A Work Project, presented as part of the requirements for the Award of a Master's degree in Finance from the Nova School of Business and Economics.

ANALYSIS OF QUANTITATIVE INVESTMENT STRATEGIES

A Market Timing Rotational Strategy based on the Dual Moving Average Crossover – a Study on Gold and the Market

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16/12/2022

Abstract

It is well-known in the financial world that investors often turn to gold as a “safe haven” during times of adverse market conditions. This study explores a rotational strategy that involves switching between gold and the market in an attempt to try to time the latter and minimize losses during these periods. Findings suggest that the effectiveness of this strategy largely varies depending on the specific macroeconomic conjuncture, showing promising results during the Covid-19 Crisis of 2020, but less so during the Great Financial Crisis of 2007-09.

Keywords: Market Timing, Safe Haven, Asset Switching, Pair Switching, Dual Moving Average Crossover, Gold

This work used infrastructure and resources funded by Fundação para a Ciência e a Tecnologia (UID/ECO/00124/2013, UID/ECO/00124/2019 and Social Sciences DataLab, Project 22209), POR Lisboa (LISBOA-01-0145-FEDER-007722 and Social Sciences DataLab, Project 22209) and POR Norte (Social Sciences DataLab, Project 22209).

1. Introduction

The relationship between gold and the stock market has long been of interest to investors and financial experts alike. As gold usually holds its value for longer than most assets, it is considered a “safe haven” against inflation when buying power decreases. Literature around this topic has seen great discussion, showing mixed results. Baur and McDermott (2010) show gold works very well as a hedge for most Western markets. Hood and Malik (2013) demonstrate gold’s limitations as the safe haven asset, arguing there are better options. Finally, Batten, Ciner, and Lucey (2014) conclude the relationship between gold and the market is very sensitive to the timespan chosen, as different macroeconomic variables create different reactions in the gold market.

This paper will explore a strategy that attempts to protect investors from bear markets by switching from the market to gold, following basic trend signals. The introduction of this signal takes Maewal and Scalaton (2011)’s proposed market timing strategy based on rotational asset allocation (Asset Switching) and provides a new perspective, understanding the viability of short-term versus long-term trends indicating the correct timing for the switch.

2. Strategy and Data

2.1. Strategy

2.1.1. Economic Motivation

Baur and Kuck (2019) conducted research on the role of gold as a "safe haven" asset in the market, and found evidence to support this idea for most developed stock markets. Baur and McDermott (2010) also discovered that gold returns react quickly to extreme negative changes in the market, suggesting that gold could be used to limit losses during these difficult periods.

Historically, gold returns have shown promising results during downturns in the stock market. In the United States, during the 1973-74 bear market (the worst bear market since the Great Depression of 1929), the Dow Jones Industrial Average (DJIA) fell by 40%, while the value of gold increased by over 50%. In 1987, the same pattern repeated itself, with the US stock market falling by 22.5% and gold seeing an increase of almost 2% (Liston, 2012). This trend seems to have continued until recent history, with the Indian market (the largest market for gold consumption) as an example. During the subprime mortgage crisis of 2008-09, market indexes dropped by 35% while gold ETFs rose by 69%. The same pattern was seen during the Covid-19 crisis in 2020, with the market falling by 37% and gold ETFs increasing by 49.5%. Finally, during the recent inflationary crisis, the market fell by 12.7% while gold ETFs rose by 10.6%. The contrary movement can also be seen during the 2014 bull market, as the market rose by 23.7% while gold fell by 12.3% (Choudhary, 2022).

Given this apparent correlation between the returns of gold and the stock market, investors are understandably interested in using gold as a way to protect themselves during bear markets (Kuck, 2021).

2.1.2. Signal Construction

Glenn (2014) proposed a simple market timing algorithm that takes advantage of a persistent negative correlation between assets. This approach, which is based on Maewal and Scalaton (2011)'s idea, periodically switches positions between two negatively correlated assets based on their relative performance over a given period.

Unlike the original authors' method of comparing the performance of the two assets and going long on the one with the highest return during the ranking period, this strategy uses the Dual Moving Average Crossover as a market timing signal to identify whether the movement of an asset is being driven more by its short-term or long-term behavior, essentially working as a trend-following strategy. To apply this methodology to the pair of assets mentioned, the strategy looks at the Price Ratio of VTI (Vanguard Total Stock Market Index Fund ETF) and GLD (Gold). Buy and sell signals are triggered whenever the short-term and long-term averages of the Price Ratio cross, as follows:

$$\text{Price Ratio}_t = \text{VTI}_t / \text{GLD}_t \tag{1}$$

$$\text{SMA30} > \text{SMA100: Buy VTI, Sell GLD} \tag{2}$$

$$\text{SMA30} < \text{SMA100: Buy GLD, Sell VTI} \tag{3}$$

Where Price Ratio $_t$ represents the ratio between the prices of VTI and GLD at time t, SMA30 is the Simple 30-Day Moving Average, used as a proxy for the short-term trend, and SMA100 is the Simple 100-Day Moving Average, used as a proxy for the long-term trend.

The persistence of the negative correlation between the two assets is crucial for this strategy to be effective, as it is only under this assumption that it makes sense to bet on an increase in the Price Ratio being explained by a rise in VTI price and/or a drop in GLD price, and a decrease being explained by a fall in VTI price and/or an increase in GLD price. As previously mentioned, while both assets have shown a recent upward long-term trend, empirical evidence and economic reasoning largely support the persistence of the negative correlation between them.

For this analysis, both a long-only strategy, where the signal triggers a long reaction on one of the two assets, as well as a long-short strategy, where besides going long on one of the assets, it simultaneously goes short on the other, were tested. Whenever the signal is activated, the positions are inverted.

2.2. Data

For the development and testing of this strategy, daily adjusted close prices were retrieved from Yahoo Finance for both GLD and VTI for the period between November 2004 and October

2022. Additionally, both market factors data as well as the risk-free rate data were supplied by the Ken French data library, available online. Finally, the effective transaction costs will be fixed at 1%, based on Hasbrouck (2009)'s historical evidence of the NYSE (New York Stock Exchange), Amex, and Nasdaq.

3. Performance Analysis

In this section, an analysis on the strategy's performance will be conducted, mainly looking at its annualized returns, standard deviation, and Sharpe Ratio. Additionally, as the strategy's main goal is to explore the possibility of using Gold as a hedge to Market Risk, a comparison with the Market portfolio will be made. Finally, several performance measures from regressions such as the CAPM, Fama French 3-Factor Model (FF3), and Fama French 5-Factor Model (FF5) will be used to measure performance.

3.1. Returns, Standard Deviation and Sharpe Ratio

The proposed strategy using the Dual Moving Average Crossover activated a trade signal 52 times over the full sample. In Figure 1, we can see cumulative returns for both our long-only and long-short strategies (with and without transaction costs), as well as the Market portfolio.

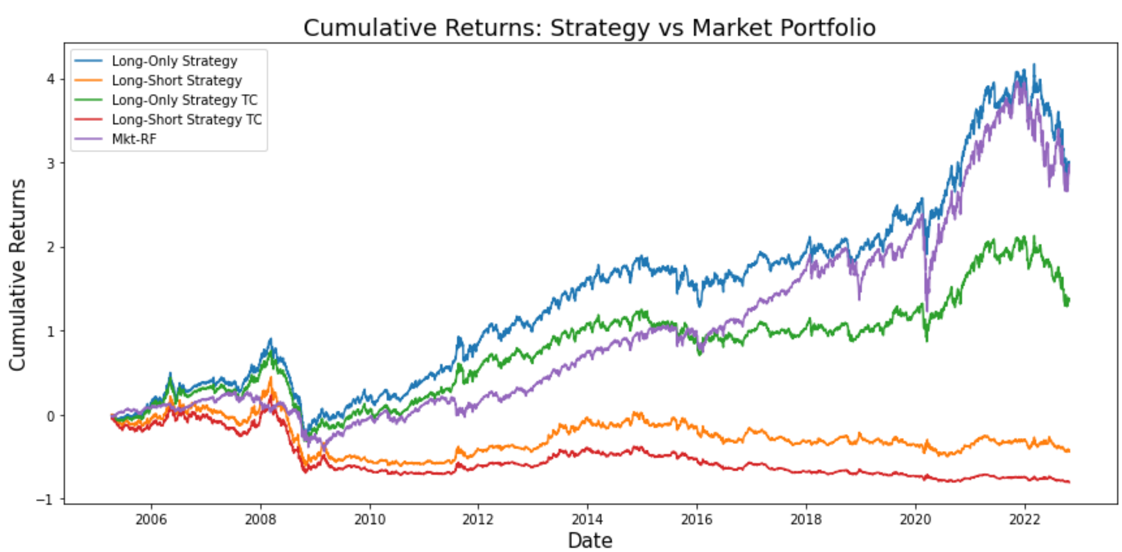

Figure 1: Asset Switching Strategy Cumulative Returns

In the Figure, one can see that the long-only strategy without transaction costs consistently beats the market ever since its cumulative returns first rise above the market's in 2006, with the long-short version looking stagnated at very low levels after the Great Financial Crisis of 2007-09.

Notably, the strategy shows promising results at the beginning of the Financial Crisis, yielding increasing cumulative returns against the market's declining cumulative returns. It is only at the end of this crisis that the strategy starts to decline, with the long-versions eventually converging with the market by the end of 2009. From this point onwards, we see a consistent widening of the gap between the long-only strategy and the market up until 2018, where the advantage fades and the gap becomes marginal.

Finally, it is important to look at the performance of the long-only strategy against the market's during the most recent severe market crashes of 2019 and 2020, where we see the market falling abruptly. In both of these periods, the decline in performance of the strategy is almost half than that of the market, thus appearing to work as a market hedge up to a certain extent.

As the only version of the strategy that consistently beats the market is the long-only strategy without transaction costs, these costs will be assumed to be zero from this point onwards.

Table 1 reports performance statistics of the long-only strategy for various periods in time, as well as for the full sample. Overall, we see very distinct performances depending on the time period being evaluated. Over the full sample, the strategy yields a mild Sharpe Ratio of 0.54, mostly driven by the strategy's performance in the first half of the sample (0.66 Sharpe Ratio against 0.36 in the second half). As the Sharpe Ratio is the quotient between the average excess return and the standard deviation of returns, it tells us the risk-adjusted performance, that is, the expected excess return per unit of volatility. Moreover, we consistently see that the maximum and minimum returns for every period are almost polar opposites, as well as large drawdowns from its best points in time, translating into the large standard deviations observed.

Finally, when looking at the two major crises present in the sample, the Great Financial Crisis of 2007-2009 and the Covid Crash Crisis of 2020, we see two very distinct performances of the strategy. While the strategy dealt well with the initial years of the Great Financial Crisis, as shown above, its sudden drop in performance at the tail-end of the crisis delivered a staggering annualized standard deviation of 22.61%, which, thanks to its annualized excess return of 5.94%, granted the strategy a very poor Sharpe Ratio of 0.26. Although weak, it still outperformed the market during this period, which reported a Sharpe Ratio of 0.08. On the other hand, when looking at the Covid Crash Crisis, we see the complete opposite reaction. With its impressive 26.26% annualized return, the strategy was able to yield a promising Sharpe Ratio of 1.34, a significant decrease in kurtosis (a quarter than that of the market), and its largest drawdown was -67.31 percentage points.

Period	Max	Min	Ann. Return	Std. Dev.	Sharpe Ratio	Largest Drawdown (pp)
2005 - 2022	7.35%	-7.43%	9.37%	17.34%	0.54	-135.11
2005 - 2014	7.35%	-7.43%	12.63%	19.13%	0.66	-113.96
2015 – 2022	4.85%	-4.39%	5.31%	14.82%	0.36	-135.11
Great Financial Crisis (07-09)	7.35%	-7.43%	5.94%	22.61%	0.26	-113.96
Covid Crash Crisis (2020)	4.85%	-4.39%	26.26%	19.63%	1.34	-67.31

Table 1: Asset Switching Strategy Performance Statistics

3.2. CAPM, Fama French 3-Factor Model, Fama French 5-Factor Model

3.2.1. CAPM

In addition to the prior performance measures, a regression analysis was made. The Capital Asset Pricing Model (CAPM) describes the relationship between the systematic risk (also known as market risk) of the strategy and its expected returns. Only this risk component is captured since, in contrast to the idiosyncratic risk, it is not diminishable through diversification - it is intrinsic to the market movement. As the strategy involves trading the market at some points, it is expected that the Market factor will be relevant for explaining the strategy's returns. The model is derived using the following regression:

$$E(r_i) = r_f + \beta_i * [E(r_m) - r_f] \quad (4)$$

Where $E(r_i)$ is the expected return of the strategy, r_f is the appropriate risk-free rate, β_i describes the sensitivity of the strategy to the market, where $\beta_i < 1$ indicates the strategy is less volatile than the market, and $\beta_i > 1$ indicates it is more volatile than the market, and $[E(r_m) - r_f]$ represents the market risk premium, where $E(r_m)$ is the expected return of the market.

Table 2 reports some of the results obtained by running the regression. As can be seen, both daily alphas (the expected abnormal return of the strategy over the market) for the long-only and long-short strategies are underwhelming (0.02% and 0.01%, respectively), with both proving to be statistically insignificant for the 95% confidence level, as their t-statistics lie inside the interval of critical values for such confidence level, -1.96 and 1.96. As expected, the Market factor revealed to be significant for both (t-statistics of 30.65 and -16.08, respectively), although the long-only strategy seems to be positively influenced by it, with a Market Beta equal to 0.0036, while the long-short version seems to behave inversely to the Market, with a Beta of -0.0031.

Finally, Table 2 also shows us the different Information Ratios, which tell us the risk-adjusted return per unit of volatility of the residuals. Both strategies seem to slightly outperform the CAPM benchmark, with Information Ratios of 0.37 and 0.12, respectively.

Strategy	CAPM Alpha	CAPM Alpha t-stat	CAPM Exp. Ret.	CAPM Info. Ratio	Mkt Beta	Mkt Beta t-stat
Long-Only	0.00023	1.54	0.00014	0.37	0.0046	30.65
Long-Short	0.00012	0.52	-0.00012	0.12	-0.0031	-16.08

Table 2: Asset Switching Strategy CAPM Regression Statistics

3.2.2. Fama-French 3-Factor Model

The Fama-French 3-Factor Model extends the CAPM by adding two other factors that could help explain returns: SMB (Small Minus Big), which captures the outperformance of small-cap stocks versus large-cap stocks, and HML (High Minus Low), which portrays high book-to-market ratio stocks' overperformance over low book-to-market ratio stocks. The model is derived using the following regression:

$$E(r_i) = r_f + \beta_{i,MKT} * \left[E(r_m) - r_f\right] + \beta_{i,SMB} * SMB + \beta_{i,HML} * HML \qquad (5)$$

Once more, Table 3 reports some of the results found from running the regression. As expected from the CAPM results, the addition of the two new factors didn't convert the regression's alphas statistically significant. However, the new model indicates a significant, although rather low, negative exposure to the HML factor for both the long-only and long-short strategies (t-statistics of -4.46 and -2.06, respectively), suggesting that the strategies behave similarly to growth stocks.

Strategy	FF3 Alpha (t-stat)	FF3 Exp. Ret.	FF3 Info. Ratio	Mkt Beta (t-stat)	SMB Beta (t-stat)	HML Beta (t-stat)
Long-Only	0.00022 (1.50)	0.00015	0.36	0.0037 (30.23)	0.000064 (0.26)	-0.00084 (-4.46)
Long-Short	0.00012 (0.50)	-0.0001	0.12	-0.0030 (-14.92)	-0.00013 (-0.31)	-0.00063 (-2.06)

Table 3: Asset Switching Strategy FF3 Regression Statistics

3.2.3. Fama-French 5-Factor Model

Finally, the Fama-French 5-Factor Model further extends the previous model by including a profitability factor (RMW), computed as the difference between the returns on diversified portfolios with high and low profitability, and an investment pattern factor (CMA), which is the return spread on conservative and aggressive re-investment firms. The model is derived using the following regression:

$$E(r_i) = r_f + \beta_{i,MKT} * \left[E(r_m) - r_f\right] + \beta_{i,SMB} * SMB + \beta_{i,HML} * HML + \beta_{i,RMW} * RMW + \beta_{i,CMA} * CMA \quad (6)$$

In Table 4, where some results of the regression are presented, we can see that once more the introduction of the new factors didn't influence the significance of the strategies' abnormal returns (alpha). However, the different factors revealed to be relevant for different strategies. As can be seen, the profitability factor proved to be significant only for the long-short strategy, with a Beta equal to -0.0016, indicating that the strategy behaves in the same way as low profitability portfolios. For the long-only strategy, the investment pattern factor gains significance, with a Beta of 0.0015, indicating positive exposure to the factor, thus following the behavior of firms with conservative investment policies, whose returns tend to be higher than firms with aggressive investment policies.

Strategy	FF5 Alpha (t-stat)	FF5 Exp. Ret.	FF5 Info. Ratio	Mkt Beta (t-stat)	SMB Beta (t-stat)	HML Beta (t-stat)
Long-Only	0.00022 (1.45)	0.00016	0.35	0.0038 (29.01)	0.000041 (0.16)	-0.0012 (-5.39)
Long-Short	0.00014 (0.57)	-0.00013	0.14	-0.0030 (-14.12)	-0.00035 (-0.86)	-0.00079 (-2.23)

	RMW Beta (t-stat)	CMA Beta (t-stat)
Long-Only	-0.00029 (-0.80)	0.0015 (3.17)
Long-Short	-0.0016 (-2.75)	0.0011 (1.36)

Table 4: Asset Switching Strategy FF5 Regression Statistics

Finally, Table 5 reports the Fama-French 5-Factor model regression results for the two halves of the sample, as well as for the Covid Crash Crisis of 2020. Both halves show, once again, statistically insignificant abnormal returns (t-statistics of 1.64 and 0.21). For the Covid Crash, despite the Sharpe Ratio of 1.34, the regression also shows these returns aren’t due to statistically significant abnormal returns, but rather that they are explained by four of the five factors included in the regression: the Market, SMB, HML, and CMA, with all of them reporting t-statistics outside the interval of critical values for the 95% confidence level, -1.96 and 1.96. Both the first half of the sample and the Covid Crash Crisis realized Information Ratios above 0.5, suggesting an outperformance against the model’s benchmark returns.

Strategy	FF5 Alpha (t-stat)	FF5 Exp. Ret.	FF5 Info. Ratio	Mkt Beta (t-stat)	SMB Beta (t-stat)	HML Beta (t-stat)
First Half	0.0004 (1.64)	0.00013	0.53	0.0040 (19.19)	-0.0006 (-1.36)	-0.0011 (-2.69)
Second Half	0.000004 (0.21)	0.00017	0.07	0.0039 (23.89)	0.0006 (2.09)	-0.0017 (-6.09)
Covid Crash	0.0004 (0.55)	0.00067	0.56	0.0028 (8.36)	0.0028 (3.25)	-0.0037 (-5.19)
	RMW Beta (t-stat)	**CMA Beta (t-stat)**				
First Half	0.0004 (0.62)	-0.0017 (-1.95)				
Second Half	-0.0009 (-2.33)	0.0038 (7.05)				
Covid Crash	0.0021 (1.45)	0.0042 (2.25)				

Table 5: Asset Switching Strategy Period-Specific FF5 Regression Statistics

4. Risk Analysis

4.1. Drawdown, Value at Risk, Skewness and Kurtosis

To better understand the risk inherent to the strategy proposed, a risk analysis was conducted, where the Drawdown, Value at Risk, Skewness and Kurtosis were analyzed. In Figure 2, the historical drawdown of both the long-only and long-short strategies are shown. A strategy's drawdown is commonly known as how far the strategy has fallen from its best point in history, computed in a running manner, where the drawdown is computed as the flat percentage point difference from the running maximum cumulative return of the strategy and the strategy's cumulative returns at each point in time. As can be seen, despite the long-only strategy exhibiting better performance results, its maximum drawdown is actually larger than the long-short strategy (135 percentage points against 106 percentage points). However, the long-only strategy does present a lower drawdown for most of the sample, it is only at the market crashes of 2009 and 2022 where it dips the most, achieving its lowest points. One final thing to note is the movement of the long-short strategy. Despite also falling abruptly with the 2009 market crash, we see a rather constant drawdown from that point onwards. Nonetheless, this behavior is explained by the strategy never recovering from that crash, as its cumulative returns were quite stable at low values since then.

Figure 2: Asset Switching Strategy Historical Drawdown

The Value at Risk (VaR) analysis' importance is linked to the fact that it tells us, with a given confidence level, the negative threshold that the strategy's losses have not exceeded, and is also known as a downside risk measure. In addition, the Conditional Value at Risk (or expected shortfall, CVaR) was also computed, representing the average of losses exceeding the Value at Risk. In Figure 3, we can see the distribution of the strategy's returns, as well as the VaR and CVaR for the 90%, 95% and 99% confidence levels. For the long-only strategy, we can see that the Value at Risk for the 95% confidence level is -1.78%, which tells us that in the worst 5% of days, the strategy's losses exceed 1.78%. Likewise, the long-short strategy's VaR(95%) is -2.51%, thus revealing that in the worst 5% of days, the strategy's losses are larger than those of the long-only strategy.

Looking at the expected shortfall, the long-only strategy showcases a CVaR(95%) of -2.71%, indicating that in the worst 5% of cases, the strategy's losses, on average, exceeded 2.71%, historically. Similarly, and according to expectations, the long-short strategy expected shortfall for the 95% confidence level is larger than that of the long-only strategy, averaging losses above 3.95% in the worst 5% of cases.

Figure 4 also shows us that independently of the confidence level, the long-short strategy's returns on the worst days and cases are always worse than those of the long-only strategy, thus indicating the strategy yields a higher risk of losses, as expected.

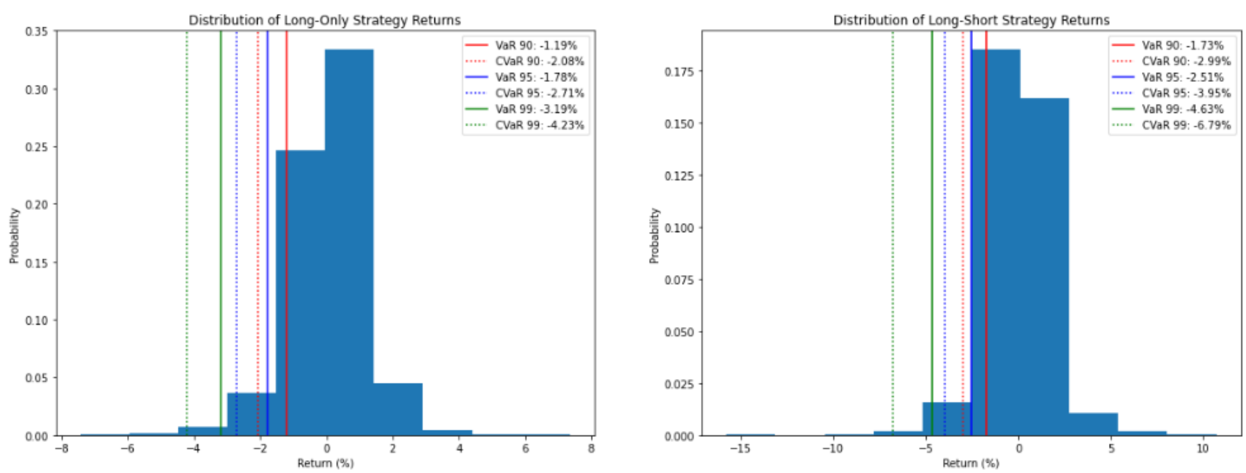

Figure 3: Asset Switching Strategy Distribution of Returns and Value at Risk

Finally, the long-only strategy seems to significantly reduce tail-risk, reporting low kurtosis values over every period analyzed, in comparison to the market. Conversely, the negative skewness values of these periods indicate the tail of the return distribution is longer on the left side, thus most outliers tend to happen on the negative side of the distribution of returns.

5. Implementation Issues

As with any theoretical analysis of an investment strategy, many limitations come in the way when it comes to trying to replicate it in practice. Obviously, one major limitation the explored Asset Switching strategy faces is transaction costs. As explained above, the version of the strategy that included transaction costs was excluded, and they were then on assumed to be zero. In real life markets, this assumption is unfeasible, as most retail investors are faced with these costs. Even most professional investors that face lower transaction costs will be affected by this change. Besides direct transaction costs, indirect trading costs further affect the feasibility of this strategy, as investors trading in real life scenarios are faced with bid-ask spreads, where one must buy the ask price for the long strategy, and sell the bid price for the short part of the strategy. As the strategy was analyzed using adjusted close prices, it doesn't represent this scenario, as investors are faced with different prices than the ones presented. Additionally, the existing bid/ask spreads can vary over time, especially growing during volatile markets, where the strategy seemed most valuable as it significantly reduced losses.

Moreover, although the gold and stock exchange markets are usually highly liquid, one can still find limitations in executing the trade when the signal is activated, which may cause slippage problems. As the strategy is fully dependent on acting upon the signal, which can be activated in consecutive days, slippage problems can have great effects on the strategy's returns.

Finally, as the common saying in Finance goes, "past performance is not indicative of future returns". Thus, one can not expect that the positive returns of the long-only strategy remain positive in the future, as these returns are dependent on the persistence of the perception of gold being a "safe haven" in times of inflation and market downturn, which can never be taken for granted. As macroeconomic conditions continuously change market reactions, as proven by the sync in gold and market prices from 2015 to 2019 (Choudhary, 2022), the negative correlation can never be a given, even if historical evidence and economic sense tell us otherwise.

6. Conclusion

Motivated by Glenn (2014) and Maewal and Scalaton (2011)'s work on market timing rotational strategies, this paper explored its viability when applied to the pair of gold and the market itself, employing a new methodology into an already existing idea.

The findings of this paper help us conclude the strategy's effectiveness largely depends on the time-period of the analysis. Its performance during the Great Financial Crisis of 2007-09 drastically contrasts the promising results during the Covid Crash of 2020, where it reported a Sharpe Ratio of 1.34. Over the full sample, the better version of the strategy, long-only, yielded a Sharpe Ratio of 0.54 and an annualized return of 9.39%. Moreover, the regression analysis conducted concluded the strategy wasn't able to produce statistically significant abnormal returns.

To summarize, the application of a market timing rotational strategy based on the movement of short-term versus long-term trends reinforces Batten, Ciner, and Lucey (2014)'s conclusion of the relationship between the two assets mostly depending on the macroeconomic scenario in place at a given time.

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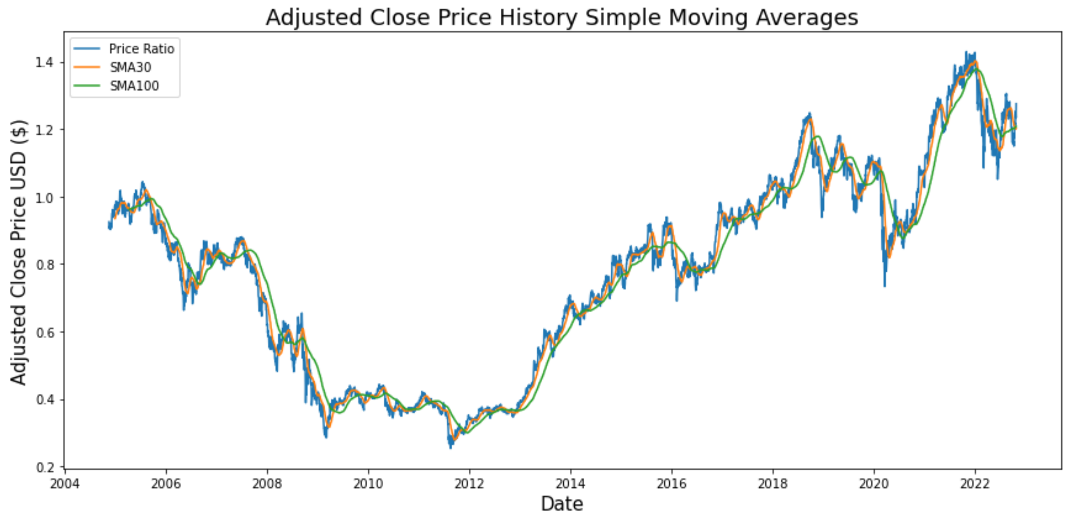
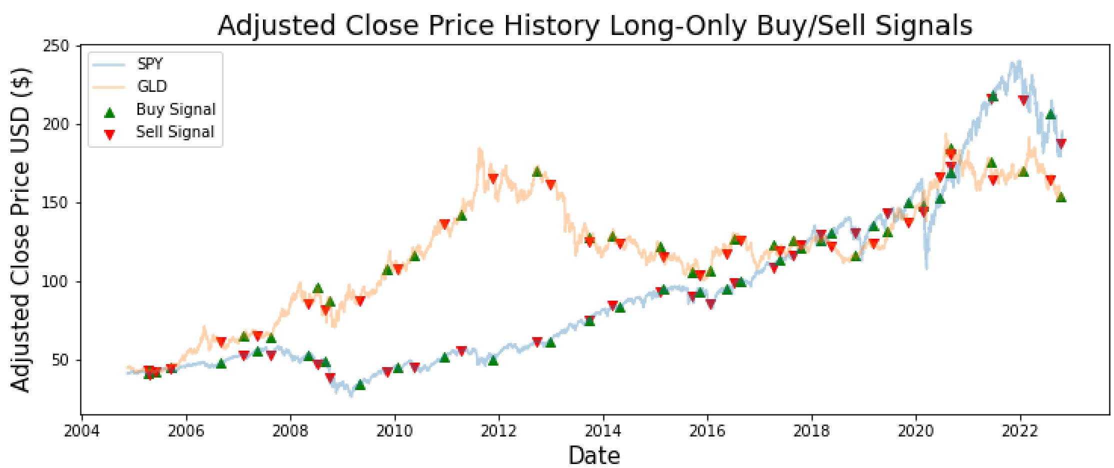
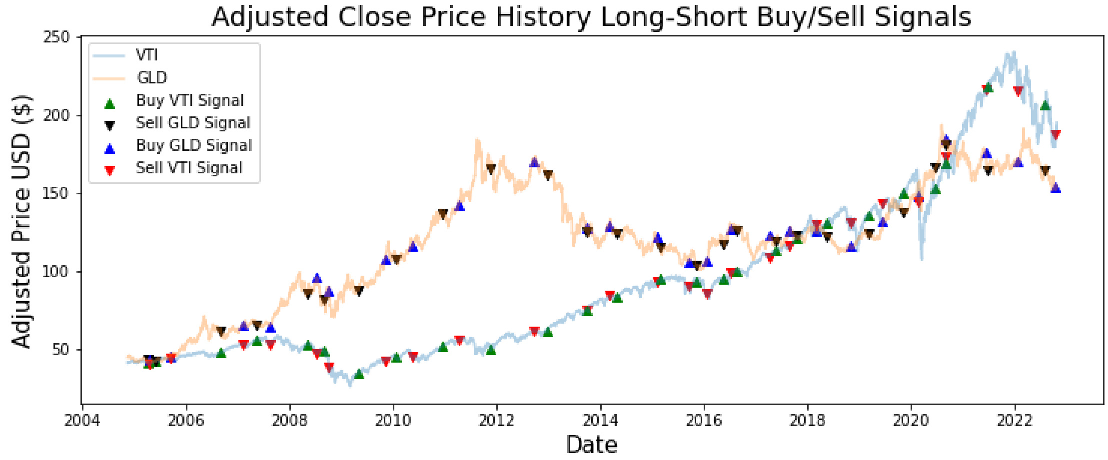
Appendix



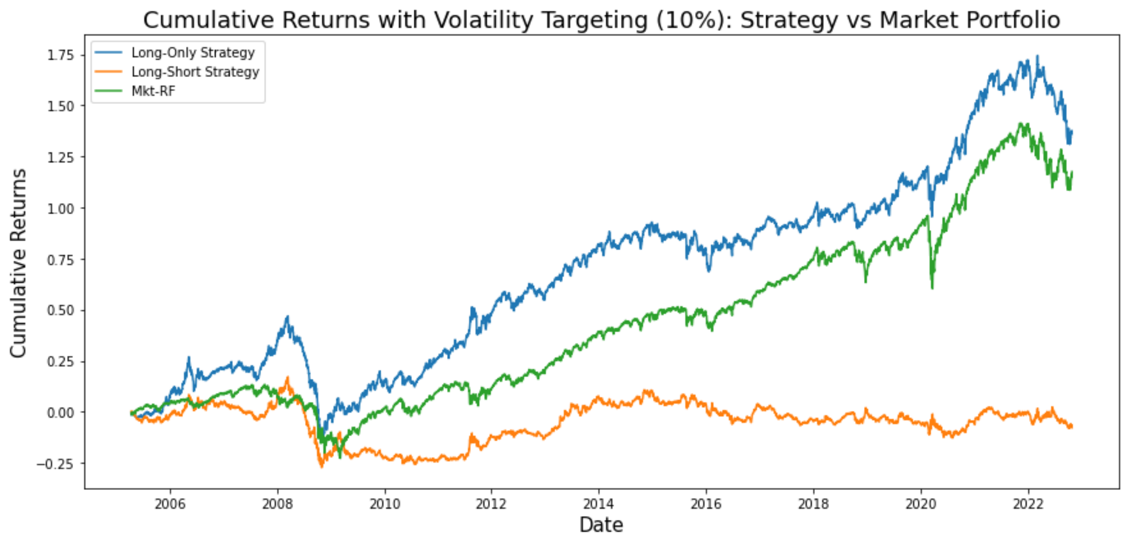
Appendix 1: Price Ratio, SMA30 and SMA100 Price History



Appendix 2: Long-Only Buy and Sell Signals



Appendix 3: Long-Short Buy and Sell Signals



Appendix 4: Cumulative Returns with Target Volatility (10%)

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                            OLS Regression Results
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Dep. Variable:     Long Excess Returns   R-squared:                       0.175
Model:                             OLS   Adj. R-squared:                  0.175
Method:                  Least Squares   F-statistic:                     939.5
Date:                 Thu, 15 Dec 2022   Prob (F-statistic):          3.14e-187
Time:                         18:11:25   Log-Likelihood:                 14113.
No. Observations:                 4418   AIC:                        -2.822e+04
Df Residuals:                     4416   BIC:                        -2.821e+04
Df Model:                            1
Covariance Type:             nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0002      0.000      1.537      0.124   -6.33e-05       0.001
Mkt-RF         0.0036      0.000     30.651      0.000       0.003       0.004
==============================================================================
Omnibus:                      679.201   Durbin-Watson:                   2.064
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             6943.188
Skew:                          -0.401   Prob(JB):                         0.00
Kurtosis:                       9.089   Cond. No.                         1.26
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

Appendix 5: Long-Only CAPM Regression Results

```
                              OLS Regression Results
=====================================================================================
Dep. Variable:     Long-Short Excess Returns   R-squared:                       0.055
Model:                                   OLS   Adj. R-squared:                  0.055
Method:                        Least Squares   F-statistic:                     258.6
Date:                       Thu, 15 Dec 2022   Prob (F-statistic):           1.36e-56
Time:                               18:11:25   Log-Likelihood:                 12016.
No. Observations:                       4418   AIC:                        -2.403e+04
Df Residuals:                           4416   BIC:                        -2.402e+04
Df Model:                                  1
Covariance Type:                   nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0001      0.000      0.516      0.606      -0.000       0.001
Mkt-RF        -0.0031      0.000    -16.081      0.000      -0.003      -0.003
==============================================================================
Omnibus:                     1008.468   Durbin-Watson:                   2.124
Prob(Omnibus):                  0.000   Jarque-Bera (JB):            11922.408
Skew:                          -0.747   Prob(JB):                         0.00
Kurtosis:                      10.908   Cond. No.                         1.26
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

Appendix 6: Long-Short CAPM Regression Results

```
                            OLS Regression Results
==============================================================================
Dep. Variable:     Long Excess Returns   R-squared:                       0.179
Model:                             OLS   Adj. R-squared:                  0.179
Method:                  Least Squares   F-statistic:                     321.1
Date:                 Thu, 15 Dec 2022   Prob (F-statistic):          1.34e-188
Time:                         18:11:27   Log-Likelihood:                 14123.
No. Observations:                 4418   AIC:                        -2.824e+04
Df Residuals:                     4414   BIC:                        -2.821e+04
Df Model:                            3
Covariance Type:             nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0002      0.000      1.501      0.133   -6.85e-05      0.001
Mkt-RF         0.0037      0.000     30.235      0.000       0.003      0.004
SMB         6.425e-05      0.000      0.259      0.796      -0.000      0.001
HML           -0.0008      0.000     -4.459      0.000      -0.001     -0.000
==============================================================================
Omnibus:                      680.818   Durbin-Watson:                   2.072
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             6675.090
Skew:                          -0.421   Prob(JB):                         0.00
Kurtosis:                       8.963   Cond. No.                         2.20
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

Appendix 7: Long-Only FF3 Regression Results

```
                            OLS Regression Results
===================================================================================
Dep. Variable:     Long-Short Excess Returns   R-squared:                     0.056
Model:                                   OLS   Adj. R-squared:                0.056
Method:                        Least Squares   F-statistic:                   87.81
Date:                       Thu, 15 Dec 2022   Prob (F-statistic):         3.48e-55
Time:                               18:11:28   Log-Likelihood:               12019.
No. Observations:                       4418   AIC:                      -2.403e+04
Df Residuals:                           4414   BIC:                      -2.400e+04
Df Model:                                  3
Covariance Type:                   nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0001      0.000      0.497      0.619      -0.000      0.001
Mkt-RF        -0.0030      0.000    -14.917      0.000      -0.003     -0.003
SMB           -0.0001      0.000     -0.314      0.754      -0.001      0.001
HML           -0.0006      0.000     -2.061      0.039      -0.001   -3.06e-05
==============================================================================
Omnibus:                     1028.016   Durbin-Watson:                   2.128
Prob(Omnibus):                  0.000   Jarque-Bera (JB):            12545.365
Skew:                          -0.759   Prob(JB):                         0.00
Kurtosis:                      11.115   Cond. No.                         2.20
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

Appendix 8: Long-Short FF3 Regression Results

```
                            OLS Regression Results
==============================================================================
Dep. Variable:     Long Excess Returns   R-squared:                       0.181
Model:                             OLS   Adj. R-squared:                  0.180
Method:                  Least Squares   F-statistic:                     195.1
Date:                 Thu, 15 Dec 2022   Prob (F-statistic):          2.67e-188
Time:                         18:11:30   Log-Likelihood:                 14128.
No. Observations:                 4418   AIC:                        -2.824e+04
Df Residuals:                     4412   BIC:                        -2.821e+04
Df Model:                            5
Covariance Type:             nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0002      0.000      1.445      0.148   -7.68e-05       0.001
Mkt-RF         0.0038      0.000     29.005      0.000       0.004       0.004
SMB         4.074e-05      0.000      0.161      0.872      -0.000       0.001
HML           -0.0012      0.000     -5.392      0.000      -0.002      -0.001
RMW           -0.0003      0.000     -0.803      0.422      -0.001       0.000
CMA            0.0015      0.000      3.165      0.002       0.001       0.002
==============================================================================
Omnibus:                      680.441   Durbin-Watson:                   2.074
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             6752.971
Skew:                          -0.415   Prob(JB):                         0.00
Kurtosis:                       9.000   Cond. No.                         4.41
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

Appendix 9: Long-Only FF5 Regression Results

```
                              OLS Regression Results
====================================================================================
Dep. Variable:     Long-Short Excess Returns   R-squared:                       0.058
Model:                                   OLS   Adj. R-squared:                  0.057
Method:                        Least Squares   F-statistic:                     54.50
Date:                       Thu, 15 Dec 2022   Prob (F-statistic):           4.28e-55
Time:                               18:11:31   Log-Likelihood:                 12023.
No. Observations:                       4418   AIC:                        -2.403e+04
Df Residuals:                           4412   BIC:                        -2.400e+04
Df Model:                                  5
Covariance Type:                   nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0001      0.000      0.574      0.566      -0.000       0.001
Mkt-RF        -0.0030      0.000    -14.125      0.000      -0.003      -0.003
SMB           -0.0004      0.000     -0.859      0.390      -0.001       0.000
HML           -0.0008      0.000     -2.232      0.026      -0.001   -9.59e-05
RMW           -0.0016      0.001     -2.752      0.006      -0.003      -0.000
CMA            0.0011      0.001      1.357      0.175      -0.000       0.003
==============================================================================
Omnibus:                     1011.122   Durbin-Watson:                   2.127
Prob(Omnibus):                  0.000   Jarque-Bera (JB):            12202.153
Skew:                          -0.743   Prob(JB):                         0.00
Kurtosis:                      11.005   Cond. No.                         4.41
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

Appendix 10: Long-Short FF5 Regression Results

OLS Regression Results

Dep. Variable:	Long Excess Returns	R-squared:	0.160
Model:	OLS	Adj. R-squared:	0.158
Method:	Least Squares	F-statistic:	92.99
Date:	Thu, 15 Dec 2022	Prob (F-statistic):	7.98e-90
Time:	18:11:34	Log-Likelihood:	7553.5
No. Observations:	2447	AIC:	-1.510e+04
Df Residuals:	2441	BIC:	-1.506e+04
Df Model:	5		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	0.0004	0.000	1.638	0.101	-7.23e-05	0.001
Mkt-RF	0.0040	0.000	19.193	0.000	0.004	0.004
SMB	-0.0006	0.000	-1.357	0.175	-0.001	0.000
HML	-0.0011	0.000	-2.685	0.007	-0.002	-0.000
RMW	0.0004	0.001	0.617	0.537	-0.001	0.002
CMA	-0.0017	0.001	-1.950	0.051	-0.003	9.48e-06

Omnibus:	395.678	Durbin-Watson:	2.049
Prob(Omnibus):	0.000	Jarque-Bera (JB):	3165.101
Skew:	-0.525	Prob(JB):	0.00
Kurtosis:	8.472	Cond. No.	5.32

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

Appendix 11: First-Half Long FF5 Regression Results

```
                            OLS Regression Results
==============================================================================
Dep. Variable:      Long Excess Returns   R-squared:                       0.248
Model:                              OLS   Adj. R-squared:                  0.246
Method:                   Least Squares   F-statistic:                     129.8
Date:                  Thu, 15 Dec 2022   Prob (F-statistic):         4.65e-119
Time:                          18:11:35   Log-Likelihood:                 6697.8
No. Observations:                  1971   AIC:                        -1.338e+04
Df Residuals:                      1965   BIC:                        -1.335e+04
Df Model:                             5
Covariance Type:              nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const       3.755e-05      0.000      0.205      0.837      -0.000       0.000
Mkt-RF         0.0039      0.000     23.878      0.000       0.004       0.004
SMB            0.0006      0.000      2.089      0.037     3.9e-05       0.001
HML           -0.0017      0.000     -6.086      0.000      -0.002      -0.001
RMW           -0.0009      0.000     -2.327      0.020      -0.002      -0.000
CMA            0.0038      0.001      7.050      0.000       0.003       0.005
==============================================================================
Omnibus:                      221.490   Durbin-Watson:                   2.120
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             1863.860
Skew:                          -0.139   Prob(JB):                         0.00
Kurtosis:                       7.756   Cond. No.                         3.87
==============================================================================

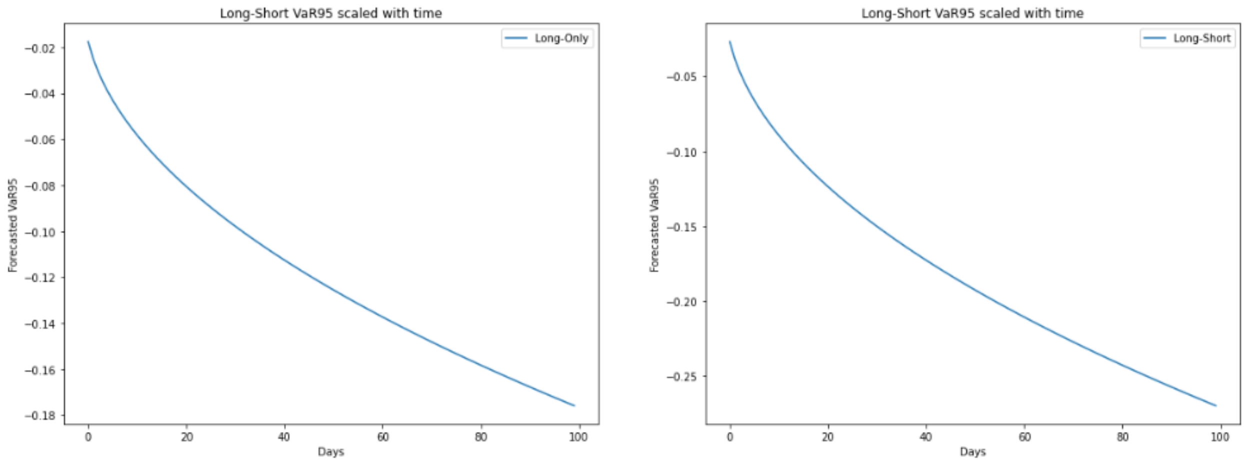
Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

Appendix 12: Second-Half Long FF5 Regression Results

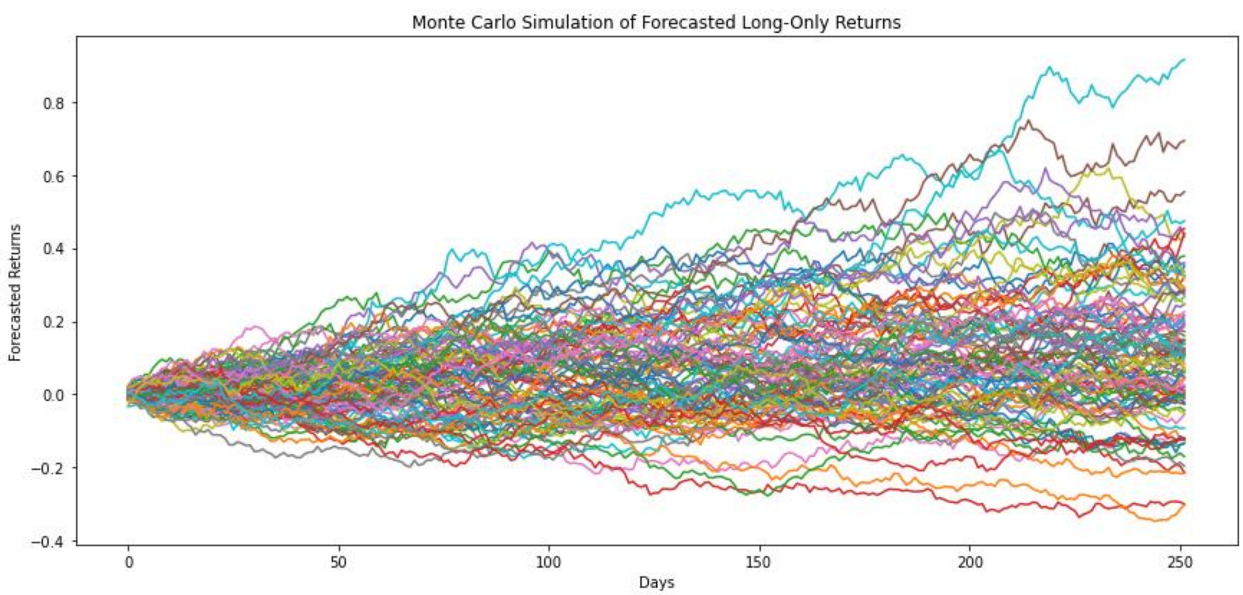
```
                            OLS Regression Results
==============================================================================
Dep. Variable:      Long Excess Returns   R-squared:                       0.254
Model:                              OLS   Adj. R-squared:                  0.239
Method:                   Least Squares   F-statistic:                     16.79
Date:                  Thu, 15 Dec 2022   Prob (F-statistic):         2.83e-14
Time:                          18:12:19   Log-Likelihood:                 789.92
No. Observations:                   253   AIC:                            -1568.
Df Residuals:                       247   BIC:                            -1547.
Df Model:                             5
Covariance Type:              nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0004      0.001      0.545      0.586      -0.001       0.002
Mkt-RF         0.0028      0.000      8.358      0.000       0.002       0.004
SMB            0.0028      0.001      3.248      0.001       0.001       0.004
HML           -0.0037      0.001     -5.192      0.000      -0.005      -0.002
RMW            0.0021      0.001      1.446      0.149      -0.001       0.005
CMA            0.0042      0.002      2.250      0.025       0.001       0.008
==============================================================================
Omnibus:                       22.358   Durbin-Watson:                   2.125
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               65.752
Skew:                          -0.295   Prob(JB):                     5.27e-15
Kurtosis:                       5.427   Cond. No.                         6.57
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

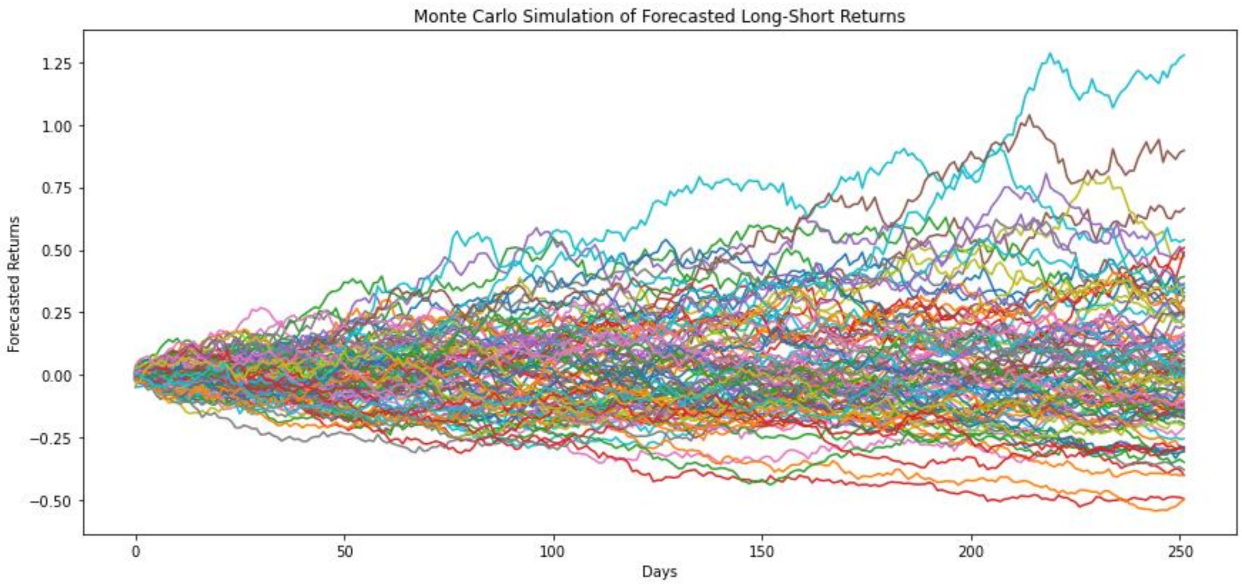
Appendix 13: Covid Crash Crisis Long FF5 Regression Results



Appendix 14: Strategy's Value at Risk Scaled with Time



Appendix 15: Monte-Carlo Simulation of Long-Only Returns



Appendix 16: Monte-Carlo Simulation of Long-Short Returns

Group Part

A Work Project, presented as part of the requirements for the Award of a Master's degree in Finance from the Nova School of Business and Economics.

ANALYSIS OF QUANTITATIVE INVESTMENT STRATEGIES

Omar Hardouf: Revisiting Value and Momentum

Francisco Perestrello: A Market Timing Rotational Strategy based on the Dual Moving Average Crossover – a Study on Gold and the Market

Jorge Gouveia: In a Quest for an Improved Momentum Strategy

Maximilian Kellerbach: Trading on ETF Mispricing - Exploiting Market Inefficiency and Liquidity in Volatile Markets

Work project carried out under the supervision of:

Nicholas Hirschey

16-12-2022

Abstract

This work project describes the strategy and results of four independently developed investment strategies. The strategies focus on value and momentum, ETF mispricing, enhanced momentum, and asset switching. The strategies are carried out in periods between 1998 and 2021. Three of the four strategies focus on the U.S. market whereas one is focused on the European market. Due to fundamental differences in their composition and execution, the strategies yield different risk and return profiles; all but one strategy underperform equity and fixed-income securities benchmark indexes. Subsequent portfolio optimization and allocation methods, with the four individual strategies as assets, improve the risk-adjusted return of a combined portfolio in excess of the benchmark indexes. However, the significance of these improved portfolio results is limited due to inconsistent treatment of transaction costs and the small sample period.

Keywords: Portfolio Management, Quantitative Investment Strategies, Financial Markets, Data Analytics

This work used infrastructure and resources funded by Fundação para a Ciência e a Tecnologia (UID/ECO/00124/2013, UID/ECO/00124/2019 and Social Sciences DataLab, Project 22209), POR Lisboa (LISBOA-01-0145-FEDER-007722 and Social Sciences DataLab, Project 22209) and POR Norte (Social Sciences DataLab, Project 22209).

1. Introduction

Global capital markets match capital providers with those that require capital in return for expected returns. Stock markets, in particular, are a well-known platform that facilitate the matching between investors and those that require capital (Hayes 2021). Naturally, investors aim to maximize their return. However, returns on the capital market are not guaranteed since investors take the positions as equity holders and thus owners of the company. Hence, the value of their investment is directly tied to the equity value of the company which depends on the actual and anticipated performance of the company (Hayes 2021). Nevertheless, despite the unpredictability compared to other predictable investment securities such as fixed-income bonds, stock markets remain very popular for investors. This is because investors are compensated for their faced volatility in the form of the equity premiums which exceed the risk-adjusted expected returns (Mehra and Prescott 1985).

On average, equity markets (in particular the S&P 500) have generated annualized returns of around 7% since the 1950s (Sullivan 2022). However, this is assuming that investors hold a portfolio precisely resembling the market portfolio according to market capitalization within the index. Indeed, market tracking portfolios have become increasingly popular in the last two decades in the form of index tracking (Seyffart 2021). Nevertheless, the allure for investors to try and beat the market persists, in particular in light of success stories from individual companies or portfolios which have generated exceptional historical returns and defied the assumptions of the efficient market hypothesis.

To elaborate, the efficient market hypothesis outlines that stock prices reflect and incorporate all relevant information and that it should not be possible to generate returns from picking and trading individual stocks (Fama 1970, Malkiel 1989). However, in light of the aforementioned violations to the efficient market hypothesis, there is a growing body of literature, even from initial proponents, suggesting that although mostly efficient, there may be irregularities in financial markets and thus in the prices of assets, which are driven by non-rational investor behavior (Malkiel 2003).

The question for willing investors then becomes how they should pick stocks and extending on the notion of how it may be applied as a consistent strategy over an extended time to generate returns greater than the market.

The aim of this work project is to answer that question by describing four different investment strategies. In addition, it will also show how the performance of the individual strategies may be further improved through optimization and allocation methods in combined portfolios.

2. Individual Strategies

The following section will describe 4 individual investment strategies. Each strategy was constructed independently and works as a standalone investment strategy. They will later serve as assets to create optimized portfolio allocations. To allow for comparisons between each other, the results for each strategy are representative of the comparison time frame between 2008-2017, unless otherwise stated.

2.1 ETF Mispricing Strategy (Strategy 1: ETF)

2.1.1 Economic Motivation

In the last two decades, ETFs have become one of the most popular investment vehicles and experienced tremendous inflows (Wursthorn 2021). Attractive through lower costs and comparable, if not superior, returns to their often actively managed mutual funds counterparts, ETFs are especially popular for their simplicity and ease of access, in particular for retail investors. In addition, high liquidity and the ability to diversify portfolios with the purchase of a single asset are additional benefits (Gastineau 2001).

By design, ETFs should (and do) for the most time trade at fair value to the underlying securities they represent (Engle and Sarkar 2006). However, there is extensive literature documenting nontrivial deviations in ETF prices to their underlying assets, in particular during periods of volatility and limited liquidity when the ETF market makers, Authorized Participants (APs), are not able or willing to exploit mispricings (Marshall, Nguyen and Visaltanachoti 2013, Kay 2009, Kaminska 2009). Furthermore, it has also been demonstrated that ETF ownership has contributed to greater systemic equity market risk, attributable to the "greater cross sectional trading commonality" (Sullivan and Xiong 2012). Hence, following this line of reasoning, shocks in the markets are amplified and thus volatility in markets should be increased through the growth in global ETF ownership.

Hence, given the current context of increased market volatility following the preceding decade-long period after the financial crisis of stable upwards-trending markets, there might be new opportunities to exploit the mispricing in ETFs. Particularly as previous research, such as by Kreis and Licht (2018) and Petajisto (2017), has already demonstrated the feasibility and profitability of such strategies.

2.1.2 Strategy

The trading strategy is built upon the framework of Kreis and Licht (2018) and extends their findings beyond their initial 2008-2015 time horizon up to October 2022. By doing so, we can evaluate whether their findings of positive net returns through a trading strategy capitalizing on ETF mispricing in periods of volatility can be replicated. Especially within the context of market volatility induced by global shocks such as the coronavirus pandemic and the recent war in Ukraine from 2020 onwards.

Furthermore, the practical findings from Kreis and Licht (2018) are not only extended to a new time period but are also embedded within the body of the theoretical framework on the effects of ETF ownership on market volatility (Sullivan and Xiong 2012). Hence, through the effectiveness of the strategy, the impact of ETF ownership on equity market risk can be deduced.

The strategy itself is a replication of the original execution by Kreis and Licht (2018). This entails that a long- and short position on the ETFs is taken according to a price-to-fundamental value ratio, which serves as a signal and reflects the potential mispricing of the respective ETF. The signal is calculated through the closing price on day_t and the fundamental value derived through the iNAV of ETF_i at the same time. The iNAV value was chosen because it gives a representation of the value of all the fund's underlying assets minus its liabilities and is relatively accurate since it is calculated in 15-second intervals every day. Hence, the trading signal is as follows:

$$Price/iNAV\ ratio_{i,t} = \frac{Price_{i,t}}{iNAV_{i,t}} \quad (1)$$

Signal values of >1 are indicative of an ETF trading at a premium as the price is higher than the fundamental value. Correspondingly, signal values of <1 suggest that an ETF is trading at a discount. According to this methodology, a long position is taken every day for the one ETF that has the highest discount and a short position for the one ETF with the highest premium. These positions will be held for one day and then the position will be closed as the trades are reversed. As outlined by Kreis and Licht (2018), the strategy should prove profitable if a) the mispricing through the premiums is not persistent and b) the reversion is driven through changes in the price of the ETF.

The sample of 19 sector-specific STOXX Europe 600 ETFs used for the strategy is the same as in the original implementation. However, it is reduced to 17 because of missing data for two ETFs. The sample period is from January 2008 until September 2022.

Crucial for the implementation of the strategy is also the inclusion of trading costs. Direct trading costs are equal to 0.48bp of every transaction. Indirect trading costs are also accounted for by using the Xetra Liquidity Measure (XLM). The XLM value is provided monthly by the Xetra marketplace where the ETFs are traded and gives investors an insight into indirect costs by considering the liquidity of the asset through the bid-ask spread and possible adverse price movement through the bid-ask basket size (Gomber and Schweickert 2002). However, the XLM value is calculated only for a hypothetical round trip investment of 25.000€. Hence, the order volume for each position is limited to 25.000€ in the strategy. Nevertheless, the transaction costs in each transaction are accounted for through the following formula:

$$Transaction\ Costs = 50 * XLM_{i,t} + 0.48bp \quad (2)$$

2.1.3 Performance Overview

Figure 1 indicates that as in the original study, the strategy performs well between 2008 and 2010. From 2011 until 2017 the returns are net negative. However, between 2018 and 2022, the strategy generates positive net returns again with the exception of 2021. Thus, similar to the original study it appears that the strategy works in specific periods and it thus splits into polar periods where it either performs well or poorly. In addition, it also shows that the downturn in net profits is due to transaction costs (explicit and implicit) since the strategy is extremely profitable when excluding them.

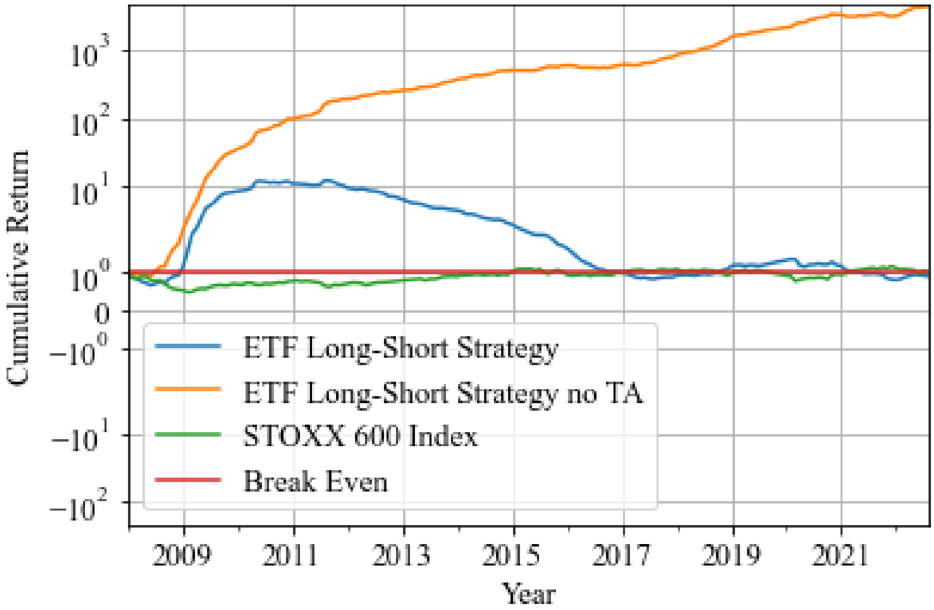


Figure 1: Long-Short Strategy Cumulative Returns

Table 1 confirms these initial observations. The strategy yielded very high net returns in the first half of the sample period after the financial crisis with an average yearly return of 46.35%. However, the standard deviation is also very high at 41.04% which leads to a Sharpe ratio of 1.13. Thus, for every 1.13% return, there is a 1% in volatility (Sharpe 1998). Noteworthy is that 50% of the months in the first half returned positive results and that the best month returned 41.19%. Hence, the results are driven by consistent positive and at times extremely high return months.

	First Half Comparison Sample	Second Half Comparison Sample	Comparison Sample *(2008-2017)*
Total Return	585.57%	-87.29%	-12.86%
Average Yearly Return	46.35%	-40.10%	3.13%
Standard Deviation	41.04%	9.45%	32.19%
Sharpe Ratio	1.13	-4.24	0.10
Excess Kurtosis	-1.04	-3.49	3.67
Skewness	1.57	0.38	2.49
Best Month	41.19%	3.58%	41.19%
Worst Month	-8.75%	-8.04%	-8.75%

	First Half Comparison Sample	Second Half Comparison Sample	Comparison Sample *(2008-2017)*
Positive Months	50%	13.33%	31.67%
Maximum Drawdown	-43.81%	-87.31%	-93.27%

Table 1: ETF Strategy Summary Statistics

Over the second half of the comparison sample, there are much fewer positive return months and they are insufficient to offset the vast majority of negative months. For instance, the best month in the second half only had a return of 3.58%. Interestingly the maximum monthly loss is slightly less drastic than in the high-performing first half of the sample. Nevertheless, the initially strong strategy returns of the sample period are reduced by an overwhelming amount of consistent losses. This is especially evident by the extreme maximum drawdown of -93.27%.

The low average yearly return in conjunction with the high standard deviation and excess kurtosis over the comparison sample period suggest that the strategy loses money consistently but occasionally yields volatile extreme returns during periods that coincide with market volatility.

To test for this hypothesis, a regression between the spread of the premium of the short positions_S to the discount of the long position_L at trading day_t and the subsequent net-returns the following day_{t+1} from the strategy in the full sample (2008-2022) is run:

$$Net\ Return_{t+1} = \beta_0 + \beta_1 * \left(\frac{Price_{S,t}}{iNAV_{S,t}} - \frac{Price_{L,t}}{iNAV_{L,t}}\right) \quad (3)$$

$$= -0.0020^{***} + 0.2516^{***} * \left(\frac{Price_{S,t}}{iNAV_{S,t}} - \frac{Price_{L,t}}{iNAV_{L,t}}\right) \quad (4)$$

The explanatory power of the model (R^2) is 15%. The regression confirms that the strategy is most profitable in periods of volatility. When there is no mispricing, due to the lack of volatility and thus efficient pricing from APs, the strategy generates daily losses of 0.02%.

When mispricing occurs the returns turn positive due to the high sensitivity as indicated by β_1 of 0.2516.

To summarize, the performance analysis of the ETF trading strategy suggests that the strategy is best employed in periods of volatility. Even in favorable periods, it yields very polar but net profitable returns. Hence, it may be used as a limited hedging mechanism within a portfolio against periods of uncertainty and volatility.

2.2 Value and Momentum Strategy (Strategy 2: V&M)

2.2.1 Economic Motivation

Both value and momentum strategies have been shown to be effective in generating returns for investors. For example, a study by Fama and French (1998) found that a portfolio of value stocks outperformed the market by 3.5% per year, while a study by Jegadeesh and Titman (1993) found that a momentum portfolio generated returns of 2.5% per year above the market.

In most instances, keeping momentum steady yields a more successful value approach. This implies that the value strategy performs best when it is not obliged to short the successful momentum approach. Akin to that, maintaining value constant leads to a momentum approach that is typically superior (Asness et al. 1998).

Value and momentum strategies can often complement each other because they are based on different assumptions about how markets work. Value investors believe that markets are inefficient and that securities can be mispriced, while momentum investors believe that markets are efficient and that securities that are rising in price are likely to continue to do so. As a result, value and momentum strategies can be used together to provide a more balanced approach to investing. Recent research points increasingly towards how value and momentum strategies can offer advantageous returns depending on different combination methods. Value works, in general, but largely fails for firms with strong momentum. Momentum works, in general, but is particularly strong for expensive firms (Asness et al. 1998). Therefore, the motivation behind this paper is to build a strategy that manages to counteract this effect.

2.2.2 Strategy

Starting from the stock price, book value and market capitalization data collected from the Bloomberg database of members of the Russell 100 index, the strategy builds value and momentum factors as follows. Considering Asness and Frazzini (2013) which proved that updating prices monthly in the construction of the value factor was proven to provide advantageous returns and reduce the negative correlation between value and momentum, the value measure (here denoted as *bm* for book to market) is built as described below:

$$bm_t = \; log(\, B_t \,/\, P_t \,) \qquad (5)$$

Where B_t refers to the book value per share at time t and P_t to the price at time$_t$ for each stock in the universe. The book to market ratio is further logged to reduce skewness and insure a normal distribution of the ratio. Contrary to Asness and Frazzini (2013), book value is also updated monthly. This was done in the hopes of testing whether an even more current value measure would affect returns and the correlation between value and momentum. Momentum is defined as the return over the previous 12 months, omitting the most recent month.

The strategy is based on the scoring system used by Fisher (2014) where stocks are assigned a score based on the percentage of the market capitalization of stocks with lower or equivalent factor (value or momentum) values which is detailed as follows:

$$Score(x) = (\, \sum Cap_{x\, if\; f(x) \leq f(y)} \;/\; \sum Cap_{row}\,) \; * \; 100, \textit{for each j in row} \qquad (6)$$

Where *row* is the list of available stock in the universe for a given month with *x* the selected stock to score and *j* is every other stock ($\boldsymbol{x} \neq \boldsymbol{y}$). *Cap* refers to the market capitalization of a given security and the function *f(x)* returns the factor of stock *x* (value or momentum factors). All securities in the universe are then scored based on the previous function.

This strategy buys or sells stocks only when both value and momentum scores are favorable as opposed to combining value and momentum strategies in a 50/50 equal combination manner as per Asness, Moskowitz, and Pedersen (2013). Inimically to Fisher, Shah, and Titman (2014) that buy or sell stocks according to fixed buy and sell thresholds, the strategy defines variable buy and sell thresholds as a function of available securities in the universe. This approach helps reduce route dependence when buying or selling stocks. Additionally, since the momentum signal decays far more quickly than the value signal, the strategy emphasizes value more. Since momentum is required to initiate trades but never acts alone to launch a transaction, this way of adding momentum exposure to a portfolio reduces portfolio turnover.

2.2.3 Performance Overview

Additionally to the strategy, a conventional combination portfolio is formed using the same scoring system. Denoted as *70/30*, this strategy is invested at 70% in a value portfolio and 30% in a momentum portfolio. As shown in Figure 2, the primary strategy, named *Value | M* is pitted against the *70/30* strategy and a market benchmark. The strategy manages to outperform both the benchmark and the normal value and momentum combination portfolio over the first half of the sample but stagnates over the second half. Ultimately, the strategy is shown to start decaying starting in 2020. The effects of the 2008 market crash are easily noticeable on both the benchmark and the *70/30* but interestingly enough, the strategy seems to generate its best returns during this period.

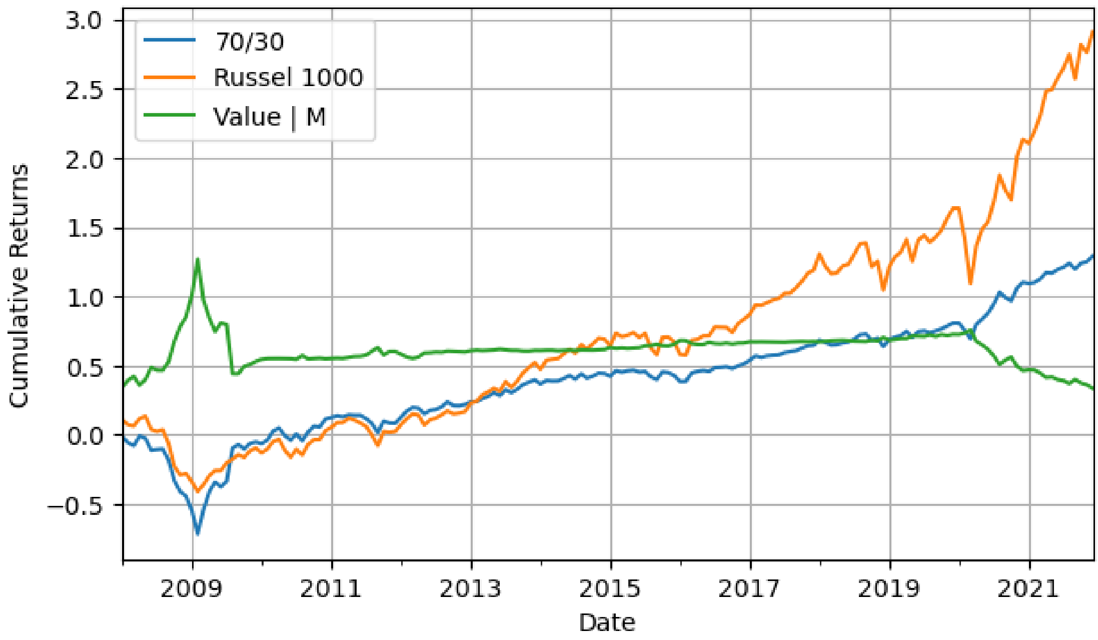


Figure 2: Value and Momentum Strategy Cumulative Returns

The performance of the strategy can be explained by the statistical analysis shown in Table 2. The first-half comparison sample shows a higher total return but a lower Sharpe ratio than the second half. This can be explained by the excessive standard deviation of the strategy during the first half of 22.4%. Even with more than 50% of positive months over both sample halves, the strategy still only manages to produce a Sharpe ratio of 0.18 over the full sample period. This can be explained by the discrepancy between the percentage of positive months and the average yearly return, which suggests that the strategy generates barely enough positive returns for the volatility it incurs. This is consistent with the maximum drawdown of -51.97%.

	First Half Comparison Sample	Second Half Comparison Sample	Comparison Sample *(2008-2017)*
Total Return	10.96%	5.53%	17.09%
Average Yearly Return	4.75%	1.11%	2.93%
Standard Deviation	22.4%	2.67%	15.89%
Sharpe Ratio	0.21	0.42	0.18
Excess Kurtosis	5.31	2.69	14.98

	First Half Comparison Sample	Second Half Comparison Sample	Comparison Sample *(2008-2017)*
Skewness	-1.46	-0.49	-1.90
Best Month	19.42%	2.66%	19.42%
Worst Month	-28.08%	-3.04	-28.08%
Positive Months	61.67%	55.0%	58.33%
Maximum Drawdown	-51.97%	-4.31%	-51.97%

Table 2: Value and Momentum Summary Statistics

Notwithstanding poor returns, the strategy's behavior during market crashes provides an interesting outlook on the importance of value and momentum strategies. More specifically, how a different combination of both value and momentum can have a drastic change on the strategy. Furthermore, the strategy's relatively low correlation with the market is a net advantage. This strategy could prove very beneficial in down times, but more tests are necessary to understand the extent of these results.

2.3 Enhanced Momentum Strategy (Strategy 3: EM)

2.3.1 Economic Motivation

Momentum strategies are one of the most studied anomalies in academia. At its core, it explores the relationship between the worst performers in the market and the best performers. Jegadeesh and Titman (1993) found that in the medium term (between 3 and 12 months), stocks that have performed well in the past keep outperforming stocks that performed poorly. Jegadeesh and Titman (2001) discovered that this anomaly remained significant in the 90s. Finally, Barroso and Santa-Clara (2015) also learned that between 1927 and 2011, momentum recorded a higher Sharpe ratio compared with the three Fama and French risk factors.

Some behavioral models try to explain why the anomaly exists. Barberis et al. (1998) explain that the root of the problem can be the underreaction to news in the short/medium term. Alternatively, Hong et al. (2000) argue that the medium-term momentum effect may be because the information about some firms is less diffused than in other firms, and it is in these firms where momentum is more pronounced. Therefore, momentum strategies that focus on small firms are more profitable. These two explanations seem to complement each other.

Despite all the evidence, there are doubts regarding the profitability of momentum strategies. Hwang and Rubesam (2015) found that momentum premium vanished in the late 1990s, an outcome delayed by the dot-com bubble. Daniel and Moskowitz (2016) also point out that while momentum strategies perform strongly historically, in periods of high market volatility (e.g. following a market crash), these strategies underperform. They explain that because momentum strategies go long in past winners and short the losers, the long leg of the portfolio has a low beta, while the short has a high beta. Therefore, when the market rebounds, the losers have higher expected returns. As a result, momentum is prone to crash in these environments. Likewise, Stivers and Sun (2010) and Wang and Xu (2015) discovered a negative correlation between momentum returns and market volatility. Finally, Baltas and Kosowski (2020) claim that momentum strategies have a signal too simple (either +1 or -1). Thus, one may be over-investing in highly volatile stocks that barely show a price trend. Hence, if one can improve on these flaws, there's room to create a robust strategy that can consistently beat the benchmark.

2.3.2 Strategy

The strategy consists of a dual momentum strategy (it combines momentum and trend-following strategies). It follows a similar methodology to Baltas and Kosowski (2020):

$$r_{t+1} = \sum_{i=1}^{N_{Long}} w_t^i * \frac{\sigma_{tgt}}{\sigma_t^i} * r_{t+1}^i + \sum_{i=1}^{N_{Short}} -w_t^i * \frac{\sigma_{tgt}}{\sigma_t^i} * r_{t+1}^i, \quad (7)$$

Where w_t^i is the individual weight of each stock at period t, σ_{tgt} is the target level of volatility, σ_t^i is the standard deviation forecast for stock i, in period t, and r_{t+1}^i is the monthly return for that stock at the period t+1. The strategy return is the sum of the long and short positions. Unlike the original paper, which uses the Newey and West (1987) t-statistics of the daily log returns from the last 12 months as the individual weights, this strategy uses a risk-parity weighting scheme in each leg of the portfolio. The weights are optimized so that each asset has the same Marginal Contribution to Risk (MCR):

$$MCR_i = w_i * \frac{\delta \sigma_p}{\delta \sigma_i} \tag{8}$$

Where $MCR_1 = MCR_2 = \ldots = MCR_i$, and $w_1 + w_2 + \ldots + w_i = 1$

Finally, the strategy also has a volatility filter based on the VIX index (commonly known as the "fear index"). If the value is higher than 30, the market's future is very uncertain, and one invests in the one-month US Treasury bills and earns the risk-free rate. Otherwise, it uses the weights found above.

This strategy tries to solve the problems described in the previous section. The risk-parity rule creates a continuous signal instead of a binary one. It also minimizes the exposure to the riskiest assets, thus reducing the influence of these stocks. As for market crashes, the volatility target should solve the problem. Hanauer and Windmüller (2022) explain that ”volatility scaling lowers the overall ex-post volatility (named volatility smoothing) and heightens strategy returns due to negative correlation between volatility and returns (named volatility timing)”. Barroso and Santa-Clara (2015) also found that momentum risk declines if one targets volatility. Finally, the volatility filter avoids investing in periods of high volatility and uncertainty.

The data used to test the strategy encompasses the period between January 1997 and March 2022 and all the stocks with a Share Code of 10 or 11, listed at either the NYSE, the AMEX, or the NASDAQ. The bottom quartile, in terms of size, was filtered, and the volatility target has a value of 18%. All the data comes from the CRSP database, made available by WRDS.

For a more detailed description of the methodology, check the individual report.

2.3.3 Performance Overview

As this is a momentum strategy, it makes sense to use the momentum risk factor as the benchmark. Additionally, it's useful to compare its performance with the market risk premium.

In Figure 3, one can see that both the market achieved significantly higher returns than the strategy and the momentum factor. The strategy is relatively stable throughout the period, unlike the momentum factor that crashed after the Great Financial Crisis of 2008 and is yet to recover. Additionally, it’s possible to see that strategy’s performance has been improving lately, and, although it never experiences extensive gains, it rarely suffers sizable losses. This is an improvement to both the market and the momentum factor, which suffered a large crash after the Great Financial Crisis.

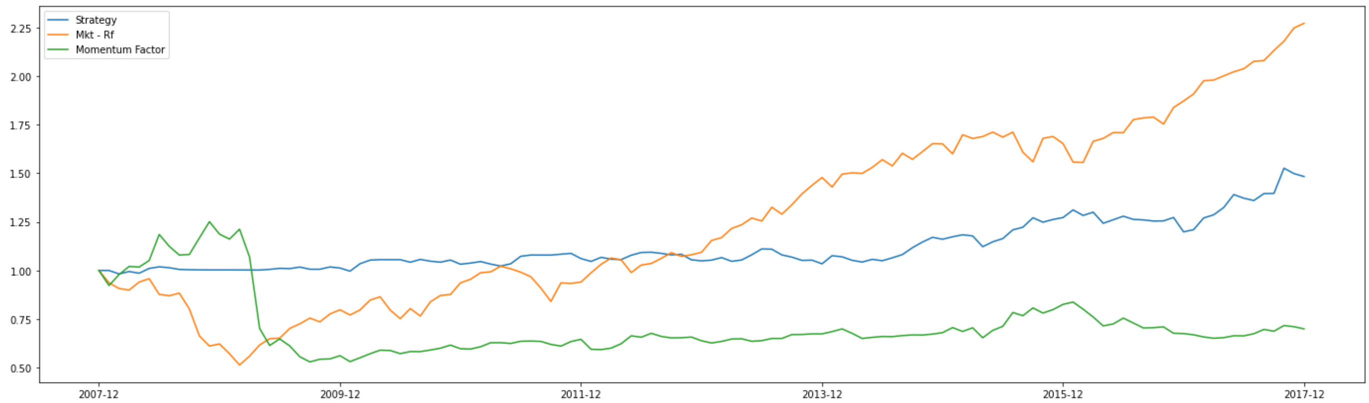


Figure 3: Enhanced Momentum Strategy Cumulative Returns

	First Half Comparison Sample	Second Half Comparison Sample	Comparison Sample *(2008-2017)*
Total Return	6.91%	42.73%	52.60%
Average Yearly Return	1.43%	7.50%	4.47%
Standard Deviation	4.37%	8.66%	6.88%
Sharpe Ratio	0.33	0.87	0.65
Excess Kurtosis	-1.59	-1.00	0.59
Skewness	0.62	0.37	0.66
Best Month	3.86%	9.37%	9.37%
Worst Month	-2.63%	-5.78%	-5.78%
Positive Months	41.67%	60.00%	50.83%
Maximum Drawdown	-4.00%	-8.41%	-8.41%

Table 3: Enhanced Momentum Strategy Summary Statistics

Table 3 summarizes the performance statistics of the strategy for different periods of the sample. Overall, it performed reasonably, reporting a Sharpe ratio of 0.65. The average annual return for the whole period was 4.47%, with a standard deviation of 6.88%. In the best month, it achieved a return of 9.37%, while in the worst, it registered a loss of 5.78%. In terms of risk management, it also did a good job. It significantly reduced the tail risk, and the largest drawdown was 5.78%.

The performance in the first half was disappointing, with a Sharpe ratio of 0.33 and just 1.43% average annual returns, with a standard deviation of 4.73%. In the second half, the strategy drastically improved (returns increased to 7.50% and standard deviation rose to 8.66%), reporting a Sharpe ratio of 0.70 in this period. For a more in-depth analysis of both the Strategy and the two risk factors, look at the individual report.

One last thing that is worth looking at is the results from the factor regression. After regressing the strategy excess returns against the three Fama French factors, and against the momentum factor, the Strategy achieved an Information Ratio of 1.05, and alpha returns of

0.6% per month. By applying a sensitivity analysis, it is also possible to see that the strategy would produce statistically significant returns as long as the average transaction costs for the whole period were lower than 3.63%.

2.4 Market Timing Rotational Strategy (Strategy 4: Asset Switching)

2.4.1 Economic Motivation

The relationship between Gold and the Stock Market has been of interest to the financial world for a long time. Baur and Kuck (2019) explored the role of Gold as a “safe haven” asset in the market, finding evidence supporting this claim for most developed stock markets. Baur and McDermott (2010) also found that Gold returns react fast to extreme negative changes in the Market, suggesting the possibility of using Gold as a way to limit losses during these negative periods.

Historically, Gold returns have shown promising results in periods of downturn in the Stock Market. In the United States, during the 1973-74 bear market (the worst bear market since the Great Depression of 1929), while the Dow Jones Industrial Average (DJIA), a stock market index composed of the 30 most prominent companies listed on the stock exchanges market, fell by 40%, the Gold value increased by more than 50%. In 1987, the story repeated itself, with the US Stock Market falling by 22.5% and Gold seeing an increase of almost 2% (Liston, 2012). This trend seems to persist until recent history, where we see the Indian example (the largest market for gold consumption), with Market Indexes dropping by 35%, and Gold ETFs rising by 69% during the Subprime Mortgage Crisis of 2008-09, and the same behavior for the 2020 Covid-19 Crisis, where the Market fell by 37% and Gold ETFs were up by 49.5%. Finally, the recent Inflationary Crisis saw the Market fall by 12.7% against the rise of 10.6% in Gold ETFs. Conversely, in the 2014 bull market, we saw the Market rising by 23.7% and Gold falling by 12.3% (Choudhary, 2022).

Naturally, this apparent correlation between the returns of the two is of high interest to investors looking to hedge themselves during bear markets (Kuck, 2021).

2.4.2 Strategy

As proposed by Glenn (2014), one could try to take advantage of an existing persistent negative correlation through a simple market timing algorithm that takes Maewal and Scalaton (2011)'s idea of periodically switching positions between the two negatively correlated assets based on their relative performance over a period immediately before the switch.

Unlike the original authors' method of looking at the performance of the two assets and going long on the one with the highest return during the raking period, this strategy proposes the Dual Moving Average Crossover as a market timing signal. This signal serves as a trend-following strategy, in an attempt to identify if the movement of a said asset is being driven more by their short-term behavior, or their long-term behavior. To apply this methodology to our pair of assets, the strategy looks at the Price Ratio of VTI (Vanguard Total Stock Market Index Fund ETF) and GLD (Gold). Using this method, buy and sell signals are triggered whenever the short-term and long-term averages of the Price Ratio cross, and are generated in the following way:

$$Price\ Ratio_t = VTI_t\ /\ GLD_t \tag{9}$$

$$SMA30\ >\ SMA100: Buy\ VTI, Sell\ GLD \tag{10}$$

$$SMA30\ <\ SMA100: Buy\ GLD, Sell\ VTI \tag{11}$$

Where Price Ratio $_t$ represents the ratio between the price of VTI and GLD at time t, SMA30 is the Simple 30-Day Moving Average used as a proxy for the short-time trend, and SMA100 is the Simple 100-Day Moving Average used as a proxy for the long-term trend.

The persistence of the negative correlation between the two assets is crucial for the strategy to be sound, as only under this assumption it makes sense to bet on an increase in the Price Ratio being explained by a rise in VIT price and/or a drop in GLD price, and a decrease being explained by a fall in VIT price and/or an increase in GLD price. As seen before, although both have shown a recent upward long-term trend, empirical evidence and economic sense are largely in favor of persistence of the negative correlation between the two.

This strategy was tested using daily data from Yahoo Finance for the period between November 2004 and October 2022, and Transaction Costs are assumed to be 0. For more discussion on the Transaction Costs, please refer to the individual report “A Market Timing Rotational Strategy based on the Dual Moving Average Crossover – a Study on Gold and the Market”. All returns are presented as excess returns, taking the Fama French Data from the Ken French data library, and subtracting the corresponding risk-free rate from the strategy returns for each trading day. Moreover, the daily returns were converted to monthly returns using compounded returns for each month, so the strategy could be combined with other monthly strategies. Finally, both a long-only and a long-short strategy were tested with the proposed strategy.

2.4.3 Performance Overview

To understand if the strategy does serve as a hedge against market crashes, let us take a look at the cumulative returns of both the long-only strategy and the market itself, measured by the Fama French data. The long-only strategy was chosen for this comparison as it is the one that performs best between the two options. Figure 4 shows these cumulative returns.

In the Figure, it is possible to see that the strategy consistently beats the market in the sample, with its cumulative returns being superior to those of the market ever since it first crossed above it in 2006.

Notably, it can be seen that during the beginning of the Great Financial Crisis (2007-2009), the Asset Switching strategy yielded increasing cumulative returns, against the declining returns of the market. It is only at the end of this crisis that the strategy drops and converges with the market. From 2009 onwards, we see a steady increase in the gap between the strategy's and the market's cumulative returns, until finally from 2018 onwards we see a tightening of this gap and the outperformance becoming minimal.

One final thing to note is the reaction of the strategy to the severe market crashes in 2019 and 2020, where we see the market falling almost twice as much as the Asset Switching strategy.

Figure 4: Asset Switching Strategy Cumulative Returns

Table 4 presents the performance statistics of the Asset Switching strategy for the different halves of the sample, as well as for its full comparison sample.

Overall, it can be seen that the strategy's performance lives and dies by the period that is being analyzed. Over the full comparison sample, it reports a mediocre Sharpe ratio of 0.44, mostly due to the large annualized standard deviation of 18.58% when compared to an annualized return of 8.19%. On its best month, the strategy yielded a return of 12.57%, a polar opposite of its worst month of -30.41% return. This behavior is experienced in every period analyzed, which translates into the large standard deviations present in the table below. It can also be seen that the strategy significantly reduces tail risk, as it reports a kurtosis of around half of that of the market (5.72 against 11.32). Its largest drawdown was -57.23%.

When comparing the two halves of the comparison sample, though, we can see an overperformance of the second half when compared to the first half of the sample. With a standard deviation of less than half of the first half of the sample (11.22% compared to 23.88%) and a similar annualized return of 8.16% and 8.22%, respectively, the second half of the comparison sample produces a Sharpe ratio of more than double the first half (0.73 against 0.34). The second half of the sample also reduces tail risk even further, reporting a kurtosis of -2.15.

Additionally, we see that although the second half's best month has a slighter lower return than the first half, 10.93% and 12.57% respectively, its worst month is a lot smoother, reporting just a -7.65% loss when compared to the -30.41% loss of the first half. Finally, we see that the Maximum Drawdown, that is, how far the strategy has fallen from its best point in history, is also worse in this half of the sample, as it reports a Maximum Drawdown of -57.23%, while the second half of the sample reports just an -18.51% Maximum Drawdown.

	First Half Comparison Sample	Second Half Comparison Sample	Comparison Sample *(2008-2017)*
Total Return	29.45%	45.68%	88.59%
Average Yearly Return	8.22%	8.16%	8.19%
Standard Deviation	23.88%	11.22%	18.58%
Sharpe Ratio	0.34	0.73	0.44
Excess Kurtosis	2.82	-2.15	5.72
Skewness	-1.58	0.14	-1.61
Best Month	12.57%	10.93%	12.57%

	First Half Comparison Sample	Second Half Comparison Sample	Comparison Sample *(2008-2017)*
Worst Month	-30.41%	-7.65%	-30.41%
Positive Months	63.33%	63.33%	63.33%
Maximum Drawdown	-57.23%	-18.51%	-57.23%

Table 4: Asset Switching Strategy Comparison Sample Summary Statistics

3. Combined Strategy

Following the analysis of the individual investment strategies, the goal in this section is to use the strategies in unison, rather than individually, to further optimize the return performance to investors. Specifically, the performances will be evaluated on the basis of the Sharpe ratio. As evident in the analysis of the individual strategies, there are significant differences in return and risk profiles between the various investment schemes. Using the Sharpe ratio as the key performance metric, various combinations and allocations between the individual strategies will be constructed and evaluated in which the returns from each strategy will serve as individual assets. Ultimately, we aim to find a portfolio that performs better than the strategies individually within the sample time horizon, and compare it against an out-of-sample period.

3.1 Strategies Summary

There are fundamental differences in the performance statistics of the individual strategies developed and described in the previous section 2. The following Table 5 highlights these differences. It also includes the SPY and AGG ETFs as references to the performance of the S&P 500 and U.S. Aggregate Bond respectively in the sample period between January 2008 and December 2017.

	Strategy 1	Strategy 2	Strategy 3	Strategy 4	SPY	AGG
Total Return	-12.86%	17.09%	52.60%	88.59%	125.00%	47.03%
Annual Return	3.13%	2.93%	4.47%	8.19%	9.30%	3.94%
Standard Deviation	32.19%	15.89%	6.88%	18.58%	15.18%	3.98%
Sharpe Ratio	0.10	0.18	0.65	0.44	0.61	0.99
Excess Kurtosis	3.67	14.98	0.59	5.72	-1.37	2.63
Skewness	2.49	-1.90	0.66	-1.61	-0.69	1.02
Best Month	41.19%	19.42%	9.37%	12.57%	11.49%	6.28%
Worst Month	-8.75%	-28.08%	-5.78%	-30.41	-16.04%	-2.57%
Positive Month	31.67%	58.33%	50.83%	63.33%	62.5%	62.5%
Maximum Drawdown	-93.27%	-51.97%	-8.41%	-57.23%	-46.32%	-4.41%

Table 5: Individual Strategy and Reference Index Summary Statistics (2008-2017)

When comparing the strategies, there appear to be two distinct groups in which they may be grouped.

As evident in Table 5, although Strategy 4 has almost double the return of Strategy 3, Strategy 3 has a better Sharpe ratio (0.65 versus 0.44 for Strategy 4). This is because Strategy 4 has more than double the standard deviation. Hence, although Strategy 4 appears to perform better when considering total return and annual return, on a risk-adjusted basis (that is considering the Sharpe Ratio) Strategy 3 is superior. Specifically, because for every unit of volatility exposed to, investors receive more return. Nevertheless, given that both strategies generate yearly positive returns with Sharpe Ratios close to or above 0.5, they will be in the group of strategies that contribute to consistent returns throughout the sample period.

Contrastingly, Strategy 1 and 2 both have a Sharpe Ratio of around 0 due to low annual returns. Even though Strategy 1 has a slightly positive average yearly return, the extremely high standard deviation of 31.19% indicates that investors are not compensated for the amount of risk they are exposed to, thus leading to the Sharpe Ratio of just 0.1. Likewise, Strategy 2 has a marginally lower annual return but is coupled with a reduced but still significant level of standard deviation of 14.46%. Consequently, the Sharpe Ratio of strategy 2 is also close to 0. Given that both Strategy 1 and 2 have annual returns close to 0% and low Sharpe ratios, they constitute the second group that contributes with sporadic returns in the portfolio.

Considering the two reference indexes, the SPY as the market index and AGG as an index for fixed income securities, they both have better Sharpe Ratios than all but one of the four constructed strategies. Strategy 3 has a better risk-adjusted return than the SPY due to lower annual returns but a significantly reduced standard deviation. Also noteworthy is the performance of the AGG index. The fixed-income ETF unsurprisingly generates lower returns than the equity-based strategies or an equity index with an annualized return of 3.94%. However, it also has a significantly reduced standard deviation of only 3.98%. Thus, from the perspective of maximizing the Sharpe Ratio, from the six possibilities presented, the AGG has the best risk-adjusted performance to offer to investors over the 2008-2017 period.

However, it needs to be noted that the SPY and the AGG are well diversified indexes unlike the four strategies considered as individual assets here. For that reason, owing to the differences between the strategies, combinations between them within a single portfolio should lead to significantly better returns, ascertained through improvements in the Sharpe Ratio.

This builds on the fundamental work of Markowitz (1952), who set the framework with his Modern Portfolio Theory for investors to achieve similar returns with lower volatility by mixing uncorrelated assets in a diversified portfolio. The implications of Markowitz can not be understated as Dalio (2018) put it: "That simple chart [diversification] struck me with the same force I imagine Einstein must have felt when he discovered $E=mc^2$… I could dramatically reduce my risks without reducing my expected returns… *I called it the "Holy Grail of Investing" because it showed the path to making a fortune.*”.

As a consequence, the framework of Markowitz (1952) will serve as a guiding principle for optimizing the initial four strategies and constructing a better-performing portfolio which will also serve as a benchmark for other allocation methods.

3.2 Correlation

Table 6 shows the correlation matrix of the 4 individual strategies in the period between 2008-2017. The maximum value is only 0.032, and the minimum value is -0.15, which means that neither strategy has a high degree of correlation with the other.

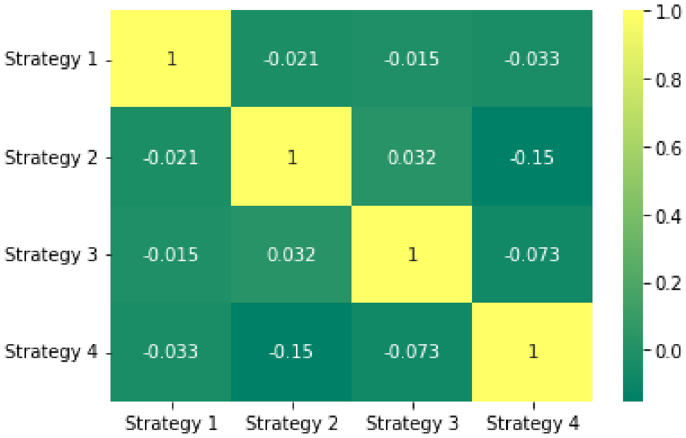


Table 6: Correlation Matrix (2008-2017)

3.3 Portfolio Optimization

Portfolio optimization will follow the methodology created by Markowitz (1952), also known as Modern Portfolio Theory. This model aims to maximize portfolio expected returns for a given level of risk. As Iyiola et al. (2012) explain, *"The fundamental concept behind the MPT is that assets in an investment portfolio should not be selected individually, each on their own merits. Rather, it is important to consider how each asset changes in price relative to how every other asset in the portfolio changes in price".*

The great advantage of using this model is that it incorporates the concept of diversification and its benefits (*"the only free lunch in Finance"*). By combining the four strategies, we can expect returns to increase without taking extra risks because, as one can see in Table 6, the 4 strategies are uncorrelated.

Nevertheless, the model has some flaws that cannot be overlooked. To find the optimal portfolio, expected returns and standard deviations are forecasted. Given the complexity of this task, the most viable and used option is to use historical data. This comes with some risks. By doing it, we assume that the past will replicate in the future, which rarely happens. Similarly, this theory also assumes that the correlation matrix will remain constant in the future. It is a wrong assumption, even more in periods of high market turmoil, where the correlation between assets tends to increase. These issues often end in overfitting, with the solution to the problem only working for the time frame where the analysis was done. To avoid misleading results, we divided the sample into two periods: a training period and a testing period.

The former includes the years from 2008 to 2017 and it is where the optimal weights are calculated. To find them, we first simulated 5000 random weight allocations and found the two portfolios that return the higher Sharpe ratio and the minimum variance. From there, the efficient frontier is generated by fitting all these points to a single line, and the capital market line is created by using the risk-free rate and the market portfolio. The intersection between these two is the tangency portfolio that we are looking for. In **Figure 5** it’s possible to see its graphical representation.

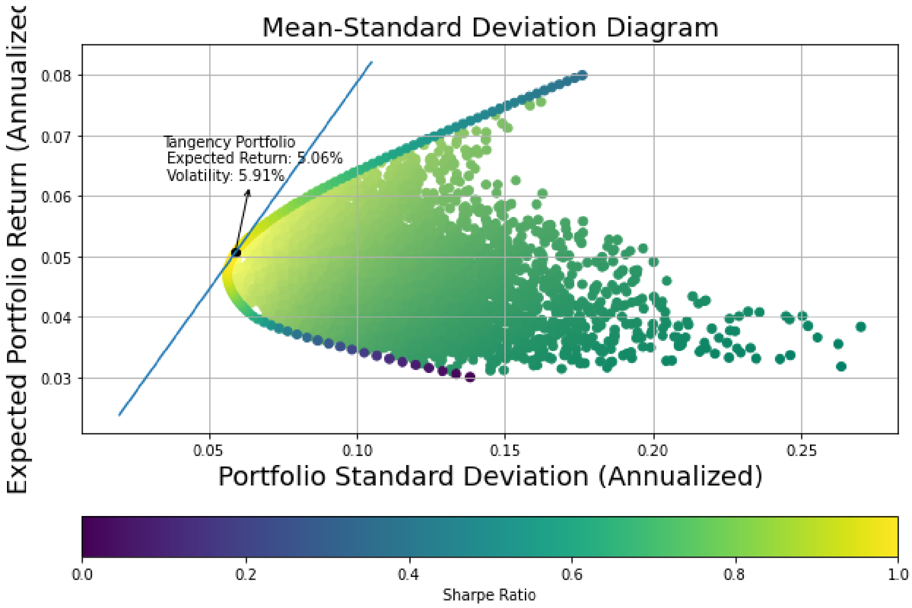


Figure 5: Tangency Portfolio

The resulting portfolio has the following composition:

	Strategy 1	Strategy 2	Strategy 3	Strategy 4
Sharpe Ratio *(2008-2017)*	0.10	0.18	0.65	0.44
Weight	2.54%	9.68%	66.87%	20.91%

Table 7: Tangency Portfolio Weights (2008-2017)

One can see that Strategy 3 and Strategy 4 make up almost 90% of the composition of the Tangency Portfolio. As explained in the previous section, they are the two strategies that can be seen as consistent throughout the sample period while the other two strategies have more sporadic returns. The Sharpe Ratios for the period complement this.

For the remaining sample (2018 to 2021), we will use these weights and find whether our combined strategy can produce robust returns in a non-controlled environment. It will answer the question of whether we overfitted our data. The results for both periods are available in the following *Performance Overview* section.

4. Performance Overview

After combining all the individual strategies in a single portfolio that is optimal for the period between 2008 and 2017, in this section, it will be possible to analyze the summary statistics for that period but also the remaining four years of the sample. Additionally, we will compare our portfolio performance to other relevant portfolio allocations and assets.

First, we will compare it to a 60/40 portfolio. It is one of the most conventional investments available. To create it, one needs to invest 60% in equity and 40% in bonds. The advantage of using it is its simplicity; however, it is a very conservative allocation of capital, and in the long-term it usually underperforms relative to other strategies that have a higher share of equity. To build it, we will use the SPY ETF as equity portion, and the AGG ETF as fixed income portion.

It's also useful to compare the tangency portfolio to other weight distributions, like equal weighting and risk-parity weighting. In the equal-weighted portfolio, each of the strategies has the same weight. As for the latter, each weight is optimized so that each strategy contributes the same risk to the overall portfolio risk (the Marginal Contribution to Risk of every strategy is the same). For the training period, the tangency portfolio should perform better than these two, given that it is the optimal distribution of weights for that period. However, it will be interesting to compare the results out of the sample and check whether our portfolio is still able to overperform them.

Finally, the AGG and SPY ETFs will also serve as benchmarks against the tangency portfolio.

4.1 In-sample

The in-sample period includes the time frame between 2008 and 2017. **Figure 6** shows the cumulative returns of the tangency portfolio and the three other portfolios that serve as benchmark.

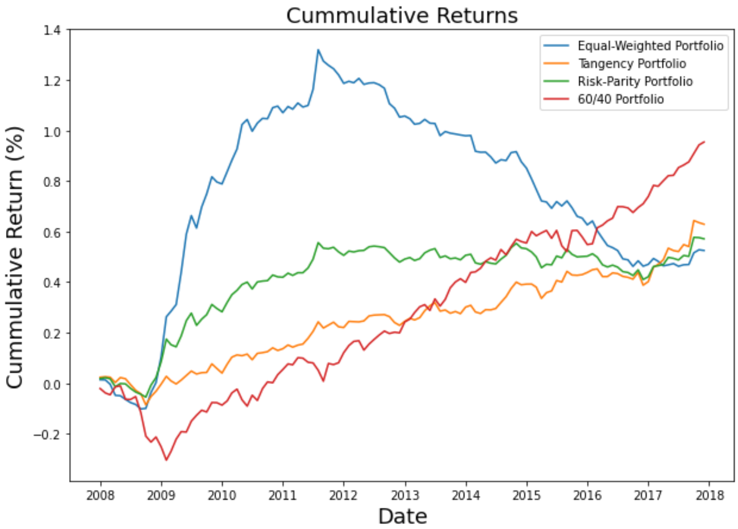


Figure 6: Training Period (2008-2017) Cumulative Returns

One can see that the 60/40 portfolio was the one that achieved the highest returns, with the other three earning similar profits. The tangency portfolio seems to be a stable investment, consistently attaining small gains and avoiding significant losses. The risk-parity and the equal-weighted portfolios have a similar pattern - they both peaked around 2012, and after that, they became obsolete. This is more drastic with the equal-weighted portfolio, which lost more than half of the cumulative returns in the last six years of the period. The reason for this sudden break is found in the performance of Strategy 1.

The strategy also peaked around 2012 and collapsed in the following years. Unlike the tangency portfolio, these two different weighting schemes give higher importance to this strategy. Finally, the 60/40 portfolio crashed during the Great Financial Crisis but enjoyed significant growth in the remaining sample.

Table 8 confirms these comments. The three portfolios that combine the four strategies obtain very similar returns with, however, different levels of risk. The tangency portfolio is the one that reports the best results, with a Sharpe Ratio of 0.86, versus 0.48 for the equal-weights and 0.74 for the risk-parity portfolio. As for the 60/40 portfolio, although it is the one that achieves the highest total return in the period, these returns come at the expense of higher risk. In the end, the portfolio reached a Sharpe ratio of 0.77, which is lower than the tangency portfolio. Finally, compared with the two ETFs, the tangency portfolio positions right in the middle. It performed significantly better than the SPY (Sharpe ratio of 0.61), while performing worse than the AGG (Sharpe ratio of 0.99).

In terms of risk management, our strategy also performed well. The worst loss was only 4.57% and the maximum drawdown was 10.84%. Likewise, tail risk is controlled (excess kurtosis of -1.96), and, as discussed before, it is consistent in obtaining small gains (60% of the months the strategy got positive returns). These results confirm the benefits of diversification. Although neither strategy performed particularly well in the period (highest Sharpe ratio of 0.65 and average of 0.34), the combined strategy still reported a reasonable Sharpe Ratio that is considerably higher than all four strategies.

	Tangency Portfolio	Equal Weights	60/40	Risk Parity	SPY	AGG
Total Return	62.91%	52.53%	95.42%	57.14%	125.00%	47.03%
Average Return	5.06%	4.68%	7.15%	4.73%	9.30%	3.94%
Standard Deviation	5.91%	9.72%	9.32%	6.40%	15.18%	3.98%
Sharpe Ratio	0.86	0.48	0.77	0.74	0.61	0.99
Excess Kurtosis	-1.96	4.15	-0.60	-0.04	-1.37	2.63
Skewness	0.2	2.31	-0.83	1.18	-0.69	1.02
Best Month	6.68%	14.18%	6.95%	8.23%	11.49%	6.28%
Worst Month	-4.57%	-4.42%	-10.54%	-3.80%	-16.04%	-2.57%

	Tangency Portfolio	Equal Weights	60/40	Risk Parity	SPY	AGG
Positive Month	60.00%	44.17%	62.50%	52.5%	62.5%	62.50%
Maximum Drawdown	-10.84%	-36.94%	-29.71%	-9.38%	-46.32%	-4.41%

Table 8: Training Period (2008-2017) Summary Statistics

4.2 Out-of-sample

It's possible to see in Figure 7 that one more time it is the 60/40 portfolio that generates the highest returns. It starts slow again but, in the second half, it grows at an impressive rate, which is not surprising given the strong performance of equity instruments after the Covid-19 market crash. The almost equal trajectory of the equal-weighted and risk-parity portfolios also seems to imply that, in this period, the four strategies had very similar risk profiles which resulted in very similar weights in the risk-parity strategy.

Finally, the tangency portfolio had a strong performance in most of the sample period, except in 2020. This can be explained by the bad performance of Strategy 3 in this period, which is aggravated by the large weight that this strategy has in the overall portfolio. Nevertheless, it still achieved larger profits than the two other weighting schemes and was more stable than the 60/40 portfolio.

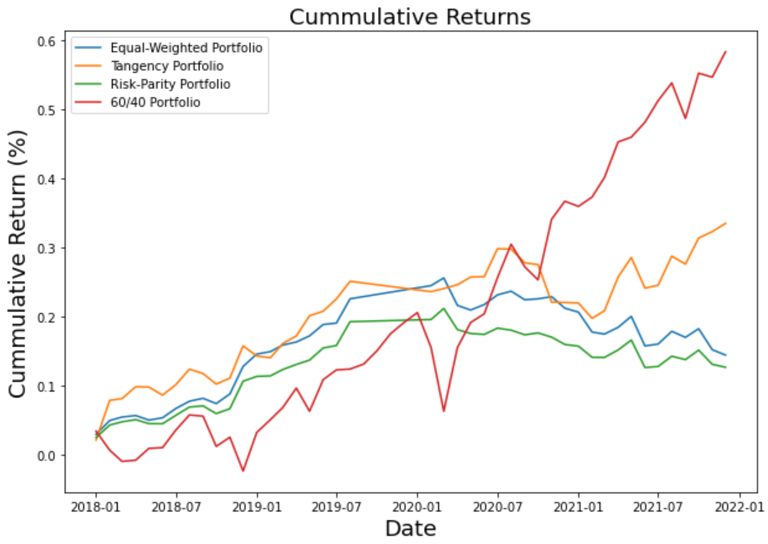


Figure 7: Testing Period (2018-2021) Cumulative Returns

Table 9 gives substance to these comments. First, the performance of the equal-weighted and the risk-parity portfolios are identical (Sharpe ratio of 0.75 vs 0.76), with the former achieving slightly more returns, at the expense of a little more risk. Second, despite the disappointing performance in 2020, the tangency portfolio still achieved the highest Sharpe ratio (1.24) of all the instruments that make the benchmark, even after including the two ETFs. It also did a good job in terms of risk management. The excess kurtosis was -2.25, the largest loss was only -4.26%, and the maximum drawdown was 7.77%. Again, it also achieved positive returns in most of the months (62.79%), which reinforces the idea that this portfolio, although incapable of generating large gains in a single month, slowly accumulates significant returns over time. Finally, one can see that despite reporting almost 4% more annual returns than our strategy, the 60/40 portfolio also takes much more risk (10.64% vs 6.72%) and is much more prone to crashes.

	Tangency Portfolio	Equal Weights	60/40	Risk Parity	SPY	AGG
Total Return	33.45%	14.38%	58.32%	12.61%	91.06%	14.90%
Average Return	8.30%	3.88%	12.1%	3.42%	17.80%	3.54%
Standard Deviation	6.72%	5.15%	10.64%	4.51%	17.49%	3.54%
Sharpe Ratio	1.24	0.75	1.14	0.76	1.02	1.0
Excess Kurtosis	-2.25	-1.96	-1.83	-1.30	-2.02	-3.28
Skewness	0.04	-0.53	-0.40	-0.14	-0.53	0.48
Best Month	5.63%	3.67%	8.70%	3.74%	13.36%	2.78%
Worst Month	-4.26%	-3.57%	-8.01%	-3.42%	-13.0%	-1.68%
Positive Month	62.79%	69.77%	72.92%	60.47%	72.92%	52.08%
Maximum Drawdown	-7.77%	-8.90%	-11.87%	-7.07%	-19.89%	-3.54%

Table 9: Testing Period (2018-2021) Summary Statistics

One interesting takeaway from this analysis is that instead of performing worse in this period, as one would expect, the tangency portfolio actually increased its Sharpe ratio, despite its weights being optimized for another period. By looking at Table 10, it's possible to understand why. Strategies 3 and 4 increased their Sharpe Ratio in this period significantly, compared with the first ten years, and although Strategy 2 suffered a dramatic decrease (0.20 to -0.90), it only accounts for around 10% of the total weight of the portfolio. Likewise, the decrease in the Sharpe ratio for Strategy 1 is negligible due to the low weight in the tangency portfolio of only 2.54%.

	Strategy 1	Strategy 2	Strategy 3	Strategy 4
Sharpe Ratio *(2008-2017)*	0.10	0.18	0.65	0.44
Sharpe Ratio *(2018-2021)*	-0.06	-1.06	0.82	1.17
Tangency Portfolio Weight	2.54%	9.68%	66.87%	20.91%

Table 10: Sharpe Ratio Comparison between the two Periods

5. Regression

Differences in returns may be attributed to different exposure to risk factors. To account for such differences, a time-series regression following the Capital Asset Pricing Model (CAPM) by Black et. al (1972) will be considered. Through the following regression, the exposure of the monthly tangency portfolio returns r_t to market risk $r_{t,Mkt-Rf}$ may be evaluated:

$$CAPM: r_t = \alpha + \beta * r_{t,Mkt-Rf} \quad (12)$$

In addition, a regression using the three Fama-French Factors (FF3) (Fama and French 1993) was also run. The additional risk factors may help in explaining the returns of stocks beyond just the single factor of market risk, as Fama and French (1993) show that value (HML) and size (SMB) of an equity asset significantly contribute to returns. Hence the regression using the FF3 is an extension of the original CAPM and is as follows:

$$FF3: r_{i,t} = \alpha_i + \beta_{i,RMRF} r_{t,RMRF} + \beta_{i,SMB} r_{t,SMB} + \beta_{i,HML} r_{t,HML} + \varepsilon_{i,t} \quad (13)$$

The factor outputs of the two models are shown in Table 11. The CAPM regression suggests that throughout the whole sample, the tangency portfolio yields a monthly excess return of

0.41%, as evident through the α. Similar monthly returns are achieved in the first and second half.

CAPM	Full Sample *(2008-2017)*	First Half Sample	Second Half Sample	Out-of-Sample *(2018-2021)*
α	0.0041**	0.0036	0.0044*	0.0075**
Mkt-Rf	0.0145	0.0077	0.0344	-0.0457
N	120	60	60	43
R^2	0.001	0.001	0.003	0.017
FF3				
α	0.0037**	0.0036	0.0041*	0.0074**
Mkt-Rf	0.0718*	0.0765	0.0540	0.0014
SMB	-0.0719	-0.0944	-0.0476	-0.2931***
HML	-0.2170***	-0.2082***	-0.2378**	0.0169
N	120	60	60	43
R^2	0.128	0.137	0.120	0.192

Notes: *, **, *** significant at 10, 5 and 1 percent levels respectively.

Table 11: CAPM & FF3 Model Regression with Monthly Tangency Portfolio

In addition, as shown by the Mkt-Rf coefficient of 0.0145 for the whole sample, the strategy is only very weakly exposed to market risk and is consistent throughout the first and second half of the sample. However, it needs to be pointed out that this factor coefficient is not statistically significant. Moreover, the R^2, the extent to which the variance in the model is explained by the implemented factors, is also very low at just 0.001. As a consequence, although the excess return explained by the model is statistically significant, the FF3 implementation might provide more meaningful insights.

The regression with the additional FF3 factors confirms the initial observation regarding the monthly excess return achieved by the tangency portfolio for the whole sample period. Again, the α indicates monthly excess returns close to 0.4%. Furthermore, the FF3 model also finds very weak exposure of the combined portfolio to market risk with a coefficient of 0.0718. For SMB, there is a small, statistically insignificant, negative correlation of -0.0719 over the full sample period. This negative risk factor suggests that the portfolio is ever so slightly biased toward bigger companies. Lastly, the model also shows a significant negative exposure to the HML factor. The coefficient of -0.2170 implies that the portfolio favors growth stocks. The coefficients for the first and second half of the sample period are virtually identical to the whole sample. Hence, there appears to be no variation in the risk factor exposure. Also, all the factor coefficients described from the FF3 are statistically significant, with the exception of the SMB factors. Likewise, the R^2 of the model is also higher compared to its CAPM counterpart with a value of 0.128, making it better a predictor of the variance in returns explained by the model.

All in all, The FF3 regression suggests that the tangency portfolio has no significant exposure to market risk. Furthermore, there is also no relevant correlation with size risk factors. However, there is a statistically significant exposure to growth stocks in the model which explains a small portion of the returns.

6. Limitations

There are some limitations to the analysis conducted in this paper. First and foremost, there are discrepancies in the way of incorporating transaction costs into the individual strategies, which all require frequent rebalancing. As a consequence, the returns reflected between the individual strategies might be distorted according to the accuracy of how well implicit and explicit transaction costs are accounted for. Since the returns of the individual strategies contribute to the combined portfolios, the feasibility of carrying out those portfolios and their returns is also not certain. Only the reference indexes SPY and AGG in this paper truly reflect real net returns since they require no rebalancing.

Another limitation of this work project is the short time horizon considered for the strategies and subsequent analysis. Although the individual strategies contain larger sample periods, the comparison period and portfolio construction out of the strategies is limited to overlapping return periods and thus potentially not representative of long-term market conditions. Given that we find optimized portfolios from the returns of the individual strategies in a time period that contains two significant financial crises (GFC, Covid) a longer time horizon would have been preferred to draw more representative conclusions about the performance of the portfolios.

7. Conclusion

Quantitative investment strategies revolve around eliminating human bias to maximize profit. This can be traced to how a part of the returns available in the market stem from investor behavior. Although the results of the group portfolio are not indicative of future returns, the optimized portfolio constructed from each individual strategy offers a diverse strategy that outperforms the market, on average. The tangency portfolio of our combined strategies performs reasonably well in-sample, compared to other portfolios as well as different benchmarks. Surprisingly, the portfolio manages to perform better out of the sample period even if its weights were constructed for a different period. Adequate risk management and low exposure to the market help explain this result with the portfolio maintaining relatively low volatility over the full sample period.

Additionally, the portfolio’s comparatively low drawdown and steady returns help shield potential investors against uneasy market conditions. Specifically, the portfolio’s resilience to market crashes can be explained by the weights attributed to each strategy. These weights allow the portfolio to be adequately invested in each strategy based on how one performed during the training period from 2008 to 2017.

Finally, after a sound analysis of the strategy, we show that the strategy is implementable considering other factors. Namely, combining four separate strategies does complicate streamlining transaction costs and liquidity estimations as each strategy invests in different asset classes with different sizes. Ultimately, we believe our analysis managed to resolve some limitations of each individual strategy and generate a more consistent strategy.

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