Convertible bond underpricing in the French market

An empirical study

Josefine Durr

Dissertation written under the supervision of Florian Nagler

Dissertation submitted in partial fulfilment of requirements for the MSc in Finance, at Universidade Católica Portuguesa and for the MSc in Accounting, Financial Management and Control at Bocconi University, 31.05.2022

“Convertible bond underpricing in the French market”, submitted by Josefine Durr

The pricing of convertible bonds is a fairly unstudied field of asset pricing due to the instruments’ complex nature and its niche character. The aim of this dissertation is to compute model implied prices for convertible bonds and compare it to their market value in order to determine whether the market truly underprices convertible bonds, a financial theory that has been discussed broadly in the academic community. As a pricing model I applied a Monte-Carlo simulation for stock prices and determined the optimal exercise strategy through the Least-Squares method. With this methodology I priced 34 convertible bonds in the French market and obtained an average underpricing of 4.17%, which reduces to 2.72% when excluding outliers. The results align with previous conducted studies of the French market but are in contrast with some other empirical results in the United States, but due to the substantial difference in convertible bond markets worldwide a direct comparison is not appropriate. Although the finding supports the general claim of convertible bond underpricing and encourages investors to engage in hedging strategies, the lack of substantial research in the European market calls for further empirical studies and improvements of the work presented.

A valorização de obrigações convertíveis é um campo muito pouco estudado da valorização de ativos devido à complexidade dos instrumentos e ao seu carácter de nicho. O objetivo desta dissertação é calcular os preços implícitos de obrigações convertíveis e compará-los com o seu valor de mercado a fim de determinar se o seu mercado está realmente subvalorizado, um fenómeno frequentemente descrito por outros autores. Como modelo de preços, apliquei uma simulação Monte-Carlo para os preços das ações e determinei a estratégia ótima de exercício através do método de Least-Squares. Com esta metodologia, fixei o preço de 34 obrigações convertíveis no mercado francês e obtive uma subvalorização média de 4,17%, que reduz para 2,72% ao excluir os outliers. Os resultados estão em linha com estudos anteriores realizados no mercado francês, mas contrastam com outros resultados empíricos nos Estados Unidos; no entanto, devido à diferença substancial nos mercados de obrigações convertíveis em todo o mundo, uma comparação direta não é apropriada. Embora a conclusão apoie a alegação geral de subvalorização de obrigações convertíveis e encoraje os investidores a adotar estratégias de cobertura, a falta de investigação substancial no mercado europeu requer mais estudos empíricos e melhorias do trabalho apresentado.

Keywords: Convertible bonds; Pricing; French market; Simulation; Least-squares method

Table of Contents

1. Introduction

Convertible bonds are a hybrid financial instrument with both equity and debt features that are increasingly used as a financing tool. Its presence in global financial markets is growing and more and more firms are choosing this instrument instead of conventional ones such as debt or equity issuance. Although most of the issuance volumes are generated in the United States, in recent years issuers in Europe have been tapping the market and generating important volumes.

From an investor perspective, convertible bonds offer a singular and interesting risk and return profile. Since the conversion feature allows investors to convert the bond into a pre-determined number of shares at a pre-determined price there is unlimited upside potential as well as downside protection from the straight bond component, where coupons get paid periodically (if stipulated) and the face value is returned at maturity.

Despite their greater presence as a financing tool, convertible bonds have not been as studied in academia as regular debt and equity. A particular emphasis in modern financial theory has always been placed on the pricing of securities and the market no-arbitrage criteria, and theories in the fields of pure debt and equity have been studied intensively after the 1950's. Nevertheless, theorical research in the pricing of convertible bonds was not initiated until the 1970's and 1980's, when modern option pricing theories surged. Since convertible bonds are a complex financial instrument with features of bonds, equity and derivatives, its valuation has been identified as a key research point in the field of hybrid instrument research. Its particular characteristics call for non-traditional pricing models and the inclusion of several implicit variables, such as the behaviour of the underlying stock, credit risk and interest rates. The multidimensionality and time varying behaviour of these variables make the pricing complex.

The first theorical models proposed where contingent claim approaches based on firm value, which priced the convertible bond through valuation equations and closed-end solutions to partial differential equations. Early on another stream of research surged which used the underlying stock price as driver and quite soon the computation methods became insufficient to tackle the multidimensionality of convertible bonds. This is the primary reason why in recent years a higher emphasis was placed on more dynamic pricing approaches, mainly simulation-based models.

As these theorical pricing models emerged, they were applied to empirically study convertible bond markets. Early on a phenomenon which is still referred to in academia today was observed: convertible bond underpricing and arbitrage. This refers to the fact that convertible bonds fundamental value is above the price at which they are trading in the market. Empirical research conducted in the 1990's all pointed towards underpricing existing in the market and led to the emergence of arbitrageurs taking advantage of the apparent mispricing observed (Batten et al. 2014). One central question posed in more recent research is whether the underpricing – traditionally observed with traditional pricing models, mainly lattice-based or tree based – is still present when applying more flexible simulation-based approaches. Traditional models are often regarded as computationally not efficient enough to include a high number of variables and path-dependencies that are needed for an accurate theoretical pricing of a convertible bond (Batten et al. 2014).

The purpose of this work is to evaluate whether the phenomenon of underpricing is still present when using a dynamic pricing model based on Monte Carlo simulation and the Least Squares Method (LSM) proposed by Longstaff and Schwartz (2001). The pricing studies conducted in recent years were mainly located in the United States market and the European market has been neglected almost entirely in empirical research. The growing issuance volumes in Europe as well as the higher demand and interest from capital markets for this instrument calls for a deeper study of the underpricing in Europe. Results from the US markets cannot be inferred to Europe since the market structure, issuance guidelines and liquidity are entirely different. The same differences persist among the countries in the European Union, which is why one specific country was chosen to conduct the empirical analysis. France is the most liquid market with the highest number of convertible bonds outstanding and available market prices and therefore the market is suited for an empirical study. Our sample for the empirical analysis is thus composed of 34 French convertible bonds to which we apply a Monte Carlo LSM model to derive theorical prices which are then compared to observed market prices to determine whether underpricing exists. The main finding is that on average, the market underprices the convertible bonds by 4.18%, which reduces to 2.71% when excluding outliers. These results are in line with theorical expectations and other studies conducted.

The contributions of this work are plentiful. Firstly, it fills the gap of lacking empirical research in Europe and thus enhances the understanding of convertible bond underpricing on a global scale. European convertible bonds have only been studied by Ammann et al. (2003) in his binomial-tree model, which yielded an underpricing of ca. 3%. Our study adds to his conclusion but employs a more dynamic approach with our simulation-based model, which is often considered a better approach to pricing convertible bonds (Batten et al. 2014). Additionally, it also adds to general studies conducted with a simulation-based approach, which are not plentiful yet. Empirical studies on underpricing in Europe as mentioned are rare, and even more so when using a simulation-based approach to derive theorical prices which is why this work is a relevant contribution to the literature.

To answer the central question on whether underpricing exists in the French market, the paper is organized as follows: Chapter 2 reviews the literature on convertible bonds, describing the instrument and its main characteristics, and review existing pricing models and approaches. In Chapter 3 I describe the data and input parameters used (including a critical discussion of why we chose them) and describe the modelling methodology including a numerical example to describe the procedure in a clear and simple way. Chapter 4 discusses the results obtained, comparing it to findings in literature as well as analysing other pricing relevant factors. We also discuss some of the limitations our model faces in light of pre-existing literature. The main consequence of underpricing, convertible bond arbitrage, is analysed as an investment consequence. Chapter 5 contains a summary of the work and a conclusion.

2. Literature review

The literature review will be split up among a short introductory part on convertible bonds, their main characteristics and why pricing them poses such a challenge, followed by a presentation of existing pricing approaches and empirical studies that have been conducted in the field.

2.2. Convertible bonds

Convertible bonds are a hybrid instrument with both characteristics of stocks and traditional fixed income. They provide the upside potential of stock markets from the equity feature and income and a downside risk protection from the debt features (Calamos 2022).

Investors are entitled to fixed coupon payments (if the issuer stipulates so) and repayment of the principal at maturity, just like in a regular bond. In its simplest form the equity component is given by the conversion option, which allows investors to convert the bond into stock of the issuing company at a predetermined rate at some point in the future. However, in recent years convertible bonds have evolved to include other specific conversion options and embedded options such as put and call features, making them truly complex in nature. Convertible bonds for example can be mandatory (if conversion is forced), convertible into preferred stock, exchangeable (if the underlying stock is not the issuers shares but those of a third party), synthetic (created by third party other than the company whose stock determines the conversion feature) or contingent (automatically converts subject to an event). For the purpose of our study, we will only focus on plain vanilla convertible bonds with no additional embedded options.

The reasons for issuing this type of instrument are often debated in academia. Loncarski et al. (2006) propose that in practice, it is believed to allow debt issuance at a lower cost (since coupon can be lower due to the conversion option and potential derived upside) or that it offers the possibility of equity issuance at a price higher than the current stock price (since conversion price is set at a premium). In their analysis they highlight that both these views in practice are deceptive and that theorical motivations – such as tax-based motivations, risk shifting or delayed equity issuance – cannot be corroborated with their study either, highlighting the need for more investigations on the rationale behind convertible bond issuance. The results from surveys on managers also highlight that delaying equity issuance and issuing at a higher price as well as taking advantage of "cheaper" debt are the main reasons, as displayed in Table 1.

Rationale	O'Neil and Pilcher (1956)	Brigham (1966)	Hoffmeister (1977)
Delayed Equity	82%	68%	40%
Sweeten debt	9%	27%	37%
Other	9%	5%	23%

Table 1: Why do firms issue convertibles? Adapted from Nyborg (1996)

Spiegeleer et al. (2013) further describe the main reasons as cost of capital considerations (cheaper financing), monetization of risk (conversion feature from convertible bonds is worth more in cases of a riskier underlying stock), postponement of dilution of earnings until the convertible bond is converted into shares as well as tax considerations (dividends on new issued shares are after-tax while interests of debt are tax-deductible).

Investors find the attractive risk-return potential from convertible bonds appealing since they offer unlimited upside and a limit to losses. If the company performs well on the stock market the investor will convert the bond into shares at a lower price than current market prices and if the company does poorly it will have the downside protection from a fixed repayment at maturity and coupons, if applicable. It has to be noted that a convertible bond is subject to the same default risk of the company as conventional debt. Due to its unique risk-return profile it's also an attractive instrument for portfolio optimization (Spiegeleer et al. 2013).

Due to the fact that it has both equity and debt features the academic research has categorized these instruments as rather complex in valuation. The main complexity to pricing is that they do not only depend on the debt part (interest rates and credit risk) but also on the variables related to the underlying stock. The conversion option adds another challenge. In its most basic form a convertible bond can be decomposed into a regular bond and an option component. Batten et al. (2014) describes that the theoretical value is the maximum of the bond component (investment value) or conversion value (value if converted into shares). In a risk-neutral environment the convertible bond value is independent of the conversion value and is limited on the downside by the straight bond value (investment value). In reality, the value is directly related to the conversion feature and the convertible bond is subject to both interest and credit risk. Figure 1 offers a depiction of payoffs and value of the convertible bonds as well as predominance of each of the sub-components of the convertible depending on the underlying stock price value.

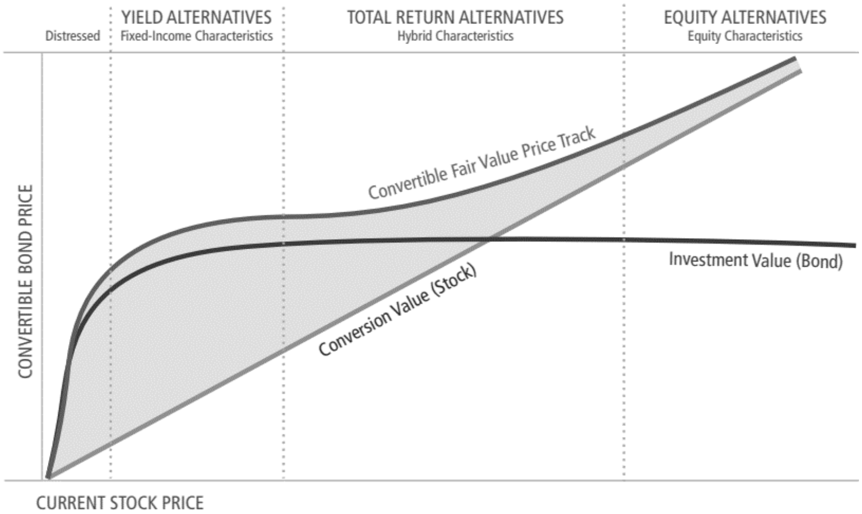


Figure 1: theorical value of a risky convertible bond with the shaded area representing the conversion premium. From Calamos (2022).

The value of a convertible bond is found by pricing the two parts – equity and debt – separately but the complexity of pricing convertible bonds comes from the interaction of several variables related to both equity and debt components, their interaction among them and with the market and the impact of the moneyness, coupon, tenor, conversion ratio, credit risk and face value.

Historically the difficulty in valuation has been approached with several pricing models. Three branches of valuation models have emerged which will be explained in the next chapter.

2.3. Pricing models

As mentioned before, the dual nature of the instrument makes valuation difficult. In theory, the convertible bond needs to be dissected into its main parts to be valued: the straight bond component (investment value) and its equity component (conversion value). In practice, pricing the whole security is more complex.

In the classic literature we can categorize two approaches: structural or firm based approach and reduced or stock based approach, differing both models mainly in the input variables (company specific vs. market based) (Batten et al. 2014). Due to the substantial difference to previous approaches, a recently a more dynamic approach based on simulation was added as a third approach to valuation by some authors (Ammann et al. 2008). Additionally, the computational and numerical solution can be classified into finite difference, finite element, lattice-based and simulation methods.

Followingly, I will describe both approaches and associated numerical solutions used, main research contributions in the field and put an emphasis on pre-existing literature on simulation-based approaches.

Structural approaches

The first theorical models were based on contingent claim approach and option pricing models, making the convertible bond price solely driven by the asset value of the issuer (firm value). The convertible bond was split up among its straight bond component, valued traditionally, and the option component, valued through the option pricing models. The value computed depended heavily on the capital structure assumptions made in the model.

The option components valuation were based on the modern option pricing theories by Black and Scholes (1973) and Merton (1974). These authors did not price convertible bonds but proposed a model to value liabilities, which were defined as contingent claims on firm value. The default was implicit and occurred when the firm's asset value hit a certain barrier. Although the capital structure is very simple, the approach marked a before and after to not only pricing options but also to pricing convertible bonds. Their contribution encouraged extensive research in the field and still has important influences on today's pricing models.

Ingersoll (1977) was the first to use Black Scholes and Merton's contribution in his convertible bond pricing under the structural or firm-value based approach. He proposed a partial differential equation to determine the price and analytically found a closed-form solution under the assumption of only the convertible bond and stock in the capital structure, a perfect market environment, no coupons and no dividends.

Brennan and Schwartz (1977) enhanced this model to incorporate discrete coupons and dividends, early conversion and a call feature. In extension of their work they later propose a two-factor model including stochastic behaviour of firm value and interest rates as well as other senior debt in the capital structure (Brennan and Schwartz 1980). In 1996, Nyborg further included a put feature, floating coupons and a more complex capital structures.

To solve the partial differential equations, the authors use either the finite difference or finite element numerical methods. These approximate a close-end solution for the partial equations proposed in the pricing model. They do not differ much in pricing results and finite element only offers a slight computational efficiency improvement.

As mentioned, this branch in the literature is based on the whole firm value, composed of equity and the convertible bond, and modelled as a Brownian motion. This model was quickly substituted by an approach based on the stock value and the convertible bond to be contingent on stock dynamics. The non-observable firm-value and volatility, the simple capital structure and the high complexity in computing were some of the factors that contributed to the decreasing importance of this approach. Furthermore, closed-end solutions do not allow for the inclusion of time varying and stochastic variables (Batten et al. 2014).

Reduced-form approaches

In response to the mentioned critiques, Mcconnell and Schwartz (1986) were among the first ones to use a one-factor model with equity prices as driving variable. They proposed the convertible bond to be contingent on the stock price, which follows a Brownian motion, and the value of the convertible corresponds either to conversion or par value, whichever one is higher. As opposed to an endogenous credit risk like in the firm-value approach, risk of default was incorporated exogenously by discounting the debt portion, subject to default, at risk-adjusted higher rate.

Further models in this approach were mainly based on the inclusion of new and improved variables, mainly credit risk. Goldman Sachs (1994) for example propose a different approach to include credit risk based on a binomial tree (in the fashion of Cox et al. (1979)) and included credit risk as an average of a risk-adjusted rate and the standard risk free rate for discounting the risky cash flows. Tsiveriotis and Fernandes (1998) improve the model to more specifically account for credit risk by separating the bond and equity component and discounting them at a different interest rate. Other extensions of the models are the ones proposed by Ho and Pfeffer (1996), who introduced a different approach to interest rates, and Yigitbasioglu (2001), who introduced foreign exchange risk.

To solve the pricing model structural approaches mainly use lattice-based and simulation methods. Different from the two methods mentioned before (finite differences and finite elements), lattice-based computations do not intend to solve partial equations and are thus less complicated. It consists of analysing the evolution of asset prices based on backward induction starting at the final node of the stock price tree and computing backwards to the starting point. After the use of pricing trees in option pricing theory, such as the one proposed by Cox et al. (1979), the trees were adapted for convertible bond valuation. Ho and Pfeffer (1996) were among the first ones to use a binomial tree to price the convertible bond algorithmically, and later extensions into trinomial trees had great repercussions in academia.

Although finite differences and element methods as well as lattice-based numerical methods are the most present in literature, some authors point out that these are only appropriate in a non-multivariate environment. As multidimensionality increases, the models fail to provide a solution or becomes too complicated from a computation point of view (Spiegeleer et al. 2013). The inclusion of stochastic and time varying variables is also not possible (Longstaff and Schwartz 2001, Wilde and Kind 2005).

Due to these reasons, some authors started to propose simulation-based computations as an alternative, since it offers a dynamic approach and allows for the inclusion of various variables. Despite its many benefits it is still among the least studied methods to compute and price a convertible bond and some authors suggest simulation-based models as a new, third approach to pricing, although in theory it can be categorized as a reduced-form approach due to the use of the underlying stock as pricing driver.

Simulation based approaches were first introduced in the 1990's to value options by authors such as Tilley (1993) and Broadie and Glasserman (1997). Most of the authors that used simulation for option pricing used either a dynamic programming model to determine the optimal exercise strategy, backwards induction to determine the continuation value on which the exercise strategy is based, simulation trees to generate boundaries or mesh-methods. One of the greatest contributions was made by Longstaff and Schwartz (2001), who priced options with their Monte-Carlo Least Squares Method (LSM). This approach found great acceptance among the academic community and is still widely used today.

Although in option pricing simulation is widely used, the valuation of convertible bonds with simulation techniques is not that widespread. Popularity has been increasing and so has the number of papers focused pricing approach. In general terms, simulation-based computations generate paths of the underlying stock and allow to determine the optimal exercise strategy based on a comparison of continuation value and exercise value. Although the simulation of the underlying stock is carried out in all simulation-based pricing studies similarly, the methods to determine the optimal exercise strategy – among them the LSM method or the Garcia algorithm adopted by Ammann et al (2008) as the most prominent ones – lead to pricing differences in the model results. Often dynamic programming is used as tool to derive pricing results and some of the models and approaches as well as main characteristics are presented below.

Buchan (1997) was among the first ones to use Monte-Carlo simulation to price convertible bonds, who employed a backwards induction technique and a parametric representation of the exercise strategy. Her approach also belongs to the structural models with firm value as underlying and she viewed the option component as European rather than American, even though most convertible bonds option component is of American nature.

First algorithms to price convertible bonds were implemented by Lvov et al. (2004) in their paper “Pricing convertible bonds by simulation”. They based the algorithm on the use of Monte Carlo valuation of American options and propose a model that enables path dependent features and a multivariate environment. They use the Longstaff and Schwartz (2001) backwards induction method to determine the optimal exercise strategy. The algorithm proposed caters for a wide range of embedded features but in the case of a non-callable non-puttable convertible bond it determines the optimal conversion strategy by comparing the payoff from conversion with the expected present value of payoffs from holding it until the next time step. They propose that the optimal strategy is dependent on the expected discounted cashflow from holding the bond and arrive at this value by using Longstaff and Schwartz regression approach.

Kimura and Shinohara (2006) analyse the value of convertible bonds with embedded reset clauses, which allows the conversion ratio to be adjusted upwards if the stock price does not meet certain trigger prices. They model stock prices according to a Monte Carlo simulation and determine the conversion value through a Grant-Vora-Week method, which combines backward and forward dynamic programming to determine the stopping time for American options.

Ammann et al. (2008) propose a two-stage pricing model based on Monte Carlo simulation and an algorithm which is an extension of an American option pricing method proposed by García (2003). It consists of an optimization process based on Monte Carlo generated price simulations to determine the optimal exercise strategy and a valuation stage which applies the optimal exercise strategy to another set of price paths to determine the convertible bond value. They state the algorithm as preferable to other methods to determine the exercise strategy, such as the LSM, because the algorithm can be stopped once a desirable result is reached.

Crépey and Rahal (2011) propose a mathematical model to price convertible bonds with call protection, a special feature which prevents issuers calling the bond. They simulating the issuers underlying stock and derive a price for the convertible bond by numerically solving stochastic differential equations through a discrete time approximation scheme.

Beveridge and Joshi (2011) price game options, which are a class of options granting early exercise rights to both issuer and investor, and convertible bonds are classified as one of the most relevant instrument among game options. They extend previous pricing algorithms proposed by Lvov et al. (2004) and Crépey and Rahal (2011) to include more defined upper and lower bounds for the price and include game option features thus extending previous approaches which only contained option features where only one party can exercise.

Pang et al. (2011) value convertible bond with complex call features where the call option is dependent on a target stock price level over a pre-defined period. Due to path dependence, they use the LSM model based on Monte Carlo simulations. Their algorithm generates random paths for the underlying, checks whether the call conditions are met and if they are and eliminate these “non-surviving” paths. For the remaining paths they employ LSM recursive least square regression approach to determine the continuation value and compare this with the conversion payoff and put value and determine the optimal exercise strategy. The price is then determined for the non-terminated paths as the average of the expected present value in a risk-neutral environment.

Dubrov (2015) propose a machine learning algorithm to price American Options and convertible bonds, arguing that this outperforms traditional methods such as the LSM. They use a Markov Decision Process, a discrete stochastic control process, to find the optimal exercise strategy.

Park et al. (2017) also used the LSM to value a callable convertible bond and include an adjustment in case of default which resembles the same termination process as when the bond is called or when the investor converts the bond. They argue that in previous pricing models based on stock prices credit risk is not modelled accurately enough and propose to reduce the cash flows resulting from the discounted face value right away when default occurs, resulting in a discounting procedure at a lower, adjusted discount rate. They extend this model with stochastic rates and specify the default conditions more in their work in 2018.

As can be seen the pricing approaches used historically are quite diverse and simulation techniques used in more recent years have started to develop more intensively. In terms of evaluation of each of the pricing models, Batten et al. (2014) analysed the most relevant empirical pricing results for each of the main approaches and computation methods and results are displayed in Table 2.

Author	Numerical method	Sample	Pricing error
Structural approaches			
King (1986)	Finite difference	103 US callable convertible bonds at two valuation dates	Underpricing of 3.75%
Carayannopoulos (1996)	Finite difference	30 US convertible bonds from Q4 1989 to Q3 of 1990 (monthly)	Underpricing of 12.9%
Reduced approach			
McConnell and Schwartz (1986)	Finite difference	1 US zero-coupon, callable, putable convertible bond	Underpricing
Ho and Pfeffer (1996)	Binomial tree	7 US callable convertible bond on 1 valuation date	Underpricing
Bailey et al. (1996)	Closed-form solution	4 Eurobonds in Korea over ca. 3 years	Overpricing
Landskroner and Raviv (2002)	Binomial tree	26 Israeli inflation indexed convertible bonds	Underpricing of 1.94%
Barone-Adesi et al. (2003)	Finite element	1 UK callable convertible bond over 9 months (daily)	Overpricing of 5%
Ammann, Kind and Wilde (2003)	Binomial tree	21 callable and puttable French convertible bons over 1.5 years (daily)	Underpricing of 3.24%
Carayannopoulos and Kalimipalli (2003)	Trinomial tree	25 US callable convertibles over ca. 1.5 years (monthly)	Overpricing of 0.58%.
Guschin and Curien (2008)	Trinomial tree	1,500 global convertible bonds	Underpricing of 3.7%
Rotaru (2006)	Trinomial tree	233 US callable CB	Underpricing of 4.84%
Ammann et al. (2008)	Simulation	32 US callable bonds over ca. 6 years (daily)	Overpricing of 0.36%

Table 2: Overview of most relevant historical empirical studies conducted and approaches and computational solutions used. Adapted from Batten et al. (2014).

As can be seen, almost all studies were conducted in the United States and tree-based computation methods were among the most popular ones. Simulation based models have only been implemented and presented in recent years and are thus not as present in empirical valuation studies.

The factor that is most notable is than most of the studies do agree on the presence of underpricing in the markets. This had led to a niche category of investors to engage in arbitrage strategies in convertible bond markets, mainly hedge funds. The aspects concerning these investment consequences will be detailed in Chapter 4.4.

3. Methodology and data

This chapter is going to be split up among the data description, showing our sample of French convertible bonds, the input parameters necessary for pricing the bonds and the methodology and model used in the pricing process.

3.2. Data

I chose a list of French outstanding bonds in the market as of 31. January 2022. Due to country wide differences in convertible bonds mechanisms and listing criteria, the selection of convertible bonds was limited to convertible bonds in France, which represents the most active market in convertible bond issuance in Europe as seen in Chart 1. The differences in legal forms in Europe due to different tax laws and conditions in equity and outside capital issuance makes it difficult to study the European market as a whole (Davis and Lischka 2002). Another reason in choosing the French market is the comparability of results to the only other study that has been conducted in the European market (Ammann et al. 2003). Their paper is focused in France due to availability of market prices, its large size compared to other countries, a high ratio of domestic issues and liquidity of the market. Furthermore, France is one of the longest established markets in this asset class in Europe and is a largely domestic market with little cross currency presence (Noddings et al. 1998). This represents a desirable characteristic for our pricing analysis and in overall the French subsample is considered suited for a pricing test (Ammann et al. 2003).

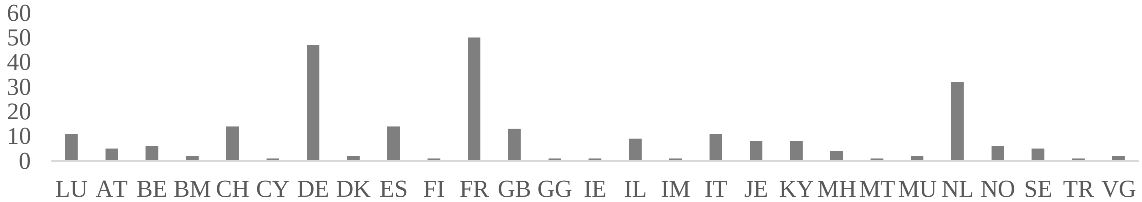


Chart 1: Distribution of active convertible bonds per country outstanding as of 31.01.2022. Retrieved from Thompson Reuters Eikon 2022

The convertibles chosen have no call and no put features and can thus be considered plain vanilla convertible bonds. To guarantee a high-quality database we excluded firms with a market cap below EUR 100 mn., cross-currency convertibles, issue sizes below EUR 50 mn, exchangeable bonds as well as defaulting bonds. As a result, our database is comprised of 34 convertible bonds. Table 3[1] gives an overview of the analysed convertible bonds with the main features of the issuing company and the details of the bond.

[1] For an extended version of the table view Appendix 1

Convertible bond	Market cap	Issue size	Issuance	Maturity	Tenor	Coupon	Conv. ratio	Par
Accor 0.7% 27	7,408	500	07/12/2020	07/12/2027	7	0.70	1.00	48.12
Atos 0% 24	2,643	500	06/11/2019	06/11/2024	5	0.00	1397.62	100,000
Big Ben Int. 1.125% 26	292	87	19/02/2021	19/02/2026	5	1.13	10416.00	100,000
Carrefour 0% 24	14,699	403	27/03/2018	27/03/2024	6	0.00	7966.16	200,000
Carrefour 0% 23	14,699	443	14/06/2017	14/06/2023	6	0.00	5964.12	200,000
EDF 0% 24	31,166	2400	14/09/2020	14/09/2024	4	0.00	1.09	10.93
Edenred 0% 28	11,401	400	14/06/2021	14/06/2028	7	0.00	1.00	64.79
Edenred 0% 24	11,401	500	06/09/2019	06/09/2024	5	0.00	1.00	61.13
Elis 0% 23	2,801	400	06/10/2017	06/10/2023	6	0.00	1.04	31.85
Engie 0% 24	28,517	290	02/06/2021	02/06/2024	3	0.00	1.00	78.25
Figeac Aero 1.125% 22	173	100	18/10/2017	18/10/2022	5	1.13	1.00	25.72
Fnac Darty 0.25% 27	1,242	200	23/03/2021	23/03/2027	6	0.25	1.02	81.03
Genfit 3.5% 22	178	180	16/10/2017	16/10/2022	5	3.50	5.50	29.60
Kering 0% 22	72,771	550	30/09/2019	30/09/2022	3	0.00	1084.95	100,000
Korian 0.875% 27	2,101	400	06/03/2020	06/03/2027	7	0.88	1.10	61.53
MDM 0.125% 23	785	200	06/12/2017	06/12/2023	6	0.13	1.01	48.78
Michelin 0% 23	21,424	501	10/01/2018	10/11/2023	5	0.00	971.14	200,000
Neoen 2% 25	4,357	170	02/06/2020	02/06/2025	5	2.00	1.08	46.20
Neoen 1.875% 24	4,357	200	07/10/2019	07/10/2024	5	1.88	1.08	30.17
Nexity 0.25% 25	1,716	200	02/03/2018	02/03/2025	7	0.25	1.24	68.91
Nexity 0.875% 28	1,716	240	19/04/2021	19/04/2028	7	0.88	1.05	59.81
Orpea 0.375% 27	2,330	500	17/05/2019	17/05/2027	8	0.38	1.02	146.50
Safran 0% 28	43,963	730	14/06/2021	01/04/2028	7	0.00	1.00	180.89
Safran 0.875% 27	43,963	1000	15/05/2020	15/05/2027	7	0.88	1.00	108.23
Schneider 0% 26	87,096	650	24/11/2020	15/06/2026	6	0.00	1.00	176.44
Soitec 0% 25	5,949	325	01/10/2020	01/10/2025	5	0.00	1.00	174.34
TotalEnergies 0.5% 22	119,423	1134	02/12/2015	02/12/2022	7	0.50	3489.30	200,000
Ubisoft Ent. 0% 24	5,289	500	24/09/2019	24/09/2024	5	0.00	1.00	114.63
Veolia Ent. 0% 25	19,690	700	12/09/2019	01/01/2025	6	0.00	1.03	30.41
Vinci 0.375% 22	52,179	679	16/02/2017	16/02/2022	5	0.38	2394.71	200,000
Voltalia 1% 25	1,881	200	13/01/2021	13/01/2025	4	1.00	1.00	31.83
VDM 3% 28	334	55	09/07/2021	09/07/2028	7	3.00	1.00	85.00
Worldline 0% 25	10,952	600	30/07/2020	30/07/2025	5	0.00	1.00	119.44
Worldline 0% 26	10,952	800	30/07/2019	30/07/2026	7	0.00	1.00	103.20
Average	**18,819**	**492**			**6**	**0.55**	**991.63**	**38,294.6**

Table 3: Overview of selected French outstanding convertible bonds. Market cap and issue size in EUR million, tenor in years, coupon in % and par value in EUR.

For the French market, the main features of our dataset are that the mean coupon is 0.55%, which is lower than for the general European sample, mainly due to the fact that there is a significantly higher proportion of pure discount bonds (51%). The maturity ranges from 3 to 10 years with an average tenor of ca. 6 years which highlights the more long-term financing nature of this instrument. On average, the bonds selected have a time to maturity of 3.28 years, with the closest expiry being in February 2022 and the longest maturity up until July 2028.

The conversion feature can either be described as a conversion price or a ratio and the terms can be used interchangeably if adapted. For the purpose of this study, we will make use of the conversion ratio, which gives the number of shares received at the time of conversion for each convertible bond. The conversion ratio stays constant over time, unless there is a capital transaction which alters the nominal value of shares, extraordinary dividends and other operations with a direct impact on the stock price (Ammann et al. 2003). On average in our sample, the conversion ratio is ca. 991 meaning that an average holder of the convertible bond in our sample will receive 991 common shares at the time of conversion. Notable is the presence of conversion ratios of 1 and the associated lower par value of the convertible bonds. This fact is due to the uniqueness of the French market in setting issue price, par value and conversion price equal: conversion price is set at a premium above the share price and par value and bond value are equally priced leading to a ratio of one with (Noddings et al. 1998). In other European countries conversion prices are not set this way, leading to a much higher conversion ratio than in the French sample (for example ca. 2,000 for German convertibles vs. the mentioned 991 for French convertible bonds).

Par values range from the traditional investor tranches sizes of EUR 100,000 to EUR 200,000, highlighting that convertible bond most of the time are acquired by large institutional investors, to par values corresponding to the conversion price which is derived from the stock price level. Convertible bond prices are represented in "cash" terms (money to pay per bond) in our sample vs. the traditional clean presentation form as a percentage over the face value, being this fact distinctive among French bonds when compared to the traditional representation form. Sector wise, Utilities is the most active convertible bond sector with ca. 18% of issued convertibles followed by IT with ca. 15% and retail with ca. 12%[2].

[2] For an industry Split view appendix 2

3.3. Input parameters

Three sources of randomness exist in the convertible bond valuation: the stock price, the interest rate and the credit spread (Xiao 2013). In terms of directly observable data, we used the underlying stock prices and risk-free rates, which were obtained from Thomson Reuters Eikon (2022). Indirect parameters used were credit spreads and volatility of the underlying stock. The specificities in data collection and elaboration are discussed in each of the categories listed below.

Stock price

For each of the respective convertible bonds we collected the underlying stock price performance over 10 years (or the longest time period available) of data, so a total of ca. 3,000 data points were collected for the underlying stock prices of the issuer.

Table 4 presents the performance, as measured by annualized average return and standard deviation of the convertible bonds compared to the underlying stock since issuance of the convertible bond.

	Underlying stock			Convertible bond		
	Mean	**Volatility**	**Sharpe ratio**	**Mean**	**Volatility**	**Sharpe ratio**
Accor 0.7% 27	6.83%	29.60%	0.23	2.93%	23.90%	0.12
Atos 0% 24	-2.26%	16.85%	-0.13	-26.56%	39.24%	-0.68
Bigben Int.1.125% 26	-5.36%	32.78%	-0.16	-11.45%	11.19%	-1.02
Carrefour 0% 24	3.95%	28.23%	0.14	3.09%	10.12%	0.31
Carrefour 0% 23	0.75%	27.54%	0.03	-0.03%	5.41%	-0.01
EDF 0% 24	7.83%	38.07%	0.21	-0.42%	23.87%	-0.02
Edenred 0% 28	-40.04%	23.58%	-1.70	-5.04%	6.43%	-0.78
Edenred 0% 24	-0.97%	31.85%	-0.03	-2.14%	12.80%	-0.17
Elis 0% 23	0.73%	41.95%	0.02	-1.16%	8.59%	-0.14
Engie 0% 24	15.17%	17.22%	0.88	15.53%	9.43%	1.65
Figeac Aero 1.125% 22	-15.47%	44.85%	-0.34	-2.39%	18.26%	-0.13
Fnac Darty 0.25% 27	4.18%	25.40%	0.16	7.29%	20.97%	0.35
Genfit 3.5% 22	-17.85%	62.50%	-0.29	6.61%	42.49%	0.16
Kering 0% 22	22.65%	33.76%	0.67	1.42%	13.01%	0.11
Korian 0.875% 27	-23.85%	37.61%	-0.63	-7.52%	19.10%	-0.39
MDM 0.125% 23	-1.91%	44.45%	-0.04	1.54%	23.96%	0.06
Michelin 0% 23	7.22%	28.94%	0.25	2.79%	11.47%	0.24
Neoen 2% 25	8.03%	39.98%	0.20	6.10%	27.82%	0.22
Neoen 1.875% 24	23.96%	41.09%	0.58	15.58%	30.39%	0.51
Nexity 0.25% 25	-2.86%	30.51%	-0.09	0.01%	17.05%	0.00

Nexity 0.875% 28	-21.03%	20.09%	-1.05	-9.10%	9.91%	-0.92
Orpea 0.375% 27	-26.94%	38.33%	-0.70	-7.84%	13.90%	-0.56
Safran 0% 28	-16.35%	33.63%	-0.49	-5.40%	8.94%	-0.60
Safran 0.875% 27	28.36%	37.46%	0.76	11.46%	17.37%	0.66
Schneider Electric 0% 26	21.48%	22.03%	0.98	9.88%	9.34%	1.06
Soitec 0% 25	21.35%	35.98%	0.59	12.95%	20.59%	0.63
TotalEnergies 0.5% 22	6.59%	13.27%	0.50	7.65%	3.65%	2.10
Ubisoft Ent. 0% 24	10.97%	34.93%	0.31	-2.60%	10.21%	-0.25
Veolia Env. 0% 25	21.94%	31.40%	0.70	8.19%	12.26%	0.67
Vinci 0.375% 22	7.39%	31.37%	0.24	-0.90%	19.08%	-0.05
Voltalia 1% 25	-31.47%	35.05%	-0.90	-7.03%	29.75%	-0.24
VYM 3% 28	26.47%	34.34%	0.77	38.77%	61.83%	0.63
Worldline 0% 25	-26.64%	34.38%	-0.77	-4.76%	7.97%	-0.60
Worldline 0% 26	-8.45%	40.17%	-0.21	-3.37%	13.07%	-0.26
Average	**0.13%**	**32.92%**	**0.02**	**1.59%**	**18.04%**	**0.08**
Positive return stocks	**12.94%**	**31.19%**	**0.43**	**4.25%**	**17.86%**	**0.46**
Negative return stocks	**-16.10%**	**33.23%**	**-0.43**	**-11.24%**	**19.56%**	**-0.38**

Table 4: underlying stock and convertible bond price development since issuance. Average return, volatility (as given by standard deviation) was annualized and Sharpe Ratio computed with long-term risk-free rate

The convertible bonds on average have had a return higher than that of the underlying stock while having less risk measured by standard deviation leading to a better overall risk-adjusted performance. For stocks with a positive return the average return was 12.94% while the average convertible bond return was 17.86%. What is notable is that for stocks with a negative average return, the convertible bond outperformed the underlying performance by far (-16.10% and 19.56%). The Sharpe ratio for all cases showed a better risk-adjusted performance for the convertible bond. With regards to volatility, the stock varies more drastically than the convertible bond which is due to the embedded debt component of the bond which reduces the risk the investor faces.

This contrasts with the characteristics of a convertible bond highlighted in the introductory chapter: lower risk compared to stocks due to the fixed component of the face value and coupons, and lower associated returns. In our subsample on average the convertible bonds performed better and at a lower risk. One of the facts that might explain is the extraordinarily low performance of the stocks due to COVID-19, while the convertible bonds performance was not harmed as much. A large proportion of the convertible bonds were issued after the 01/01/2020, more specifically 47% of the sample, and the large drop on stock markets happened during February and March of 2020. Calamos (2022) also described that during bull markets equities outperformed convertible bonds while in bear markets they offer less severe negative returns. He also found that during the COVID-19 reflation (01/04/2020 – 31/06/2021) convertible bonds offered a return of 60.81% while equities recovered with a 52.76% return, in line with what I observed in the sample. To highlight this behaviour graphically, Chart 2 shows the rebased underlying and convertible bond performance since issuance of the exemplary TotalEnergies 2022 convertible bond.

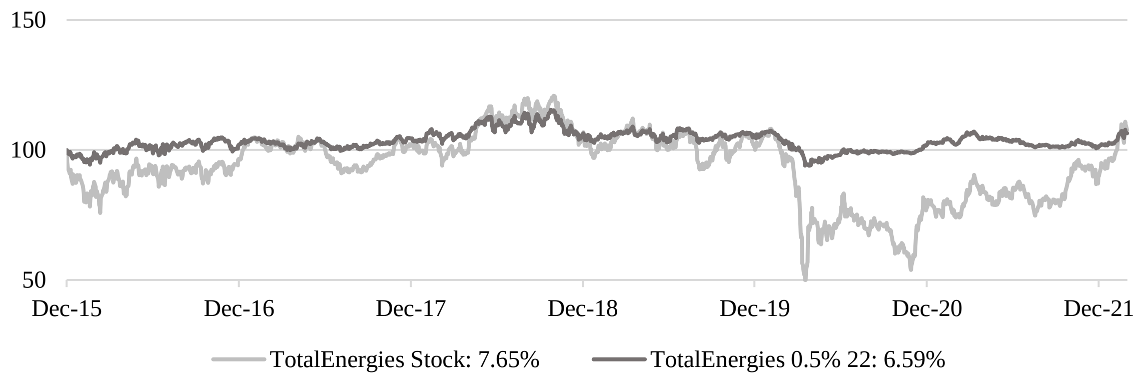


Chart 2: TotalEnergies exemplary underlying and convertible bond price development history

An additional fact to keep in mind is the phenomenon of negative abnormal stock returns after a convertible bond issuance. Duca et al. (2012) for example attribute an induced negative stock return of 4.59% from 2000 – 2008 to the shift of traditional long-only investors to convertible arbitrage funds which short the underlying and cause downward pressure on the stock. This could be another reason for the observed worse performance of the underlying stock.

Interest rates

For the risk-free rate, I use the spot yield curve for French government bonds which was retrieved from Thomson Reuters Eikon. To obtain the continuous term structure of the interest rates, and thus be able to use risk-free rates for the maturity of each convertible bond, I used linear interpolation.

In academia there has been a persistent debate on whether modelled (stochastic) rates should be used instead of constant rates as input in pricing models. In this case, the reason for using static rates is that the effect of stochastic interest rate on convertible bond prices is neglectable according to Brennan and Schwartz (1980) and other authors, such as Amman et al. (2008). They suggest that the cost-benefit of modelling stochastic rates points towards static interest rates being enough for the purpose of pricing convertible bonds. This is why most practitioners and academia do not use stochastic interest rates (Xiao 2013). However, other authors preferred stochastic rates under certain circumstances, for example, when there was a high correlation between stock price and interest rate changes (Ho and Pfeffer 1996).

Volatility

In terms of volatility, in academia there are several different approaches to estimate the underlying stocks volatility and there is no consensus on which model is the most appropriate for forecasting volatility. Among the most popular methods used is the deterministic rate based on historical stock volatility or the implied volatility concept derived from option pricing formulas, but as stated by Ammann et al. (2008) this is not appropriate due to the shorter maturity of options, the lack of traded options for many companies and studies showing that it's not an unbiases estimator of realized volatility (Figlewski 1997). Other authors have preferred the use of the GARCH(1,1) model as discussed by Bollerslev (1986) and Duan (1995).

For the purpose of the study we will use the historical standard deviation of stock prices over the mentioned period and compound it to the appropriate frequency for each specific bond as proposed by Ammann et al. (2003).

Credit spread

Assuming a risk-neutral environment, credit risk (that is the risk of borrowers default and inability to repay coupons or principal) should not affect asset pricing (Hull 2003). Nevertheless, recent studies have demonstrated that credit risk has indeed a significant impact on asset prices and that risky asset pricing differs from risk-neutral pricing. Convertibles tend to be issued by growth and smaller companies, while more mature firms rely on other forms of financing, and this fact points even more towards the importance of credit risk in pricing (Xiao 2013). Specifically for convertibles, a higher associated credit risk is less appealing for investors and thus reduces its price (Gushchin and Curien 2008).

Therefore, additional parameters included in the pricing were convertible bond rating (if applicable), issuer rating and associated credit spread. It has to be noted that for a European issuance, as compared to US ones, bonds are not always rated due to the comparably "low" popularity and underdevelopment in the market. When conducting a credit risk adjustment in the pricing of our convertibles this might lead to some issues, which will be discussed in the modelling part.

In academia the issue of incorporating credit risk in valuation has been tackled with different methods. One approach taken by Goldman Sachs (1994) and Hull (2003) consists of pricing binomial trees with a discount rate given by the arithmetic average between the risk free rate and the risky rate (obtained as credit spread added to risk free rate). McConnell and Schwartz (1986) use a constant spread to capture credit risk. Whilst the approach is widely used among practitioners it has some drawbacks as discussed by Batten et al., who argues that credit spreads are not constant over time (2014).

A more recent stream of methodologies is in line with Tsiveriotis and Fernandes (1998) proposal. They propose splitting the convertible in a cash and stock component. The stock component – the underlying equity of the issuer – has no credit risk since the issuer can always deliver its stock. Oppositely, the cash component – composed of the coupon and principal repayment – is subject to credit risk. Therefore, both components are discounted at different rates. In general terms, this approach has somewhat become a consensus among researchers such as Ho and Pfeffer (1996), Ammann et al. (2008), Zabolotnyuk et al. (2010) or Gushchin and Curien (2008).

For the purpose of this study, I will also use the approach proposed by Tsiveriotis and Fernandes (1998) and use a fixed credit spread. We use the convertible bonds rating if it possesses one or the issuer rating, and if both of them are unavailable (only for a small proportion of the convertibles) we use a model implied Thomson Reuters Eikon rating. Afterwards, we use the Damodaran (2022) equivalence table of credit spreads and company ratings to derive the associated credit spreads to the ratings. In Table 5, we can observe the used ratings and implied credit spreads.

Convertible Bond	Rating type	Rating category	Rating	Implied credit spread
Accor 0.7% 27	Issuer	S&P Senior Unsecured	BB+	2.31%
Atos 0% 24	Issuer	S&P Senior Unsecured	BBB-	1.71%
Bigben Int. 1.125% 26	Model implied	n.a.	BB+	2.31%
Carrefour 0% 24	Issuer	S&P Senior Unsecured	BBB	1.71%
Carrefour 0% 23	Issuer	S&P Senior Unsecured	BBB	1.71%
EDF 0% 24	Issuer	S&P Senior Unsecured	BBB	1.71%
Edenred 0% 28	Issuer	S&P Senior Unsecured	BBB+	1.71%
Edenred 0% 24	Issuer	S&P Senior Unsecured	BBB+	1.71%
Elis 0% 23	Issuer	S&P Senior Unsecured	BB+	2.31%
Engie 0% 24	Bond	n.a.	BBB+	1.71%
Figeac Aero 1.125% 22	Model implied	n.a.	B	4.86%
Fnac Darty 0.25% 27	Issuer	S&P Senior Unsecured	BB+	2.31%
Genfit 3.5% 22	Model implied	n.a.	BB-	2.77%
Kering 0% 22	Issuer	S&P Senior Unsecured	A-	1.33%
Korian 0.875% 27	Model implied	n.a.	B	4.86%

MDM 0.125% 23	Model implied	n.a.	BBB-	1.71%
Michelin 0% 23	Bond	n.a.	A-	1.33%
Neoen 2% 25	Model implied	n.a.	BB-	2.77%
Neoen 1.875% 24	Model implied	n.a.	BB-	2.77%
Nexity 0.25% 25	Model implied	n.a.	BB+	2.31%
Nexity 0.875% 28	Model implied	n.a.	BB+	2.31%
Orpea 0.375% 27	Model implied	n.a.	CCC+	9.46%
Safran 0% 28	Issuer	S&P Senior Unsecured	BBB+	1.71%
Safran 0.875% 27	Issuer	S&P Senior Unsecured	BBB+	1.71%
Schneider Electric 0% 26	Issuer	S&P Senior Unsecured	A-	1.33%
Soitec 0% 25	Model implied	n.a.	BBB+	1.71%
TotalEnergies 0.5% 22	Bond	n.a.	A	1.18%
Ubisoft Ent. 0% 24	Model implied	n.a.	BBB-	1.71%
Veolia Env. 0% 25	Bond	n.a.	BBB	1.71%
Vinci 0.375% 22	Issuer	S&P Senior Unsecured	A-	1.33%
Voltalia 1% 25	Model implied	n.a.	BB-	2.77%
VYM 3% 28	Model implied	n.a.	BB+	2.31%
Worldline 0% 25	Issuer	S&P Senior Unsecured	BBB	1.71%
Worldline 0% 26	Issuer	S&P Senior Unsecured	BBB	1.71%

Table 5: ratings and associated credit spread overview for convertible bonds in the sample

Overall, our sample is composed of investment grade convertible bond (except for one bond whose rating was model implied) being the rating category most present a BB+ rating.

3.4. Methodology

The model used to price the convertible bonds is based on the Longstaff-Schwartz algorithm, which, as discussed in the literature review on pricing mechanisms, consists of determining the optimal exercise strategy and derived payoffs by simulating the underlying stock prices with Monte Carlo simulation and derive the associated exercise strategy through backwards induction and least squares regression. We will first describe the model and then display a numerical example with one of our sample bonds to clearly display the modelling process.

Instead of using firm value as driver of convertible bond prices, as done by Mcconnell and Schwartz (1986), Tsiveriotis and Fernandes (1998), Takahashi et al. (2001) and Ayache et al. (2003) – just to name a few discussed in the literature review – we use a reduced dynamic framework as adapted version of the Least squares Monte Carlo method based on stock value as pricing driver. According to Park et al. (2018) this approach, compared to the traditional finite difference method is computationally more efficient for valuing convertibles with multiple state variables (Ammann et al. 2008).

A plain vanilla convertible bond, which is considered for the purpose of this study, offers the investor two options: exercise the conversion option or hold the convertible until maturity to get the redemption value.

Let $\mathrm{C}(\omega, s;\ \mathrm{t}, \mathrm{T})$ describe the cash flows of the convertible at time s, under realization of strategy ω and conditional on not being exercised prior to t as well as the investor following the optimal stopping strategy for all s, i.e. $\mathrm{t} < \mathrm{s} \leq \mathrm{T}$. In accordance with Kind and Wilde (2003) explanation, in case of conversion the investor receives nSt where n is the conversion ratio and St the underlying price at time t. In general terms, a bondholder exercises its conversion option at final exercise date/maturity if it is in the money. The payoff thus depends on whether the investor decides to terminate the convertible by exercising the option (Kind and Wilde 2003). Prior to this date, investors compare immediate conversion value with expected cash flows from continuing and exercising only if it is more valuable to do so (Longstaff and Schwartz 2001). A convertible bond holder (investor) always follows a conversion policy to maximize his payoff and the value of the convertible bond:

$$Convertible\ bond\ value_{t=T} = \mathrm{CF}(\omega, \mathrm{s}\ =\ \mathrm{T}; \mathrm{t}, \mathrm{T}) = \max\,(nSt, kFV)$$

Formula 1: Investor profit maximization behaviour at maturity "T"

Where n corresponds to the conversion ratio, St to the underlying stock price at time t, k is the final redemption ratio and FV the face value of the convertible bond. The final redemption ratio is usually 100% but some issuers repay the face value at a premium. The bond will only be kept alive if the continuation value is higher than the profit from immediate conversion:

$$C_t(S_t, t) \geq nSt$$

Formula 2: decision rule of continuation vs. immediate conversion

For any step prior to maturity (t_k) the exercise strategy is more complicated, since the continuation value $V't$ of the convertible bond must be determined. This is the conditional expected value of continuation, the value of holding the convertible bond for one more period instead of exercising immediately. It is the expected cash flows from holding and not exercising at point t_k. The payoffs $\mathrm{CF}(\omega, t_k; t_k, \mathrm{T})$ at time t_k are determined through LSM backward induction and at each time step, the payoffs can be as follows:

Strategy	Payoff	Condition
Conversion	nSt	$nSt > V't$
Redemption	kFV	t = T (maturity) and $nSt \leq kFV$
Continuation	0	otherwise

Table 6: exercise options for convertible bond holder

The bondholder either ends the bond by conversion when the conversion value exceeds continuation value or keeps the convertible alive thus comparing at any time step, payoff from exercise with expected value from continuation. At each simulation path "i" the optimal stopping time τ_i^* is defined through LSM induction.

If the bond is not converted the investor, apart from the redemption value, receives stipulated coupon payments until the optimal stopping time (unless it's a zero-coupon bond). Thus, the payoff from the bond must be adjusted to the cash flows under the exercise strategy and the present value of the coupon payments $c(\tau^*)$:

$$Adj.\, convertible\ bond\ value = \mathrm{CF}_{tot} = \mathrm{CF}(\omega, \tau^*; t_k, \mathrm{T}) + c(\tau^*)$$

Formula 3: Adjusted cash flows from convertible bond including accrued interests

Therefore, once simulated the underlying stock price, derived the optimal exercise strategy along each of the time steps modelled and valued the convertible and cash flows under the associated strategy (stipulated through LSM backwards induction) we can derive the theorical model price by discounting all cash flows under risk-neutral pricing measure Q and averaging these over all the simulation paths "n":

$$V_0 = \frac{1}{n}\sum_{i=1}^{n} e^{-\int_{t_0}^{\tau_i^*} r(\omega_i, s)ds}\mathrm{CF}_{tot}(\omega_i, \tau_i^*; t_0, \mathrm{T})$$

Formula 4: theorical model price

Where V_0 is the price of a convertible at time t_0, τ_i^* is the optimal exercise time for path "I", CF_{tot} is the adjusted convertible bond value from Formula 3 and $r(\omega_i, s)$ is the risk-free interest rate applicable during period from t0 until the stopping time τ_i^* for path "i". Ultimately, the goal is to compare that theoretical model value to the observed market price and determine whether the market over- or underprices the convertible bond.

In Table 7 I explain practical steps followed and consequently I will describe the practical implementation with the TotalEnergies 0.5% 2022 convertible bond (from our sample) along the lines of Longstaff and Schwarz's (2001) numerical example.

Step	Process
1	Simulate stock prices through Monte Carlo simulation for "t" paths
2	Adjust algorithm to individual bond specificities
3	Starting at t=n (last step) obtains bond values from simulated stock prices, conversion value and face value
4	Retrospective algorithm from t=n-1 to t=0 to find optimal exercise through LSM regression
6	Obtain optimal stopping point and discount associated cash flows
7	Compare theoretical model price to observed market price

Table 7: Longstaff and Schwartz proposed modelling steps in Monte-Carlo LSM algorithm

Simulated stock prices

The first step of the modelling consists of a simulation of the stock prices for 30-time steps (t=30) with a Brownian motion. Since the stock price is a variable which changes in an uncertain way over time, it follows a stochastic process and due to the timing nature of our analysis it is in discrete time. Therefore, we can apply a Brownian motion as defined by Hull (2003) and used by Longstaff and Schwartz (2001) covering 5,000 paths over t=30 steps:

$$d\,Ln(St) = \left(\mu - \frac{1}{2}\sigma^2\right)dt + \sigma dW(t)$$

Formula 5: Brownian motion as defined by Hull (2003)

More specifically, the formula can be stated as:

$$\ln\left(\frac{S_t}{S_{t-1}}\right) \sim \Phi\left[\left(\mu - \frac{\sigma^2}{2}\right)T, \sigma\sqrt{T}\right]$$

Formula 6: specification of Brownian motion as defined by Hull (2003)

Which corresponds to the discrete-time model known as geometric Brownian motion but adapted under Ito's Lemma for lognormal conditions. S_t corresponds to the stock price at time t, S_{t-1} to the price at t-1 and Φ(m, v) a normal distribution with mean *m* and variance *v*. The formula simply states that the periodic continuous return between the time steps is approximately normal with a certain drift and volatility scaled by the square root of time.

The deterministic component is the constant drift rate $\mu - \frac{\sigma^2}{2}$ and a random shock scaled by volatility. The drift rate is determined by the average return and the standard deviation, which were derived from historical stock price development over the last 10 years. The random component for the stochastic modelling dW(t) corresponds to a Wiener process.

3.5. Numerical example

Applying the mathematical model and the simulation to our TotalEnergies bond I display a numerical exam of how the procedure and pricing is conducted. First, I obtain simulated prices along each of the 30-time steps.

Path	S_0	Step 1	Step 2	….	Step 29	Step 30
1	50.22	50.36	48.73	…	60.40	58.45
2	50.22	48.80	50.59	…	86.01	89.83
…	…	…	…	…	…	…
4,999	50.22	53.60	52.06	…	64.27	61.49
5,000	50.22	48.98	48.83	…	48.48	54.18

Table 8: selected simulated stock price path of TotalEnergies stock

Graphically, for 10 selected paths this is shown as:

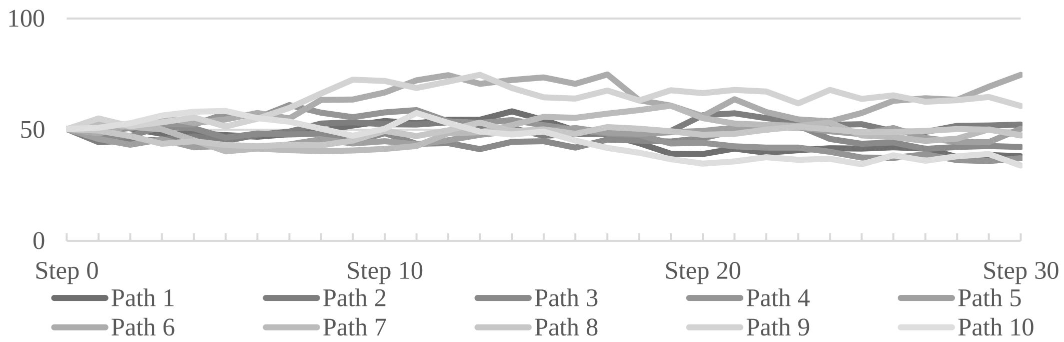


Chart 3: graphical representation of selected simulated stock prices along 30-time steps of TotalEnergies stock

From this simulation I adjust the input data to the specific bond and start the recursive algorithm. Starting from the last step t=30, we compare conversion value (obtained as $simulated\ stock\ price_{t=30} * conversion\ ratio$) to the face value obtained at maturity. The latter would be identical to the respective European bond cash flow, in which the holder will exercise only if the value derived from doing so is higher than the face value obtained from holding the convertible until maturity:

$$Payoff_{t=30} = \max\,(conversion\ value, face\ value)$$

Path	Simulated price	Conversion value	Maturity value	Payoff
1	58.45 €	203,644.72 €	200,000.00 €	203,644.72 €
2	89.83 €	312,975.01 €	200,000.00 €	312,975.01 €
…	…	…	…	…
4,999	61.49 €	214,257.75 €	200,000.00 €	214,257.75 €
5,000	54.18 €	188,779.49 €	200,000.00 €	200,000.00 €

Table 9: Payoff structure at t=30

Moving to the previous step t=29, we make use of the Least Squares regression method ("LSM") implemented by Longstaff and Schwartz to determine the optimal strategy of holding or exercising. Since we have the simulated stock price at t=29 and thus the conversion value, we must determine the cash flow at t=29 and this is done through the LSM regression. If the payoff at t=30 is the face value, i.e. the convertible bond is redeemed, the payoff at t=29 will be the discounted face value and accrued coupons. If it is converted, the payoff will be the discounted conversion value at t=30. To determine the optimal exercise strategy at t=29 we will infer the continuation value from the cashflows at t=30 through regression. This regression, in its simplest form, will look as follows:

$$Discounted\ CF_{t=30} = \beta_0 + \beta_1 simulated\ stock\ price_{t=29} + \beta_2 simulated\ stock\ price_{t=29}^2$$

Where the $Discounted\ CF_{t=30}$ is either redemption or conversion value at maturity. When obtaining the coefficients, we will be able to estimate the expected cash flow from continuing the bond conditional on stock price at t=29. The fitted values obtained are an estimate of the conditional expectation function and allows to estimate the optimal stopping time and exercise strategy of the option component and convertible bond behaviour. For our numerical example, results are displayed in Table 10 and Table 11.

Path	$\boldsymbol{Discounted\ CF_{t=30}}$	$\boldsymbol{simulated\ stock\ price_{t=29}}$	$\boldsymbol{simulated\ stock\ price^2_{t=29}}$
1	203,686.65 €	60.40 €	3647.98
2	313,039.45 €	86.01 €	7398.21
…	…	…	…
4,999	214,301.86 €	64.27 €	4130.23
5,000	200,009.15 €	48.48 €	2350.37

Table 10: data inputs to estimate regression parameters at t=29

Regression parameter	**Estimated value**
$\boldsymbol{\beta_0}$	245,594.88
$\boldsymbol{\beta_1}$	-2,673.56
$\boldsymbol{\beta_2}$	36.39

Table 11: estimated regression parameters at t=29

We can now estimate the cash flow from continuation with the conditional expectation function and the parameters obtained through the regression:

$$Discounted\ CF_{t=29} = 245{,}594 - 2{,}673 * simulated\ stock\ price_{t=29} * simulated\ stock\ price^2_{t=29}$$

Path	Exercise value	Continuation value	Strategy
1	210,444.91 €	216,850.46 €	Continue
2	299,692.16 €	284,824.03 €	Exercise
…	…	…	…
4,999	223,923.15 €	224,055.14 €	Continue
5,000	168,919.77 €	201,499.11 €	Continue

Table 12: optimal exercise strategy at t=29 under Formula 2.

Proceeding recursively, for each time step t=29 to t=1 we will repeat this algorithm and this will yield the optimal strategy for each time step. After having completed this, we can define a stopping rule (0 = no conversion, 1 = conversion for each time step). In the next step we derive the respective cash flows from the rule and discounted them back to t=0 and if there is conversion in the respective path, that will be the value of the convertible and if not, its value will be the discounted cash flows (principal and coupon).

Path	Value with conversion	Maturity value
1	211,909.33	211,909.33
2	226,047.83	226,047.83
…	…	…
4,999	211,686.55	211,686.55
5,000	0.00	200,272.93

Table 13: comparison of modelled prices

The model price is determined by averaging the maturity value given by our model with the market observed ask price. For the case of TotalEnergies, we observe a model value of 207,260.33 € and a market price of 212,968.00 €, yielding to an overpricing of 2.75 %. Of the 5,000 paths used, 45.7% were converted.

It has to be noted that for the discount factor we deploy a method that takes into account credit risk as described in the data input section. This means that face values and coupons are subject to credit risk and thus discounted taking into account the risk-free rate and an adjusting factor for the credit spread. For example, in t=30 when discounting the CF, if the bond was converted, we discount the exercise value at the regular risk-free rate, while if the holder does not exercise, we discount the face value and the corresponding coupon for that step at the adjusted risk-free rate, thus taking into account the credit risk inherent to the bond.

4. Empirical study

After having conducted the analysis to obtain the theoretical model price I compare it to the observed market prices to determine whether there is an under- or overpricing. I then extend the study by analysing several bond characteristics that potentially have an impact on the pricing and discuss some limitations our model could present as well as enhancements it could benefit from.

4.2. Results

The results obtained overall and for each individual bond are summarized in Table 14.

Convertible bond designation	Result	%
Accor 0.7% 27	Overpriced	4.23%
Atos 0% 24	Overpriced	6.50%
Bigben Interactive 1.125% 26	Underpriced	-21.41%
Carrefour 0% 24	Overpriced	8.81%
Carrefour 0% 23	Overpriced	0.58%
Electricite de France 0% 24	Overpriced	14.68%
Edenred 0% 28	Underpriced	-11.82%
Edenred 0% 24	Underpriced	-5.23%
Elis 0% 23	Overpriced	1.17%
Engie 0% 24	Underpriced	-3.84%
Figeac Aero 1.125% 22	Underpriced	-10.63%
Fnac Darty 0.25% 27	Underpriced	-7.71%
Genfit 3.5% 22	Underpriced	-9.30%
Kering 0% 22	Overpriced	0.62%
Korian 0.875% 27	Overpriced	3.33%
Maisons du Monde 0.125% 23	Underpriced	-0.19%
Michelin 0% 23	Overpriced	0.42%
Neoen 2% 25	Underpriced	-33.29%
Neoen 1.875% 24	Underpriced	-12.20%
Nexity 0.25% 25	Underpriced	-6.35%
Nexity 0.875% 28	Underpriced	-15.54%
Orpea 0.375% 27	Overpriced	6.48%
Safran 0% 28	Underpriced	-4.77%
Safran 0.875% 27	Underpriced	-13.37%
Schneider Electric 0% 26	Underpriced	-6.74%
Soitec 0% 25	Overpriced	0.87%
TotalEnergies 0.5% 22	Overpriced	3.63%
Ubisoft Entertainment 0% 24	Underpriced	-2.97%
Veolia Environment 0% 25	Overpriced	4.00%
Vinci 0.375% 22	Underpriced	-7.36%

Voltalia 1% 25	Underpriced	-5.68%
Voyageurs du Monde 3% 28	Underpriced	-10.67%
Worldline 0% 25	Underpriced	-0.90%
Worldline 0% 26	Underpriced	-7.18%
	Average	**-4.17%**
	Median	**-4.31%**
	Average excl. Outliers	**-2.72%**

Table 14: pricing overview

On average, bonds in the market appear to be underpriced by 4.17% although if I take out outliers (+/-2 standard deviations) the underpricing gets reduced to 2.72% in our sample. This result is in line with other results obtained in other pricing studies, although some important differences are found among these studies and ours so results are not entirely comparable.

Pricing analysis and comparison to literature

One of the only empirical studies analysing the convertible bond pricing in France is the one carried out by Ammann et al. (2003), analysing a sample of 21 French convertible bonds over a period of 18 months. They use a stock-based binomial-tree with exogeneous credit risk and the main finding is that on average the market underprices the convertible bonds by 3.7%. Furthermore, they find that out of the money convertible bonds have a more severe underpricing than in or at the money instruments and that the mispricing is decreasing with shorter time to maturity.

The methodology is similar to ours in that it is also stock value based (instead of the traditional firm value-based approaches). As opposed to our model, they use a univariate binomial tree with 100-time steps which allows for inclusion of non-plain convertible bonds with more complex characteristics than our sample, including call and put options as well as triggers. Another divergence from our model is the selection criteria for the convertible bonds: their criteria are market capitalization above USD 75mn and a minimum exchange-based trading volume of USD 75 mn. as well as spreads below 2 percentage points by trading of minimum 3 market makers out of the top 10. These limitations cannot be taken into consideration for our study since their data sample consists of private data obtained by Mace Advisers, while in our case we rely on Thomson Reuters Eikon as public database. They also compute under-pricing for the component and Magrabe models, finding an average under-pricing of 8.74% and 5.6%. They point this to the fact that the call feature is not considered in these alternative models and that the call feature reduces the upside-potential of the equity component in the bond and has a negative impact on prices. Since our sample does not consider callable or puttable bonds, these results are not directly comparable.

The remaining empirical studies on convertible bond mispricing are carried out in other regions (mainly the US) and are thus not directly comparable to our results but in the next paragraphs the most relevant studies will be briefly described and contrasted to our methodology and findings.

King (1986) used an option pricing model with the value of the firm as underlying to value a sample of 103 US convertible bonds and found that on average theorical prices exceeded market prices by 3.75%, thus obtaining the same results to our study. Results were not significantly significant and 90% of model predictions laid within 10% of market values retrieved. The pricing model used relies on the contingent claims valuation models by Brennan and Schwartz (1980) and Ingersoll (1977) and apply their finite difference approximation algorithm.

Carayannopoulos (1996) investigates convertible bond valuation under contingent claim approach (with firm value as driver) and stochastic interest rates. His analysis focuses only the American market by valuing 30 convertible bonds from Q4 of 1989 to Q4 of 1990 using 12 days as data points. Results observed were quite wide: 50% of the bonds pricing within 10% of the mean observed price, ca. 10% pricing within 15% and the remaining bonds priced at more than 115% of the mean market price. Overall, he finds convertible bonds to be underpriced by an average 12.9% but it is clear that the diversity of results calls for further analysis and adjustments of the model. Although it does provide evidence of underpricing, neither the methodology nor the region is comparable to our results.

Loncarski et al. (2009) price a sample of Canadian convertible bonds in the context of an assessment of convertible bond arbitrage and find that at issuance convertible bonds are underpriced. The effect is more pronounced for equity-like convertibles than for debt-like convertibles. As will be discussed later, it is important to note that the US market can be considered more equity-like than the European so contrasting these results directly to our study should be avoided. Their results were that equity-linked issues were underpriced by 27% and that debtlike issues were underpriced by 7%. In the same fashion, Henderson (2005) also analyses convertible bond arbitrage in the US and finds that recently issued convertibles are underpriced. Both of these studies referred to underpricing differently than to other pricing studies (were model and market prices are compared) in that they view a convertible bond as underpriced if it has positive risk-adjusted return in initial secondary market trading. Again results to theoretical pricing studies are not directly comparable but provide evidence of underpricing in the Canadian and US market.

Chan and Chen (2007) empirically analyse US underpricing by using models comparatively: Mcconnell and Schwartz (1986) approach, Tsiveriotis and Fernandes (1998) extension and the Takahashi et al. (2001) model. They find a mean underpricing in Tsiveriotis and Fernandes (1998)/ Mcconnell and Schwartz (1986) of 9.37% and a underpricing of 9.18% in the Takahashi et al. (2001) approach. They further analyse the impact of bond ratings, stock price declines and pricing errors over time.

Greiner et al. (2002) study the mispricing of callable convertible bonds and find evidence of substantial underpricing. They collect a sample of 1,357 Japanese bonds trading on the Tokyo Stock Exchange during 1982 – 1992 and find out that they are underpriced and present arbitrage opportunities.

Buchan (1997) is the first author to use Monte Carlo simulations to price a sample of 35 Japanese convertible bonds. She uses a firm-value based simulation approach with a CIR term structure to price the bonds, and encounters that bonds are slightly overpriced by 1.7% but results are not significant. Ammann et al. (2008) also use a stock-based simulation based pricing model to evaluate the underpricing. They use a Monte Carlo Simulation model based on Longstaff and Schwartz (2001) approach and Garcia (2003) algorithm. They find an overpricing of 0.36%, which contrasts with previous results. Both of these studies are more comparable in terms of methodology, but have a different geographical focus which makes a comparison of results not appropriate.

The most relevant takeaway from the analysis of these results is that a direct comparison is impossible due to the inherent differences between the European and American convertible bond market (where most studies have been conducted). Firstly, in the US convertibles behave in fact more like equity-linked products than their German counterpart, which show more debt related characteristics (Dutordoir and van de Gucht 2004). The differences might be explained by why firms issue convertible bonds, which in the literature are often classified as either reduce the cost of debt or delay the equity issuance (Mayers 1998; Stein 1992). Another difference is the construction of the capital markets: in the United States the conservation of shareholder rights is a much more relevant factor than in other countries and Lee et al. (2009) empirically proved that countries with higher shareholder rights had higher convertible bond issuance. Apart from the difference in issuance and design of convertible bonds among Europe and the

United States, the characteristics of the issuing firms could also not be more different. While in the US issuers tend to be high-growth and smaller companies (Lewis et al. 2003) issuers in Europe are more mature, large and balance sheet strong companies (Bancel and Mittoo 2004). Another consideration when contrasting the results is that the methodology behind the studies differs largely and most of conducted studies only take into account a very small number of data points per convertible bond due to the computational effort. Moreover, they are obtained using firm-value models, which are inherently difficult to parameterize because the firm value is not observable. These further considerations will be discussed in the 4.3 model limitations chapter following.

Option behaviour and relation to mispricing

Another relevant factor broadly discussed in convertible bond pricing studies is the impact of the moneyness on the mispricing. In line with Ammann et al. (2003) and Ammann et al. (2008) we estimate the moneyness and analyse the impact of it on the theoretical pricing of the convertible bonds. A moneyness of 1 indicates that the convertible bond is at-the-money and its conversion value equals the straight debt component (value of the bond hypothetically assuming that there is no conversion option). Descriptive results are displayed in Table 15.

Convertible bond designation	%	Moneyness
Accor 0.7% 27	4.23%	0.67
Atos 0% 24	6.50%	0.44
Bigben Interactive 1.125% 26	-21.41%	1.78
Carrefour 0% 24	8.81%	0.83
Carrefour 0% 23	0.58%	0.56
Electricite de France 0% 24	14.68%	0.77
Edenred 0% 28	-11.82%	0.58
Edenred 0% 24	-5.23%	0.62
Elis 0% 23	1.17%	0.53
Engie 0% 24	-3.84%	0.17
Figeac Aero 1.125% 22	-10.63%	0.24
Fnac Darty 0.25% 27	-7.71%	0.66
Genfit 3.5% 22	-9.30%	0.67
Kering 0% 22	0.62%	7.12
Korian 0.875% 27	3.33%	0.35
Maisons du Monde 0.125% 23	-0.19%	0.42
Michelin 0% 23	0.42%	0.87
Neoen 2% 25	-33.29%	0.73
Neoen 1.875% 24	-12.20%	1.12
Nexity 0.25% 25	-6.35%	0.67

Nexity 0.875% 28	-15.54%	0.65
Orpea 0.375% 27	6.48%	0.27
Safran 0% 28	-4.77%	0.59
Safran 0.875% 27	-13.37%	0.99
Schneider Electric 0% 26	-6.74%	0.85
Soitec 0% 25	0.87%	0.92
TotalEnergies 0.5% 22	3.63%	0.88
Ubisoft Entertainment 0% 24	-2.97%	0.44
Veolia Environment 0% 25	4.00%	1.08
Vinci 0.375% 22	-7.36%	1.16
Voltalia 1% 25	-5.68%	0.52
Voyageurs du Monde 3% 28	-10.67%	1.07
Worldline 0% 25	-0.90%	0.36
Worldline 0% 26	-7.18%	0.41
Average		**0.88**

Table 15: moneyness and convertible bond mispricing

On average, the bonds in our sample are out-of-the money and in terms of relation with the mispricing, both variables seem rather unrelated with a correlation of 0.0058. When categorizing the moneyness of the convertible bonds we can see that for out-of-the money bonds (moneyness below 1) the underpricing seems less severe than for bonds that are in- or at-the-money. Nevertheless, there result should be evaluated carefully as can be seen that the sample of bonds used for in-the-money convertible bonds is quite reduced since most bonds traded out-of-the money, as already indicated by the average mean of moneyness present in our sample.

Moneyness	Mean underpricing	Count of bonds
< 0.5	-1.04%	9
0.5 – 0.8	-6.08%	13
0.8 – 1.0	-1.06%	6
1.0 – 1.2	-6.56%	4
1.2 – 2.0	-21.41%	1
> 2.0	0.62%	1

Table 16: underpricing per moneyness category

Despite the descriptive indication of higher moneyness leading to more underpricing, we do not find empirical evidence of this. On average there is indication that as the bond gets in the money, there is less underpricing but this effect is not significant at all with extremely high p-values.

	Coefficients	Standard Error	P-value
Intercept	-0.0421	0.0203	0.0463*
Moneyness	0.0005	0.0142	0.9741

Table 17: regression results of moneyness on mispricing without control variables

A categorization of into out-of-the money and in the money, we find the same results: out of-the-money bonds are less underpriced when compared to in-the-money convertible bonds yet the effect is not significant.

	Coefficients	Standard Error	t Stat	P-value
Intercept	0.0062	0.0918	0.0676	0.9466
Out-of-the money	-0.0401	0.0934	-0.4290	0.6709
In-the-money	-0.1015	0.1005	-1.0097	0.3205

Table 18: regression of moneyness categories on mispricing without control variables

	Coefficients	Standard Error	t Stat	P-value
Intercept	0.0062	0.0898	0.0691	0.9454
< 0.5	-0.0166	0.0946	-0.1759	0.8617
0.5 – 0.8	-0.0670	0.0932	-0.7190	0.4781
0.8 – 1.0	-0.0168	0.0970	-0.1736	0.8634
1.0 – 1.2	-0.0718	0.1004	-0.7150	0.4805
1.2 – 2.0	-0.2203	0.1270	-1.7350	0.0937*

Table 19: regression of moneyness subcategories on mispricing without control variables

Contrasting these results with previous literature, Ammann et al. (2003) in their French study finds that at-the-money and out-of-the-money are underpriced while in-the-money bonds are overpriced. He also notes that the dispersion is rather high and that the relationship is non-linear. For convertibles moving in-the-money the average mispricing reduces, a fact which he explains to the high probability of conversion and the time value of the option becoming small thus making the pricing easier.

In their simulation based pricing study Ammann et al. (2008) also categorize the moneyness of the convertible bonds and find that the accuracy of the pricing measured by standard deviation of the pricing error is high for in-the-money convertibles compared to lower at- and out-of-the-money. Generally, our results are in line with this study since moneyness also proves to have no significant impact on the pricing deviation.

Older studies, such as the one by King (1986) find a negative relationship between moneyness and the underpricing while Carayannopoulos (1996) finds that in-the-money bonds are slightly underpriced but that the pricing differences vary significantly and that further analysis is necessary to assess the accuracy.

Further convertible bond details and relation to mispricing

I extend the analysis to find out if there are any other characteristics that have an effect on the mispricing in a systematic way. When controlling for these other pricing relevant variables we still find that higher moneyness has a negative impact on the mispricing, but the effect is not significant. The rating category, coupon and years to maturity also have a negative effect on mispricing while on average, the convertible bonds tenor and the issue size leads to a positive impact on the pricing deviation. Only coupon and issue size have a significant impact on the mispricing while all other variables don't have a significant effect.

	Coefficients	Standard Error	t Stat	P-value
Intercept	-0.0683	0.1024	-0.6670	0.5105
Moneyness	-0.0027	0.0134	-0.2014	0.8419
Rating category	-0.0026	0.0073	-0.3608	0.7211
Tenor	0.0123	0.0156	0.7842	0.4397
Coupon (%)	-0.0425	0.0171	-2.4873	0.0193*
Years to maturity	-0.0123	0.0098	-1.2529	0.2210
Issue size (EUR mn)	0.0001	0.0000	2.1931	0.0371*

Table 20: regression results of moneyness and control variables on mispricing

Ammann et al. (2003) also analyse the impact of other bond characteristics on the pricing deviation between market prices and theoretical prices generated by the model. They discover a relationship between overpricing and maturity: the longer the time to maturity, the more convertibles tend to be overpriced. For our case this positive relationship only holds for the initial tenor but not to the time to maturity remaining, although it has to be remembered that both factors are non-significant in our analysis while for Ammann's case the result are significant. They also find that bonds with a maturity in excess of 2,500 days (ca. 7yrs) tare underpriced by 6.8% at a significance level of 1%. In our sample none of the bonds has such a long time to maturity remaining, so this result cannot be contrasted with our data. For bonds with less than 500 days remaining to maturity, the underpricing disappears. King (1986) finds an increased mispricing for shorter time to maturity bonds. In our sample our data is inconclusive of this aspect.

The low statistical significance I encountered in the analysis what was found in Ammann et al. (2008) cross-sectional analysis as well. While the coupon had a negative impact on the mispricing, no significance was reported. The tenor also had a positive impact but wasn't significant either. What differs from our results, as mentioned above, is their positive finding on the impact of moneyness on the pricing but they also find no statistical evidence of this fact.

Zabolotnyuk et al. (2010) in their empirical comparison study of different convertible bond pricing models relate the impact of bond characteristics on the model over- or underpricing to the pricing errors obtained. They find that errors for deep in-the-money convertible bonds are lower or non-significant. For the coupon they find that higher coupon bonds tend to be overpriced under one pricing model and the opposite effect for the other two models used in his comparative analysis. Regarding the time to maturity, they find a higher mispricing error for bonds with longer time to maturity and non-significance in two other models.

4.3. Model limitations and potential extensions

Some aspects of our model need to be reviewed further in light of other pricing studies and the literature analysis. Especially the data availability and the non-explicit input factors for my model are points that could be further analysed and might represent a model limitation.

First, one central aspect that needs to be considered is the data retrieval which is severely limited in comparison to other much more liquid markets, such as the United States, where most of pricing studies have been conducted. This leads to some of the more stringent criteria used for bond selection in academia not being used in our study due to limited data. For example, the liquidity criteria used in Ammann et al. (2008) for a bid-ask spread lower than 2% was not considered when choosing our data sample. Furthermore, the minimum average exchanged-based trading volume of min. 75 mn. was also not considered. Another point that has arisen in literature is the short time period taken into account: King (1986) analysed 103 bonds over 2 days, Buchan (1997) analysed only prices over 1 day and Ammann et al. (2003) over 1.5 years. In our study we only compare modelled prices to market prices on one day, so an extension of our model to a longer time frame would yield more accurate results and be more in line and comparable to other previously conducted studies. As noted by Batten et al. (2014), empirical research in Europe can benefit from samples that are greater and thus increase statistical inference. Due to the fact that the European convertible bond market is not as large in issuance and volumes as the American one, a direct comparison is difficult. Regarding sample data, an interesting adaptation in our model would be the inclusion of callable and puttable convertible bonds since this would increase sample size.

With regards to timing, two facts could lead to biases in the results. The known January effect due to the seasonality in the underlying stock carries over to convertible bond performances, as highlighted by Ma et al. (1988). Since our data collection period ends on the 31st of January 2022 an anomalous effect in the prices might need to be taken into account. Another timing effect which could be taken into account when interpreting our results is the presence of COVID-19 impacts in the financial markets. Throughout the pandemic volatility has been higher than in previous periods and issuances in 2020 were lower than usual while 2021 was a year of significantly higher issuance in the equity-linked market. A separate analysis of the impact of the pandemic on the mispricing of convertible bonds and the separation of pre- and post-covid periods would be an enhancement to our model and yield more specific conclusions on mispricing behaviour.

As already introduced in the methodology chapter, a central discussion point in pricing studies is the input variables used in modelling. The discussions mostly focus on risk-free rates, credit spreads and measures of volatility.

Firstly, the debate on risk-free rates focuses on the use of constant or stochastic interest rates. On the one hand, some authors such as Yigitbasioglu (2001) propose a model in which interest rates are modelled stochastically according to the Cox et al. (1985) model (CIR), arguing that capturing stochastic behaviour of many variables will increase pricing accuracy. Ho and Pfeffer (1996) argue that using stochastic rates versus not using them can have an impact if there is correlation between stock price and interest rate changes. This argument is followed by other authors that include a stochastic component to model credit risk. Among them are Barone-Adesi et al. (2003) who model the interest rate term structure with a mean reverting spot rate process and find that the convertible bond value is negatively related to spot rates.

On the other hands, other authors prove that the benefit added from using stochastic rates is rather small. Brennan and Schwartz (1980) consider the effect of use of stochastic rates to be negligible in pricing a convertible bond for a range of rates and Carayannopoulos (1996) compares the use of constant and stochastic rates in their valuation model and find that pricing differences between a more complex model using stochastic rate and a simple model using constant rates to be small. The use of constant rates exceeds the pricing with stochastic rates by only 1 percent. Additionally, Ammann et al. (2008) compared the use of both methods and their conclusion was that pricing deviations are smaller than half a percentage point and that the difference between prices increase for at-the-money convertibles. The small pricing deviation and the fact that most of the convertible bonds of our sample are out-of-the-money led us to choose a constant interest rate approach as literature seems to prove that pricing differences are neglectable. Nevertheless, using stochastic rates and evaluating the pricing differences with the current model used could be object of further analysis.

Secondly, the implementation of credit risk in empirical studies also differs greatly. The issue revolves mainly around what parameters measures credit risk and how it can be included in the modelling process. Whereas firm-value based pricing models implicitly assume credit risk to be included, the more recent studies based on the stock-value include credit risk separately. McConnell and Schwartz (1986) were the first ones to use an adjusted risk-free rate which included a constant credit spread to include default risk in the pricing. Other authors argued that this approach is not correct due to varying credit spreads over time (Batten et al. 2014). This criticism led authors such as Tsiveriotis and Fernandes (1998) and Buchan (1998) to change the way to approach credit risk inclusion in pricing, mainly by differentiating the debt component of the convertible bond and the equity component and discounting them at different rates.

Another part of the academic community models credit risk through a dynamic component. Takahashi et al. (2001) used Duffie and Singleton (1999) approach and modelled credit risk as jump process and discrete movement where the stock price jumps to zero at default and the fractional loss is thus assumed to be fixed. Ayache et al. (2003) incorporates credit risk by assuming a Poisson distribution process in which the share price drops by a fixed percentage at default. Another popular approach is the one used by Hung and Wang (2002) and Chambers and Lu (2007), which use the Jarrow and Turnbull credit risk model under which stock price jumps to zero when defaulting. Zabolotnyuk et al. (2010) compare the three main models used to incorporate credit risk (Brennan and Schwartz, Tsiveriotis and Fernanes and Ayache) and finds that in terms of measures of fit, the Brennan Schwartz model underperforms the other two.

As can be seen the approaches that are used to incorporate credit risk into the model are very diverse. The unavailability of credit spreads due to lack of straight bonds outstanding led me to use the approach of taking the bonds rating – or if lack thereof, the issuers rating or model implied rating – and use the equivalent credit spread as proposed by Damodaran (2022). Ammann et al. (2008) use a similar approach although more specific since they extract time series of spreads for several rating categories instead of using pre-defined rates provided by the Damodaran database. They note that this procedure can limit the model and derived pricing due to several facts: (1) the spreads are averages for rating categories, (2) ratings change over time and thus the approach is too static and (3) omission of potential time lags and rating assessment issues. They also point that these drawbacks might have a strong impact since most bonds are lower rated and thus have a higher credit risk and spreads. Since our sample is also composed of lower rated bonds these drawbacks also apply to our implementation of credit spreads and in light of the broad academic discussion our chosen approach could need further analysis on how choosing different credit spreads could lead to pricing differences. Furthermore, the lack of inclusion of probability of default and recovery rates are another drawback of the Tsiveriotis and Fernandes (1998) model, which was used in our model, as pointed out by Ayache et al. (2003).

Lastly, the parameter of volatility is probably among the most discussed in academia since a broad range of approaches have been used in academia. Historically the use of deterministic volatility based on historical stock returns was among the most used procedure as seen in King (1986), Carayannopoulos (1996) and Mcconnell and Schwartz (1986). Even in more recent studies, among them Ammann et al. (2003) and Gushchin and Curien (2008), the use of historical stock returns to derive volatility through standard deviation over a certain, varying time period is present.

Since the assumption of constant volatility is debated, some authors incorporate volatility in other ways. Barone-Adesi et al. (2003) proposed the use of implied volatility derived from option pricing. Ammann et al. (2008) in his simulation based study discuss some of the drawbacks that are derived from using IVOL, mainly that maturities of option and convertibles differ, many companies have no traded options and that IVOL is not an unbiased estimator. They also state that constant volatility is not able to fully exploit the benefits of the dynamic model and due to these drawbacks and the clustering of volatility they propose the GARCH(1,1) model as an alternative to estimate volatility.

The preference from practitioners for using historical volatility is due to the fact that it is modelling free (Poon and Granger 2005), the complexity is low and that there are no ex-ante assumptions about market efficiency (Rotaru 2006). Since it has been a preferred practitioners approach, I incorporate the volatility in our model in this rather simplistic way through historical standard deviation of returns. Studies on how using different volatility approaches affects pricing have not yet been conducted, so a preference for one or the other option is difficult to manifest and pose an opportunity for further research.

A comparative difference study on how these input factors – interest rates, credit risk and volatility – could be of interest and would extend and confirm the observed underpricing and results could be further compared to other research on the evaluation of parameter variations on the pricing model. For example, implementing stochastic rates, different approaches to credit risk inclusion and other approaches to volatility could be implemented and pricing discrepancies observed could pose further research questions.

4.4. Investment consequences

One of the key consequences of the underpricing historically present in theory and to some extent in empirical studies, such as ours, is convertible bond arbitrage. This strategy is based on the belief that convertible bond pricing is uncertain and arbitrage opportunities exist. Niche investors, such as particular convertible bond funds or hedge funds take advantage of this and engage in trading activities with the convertible bond and underlying shares to make a profit. In the last years there has been an important rise of convertible arbitrage hedge funds up to the point that these constitute ¾ of the convertible bond market (Mitchell et al. 2007). Additionally, half of trading in the secondary market are due to convertible arbitrage strategies (Lhabitant and Learned 2002). In the primary markets hedge funds purchases account for 70 – 80% of primary market transactions. Convertible bond investors and arbitragers are thus an important source of liquidity in the markets.

One of the most prominent strategies is the delta hedge strategy, in which investors take a long position in the convertible bond and short the underlying stock. The number of shares sold short are affected by the conversion ratio, the delta (sensitivity of option to changes in price of the underlying stock) and the sensitivity of delta to changes in the underlying stock (gamma) (Calamos 2013). In general terms, the number of shares shorted equivale the delta ratio. In one of the rare empirical studies on return characteristics of convertible arbitrage strategies, Agarwal et al. (2011) describe convertible arbitrageurs as intermediary of capital provision to issuers and using delta hedge (buy-and-hedge strategy) as a way to transmit the equity component risk to the equity market.

Calamos (2013) notes that equity-like convertible bonds – those that have a delta measure close to one and are thus highly reactive to changes in the price of the underlying stock – are the ones most desired for investors looking for arbitrage opportunities. Other characteristics that make convertible bonds attractive for arbitrage are issuers with low or no dividend pay-out, undervalued stock and high market liquidity so that they can be sold easily. They should offer some sort of coupon as well because this offers an additional cash inflow for investors.

Although we observe an underpricing in our sample, most of the convertible bonds analysed in our sample are zero-coupons and are more debt-like than equity like (due to the nature of the European market) so the benefit of an arbitrage strategy for our sample would need to be analysed further. Nevertheless, results obtained in the field of empirical analysis of underpricing and derived arbitrage strategies are insightful and prompt an interest for further continuation of this study in the field of arbitrage opportunities.

Loncarski et al. (2009) for example analyse the convertible bond arbitrage in the Canadian market from 1998 to 2007, providing one of the most comprehensive empirical studies in the field of arbitrage due to underpricing. They apply a Tsiveriotis and Fernandes (1998) pricing model and find that on average, there is an underpricing of 10% at issuance for a sample of non-zero-coupon bonds (due to the fact that they have no additional cash flow for investors). They also analyse the underpricing by different classes of delta measures, finding that convertible bonds are more underpriced the higher their delta values are (i.e., the higher the sensitivity of the convertible bond to price changes in the underlying stock). They distinguish equity-like convertibles as having a delta higher than 0.5 and find that they are underpriced by 26.8% while debt-like convertible bonds only are underpriced by an average 7%. With regards to capital market reactions, they highlight that the short positions taken due to arbitrage affect the underlying stock returns negatively over the issuance date. Apart from their pricing and arbitrage analysis, they also point out the fact that in practice the arbitrage strategy is difficult to implement and that its performance has declined in recent years. They do not study the reasons for this, but attribute the performance decline to steady equity markets with lower volatility, higher interest rates, increased competition among hedge funds and a shift in the issuers universe and in their issuance structures. Although some of the criteria might still be a cause for diminishing performance, it is obvious that in light of the volatile capital markets in the last couple of years and the current circumstances of geopolitical disorders, changes in central bank behaviours and macro-economic shifts these arbitrage strategies would need further analysis.

Another relevant contribution in the field of convertible bond arbitrage is Ammann et al.'s (2010) empirical study on performance drivers of convertible bond funds. They analyse several variables and find that there seems to be a relation between return and holdings of convertible bond and stocks, which is interpreted as a positive sign that some funds engage in arbitrage ctivities of dynamic nature.

Xiao (2013) poses a contrary view to arbitrage due to underpricing stating that the driver of profitability of a convertible bond arbitrage strategy is not solely underpricing but rather a large positive gamma (gamma is the sensitivity of the convertible bonds delta to the underlying stock price change). He draws the conclusion due to not finding empirical evidence of underpricing, and the fact that most arbitrageurs take a delta-neutral position (by shorting the underlying stock), which makes the sign of gamma very relevant. A large gamma makes the portfolio very profitable, especially due to large underlying movements which are common for the type of issuing companies (start-up and small companies).

Although the pricing models and an apparent underpricing in the markets allows arbitrage investors to find mispriced instruments, the holding of a hedged portfolio which allows to make a profit is difficult. For a delta hedge strategy for example, the arbitrageur must constantly rebalance the portfolio to the changing delta ratio when stock prices change (Choi et al. 2008). Additionally, even though the theory of convertible bond arbitrage is based on underpricing, this underpricing is not steadily present in all markets as shown by the literature review and analysis of empirical studies conducted. This is why investment professionals engaging in convertible bond arbitrage must decide whether there is an underpricing in the market, define the instruments which are mispriced and determine the appropriate strategy to make a profit out of underpriced instruments. Additionally, depending on the type of strategy they engage, a consistent observation of the market and the convertible bond and underlying performance is needed in order to rebalance the arbitrage portfolio.

5. Conclusion

This work represents the first simulation-based pricing study on convertible bonds conducted in the French market. Although simulation-based models have been gaining popularity in recent years, empirical studies have been focused almost solely in the US market, so our study adds to the research on convertible bond underpricing.

The model used simulates stock prices through a Monte Carlo approach and uses the Longstaff and Schwarz method to evaluate the optimal exercise strategy. This allows to evaluate and price the sample of 34 vanilla convertible bonds in a dynamic and multivariate environment, which is one of the main benefits of using stock-based simulation methods versus traditional computation methods.

The pricing study yielded an average underpricing of 4.17%, which reduces to 2.72% when excluding outliers. This means that on average, the theoretical bond value derived from our model are higher than observed market prices. Although most studies were realized in the US, our study seems to confirm the phenomenon of convertible bond underpricing. When compared to previous literature, we found that the results were in line with Ammann et al.'s (2003) binomial-tree based pricing study of the French market. He found an average underpricing of 3.24% for a sample of 21 convertible bonds. Except for the pricing model used and the significantly shorter time period observed in our model, his results are the ones most comparable to our study. Other simulation-based studies in the US market have led to both conclusion on over- and underpricing, so an interesting extension to our model would be a geographical implementation in the US to have contrastable results.

When assessing relationships among the bond characteristics and the mispricing, we found that the more a bond is in-the-money, the higher the underpricing. The coupon, rating category and years to maturity left implied a higher underpricing, although none of the variables are significant in our analysis. This is in fact with previous pricing studies, where variables effect on mispricing were found to be insignificant or inconclusive.

Overall, the use of a simulation-based model entails many benefits for pricing convertible bonds, mainly the dynamic and higher flexibility they offer. Although this fact has been described in recent literature, empirical studies on simulation-based pricing is still lagging behind. Our work contributes to filling this gap since it provides results for a modelling approach not used broadly in academia – the Monte Carlo and LSM method for plain vanilla convertible bonds – and it adds to mispricing studies in Europe, where almost no studies have been conducted, not to mention simulation-based studies. When compared to previous literature, our model could be enhanced in several ways to have higher pricing accuracy and comparability. Among the extensions proposed could be a longer time horizon observed, since we are only observing mispricing on one single day. The inclusion of callable and puttable bonds would increase our sample size and to make the study more comparable to previous US mispricing findings an extension of the model to convertible bonds to the US market would be additions to the literature and enhancements to the work proposed.

6. Appendix

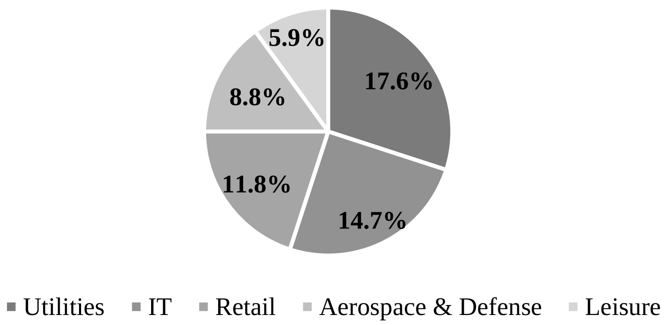
Appendix 1: extended version of Table 3

Company	Industry	Market cap	Designation	Issue size	Issuance date	Maturity	Tenor	Coupon	Coupon Frequency	Conversion price	Conversion ratio	Par value	Uts	Days to maturity	Years
Accor	Leisure	7408	Accor 0.7% 27	500	07/12/2020	07/12/2027	7	0.7	Annual	48.12	1.00	48.12	Gross	2136	5.93
Atos	IT	2643	Atos 0% 24	500	06/11/2019	06/11/2024	5	0		71.55	1397.62	100000.00	Clean	1010	2.81
Bigben Interactive	Durables	292	Big Ben Int. 1.125% 26	87	19/02/2021	19/02/2026	5	1	Semi	9.60	10416.00	100000.00	Clean	1480	4.11
Carrefour	Retail	14699	Carrefour 0% 24	403	27/03/2018	27/03/2024	6	0		20.37	7966.16	200000.00	Clean	786	2.18
Carrefour	Retail	14699	Carrefour 0% 23	443	14/06/2017	14/06/2023	6	0		29.97	5964.12	200000.00	Clean	499	1.39
Electricite de France	Utilities	31166	EDF 0% 24	2400	14/09/2020	14/09/2024	4	0		10.49	1.09	10.93	Clean	957	2.66
Edenred	IT	11401	Edenred 0% 28	400	14/06/2021	14/06/2028	7	0		64.79	1.00	64.79	Clean	2326	6.46
Edenred	IT	11401	Edenred 0% 24	500	06/09/2019	06/09/2024	5	0		61.13	1.00	61.13	Clean	949	2.64
Elis	Textile service	2801	Elis 0% 23	400	06/10/2017	06/10/2023	6	0		30.48	1.04	31.85	Clean	613	1.70
Engie	Utilities	28517	Engie 0% 24	290	02/06/2021	02/06/2024	3	0		78.25	1.00	78.25	Clean	853	2.37
Figeac Aero	Aerospace & Defense	173	Figeac Aero 1.125% 22	100	18/10/2017	18/10/2022	5	1.125	Semi	25.72	1.00	25.72	Gross	260	0.72
Fnac Darty	Retail	1242	Fnac Darty 0.25% 27	200	23/03/2021	23/03/2027	6	0.25	Annual	79.52	1.02	81.03	Gross	1877	5.21
Genfit	Biotech	178	Genfit 3.5% 22	180	16/10/2017	16/10/2022	5	3.5	Semi	5.38	5.50	29.60	Gross	258	0.72
Kering	Luxury	72771	Kering 0% 22	550	30/09/2019	30/09/2022	3	0		92.17	1084.95	100000.00	Clean	242	0.67
Korian	Health Care	2101	Korian 0.875% 27	400	06/03/2020	06/03/2027	7	0.875	Annual	55.84	1.10	61.53	Gross	1860	5.17
Maisons du Monde	Retail	785	MDM 0.125% 23	200	06/12/2017	06/12/2023	6	0.125	Annual	48.20	1.01	48.78	Gross	674	1.87
Michelin	Automotive	21424	Michelin 0% 23	501	10/01/2018	10/11/2023	5	0		169.89	971.14	200000.00	Clean	648	1.80
Neoen	Utilities	4357	Neoen 2% 25	170	02/06/2020	02/06/2025	5	2	Semi	42.98	1.08	46.20	Gross	1218	3.38
Neoen	Utilities	4357	Neoen 1.875% 24	200	07/10/2019	07/10/2024	5	1.875	Semi	28.07	1.08	30.17	Gross	980	2.72
Nexity	Real Estate	1716	Nexity 0.25% 25	200	02/03/2018	02/03/2025	7	0.25	Semi	55.44	1.24	68.91	Gross	1126	3.13
Nexity	Real Estate	1716	Nexity 0.875% 28	240	19/04/2021	19/04/2028	7	0.875	Semi	57.18	1.05	59.81	Gross	2270	6.31
Orpea	Health Care	2330	Orpea 0.375% 27	500	17/05/2019	17/05/2027	8	0.375	Annual	143.63	1.02	146.50	Gross	1932	5.37
Safran	Aerospace & Defense	43963	Safran 0% 28	730	14/06/2021	01/04/2028	7	0		180.89	1.00	180.89	Clean	2252	6.26
Safran	Aerospace & Defense	43963	Safran 0.875% 27	1000	15/05/2020	15/05/2027	7	0.875	Annual	107.80	1.00	108.23	Gross	1930	5.36
Schneider Electric	Electrical technology	87096	Schneider 0% 26	650	24/11/2020	15/06/2026	6	0		176.44	1.00	176.44	Clean	1596	4.43
Soitec	Semiconductors	5949	Soitec 0% 25	325	01/10/2020	01/10/2025	5	0		174.34	1.00	174.34	Clean	1339	3.72
TotalEnergies	Oil and gas	119423	TotalEnergies 0.5% 22	1134	02/12/2015	02/12/2022	7	1	Semi	57.32	3489.30	200000.00	Clean	305	0.85
Ubisoft Entertainment	Entertainment	5289	Ubisoft Ent. 0% 24	500	24/09/2019	24/09/2024	5	0		114.63	1.00	114.63	Clean	967	2.69
Veolia Environnement	Utilities	19690	Veolia Ent. 0% 25	700	12/09/2019	01/01/2025	6	0		29.50	1.03	30.41	Clean	1066	2.96
Vinci	Construction	52179	Vinci 0.375% 22	679	16/02/2017	16/02/2022	5	0	Semi	83.52	2394.71	200000.00	Clean	16	0.04
Voltalia	Utilities	1881	Voltalia 1% 25	200	13/01/2021	13/01/2025	4	1	Annual	31.83	1.00	31.83	Gross	1078	2.99
Voyageurs du Monde	Leisure	334	VDM 3% 28	55	09/07/2021	09/07/2028	7	3	Annual	85.00	1.00	85.00	Gross	2351	6.53
Worldline	IT	10952	Worldline 0% 25	600	30/07/2020	30/07/2025	5	0		119.44	1.00	119.44	Clean	1276	3.54
Worldline	IT	10952	Worldline 0% 26	800	30/07/2019	30/07/2026	7	0		103.20	1.00	103.20	Clean	1641	4.56
Average		**18819**		**492**			**5.71**	**0.55**			**991.63**	**38294.64**		**1199.147**	**3.33**

Appendix 2: industry split of sample

Industries	Convertible bonds
Utilities	6
IT	5
Retail	4
Aerospace & Defense	3
Leisure	2
Real Estate	2
Luxury	1
Durables	1
Health Care	1
Automotive	1
Construction	1
Health Care	1
Electrical technology	1
Semiconductors	1
Oil and gas	1
Textile service	1
Entertainment	1
Biotech	1
Grand Total	**34**

Graphical representation of top 5 industries:



7. Publication bibliography

Agarwal, Vikas; Fung, William H.; Loon, Yee Cheng; Naik, Narayan Y. (2011): Risk and return in convertible arbitrage: Evidence from the convertible bond market. In *Journal of Empirical Finance* 18 (2), pp. 175–194. DOI: 10.1016/j.jempfin.2010.11.008.

Ammann, Manuel; Kind, Axel; Seiz, Ralf (2010): What drives the performance of convertible-bond funds? In *Journal of Banking & Finance* 34 (11), pp. 2600–2613. DOI: 10.1016/j.jbankfin.2010.04.016.

Ammann, Manuel; Kind, Axel; Wilde, Christian (2003): Are convertible bonds underpriced? An analysis of the French market. In *Journal of Banking & Finance* 27 (4), pp. 635–653. DOI: 10.1016/S0378-4266(01)00256-4.

Ammann, Manuel; Kind, Axel; Wilde, Christian (2008): Simulation-based pricing of convertible bonds. In *Journal of Empirical Finance* 15 (2), pp. 310–331. DOI: 10.1016/j.jempfin.2006.06.008.

Ayache, E.; Forsyth, P. A.; Vetzal, Kenneth R. (2003): Valuation of Convertible Bonds With Credit Risk. In *JOD* 11 (1), pp. 9–29. DOI: 10.3905/jod.2003.319208.

Bancel, Franck; Mittoo, Usha R. (2004): Why Do European Firms Issue Convertible Debt? In *Eur Financial Management* 10 (2), pp. 339–373. DOI: 10.1111/j.1354-7798.2004.00253.x.

Barone-Adesi, Giovanni; Bermudez, Ana; Hatgioannides, John (2003): Two-factor convertible bonds valuation using the method of characteristics/finite elements. In *Journal of Economic Dynamics and Control* 27 (10), pp. 1801–1831. DOI: 10.1016/S0165-1889(02)00083-0.

Batten, Jonathan A.; Khaw, Karren Lee-Hwei; Young, Martin R. (2014): Convertible bond pricing models. In *Journal of Economic Surveys* 28 (5), pp. 775–803. DOI: 10.1111/joes.12016.

Beveridge, Christopher; Joshi, Mark (2011): Monte Carlo Bounds for Game Options Including Convertible Bonds. In *Management Science* 57 (5), pp. 960–974. DOI: 10.1287/mnsc.1110.1319.

Black, Fischer; Scholes, Myron (1973): The Pricing of Options and Corporate Liabilities. In *Journal of Political Economy* 81 (3), pp. 637–654. DOI: 10.1086/260062.

Bollerslev, Tim (1986): Generalized autoregressive conditional heteroskedasticity. In *Journal of Econometrics* 31 (3), pp. 307–327. DOI: 10.1016/0304-4076(86)90063-1.

Brennan, M. J.; Schwartz, E. S. (1977): Convertible bonds: Valuation and optimal strategies for call and conversion. In *The Journal of Finance* 32 (5), pp. 1699–1715. DOI: 10.1111/j.1540-6261.1977.tb03364.x.

Brennan, Michael J.; Schwartz, Eduardo S. (1980): Analyzing Convertible Bonds. In *Journal of Financial and Quantitative Analysis* 15 (4), p. 907. DOI: 10.2307/2330567.

Brigham, Eugene F. (1966): An analysis of convertible debentures: theory and some empirical evidence. In *The Journal of Finance* 21 (1), pp. 35–54. DOI: 10.1111/j.1540-6261.1966.tb02953.x.

Broadie, Mark; Glasserman, Paul (1997): Pricing American-style securities using simulation. In *Journal of Economic Dynamics and Control* 21 (8-9), pp. 1323–1352. DOI: 10.1016/S0165-1889(97)00029-8.

Buchan, M. J. (1997): Convertible bond pricing: Theory and evidence. Doctoral thesis. Harvard University.

Calamos, Nick P. (2013): Convertible arbitrage. Insights and techniques for successful hedging. Hoboken, N.J.: Wiley.

Calamos, Nick P. (2022): Convertible securities. Available online at https://www.calamos.com/globalassets/media/documents/market-insights/convertiblesecurities.pdf.

Carayannopoulos, Peter (1996): Valuing convertible bonds under the assumption of stochastic interest rates: An empirical investigation. In *Quarterly Journal of Business and Economics* 35 (3), pp. 17–31.

Chambers, Donald R.; Lu, Qin (2007): A Tree Model for Pricing Convertible Bonds with Equity, Interest Rate, and Default Risk. In *The Journal of Derivatives* 14 (4), pp. 25–46. DOI: 10.3905/jod.2007.686421.

Chan, Alex W. H.; Chen, Nai-fu (2007): Convertible Bond Underpricing: Renegotiable Covenants, Seasoning, and Convergence. In *Management Science* 53 (11), pp. 1793–1814. DOI: 10.1287/mnsc.1070.0722.

Choi, D.; Getmansky, M.; Tookes, H. (2008): Convertible bond arbitrage, liquidity externalities, and stock prices☆. In *Journal of Financial Economics* 91 (2), pp. 227–251. DOI: 10.1016/j.jfineco.2008.02.008.

Cox, John C.; Ingersoll, Jonathan E.; Ross, Stephen A. (1985): An Intertemporal General Equilibrium Model of Asset Prices. In *Econometrica* 53 (2), p. 363. DOI: 10.2307/1911241.

Cox, John C.; Ross, Stephen A.; Rubinstein, Mark (1979): Option pricing: A simplified approach. In *Journal of Financial Economics* 7 (3), pp. 229–263. DOI: 10.1016/0304-405X(79)90015-1.

Crépey, Stéphane; Rahal, Abdallah (2011): Pricing convertible bonds with call protection. In *JCF* 15 (2), pp. 37–75. DOI: 10.21314/jcf.2011.258.

Damodaran, Aswath (2022): Capital Structure DataRatings. Spreads and Interest Coverage Ratios. Available online at https://pages.stern.nyu.edu/~adamodar/.

Davis, Mark; Lischka, Fabian R. (2002): Convertible bonds with market risk and credit risk. Ammann, Manuel; Kind, Axel; Wilde, Christian. In : AMS/IP Studies in Advanced Mathematics. Providence, Rhode Island: American Mathematical Society, pp. 45–58.

Duan, Jin-Chuan (1995): THE GARCH OPTION PRICING MODEL. In *Mathematical Finance* 5 (1), pp. 13–32. DOI: 10.1111/j.1467-9965.1995.tb00099.x.

Dubrov, Bella (2015): Monte Carlo Simulation with Machine Learning for Pricing American Options and Convertible Bonds. Available online at https://dx.doi.org/10.2139/ssrn.2684523, updated on 11/2/2015.

Duca, Eric; Dutordoir, Marie; Veld, Chris; Verwijmeren, Patrick (2012): Why are convertible bond announcements associated with increasingly negative issuer stock returns? An arbitrage-based explanation. In *Journal of Banking & Finance* 36 (11), pp. 2884–2899. DOI: 10.1016/j.jbankfin.2012.03.019.

Duffie, Darrell; Singleton, Kenneth J. (1999): Modeling Term Structures of Defaultable Bonds. In *Rev Financ Stud* 12 (4), pp. 687–720. DOI: 10.1093/rfs/12.4.687.

Dutordoir, Marie; van de Gucht, Linda M. (2004): Are European Convertibles More Debt-Like than the US Issues? An Empirical Analysis. In *SSRN Journal*. DOI: 10.2139/ssrn.579681.

Figlewski, Stephen (1997): Forecasting Volatility. In *Financial Markets Inst & Instr* 6 (1), pp. 1–88. DOI: 10.1111/1468-0416.00009.

García, Diego (2003): Convergence and Biases of Monte Carlo estimates of American option prices using a parametric exercise rule. In *Journal of Economic Dynamics and Control* 27 (10), pp. 1855–1879. DOI: 10.1016/S0165-1889(02)00086-6.

Goldman Sachs (1994): Valuing convertible bonds as derivatives. In *Quantitative strategies research notes* 11.1, :30.

Greiner, Daniel; Kalay, Avner; Kato, Hideaki Kiyoshi (2002): The market for callable-convertible bonds: Evidence from Japan. In *Pacific-Basin Finance Journal* 10 (1), pp. 1–27. DOI: 10.1016/S0927-538X(01)00025-7.

Gushchin, Victor; Curien, Erwan (2008): The pricing of convertible bonds within the Tsiveriotis and Fernandes framework with exogenous credit spread: Empirical analysis. In *J Deriv Hedge Funds* 14 (1), pp. 50–65. DOI: 10.1057/jdhf.2008.7.

Henderson, Brian (2005): Convertible bonds: new issue performance and arbitrage opportunities. Unpublished PhD Thesis. University of Illinois, USA.

Ho, Thomas S.Y.; Pfeffer, David M. (1996): Convertible Bonds: Model, Value Attribution, and Analytics. In *Financial Analysts Journal* 52 (5), pp. 35–44. DOI: 10.2469/faj.v52.n5.2022.

Hoffmeister, J. Ronald. (1977): Use of convertible debt in early 1970s. Re-evaluation of corporate motives. In *Quarterly Review of Economics and Business* 17.2, 23-21.

Hull, John C. (Ed.) (2003): Options futures and other derivatives. Pearson Education. Boston, MA: Springer US.

Hung, Mao-Wei; Wang, Jr-Yan (2002): Pricing Convertible Bonds Subject to Default Risk. In *The Journal of Derivatives* 10 (2), pp. 75–87. DOI: 10.3905/jod.2002.319197.

Ingersoll, Jonathan E. (1977): A contingent-claims valuation of convertible securities. In *Journal of Financial Economics* 4 (3), pp. 289–321. DOI: 10.1016/0304-405X(77)90004-6.

Kimura, Toshikazu; Shinohara, Toshio (2006): Monte Carlo analysis of convertible bonds with reset clauses. In *European Journal of Operational Research* 168 (2), pp. 301–310. DOI: 10.1016/j.ejor.2004.07.008.

Kind, Axel; Wilde, Christian (2003): A simulation-based pricing method for convertible bonds. Working paper series: Derivatives Research Project. In *Salomon Center for the Study* *of Financial Institutions* (March). Available online at http://web-docs.stern.nyu.edu/salomon/docs/derivatives/s-drp-03-07.pdf.

King, Raymond (1986): Convertible bond valuation: an empirical test. In *Journal of Financial Research* 9 (1), pp. 53–69. DOI: 10.1111/j.1475-6803.1986.tb00435.x.

Lee, Cheng-Few; Lee, Kin-Wai; Yeo, Gillian Hian-Heng (2009): Investor protection and convertible debt design. In *Journal of Banking & Finance* 33 (6), pp. 985–995. DOI: 10.1016/j.jbankfin.2008.10.010.

Lewis, Craig M.; Rogalski, Richard J.; Seward, James K. (2003): Industry conditions, growth opportunities and market reactions to convertible debt financing decisions. In *Journal of Banking & Finance* 27 (1), pp. 153–181. DOI: 10.1016/S0378-4266(01)00212-6.

Lhabitant, François-Serge; Learned, Michelle (2002): Hedge Fund Diversification. In *JAI* 5 (3), pp. 23–49. DOI: 10.3905/jai.2002.319062.

Loncarski, Igor; Horst, Jenke R. ter; Veld, Chris H.: Why do Companies Issue Convertible Bonds? A Review of Theory and Empirical Evidence. In : Advances in Corporate Finance and Asset Pricing, pp. 311–339.

Loncarski, Igor; Horst, Jenke ter; Veld, Chris (2009): The Rise and Demise of the Convertible Arbitrage Strategy. In *Financial Analysts Journal* 65 (5), pp. 35–50. DOI: 10.2469/faj.v65.n5.1.

Longstaff, Francis A.; Schwartz, Eduardo S. (2001): Valuing American Options by Simulation: A Simple Least-Squares Approach. In *Rev Financ Stud* 14 (1), pp. 113–147. DOI: 10.1093/rfs/14.1.113.

Lvov, D.; Yigitbasioglu, A.; El Bachir N. (2004): Pricing Convertible Bonds by Simulation. In *Second IASTED international conference*. Available online at https://www.actapress.com/pdfviewer.aspx?paperid=17657.

Ma, Christopher K.; Rao, Ramesh P.; Weinraub, Herbert J. (1988): The Seasonality in Convertible Bond Markets: A Stock Effect or Bond Effect? In *Journal of Financial Research* 11 (4), pp. 335–347. DOI: 10.1111/j.1475-6803.1988.tb00093.x.

Mayers, David (1998): Why firms issue convertible bonds: The matching of financial and real investment options. In *Journal of Financial Economics* 47 (1), pp. 83–102. DOI: 10.1016/S0304-405X(97)00038-X.

Mcconnell, John; Schwartz, Eduardo (1986): LYON Taming. In *The Journal of Finance* 41 (3), pp. 561–576. DOI: 10.1111/j.1540-6261.1986.tb04516.x.

Merton, Robert C. (1974): On the Pricing of Corporate Debt: The Risk Structure of Interest Rates. In *The Journal of Finance* 29 (2), p. 449. DOI: 10.2307/2978814.

Mitchell, Mark; Pedersen, Lasse Heje; Pulvino, Todd (2007): Slow Moving Capital. In *American Economic Review* 97 (2), pp. 215–220. DOI: 10.1257/aer.97.2.215.

Noddings, Thomas C.; Christoph, Susan C.; Noddings, John G. (1998): The international handbook of convertible securities. A global guide to the convertible market. London: Fitzroy Dearborn.

Nyborg, K. G. (1996): The use and pricing of convertible bonds. In *Applied Mathematical Finance* 3 (3), pp. 167–190. DOI: 10.1080/13504869600000009.

O'Neil, John T.; Pilcher, C. James (1956): Raising Capital with Convertible Securities. In *Southern Economic Journal* 23 (1), p. 103. DOI: 10.2307/1053635.

Pang, Huanpeng; Wang, An; Li, Shenghong: Pricing the convertible bonds under complex call trigger condition with Longstaff and Schwartz Model. 2011. In : International Conference on Computer, vol. 2011, pp. 2079–2082.

Park, Keehwan; Jung, Mookwon; Lee, Sangki (2017): Pricing a Defaultable Convertible Bond by Simulation. In *KJFS* 46 (4), pp. 947–965. DOI: 10.26845/KJFS.2017.09.46.4.947.

Park, Keehwan; Jung, Mookwon; Lee, Sangki (2018): Credit ratings and convertible bond prices: a simulation-based valuation. In *The European Journal of Finance* 24 (12), pp. 1001–1025. DOI: 10.1080/1351847X.2017.1368682.

Poon, Ser-Huang; Granger, Clive (2005): Practical Issues in Forecasting Volatility. In *Financial Analysts Journal* 61 (1), pp. 45–56. DOI: 10.2469/faj.v61.n1.2683.

Rotaru, Camelia (2006): Underpricing of new convertible debt issues of US firms: 1980-2003-Empirical analysis. In *Journal of Financial Management & Analysis* 19.1, p. 45.

Spiegeleer, Jan de; Schoutens, Wim; Jabre, Philippe (2013): The handbook of convertible bonds. Pricing, strategies and risk management. Hoboken, N.J.: Wiley.

Stein, Jeremy C. (1992): Convertible bonds as backdoor equity financing. In *Journal of Financial Economics* 32 (1), pp. 3–21. DOI: 10.1016/0304-405X(92)90022-P.

Takahashi, Akihiko; Kobayashi, Takao; Nakagawa, Naruhisa (2001): Pricing Convertible Bonds with Default Risk. In *The Journal of Fixed Income* 11 (3), pp. 20–29. DOI: 10.3905/jfi.2001.319302.

Thompson Reuters Eikon (2022): Database.

Tilley, James A. (1993): Valuing American options in a path simulation model: Transactions of the Society of Actuaries.

Tsiveriotis, Kostas; Fernandes, Chris (1998): Valuing Convertible Bonds with Credit Risk. In *JFI* 8 (2), pp. 95–102. DOI: 10.3905/jfi.1998.408243.

Wilde, Christian; Kind, Axel H. (2005): Pricing Convertible Bonds with Monte Carlo Simulation. In *SSRN Electronic Journal*. DOI: 10.2139/ssrn.676507.

Xiao, Tim (2013): A simple and precise method for pricing convertible bond with credit risk. In *J Deriv Hedge Funds* 19 (4), pp. 259–277. DOI: 10.1057/jdhf.2014.5.

Yigitbasioglu, Ali Bora Bora (2001): Pricing Convertible Bonds with Interest Rate, Equity, Credit, and FX Risk. In *SSRN Journal*. DOI: 10.2139/ssrn.294464.

Zabolotnyuk, Yuriy; Jones, Robert; Veld, Chris (2010): An Empirical Comparison of Convertible Bond Valuation Models. In *Financial Management* 39 (2), pp. 675–706. DOI: 10.1111/j.1755-053X.2010.01088.x.