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# Simulated annealing algorithm for facility layout problem with fixed machines and multiple process routes 

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## 1. Introduction

A good placement of facilities contributes to the overall efficiency of operations and may reduce the total operating expenses up to $50 \%$ [Tompkins et al. (1996)]. Due to the variety of considerations found in the articles, researchers do not agree about a common and exact definition of layout problems. In its most general form, the FLP is defined as follows: Given $N$ departments with known area requirements. In general, the objective is minimizing the total material handling cost which expressed as

$$
\begin{equation*}
Z=\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} c_{i j} f_{i j} d_{i j} \tag{1}
\end{equation*}
$$

[Konak et al, Kulturel-Konak, Norman, \& Smith (2006)], in which $d_{i j}$ is the distance between departments $i$ and $j$ for a specific distance metric, $f_{i j}$ is the amount of material flow, and $c_{i j}$ is the material handling cost per unit flow per unit distance traveled between departments $i$ and $j$. The constraints of the problem include satisfying the area requirements of the departments and the boundaries of the layout. There are many researches that has formulated and solved the facility layout problem. Some recent researches on this problem are surveyed in the following: Kirkpatrick, Gelatt, and Vecchi (1983) firstly suggested that simulation annealing algorithm for combinatorial optimization problems is firstly used by Lacksonen and Enscore (1993). Chwif, Barretto and Moscato (1998) suggest a solution to the facility layout problem using simulated annealing. Baykasoglu and Gindy (2001) considered SA for dynamic layout problem with equally sized facilities. Mir and Imam (2001) developed a hybrid optimization approach for the layout design of un-equal-area facilities. Mckendall, Shang, and kuppusamy (2006) developed SA for dynamic layout problem.

In the recent researches Solimanpur and Kamran (2010) formulated and solved the facilities location problem in the presence of alternative processing routes with genetic algorithm. Ramezan Sahin (2011) considered simulated annealing algorithm for solving the bi-objective facility layout problem.

In a general perspective, the researches on facilities location problem available in the literature are classified in two categories. In the first category it is assumed that the locations are known in advance and the problem is to assign facilities to different locations. In second category it is assumed that locations are not known preferment and must be determined in a continuous area. The problem studied in this paper refers to the first category.

In fact, often products in a manufacturing system can be produced by different process routes by several machines. Imagine a manufacturing system that found beforehand. In this system there are two machine groups. In the first group there are machines that have displaciment capability and can locate in any location, and in the second group there are machines that don't have displaciment capability because of the displacing of these machines is impossible or has heavy cost for system. Therefore arrangement of facilities is affect by the fixed machines and process routes. For creating a new layout for this system machines of the first group must adapt themselves with machines of second group by shifting among themselves to minimizing the total distance traveled by the materials. This problem has not been considered in the existing literature. A majority of the researches reviewed above have formulated this problem as a QAP model without any focus on the fix machines. For this problem without fixed machines an integer linear model was suggested and showed that this problem is NP-hard [Solimanpur and Kamran, (2010)]. Because the increasing fixed machines increase the constraints of the suggested model, the new model is NP-hard too. Despise of this complexity, improved simulated annealing is efficiency and can solve the problem in reasonable time.

## 2. Mathematical formulation

Suppose that a manufacturing system produces $P$ kinds of different products. The assumptions are as follow: There are $I_{p}$ process routes to produce product $p$. There are $M$ machines in this manufacturing system so that $K$ machines are fixed in $K$ apparent location and ( $M-K$ ) machines must be located in ( $L-K$ ) existing locations ( $L \geq M$ ). The distance between locations $l$ and $l^{\prime}$ is assumed to be $d_{l l}$. Let $D_{p}$ denote the production volume of product $p$. The problem is to determine the process route of each product and location of each not fixed machine. Other notations and mathematical model are defined as follows:

Set $\mathrm{J}=\{(\mathrm{i}, \mathrm{j}) \mid$ machine i fixed in location j for all fixed machines $\}$
$\mathrm{I}=$ total number of process routes $=\sum_{\mathrm{p}=1}^{\mathrm{p}} I_{p}$
$\boldsymbol{a}_{\text {mm }{ }^{\prime} p i}=\left\{\begin{array}{l}1 \text { if machine } \mathrm{m} \text { ' is needed after machine } \mathrm{m} \text { in the process route } \mathrm{i} \text { of product } \mathrm{p} \\ 0 \\ \text { otherwise }\end{array}\right.$
$X_{m l}=\left\{\begin{array}{lc}1 & \text { if machine } \mathrm{m} \text { located in location l } \\ 0 & \text { otherwise }\end{array}\right.$
$Y_{i p}=\left\{\begin{array}{lc}1 & \text { if product } \mathrm{p} \text { is processed by route } \mathrm{i} \\ 0 & \text { otherwise }\end{array}\right.$

The variable $Z_{m m^{\prime} l l^{\prime} i p}$ equals 1 only when all the three $X_{m l}, X_{m^{\prime} l^{\prime}}$ and $Y_{i p}$ variables are 1 and it equals 0, otherwise [Solimanpur and Kamran, (2010)].

The total distance traveled by the products is computed as follows [Solimanpur and Kamran (2010)]:
$\operatorname{Min} \sum_{m=1}^{M} \sum_{m^{\prime}=1}^{M} \sum_{l=1}^{L} \sum_{l^{\prime}=1}^{L} \sum_{p=1}^{P} \sum_{i=1}^{I_{P}} Z_{m m^{\prime} l l^{\prime} i p} \times a_{m m^{\prime} l l^{\prime}} \times D_{P} \times d_{l l^{\prime}}$

So, the linear model can be written as follows: constraints (3)-(7) suggested by Solimanpur and Kamran (2010).
$\sum_{l=1}^{L} X_{m l}=1 \quad \forall m=1,2, \ldots, M$
$\sum_{m=1}^{M} X_{m l} \leq 1 \quad \forall l=1,2, \ldots, L$
$\sum_{i=1}^{I_{p}} Y_{i p}=1 \quad \forall p=1,2, \ldots, P$
$\left(X_{m l}+X_{m^{\prime} l^{\prime}}+Y_{i p}-2\right) \leq Z_{m m^{\prime} l l^{\prime} i p} \quad \forall m, m^{\prime}, l, l^{\prime}, i, p ; m \neq m^{\prime}, l \neq l^{\prime}$
$X_{i j}=1 \quad \forall(i, j) \in J$
$Z_{m m^{\prime} l l^{\prime} i p} \geq 0$
$X_{m l} \in\{0,1\}$
$Y_{i p} \in\{0,1\}$

Constraint (3) ensures that each not fixed machine must be assigned to one location. Constraint (4) guarantees that the maximum assigned machine in each location is 1. Constraint (5) ensures that only one process route will be selected for each product and constraint (6) ensures that fixed machines don't move from the first their locations.

## 3. Simulated annealing algorithm

### 3.1. Normal simulated annealing algorithm scheme

SA is a casual search method, which refer to the physical annealing of solid, for finding solution to combinatorial optimization problems. In the physical annealing, solid is heated until it melts and then with a proper annealing schedule it gets cold till it reaches the least
energy point. If the initial temperature is not selected high enough or cooling process is very fast, at the low energy state, there can be a deformation in the solid. SA algorithm for combinatorial optimization problems is firstly used by Kirkpatrick, Gelatt, and Vecchi (1983).

Several simulated annealing algorithms are proposed for facility layout problems [Leonardo Chwif, Marcos R. Pereira Barretto and Lucas Antonio Moscato (1998)], [Alan R. McKendall, Jr. Jin Shang and Saravanan Kuppusamy (2005)] and [Ramezan Sahin (2011)]. SA has an advantage over the other meta-heuristic algorithms in terms of the ease of implementation and gives reasonably good solutions for many combinatorial problems. In this paper, whereas the problem is NP-hard [Solimanpur and Kamran (2010)] we purpose SA algorithm to solve facility layout problem with fixed machines and multiple process routes. Steps of normal SA algorithm for the solution of this problem are given below:

Step 1: Read the input data (fixed machines and their locations, process routes of each product, Production volume of products and the distance between different locations) and the parameters of simulated annealing $\left(T_{\max }=\right.$ initial temperature, $\alpha=$ cooling rate, $\mathrm{T}_{\text {min }}=$ terminal temperature and $\mathrm{IT}=$ size of iterations in each temperature).

Step 2: Start temperature counter: el $=0$.
Step 3: Create a random initial solution $\left(S_{0}\right)$ and calculate the objective value of initial solution
$\left(E_{0}\right)$.
$S_{\text {best }}=S_{c}=S_{0}, E_{\text {best }}=E_{c}=E_{0}$

Step 4: Make the iteration counter 0 at each temperature level: il $=0$
Step 5: Increase the iteration counter one at each temperature level: il = il + 1.

Step 6: find a neighbor solution ( $S_{i l}$ ). Then calculate the value of objective of neighbor solution $\left(E_{i l}\right)$.

Step7: Calculate the change in objective value: $\Delta E=E_{i l}-E_{c}$.

If $\Delta E<0$ or $\Delta E \geq 0$ and $r=\operatorname{random}[0,1]<P(\Delta E)=\exp (-\Delta E / T)$, accept the change which means the neighbor solution and, $S_{c}=S_{i t}, E_{c}=E_{i t}$

Step 8: If $\left(E_{c}<E_{\text {best }}\right)$ Set to $S_{\text {best }}=S_{c}$ and $E_{\text {best }}=E_{c}$. If it is not, go to next step.

Step 9: If (il > IT) go to next step. If it is not, go to Step 5.
Step 10: el $=$ el +1.
Step 11: $T_{e l+1}=\alpha T_{e l}$.

Step 12: If $T_{e l+1}<T_{\text {min }}$ go to next step. If it is not, go to Step 4.

Step 13: Stop algorithm and print the solutions.

### 3.2. Improved simulated annealing scheme

In this paper because our problem is species, we propose new approach of SA that has one step more than normal SA. To clarify the issue first we explain the shape of output answer of SA. The configuration of answers have two segments. Segment 1 shows the location of machines and segment 2 shows the process route for each product. For example, let us consider a problem with five products and eight machines to be located in eight locations. Assume that products 1, 2, 3, 4 and 5 have 5, 2, 3, 4 and 3 process routes respectively, and machines 2 and 5 have been fixed in 3 and 7 location respectively. A typical solution for this problem can be represented by the following. In this solution, machines 3,1,2,8,4,6,5 and 7 have been located in locations 1-8 respectively. Similarly, for products 1 to 5 the process routes 2, 2, 1, 4 and 3 have been selected.


In step 6 in SA we create a neighbor by swapping the places of two not fixed machines randomly and repeat this creating for $I T$ iteration in each temperatura; but in this problem swapping is not effective because the shape of the solution doesn't let to swapping. Therefore we have two kinds of neighbors. The first kind of neighbor is created by swapping the places of two not fixed machines randomly in first segment and second kind of neighbor is created by changing the processing route of one product randomly. In this approach for each temperature we have $I T$ iteration that in each iteration first we create the first kind of neighbor for itl iteration and evaluate each neighbor and select the best of them, then create second kind of neighbor for it2 iteration and evaluate each neighbor and select the best of them. This approach helps us attain the optimal solution quickly.

## 4. Evaluation and computational results

The proposed simulated annealing algorithm was coded in MATLAB and run on Intel Core ${ }^{\mathrm{TM}}$ i5 CPU with 2.40 GHz speed and 4 GB of RAM computer. To evaluate the effectiveness and efficiency of the proposed algorithm, ten test problems of different sizes, given by Solimanpur and Kamran (2010), were tested, and the results were compared with the Lingo 8 outputs. The results show that the proposed SA approach is very quick to solve the problems.

### 4.1. Test problems 1 and 2

These problems includ four products and five machines including one fixed machines and four not fixed machines which are to be located in four locations. Table 1 shows the order of machines at each process routes of each product. Production volume of parts $1-4$ is assumed as 1500 , 5000, 200 and 40,000, respectively. The distance between different locations is expressed by the following matrix.

Problem 1: Assume that machine 3 fixed in location 3.

Problem 2: Assume that machine 4 fixed in location 1.

$$
\text { Distance matrix between locations }=\left[\begin{array}{ccccc}
0 & 6 & 17 & 4 & 10 \\
6 & 0 & 8 & 5 & 11 \\
17 & 8 & 0 & 14 & 2 \\
4 & 5 & 14 & 0 & 13 \\
10 & 11 & 2 & 13 & 0
\end{array}\right]
$$

Table 1. Process routes of each product in Problems 1 and 2

| Product | Process route |  |  |
| :--- | :--- | :--- | :--- |
|  | 1 | 2 | 3 |
| 1 | $1,2,3$ | $1,2,4$ | $1,2,5$ |
| 2 | $1,2,3,4$ | $2,3,4,5$ | - |
| 3 | $1,2,3,4$ | $1,3,4,5$ | - |
| 4 | $1,2,3,4,5$ | - | - |

These problems solved with SA approach under these parameters:

$$
T_{\max }=100, T_{\min }=40, \alpha=0.99, I T=5, i t 1=15 \text { and } i t 2=15 .
$$

The final solution obtained by the proposed solution of SA is as follows.

Problem 1:


This solution indicates that machines $1,4,3,5$ and 2 are assigned to locations $1-5$, respectively and process routes $3,2,1$ and 1 are selected for products $1-4$, respectively.

Problem 2:


This solution indicates tha machines $4,3,2,5$ and 1 are assigned to locations $1-5$, respectively and process routes $1,1,1$ and 1 are selected for products $1-4$, respectively.

### 4.2. Test problems 3 and 4

These problems includ eight products and eight machines include two fixed machines and six not fixed machines which to be located in six locations. Table 2 shows the order of machines at each process route of each product. Production volume of parts 1 to 8 is assumed as 500, 140, 600, 100, 50, 160, 200 and 400, respectively. The distance between different locations is expressed by the following matrix.
problem 3: Assume that machines 1 and 5 fixed in locations 3 and 4 respectively.
problem 4: Assume that machines 7 and 2 fixed in locations 3 and 5 respectively.

$$
\text { Distance matrix between locations }=\left[\begin{array}{cccccccc}
0 & 9 & 5 & 2 & 8 & 7 & 7 & 4 \\
9 & 0 & 2 & 5 & 5 & 6 & 9 & 8 \\
5 & 2 & 0 & 8 & 8 & 7 & 9 & 5 \\
2 & 5 & 8 & 0 & 6 & 8 & 9 & 10 \\
8 & 5 & 8 & 6 & 0 & 11 & 9 & 8 \\
7 & 6 & 7 & 8 & 11 & 0 & 12 & 9 \\
7 & 9 & 9 & 9 & 9 & 12 & 0 & 8 \\
4 & 8 & 5 & 10 & 8 & 7 & 8 & 0
\end{array}\right]
$$

Table 2. Process routes of each product in problems 3 and 4

| Product | Process route |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 |  |  |  |
| 1 | $1,2,3,4,5,6$ | $1,2,3,4,5,7$ | $1,2,3,4,5,8$ | - |  |  |  |
| 2 | $1,2,3,4,5,8$ | $1,3,4,5,6,7,8$ | $1,3,4,5,7,8$ | - |  |  |  |
| 3 | $1,2,3,5,6,7$ | $1,3,4,5,6$ | $1,2,3,4,6,7$ | - |  |  |  |
| 4 | $2,4,5,6,7,8$ | $1,2,3,4,5,6,7$ | $2,3,4,5,6,8$ | $1,2,3,5,7,8$ |  |  |  |
| 5 | $1,2,3,6$ | $1,2,3,7$ | $1,2,6,7$ | $1,2,3,8$ |  |  |  |
| 6 | $1,3,5,7,8$ | $1,2,4,5,8$ | $3,4,5,6,8$ | - |  |  |  |
| 7 | $1,2,4,5,6,7,8$ | $1,4,5,6,7,8$ | $1,2,4,5,6,7$ | $1,3,4,5,6,7$ |  |  |  |
| 8 | $1,2,3,4,5,6,7,8$ | - | - | - |  |  |  |

These problems solved with SA approach under these parameters:

$$
T_{\max }=100, T_{\min }=20, \alpha=0.99, I T=10, i t 1=15 \text { and } i t 2=15 .
$$

The final solution obtained by the proposed SA is as follows.
Problem 3:


This solution indicates that machines $6,3,1,5,4,2,8$ and 7 are assigned to locations $1-8$, respectively and process routes $1,2,1,3,3,3,4$ and 1 are selected for products $1-8$, respectively.

Problem 4:


This solution indicates that machines $5,3,7,4,2,8,1$ and 6 are assigned to locations $1-8$, respectively and process routes $1,1,2,3,2,3,4$ and 1 are selected for products $1-8$, respectively.

### 4.3. Test problems 5, 6 and 7

These problems includ three products and fifteen machines include four fixed machines and eleven not fixed machines which to be located in eleven locations. Table 3 shows the order of machines at each process route of each product. Production volume of parts $1-3$ is assumed as 300,400 and 200, respectively. The distance between different locations is expressed by the following matrix.

Problem 5: Assume that machines 2,5,7 and 10 fixed in locations 1,4,9 and 15 respectively.
Problem 6: Assume that machines $6,8,11$ and 15 fixed in locations $4,5,11$ and 15 respectively.

Problem 7: Assume that machines 1,2,3 and 4 fixed in locations 1,2,3 and 4 respectively.
Distance matrix between location $=\left[\begin{array}{ccccccccccccccc}0 & 1 & 11 & 4 & 9 & 2 & 8 & 5 & 12 & 7 & 8 & 3 & 10 & 4 & 8 \\ 1 & 0 & 6 & 1 & 7 & 10 & 3 & 11 & 6 & 8 & 3 & 4 & 12 & 5 & 9 \\ 11 & 6 & 0 & 2 & 7 & 12 & 5 & 9 & 4 & 3 & 7 & 6 & 11 & 7 & 1 \\ 4 & 1 & 2 & 0 & 11 & 3 & 12 & 6 & 9 & 2 & 8 & 6 & 3 & 13 & 4 \\ 9 & 9 & 7 & 11 & 0 & 2 & 7 & 3 & 4 & 10 & 4 & 9 & 8 & 1 & 7 \\ 2 & 10 & 12 & 3 & 2 & 0 & 10 & 12 & 55 & 3 & 7 & 3 & 9 & 10 & 6 \\ 8 & 3 & 5 & 12 & 7 & 10 & 0 & 1 & 9 & 13 & 5 & 7 & 3 & 9 & 12 \\ 5 & 11 & 9 & 6 & 3 & 2 & 1 & 0 & 5 & 8 & 1 & 10 & 4 & 12 & 6 \\ 12 & 6 & 4 & 9 & 4 & 55 & 9 & 5 & 0 & 9 & 3 & 8 & 7 & 12 & 5 \\ 7 & 8 & 3 & 2 & 10 & 3 & 13 & 8 & 9 & 0 & 4 & 9 & 12 & 3 & 7 \\ 8 & 3 & 7 & 8 & 4 & 7 & 5 & 1 & 3 & 4 & 0 & 7 & 11 & 2 & 9 \\ 3 & 4 & 6 & 6 & 9 & 3 & 7 & 10 & 8 & 9 & 7 & 0 & 4 & 8 & 3 \\ 10 & 12 & 11 & 3 & 8 & 9 & 3 & 4 & 7 & 12 & 11 & 4 & 0 & 1 & 9 \\ 4 & 5 & 7 & 13 & 1 & 10 & 9 & 12 & 12 & 3 & 2 & 8 & 1 & 0 & 12 \\ 8 & 9 & 1 & 4 & 7 & 6 & 12 & 6 & 5 & 7 & 9 & 3 & 9 & 12 & 0\end{array}\right]$

Table 3. Process routes of each product in problems 5, 6 and 7

| Product | Process route |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 |  |  |  |
| 1 | $1,2,3,4,5,6$ | $1,2,3,4,5,7$ | $1,2,3,4,5,8$ | - | - |  |  |  |
| 2 | $1,2,3,4,5,7,8$ | $1,3,4,5,6,7,8$ | $1,3,4,5,7,8,9$ | - | - |  |  |  |
| 3 | $1,4,10,11$ | $1,4,10,12$ | $1,4,10,13$ | $1,4,10,14$ | $1,4,10,15$ |  |  |  |

These problems solved with SA approach under these parameters:

$$
T_{\max }=100, T_{\min }=5, \alpha=0.99, I T=20, i t 1=25 \text { and it } 2=5 .
$$

The final solution obtained by the proposed SA is as follows.
Problem 5:

| 2 | 6 | 12 | 5 | 15 | 3 | 11 | 9 | 7 | 4 | 8 | 1 | 14 | 3 | 10 | 1 | 2 | 2 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Problem 6:

| 1 | 2 | 9 | 6 | 8 | 10 | 3 | 4 | 12 | 13 | 11 | 14 | 5 | 7 | 15 | 2 | 1 | 3 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Problem 7:

| 1 | 2 | 3 | 4 | 8 | 11 | 9 | 6 | 13 | 10 | 15 | 12 | 5 | 7 | 14 | 2 | 1 | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

### 4.4. Test problems 8,9 and 10

These problems includ eight products and fifteen machines four fixed machines and eleven not fixed machines which to be located in eleven locations. Table 4 shows the order of machines at each process route of each product. Production volume of parts $1-8$ is assumed as $300,400,200,500,100,150,250$ and 450 , respectively. The distance between different locations is expressed by the following matrix.

Problem 8: Assume that machines 2,5,7 and 10 fixed in locations $1,4,9$ and 10 respectively.

Problem 9: Assume that machines 4,9,11 and 14 fixed in locations 5,8,9 and 13 respectively.
Problem 10: Assume that machines 3,7,11 and 12 fixed in locations 5,10,11 and 14 respectively.

Distance matrix between location $=\left[\begin{array}{ccccccccccccccc}0 & 1 & 11 & 4 & 9 & 2 & 8 & 5 & 12 & 7 & 8 & 3 & 10 & 4 & 8 \\ 1 & 0 & 6 & 1 & 9 & 10 & 3 & 11 & 6 & 8 & 3 & 4 & 12 & 5 & 9 \\ 11 & 6 & 0 & 2 & 7 & 12 & 5 & 9 & 4 & 3 & 7 & 6 & 11 & 7 & 1 \\ 4 & 1 & 2 & 0 & 11 & 3 & 12 & 6 & 9 & 2 & 8 & 6 & 3 & 13 & 4 \\ 9 & 9 & 7 & 11 & 0 & 2 & 7 & 3 & 4 & 10 & 4 & 9 & 8 & 1 & 7 \\ 2 & 10 & 12 & 3 & 2 & 0 & 10 & 12 & 55 & 3 & 7 & 3 & 9 & 10 & 6 \\ 8 & 3 & 5 & 12 & 7 & 10 & 0 & 1 & 9 & 13 & 5 & 7 & 3 & 9 & 12 \\ 5 & 11 & 9 & 6 & 3 & 12 & 1 & 0 & 5 & 8 & 1 & 10 & 4 & 12 & 6 \\ 12 & 6 & 4 & 9 & 4 & 55 & 9 & 5 & 0 & 9 & 3 & 8 & 7 & 12 & 5 \\ 7 & 8 & 3 & 2 & 10 & 3 & 13 & 8 & 9 & 0 & 4 & 9 & 12 & 3 & 7 \\ 8 & 3 & 7 & 8 & 4 & 7 & 5 & 1 & 3 & 4 & 0 & 7 & 11 & 2 & 9 \\ 3 & 4 & 6 & 6 & 9 & 3 & 7 & 10 & 8 & 9 & 7 & 0 & 4 & 8 & 3 \\ 10 & 12 & 11 & 3 & 8 & 9 & 3 & 4 & 7 & 12 & 11 & 4 & 0 & 1 & 9 \\ 4 & 5 & 7 & 13 & 1 & 10 & 9 & 12 & 12 & 3 & 2 & 8 & 1 & 0 & 12 \\ 8 & 9 & 1 & 4 & 7 & 6 & 12 & 6 & 5 & 7 & 9 & 3 & 9 & 12 & 0\end{array}\right]$

Table 4. process routes of each product in problems 8, 9 and 10

| Product | Processing route |  |  |
| :--- | :--- | :--- | :--- |
|  | 1 | 2 | 3 |
| 1 | $1,2,3,4,5,6$ | $1,2,3,4,5,8,9,10$ | - |
| 2 | $1,2,3,5,7,8,10,11$ | $3,4,5,6,7,8,9,10$ | $1,4,9,12,13,14$ |
| 3 | $1,4,10,11$ | $1,4,10,12,13,14$ | $1,4,9,12,13,14$ |
| 4 | $1,2,3,5,6,7,8,9,10,11,12,13,14$ | - | - |
| 5 | $4,5,6,7,8,14,15$ | $4,5,6,7,12,13,14,15$ | $4,5,7,9,10,11,12,13,14,15$ |
| 6 | $2,3,4,5,6$ | $2,3,4,5,7,8$ | $2,3,4,5,7,10$ |
| 7 | $4,5,7,8,10,11,12$ | $4,5,8,9,10,12,14$ | $4,5,8,10,11,12,13,15$ |
| 8 | $1,2,3,4,5,6,7,8,9,10,12$ | $1,2,3,4,5,6,7,8,9,10,11,13,14$ | - |

These problems solved with SA approach under these parameters:

$$
T_{\max }=100, T_{\min }=5, \alpha=0.995, I T=15, i t 1=20 \text { and it } 2=20 .
$$

The final solution obtained by the proposed SA is as follows.

Problem 8:

| 2 | 1 | 6 | 5 | 8 | 9 | 14 | 13 | 7 | 10 | 12 | 3 | 15 | 11 | 4 | 1 | 2 | 1 | 1 | 2 | 1 | 2 | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Problem 9:


Problem 10:

| 10 | 9 | 6 | 8 | 3 | 4 | 1 | 2 | 15 | 7 | 11 | 14 | 13 | 12 | 5 | 1 | 2 | 1 | 1 | 1 | 1 | 2 | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

### 4.5. Comparison of results

These ten problems that attempted in the previous subsections were solved by Lingo 8.0 to obtain the globally optimum solution of each problem. The results of both simulated annelling algorithm and Lingo 8.0 for all problems are compared in Table 5. Due to the probabilistic nature of the computation process in SA, each problem has been solved by SA five times. The best objective function value obtained in each run of SA is shown in Table 5. As seen in this table, the standard deviation of the final solution is less than $8 \%$ for all the attempted problems which are indicated thorough inspection of the solution space by the proposed algorithm. The Lingo Software has solved the tested problems by branch-andbound technique to determine the global optimum solution of each problem. As seen in Table 5, the proposed SA has obtained the globally optimum solution of all Examples. The results show that the proposed SA can get the optimum solution in a short time whereas Lingo coudn't solve problems 6, 7, 8, 9 and 10 in resonable time and you can see that the SA run times is shorter than Lingo run times in all problems.

Table 5. Comparison of the results obtained by SA with the global optimum solutions.

|  | Run | Prob 1 | Prob 2 | Prob 3 | Prob 4 | Prob 5 | Prob 6 | Prob 7 | Prob 8 | Prob 9 | Prob 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| SA algorithm | 1 | 1097000 | 898200 | 57210 | 57200 | 14500 | 9400 | 11300 | 57850 | 47800 | 53050 |
|  | 2 | 1097000 | 898200 | 57210 | 57200 | 13900 | 9700 | 11300 | 57350 | 47300 | 53000 |
|  | 3 | 1097000 | 898200 | 56880 | 57950 | 13900 | 9700 | 11300 | 57500 | 47300 | 54950 |
|  | 4 | 1097000 | 898200 | 57210 | 57200 | 13900 | 9400 | 11300 | 57850 | 47300 | 53050 |
|  | 5 | 1097000 | 898200 | 56880 | 57200 | 13900 | 9400 | 11300 | 57500 | 47800 | 53000 |
| Best solution |  | 1097000 | 898200 | 56880 | 57200 | 13900 | 9400 | 11300 | 57350 | 47300 | 53000 |
| Worst solution | 1097000 | 898200 | 57210 | 57950 | 14500 | 10300 | 11300 | 57850 | 47800 | 55250 |  |
| Mean | 1097000 | 898200 | 57078 | 57350 | 14020 | 9520 | 11300 | 57610 | 47500 | 53410 |  |
| Variance | 0 | 0 | 792 | 1200 | 960 | 720 | 0 | 960 | 1200 | 3080 |  |
| SD(\%) | 0 | 0 | 1 | 2 | 7 | 8 | 0 | 2 | 3 | 6 |  |
| Lingo | 1097000 | 898200 | 56880 | 57200 | 13900 | 9400 | 11300 | 57350 | 47300 | 53000 |  |
| SA run time (sec) | 0 | 0 | 3 | 3 | 6 | 6 | 6 | 15 | 15 | 15 |  |

## 5. Conclusion

This paper presents an integer linear programming formulation to find optimal solution for the facility layout problem with fixed machines and multiple process routes for each products. A simulated annealing algorithm was proposed to solve the formulated problem so that the distance traveled by the materials is minimized. Computational results indicate that the proposed SA provides very good solutions for the problems in a very small priod of time. The approach presented in this paper can be further extended in future researches to overcome limitations of this study. For example, it has been assumed in this paper that the locations to which machines are to be assigned are known in advance. This assumption can be relaxed to consider arrangement of machines in a continuous area. Availability of multiple machines of each type can be captured in subsequent researches. Because the problem is NPhard, application of other meta-heuristics such as neural networks, Tabu search, ant colony optimization, etc. can be attempted in the future.

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