On the diversification benefits of cryptocurrencies

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Abstract

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Whether cryptocurrencies are to be considered as a means of payment or a speculative asset and whether they are best employed as a diversifier or hedge is still a *vexata quaestio*. In this paper I question the diversification abilities of cryptocurrencies by adding them to portfolios of different instruments and comparing the performance of the portfolios with and without cryptocurrencies. At the end I add transaction costs to see if, after accounting for them, the same results would hold. I find that adding cryptocurrencies does not improve diversification. For portfolios formed with the 1/N and optimized Sharpe ratio strategies adding cryptocurrencies does not reduce risks but it improves the risk-return tradeoff. Even after accounting for transaction costs, this improvement remains. For the global minimum variance portfolios adding cryptocurrencies reduces risks but does not improve the risk return tradeoff.

Keywords: Cryptocurrencies; Diversification; Risk-return tradeoff; Transaction costs

Resumo

Título: Sobre os benefícios da diversificação das moedas criptográficas

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Se as moedas criptográficas devem ser consideradas como um meio de pagamento ou um ativo especulativo e se são melhor empregues como um diversificador ou uma cobertura é ainda uma *vexata quaestio*. Neste artigo questiono as capacidades de diversificação das moedas criptográficas ao adicioná-las às carteiras de diferentes instrumentos e ao comparar o desempenho das carteiras com e sem moedas criptográficas. No final, adiciono custos de transação para ver se, após a contabilização dos mesmos, os mesmos resultados se manteriam. Concluo que adicionar moedas criptográficas não aumenta a diversificação. Para as carteiras formadas com as estratégias de rácio 1/N e Sharpe otimizado, adicionar moedas criptográficas não reduz os riscos, mas melhora o tradeoff risco-retorno. Mesmo após a contabilização dos custos de transação, esta melhoria mantém-se. Para as carteiras de variação mínima global, a adição de moedas criptográficas reduz os riscos mas não aumenta o tradeoff risco-retorno.

Palavras-chave: Moedas criptográficas; Diversificação; Tradeoff risco-retorno; Custos de transação.

Acknowledgments

As Isaac Newton wrote in a 1675 letter to Robert Hooke "If I have seen further, it is by standing on the shoulders of giants".

I want to thank all the people that throughout my academic career have taught me something and helped me in this journey. I am thankful to my mother and sister for everything they taught me and who continuously gave me support and motivation. I want to thank my friends and colleagues for the amazing time spent together.

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1. Introduction

Since their invention in the 3rd of January 2009 cryptocurrencies have seen a tremendous rise in popularity and usability, they have been revolutionizing the world by constituting a new approach for a trusted and dynamic financial system that has no regulatory controls from governments. They became an alluring investment for the people who had lost trust in the financial institutions after the 2008 crisis. Originally invented as a transaction device they increasingly became perceived and used as an asset.

This study is meant to assess if investment strategies using cryptocurrencies to diversify show lower risks and/or higher returns. I compare the performance of portfolios with and without cryptocurrencies by using conventional measures such as the Sharpe ratio, the Treynor ratio and Jensen's alpha. I compare 3 different allocation strategies for each portfolio: 1/N, Global minimum variance, Optimized Sharpe ratio. I then add transaction cost to see whether, after accounting for them, the same conclusions would hold. I find that for the 1/N and the optimized Sharpe ratio portfolios adding cryptocurrencies is not beneficial in terms of diversification, since, despite there is a higher risk return tradeoff, there is no decrease in the standard deviation. The improvement in the risk return tradeoff remains even after accounting for transaction costs. For the global minimum variance portfolios cryptocurrencies allow to reduce the standard deviation, but the decrease in volatility also entails a decrease in return. I conclude that cryptocurrencies are not a good diversifier and that investors should carefully consider whether or not to include them in their portfolio.

The rest of the paper is organized as follows. In the remaining part of Section 1 I highlight the most relevant facts regarding cryptocurrencies and provide a review of the main articles examining their properties. In section 2 I describe the data I use throughout the analysis and provide summary statistics for all the variables. Section 3 gives an explanation of the methodology used in the analysis. In section 4 I provide the results of my analysis, check for their robustness to implementation costs, and point out the limitations that are present in my work. Finally, section 5 presents my conclusions.

1.1 Some facts on Cryptocurrencies

The digital currency concept emerged numerous years before Bitcoin. Digi Cash was the first company to use electronic currency, which was established in 1990. Then more money transfer companies were founded, such as PayPal, that allowed users to transfer money through online services over the world. In 2008, anonymous programmers working under the pseudonym "Satoshi Nakamoto" released the Bitcoin software in January 2009 as open secure code. He intended it to be a purely peer-to-peer version of electronic cash that would allow online payments to be sent directly from one party to another without going through a financial institution. Cryptocurrencies quickly escalated from being an obscure technology to a multi-billion dollar market. As of now, the number of cryptocurrencies is up to 9351 and the total market value is as high as $851,36 billion with a trading volume per day of $ 111,13 bn[1]. There are currently 38900 Crypto ATMs around the world[2]. There are 4 key characteristics of cryptocurrencies: decentralization, immutability and irreversibility (no one can transfer their digital assets other than the owner of the private key, and transactions cannot be changed once they have been stored in the system), anonymity (no need for a central authority) and limitation of supply. These currencies use only cryptography and an internal incentive system[3] to control transactions, manage the supply, and prevent fraud. The steady addition of a constant amount of new coins is analogous to gold miners expending resources to add gold to circulation. In this case, the resources are CPU time and electricity. Payments are validated by a decentralized network. Once confirmed, all transactions are stored digitally and recorded in a public "blockchain," which can be thought of as an accounting system. Due to the high volatility of cryptocurrencies, it is very difficult, if not impossible, to accurately predict future price changes. However, is very important for crypto investors to be aware that crypto winters happen. Crypto winter is a common expression that is used to refer to a poorly performing cryptocurrencies market. It is comparable to what a bear market is for the stock market. The term was first used in 2018 when Bitcoin lost more than half of its value, and other cryptos,

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[1] According to investing.com (https://cn.investing.com/crypto/) Accessed on 01/12/2022.
[2] According to coinatmradar.com (https://coinatmradar.com) Accessed on 01/12/2022.
[3] Each miner is awarded a block, i.e. a group of currencies. For Bitcoin the block reward as of now is 6.25 BTC. The block reward is designed to "halve" for every 2,016 blocks mined. It is called the "halving" process and happens every four years. The most recent halving happened in May 2020. Below are the historical block rewards, dating back to 2012: 2012: 25 BTC; 2016: 12.50 BTC; 2020: 6.25 BTC. The halving will continue until the last block and coin are mined. There are 30 more halvings before it goes to 0. With each block of Bitcoin being mined in 10 minutes, the last coin is predicted to be mined in 2140. In 2140 the block reward will be a mere 0.000000011641532 BTC per block. As of now on average 900 new bitcoins are mined per day.

such as Ethereum and Litecoin, dropped sharply. There are still no specific guidelines for how far cryptocurrency prices must fall in order for it to be considered a crypto winter. This year bankruptcy of FTX, which in a matter of few days passed from being one of the top companies in the crypto world with a valuation of 32 billion dollars to a bankrupt firm, led several exchanges such as Gemini, and lending platforms, including Genesis, to prevent customers from withdrawing their funds. From late 2017 to December 2020, crypto prices fell and hovered far off from prior peak prices. However, in December 2020, prices exploded to record highs in a significant crypto bull market. But in 2022 it has reached more than a 75% drawdown since the all-time high in 8 November 2021 when the price of Bitcoin was $67617,01.

1.2 Literature review

In the last years a rich collection of articles examining the properties and characteristics of cryptocurrencies has been published. In this section, I mention the most important ones related to my analysis.

Urquhart (2016) followed by Tiwari et al. (2018) reach the important conclusion that, although the Bitcoin market started by being inefficient, it has been moving towards becoming efficient. This is consistent with the idea that as more investors trade and analyze cryptocurrencies, along with their increasing world-wide adoption, their market will become more efficient.

Brière et al. (2015) investigate the potential of Bitcoin as diversification tool for a portfolio of traditional and alternative assets. They conclude that if Bitcoin is included in a portfolio, even in a small proportion, it may significantly improve the risk and return profile. Symitsi and Chalvatzis (2019) confirm this by finding that the inclusion of Bitcoin, provides significant statistical benefits when it comes to diversification. They suggest the reduction in risk comes from the low levels of correlation between Bitcoin and the other assets comprised in the portfolio.

Kajtazi and Moro (2019) consider the US, EU, and Chinese markets. They analyze the performance of the 1/N, long only, and long-short portfolios, with the inclusion of Bitcoin, on a set of daily data, from February 2012 to January 2017. They show that Bitcoin plays an active role in portfolio diversification, primarily by increasing the returns rather than in reducing the risk.

Guesmi et al. (2019) compare two hedging strategies, one involving only gold, oil and equities and one which also includes Bitcoin. They find that the portfolio's risk reduce more under the strategy which includes Bitcoin. Brauneis et al. (2019) document a substantial potential for risk reduction, when several cryptocurrencies are mixed.

Yermack (2015) argues that Bitcoin is not yet a proper currency according to the criteria widely used by economists, in which a currency has to be able to function as a medium of exchange, a unit of account, and a store of value. He suggests that due to the excessive volatility, Bitcoin is more consistent with the behaviour of a speculative investment rather than that of a currency. Consistent with these results Baur et al. (2017) find empirical evidence that Bitcoin and other cryptocurrencies should be viewed as a speculative asset.

Using the mean-variance optimization, and the risk-parity on a data of cryptocurrencies, spanning from January 2015 to December 2017, Petukhina et al. (2018) observe that cryptocurrencies provided higher returns, but did not play an active role in minimizing variance.

2. Data

I choose the five most market capitalized cryptocurrencies–Bitcoin (BTC), Ripple (XRP), Litecoin (LTC), Tether (USDT) and Ethereum (ETH). I use daily closing price[4] data retrieved from coinmarketcap.com from 10 August 2015 to 1 December 2022 (a total of 1842 observations). Although the Bitcoin, Ripple and Litecoin data goes back to 2013, I start the analysis in 2015 because the second and third largest component of the cryptocurrencies market, Ethereum and Tether respectively, were not capitalized until then. Unlike stocks, Bitcoin can be traded on many different exchanges. Binance has more than 50% of the entire crypto market, and, as a result, it sets the price of Bitcoin and other cryptocurrencies. In order to buy cryptocurrencies, traders must convert fiat money, into a stablecoin[5] like tether. Bitcoin-tether has by far the largest volume of all products on Binance, and because one dollar usually equals one tether, trading on bitcoin-tether sets the dollar price of bitcoin. But when bitcoin crashes, so does the entire crypto ecosystem.

[4] Technically, cryptocurrencies and currencies markets never close; the terminology "closing price" is still used in reference to the last price of a day, where days are customary defined on UTC time.

[5] A crypto whose price is pegged 1:1 to the US dollar or another "fiat" currency.

Tether is an “asset-backed”[6] stablecoin that was launched by the company Tether Limited Inc. in 2014. Ripple is a monetary system that works as a cryptocurrency and an online payment network at the same time. It was co-founded by Chris Larsen. Ripple’s main function is to act as a payment settlement capital transfer and remittance method, similar to the SWIFT (“Society for Worldwide Interbank Financial Telecommunications”) system for international money and safeness transfers. Unlike most cryptocurrencies, XRP is pre-mined, with a maximum token supply of 100 billion. XRP offers a more centralized medium of security mechanism supported by a customized algorithm based on the Byzantine Consensus Protocol known as RPCA (Ripple Protocol Consensus Algorithm). It therefore operates in a different way than the rest of cryptocurrencies, raising questions about its true decentralized nature. Ethereum was suggested by Vitalik Buterin, a programmer, in 2013. The network went live on July 30, 2015, with an initial supply of 72 million coins. There is no real cap on the total number of ETH than can come into existence like there is with other cryptocurrencies. Ethereum allows anyone to deploy permanent and immutable decentralized applications onto it, with which users can interact. It also allows users to create and exchange Non-Fungible Tokens (NFTs), which are unique tokens representing ownership of an associated asset or privilege, as recognized by any number of institutions. Litecoin was among the earliest altcoins[7], starting in October 2011. It is a clone of Bitcoin and shares a slightly modified codebase. The practical effects of those codebase differences are lower transaction fees, faster transaction confirmations[8], and faster mining difficulty retargeting[9]. Due to its underlying similarities to Bitcoin, Litecoin has historically been referred to as the “silver to Bitcoin's gold”.

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[6] Tether Limited has stated that it maintains $1 of asset reserves for each USDT issued, but in 15/10/2021 it has been fined by the Commodity Futures Trading Commission for failing to do this and for failing to present audits showing sufficient asset reserves.

[7] Tokens, cryptocurrencies, and other digital assets other than Bitcoin.

[8] The targeted block time for Litecoin is every 2.5 minutes, as opposed to Bitcoin's 10 minutes. This allows Litecoin to confirm transactions four times faster than Bitcoin.

[9] Litecoin has a maximum circulating supply of 84,000,000 LTC, which is four times larger than Bitcoin's maximum circulating supply of 21,000,000 BTC. Both Litecoin and Bitcoin retarget their mining difficulty every 2016 blocks. However, due to the faster block speed for Litecoin, mining difficulty retargets occur approximately every 3 and a half days. This compares to approximately every 14 days for Bitcoin.

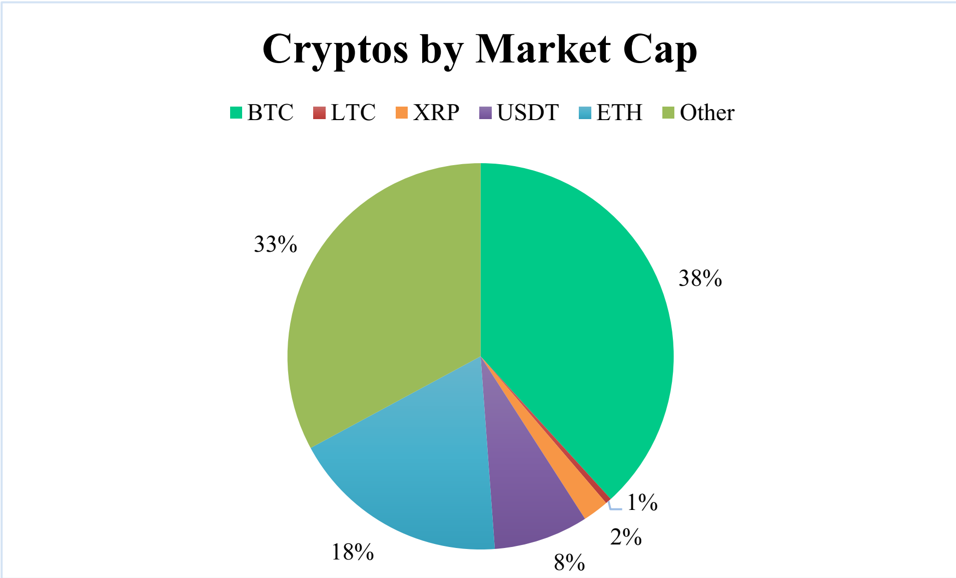

Figure 1: Cryptos by Market Cap as of 01 December 2022

The Market Cap of a cryptocurrency is defined as: Market Cap = Current Price × Circulating Supply. The Market Cap data have been retrieved from coinmarketcap.com (https://coinmarketcap.com) accessed on 01/12/2022.

I retrieve the daily closing price data for stocks, commodities and currencies from Refinitiv. For the portfolio of tech stocks I select seven stocks–Apple Inc. (AAPL), Meta Platforms Inc. (META), Microsoft Corporation (MSFT), Amazon (AMZN), Alphabet Inc Class A (GOOGL), IBM (IBM) and Intel Corporation (INTC). For the portfolio of stocks I select JP Morgan Chase & Co. (JPM), Johnson & Johnson (JNJ), ExxonMobil Corporation (XOM), Visa Inc. (V), Berkshire Hathaway Inc Class A (BRKa), Walmart Inc. (WMT) and Procter & Gamble (PG). For the portfolio of commodities I select the price of the futures on coffee (CF), copper (CPR), gold, silver, gas, oil and cocoa. For the portfolio of currencies I select the US Dollar/ Euro (EUR), British Pound (GBP), Japanese Yen (JPY), Australian Dollar (AUD), Canadian Dollar (CAD), Swiss Franc (CHF) and Chinese Renminbi (CNY) spot rates. As proxy for the market I use the S&P 500 index[10]. As proxy for the risk free rate I use the one-month Treasury bill rate from Ibbotson Associates downloaded from the Kenneth R. French Data Library.

[10] Data taken from splglobal.com (https://www.spglobal.com/spdji/en/indices/equity/sp-500/#overview). Accessed on 01/12/2022.

Unlike conventional assets, which are traded on weekdays only, cryptocurrencies and FX are traded every day. I filter out cryptocurrencies and FX data during periods when the stock markets are closed[11].

In the following tables I report the descriptive statistics for all of the variable used in the analysis.

Descriptive statistics: Cryptocurrencies									
	Mean	**Median**	**Max**	**Min**	**St. Dev.**	**Skewness**	**Ex. Kurt.**	**JB**	**p-value**
BTC	24,61	0,11	12,47	-18,84	32,87	-0,68	8,19	5290,98	0,00
XRP	22,77	-0,07	27,12	-23,87	52,44	1,29	14,28	16152,22	0,00
LTC	17,43	0,03	31,30	-20,47	45,67	0,63	14,54	16350,40	0,00
USDT	0,003	0,00	5,50	-12,31	7,797	-6,86	252,12	4893195,57	0,00
ETH	42,11	0,07	17,69	-24,45	50,78	-0,03	6,97	3728,24	0,00

Table 1: Descriptive statistics for Cryptocurrencies. For each cryptocurrency I report the annualized mean and standard deviation (St. Dev.), the median, maximum (Max), minimum (Min), skewness, excess kurtosis (Ex. Kurt.), Jarque-Bera test statistics (JB) and its corresponding p-value.

Descriptive statistics: Tech stocks									
	Mean	**Median**	**Max**	**Min**	**St. Dev.**	**Skewness**	**Ex. Kurt.**	**JB**	**p-value**
AAPL	9,51	0,04	4,91	-5,98	13,20	-0,23	5,35	2209,15	0,00
META	1,46	0,04	7,04	-13,31	16,91	-2,12	28,59	64098,65	0,00
MSFT	9,99	0,04	5,77	-6,92	12,28	-0,20	8,18	5147,76	0,00
AMZN	7,68	0,06	5,51	-6,58	14,38	-0,06	5,35	2201,59	0,00
GOOGL	6,62	0,05	3,99	-5,37	12,25	-0,23	4,69	1702,74	0,00
IBM	-0,02	0,02	4,65	-5,97	11,01	-0,71	9,95	7751,01	0,00
INTC	0,04	0,02	7,74	-8,64	14,61	-0,74	13,53	14210,97	0,00

Table 2: Descriptive statistics for Tech stocks. For each Tech stock I report the annualized mean and standard deviation (St. Dev.), the median, maximum (Max), minimum (Min), skewness, excess kurtosis (Ex. Kurt.), Jarque-Bera test statistics (JB) and its corresponding p-value.

[11] According to Urquhart et al. (2019) the trading volume of Cryptocurrencies and FX is extremely small during the weekends. Therefore, filtering out the data for cryptocurrencies and FX does not affect the results in a significant way.

Descriptive statistics: Stocks									
	Mean	**Median**	**Max**	**Min**	**St. Dev.**	**Skewness**	**Ex. Kurt.**	**JB**	**p-value**
JPM	4,05	0,00	7,19	-7,04	12,71	-0,04	12,89	12762,18	0,00
JNJ	3,46	0,01	3,34	-4,59	8,25	-0,47	10,26	8139,54	0,00
XOM	2,03	0,01	5,19	-5,66	12,98	-0,16	6,37	3125,26	0,00
V	6,36	0,06	5,63	-6,32	11,54	-0,08	9,05	6282,02	0,00
BRKa	4,73	0,02	4,64	-3,88	8,75	-0,28	8,37	5394,13	0,00
WMT	4,54	0,02	4,64	-5,24	9,78	-0,16	15,47	18368,13	0,00
PG	3,98	0,03	4,93	-3,97	8,58	0,01	11,67	10459,66	0,00

Table 3: Descriptive statistics for Stocks. For each stock I report the annualized mean and standard deviation (St. Dev.), the median, maximum (Max), minimum (Min), skewness, excess kurtosis (Ex. Kurt.), Jarque-Bera test statistics (JB) and it s corresponding p-value.

Descriptive statistics: Commodities									
	Mean	**Median**	**Max**	**Min**	**St. Dev.**	**Skewness**	**Ex. Kurt.**	**JB**	**p-value**
CPR	2,53	0,01	3,13	-3,01	9,7127	-0,02	1,72	226,80	0,00
CF	1,35	-0,01	4,15	-3,92	13,906	0,21	1,32	146,23	0,00
GOLD	2,91	0,02	2,51	-2,22	6,4336	-0,08	4,53	1575,80	0,00
SILVER	2,33	0,02	3,86	-5,38	12,481	-0,53	6,50	3332,34	0,00
GAS	5,13	0,01	16,58	-13,05	25,79	0,27	10,46	8421,16	0,00
OIL	3,23	0,09	8,29	-12,15	18,79	-1,34	18,53	26896,11	0,00
COCOA	-0,96	0,00	4,99	-4,09	12,47	-0,08	1,95	292,77	0,00

Table 4: Descriptive statistics for Commodities. For each commodity I report the annualized mean and standard deviation (St. Dev.), the median, maximum (Max), minimum (Min), skewness, excess kurtosis (Ex. Kurt.), Jarque-Bera test statistics (JB) and it s corresponding p-value.

Descriptive statistics: Currencies and S&P 500									
	Mean	**Median**	**Max**	**Min**	**St. Dev.**	**Skewness**	**Ex. Kurt.**	**JB**	**p-value**
EUR	-0,42	0,00	1,32	-1,05	3,27	0,12	2,15	358,00	0,00
GBP	-1,51	0,00	1,34	-3,65	4,21	-1,34	18,42	26588,62	0,00
JPY	-0,4	-0,01	1,66	-1,37	3,72	0,49	6,27	3095,71	0,00
AUD	-0,29	0,01	1,25	-1,70	4,31	-0,27	2,27	419,76	0,00
CAD	0,11	0,00	0,85	-0,92	3,27	0,03	1,27	124,06	0,00
CHF	0,26	0,00	1,22	-0,80	3,16	0,36	2,08	370,11	0,00
CNY	-0,52	0,00	0,70	-0,80	1,71	-0,13	5,56	2378,45	0,00
S&P 500	3,93	0,03	3,89	-0,06	8,42	-0,83	15,29	17373,63	0,00

Table 5: Descriptive statistics for Currencies and S&P 500. For each currency and the S&P 500 I report the annualized mean and standard deviation (St. Dev.), the median, maximum (Max), minimum (Min), skewness, excess kurtosis (Ex. Kurt.), Jarque-Bera test statistics (JB) and it s corresponding p-value.

As any high frequency financial return series, the value of kurtosis for all of the return series of cryptocurrencies indicates a typical leptokurtic distribution. This means that the return series is more peaked around the mean with thicker tails compared to the normal distribution. Furthermore, the Jarque-Bera test statistics and the corresponding p-values reinforce the above findings by rejecting for all the variables the null hypothesis of normality at the 1 percent level of significance. Almost all of the variables are negatively skewed and present quite high a standard deviation. The group of cryptocurrencies exhibit the highest standard deviation, with the exception of Tether, that, by construction has a low variance. Cryptocurrencies, excluding Tether, are also the assets with the highest average return.

3. Methodology

In this section I describe the strategies and performance measures used to compare the different portfolios. I form 4 portfolios, one with tech stocks, one with stocks, one with commodities and one with currencies. Each of the 4 portfolios is formed 3 times, using different allocation strategies: 1/N, global minimum variance and optimized Sharpe ratio. Then I form new portfolios by adding the 5 cryptocurrencies to each of the previous portfolios.
After collecting the daily data of each asset used in the analysis I compute the log-returns by using the formula:

$$r_{i,t} = log\left(\frac{P_{i,t}}{P_{i,t-1}}\right)$$

Where $P_{i,t}$ is the price of the ith asset at time t, $P_{i,t-1}$ is the price of the same asset at time t-1 – on the previous day –, and $r_{i,t}$ is the log-return[12] on asset i at time t.

I then calculated the average return μ_i and standard deviation σ_i for asset i using the following formulae:

$$\mu_i = \frac{\sum_{t=1}^{n} r_i}{n}$$

[12] I use the log-returns as they allow to smooth out extreme values.

$$\sigma_i = \sqrt{\frac{\sum_{t=1}^{n}(r_{i,t}-\mu_i{}^2)}{n-1}}$$

Where t is the number of observations for asset i. I then annualize the values, as conventional, multiplying the daily mean by 252[13] and the daily standard deviation by the square root of 252.

I then compute the average return μ_{P_i} and standard deviation σ_{P_i} of each portfolio i by using the following formulae:

$$\mu_{P_i} = \sum_{j=1}^{m} w_j \mu_j$$

where w_j is the weight of asset j in the portfolio and μ_j is the expected return on asset j

$$\sigma_{P_i} = \sqrt{\boldsymbol{w}^T \sum \boldsymbol{w}}$$

where $\boldsymbol{\Sigma}$ is the variance-covariance matrix of the assets in the portfolio, $\boldsymbol{w} = (w_1, w_2, \ldots, w_n)$ is the vector of weights and $\boldsymbol{w}^T$ is its transpose.

3.1 Performance measures

3.1.1 Sharpe Ratio

The Sharpe ratio is a measure of an investment's risk-adjusted performance, calculated by comparing its return to that of a risk-free asset. It was proposed in 1966 by the economist William F. Sharpe as an outgrowth of his work on the capital asset pricing model (CAPM), calling it the reward-to-variability ratio.
The numerator is the difference over time between realized, or expected, returns and a benchmark such as the risk-free rate of return or the performance of a particular investment category. Its denominator is the standard deviation of returns over the same period of time, a measure of volatility and risk.

[13] Which is the number of business days in a year.

$$\text{SR}=\frac{E\left(R_{P_i}\right)-R_f}{\sigma_{P_i}}$$

3.1.2 Treynor ratio

The Treynor ratio is similar to the Sharpe ratio in that it standardizes the portfolio return. However, instead of standardizing using the standard deviation as in the Sharpe ratio, the Treynor ratio uses the portfolio beta as a volatility measure so that it equals:

$$\text{TR}=\frac{E\left(R_{P_i}\right)-R_f}{\beta_{P_i}}$$

Where $\beta_{P_i} = \sum_{j=1}^{m} w_j\beta_j$, where w_j is the weight of asset j and $\beta_j = \frac{\sigma_{jm}}{\sigma_m^2}$, where σ_m^2 is the variance of the market and σ_{jm} is the covariance between asset j and the market.

3.1.3 Jensen's Measure

The Jensen's measure, or Jensen's alpha, is a risk-adjusted performance measure that represents the average return on a portfolio or investment, above or below that predicted by the capital asset pricing model, given the portfolio's or investment's beta and the average market return. This metric is also commonly referred to as simply alpha.

$$\alpha = R_P - \left(R_f + \beta_P(R_M - R_f)\right)$$

3.2 Portfolio allocation strategies

3.2.1 Equal-weighted portfolios

This portfolio is formed by investing an equal amount of wealth in each asset. It is also known as the naïve strategy or the 1/N, since each asset is given a weight $w_i = 1/\text{N}\ \ \forall\ i$ where N is the number of assets forming the portfolio.

According to the study by Demiguel, Garlappi and Uppal (2007) in which they evaluate the OOS (Out-Of-Sample) performance of the sample-based mean-variance model, and its extensions designed to reduce estimation error, relative to the naive 1/N portfolio, out of the 14 models, none is consistently better than the 1/N rule which indicates that, out of sample, the gain from optimal diversification is more than offset by estimation error. Based on parameters calibrated to the US equity market, our analytical results and simulations show that the estimation window needed for the sample-based mean-variance strategy and its extensions to outperform the 1/N benchmark is around 3000 months for a portfolio with 25 assets and about 6000 months for a portfolio with 50 assets. This suggests that there are still many "miles to go" before the gains promised by optimal portfolio choice can actually be realized out of sample.

3.2.1 Constrained optimization

Global Minimum variance portfolio

Under this allocation strategy I find the weights that minimize the portfolio variance. Hence, I use w* s.t.

Min $\boldsymbol{w}^T \sum \boldsymbol{w}$

subject to the constraints:

$\boldsymbol{w}^T \sum \boldsymbol{w} \geq 0, \sum_{i=1}^{n} w_i = 1$ and $w_i \geq 0 \ \forall \ i$

The latter constraint does not allow for short positions.

Optimized Sharpe ratio portfolio

For this allocation strategy I select the weights that maximize the risk-return tradeoff. Each asset receives a weight w* s.t. Max $\frac{\boldsymbol{E}\left(R_{P_i}\right)-R_f}{\sigma_{P_i}}$ subject to the constraints:

$\sum_{i=1}^{n} w_i = 1$ and $w_i \geq 0 \ \forall \ i$

3.2.2 Unconstrained optimization

Global Minimum variance portfolio

Under this allocation strategy I find the weights that minimize the portfolio variance. Hence, I use w* s.t.

Min $\boldsymbol{w}^T \sum \boldsymbol{w}$

subject to the constraints:

$\boldsymbol{w}^T \sum \boldsymbol{w} \geq 0, \sum_{i=1}^{n} w_i = 1$

Optimized Sharpe ratio portfolio

For this allocation strategy I select the weights that maximize the risk-return tradeoff. Each asset receives a weight w* s.t. Max $\frac{\boldsymbol{E}\left(R_{P_i}\right)-R_f}{\sigma_{P_i}}$ subject to the constraints:

$\sum_{i=1}^{n} w_i = 1$

4. Results

4.1 Equal-weighted portfolio

For all portfolios, after adding the group of cryptocurrencies there are significant improvements in the Sharpe ratios and alpha meaning respectively that there is a higher tradeoff between risks and returns and higher returns with respect to the ones predicted by the CAPM.

For all portfolios except the commodities one, for which the Treynor ratio stays the same, there was also an increase in the Treynor ratio meaning that the excess returns earned for each unit of systematic risk taken was higher. The increase in return however is matched by an increase in risk. We can state that for the equal weighted portfolio adding cryptocurrencies increases the risk-return tradeoff. However, they are not efficient as a diversifier since the standard deviation of the portfolio almost doubles. A risk averse investor would not include cryptocurrencies in her portfolio.

	1/N	
	TS	**TS + CC**
Mean	5,04	11,85
St. Dev	10,66	15,00
β	1,14	1,03
Sharpe ratio	0,40	0,74
Treynor ratio	0,04	0,11
α	0,69	7,83

Table 6: Results for the 1/N allocation strategy of the portfolios of Tech Stocks only (TS) and the Tech Stocks and Cryptocurrencies (TS + CC) one. I report the annualized mean, annualized standard deviation (St. Dev.), beta (β), Sharpe ratio, Treynor ratio and Jensen's measure (α).

	1/N	
	S	**S + CC**
Mean	4,16	11,34
St. Dev	7,59	13,91
β	0,81	0,84
Sharpe ratio	0,44	0,76
Treynor ratio	0,04	0,13
α	0,84	7,92

Table 7: Results for the 1/N allocation strategy of the portfolios of Stocks only (S) and the Stocks and Cryptocurrencies (S + CC) one. I report the annualized mean, annualized standard deviation (St. Dev.), beta (β), Sharpe ratio, Treynor ratio and Jensen's measure (α).

	1/N	
	CMD	**CMD + CC**
Mean	2,36	10,29
Std. Dev	6,45	13,09
β	0,07	0,41
Sharpe ratio	0,24	0,72
Treynor ratio	0,23	0,23
α	1,35	8,22

Table 8: Results for the 1/N allocation strategy of the portfolios of Commodities only (CMD) and the Commodities and Cryptocurrencies (CMD + CC) one. I report the annualized mean, annualized standard deviation (St. Dev.), beta (β), Sharpe ratio, Treynor ratio and Jensen's measure (α).

	1/N	
	C	C + CC
Mean	-0,39	8,68
Std. Dev	2,48	12,52
β	0,05	0,39
Sharpe ratio	-0,48	0,63
Treynor ratio	-0,25	0,20
α	-1,34	6,65

Table 9: Results for the 1/N allocation strategy of the portfolios of Currencies only (C) and the Currencies and Cryptocurrencies (C + CC) one. I report the annualized mean, annualized standard deviation (St. Dev.), beta (β), Sharpe ratio, Treynor ratio and Jensen's measure (α).

4.2 Constrained optimization

For the optimized Sharpe ratio portfolios adding cryptocurrencies does not improve diversification, however, the risk return tradeoff and the alpha improve.
For all portfolios except the commodities one the Treynor Ratio increases.
For all the global minimum variance portfolios adding cryptocurrencies allows to reduce the risk[14], but it also reduces the returns, resulting in an overall reduction of the Sharpe ratio, alpha and Treynor ratio.

[14] This reduction is mainly coming from the inclusion of Tether, which is one of the assets with lower standard deviation in all the sample.

	GMV		SR*	
	TS	**TS + CC**	**TS**	**TS + CC**
Mean	3,51	1,60	9,86	16,41
ST. Dev	9,59	6,10	11,81	14,85
β	1,00	0,41	1,21	1,18
Sharpe ratio	0,28	0,13	0,77	1,05
Treynor ratio	0,03	0,02	0,07	0,13
α	-0,41	-0,48	5,27	11,91

Table 10: Results for the global minimum variance (GMV) and optimized Sharpe ratio (SR*) constrained allocation strategies of the portfolios of Tech Stocks only (TS) and the Tech Stocks and Cryptocurrencies (TS + CC) one. I report the annualized mean, annualized standard deviation (St. Dev.), beta (β), Sharpe ratio, Treynor ratio and Jensen's measure (α).

	GMV		SR*	
	S	**S + CC**	**S**	**S + CC**
Mean	3,72	2,23	5,15	16,12
ST. Dev	6,78	5,15	8,08	14,87
β	0,64	0,37	0,84	0,92
Sharpe ratio	0,43	0,28	0,54	1,03
Treynor ratio	0,05	0,04	0,05	0,17
α	0,94	0,27	1,71	12,44

Table 11: Results for the global minimum variance (GMV) and optimized Sharpe ratio (SR*) constrained allocation strategies of the portfolios of Stocks only (S) and the Stocks and Cryptocurrencies (S + CC) one. I report the annualized mean, annualized standard deviation (St. Dev.), beta (β), Sharpe ratio, Treynor ratio and Jensen's measure (α).

	GMV		SR*	
	CMD	**CMD + CC**	**CMD**	**CMD + CC**
Mean	2,35	1,95	3,07	10,64
Std. Dev	5,06	4,19	5,63	10,43
β	0,03	0,04	0,03	0,29
Sharpe ratio	0,31	0,27	0,40	0,94
Treynor ratio	0,48	0,31	0,87	0,34
α	1,45	1,04	2,19	8,93

Table 12: Results for the global minimum variance (GMV) and optimized Sharpe ratio (SR*) constrained allocation strategies of the portfolios of Commodities only (CMD) and the Commodities and Cryptocurrencies (CMD + CC) one. I report the annualized mean, annualized standard deviation (St. Dev.), beta (β), Sharpe ratio, Treynor ratio and Jensen's measure (α).

	GMV		SR*	
	C	**C + CC**	**C**	**C + CC**
Mean	-0,38	-0,56	0,11	33,55
Std. Dev	1,64	1,61	3,33	37,88
β	0,02	0,02	0,16	1,09
Sharpe ratio	-0,72	-0,84	-0,21	0,86
Treynor ratio	-0,51	-0,60	-0,04	0,30
α	-1,26	-1,43	-1,21	29,35

Table 13: Results for the global minimum variance (GMV) and optimized Sharpe ratio (SR*) constrained allocation strategies of the portfolios of Currencies only (C) and the Currencies and Cryptocurrencies (C + CC) one. I report the annualized mean, annualized standard deviation (St. Dev.), beta (β), Sharpe ratio, Treynor ratio and Jensen's measure (α).

4.3 Unconstrained optimization

The unconstrained optimized Sharpe ratio portfolios of Tech stocks and Currencies produces extreme results with weights that are close to impossible to implement, and with risk levels that no investor would ever bear. For the Tech Stocks and Commodities portfolios adding cryptocurrencies improves the risk return tradeoff and the alpha but does not benefit in terms of diversification.
For the Commodities portfolio the Treynor ratio decreases, while for the Stocks one it increases.
For the global minimum variance portfolios there is no improvement in terms of diversification, however, the standard deviation decreases, but there is also a decrease in returns leading to a lowering of the alpha and of the Sharpe and Treynor ratio.

	GMV		SR*	
	TS	**TS + CC**	**TS**	**TS + CC**
Mean	3,51	1,48	140046868598,28	121,53
Std. Dev	9,59	6,06	117180748686,30	80,52
β	1,00	0,40	1913638702,88	2,83
Sharpe ratio	0,28	0,11	1,20	1,50
Treynor ratio	0,03	0,02	0,73	0,43
α	-0,41	-0,57	134058595648,23	111,87

Table 14: Results for the global minimum variance (GMV) and optimized Sharpe ratio (SR*) unconstrained allocation strategies of the portfolios of Tech Stocks only (TS) and the Tech Stocks and Cryptocurrencies (TS + CC) one. I report the annualized mean, annualized standard deviation (St. Dev.), beta (β), Sharpe ratio, Treynor ratio and Jensen's measure (α).

	GMV		SR*	
	S	**S + CC**	**S**	**S + CC**
Mean	3,77	2,20	7,47	29,01
Std. Dev	6,73	5,10	10,08	22,87
β	0,61	0,35	0,75	1,02
Sharpe ratio	0,44	0,27	0,66	1,23
Treynor ratio	0,05	0,04	0,09	0,28
α	1,08	0,31	4,33	25,03

Table 15: Results for the global minimum variance (GMV) and optimized Sharpe ratio (SR*) unconstrained allocation strategies of the portfolios of Stocks only (S) and the Stocks and Cryptocurrencies (S + CC) one. I report the annualized mean, annualized standard deviation (St. Dev.), beta (β), Sharpe ratio, Treynor ratio and Jensen's measure (α).

	GMV		SR*	
	CMD	**CMD + CC**	**CMD**	**CMD + CC**
Mean	2,35	1,82	3,99	19,33
Std. Dev	5,06	4,18	7,25	17,81
β	0,03	0,04	0,02	0,40
Sharpe ratio	0,31	0,24	0,44	1,04
Treynor ratio	0,48	0,29	1,48	0,46
α	1,45	0,91	3,12	17,27

Table 16: Results for the global minimum variance (GMV) and optimized Sharpe ratio (SR*) unconstrained allocation strategies of the portfolios of Commodities only (TS) and the Commodities and Cryptocurrencies (TS + CC) one. I report the annualized mean, annualized standard deviation (St. Dev.), beta (β), Sharpe ratio, Treynor ratio and Jensen's measure (α).

	GMV		SR*	
	C	**C + CC**	**C**	**C + CC**
Mean	-0,34	-0,49	116010420,68	305022099948,77
Std. Dev	1,60	1,56	265494168,55	288855367420,66
β	0,01	0,01	9327061,69	3577608601,57
Sharpe ratio	-0,71	-0,83	0,44	1,06
Treynor ratio	-1,01	-1,04	0,12	0,85
α	-1,17	-1,33	86823619,29	293826832638,91

Table 17: Results for the global minimum variance (GMV) and optimized Sharpe ratio (SR*) unconstrained allocation strategies of the portfolios of Tech Stocks only (TS) and the Tech Stocks and Cryptocurrencies (TS + CC) one. I report the annualized mean, annualized standard deviation (St. Dev.), beta (β), Sharpe ratio, Treynor ratio and Jensen's measure (α).

4.4 Transaction costs

One very important consideration is the sensitivity of my analysis to implementation costs (e.g., transaction costs[15], rebalancing costs, borrowing costs, margin requirements and management fees). I add transaction costs to the previous portfolios and see whether the same results hold.
I analyze 2 different scenarios: neutral, pessimistic.
For the neutral scenario I assume that the annual costs for holding a portfolio are 50 basis points per annum and 75 for the portfolios that include cryptocurrencies. In addition, I assume 100 basis points per annum for the shorting costs.
For the pessimistic scenario I assume 100 basis points per annum and 150 for the portfolios that include cryptocurrencies. In addition, I assume 200 basis points per annum for the shorting costs.

4.4.1 Equal-weighted

For both the neutral and the pessimistic scenarios, even after accounting for transaction costs, the improvement in the risk return tradeoff remains.

[15] This includes the brokers' commissions and spreads, which are the differences between the price the dealer paid for a security and the price the buyer pays.

Neutral

	1/N	
	TS	**TS + CC**
Mean	4,54	11,10
St. Dev	10,66	15,00
β	1,14	1,03
Sharpe ratio	0,35	0,69
Treynor ratio	0,03	0,10
α	0,19	7,08

Table 18: Results for the 1/N allocation strategy of the portfolios of Tech stocks only (TS) and the Tech Stocks and Cryptocurrencies (TS + CC) one after accounting for transaction costs. I report the annualized mean, annualized standard deviation (St. Dev.), beta (β), Sharpe ratio, Treynor ratio and Jensen's measure (α).

	1/N	
	S	**S + CC**
Mean	3,66	10,59
St. Dev	7,59	13,91
β	0,81	0,84
Sharpe ratio	0,38	0,70
Treynor ratio	0,04	0,12
α	0,34	7,17

Table 19: Results for the 1/N allocation strategy of the portfolios of Stocks only (S) and the Stocks and Cryptocurrencies (S + CC) one after accounting for transaction costs. I report the annualized mean, annualized standard deviation (St. Dev.), beta (β), Sharpe ratio, Treynor ratio and Jensen's measure (α).

	1/N	
	CMD	**CMD + CC**
Mean	1,86	9,54
Std. Dev	6,45	13,09
β	0,07	0,41
Sharpe ratio	0,16	0,67
Treynor ratio	0,16	0,22
α	0,85	7,47

Table 20: Results for the 1/N allocation strategy of the portfolios of Commodities only (CMD) and the Commodities and Cryptocurrencies (CMD + CC) one after accounting for transaction costs. I report the annualized mean, annualized standard deviation (St. Dev.), beta (β), Sharpe ratio, Treynor ratio and Jensen's measure (α).

	1/N	
	C	C + CC
Mean	-0,89	7,93
Std. Dev	2,48	12,52
β	0,05	0,39
Sharpe ratio	-0,68	0,57
Treynor ratio	-0,35	0,18
α	-1,84	5,90

Table 21: Results for the 1/N allocation strategy of the portfolios of Currencies only (C) and the Currencies and Cryptocurrencies (C + CC) one after accounting for transaction costs. I report the annualized mean, annualized standard deviation (St. Dev.), beta (β), Sharpe ratio, Treynor ratio and Jensen's measure (α).

Pessimistic

	1/N	
	TS only	**TS + CC**
Mean	4,04	10,35
ST. Dev	10,66	15,00
β	1,14	1,03
Sharpe ratio	0,30	0,64
Treynor ratio	0,03	0,09
α	-0,31	6,33

Table 22: Results for the 1/N allocation strategy of the portfolios of Tech stocks only (TS) and the Tech Stocks and Cryptocurrencies (TS + CC) one after accounting for transaction costs. I report the annualized mean, annualized standard deviation (St. Dev.), beta (β), Sharpe ratio, Treynor ratio and Jensen's measure (α).

	1/N	
	S	**S+ CC**
Mean	3,16	9,84
ST. Dev	7,59	13,91
β	0,81	0,84
Sharpe ratio	0,31	0,65
Treynor ratio	0,03	0,11
α	-0,16	6,42

Table 23: Results for the 1/N allocation strategy of the portfolios of Stocks only (S) and the Stocks and Cryptocurrencies (S + CC) one after accounting for transaction costs. I report the annualized mean, annualized standard deviation (St. Dev.), beta (β), Sharpe ratio, Treynor ratio and Jensen's measure (α).

	1/N	
	CMD	**CMD + CC**
Mean	1,36	8,79
Std. Dev	6,45	13,09
β	0,07	0,41
Sharpe ratio	0,09	0,61
Treynor ratio	0,08	0,20
α	0,35	6,72

Table 24: Results for the 1/N allocation strategy of the portfolios of Commodities only (CMD) and the Commodities and Cryptocurrencies (CMD + CC) one after accounting for transaction costs. I report the annualized mean, annualized standard deviation (St. Dev.), beta (β), Sharpe ratio, Treynor ratio and Jensen's measure (α).

	1/N	
	C	C + CC
Mean	-1,39	7,18
Std. Dev	2,48	12,52
β	0,05	0,39
Sharpe ratio	-0,89	0,51
Treynor ratio	-0,46	0,16
α	-2,34	5,15

Table 25: Results for the 1/N allocation strategy of the portfolios of Currencies only (C) and the Currencies and Cryptocurrencies (C + CC) one after accounting for transaction costs. I report the annualized mean, annualized standard deviation (St. Dev.), beta (β), Sharpe ratio, Treynor ratio and Jensen's measure (α).

4.4.2 Constrained optimization

For both scenarios the global minimum variance portfolios the alpha and the Sharpe and Treynor ratios further decrease after accounting for transaction costs. For both scenarios the optimized Sharpe ratio portfolios the improvement in the risk return tradeoff does not vanish.

Neutral

	GMV		SR*	
	TS	**TS + CC**	**TS**	**TS + CC**
Mean	3,01	0,85	9,36	15,66
ST. Dev	9,59	6,10	11,81	14,85
β	1,00	0,41	1,21	1,18
Sharpe ratio	0,23	0,01	0,72	1,00
Treynor ratio	0,02	0,00	0,07	0,13
α	-0,91	-1,23	4,77	11,16

Table 26: Results for the global minimum variance (GMV) and optimized Sharpe ratio (SR*) constrained allocation strategies of the portfolios of Tech stocks only (TS) and the tech stocks and Cryptocurrencies (TS + CC) one. I report the annualized mean, annualized standard deviation (St. Dev.), beta (β), Sharpe ratio, Treynor ratio and Jensen's measure (α).

	GMV		SR*	
	S	**S + CC**	**S**	**S + CC**
Mean	2,90	1,48	4,65	15,37
ST. Dev	6,78	5,15	8,08	14,87
β	0,64	0,37	0,84	0,92
Sharpe ratio	0,31	0,13	0,48	0,98
Treynor ratio	0,03	0,02	0,05	0,16
α	0,11	-0,48	1,21	11,69

Table 31: Results for the global minimum variance (GMV) and optimized Sharpe ratio (SR*) constrained allocation strategies of the portfolios of Stocks only (S) and the Stocks and Cryptocurrencies (S + CC) one. I report the annualized mean, annualized standard deviation (St. Dev.), beta (β), Sharpe ratio, Treynor ratio and Jensen's measure (α).

	GMV		SR*	
	CMD	**CMD + CC**	**CMD**	**CMD + CC**
Mean	1,85	1,20	2,57	9,89
Std. Dev	5,06	4,19	5,63	10,43
β	0,03	0,04	0,03	0,29
Sharpe ratio	0,21	0,10	0,31	0,87
Treynor ratio	0,32	0,11	0,68	0,31
α	0,95	0,29	1,69	8,18

Table 28: Results for the global minimum variance (GMV) and optimized Sharpe ratio (SR*) constrained allocation strategies of the portfolios of Commodities only (CMD) and the Commodities and Cryptocurrencies (CMD + CC) one. I report the annualized mean, annualized standard deviation (St. Dev.), beta (β), Sharpe ratio, Treynor ratio and Jensen's measure (α).

	GMV		SR*	
	C	C + CC	C	C + CC
Mean	-0,88	-1,16	-0,39	33,55
Std. Dev	1,64	1,75	3,33	37,88
β	0,02	0,02	0,16	1,09
Sharpe ratio	-1,03	-1,12	-0,36	0,86
Treynor ratio	-0,73	-0,99	-0,07	0,30
α	-1,76	-2,02	-1,71	29,35

Table 29: Results for the global minimum variance (GMV) and optimized Sharpe ratio (SR*) constrained allocation strategies of the portfolios of Currencies only (C) and the Currencies and Cryptocurrencies (C + CC) one. I report the annualized mean, annualized standard deviation (St. Dev.), beta (β), Sharpe ratio, Treynor ratio and Jensen's measure (α).

Pessimistic

	GMV		SR*	
	TS	TS + CC	TS	TS + CC
Mean	2,51	0,10	8,86	14,91
ST. Dev	9,59	6,10	11,81	14,85
β	1,00	0,41	1,21	1,18
Sharpe ratio	0,18	-0,11	0,68	0,95
Treynor ratio	0,02	-0,02	0,07	0,12
α	-1,41	-1,98	4,27	10,41

Table 30: Results for the global minimum variance (GMV) and optimized Sharpe ratio (SR*) constrained allocation strategies of the portfolios of Tech stocks only (TS) and the tech stocks and Cryptocurrencies (TS + CC) one. I report the annualized mean, annualized standard deviation (St. Dev.), beta (β), Sharpe ratio, Treynor ratio and Jensen's measure (α).

	GMV		SR*	
	S	S + CC	S	S + CC
Mean	2,40	2,23	4,15	14,62
ST. Dev	6,78	5,15	8,08	14,87
β	0,64	0,37	0,84	0,92
Sharpe ratio	0,24	0,28	0,42	0,93
Treynor ratio	0,03	0,04	0,04	0,15
α	-0,39	0,27	0,71	10,94

Table 31: Results for the global minimum variance (GMV) and optimized Sharpe ratio (SR*) constrained allocation strategies of the portfolios of Stocks only (S) and the Stocks and Cryptocurrencies (S + CC) one. I report the annualized mean, annualized standard deviation (St. Dev.), beta (β), Sharpe ratio, Treynor ratio and Jensen's measure (α).

	GMV		SR*	
	CMD	**CMD + CC**	**CMD**	**CMD + CC**
Mean	1,35	0,45	2,07	9,14
Std. Dev	5,06	4,19	5,63	10,43
β	0,03	0,04	0,03	0,29
Sharpe ratio	0,11	-0,08	0,23	0,80
Treynor ratio	0,17	-0,10	0,49	0,29
α	0,45	-0,46	1,19	7,43

Table 32: Results for the global minimum variance (GMV) and optimized Sharpe ratio (SR*) constrained allocation strategies of the portfolios of Commodities only (CMD) and the Commodities and Cryptocurrencies (CMD + CC) one. I report the annualized mean, annualized standard deviation (St. Dev.), beta (β), Sharpe ratio, Treynor ratio and Jensen's measure (α).

	GMV		SR*	
	C	**C + CC**	**C**	**C + CC**
Mean	-1,38	-1,91	-0,89	32,05
Std. Dev	1,64	1,75	3,33	37,88
β	0,02	0,02	0,16	1,09
Sharpe ratio	-1,33	-1,55	-0,51	0,82
Treynor ratio	-0,95	-1,37	-0,10	0,29
α	-2,26	-2,77	-2,21	27,85

Table 33: Results for the global minimum variance (GMV) and optimized Sharpe ratio (SR*) constrained allocation strategies of the portfolios of Currencies only (C) and the Currencies and Cryptocurrencies (C + CC) one. I report the annualized mean, annualized standard deviation (St. Dev.), beta (β), Sharpe ratio, Treynor ratio and Jensen's measure (α).

4.4.3 Unconstrained optimization

For both scenarios the global minimum variance portfolios the alpha and the Sharpe and Treynor ratios further decrease after accounting for transaction costs. For both scenarios the optimized Sharpe ratio portfolios the improvement in the risk return tradeoff does not vanish.

Neutral

	GMV		SR*	
	TS	TS + CC	TS	TS + CC
Mean	2,01	-0,27	140046868596,78	119,78
ST. Dev	9,59	6,06	117180748686,30	80,52
β	1,00	0,40	1913638702,88	2,83
Sharpe ratio	0,13	-0,18	1,20	1,48
Treynor ratio	0,01	-0,03	0,73	0,42
α	-1,91	-2,32	134058595646,73	110,12

Table 34: Results for the global minimum variance (GMV) and optimized Sharpe ratio (SR*) unconstrained allocation strategies of the portfolios of Tech Stocks only (TS) and the Tech Stocks and Cryptocurrencies (TS + CC) one. I report the annualized mean, annualized standard deviation (St. Dev.), beta (β), Sharpe ratio, Treynor ratio and Jensen's measure (α).

	GMV		SR*	
	S	S + CC	S	S + CC
Mean	2,27	-1,75	5,97	27,26
ST. Dev	6,73	5,10	10,08	22,87
β	0,61	0,35	0,75	1,02
Sharpe ratio	0,22	-0,50	0,51	1,16
Treynor ratio	0,02	-0,07	0,07	0,26
α	-0,42	-3,65	2,83	23,28

Table 35: Results for the global minimum variance (GMV) and optimized Sharpe ratio (SR*) unconstrained allocation strategies of the portfolios of Stocks only (S) and the Stocks and Cryptocurrencies (S + CC) one. I report the annualized mean, annualized standard deviation (St. Dev.), beta (β), Sharpe ratio, Treynor ratio and Jensen's measure (α).

	GMV		SR*	
	CMD	CMD + CC	CMD	CMD + CC
Mean	0,85	0,07	2,49	17,58
Std. Dev	5,06	4,18	7,25	17,81
β	0,03	0,04	0,02	0,40
Sharpe ratio	0,01	-0,17	0,23	0,94
Treynor ratio	0,02	-0,20	0,78	0,42
α	-0,05	-0,84	1,62	15,52

Table 36: Results for the global minimum variance (GMV) and optimized Sharpe ratio (SR*) unconstrained allocation strategies of the portfolios of Commodities only (CMD) and the Commodities and Cryptocurrencies (CMD + CC) one. I report the annualized mean, annualized standard deviation (St. Dev.), beta (β), Sharpe ratio, Treynor ratio and Jensen's measure (α).

	GMV		SR*	
	C	C + CC	C	C + CC
Mean	-1,84	-2,24	116010419,18	305022099947,02
Std. Dev	1,60	1,56	265494168,55	288855367420,66
β	0,01	0,01	9327061,69	3577608601,57
Sharpe ratio	-1,65	-1,94	0,44	1,06
Treynor ratio	-2,34	-2,45	0,12	0,85
α	-2,67	-3,08	86823617,79	293826832637,16

Table 37: Results for the global minimum variance (GMV) and optimized Sharpe ratio (SR*) unconstrained allocation strategies of the portfolios of Currencies only (C) and the Currencies and Cryptocurrencies (C + CC) one. I report the annualized mean, annualized standard deviation (St. Dev.), beta (β), Sharpe ratio, Treynor ratio and Jensen's measure (α).

Pessimistic

	GMV		SR*	
	TS	TS + CC	TS	TS + CC
Mean	0,51	-2,02	1,40047E+11	118,03
ST. Dev	9,59	6,06	117180748686,30	80,52
β	1,00	0,40	1913638702,88	2,83
Sharpe ratio	-0,03	-0,46	1,20	1,46
Treynor ratio	0,00	-0,07	0,73	0,41
α	-3,41	-4,07	134058595645,23	108,37

Table 38: Results for the global minimum variance (GMV) and optimized Sharpe ratio (SR*) unconstrained allocation strategies of the portfolios of Tech Stocks only (TS) and the Tech Stocks and Cryptocurrencies (TS + CC) one. I report the annualized mean, annualized standard deviation (St. Dev.), beta (β), Sharpe ratio, Treynor ratio and Jensen's measure (α).

	GMV		SR*	
	S	S + CC	S	S + CC
Mean	0,77	-3,50	4,47	25,51
ST. Dev	6,73	5,10	10,08	22,87
β	0,61	0,35	0,75	1,02
Sharpe ratio	0,00	-0,84	0,36	1,08
Treynor ratio	0,00	-0,12	0,05	0,24
α	-1,92	-5,40	1,33	21,53

Table 39: Results for the global minimum variance (GMV) and optimized Sharpe ratio (SR*) unconstrained allocation strategies of the portfolios of Stocks only (S) and the Stocks and Cryptocurrencies (S + CC) one. I report the annualized mean, annualized standard deviation (St. Dev.), beta (β), Sharpe ratio, Treynor ratio and Jensen's measure (α).

	GMV		SR*	
	CMD	CMD + CC	CMD	CMD + CC
Mean	-0,65	-1,68	0,99	15,83
Std. Dev	5,06	4,18	7,25	17,81
β	0,03	0,04	0,02	0,40
Sharpe ratio	-0,29	-0,59	0,03	0,84
Treynor ratio	-0,45	-0,69	0,09	0,37
α	-1,55	-2,59	0,12	13,77

Table 40: Results for the global minimum variance (GMV) and optimized Sharpe ratio (SR*) unconstrained allocation strategies of the portfolios of Commodities only (CMD) and the Commodities and Cryptocurrencies (CMD + CC) one. I report the annualized mean, annualized standard deviation (St. Dev.), beta (β), Sharpe ratio, Treynor ratio and Jensen's measure (α).

	GMV		SR*	
	C	C + CC	C	C + CC
Mean	-3,34	-3,99	116010417,68	305022099945,27
Std. Dev	1,60	1,56	265494168,55	288855367420,66
β	0,01	0,01	9327061,69	3577608601,57
Sharpe ratio	-2,59	-3,06	0,44	1,06
Treynor ratio	-3,67	-3,86	0,12	0,85
α	-4,17	-4,83	86823616,29	293826832635,41

Table 41: Results for the global minimum variance (GMV) and optimized Sharpe ratio (SR*) unconstrained allocation strategies of the portfolios of Currencies only (C) and the Currencies and Cryptocurrencies (C + CC) one. I report the annualized mean, annualized standard deviation (St. Dev.), beta (β), Sharpe ratio, Treynor ratio and Jensen's measure (α).

4.5 Limitations

There are many limitations in this analysis. As highlighted by Platanakis and Urquhart (2019) given the more elevated potential estimation error in a portfolio of cryptocurrencies, portfolio theory might not work in a seamless way. Didisheim and Somoza (2022) find that Crypto-oriented retail investors, on average, have few assets, are young, male, active, and keener on taking risks. Their findings are consistent with the literature (Hackethal et al., 2022). The inflow of short-term investors and noise traders is magnified by the availability of Cryptocurrencies in very small fractions (up to 8 decimal places) without requiring capital in large bundles as happens with other economic instruments. Since cryptocurrencies witness huge price fluctuations, investors must exercise caution and limit their exposure, as they may not essentially lead to improvements in portfolio performance. When analyzing portfolio performance, one wants a wide set of assets with a lot of observations. However, this can become tricky when considering cryptoassets. When looking at the Top 200 cryptoassets in

terms of market capitalization as of the beginning of 2021, only 16 date back to the beginning of 2016; furthermore only 12 in the Top 100 and only 8 in the Top 30 date back to the beginning of 2016. This makes any analysis susceptible to survival bias[16] because if one selects the set of interest with respect to the current position in the cryptoasset ranking and then filters with respect to the data availability, the final set will consist only of the winners that made it and survived to the top ranks.
It must be pointed out that using different techniques, variables, and time periods, may results in contradictory results. Moreover, results may change when more recent data is used, due to the evolving dynamic features of cryptocurrencies.

5. Conclusion

The emergence of such a prominent technologic-intensive market can't be ignored, with countless institutional organizations already holding cryptocurrencies in their balance sheets and/or are exposed to cryptocurrency-related products (e.g., CME's Bitcoin Futures, Bitcoin Futures ETFs, Spot Bitcoin ETF, among others). Two important lessons can be inferred from my analysis. First, including cryptocurrencies in a portfolio does not improve diversification, second, in most of the cases it is likely to improve the risk return tradeoff and this improvement remains even after accounting for transaction costs. The crypto winter and this year's collapse of FTX leading to billions in losses for investors have brought a lot of attention to the cryptocurrencies market. All of this has put into question whether decentralized finance will survive.

We conclude that investors should carefully ponder all the pros and cons of including cryptocurrencies in their portfolio. My framework is intended to help them make that decision. As a final consideration, among the many opportunities to improve upon my admittedly but intentionally simple approach, one could scale returns for each asset and see whether after accounting for higher moments of the distribution the same conclusions would hold. It could also be tested whether in an out of sample analysis, using a rolling window, the results for each allocation strategy remain unaltered. Another alternative could also be to repeat the analysis with a different frequency, e.g. working with hourly data, this could better capture the highly

[16] Survival bias is the tendency to view the performance of existing stocks or funds in the market as a representative comprehensive sample without regarding those that have gone bust. This can result in the overestimation of historical performance and general attributes.

volatile nature of cryptocurrencies. It could also be interesting to repeat the analysis with a different time period and with different assets.

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7. Appendix

A.1 Weights of the Constrained portfolios

	GMV			SR*	
	TS	TS + CC		TS	TS + CC
AAPL	0,10	0,11	AAPL	0,28	0,2
META	0,01	0,01	META	0	0
MSFT	0,05	0	MSFT	0,72	0,53
AMZN	0,11	0,04	AMZN	0	0
GOOGL	0,19	0,03	GOOGL	0	0
IBM	0,53	0,21	IBM	0	0
INTC	0,02	0	INTC	0	0
BTC		0	BTC		0,12
XRP		0	XRP		0
LTC		0	LTC		0
USDT		0,60	USDT		0
ETH		0	ETH		0,15

Table A1: Weights in percentage for each asset.

	GMV			SR*	
	S	S + CC		S	S + CC
JPM	0	0	JPM	0	0
JNJ	0,30	0,15	JNJ	0	0
XOM	0,04	0,02	XOM	0	0
V	0	0	V	0,44	0,3
BRKa	0,23	0,14	BRKa	0,19	0,05
WMT	0,21	0,12	WMT	0,14	0,07
PG	0,22	0,14	PG	0,23	0,23
BTC		0	BTC		0,18
XRP		0	XRP		0
LTC		0	LTC		0
USDT		0,43	USDT		0
ETH		0	ETH		0,17

Table A2: Weights in percentage for each asset.

	GMV			SR*	
	CMD	CMD + CC		CMD	CMD + CC
CPR	0,17	0,11	CPR	0,16	0,13
CF	0,07	0,05	CF	0	0
GOLD	0,53	0,38	GOLD	0,66	0,51
SILVER	0	0	SILVER	0	0
GAS	0,03	0,02	GAS	0,09	0,06
OIL	0,07	0,05	OIL	0,10	0,05
COCOA	0,12	0,09	COCOA	0	0
BTC		0,01	BTC		0,13
XRP		0	XRP		0
LTC		0	LTC		0
USDT		0,29	USDT		0
ETH		0	ETH		0,12

Table A3: Weights in percentage for each asset.

	GMV			SR*	
	C	C + CC		C	C + CC
EUR	0	0	EUR	0	0
GBP	0	0	GBP	0	0
JPY	0,10	0,18	JPY	0	0
AUD	0	0	AUD	0	0
CAD	0,12	0,21	CAD	1	0
CHF	0,06	0,12	CHF	0	0
CNY	0,72	0,42	CNY	0	0
BTC		0	BTC		0,49
XRP		0	XRP		0
LTC		0	LTC		0
USDT		0,06	USDT		0
ETH		0	ETH		0,51

Table A4: Weights in percentage for each asset.

A.2 Weights of the Unconstrained Portfolios

	GMV			SR*	
	TS	**TS + CC**		**TS**	**TS + CC**
AAPL	0,10	0,03	AAPL	4513728244	2,61
META	0,01	0,01	META	-4263214024	-2,41
MSFT	0,05	0,04	MSFT	10167909651	5,32
AMZN	0,11	0,05	AMZN	127034671	0,13
GOOGL	0,19	0,05	GOOGL	118089620	0,24
IBM	0,53	0,22	IBM	-5676738204	-2,21
INTC	0,02	0	INTC	-4986809957	-2,83
BTC		0,02	BTC		0,98
XRP		0	XRP		0,09
LTC		0	LTC		-0,69
USDT		0,60	USDT		-0,97
ETH		-0,01	ETH		0,72

Table A5: Weights in percentage for each asset.

	GMV			SR*	
	S	**S + CC**		**S**	**S + CC**
JPM	-0,10	-0,08	JPM	-0,36	-0,45
JNJ	0,29	0,14	JNJ	-0,04	0,02
XOM	0,06	0,04	XOM	-0,43	-0,63
V	0	0,01	V	0,61	0,59
BRKa	0,32	0,20	BRKa	0,85	0,98
WMT	0,20	0,12	WMT	0,14	0,09
PG	0,22	0,14	PG	0,22	0,26
BTC		0,01	BTC		0,46
XRP		0	XRP		0,04
LTC		0	LTC		-0,31
USDT		0,43	USDT		-0,33
ETH		-0,01	ETH		0,28

Table A6: Weights in percentage for each asset.

	GMV			SR*	
	CMD	CMD + CC		CMD	CMD + CC
CPR	0,17	0,11	CPR	0,21	0,25
CF	0,07	0,05	CF	0,01	-0,01
GOLD	0,53	0,37	GOLD	0,84	0,91
SILVER	0,00	0	SILVER	-0,04	-0,12
GAS	0,03	0,02	GAS	0,10	0,09
OIL	0,07	0,05	OIL	0,12	0,08
COCOA	0,12	0,09	COCOA	-0,23	-0,27
BTC		0,02	BTC		0,34
XRP		0	XRP		0,04
LTC		0,00	LTC		-0,20
USDT		0,29	USDT		-0,33
ETH		-0,01	ETH		0,23

Table A7: Weights in percentage for each asset.

	GMV			SR*	
	C	C + CC		C	C + CC
EUR	0,03	0,03	EUR	-35900711	10297869079
GBP	-0,01	-0,01	GBP	-53687091	-19841008407
JPY	0,11	0,10	JPY	-10687698	-551081757
AUD	-0,12	-0,12	AUD	28072275,8	-8627226250
CAD	0,21	0,21	CAD	53344742,2	1965931049
CHF	0,07	0,07	CHF	35355372,7	11828900968
CNY	0,72	0,68	CNY	-16496890	-4590425857
BTC		0	BTC		6919325191
XRP		0	XRP		827650960
LTC		0	LTC		-3565359563
USDT		0,04	USDT		2000605901
ETH		0	ETH		3334818688

Table A8: Weights in percentage for each asset.