fire

Article

A Model for Fire Departments' Performance Assessment in Portugal

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Abstract: Fire departments' performance assessment (FDPA) is an important task for fire protection service (FPS) authorities to evaluate fire departments' (FDs) efficiencies, identify the efficient FDs, and identify areas for improvement of the inefficient units. Therefore, many countries all around the world conducted FDPA research and published its results. Although Portugal is a country with many old cities and buildings, to the best of our knowledge, no FDPA study has been conducted. Hence, the objective of this paper is to conduct a FDPA on Portuguese (PT) FDs using the general FDPA framework and slack-based data envelopment analysis. The results of analyzing the 376 PT-FDs' data in 2020 showed that only 22 out of 376 FDs were efficient; in addition, in most districts in Portugal, less than 10% of FDs were efficient, and a high percentage of FDs had less than 50% efficiency. Further details and findings are discussed, and some recommendations are provided throughout the paper. The paper's findings could help the FPS decision maker have a better view of the FDs' performances and take corrective actions to improve efficiencies. It can also help the FDPA evaluator to have a better understanding of how a FDPA can be conducted and discussed.

Keywords: performance assessment; fire protection services; data envelopment analysis; Portuguese fire departments; geographical information system

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1. Introduction

Providing efficient fire protection services (FPS), whose main objectives are to reduce the number of fire incidents and casualties, has always been an important part of public management and has been in the spotlight due to the indisputable importance of the FPSs to the safety of both people and the environment [1]. Therefore, countries and local governments are required to constantly evaluate the performance of their fire departments (FDs) [2–4].

Due to the importance of the FPS for public and private sectors, fire departments' performance assessment (FDPA) is a significant challenge for the FPS authorities, and lots of research has been conducted to provide methodologies for FDPA [5]. However, because FPS is a multi-factor activity with many variables interfering with its performance, FDPA is a complex evaluation [3,6].

FPS performance assessment and FDPA have been conducted in many countries for decades [5] to identify the FPS efficiency at the international, national, or regional level. For instance, FPS performance assessment at the national level in eight countries [7] or FDPA at the national level in countries such as Sweden [3], Spain [8], Estonia [9], the UK [4], the US [10], Australia [11], Taiwan [6], South Korea [12], and China [13].

Portugal is one of the European countries that is facing challenges in providing adequate FPS [14], especially to residential areas due to having a lot of old buildings and neighbourhoods in its cities [15]. Therefore, the Portuguese FPS authorities should monitor the FDs' proficiency to make sure they provide the utmost level of FPS to the people, especially urban residents [5], to reduce the number of incidents and decrease fire losses. However, to the best of our knowledge, although there are some studies about fire risk assessments in old cities [15–17] and a comparative study about fire performance indicators in Portugal, Wales, Scotland, and England [14], no research has been conducted on FDPAs in Portugal. Hence, the objective of this study is to conduct a Portuguese (PT) FDPA, find the FPS efficiency of the PT FDs in urban and residential areas, and provide a set of recommendations and more insights for improving the FPS and FDPA process in Portugal and the efficiency of PT FDs. The findings of this paper are an important part of FD resource allocation strategies (i.e., allocating constrained resources such as financial budgets, firefighters, and fire engines to the FDs), taking corrective actions at the operational level (e.g., reducing response time and choosing better suppression scenarios to save more lives and properties) [5], and changing the size and optimizing the location of FDs and fire stations [1] to improve their performance.

The most similar research among the aforementioned country-wide FDPA studies is the FPS performance assessment in the US [10], which provides FPS efficiencies at the state level but not at the FD level. In this study, both FD-level and district-level PAs are performed, and their results are discussed in detail.

The general FDPA framework (GFDPAF) and data envelopment analysis (DEA) [5] are used in this study for PT-FDPA and are explained in detail in the next four sections. The research methodology and the DEA model are described in Section 2, and the PT FDPA findings and some recommendations are provided in Section 3. Further analysis of the findings is presented and discussed in Section 4, and finally, the paper is concluded in Section 5.

2. Research Methodology

This research used the four-stage methodology suggested by Eslamzadeh et al. (2022) [1] for resource allocation in fire departments. An important part of this methodology is the general FDPA framework (GFDPAF) [5], which was used as a guide for selecting input and output variables and the FDPA analysis method. As shown in Figure 1, the four stages of the research methodology are data gathering, processing, analysis, and reporting. All the stages are covered in more detail in the subsequent sections. As the authors have found some limitations in the model implementation from the original GFDPAF, some recommendations for evolution are also presented.

The abbreviations used in this paper are summarized in Table 1.

Table 1. List of abbreviations used in the paper.

Abbreviation	Full Phrase	Abbreviation	Full Phrase
ANEPC	Autoridade Nacional de Emergência e Proteção Civil	FSI	Fire risk and socioeconomic index
BCC	Banker, Charnes, and Cooper	GFDPAF	General fire departments' performance assessment framework
CCR	Charnes, Cooper, and Rhodes	OTE	Overall technical efficiency
CRS	Constant return to scale	PA	Performance sssessment
DEA	Data envelopment analysis	PT	Portuguese/Portugal
DMU	Decision-making unit	PTE	Pure technical efficiency
FD	Fire department	SBM	Slack-based model
FDPA	Fire departments' performance assessments	SE	Scale efficiency
FPS	Fire protection services	VRS	Variable returns to scale

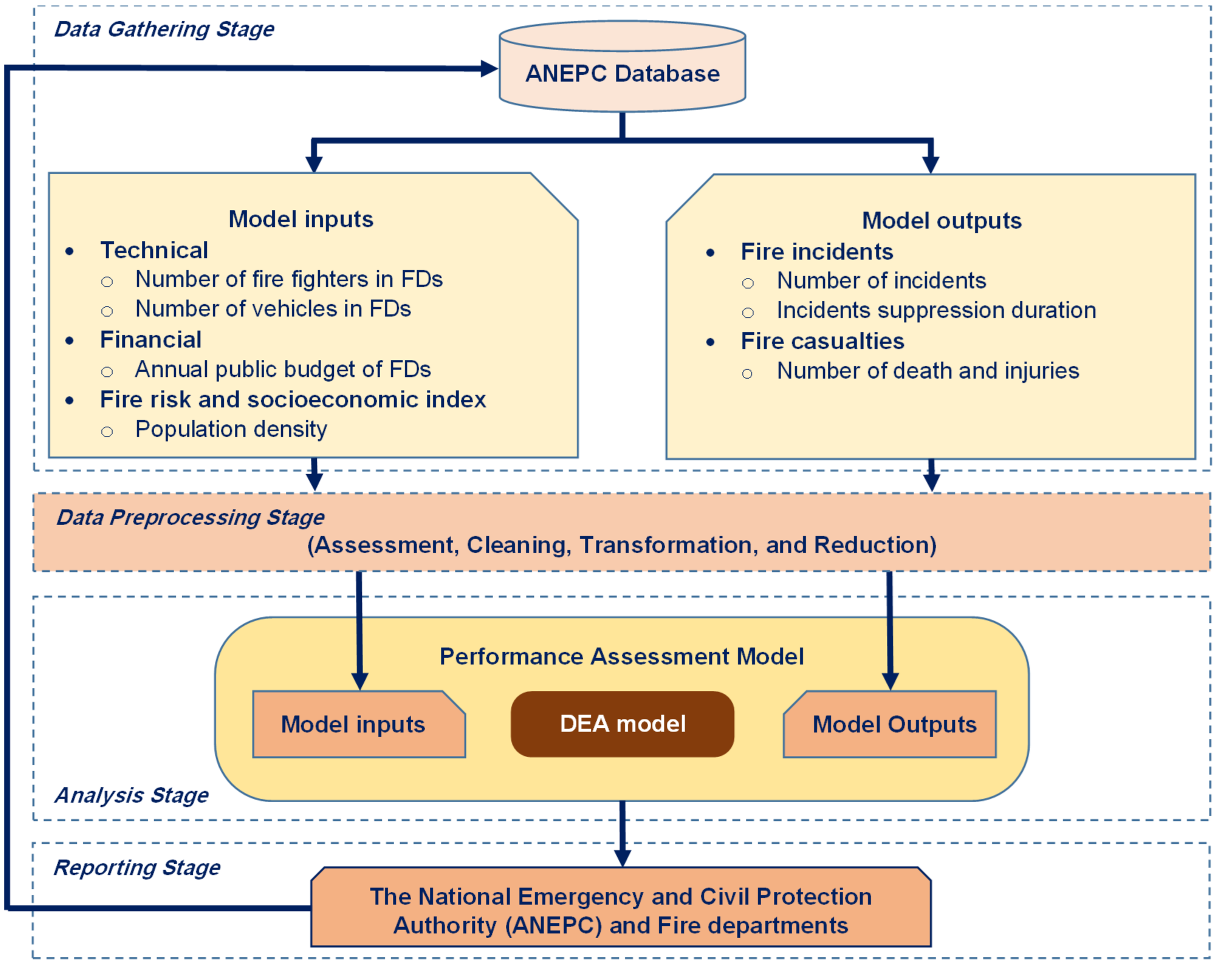


Figure 1. The PT-FDPA model-2020.

2.1. *Data-Gathering Stage*

The National Emergency and Civil Protection Authority (Autoridade Nacional de Emergência e Proteção Civil-ANEPC) provided all the publicly available and private but accessible data related to the fire incidents that occurred between 2012–2020 in Portugal. The FPS experts for verifying the research variables and results also were from the ANEPC and consisted of the former director of the ANEPC and the current director of the National School of Firefighters operational manager, the national senior chief technician, and two chief commanders of FDs. Because the goal of this study was FDPA in the domain of urban incidents, 72,174 urban fire records were chosen, and wild and green fire incident data were excluded.

The provided datasets contain incident data; however, according to the GFDPAF [5], the incident data are only part of the output variables required for building the FDPA model. Because of that, with the help of the ANEPC, more data from public and private but reliable resources were collected to create a set of necessary variables for the Portuguese (PT) FDPA model.

The year 2020 has a complete set of data for FDPA; therefore, considering the available resources for the model (incident data, FDs' financial data, and the population density), it was decided and confirmed by the experts to use only the 7038 fire incident records that happened in 2020 to have the latest FDs' efficiency results in Portugal. The evaluated data in the PT-FDPA model 2020 contained the following variables:

Inputs

In accordance with the GFDPAF [5] and as illustrated in Figure 1, four variables were used as the technical, financial, and socioeconomic inputs for PT-FDPA. These input variables are:

- Technical inputs: The available variables for this category in the acquired datasets are *the number of fire fighters in FDs* and *the number of vehicles (engines) in FDs* that are compliant with the technical category of inputs in the GFDPAF. Since many of the Portuguese FDs have only one fire station, the number of fire stations was not considered a proper indicator for the PT-FDPA.

Recommendation 1: *The number of water hydrants* in the FDs' jurisdiction areas is one of the suggested inputs in GFDPAF because it is one of the most frequently used variables in the FDPA papers and because water hydrant accessibility increases the success rate of suppression operations. The data for this indicator were not recorded in the ANEPC datasets. This study suggests that PT FDs record and update this variable data.

- Financial input: The PT FDs have a variety of financial resources that are considered internal data and are not accessible; therefore, with the experts' confirmation, the most relevant and available data were used, which are on *the annual public budget of FDs* and were granted to them by the ANEPC.

Recommendation 2: The granular details about the FDs' financial resources such as the cost of personnel, maintenance, equipment, and stations; the cost of prevention and suppression activities; and the cost of residential, industrial, wild, and green fires were not accessible during the data gathering stage, and it is recommended to keep a record of these financial details for future FDPAs.

- Fire risk and socioeconomic index (FSI): From this input of the GFDPAF, *the population density* was acquired during the domain analysis stage. At the time this study was conducted, the most recent information on the FD population from 2017 was still valid.

Recommendation 3: Two important FSI variables are *the number of high-risk buildings* and *the portion of high-risk population* in the FDs' jurisdictional area. Since these two variables are dependent on the other FSI metrics (e.g., low-income populations, single or single-parent families, old and unprotected buildings against fire, and strategic infrastructures) [5], this study advises the ANEPC decision makers and PT-FDPA evaluators to prepare this information for future PT-FDPAs.

Outputs

The GFDPAF includes evaluating the effectiveness of the FDs using FPS outcomes (e.g., number of incidents responded to within the target time or number of civilians saved) [5]. Although there are more data and variables available in the ANEPC's and the other authorities' databases, gathering them needed more time, which was outside the scope and timeframe of this research. Therefore, the PT-FDPA model was created based on the following data that was accessible and gathered during the domain analysis stage:

- Fire incidents: The applied variables from this category were *the number of incidents* and *the total suppression duration of incidents*. However, as mentioned earlier, the response time data, the type of building, and some other frequent FDPA variables were not available for the data collection of this research during the domain analysis stage. Therefore, for building the PT-FDPA model, the first two variables mentioned in this category were selected. In addition, to limit the outliers and incorrect variables, the incidents with negative, less than 3 min, or more than 7200 min durations were excluded from the data processing stage.

Recommendation 4: Having an integrated database is an important part of review investigations [1,18], and it is highly dependent on the fire incident official reports and the onsite collected data. For instance, *the incident response time* (from alarm until arrival at the fire scene) data were gathered by the FDs but were not integrated into the provided datasets by the ANEPC during the gathering stage, and with the experts' confirmation, they were replaced by the total incident duration (from alarm until units came back to the station); therefore, it is recommended for future FDPAs to provide all the required data in an integrated dataset to avoid any missing variables.

- Fire casualties: Among the suggested desirable and undesirable output and outcome variables by the GFDPAF, *the number of deaths*, *number of serious injuries*, and *number of light injuries*, were available in the provided datasets by the ANEPC and were used in the PT-FDPA model.

Recommendation 5: The two desirable outputs and undesirable outcomes of the suppression activities based on GFDPAF that could be gathered in future PT-FDPAs are *the number of civilians saved* and *the total property losses*, respectively.

Recommendation 6: An important but missing aspect of the FPS activities in many FDPA research studies is the fire prevention activities and related variables, such as the cost of prevention activities, the number of inspected buildings, the number of installed fire sensors or protectors, and the number and cost of educational campaigns [5]. Since this information is not available and acquired by the ANEPC or the FDs, this research highly advocates for acquiring and retaining this information for different managerial assessments such as FDPAs and strategies such as RAFD.

2.2. Data Preprocessing Stage

The acquired data had to be processed through the following four steps (data cleansing, integration, reduction, and transformation) to be prepared for the evaluation processes in the analysis stage [19,20] because the provided datasets by the ANEPC were unconstructed and contained incomplete, incorrect, missing, and outlier values that were stored in various datasets with granular details.

- Data Cleansing: In this step, to improve the quality of the data, missing, zero, or unreasonable values were identified and recovered from the ANEPC databases. If any incidents were important but irrecoverable, missing, or noisy data, they were removed. Here is a brief report on the removed incident records:
- Four incidents with suppression durations of more than 7200 min were considered outliers and, after expert confirmation, were excluded;
- Data Integration: Because the incident data supplied to the ANEPC came from several departments, they were kept in various datasets. As a result, in this stage, they were all combined into a single, comprehensive dataset of the 2020 fire occurrences in Portugal that contains all of the necessary information for the PT-FDPA;
- Data Reduction: The ANEPC datasets have some granular details about the incidents that are not useful for the PT-FDPA (e.g., incidents' exact time and coordinates). Therefore, in these steps, the irrelevant details were removed.

Furthermore, because the goal of this research was a PT-FDPA in 2020, all occurrences in each FD's jurisdiction were aggregated into a smaller dataset containing the aggregated 2020 incident data at the FD level.

The majority of PT-FDs (443 out of 469) are voluntary, with the other FDs being municipal and private. Therefore, only the voluntary FDs were taken into account for the PT-FDPA since the management structure and resources of the nonvoluntary FDs are different and inaccessible. In the ANEPC datasets, 376 FDs had sufficient data to go to the assessment step, whereas the other FDs were excluded for at least one of the following reasons with the approval of the ANEPC's experts:

- 32 FDs had no data in the provided datasets;
- 43 FDs has no financial data or had zero or one incident in one year;
- 18 FDs had no population data.

Recommendation 7: Having accurate and complete data is an essential part of data mining [18] and FDPA investigations. Therefore, there should be a reminder to FD commanders and firefighters to record incident and managerial information with the utmost accuracy and completeness to not lose any FDs in future FDPAs due to missing values or incomplete data.

- Data Transformation: In this last step of the preprocessing stage, the cleaned, integrated, and reduced dataset was turned into comma-separated formats for further processing in the analysis stage.

2.3. Analysis Stage

This research used the GFDPAF for the analysis stage and FPS performance assessment in PT-FDs [5]. The GFDPAF suggests two DEAs for evaluating the financial and technical resource efficacies of the FDs and one for assessing their effectiveness. However, in this research, the framework was modified to reflect the available and accessible data in Portugal. As shown in Figure 1, the PT-FDPA model used one DEA model since the suggested variables in GFDPAF were not available or accessible in Portugal. Further details about the DEA model are explained in the following section.

2.4. Data Envelopment Analysis (DEA) Models

Introduced by Charnes, Cooper, and Rhodes (1978) [21] by generalizing Farrell's measure to multiple-input and multiple-output situations, the DEA method is a frontier analysis and nonparametric mathematical programming approach that is widely used for assessing organizational performances [9,13,22], and according to Eslamzadeh et al. (2022) [5], DEA is the most frequent method for FDPAs, which was applied in 1998 by Athanassopoulos [23] for the first time to assess the efficiencies of 25 FDs in the UK.

In DEA, as a frontier analysis, an individual DMU is compared to the "best practice set" of the sample rather than to the sample mean. In other words, the efficient DMUs are those units (in this research FDs) that are found on the frontier, while less efficient units are those below the frontier, and their inefficiency is the distance from the frontier [24,25].

The efficiency frontier in DEA is identified through the returns to scale evaluation, which refers to the ratio of the change in the output to the change in the input [7]. Basic DEA models are divided into the CCR (Charnes, Cooper, and Rhodes) model [21] and the BCC (Banker, Charnes, and Cooper) model [26]. The CCR model considers the constant return to scale (CRS) to create an efficiency frontier and identify the overall technical efficiency (OTE) of the DMUs, but BCC uses the variable returns to scale (VRS) [7,27]. As shown in Figure 2, CRS creates a straight efficiency line in the CCR model and ignores the differences in the scales of the DMUs' operation; it considers all DMUs the same to calculate the overall technical efficiency and helps to rank all DUMs. In contrast, VRS is a convex line used by the BCC model and considers the operation of the DMUs at different scales. The BCC model calculates the pure technical efficiency (*PTE*) and scale efficiency (*SE*) to evaluate DMUs by their technique (how the resource is used) and scale (the amount of resource used). The *SE* is calculated via dividing the *OTE* by the PTE, and it means, to have scale efficiency, both the CCR and BCC version of the DEA must be calculated [7,28].

$$SE = \frac{OTE}{PTE} \tag{1}$$

This relation says that a BCC efficient DMU surely is RCC efficient, but the reverse is not true. The BCC results and the comparison between the *SE* and the *PTE* can help to find the source of inefficiency in the given DMU, and the RCC can help rank DMUs by their efficiencies [7,27].

Other classifications of DEA models are "input-oriented" and "output-oriented" models. For example, considering DMU 4 in Figure 2 as an inefficient DMU, the input-oriented DEA model is about measuring the potential input savings (input reduction from C to B) if DMU 4 operates efficiently or produces the same output (A) by using the efficient input (B). On the contrary, in output-oriented DEA, a DMU calculates the maximization of the output (from C to E) using the same amount of input (D) [27].

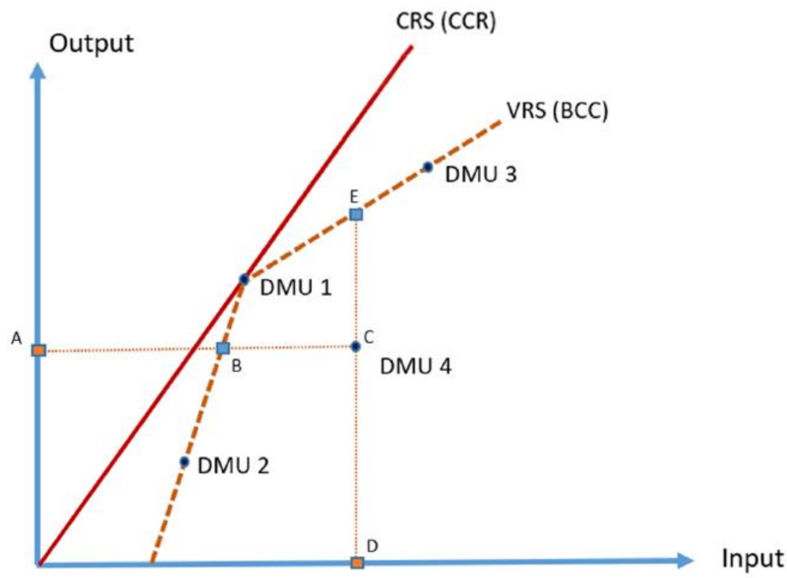


Figure 2. An illustration of DEA and its diversification, adapted [27].

Classical DEA methodology evaluates the relative efficiencies of a DMU in maximizing output levels using the same level of inputs (output-oriented) and/or minimizing input levels using the same level of outputs (input-oriented) [10,24]. It works with desirable inputs and outputs where a DMU is more efficient if it minimizes the input and/or maximizes the output. Therefore, in traditional DEA, the isotonic condition in the input–output relation should be fulfilled by testing variables isotonicity via a technique such as correlation analysis; this means that an increase in the input should not result in a decrease in the output [29].

Let us assume that:

n is the number of FDs that were evaluated with respect to one another;
m is the number of inputs;
s is the number of outputs;
y_{ip} is the value ($\geq$0) of output measure i (i = 1, . . . , m) for DMU_p (P = 1, . . . , n);
x_{jp} is the value ($\geq$0) of input measure j (j = 1, . . . , s) for DMU_p;
u_{ip} is an unknown weight *of* input measure i for DMU_p;
v_{jp} is an unknown weight *of* output measure j for DMU_p;
ε is a very small positive value ($0 < \varepsilon << 1$) to prevent zero weights.

Then, for each FD (FD_q), the relative efficiency in the CCR model is calculated as follows [13,21]:

$$\begin{aligned} &\text{MAX}\left(\sum_{j=1}^{s} v_{jp}y_{jq}\right)\bigg/\left(\sum_{i=1}^{m} u_{ip}x_{iq}\right) \\ s.t.\ &\left(\sum_{j=1}^{s} v_{jp}y_{jp}\right)-\left(\sum_{i=1}^{m} u_{ip}x_{ip}\right)\leq 0;\ p=1,\ldots,n \\ &u_i,\ v_j \geq \varepsilon \text{ for all } i \text{ and } j \end{aligned} \tag{2}$$

The value of the efficiency score is between zero and one, so an FD is considered efficient if its score is one [10].

For (1), the linear program dual problem is as follows, where θ is the *OTE* [7]:

Min θ

$$\begin{aligned} s.t.\ &\sum_{p=1}^{n} \lambda_p x_{ip} - \theta x_{iq} \leq 0;\ i=1,\ldots,m \\ &\sum_{p=1}^{n} \lambda_p y_{jp} - y_{iq} \geq 0;\ j=1,\ldots,s \\ &\lambda_p \geq 0;\ p=1,\ldots,n \end{aligned} \tag{3}$$

As explained earlier in this section, the BCC-DEA model calculates the *PTE*, which is α in the following linear program model [7]; then, the SE is achieved using formula (1) [28] or the result of $\frac{\alpha}{\theta}$.

Min α

$$\begin{array}{c} s.t.\ \sum\limits_{p=1}^{n} \lambda_p x_{ip} - \alpha x_{iq} \leq 0;\ i = 1, \ldots, m \\ \sum\limits_{p=1}^{n} \lambda_p y_{jp} - y_{iq} \geq 0;\ j = 1, \ldots, s \\ \sum\limits_{p=1}^{n} \lambda_p = 1 \\ \lambda_p \geq 0;\ p = 1, \ldots, n \end{array} \tag{4}$$

Although the above traditional DEA models work with desirable variables, in real-life cases, sometimes the variables are undesirable and the goal is to maximize the inputs and/or minimize the outputs [30]. For instance, in recycling processes, one goal is to use the maximum amount of input waste and recycle them [27], and fire protection services usually pursue both desirable (maximize rescues) and undesirable (minimize losses) variables [7].

Some studies tried to address the constraint of undesirable outputs in the DEA method by using original data in their models or by applying data transformations before using them in the model [7,10]. Examples of using original data could be using radial measures and involving both desirable and undesirable variables [30], adopting a directional distance function to maximize desirable and minimize undesirable outputs at the same time [31], or using two disposability (natural and managerial) concepts [32,33], which are two different strategies in response to undesirable outputs; one is decreasing undesirable outputs by reducing the operation size or input (natural disposability), and another is increasing the inputs to reduce the undesirable outputs (managerial disposability). For data transformations, a variety of techniques are proposed such as the ADD approach (*f(U)*= $-U$) [34], multiplicative inverse (*f(U)*= *1/U*) [35], or reversing the traditional BCC model and adding a large positive number or a big enough scaler to inverse the undesirable number (*f(U)*= $-U + C$) [36], which is used in many studies [7]. More about the DEA method and its variants can be found in works by Coelli (1995) [37], Mariz et al. (2018) [38], and Australia et al. (1997) [39].

Another shortcoming of DEA is its sensitivity to the sample and not considering all the DMUs in the sample as efficient DUMs, even if they are efficient in the experts' opinions. It is caused by the nature of DEA computation, which compares the DMUs with the best samples and creates the frontier based on them. Therefore, the data error, variables, and model specifications are important and should be carefully selected to reduce the problems [24].

The aforementioned models could address input excesses and output deficits (slacks) concurrently, but they were unable to produce an efficiency score comparable to that of the CCR and BCC radial models. Tone (2001) [40] suggested a DEA model called the slacks-based measure (SBM) of efficiency to solve this deficiency. Unlike conventional radial efficiency models, the SBM model calculates the efficiency ratings of the inspected DMUs while taking into account all of their slack [40,41]. In this research, the SBM model was used to simultaneously perform the FDPA and calculate the *slacks*, which are the input excesses and output shortfalls.

Therefore, let FD = {FD_1, . . . , FDn} be a set of n DMUs, all of them having *i* inputs and *j* outputs. X is the input and Y is the output variable of the reference set FD, and eventually, P is the production possibility set defined by the FD. An FD that has *m* inputs and *s* outputs is a pair of nonnegative vectors ($\mathbf{x}$, $\mathbf{y}$), where $\mathbf{x} \in \mathbb{R}^m_+$ are the *input* and $\mathbf{y} \in \mathbb{R}^s_+$ are the *output* *vectors*, and the SBM efficiency score (with respect to the FD) of an activity (**x**, **y**) can be defined as:

$$\begin{gathered}\rho^*(\mathbf{x},\mathbf{y}) = \underset{\lambda,s^-,s^+}{\text{MIN}}\,\rho(\mathbf{x},\mathbf{y},\mathbf{s}^-,\mathbf{s}^+)\left(1-\frac{1}{m}\sum_{i=1}^{m}\frac{s_i^-}{x_i}\right)/\left(1-\frac{1}{s}\sum_{j=1}^{s}\frac{s_j^+}{y_j}\right)\\ s.t.\ \mathbf{x} = X\boldsymbol{\lambda}+\mathbf{s}^-,\\ \mathbf{y} = Y\boldsymbol{\lambda}+\mathbf{s}^+,\\ \boldsymbol{\lambda}\in\mathbb{R}_+^n,\ s^-\in\mathbb{R}_+^m,\ s^+\in\mathbb{R}_+^s,\end{gathered} \tag{5}$$

In program (5), the vectors $\mathbf{s}^+$ and $\mathbf{s}^-$ are *inefficiency slack* vectors [42], and based on the SBM efficiency model [41], $\rho^*(\mathbf{x},\mathbf{y})$ is the score that the model would assign to a new DMU with (**x**,**y**). According to Tone (2001) [41], an FD (DMU) is CCR-efficient, if $\rho^*(\mathbf{x},\mathbf{y}) = 1$ and the optimal slacks, $\mathbf{s}^{+*}$ and $\mathbf{s}^{-*}$, are zero for every optimal solution of the CCR.

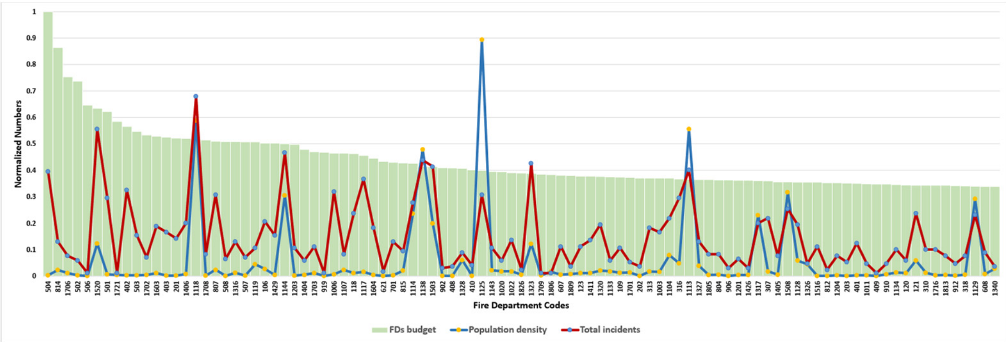
As mentioned in this section and more specifically under the Analysis Stage subsection, the SBM efficiency model of the VRS DEA method was used for the PT-FDPA, with multiple nonnegative desired inputs and multiple nonnegative undesired outputs. However, the FDPA evaluators have more alternatives according to their objectives [5]. Since the expert confirmed that the input and output weights should be treated equally, their weights were set to one to maintain their values. DeaR-Shiny is an online analytical software that was used for the PT-FDPA calculations. Section 3 provides the results of the PT-FDPA 2020.

3. Findings

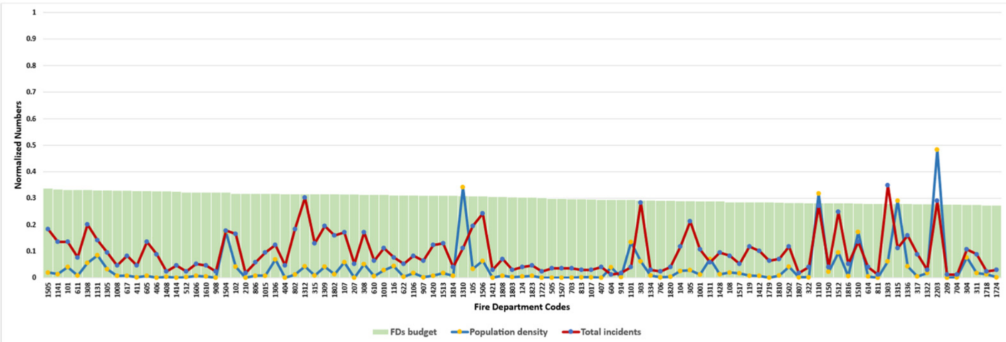
After the data preprocessing stage, it was possible to compare the FDs' financial income, the population density in their jurisdiction, and the total fire occurrences during the year 2020. The results depicted in Figure 3a–d show the changes in the normalized values of the FDs' financial resources (spent on prevention and suppression strategies) and population density, an important socioeconomic index variable [1,5], and the number of fire incidents in the same region. There is an obvious relation between the total incidents in the FDs' areas and their total financial budgets, and it shows that while the FDs' budgets decrease, the number of incidents also decreases, but there are many FDs that receive a much higher financial income while they do not have many incidents. The results of the Pearson correlation, calculated by the online software Statistics Kingdom [43], indicated that there is a significant medium positive relationship between the *FDs' budgets* and the *total incidents* (r(376) = 0.419, $p < 0.001$) and between the *population density* and the *total incidents*, (r(376) = 0.425, $p < 0.001$) in PT FDs. This could be an interesting result for the ANEPC decision makers and other FDPA investigators to control with their additional information in different levels to make sure that this type of correlation is happening between other variables also.

The effect of these differences on the FDs' performances is further discussed in Section 4 and could help FPS and the ANEPC decision makers compare the controllable and uncontrollable but effective variables influencing fire occurrences in their respective regions.

The PT-FDPA calculation was conducted using the online software DeaR-Shiny (https://rbensua.shinyapps.io/deaR accessed on 23 November 2022) [44] by selecting the VRS version of the SBM DEA [29]. As shown in Table 2 and illustrated in Figure 4a,b, among the 376 FDs in the evaluated dataset by the PT-FDPA model, 22 FDs were identified as efficient FDs, and the majority of the 354 nonefficient FDs (229 FDs) had less than a 50% efficiency score. This is important information for the decision makers in the ANEPC to reconsider their resource allocation and FPS strategies according to the PT-FDPA model 2020 (please see Figure 1).



(**a**)



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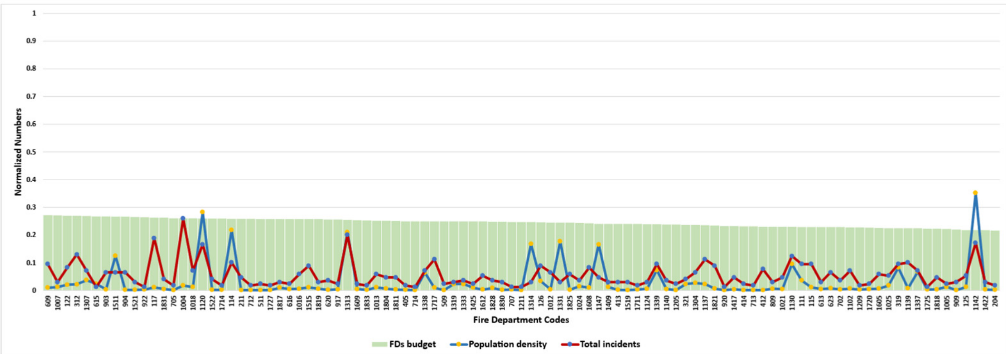

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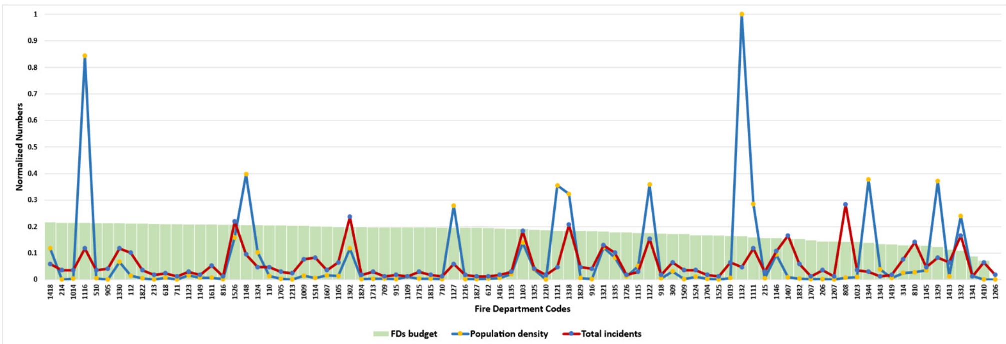


(**d**)

Figure 3. (**a**–**d**). FDs' budgets, populations densities, and number of incidents in 2020.

Table 2. Efficiencies of the Portuguese FDs in 2020.

FD Code	FD Name	Efficiency	FD Code	FD Name	Efficiency	FD Code	FD Name	Efficiency	FD Code	FD Name	Efficiency
118	Águeda	0.244	607	Soure	0.251	1148	Pontinha	0.769	1405	Rio.Maior	0.384
109	Albergaria	0.280	613	VN.Oliveirinha	0.440	1147	Caneças	0.571	1412	Salvat.de.Magos	0.443
117	Anadia	0.439	614	Tábua	0.379	1116	Algés	1.000	1422	Sardoal	0.541
124	Arouca	0.343	618	VN.de.Poiares	0.422	1121	Dafundo	0.688	1407	Tomar	0.584
106	Aveiro.Novos	0.256	713	Alandroal	0.875	1101	Barcarena	0.502	1411	Torres.Novas	0.312
101	Aveiro.Velhos	0.344	705	Arraiolos	0.553	1122	Carnaxide	0.611	1409	VN.Barquinha	0.503
123	Castelo.de.Paiva	0.307	709	Borba	0.639	1110	Oeiras	0.529	1507	Alcacér.do.Sal	0.657
108	Estarreja	0.316	704	Estremoz	0.411	1111	Paço.de.Arcos	0.747	1515	Alcochete	0.445
102	Ílhavo	0.362	701	Évora	0.298	1138	Agualva.Cacém	0.535	1508	Almada	0.443
112	Mealhada	0.467	703	Montemor.Novo	0.383	1144	Alg.Mem.Martins	0.358	1503	Cacilhas	0.268
110	Pampilhosa	0.446	711	Mourão	1.000	1150	Montelavar	0.570	1511	Trafaria	0.434
125	Murtosa	0.508	712	Portel	0.694	1106	Colares	0.427	1510	Barreiro	0.433
126	Fajões	0.426	710	Redondo	0.625	1128	Belas	0.366	1504	CFSS.Barreiro	0.437
105	Oliv.de.Azeméis	0.310	706	Regueng.Monsar	0.468	1125	Queluz	1.000	1516	Grândola	0.363
122	Oliv.do.Bairro	0.388	702	Vendas.Novas	0.553	1119	S.Pedro.Sintra	0.312	1512	Moita	0.475
116	Esmoriz	0.364	714	Viana.Alentejo	0.647	1107	Sintra	0.293	1522	Canha	0.432
104	Ovar	0.392	707	Vila.Viçosa.	0.529	1123	S.Monte.Agraço	0.501	1506	Montijo	0.402
111	Arrifana	0.498	814	Albufeira	0.203	1117	Torres.Vedras	0.207	1513	Palmela	0.364
121	Lourosa	0.342	812	Aljezur	0.398	1115	Alhandra	0.572	1517	Pinhal.Novo	0.404
107	Feira	0.432	815	Lagoa	0.334	1130	Alverca	0.517	1521	Águas.de.Moura	0.474
114	SJ.da.Madeira	0.470	802	Lagos	0.409	1145	Cast.Ribatejo	0.679	1525	Alvalade	1.000
119	Sever.do.Vouga	0.442	808	Loulé	0.339	1142	Póvoa.Sta.Iria	0.680	1519	Cercal.Alentejo	0.519
115	Vagos	0.400	811	Monchique	0.506	1146	Vialonga	0.671	1509	Santiago.Cacém	0.556
120	Vale.de.Cambra	0.340	810	Olhão	0.438	1102	V.Franca.Xira	0.509	1524	Santo.André	0.749
204	Aljustrel	0.608	807	Portimão	0.220	1207	Alter.do.Chão	1.000	1526	Amora	0.507

Table 2. *Cont.*

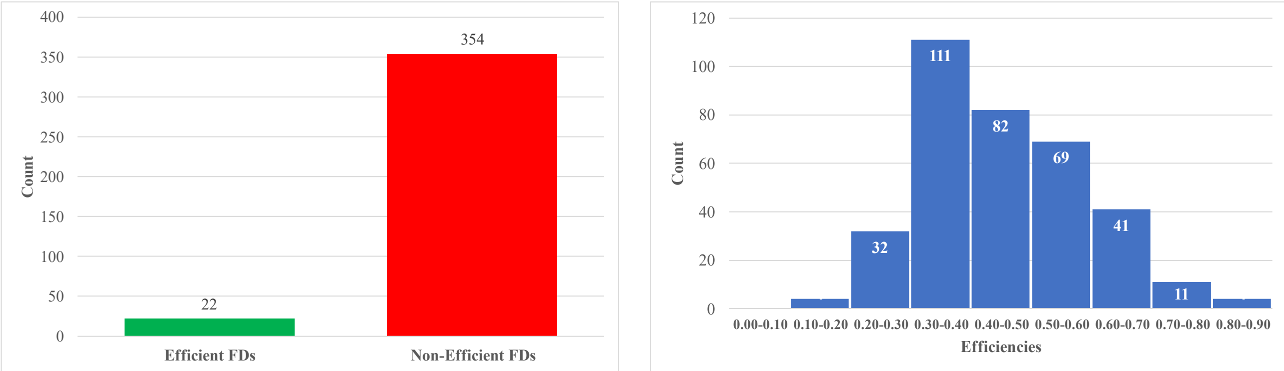
FD Code	FD Name	Efficiency	FD Code	FD Name	Efficiency	FD Code	FD Name	Efficiency	FD Code	FD Name	Efficiency
209	Almodôvar	0.760	809	S.Brás.Alportel	0.506	1211	Arronches	1.000	1520	Seixal	0.245
201	Beja	0.268	813	S.Bart.Messines	0.379	1210	Avis	1.000	1505	Sesimbra	0.347
213	Castro.Verde	0.650	806	Silves	0.343	1209	Campo.Maior	1.000	1502	Setúbal	0.438
206	Cuba	1.000	816	Vila.do.Bispo	0.615	1204	Elvas	0.334	1514	Sines	0.528
207	F.do.Alentejo	0.454	804	VR.S.António	0.345	1206	Gavião	1.000	1604	Arcos.Valdevez	0.338
203	Moura	0.355	915	Aguiar.da.Beira	0.540	1216	Marvão	0.622	1605	Caminha	0.555
202	Odemira	0.382	908	Almeida	0.444	1213	Monforte	1.000	1607	V.Praia.Âncora	0.624
215	Vila.Nova.Milfontes	0.721	912	Celorico.Beira	0.349	1205	Nisa	0.513	1612	Melgaço	0.568
210	Ourique	0.646	905	F.Cast.Rodrigo	0.665	1203	Ponte.de.Sôr	0.292	1606	Monção	0.413
211	Serpa	0.437	914	Fornos.Algodres	0.396	1201	Portalegre	0.329	1609	P.de.Coura	0.453
214	Vidigueira	0.608	903	Gouveia	0.427	1316	Amarante	0.261	1610	Ponte.da.Barca	0.478
311	Amares	0.425	918	V.Nova.de.Tazem	0.676	1340	Vila.Meã	0.358	1603	Ponte.de.Lima	0.249
312	Barcelinhos	0.300	916	Manteigas	0.594	1334	Baião	0.393	1608	Valença	0.505
305	Barcelos	0.318	906	Meda	0.419	1309	Felgueiras	0.329	1611	VN.de.Cerveira	0.547
321	Viatodos	0.441	904	Pinhel	0.428	1307	Lixa	0.374	1715	Alijó	0.567
318	Cab.de.Ba	0.413	902	Sabugal	0.339	1339	S.Pedro.da.Cova	0.378	1707	Favaios	1.000
315	Celorico.Ba	0.390	919	Soito	0.346	1313	Gondomar	0.492	1704	Sanfins.Douro	0.695
314	Fão	0.621	920	Loriga	0.516	1324	Valbom	0.465	1722	Boticas	0.437
309	Esposende	0.581	917	.Romão	0.407	1341	Melres	1.000	1702	Flaviense	0.310
307	Fafe	0.310	910	Seia	0.319	1318	Areosa.Rio.Tinto	0.659	1717	SP.Chaves	0.557
303	Guimarães	0.317	907	Trancoso	0.338	1327	Lousada	0.332	1721	Vidago	0.327
310	Póvoa.Lanhoso	0.314	922	V.Franca.Naves	0.520	1323	Moreira.da.Maia	0.318	1718	Me.Frio	0.609
322	Terras.do.Bouro	0.434	909	VN.de.Foz.Côa	0.566	1344	Pedrouços	1.000	1709	Mondim.de.Ba	0.413
317	Vieira.do.Minho	0.414	1001	Alcobaça	0.347	1320	Marco.Canaveses	0.280	1719	Montalegre	0.443
319	Riba.de.Ave	0.439	1024	Benedita	0.418	1332	Leça.do.Balio	0.841	1727	Salto	0.543

Table 2. *Cont.*

FD Code	FD Name	Efficiency	FD Code	FD Name	Efficiency	FD Code	FD Name	Efficiency	FD Code	FD Name	Efficiency
308	VN.Famalicão	0.281	1019	Pataias	0.644	1302	Matosinhos.Leça	0.524	1711	Murça	0.510
316	Famalicenses	0.252	1005	S.Martinh.Porto	0.428	1329	Leixões	1.000	1701	Peso.da.Régua	0.389
313	Vila.Verde	0.358	1012	Alvaiázere	0.410	1315	Mamede.Infesta	0.533	1725	de.Cerva	0.750
304	Vizela	0.366	1016	Ansião	0.369	1328	Freamunde	0.326	1724	Ribeira.da.Pena	0.515
409	Alfândega.da.Fé	0.415	1018	Batalha	0.318	1330	Paços.de.Ferreira	0.472	1713	Provesende	0.620
402	Bragança	0.241	1007	Bombarral	0.445	1325	Baltar	0.412	1705	Sabrosa	0.591
405	Carrazeda.Ansiães	0.494	1003	Caldas.Rainha	0.273	1322	Cête	0.452	1720	Fontes	0.600
404	Freixo.Espada.Cinta	0.388	1014	Cast.de.Pêra	0.495	1335	Lordelo	0.508	1726	Sta.M.Penaguião	0.866
403	Macedo.Cavaleiros	0.298	1011	Figueiró.Vinhos	0.343	1306	Paredes	0.396	1714	C.de.Montenegro	0.638
412	Miranda.do.Douro	0.602	1022	Leiria	0.199	1338	Rebordosa	0.412	1716	Valpaços	0.386
414	Sendim	0.539	1020	Maceira	0.299	1319	Entre.os.Rios	0.477	1708	V.Pouca.Aguiar	0.319
401	Mirandela	0.301	1025	Ortigosa	0.472	1333	Paço.de.Sousa	0.435	1706	C.Branca.V.Real	0.193
413	Torre.Dona.Chama	0.543	1004	Marinha.Grande	0.384	1305	Penafiel	0.340	1703	C.Verde.V.Real	0.286
406	Mogadouro	0.389	1013	Vieira.Leiria	0.447	2203	Portuenses	1.000	1818	Armamar	0.655
408	Torre.de.Moncorvo	0.353	1009	Nazaré	0.459	1303	Póvoa.de.Varzim	0.391	1819	Cabanas.Viriato	0.572
411	Vila.Flor	0.371	1008	Óbidos	0.326	1337	Vila.das.Aves	0.485	1821	Carregal.do.Sal	0.668
407	Vimioso	0.572	1017	Pedrógão.Grande	0.374	1304	Santo.Tirso	0.452	1803	Castro.D'Aire	0.320
410	Vinhais	0.324	1010	Peniche	0.335	1326	Tirsenses	0.367	1815	Farejinhas	0.720
510	Belmonte	0.518	1006	Pombal	0.186	1336	Trofa	0.352	1826	Cinfães	0.401
504	Castelo.Branco	0.167	1023	Juncal	0.630	1317	Ermesinde	0.400	1828	Nespereira	0.549
501	Covilhã	0.211	1021	Mira.de.Aire	0.547	1308	Valongo	0.372	1802	Lamego	0.358
503	Fundão	0.215	1015	Porto.de.Mós	0.366	1312	Vila.do.Conde	0.306	1814	Mangualde	0.375
508	Idanha.a.Nova	1.000	1149	Merceana	0.546	1321	Aguda	0.481	1813	Moimenta.Beira	0.390
506	Oleiros	0.355	1141	Alenquer	0.299	1331	Avintes	0.680	1811	Mortágua	0.387
505	Penamacor	0.548	1118	Amadora	0.392	1311	Carvalhos	0.423	1817	Canas.Senhorim	0.543

Table 2. *Cont.*

FD Code	FD Name	Efficiency	FD Code	FD Name	Efficiency	FD Code	FD Name	Efficiency	FD Code	FD Name	Efficiency
507	Proença.a.Nova	0.284	1105	Arruda.Vinhos	0.530	1314	Valadares	0.518	1809	Nelas	0.398
512	Cern.Bonjardim	0.353	1140	Alcoentre	0.512	1343	Crestuma	1.000	1816	Oliveira.Frades	0.629
502	Sertã	0.202	1139	Azambuja	0.465	1310	Coimbrões	0.549	1822	Penalva.Castelo	0.606
511	Vila.de.Rei	0.627	1124	Cadaval	0.488	1429	Abrantes	0.278	1827	Penedono	0.737
509	V.Velha.Ródão	0.661	1131	Alcabideche	0.345	1420	Almeirim	0.427	1823	Resende	0.509
612	Arganil	0.534	1129	Parede	0.498	1419	Alpiarça	0.847	1808	Santa.Comba.Dão	0.349
620	Coja	0.410	1120	Carcavelos.SDR	0.500	1404	Benavente	0.381	1824	Ervedosa.Douro	1.000
605	Cantanhede	0.272	1103	Cascais	0.553	1426	Samora.Correia	0.376	1807	SJ.Pesqueira	0.518
615	Brasfemes	1.000	1109	Bucelas	0.628	1413	Cartaxo	0.628	1804	S.Pedro.do.Sul	0.513
604	Coimbra	1.000	1127	Camarate	0.591	1421	Chamusca	0.537	1833	Sta.Cruz.Trapa	0.773
609	Condeixa.a.Nova	0.322	1135	Fanhões	0.578	1408	Constância	0.330	1831	Sátão	0.513
623	Mira	0.434	1104	Loures	0.322	1410	Coruche	0.766	1825	Sernancelhe	0.681
617	Miranda.Corvo	0.321	1132	Moscavide	1.000	1418	Entroncamento	0.600	1820	Tabuaço	0.480
611	Montemor.o.Velho	0.306	1114	Sacavém	0.380	1417	Ferreira.Zêzere	0.549	1829	Tarouca	0.569
616	Lagares.Beira	0.543	1134	Lourinhã	0.343	1416	Golegã	0.736	1832	Vale.Besteiros	0.659
608	Oliv.Hospital	0.310	1137	Ericeira	0.499	1414	Mação	0.444	1810	Tondela	0.349
621	Pampilhosa.Serra	0.335	1133	Mafra	0.344	1425	Caxarias	0.454	1830	V.Nova.de.Paiva	0.566
610	Penacova	0.293	1143	Malveira	0.335	1428	Fátima	0.382	1806	Viseu	0.315
622	Penela	0.332	1113	Odivelas	0.476	1406	Ourém	0.243	1805	Vouzela	0.326



(**a**) (**b**)

Figure 4. (**a**) Efficient/nonefficient PT FDs in 2020, (**b**) Efficiency distribution of the nonefficient FDs.

Another important result from the TP-FDPA model is finding the target FDs, which are the efficient DMUs, and checking how many inefficient FDs there are, and which ones should compare themselves with those targets and improve their performance based on the target results. Figure 5 shows the target FDs as well as the number of inefficient FDs that must increase their performance in order to meet the desired level of efficiency. The top three frequently referenced FDs have a big difference in their number of appearances in the reference sets, which is further discussed in Section 4.

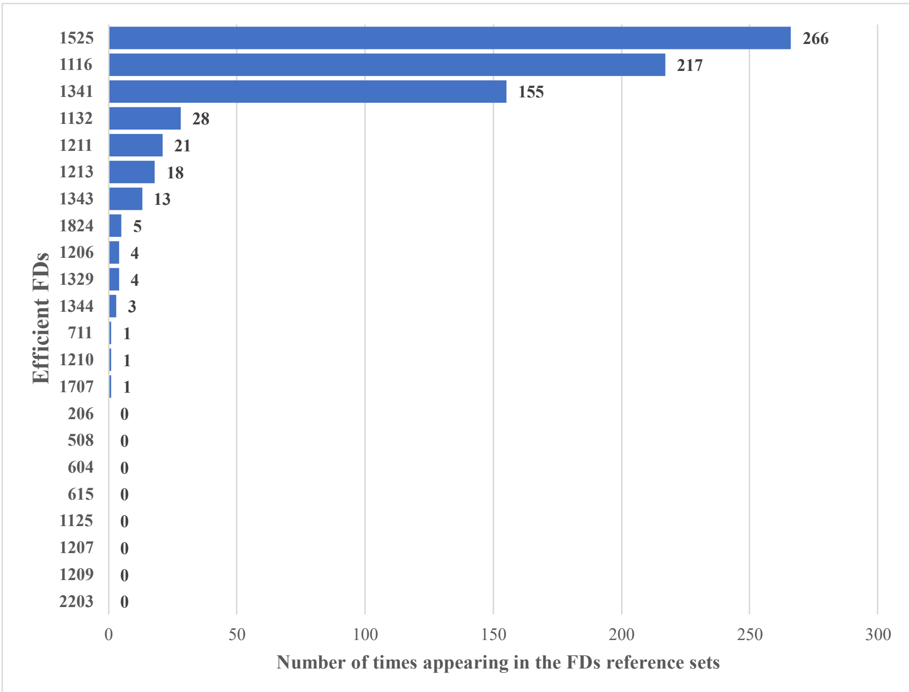


Figure 5. Efficient FDs and their number of appearances in the reference sets as targets for other FDs.

The results of the PT-FDPA model 2020 were provided and reviewed in this section. The next section contains further analysis, discussion, and recommendations for the PT FPS decision makers.

4. Discussion and Recommendations

By using the coordinates of the PT FDs and their efficiency results from Table 2, as shown in Figure 6, their efficiency status and location were plotted as places on the map to have a better understanding of their geographical distribution. This map was created by QGIS [45] to helping the ANEPC decision maker control the efficient and nonefficient

FDs in the main land of Portugal and have a better view for setting their regional and local strategies. To make it easier to locate the efficient FDs, their codes were added to the map, and their distribution is further discussed in Section 4.

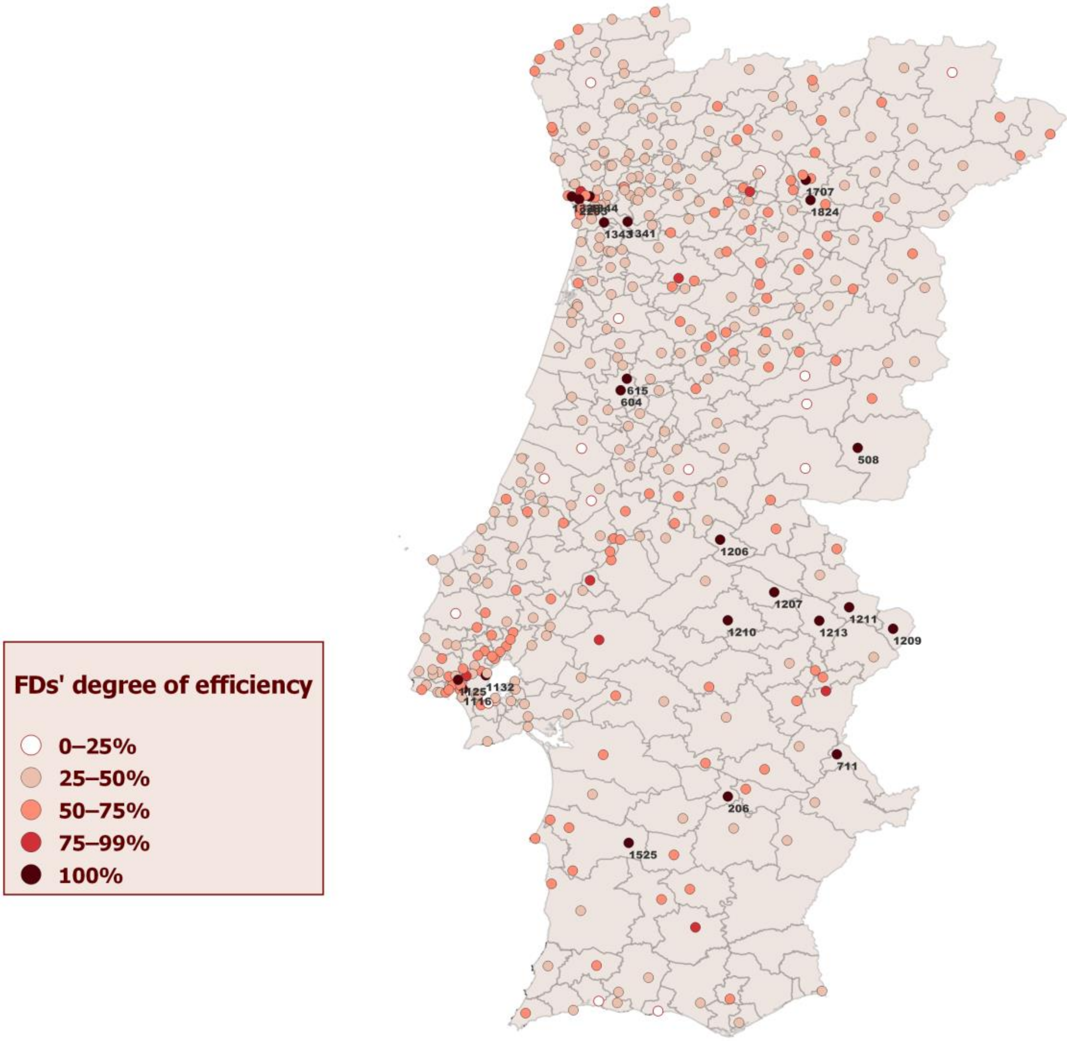


Figure 6. Efficient FDs and their number of appearances in the reference sets as targets for other FDs.

Table 3 shows a list of the 22 efficient FDs in 2020 in Portugal. Being efficient in the PT-FDPA model means that these FDs used their financial and technical resources to decrease fire incidents, their duration, and human casualties. The rest of them (the 354 FDs that can be seen in Table 2) are inefficient to some degree and have to improve their performance by making better managerial and operational decisions [5].

The FDs' performances are a multi-factor indicator [5], and as shown in Table 3, the efficient FDs had different input–output values that were not necessarily the minimum values in the variable range (the last row of the table shows the minimum values), which means their efficiency is the result of all changes in their inputs and outputs.

By checking the distribution of these efficient FDs on the map in Figure 6, it can be seen that some districts and regions have more efficient FDs than the others. Therefore, they were analyzed by their locations, and Table 4 provides more details about the number of total FDs in each district and the percentage of efficient FDs among their total FDs.

Table 3. Efficient Portuguese FDs in 2020 and their input–output values (Fin = financial budget, FF = number of firefighters, FE = number of fire engines, PD = population density, Dur = total incident durations, Inc = total incidents, D = number of deaths, SI = number of serious injuries, and LI = number of light injuries).

FD Code	District	FD Name	Fin Euro	FF	FE	PD p/km^2	Dur min	Inc	D	SI	LI
206	BEJA	Cuba	107,166.41	34	11	28.35	288	6	0	0	0
508	CASTELO BRANCO	Idanha a Nova	376,369.99	94	18	6.86	791	11	0	0	0
615	COIMBRA	Brasfemes	199,608.75	89	18	296.02	66	2	0	0	0
604	COIMBRA	Coimbra	218,623.71	76	14	497.86	81	2	0	0	0
711	ÉVORA	Mourão	155,848.61	31	11	9.56	76	2	0	0	0
1132	LISBOA	Moscavide	122,128.83	50	16	12,519.81	573	8	0	1	1
1116	LISBOA	Algés	159,553.91	63	15	10,555.39	869	20	0	0	0
1125	LISBOA	Queluz	296,119.84	85	22	11,191.53	3360	52	0	0	15
1207	PORTALEGRE	Alter do Chão	107,260.24	44	9	9.84	169	2	0	0	0
1211	PORTALEGRE	Arronches	184,260.33	27	6	10.06	61	2	0	0	0
1210	PORTALEGRE	Avis	139,682.14	43	10	7.54	153	3	0	0	0
1209	PORTALEGRE	Campo Maior	170,077.62	23	5	34.21	158	3	0	0	1
1206	PORTALEGRE	Gavião	2360.00	62	12	14.03	215	3	0	0	1
1213	PORTALEGRE	Monforte	151,916.01	40	7	7.92	400	4	0	0	0
1341	PORTO	Melres	65,939.14	21	10	174.66	184	2	0	0	0
1344	PORTO	Pedrouços	103,529.12	77	12	4722.09	370	5	0	0	0
1329	PORTO	Leixões	92,680.88	45	9	4640.32	1072	14	0	1	5
2203	PORTO	Portuenses	205,889.08	61	8	6045.54	3373	49	0	0	1
1343	PORTO	Crestuma	100,074.69	38	9	483.73	79	2	0	0	0
1525	SETÚBAL	Alvalade	123,599.20	17	6	13.70	172	2	0	0	0
1707	VILA REAL	Favaios	111,104.46	41	10	29.90	101	2	0	0	0
1824	VISEU	Ervedosa Douro	148,347.47	40	3	28.27	157	3	0	0	0
		Minimums	2360.00	17	3	6.86	61	2	0	0	0

Eight of the eighteen districts in Portugal have no efficient FDs according to the PT-FDPA data, while among the other districts, Portalegre has the greatest overall performance with six efficient FDs (54.55% of its total FDs). It could be associated with its socioeconomic, spatiotemporal, and/or FPS strategies [5,7,10]. However, it is recommended that the ANEPC's decision maker should investigate this information to determine the actual reasons why some districts have very few or no efficient FDs.

Three efficient FDs, as shown in Figure 5, appear most frequently in the reference sets of the remaining inefficient FDs. It means the other FDs should aim to emulate them to increase their performance. The decision maker can also study the inputs and outputs of the effective FDs to make better decisions on resource allocation and FPS techniques. The most frequent efficient FDs used as targets for other inefficient FDs are listed below:

- CBV Alvalade in Setúbal (FD code 1525) with 266 appearances;
- CBV Algés in Lisboa (FD code 1116) with 217 appearances;
- CBV Melres in Porto (FD code 1341) with 155 appearances.

Table 4. Number and percentage of efficient FDs in each of the 18 districts of Portugal (except islands).

District	Number of FDs	Number of Efficient FDs	% of Efficient FDs
AVEIRO	24	0	0.00%
BEJA	12	1	8.33%
BRAGA	18	0	0.00%
BRAGANÇA	14	0	0.00%
CASTELO BRANCO	12	1	8.33%
COIMBRA	18	2	11.11%
ÉVORA	13	1	7.69%
FARO	13	0	0.00%
GUARDA	18	0	0.00%
LEIRIA	24	0	0.00%
LISBOA	46	3	6.52%
PORTALEGRE	11	6	54.55%
PORTO	43	5	11.63%
SANTARÉM	22	0	0.00%
SETÚBAL	23	1	4.35%
VIANA DO CASTELO	10	0	0.00%
VILA REAL	24	1	4.17%
VISEU	31	1	3.23%

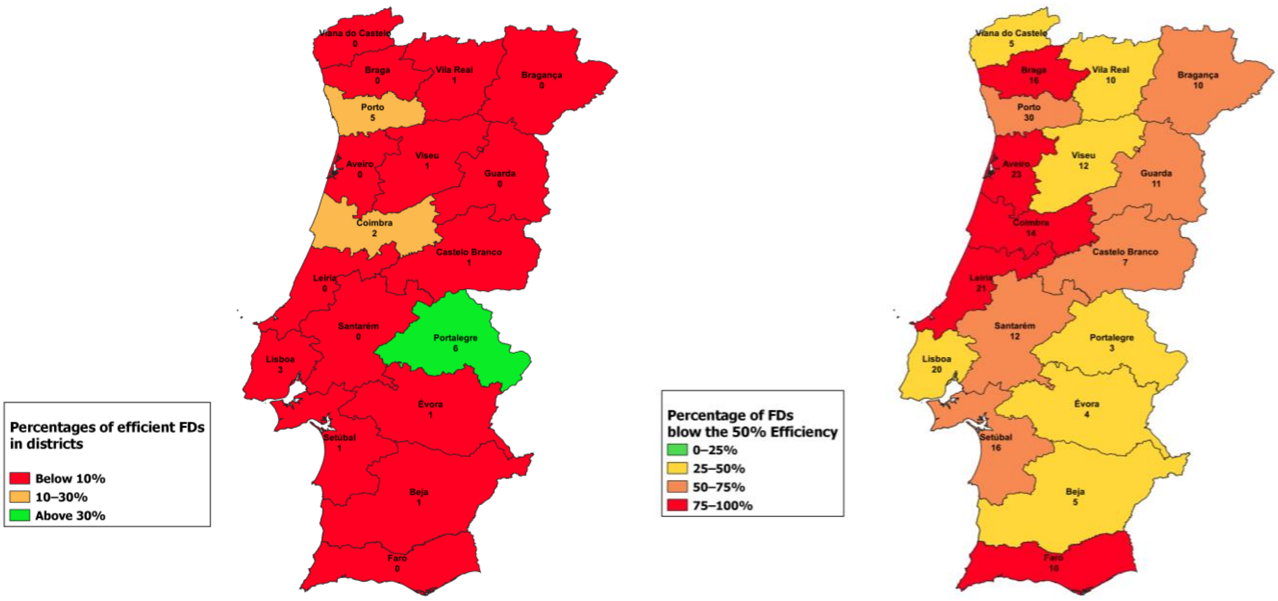
Another important finding that was provided in Section 3 Figure 4b is the number of FDs with less than 50% efficiency in comparison with the efficient target FDs. Table 5 provides the results of the statistical analysis on those findings and a better view of the number of efficient and nonefficient FDs in the Portuguese districts.

From the results of Table 5, 95.83% of Aveiro's FDs have below 50% efficiencies, followed by Braga with almost 89% and Leiria with 87.5% of their FDs having less than 50% efficiency in their FPS activities. Based on the results of Tables 4 and 5, Figure 7a,b show the efficiency status of Portuguese FDs at the district level. Figure 7a highlights the districts based on the number of efficient FDs, and it gives us a very important overall view of the FPS efficiency in Portugal in 2020. Except for Portalegre, Coimbra, and Porto, all the districts have less than 10% efficient FDs in their region. As depicted in Figure 7b, more than 75% of the FDs in Braga, Aveiro, Leiria, Coimbra, and Faro are not only inefficient, but are also operating below 50% efficiency compared to their target efficient FDs. Although having FDs with lower percentages of inefficiency is an important objective for the FPS authorities [5], at least 25% of the FDs in all the districts have less than 50% efficiency according to Table 5 and Figure 7b.

The results provided in this paper bring some important situations to the attention of the ANEPC's decision makers. The performance of FDs is a multi-factor dependent variable [1,5] that is influenced by independent variables such as socioeconomics (e.g., population size and density, resident income rate, and educational level), spatiotemporal factors (e.g., the percentage of residential, industrial, wild, and green areas, the environment, traffic levels, and climate indicators), and the FDs' prevention and suppression activities [5].

Table 5. Number and percentage of efficient FDs below and over 50% efficiency in each 18 districts of Portugal (except islands).

District	FDs with Below 50% Efficiency	% of FDs with Below 50% Efficiency	FDs with Over 50% Efficiency	%of FDs with Over 50% Efficiency
AVEIRO	23	95.83%	1	4.17%
BEJA	5	41.67%	7	58.33%
BRAGA	16	88.89%	2	11.11%
BRAGANÇA	10	71.43%	4	28.57%
CASTELO BRANCO	7	58.33%	5	41.67%
COIMBRA	14	77.78%	4	22.22%
ÉVORA	4	30.77%	9	69.23%
FARO	10	76.92%	3	23.08%
GUARDA	11	61.11%	7	38.89%
LEIRIA	21	87.50%	3	12.50%
LISBOA	20	43.48%	26	56.52%
PORTALEGRE	3	27.27%	8	72.73%
PORTO	30	69.77%	13	30.23%
SANTARÉM	12	54.55%	10	45.45%
SETÚBAL	16	69.57%	7	30.43%
VIANA DO CASTELO	5	50.00%	5	50.00%
VILA REAL	10	41.67%	14	58.33%
VISEU	12	38.71%	19	61.29%



(**a**) (**b**)

Figure 7. (**a**) Number and percentage of efficient FDs in each district. (**b**) Number and percentage of FDs with less than 50% efficiency in each district.

Recommendation 8: Since the PT-FDPA model 2020 used the ANEPC data and the approved performance variables for the analysis, the results in Tables 3 and 4 and Figure 7a,b should be considered a serious situation in Portuguese FDs that needs a close investigation by the ANEPC experts to find the main causes of the inefficiencies in the reported districts.

The slack results for the inefficient FDs in these districts and their efficiency distance from the target FDs in the PT-FDPA are two more outcomes that the SBM DEA model produced that could be reviewed and may provide further insights for corrective actions. This information is available upon readers' request.

5. Conclusions

FDPAs are an important task for FPS authorities (e.g., ANEPC in Portugal) to evaluate the FDs' efficiencies, identify the efficient ones, and identify the areas for improvement of the inefficient units. Although many countries have conducted FDPAs, to the best of our knowledge, no FDPA study has been conducted for PT FDs. Therefore, in this paper, following the GFDPAF and applying the SBM version of DEA, the PT-FDPA model 2020 was introduced and applied to the ANEPC data on the incidents and FD operations in 2020.

During the data-gathering stage of the PT-FDPA model 2020, minor limitations caused the removal of some of the suggested FDPA variables by GFDPAF from the PT-FDPA model with the confirmation of experts. The main reason for their removal was that either the variables were not available in the ANEPC datasets (e.g., number of hydrants), they were not available during the data-gathering stage (e.g., incident response time), or because granular details were not accessible due to aggregation (e.g., prevention activities budget or urban FPS budget) or due to confidentiality (e.g., FDs' other financial resources). This study provides significant recommendations for future FDPA investigations for each of these instances.

According to the PT-FDPA model 2020 efficiency results, only 22 of the 376 analyzed PT FDs were efficient. Analyzing the geographical distribution of the efficient and inefficient FDs in GIS also revealed that, in most of the districts in Portugal (15 out of 18 districts), less than 10% of their total FDs are efficient, and the percentage of FDs with less than 50% efficiency is very high. It is important for decision makers within the FPS to address the inefficiencies within the PT FDs. While factors such as the financial budget, number of incidents and casualties, and population density have an impact on the level of inefficiency within these FDs, further investigation is needed to identify the root causes and areas for improvement.

The findings of this research may assist the FPS decision maker in gaining a better understanding of the FDs' efficiencies in 2020 and in taking remedial measures to raise those efficiencies, which includes setting RAFD strategies for their limited resources, taking corrective actions in suppression scenarios, and finally optimizing the size and the location of FDs and stations. A better grasp of how an FDPA might be conducted and be discussed can also be helpful to the FDPA evaluator. Given that the SBM DEA in its generic form was employed for the PT FDPA, it is suggested that future studies define a more sophisticated DEA model for FDPAs utilizing the set of recommendations in this work. Further research could be conducted to provide more reliable findings by compiling more specific and advised factors in the article. Another idea for a future study is to use various versions of the DEA model and to compare the outcomes to determine which version is better suited for FDPAs. In addition to applying to wild and green fire data, the PT-FDPA model might also be used for PA in other domains, such as public–private organizations or commercial and industrial incorporations. The last, but not the least, suggestion for future research is to gather and use data on preventive activities for FDPAs due to their significant effect on reducing the number of fire incidents and casualties.

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