, 2021).

4.2. Dynamic Bayesian Networks

Bayesian networks (BNs) are a class of probabilistic graphical models used to easily represent the probabilistic structure of multivariate data. BNs are composed of a combination of random variables, $\boldsymbol{X} = \{X_1, X_2, \dots, X_k\}$, and the relationship between them stated by a directed acyclic graph (DAG), denoted by $G = (\boldsymbol{V}, A)$. $\boldsymbol{V}$ denotes the set of nodes, with each node $v \in \boldsymbol{V}$ associated to a variable $X_i \in \boldsymbol{X}$. A denotes the directed arcs that connect the nodes, and each $a \in A$ is a directed arc. If no arc connects two nodes, the related variables are either independent or conditionally independent depending on the rest of the variables. There is an extensive theory on how it is possible to map the nodes of a BN and its connections (Scutari and Denis, 2021). Similarly, a theoretical mathematical basis underlies the conditional independence between nodes.[4]

Given a probability distribution P over a set of variables $\boldsymbol{X}$, a BN is a DAG, $G = (\boldsymbol{X}, A)$, such that it is a minimal independency map (I-map) of P in which none of the arcs can be eliminated without destroying the independence structure. The BN is denoted $\boldsymbol{B} = (G, \boldsymbol{X})$.

Assuming that a DAG is an I-map leads us to the general formulation of the joint probability decomposition, associated with the so-called global distribution:

$$P(\boldsymbol{X}) = \prod_{i=1}^{k} P\left(X_i \middle| \Pi_{X_i}\right) \tag{1}$$

where Π_{X_i} is the set of parent nodes of X_i.

From equation (1) we can deduce that the so-called local Markov property is verified. The local Markov property tells us that each node X_i is conditionally independent, given its parents, of any node that is not one of its descendants. In other words, each node X_i is conditionally independent of node X_j, so that there is no path from X_i to X_j. The result of this property being verified is that several BNs with different sets of arcs may be encoding the same conditional independence relation and represent the same global distribution. As a consequence, the construction of equivalence classes between the

[4] Regarding how to represent the dependency or independence relationships between nodes, the interested reader can consult Holmes and Jain (2008).

different DAGs is immediate. A large number of graph theory results have been applied to BNs.[5]

A dynamic Bayesian network (DBN) is a BN in which the temporal nature of the variables studied is taken into account. A DBN is represented by different nodes over time and is obtained by expanding, under certain conditions, the interaction network defined in a BN over time. In particular, we will not have problems with possible loops, guaranteeing the acyclicity of the graph by considering that only the nodes corresponding to the past can be parents of a node at time t. A DBN with a directed acyclic graph G describes a discrete stochastic process $\boldsymbol{X} = \{X_i(t): i = 1, \dots, n\}$ that takes values in $\mathbb{R}^k$ at n points in time.

In the implementation of a DBN there are certain underlying assumptions. The first assumption is that the stochastic process that follows $\boldsymbol{X}$ is m-order Markovian. This assumption is fundamental for it to make sense to work with a DBN. The second assumption we make is that for all time points $t > 0$ the random variables $\boldsymbol{X}(t) = \{X_1(t), X_2(t), \dots, X_k(t)\}$, observed at time t, are conditionally independent given the random variables at the previous time points $\boldsymbol{X}(t-1), \boldsymbol{X}(t-2), \dots, \boldsymbol{X}(t-m)$. This assumption allows for a simpler modelling of conditional data. It should be tested carefully, as it has important implications. The third assumption is that the variables are temporally independent of each other. Thus, in no case can the time profile of a variable $\boldsymbol{X}_i = (X_i(1), X_i(2), \dots, X_i(n))$ be written as a linear combination of the rest of the profiles $(X_j(1), X_j(2), \dots, X_j(n))$ $j \neq i$. This assumption is made so that the proposed modelling does not have problems in inference by taking a higher dimensionality in the parameter set than the data allow. When the k variables are linearly independent, i.e. none of the profiles can be written as a linear combination of the others, the uniqueness of G is guaranteed. As a result, the first and second assumptions permit the existence of a DBN with graph G that contains only arcs pointing out from a variable observed at earlier times $\{t-1, t-2, \dots, t-m\}$ toward a variable observed at time t, with no arcs between concurrently observed variables.

We also assume a constant time delay for all interactions, known as time point sampling, which is characterised by the interval between successive time points in order to limit the number of parameters in the network. Allowing the presence of arcs

[5] Further details on the equivalence between DAGs and on the definitions of the different types of structures are available in Koller and Friedman (2009).

between variables observed either at the same time or at different times can definitely serve to incorporate simultaneous interactions. However, we must be careful when adding new time points, as the number of model parameters increases exponentially with the number of time points. Under the assumptions made, the probability distribution of $\boldsymbol{X}$ can be represented as a DBN with a DAG G whose arcs describe exactly the conditional dependence of the current values with respect to the variables on past values.

DBN models depend on the number of time delays selected. Extending the number of time delays can lead to spurious conclusions about the network structure. Therefore, in order to carry out the model estimation we will make one last assumption: that the process is homogeneous over time, i.e. that all network arcs and their directions are invariant over time. In other words, we assume that the network under consideration is stationary, maintaining the relationships over time. The result of making this assumption is that we now have a total of $(n - m)$ repeated measures observed for each of the variables. Thus, it will be possible to make a representation of the model parameters. Each of the past time moments is represented by a matrix of coefficients of size $k \times k$.

The homogeneity assumption allows us to perform the estimation without any problem. However, it is a strong assumption that needs to be verified. In the event that this property is not true, other strategies can be used (Scutari and Denis, 2021). Under the proposed assumptions, the compatible models are autoregressive models where, in addition, a diagonal variance-covariance matrix is considered.

4.3. Vector Autoregressive Moving Average Models

ARMA models consist of two parts, an autoregressive (AR) part and a moving average (MA) part (Box *et al.*, 2015) The notation $ARMA(p, q)$ refers to the model with p autoregressive terms and q moving average terms. The generalisation of ARMA models to the multivariate case is given by Vector ARMA (VARMA) models, where the temporal relationships are captured through multivariate models in which the coefficients are stacked in matrices.

VARMA models can be also considered in a Bayesian context. The main difference in the modelling is that the various parameters (coefficients) present in the model are no longer considered as constants but as random variables. Therefore, Bayesian modelling of VARMA models must include a distribution for each and every parameter of the VARMA model in question. Specifically, a distribution must be proposed for each of the parameter matrices of the ARMA structure, as well as for the other parameters of the proposed data distribution. VARMA models can be expressed as the following Bayesian hierarchical model given by equation (2):

$$\mu_t = A_0 + \sum_{k=1}^{p} A_k \, x_{t-k} + \sum_{j=1}^{q} B_j \, \mu_{t-j} \qquad \forall t > max(p,q) \tag{2}$$

where $\boldsymbol{x}_t = (x_{1,t}, \dots, x_{K,t})^T$ is a multivariate time series; $\mathcal{F}_t = \{\boldsymbol{x}_t, \boldsymbol{x}_{t-1}, \dots\}$ is the set of all information up to time t; μ_t is the vector of means of the Normal distribution associated with time point t; Σ is the covariance matrix of the Normal distributions; $\boldsymbol{x}_t | \mathcal{F}_t \sim N(\mu_t, \Sigma)$, $A_0 \sim P(A_0)$, $A_k \sim P(A_k)$ for $k = 1, \dots, p$, $B_j \sim P(B_j)$ for $j = 1, \dots, q$, $\Sigma \sim P(\Sigma)$ and $\mu_t \sim P(\mu_t)$ $\forall t > max(p,q)$, where A_0, A_k and B_j are the matrix of parameters of the ARMA structure; $P(A_0)$, $P(A_k)$ and $P(B_j)$ are the prior distributions of the parameters; $P(\Sigma)$ are the prior distributions of Σ; and $P(\mu_t)$ are the structures we give to the parameters μ_t for the first temporal moments, those for which we cannot give the ARMA structure due to the lack of prior information.

The particular case we deal with is that of considering vague prior distributions. Moreover, after performing a pre-treatment to contrast the importance of the different elements of the modelling using VARMA models, the final model chosen was simplified. We have implemented an autoregressive VAR model without MA components, whose covariance matrix is diagonal. The data used for the returns are stationary in mean and variance, so these VARMA models are reasonable.

4.4. Bayesian Network Learning

Model selection and estimation are known in the field of BNs as learning, as they are in the domain of machine learning and artificial intelligence. In the case of a BN, these elements are consolidated in a two-step process. First, model selection is done by Structure Learning. This first step is based on learning the structure of the DAG, in which the arcs and their directions must be contrasted (Darwiche, 2008). The second step, once the DAG structure has been defined, consists of Parameter Learning. It is based on learning about the local distributions implied by the DAG structure. This estimation process can be performed either by unsupervised learning, using only the information provided by the data set, or by supervised learning, using expert information that informs about the structure of the parameter values at a specific time (Scutari and Denis, 2021).

4.4.1. Network Structure Learning

Network Structure Learning (NSL) is the first step to be used in BN modelling, and is equivalent to model selection. The structure specification of the DAG of a BN is usually done using two different statistical approaches: hypothesis testing and information criteria.

Determining the DAG is a very complex task. On the one hand, the space of possible DAGs is very large, and its cardinality grows exponentially with the number of nodes. On the other hand, this space is complex due to the dichotomous nature of the connections. The algorithms used to perform NSL are very diverse and, in many cases, specific to the problem being addressed (Scanagatta *et al*., 2019). However, in addition to the assumptions underlying the definition of the BN itself (see subsection 4.2), there are some fundamentals on which these algorithms are based. Regardless of which NSL approach we employ, any combination of possible values of the variables must represent a valid observable event. This property must be verified for it to make sense to consider the problem in probabilistic terms. Similarly, to form a DBN we need to consider the VARMA temporal structure and both the contrasts performed to analyse the conditional independence between the variables and the information criteria used to calculate the performance (score) of the network need to be assessed in the context of the selected distribution, in our case the multivariate normality framework.

Conditional independence tests focus on the presence of individual arcs. Each arc indicates a probabilistic dependence. Conditional independence tests can be used to decide whether such probabilistic dependence is compatible with the data. If the null hypothesis of conditional independence is rejected, the inclusion of the arch in the DAG can be considered. The null hypothesis is that two of the variables are probabilistically independent conditional on past values.

The most common way to decide is to work with the exact test for partial correlations. This test can only express the marginal linear dependencies between two variables. Using conditional partial correlation, the null hypothesis is rejected if and only if $\rho_{i,j|\boldsymbol{X}(t-1),\dots,\boldsymbol{X}(t-m)}$ is significantly different from zero. Conditional partial correlations must be estimated numerically, as no closed form exists (Scutari and Denis, 2021). In particular, once an unbiased and efficient numerical estimate of the partial correlation conditional on past moments is available, the test can be solved using either a *t*-transformation of the conditional partial correlation or the Fisher's Z test (Scutari *et al.*, 2019).[6] By considering the different tests, we achieve a complete NSL.

We can use *network scores* as an alternative to conditional independence tests (De Campos and Friedman, 2006). Network scores approach the construction of the DAG structure as a whole, measuring the performance of the DAG on the observed data. Network scores are goodness-of-fit statistics that measure how well the DAG captures the dependence structure of the data. There are many network scores that are commonly used to measure the performance of a DAG, including the Bayesian Information Criterion (BIC) and the Bayesian Dirichlet equivalent uniform (BDeu or BDeu), which depend, respectively, on the conditional likelihood and on posterior probabilities, the latter considering a uniform prior distribution over the space of DAGs and parameters (Rios *et al.*, 2015).

Both BIC and BDeu assign a higher score to those DAGs that best fit the data.[7] Using this criterion, network scores are useful for completing NSL. Unfortunately, network scores generally have a high computational cost. Computing a network score for each DAG involves the calculation of a likelihood function and its optimisation, which is not immediate. As the number of possible DAGs increases exponentially with the number of nodes, it is impossible to obtain the network score for all of them. Therefore, to overcome this drawback, network scores are combined with heuristic methods.[8]

4.4.2. Parameter Learning

Once the structure of the BN has been learned (i.e., once NSL has been performed), we can move on to learning about the parameters and perform Parameter Learning (PL). Although the strategy we have used to perform NSL does not in any way determine the

[6] In addition to these two tests, other tests could be also performed, such as those based on mutual information or mutual information shrinkage, both based on a Chi-square distribution of one degree of freedom.
[7] Note that in the BN literature, the BIC is usually defined in the opposite way to how it is usually defined in the classical statistical literature (see, e.g., Scutari, 2018).
[8] Greedy search proposes an initial random network structure and adds and removes arcs until it does not find a possible improvement. Genetic algorithms are based on exploring the space of DAGs by means of crossover (combination between networks) and mutation (random alterations). Finally, simulated annealing is an algorithm that allows changes to be made both to improve the network score and to worsen it, but associating different probabilities to these changes depending on the resulting score.

approach to be used in PL, it should be noted that there are some methods to perform a joint analysis, in which NSL is performed at the same time as PL (Faruqui *et al*., 2021).

PL consists in estimating and updating the parameters of the global distribution. This can be done by either maximising the likelihood or using a penalised estimation. The simplest possibility is to perform a maximum likelihood estimation. This strategy is based on maximising the global likelihood function or, equivalently, maximising each and every local likelihood function. Under the assumptions made (whose suitability we verify in section 5), each of the local distributions can be expressed as a classical linear regression model, in which a node is explained by the past nodes. The contribution of past nodes is additive, and no interaction term is considered. Thus, the maximum likelihood maximisation strategy is trivial. Another possibility is to employ penalised regression methods, such as ridge, lasso or net elastic regression (Hastie *et al.*, 2009). These methods focus on minimising the quadratic errors by controlling the parameter values. In this way, large quadratic errors and high parameter values are penalised.

5. Results

In this section we present the findings of performing a complete analysis of the transmission of systemic risk across industrial sectors in Europe, obtained after applying the methods described in section 4 on the data presented in section 3. We begin by learning the structure of the Bayesian network, subsection 5.1, and then move on to work on learning the parameters of the resulting model, subsection 5.2. The Bayesian network learned provides valuable information on how risk is transmitted across different industrial sectors in Europe.

5.1. Network Structure Learning

The first step in constructing a DBN is to learn its structure. A structure is determined by a Directed Acyclic Graph (DAG) between its nodes. In our case, we start with 60 nodes: 10 nodes for the original series of CDS returns and another 10 for each of the five (initially) considered lagged series. The selection of lags from order one to order five responds to the need to explore possible workday/week interactions between the series. This entails $10 \cdot 2^{50}$ potential arcs[9], a quantity that must be multiplied by the number of all possible combinations of instantaneous relationships between the series.

[9] Remember that, as stated in the Methods section, we impose the constraint that arcs must be directional towards the 10 nodes of the original (non-lagged) series, as the rest of the relationships are marked by previous moments.

This results in an enormous number that prevents the NSL process from being performed by an exhaustive search in the set of all possible DAGs.

5.1.1. Reducing the complexity of the search

To reduce the number of potential DAGs, we begin by performing unconditional independence tests on each set of two series containing an original series (i.e. performing a total of 59 unconditional independence tests for each original series). This allows us to rule out some of the relationships in the full connected model. Subsequently, we perform 'global' conditional independence tests to find out the true dependence relationships present in the data.[10]

In total, we carry out 545 unconditional independence tests[11] and obtain relevant learnings for both the original and the lagged series. On the one hand, we find contemporaneous unconditional dependence relationships between almost all the series. That is, simultaneous correlations seem to be relevant in most cases, with almost all of them positive. This result agrees with what is observed in Figures 1 and 2. On the other hand, we also observe significant unconditional dependence relationships with the series lagged one and two periods (with quite a few of them being non-relevant) and unconditional independence relationships with most of the series with three, four and five lags.[12] In this case, the significant relationships are more diverse, with some of the correlations being positive, showing direct risk transmission, and others negative, showing market corrections. The intensity of the relationships obtained also varies across sectors. For example, the manufacturing and transport industrial sectors are characterised by having stronger unconditional relationships than the rest and the sovereign bond series by having less relevant unconditional relationships.

In light of these results, we reduce the complexity of our initial models and, from now on, we just consider DAGs with 30 nodes by only including instantaneous and lagged one and two relationships in the model. Despite this, the cardinality of the space of DAGs still remains too large to carry out an exhaustive search, so we move on to

[10] Unconditional independence tests only measure marginal relationships. Therefore, when considering models with more than one covariate, such as the dynamic Bayesian network we are constructing, the relationships we are interested in are not unconditional, but conditional on the information provided by the rest of the variables.
[11] We need to subtract 45 from 59×10 because of the symmetry of the tests between each pair of original series.
[12] While it is true that some of the relationships appear to be relevant, it is possible that this is due to spurious relationships. Generally speaking, the vast majority of these unconditional relationships are directly irrelevant.

performing global conditional independence tests, that is, tests in which we condition on the total set of series (original and delayed), excluding the two series for which the conditional independence relationship is being analysed. This calculation is quite manageable. It only requires the computation of 29 conditional independence tests for each of the original series, that is, a total of 245 different tests. The results of these new tests allow us to further restrict the structure of our model. On the one hand, the global conditional tests show very different relationship structures across sectors. On the other hand, and as a main result, they exhibit very weak conditional dependence relationships between the original series. The strength of the relationship between each pair of original series is significantly reduced when we condition on the previous values of the rest of the series, that is, we can further prune our structure, not allowing for contemporaneous interconnections.

By eliminating the dependence relationships between the original series, we simplify the problem significantly. Considering the possible relationships between the original series means dealing with the whole structure when performing NSL. However, by forbidding contemporaneous relationships, we gain the ability to divide NSL into chunks. We can perform NSL for each of the ten original series separately, and then combine the conclusions obtained for each of the series. For each of the sectoral CDS we can analyse how the risk is transmitted between the different sectoral CDS with one or two lags, and then define the structure of the DAG as the union of all these individual DAGs. This not only reduces the complexity of the problem, but also lessens its computational burden by allowing the use of parallelisation computational techniques. At this point, we can now perform NSL using an exhaustive search strategy that takes into account all the different techniques discussed in section 4.

5.1.2. Conditional Independence Tests

The first of the strategies followed is based on performing an exhaustive search among all possible DAGs by means of sequential conditional independence tests. This strategy ensures that the best fitting structures are found among all the allowed DAGs.[13] In our case, the process has led to a single structure, the DBN presented in Figure 3. This DAG is obtained after fully integrating all the relationships included in each of the individual structures.

[13] It should be noted that this approach could lead to more than one feasible structure, so other criteria might be necessary to choose among them.

The above search method, however, still has a high computational cost[14], which makes this approach unfeasible if a larger number of series and/or delays are involved. So, as is standard practice (Cowell, 2013), we have also carried out the strategy based on the global conditional independence tests discussed in the previous section. This proposal represents a significant simplification with respect to the exhaustive search, as we go from performing $20 \cdot 2^{20}$ conditional independence tests to only 20 tests for each of the series. Remarkably, although with this strategy there is no guarantee of arriving at the network structure that best fits the data[15], in our case we do arrive at the same structure. In our view, this reinforces the validity of the idea of using global conditional independence tests to perform NSL.[16]

As an alternative to the graphical representation of the identified DAG shown in Figure 3, Table 1 presents by rows all the relevant relationships identified through the global conditional independence tests. Despite the great diversity of singular structures, we can observe that, as expected, the returns exhibit more relationships with one-lagged series than with two-lagged ones. We also observe that all the series have a significant relationship with themselves, the majority of them with their immediately previous value (lagged one series). Some of the series, however, have particularly complex structures, requiring the relationship given by both one and two lags. We also highlight the fact that the individual structure proposed for each of the original series clearly rules out overfitting. As the relevant relationships between the original series and the lagged series are so few, we arrive at a model with a low dimensionality for the parameter space, so we will not fall into problems associated with overfitting. This is verified in the next subsection.

5.1.3. Network scores

While NSL strategies based on conditional independence tests build DAGs in a piecewise fashion, methods based on network scores offer a completely different

[14] To perform an exhaustive search, we needed to check a total of $20 \cdot 2^{20}$ possible DAGs for each original series. We needed to consider the 2^{20} structures and to perform the 20 conditional independence tests associated with the lagged series.

[15] This procedure usually leads to a simplified structure that correctly fits the data.

[16] As an intermediate solution between the use of an exhaustive search and a strategy based on global conditional independence tests, a conditional independence heuristic algorithm could be employed. A heuristic strategy works in a similar fashion than an exhaustive search, the difference being that not all possible structures are considered. The idea is to start from a structure that includes all (any) of the possible arcs, and to perform sequential conditional independence tests on successive sets to remove/add arcs, until a structure is reached in which all relationships are relevant and no other arc can be added. Again, we also arrived at the same solution with this approach.

approach by gauging how structures as a whole fit the data. As advantages, network scores (i) assess DAGs once the models have been fitted, enabling comparisons in terms of both estimation and prediction, (ii) penalise model overfitting by decreasing the score as the number of parameters increases and (iii) allow structures to be ranked, saving several good candidates rather than a single structure. On the downside, network scores depend on the metric used and are not completely comparable, except in the case of nested networks, so it is possible to reach different solutions with different criteria.

Applying network scores is quite straightforward. Once we have a candidate structure, we only have to fit and evaluate it. Depending on the complexity of the model and the number of nodes, this approach can be more or less computationally expensive. In our case, we have performed an exhaustive search[17] and saved the five structures with the highest network scores in terms of BIC and BDeu.

The results obtained are quite interesting and encouraging. Although the structures might had been markedly different from those obtained using conditional independence tests, we have found quite similar structures. On the one hand, for six of the ten original series, the structure with the best network score coincides with the structure identified by conditional independence tests. On the other hand, for the other four series, the structure identified using conditional independence tests coincides with either the second or the third best network score. Furthermore, the five best structures identified using network scores were in all cases quite similar to one another, differing only in one arc with the best-scoring network. For all these reasons, we consider the structure identified using conditional independence tests and shown graphically in Figure 3 to be a really good candidate to fit our data.

Figure 3 displays the complete structure of our DBN (see also Table 1), underlying the data. This graph depicts the full set of relevant conditional dependence relationships between our 30 nodes. Yellow arcs represent relationships between the original series and the single-lagged series, blue arcs capture relationships between the original series and the two-lagged series, and green arcs identify relationships between the original series and the one- and two-lagged series. We must emphasise that the relationships marked in the DAG are directed from the delayed series to the original series and that

[17] We must note, however, that we obtained exactly the same results after implementing a heuristic search.

the structure represented is constructed from the union of the ten individual structures corresponding to each of the original series.

5.1.4. Robustness analysis

The identification of the structure described above was performed using the sub-dataset corresponding to the COVID-19 crisis period. Therefore, as a final validation we have repeated the previously implemented NSL process using the complete data series and have found that the identified DAG coincides, for the most part, with the one plotted in Figure 3. Only in two of the original series is there a change in the structure of its ten chucks, in one of them by adding a new arc and in the other by removing a relationship already present.

5.2. Parameter Learning

As explained in subsection 4.4, the estimation of a Bayesian network consists of two steps: model selection and model estimation. Parameter Learning (PL) performs model estimation, which in our application consists in estimating the model stated in equation (2) using the structure of relationships identified in subsection 5.1. In contrast to NSL, which was implemented from a frequentist approach, we perform PL from the Bayesian statistics framework.[18]

Translating the identified DAG requires, firstly, that all the B_j matrices are null[19] and that the only non-null coefficients in the matrices A_k are the entries (i, j) corresponding to the arcs identified in subsection 5.1 and detailed in Table 1. Furthermore, the conditional independence found among the original series implies that the covariance matrix Σ must be a diagonal matrix. This means that we can define the priors for the covariance matrix by working separately with each of the diagonal elements, the variances.

For the selection of prior distributions, we can consider three different situations, depending on the availability or not of prior information. On the one hand, we could set informative priors in two scenarios, whether we have expert information or prior experience[20]. On the other hand, we could take uninformative or vague prior

[18] We want to note that we obtained similar results implementing PL using penalised regression methods in the frequentist context.
[19] We just work with VAR models of order 2 (i.e. with a maximum of two lags).
[20] For example, in the latter case, we could fit the model using the data prior to the COVID-19 financial crisis and use posterior distributions attained as priors.

distributions, leaving the model to learn only from the information enclosed in the data. This decision has the advantages of being simpler to implement and of avoiding the criticism of being subjective, but it has the disadvantage of not exploiting the full potential of the Bayesian approach.

In our application we have selected particular vague priors that have the quality of stressing the identified model even more, leaving all the estimation effort on the information contained in the data. On the one hand, we have established standard Normal distributions as a priori for the autoregressive parameters.[21] These distributions are sufficiently loose that (i) being centred at zero they leave the data to determine whether the corresponding variable has predictive power, (ii) being symmetric they allow for positive and negative effects, and (iii) being concentrated in small values in absolute terms they recognise that the effects of a shock tend to be dampened several time points later on. On the other hand, for the variances of the shocks (which must be non-negative) impacting the returns (which as we can observe in Figure A1 are indeed small), we consider a uniform distribution between zero and one. Without being totally uninformative, this distribution will be flexible enough for the posterior to be a faithful reflection of what the data report.

Once the Bayesian hierarchical model has been completely defined, we have estimated the 78 a posteriori distributions of the parameters of our model (68 autoregressive coefficients and 10 variances) using simulation methods based on Markov Chain Monte Carlo (MCMC), through Gibbs Sampling. We have worked with five MCMC chains of 50,100 simulations each, eliminating the first 100 as a burn-in period. In addition, we have thinned the simulations, keeping only one out of five values in order to avoid possible autocorrelations in the simulations. The results obtained positively surpassed all the checks[22], ensuring that the estimates obtained are correct.

At this point, a case-by-case analysis of the different parameters of the model is possible, as they explain the risk transmission relationships across the different CDS series. On the one hand, the posterior distributions of the 68 parameters corresponding to the autoregressive structure provide information on the specific transmission of

[21] The parameters of the autoregressive structure measure the propagation of risk between the different CDS series. These parameters indicate how each movement in one CDS series, in the form of a return, affects another CDS series one or two periods later.

[22] We have checked (i) the convergence of the MCMC chains, (ii) the representativeness of a posteriori distributions, (iii) the independence from initial conditions, (iv) the non-existence of pathologies in the chains, and (v) the length of the effective sample sizes.

systemic risk between each of the lagged series and the original series.[23] These parameters measure the amount of systemic risk transmitted by a given lagged series over one of the original CDS series, i.e. how shocks in the lagged series affect the sectoral CDS series. On the other hand, the posterior distributions of the 10 parameters corresponding to volatilities (variances) provide information on the residuals, that is, on the part of the sectoral CDS returns that is not explained by the returns of the different CDS series in previous periods. In other words, the estimated standard deviation for each CDS tells us the size of the unexplained risk. Therefore, by comparing the estimated standard deviations with the standard deviations of the original series, we can ascertain what proportion of the risk of each of the sectoral CDSs has been transmitted from other CDS series at earlier time points and what proportion of the risk is unexplained.

To simplify the analysis, a summary of the estimation results of the identified DBN is presented in Table 2, in which the means of the posterior distribution of each autoregressive parameter and the percentage of explained variance for each of the CDS returns series are given. The interested reader can find in Tables A3 to A12 and in Figures A2 to A11 of the Supplementary Material statistical summaries of the distributions for the full parameter set and empirical graphical representations of the posterior distributions of parameters, respectively.

By analysing the values that the posterior distributions of the parameters corresponding to the autoregressive structure (Table 2), we can get an idea of how specifically risk propagates from the lagged series to the original series. Taking into account the structure of the DBN, we could study how a shock in a given CDS series is transmitted over time in the different series or see how risk is transmitted over the long-term from one series to another, fixing the rest of the series. These two approaches, among others, can provide valuable information on the transmission of systemic risk in the European credit system.

6. Conclusions

This study lies at the intersection of two literature topics, namely systemic credit risk and Bayesian statistics, with network models being the nexus. Dynamic Bayesian

[23] These coefficients indicate the transmission of risk at a given period, although obviously one could simply understand that the transmission of shocks is longer term, taking into account that this transmission is significantly dampened and becomes less and less strong over time.

Networks (DBNs) have been used to analyse the transmission of systemic credit risk across European sectoral CDS, and both Network Structure Learning (NSL) and Parameter Learning (PL) have been employed to understand how systemic risk is transmitted across them. After considering an initial DBN with a total of 60 nodes, we have performed NSL to learn the entire relationship structure. The final structure (see Figure 3) has been obtained using conditional independence tests and has been validated from both a heuristic search and a network score search. Furthermore, working with a data set that did not coincide with the COVID-19 financial crisis, and extending the period, we have arrived at an almost identical structure, confirming that the structure holds over time. Once the NSL was performed, we have carried out PL to estimate the parameters of the model. This phase has been entirely completed in the context of Bayesian statistics. To this end, an estimation was performed using simulation methods based on Markov Chain Monte Carlo.

The most significant finding of this study is the way in which systemic risk is transmitted across European sectoral CDS. The various parameters of the model, both those associated with the autoregressive structure and those related to the variance and covariance matrix, provide information on the transmission of risk. By analysing the posterior distributions of all the parameters, we understand how risk is transmitted along the DBN, as well as what proportion of the new risk is a consequence of the transmission of risk from previous time points. By analysing the posterior distribution of the parameters associated with the standard deviation of the residuals, we have been able to understand what proportion of the risk is a consequence of risk transmission between the sectoral CDS series. Thus, we have found that the new systemic risk is explained between 5% and 40% by the transmission between the different CDS series. Furthermore, by analysing the posterior distribution of the parameters associated with the autoregressive structure of the returns (see Table 2) we can see exactly how risk is transmitted along the DBN. The posterior distributions have allowed us to understand the transmission of systemic risk across European sectoral CDS during the COVID-19 financial crisis.

Given the importance of risk transmission analysis, this study may be relevant from the perspective of policy implementation. DBNs are able to identify the sources of risk and the transmission paths resulting from exogenous shocks. This modelling should be of significant interest to regulators and to supervisory authorities as it provides them with information on market fragility hotspots and allows them to anticipate potential increases in risk in different sectors. In addition, this methodology can be used by financial institutions to carry out internal systemic stress tests of contagion risk. Similarly, this study may be of interest to any financial market participant interested in analysing and understanding the transmission of systemic risk.

Finally, the modelling of the transmission of systemic risk across CDS by means of DBNs presented in this paper opens various lines of research. The study conducted in this case considers that the variance of the residuals for each of the sectoral CDS series is constant over time; as this assumption need not be true, a dynamic model for volatility could be developed. On the other hand, we have rejected the instantaneous relationships given their lack of strength in the analysis with daily data; however, a model could be created that takes into account instantaneous relationships, especially if intraday data were used. Finally, the study of risk transmission in a broader network, including other series from other regions, in order to better understand how systemic risk is transmitted at the international level, remains to be done.

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ANNEX. Tables and Figures

Table 1. Lagged relationships identified as relevant for each original series

Delayed series	Original series									
	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)
(a): Banking	1st		2nd	2nd						
(b): Consumer		1st		1st	1st		1st		1st	1st
(c): Electricity	1st	1st	1st/2nd	1st		2nd	1st/2nd	1st/2nd		
(d): Energy	2nd		1st	2nd	1st/2nd	2nd				1st/2nd
(e): Manufacturing		1st/2nd	1st/2nd	1st	1st	1st	2nd	1st	1st	1st
(f): Other Financial	1st		1st/2nd		1st	1st			1st	
(g): Services				1st	1st		1st			1st/2nd
(h): Sovereign	1st	1st						1st/2nd		
(i): Telephony	1st	1st		1st	1st		1st		1st	
(j): Transport	1st	2nd	2nd	2nd			2nd	1st	1st	1st/2nd
Total nodes	7	7	9	8	7	4	7	6	5	8

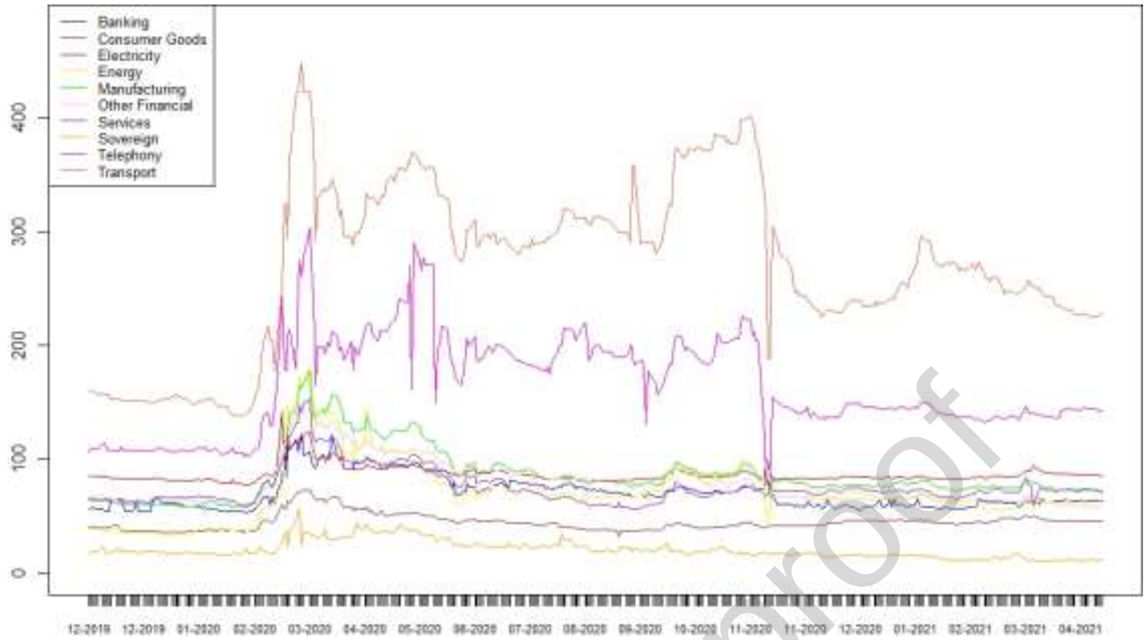
Note: Each series represents a node in the network: contemporary sectoral CDS by columns and lagged series by row. 1st and 2nd indicate if the relationship between series corresponds to one and two delays, respectively.

Table 2. A posteriori means of the parameters of the dynamic Bayesian network

Delayed series	Lag	Original series									
		(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)
(a): Banking	1	-0.387									
	2			-0.065	0.328						
(b): Consumer	1		-0.092		0.448	0.190		0.636		0.099	0.564
	2										
(c): Electricity	1	0.373	-0.006	-0.776	0.253			-0.705	0.476		
	2			-0.536			0.057	-0.731	0.613		
(d): Energy	1			-0.148	0.200	0.114					0.356
	2	0.132				0.145	0.076				0.249
(e): Manufacturing	1		0.009	0.443	-0.238	-0.237	0.217		-0.642	0.270	-0.496
	2		0.009	0.351				0.685			
(f): Other Financial	1	0.287		0.281		0.119	-0.042			0.254	
	2			0.194							
(g): Services	1				0.120	-0.027		-0.242			0.127
	2										0.144
(h): Sovereign	1	0.065	0.022						-0.304		
	2								-0.124		
(i): Telephony	1	0.204	0.045		0.247	0.192		0.531		-0.223	
	2										
(j): Transport	1	-0.143							0.402	-0.068	-0.178
	2		0.018	-0.154	-0.271			-0.490			-0.449
% Explained*		32.7	31.1	39.0	24.7	22.0	5.5	5.5	13.3	10.1	41.5

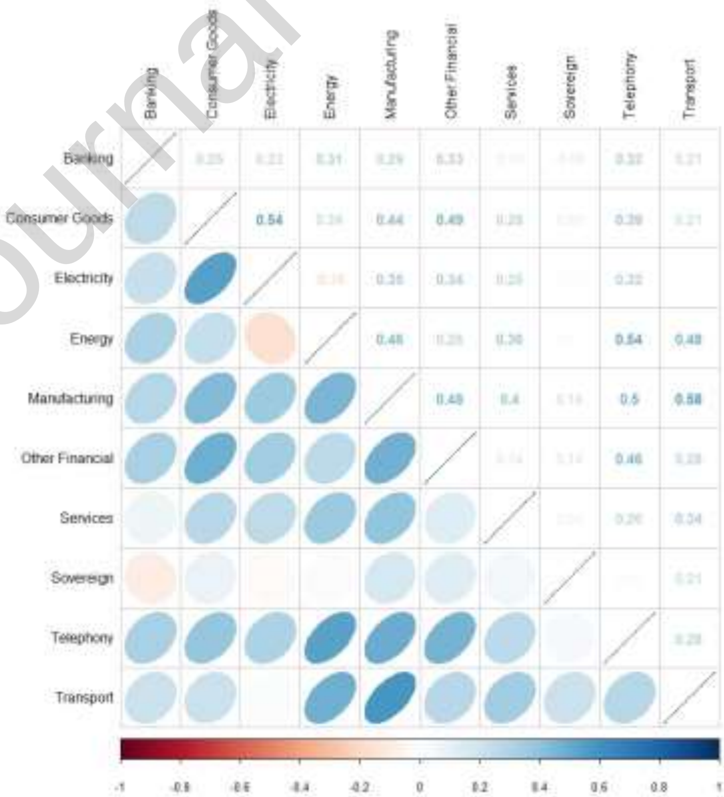
Note: The a posteriori means measure the direction and strength of the relationship defined between the nodes of the sectoral CDS Bayesian network. *'Explained' denotes the percentage of total variance of the corresponding CDS series explained by the relationships in the network.

Figure 1. Time evolution of European CDS spreads by sectors during COVID-19 crisis



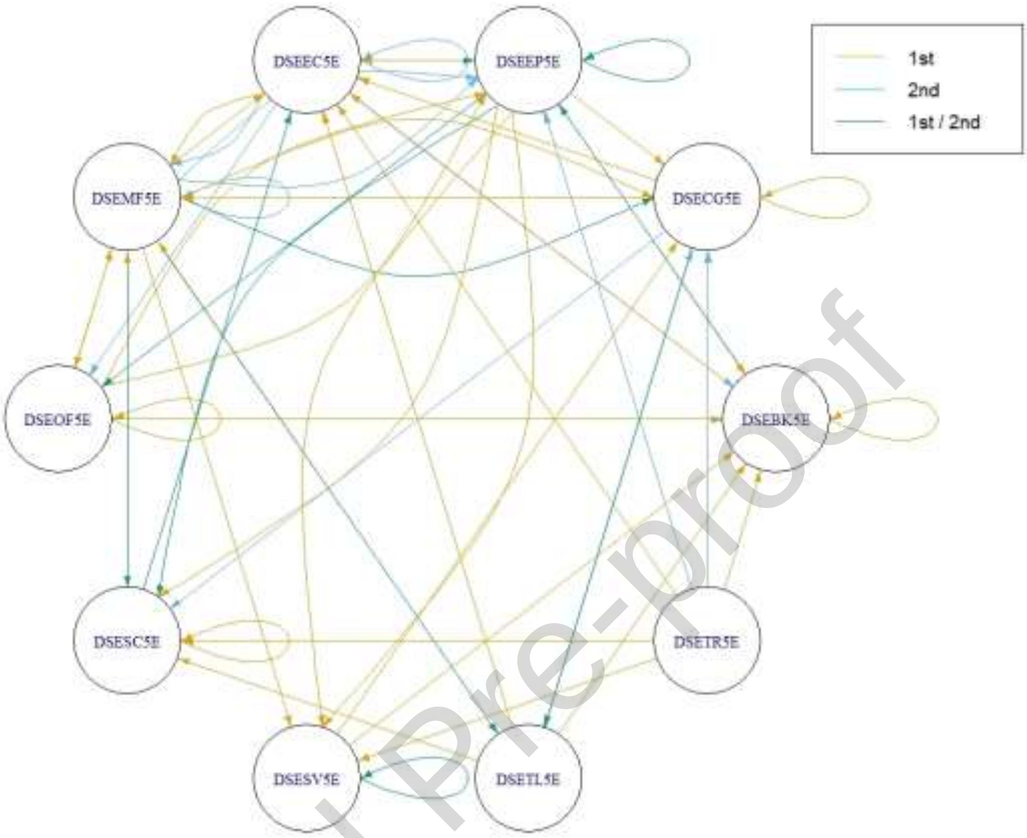
Note: Each line shows the evolution of a series of European CDS spreads from 2 December 2019 to 30 April 2021. Despite the large differences in levels of risk, co-movements between CDS sectoral series are clearly seen in the figure. The major changes in the series correspond to identifiable financial events, such as the outbreak of COVID-19 in March 2020 or the announcement of a vaccine in November 2020.

Figure 2. European sectoral CDS spread correlations



Note: The relationships have been measured using contemporaneous correlations between sectors series for the period from 2 December 2019 to 30 April 2021.

Figure 3. DAG selected by Structure Learning

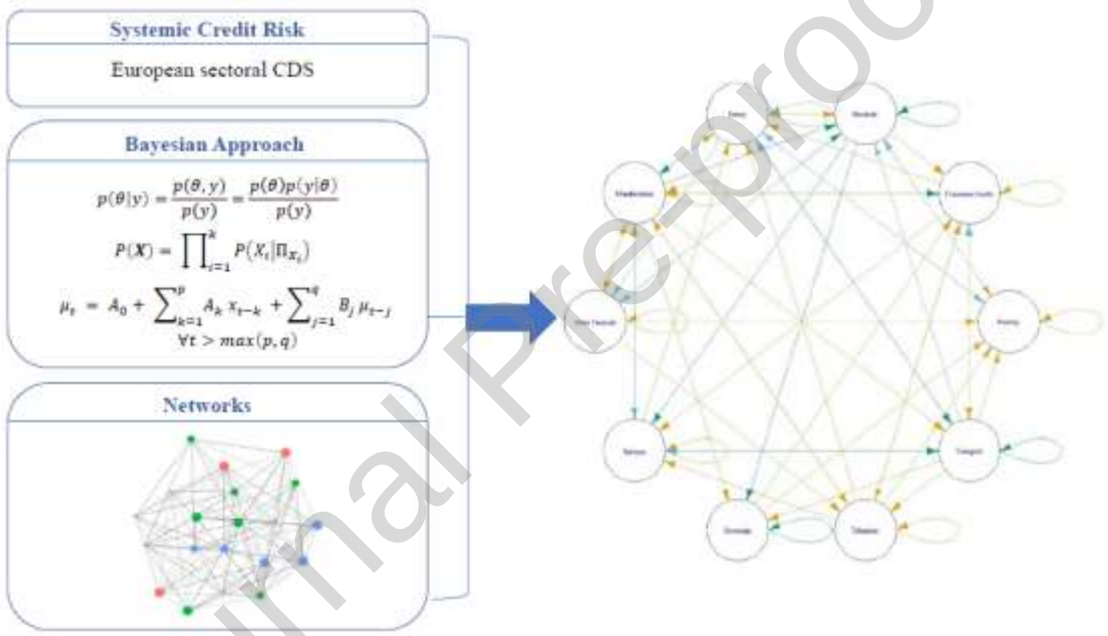


Note: Each node represents an original CDS series. A yellow arc indicates an identified relationship between the one-lagged series corresponding to the origin of the arrow and the original series corresponding to the destination of the arrow. Blue arcs and green arcs, on the other hand, identify relationships with two-lagged series and with both one- and two-lagged series, respectively. The sectoral CDS, together with their Datastream codes, correspond to the banking sector (DSEBK5E), the consumer goods sector (DSECG5E), the electricity sector (DSEEP5E), the energy sector (DSEEC5E), the manufacturing sector (DSEMF5E), the other financial firms sector (DSEOF5E), the services sector (DSESC5E), the sovereign sector (DSESV5E), the telephony sector (DSETL5E) and the transport sector (DSETR5E).

CRediT authorship contribution statement

Laura Ballester: Conceptualization; Data curation; Funding acquisition; Investigation; Project administration; Resources; Supervision; Validation; Visualization; Roles/Writing - original draft; Writing - review & editing. **Jesúa López**: Conceptualization; Data curation; Formal analysis; Investigation; Methodology; Resources; Software; Validation; Visualization; Roles/Writing - original draft; Writing - review & editing. **Jose M. Pavía:** Conceptualization; Formal analysis; Funding acquisition; Investigation; Project administration; Resources; Supervision; Validation; Visualization; Roles/Writing - original draft; Writing - review & editing.

Graphical abstract



Highlights

- This paper studies systemic credit risk transmissions among European sectoral CDS.
- Dynamic Bayesian networks shows a robust and stationary underlying risk transmission structure.
- Only relationships between original series and series delayed with one or two lags are relevant.
- The network relationships explain between 5% and 40% of single systemic risks.
- Amplifier and reducer effects of systemic risk transmission are identified.