Accepted version of the paper Two Level Credibility-limited Revisions published in:
The Review of Symbolic Logic, 15(2), 388-408.
doi:10.1017/S1755020320000283

Two Level Credibility-limited Revisions

MARCO GARAPA
Universidade da Madeira and
CIMA - Centro de Investigação em Matemática e Aplicações

Abstract. In this paper, we propose a new kind of non-prioritized operator which we call *two level credibility-limited revision*. When revising through a two level credibility-limited revision there are two levels of credibility and one of incredibility. When revising by a sentence at the highest level of credibility, the operator behaves as a standard revision, if the sentence is at the second level of credibility, then the outcome of the revision process coincides with a standard contraction by the negation of that sentence. If the sentence is not credible, then the original belief set remains unchanged. In this article, we axiomatically characterize several classes of two level credibility-limited revision operators.
Key words: Belief Change; Non-prioritized Belief Revision; Two Level Credibility-limited Revision Functions; Axiomatic Characterizations.

§1. Introduction The central goal underlying the research area of *logic of theory change* is the study of the changes which can occur in the belief state of a rational agent when he/she receives new information. One of the main contributions to the study of belief change is the so-called AGM model for belief change —named after the initial of its authors: Alchourrón, Gärdenfors and Makinson (Alchourrón et al. (1985)). In that framework, each *belief* of an agent is represented by a sentence and the *belief state* of an agent is represented by a logically closed set of (belief-representing) sentences. These sets are called *belief sets*. A change consists in adding or removing a specific sentence from a belief set to obtain a new belief set.

Although the AGM model has acquired the status of a standard model of belief change, several researchers (for an overview see Fermé & Hansson (2011) and Fermé & Hansson (2018)) have pointed out its inadequateness in several contexts and proposed several extensions and generalizations to that framework. One of the criticisms of the AGM model, that appears in the belief change literature, is related to the use of the *success* postulate to characterize both contractions and revisions. In AGM Belief Revision the new information is always accepted. "This feature appears, in general, to be unrealistic, since rational agents, when confronted with information that contradicts previous beliefs, often reject it altogether or accept only parts of it" Fermé et al. (2003). This can happen for several reasons. For example, the new information may be unreliable or may contradict previous highly entrenched beliefs.

doi:10.1017/S1755020300000000

Models in which the belief change operators considered do not always satisfy the *success* postulate (contrary to what is the case regarding the AGM model) are referred to as *non-prioritized belief change operators*. The output of a non-prioritized revision may not contain the new belief that motivated the revision. On the other hand, the outcome of a non-prioritized contraction may still contain the sentence by which the contraction was made. In Fermé & Hansson (2001), a non-prioritized contraction operator, designated by *shielded contraction*, was proposed. These operators are defined by means of a basic AGM contraction and a set of sentences R satisfying certain properties, called *set of retractable sentences*, and are such that, the outcome of the shielded contraction by a sentence that belongs to the set R coincides with the outcome of the associated basic AGM contraction. On the other hand, the shielded contraction by a sentence that does not belong to the set R leaves the original belief set unchanged. In Fermé & Hansson (2001) several classes of shielded contraction were presented. In Fermé et al. (2003) and in Garapa et al. (2018b) several classes of shielded contractions on belief bases were proposed and axiomatically characterized.

In Hansson et al. (2001) a non-prioritized revision operator designated by *credibility-limited revision* was proposed. These operators have the following behaviour (that relies on the notion of *credibility (of a sentence)*): If a sentence α is credible, then the outcome of the credibility-limited revision coincides with the outcome of the revision by that sentence through the associated basic AGM revision operator, otherwise no change is made to the belief set. Hence, a credibility-limited revision operator is induced by a (standard) revision operator and a set containing the sentences that are considered to be credible —the *set of credible sentences*. In Hansson et al. (2001) axiomatic characterizations are presented for credibility-limited revision operators induced by basic AGM revisions and by several (different) kinds of sets of credible sentences. More precisely it presents some results exposing the relation between the postulates satisfied by the credibility-limited revision operator and the properties satisfied by the underlying set of credible sentences. The model of credibility-limited revision was extended to cover iterated revision in Booth et al. (2012). In Fermé et al. (2003) an operator of credibility-limited base revision induced by a partial meet revision operator and a set of credible sentences satisfying a certain set of properties was presented. In Garapa et al. (2018a) axiomatic characterizations were presented for operators of credibility-limited base revision induced by other kinds of standard base revision functions.

In this paper we propose a non-prioritized operator that we designate by *two level credibility-limited revision*. In this model, contrary to credibility-limited revisions, there are considered two levels of credibility (and one of incredibility). When revising a belief set by a certain belief, first the degree of credibility of that sentence is analysed. If that belief is not credible, then the original belief set remains unchanged. When revising by a belief that is considered to be at the first and highest level of credibility, the two level credibility-limited revision has the behaviour of a standard revision. When revising by a belief that is considered to be at the second level of credibility, that sentence is not incorporated but all the beliefs that are inconsistent with it are removed. The intuition underlying this behaviour is that, the belief is not credible enough to be incorporated in the agent's belief set, but creates in the agent some doubt making him/her remove all the beliefs that are inconsistent with it. In the present paper we define and study several classes of two level credibility-limited revisions and present axiomatic characterizations for those classes of operators.

This paper is organized as follows: In Section §2. we introduce the notations and recall the main background concepts and results that will be needed throughout this article. We recall the definition of credibility-limited revisions and some of the properties proposed to characterize the sets of credible sentences that induce it, as well as some of the postulates proposed to characterize these operators. In Section §3. we present the definition of the two level credibility-limited revision operators and the axiomatic characterizations for several classes of such operators. In section §4. we summarize the main contributions of the paper and briefly discuss their relevance. In the Appendix we provide proofs for all the original results presented.

§2. Background

2.1. Formal preliminaries We will assume a propositional language $\mathcal{L}$ that contains the usual truth functional connectives: $\neg$ (negation), $\wedge$ (conjunction), $\vee$ (disjunction), $\rightarrow$ (implication) and $\leftrightarrow$ (equivalence). We shall make use of a consequence operation Cn that takes sets of sentences to sets of sentences and which satisfies the standard Tarskian properties, namely *inclusion, monotony* and *iteration*. Furthermore we will assume that Cn satisfies *supraclassicality, compactness* and *deduction*. We will sometimes use $Cn(\alpha)$ for $Cn(\{\alpha\})$, $A \vdash \alpha$ for $\alpha \in Cn(A)$, $\vdash \alpha$ for $\alpha \in Cn(\emptyset)$, $A \not\vdash \alpha$ for $\alpha \notin Cn(A)$, $\not\vdash \alpha$ for $\alpha \notin Cn(\emptyset)$. The letters $\alpha, \beta, \ldots$ (except for γ and σ) will be used to denote sentences of $\mathcal{L}$. Uppercase Latin letters such as $A, B, \ldots$ shall denote sets of sentences of $\mathcal{L}$. $\mathbf{K}$ is reserved to represent a set of sentences that is closed under logical consequence (i.e. $\mathbf{K} = Cn(\mathbf{K})$) — such a set is called a *belief set* or *theory*. Given a belief set $\mathbf{K}$ we will denote $Cn(\mathbf{K} \cup \{\alpha\})$ by $\mathbf{K} + \alpha$.

2.2. AGM revisions The operation of revision of a belief set consists of the incorporation of new beliefs in that set. In a revision process, some previous beliefs may be retracted in order to obtain, as output, a consistent belief set. The following six postulates, which were originally presented in Gärdenfors (1988), are commonly known as *basic AGM postulates for revision*:[1]

($\star$1) $\mathbf{K} \star \alpha = Cn(\mathbf{K} \star \alpha)$ (*i.e.* $\mathbf{K} \star \alpha$ is a belief set). (Closure)
($\star$2) $\alpha \in \mathbf{K} \star \alpha$. (Success)
($\star$3) $\mathbf{K} \star \alpha \subseteq \mathbf{K} + \alpha$. (Inclusion)
($\star$4) If $\neg\alpha \notin \mathbf{K}$, then $\mathbf{K} + \alpha \subseteq \mathbf{K} \star \alpha$. (Vacuity)
($\star$5) If α is consistent, then $\mathbf{K} \star \alpha$ is consistent. (Consistency)
($\star$6) If $\vdash \alpha \leftrightarrow \beta$, then $\mathbf{K} \star \alpha = \mathbf{K} \star \beta$. (Extensionality)

The operators that satisfy postulates ($\star$1) to ($\star$6) are known as *basic AGM revisions.*

DEFINITION 2.1. *An operator $\star$ for a belief set $\mathbf{K}$ is a basic AGM revision if and only if it satisfies postulates* ($\star$1) *to* ($\star$6).

[1] The postulates were already presented in Alchourrón et al. (1985) but with slightly different formulations.

Two other postulates were proposed to deal with the revision by disjunctions Gärdenfors (1978, 1982):

(Disjunctive Overlap) $\mathbf{K} \star \alpha \cap \mathbf{K} \star \beta \subseteq \mathbf{K} \star (\alpha \vee \beta)$.
(Disjunctive Inclusion) If $\neg\alpha \notin \mathbf{K} \star (\alpha \vee \beta)$, then $\mathbf{K} \star (\alpha \vee \beta) \subseteq \mathbf{K} \star \alpha$.

Definition 2.2. *An operator $\star$ for a belief set $\mathbf{K}$ is an AGM revision if and only if it satisfies postulates ($\star$1) to ($\star$6), disjunctive inclusion and disjunctive overlap.*

The following postulate, proposed by Alchourrón, Gärdenfors and Makinson in Alchourrón et al. (1985), introduces a factoring condition on the revision by disjunctions.

($\star V$) $\mathbf{K} \star (\alpha \vee \beta) = \mathbf{K} \star \alpha$ or $\mathbf{K} \star (\alpha \vee \beta) = \mathbf{K} \star \beta$ or $\mathbf{K} \star (\alpha \vee \beta) = \mathbf{K} \star \alpha \cap \mathbf{K} \star \beta$. (Disjunctive factoring)

The following proposition illustrates that in the presence of the six basic AGM postulates for revision, *disjunctive overlap* and *disjunctive inclusion* are equivalent to *disjunctive factoring.*

Proposition 2.3. *Gärdenfors (1988) Let $\mathbf{K}$ be a logically closed set and $\star$ be an operation for $\mathbf{K}$ that satisfies the basic AGM postulates for revision. Then $\star$ satisfies disjunctive factoring if and only if it satisfies both disjunctive overlap and disjunctive inclusion.*

2.3. AGM contractions A contraction of a belief set occurs when some beliefs are removed from it (and no new beliefs are added). The following postulates, which were presented in Alchourrón et al. (1985) (following Gärdenfors (1978, 1982)), are commonly known as *basic Gärdenfors postulates for contraction* or *basic AGM postulates for contraction*:

($\div$1) $\mathbf{K} \div \alpha = Cn(\mathbf{K} \div \alpha)$ (*i.e.* $\mathbf{K} \div \alpha$ is a belief set). (Closure)
($\div$2) $\mathbf{K} \div \alpha \subseteq \mathbf{K}$. (Inclusion)
($\div$3) If $\alpha \notin \mathbf{K}$, then $\mathbf{K} \subseteq \mathbf{K} \div \alpha$. (Vacuity)
($\div$4) If $\nvdash \alpha$, then $\alpha \notin \mathbf{K} \div \alpha$. (Success)
($\div$5) $\mathbf{K} \subseteq (\mathbf{K} \div \alpha) + \alpha$. (Recovery)
($\div$6) If $\vdash \alpha \leftrightarrow \beta$, then $\mathbf{K} \div \alpha = \mathbf{K} \div \beta$. (Extensionality)

The operators that satisfy postulates ($\div$1) to ($\div$6) are known as *basic AGM contractions.*

Definition 2.4. *An operator $\div$ for a belief set $\mathbf{K}$ is a basic AGM contraction if and only if it satisfies postulates ($\div$1) to ($\div$6).*

In addition to the six basic AGM postulates for contraction, Alchourrón, Gärdenfors and Makinson presented in Alchourrón et al. (1985) the following postulates for contraction by a conjunction:

($\div$7) $(\mathbf{K} \div \alpha) \cap (\mathbf{K} \div \beta) \subseteq \mathbf{K} \div (\alpha \wedge \beta)$. (Conjunctive overlap)
($\div$8) $\mathbf{K} \div (\alpha \wedge \beta) \subseteq \mathbf{K} \div \alpha$ whenever $\alpha \notin \mathbf{K} \div (\alpha \wedge \beta)$. (Conjunctive inclusion)

These postulates are known as the supplementary AGM postulates for contraction. The operators that satisfy postulates $\div 1$ to $\div 8$ are known as *AGM contractions*.

DEFINITION 2.5. *An operator* $\div$ *for a belief set* $\mathbf{K}$ *is an AGM contraction if and only if it satisfies postulates* $(\div 1)$ *to* $(\div 8)$.

2.4. Credibility-limited revision In this subsection, we address the *credibility-limited revision* operators. These are the non-prioritized belief revision operators that are more closely connected to the *two level credibility-limited revisions* operators that we propose. The credibility-limited revision operators were proposed in Hansson et al. (2001). When revising a belief set by a sentence by means of a credibility-limited revision, we need first to analyse whether that sentence is credible or not. When revising by a credible sentence, the operator works as a basic AGM revision operator, otherwise it leaves the original belief set unchanged. Formally:

DEFINITION 2.6. *Hansson et al. (2001) Let* $\mathbf{K}$ *be a belief set,* $\star$ *be a basic AGM revision operator on* $\mathbf{K}$ *and* C *be a subset of* $\mathcal{L}$ *(the set of credible sentences). Then* $\circledast$ *is a credibility-limited revision operator induced by* $\star$ *and* C *if and only if:*

$$\mathbf{K} \circledast \alpha = \begin{cases} \mathbf{K} \star \alpha & \text{if } \alpha \in C \\ \mathbf{K} & \text{otherwise} \end{cases}$$

This construction can be further specified by adding constraints to the structure of C (the set of credible sentences). In Hansson et al. (2001), the following properties for C were proposed:

Credibility of Logical Equivalence: If $\vdash \alpha \leftrightarrow \beta$, then $\alpha \in C$ if and only if $\beta \in C$.[2]
Single Sentence Closure: If $\alpha \in C$, then $Cn(\alpha) \subseteq C$.
Disjunctive Completeness: If $\alpha \vee \beta \in C$, then either $\alpha \in C$ or $\beta \in C$.
Negation Completeness: $\alpha \in C$ or $\neg\alpha \in C$.
Element Consistency: If $\alpha \in C$, then $\alpha \nvdash \perp$.
Expansive Credibility: If $\mathbf{K} \nvdash \alpha$, then $\neg\alpha \in C$.
Revision Credibility: If $\alpha \in C$, then $\mathbf{K} \circledast \alpha \subseteq C$.

In Garapa et al. (2018a) the following property for C (the set of credible sentences) was proposed.

Credibility lower bounding: If $\mathbf{K}$ is consistent, then $\mathbf{K} \subseteq C$.

Note that it follows from credibility lower bounding that tautologies are credible sentences.

2.4.1. Postulates for credibility-limited revisions When considering credibility-limited revisions (instead of standard revisions) the *success* postulate must be removed, and replaced by appropriate properties, capable of capturing the intuitions underlying credibility-limited revisions. The following postulates were proposed in

[2] In Hansson et al. (2001) this property was designated by *closure under logical equivalence* and was formulated as follows: If $\vdash \alpha \leftrightarrow \beta$, and $\alpha \in C$, then $\beta \in C$.

Hansson et al. (2001).

(Relative Success) $\alpha \in \mathbf{K} \circledast \alpha$ or $\mathbf{K} \circledast \alpha = \mathbf{K}$.
(Strict Improvement) If $\alpha \in \mathbf{K} \circledast \alpha$ and $\vdash \alpha \to \beta$, then $\beta \in \mathbf{K} \circledast \beta$.
(Regularity) If $\beta \in \mathbf{K} \circledast \alpha$, then $\beta \in \mathbf{K} \circledast \beta$.
(Disjunctive Distribution) If $\alpha \vee \beta \in \mathbf{K} \circledast (\alpha \vee \beta)$, then $\alpha \in \mathbf{K} \circledast \alpha$ or $\beta \in \mathbf{K} \circledast \beta$.

The following postulate is related to consistency. It was one of the postulates used in the representation theorems for credibility-limited revisions presented in Hansson et al. (2001).

(Consistency Preservation) Makinson (1997) If $\mathbf{K}$ is consistent, then $\mathbf{K} \circledast \alpha$ is consistent.

§3. Two Level Credibility-limited Revisions The *two level credibility-limited revisions* are operators of non-prioritized revision. When revising a belief set by a sentence α, we first need to analyse the degree of credibility of that sentence. When revising by a sentence that is considered to be at the first and highest level of credibility, the operator works as a standard revision operator. If it is considered to be at the second level of credibility, then that sentence is not incorporated in the revision process but its negation is removed from the original belief set. The intuition underlying this, is that the belief by which the belief set is revised is not credible enough to be incorporated but, creates in the agent sufficient doubt that forces him/her to remove the beliefs that are inconsistent with it. When revising by a non-credible sentence, the operator leaves the original belief set unchanged. The following definition formalizes this concept:

Definition 3.7. *Let $\mathbf{K}$ be a belief set, $\star$ be a basic AGM revision operator on $\mathbf{K}$ and C_H and C_L be subsets of $\mathcal{L}$. Then $\odot$ is a two level credibility-limited revision operator induced by $\star$, C_H and C_L if and only if:*

$$\mathbf{K} \odot \alpha = \begin{cases} \mathbf{K} \star \alpha & \text{if } \alpha \in C_H \\ (\mathbf{K} \star \alpha) \cap \mathbf{K} & \text{if } \alpha \in C_L \\ \mathbf{K} & \text{if } \alpha \notin (C_L \cup C_H) \end{cases}$$

In the previous definition $C_H \cup C_L$ represent the sentences that are considered to have some degree of credibility, C_H and C_L represent respectively the set of sentences that are considered to be at the first (highest) and at the second level of credibility. Note that if $\alpha \in C_L$, then $\mathbf{K} \odot \alpha = (\mathbf{K} \star \alpha) \cap \mathbf{K}$. According to the Harper identity[3] $(\mathbf{K} \star \alpha) \cap \mathbf{K}$ coincides with the contraction of $\mathbf{K}$ by $\neg\alpha$. Being $\star$ a basic AGM revision (or an AGM revision) the operator obtained from it through the Harper identity is a basic AGM contraction (respectively, an AGM contraction) Alchourrón et al. (1985); Gärdenfors (1978, 1982). Thus the second condition of the previous definition states that if α is at the second level of credibility, then the outcome of the two level credibility-limited revision by α coincides with the outcome of a standard contraction by its negation. The revision of the belief set

[3] **Harper identity:**Harper (1976) $\mathbf{K} \div \alpha = (\mathbf{K} \star \neg\alpha) \cap \mathbf{K}$

by a non-credible sentence (*i.e.* by a sentences that is not in $C_H \cup C_L$) leaves the original belief set unchanged.

This construction can be further specified by adding constraints to the structure of the sets C_H, C_L and $C_H \cup C_L$. The properties that we will consider for such sets are the ones presented in Subsection 2.4. for the sets of credible sentences that induce credibility-limit revisions with a small modification on the revision credibility property (that we will present further ahead). Additionally we will consider a condition that relates the set of credible sentences C with the revision function $\star$ that induce a same two credibility-limited operator. This condition will be designated by *condition* $(\mathbf{C} \text{ - } \star)$ and states that if a sentence α is not credible, then any possible outcome of revising the belief set $\mathbf{K}$ through $\star$ by a credible sentence contains $\neg\alpha$. The intuition underlying this property is that if α is not credible then its negation cannot be removed. Thus its negation should still be in the outcome of the revision by any credible sentence.

$$\text{If } \alpha \notin C \text{ and } \beta \in C, \text{ then } \neg\alpha \in \mathbf{K} \star \beta. \qquad (\mathbf{C} \text{ - } \star)$$

We now present a slightly reformulated version of the *revision credibility* property mentioned in Subsection 2.4. for credibility-limited revisions. This property states that the sentences in the outcome of the basic AGM revision operator $\star$, that induces a given two credibility-limited revision, by a credible sentence are also credible.[4]

Revision Credibility: If $\alpha \in C$, then $\mathbf{K} \star \alpha \subseteq C$.

We now present a proposition that relates some of the properties mentioned above for the set of credible sentences.

Proposition 3.8. *Let $\star$ be a basic AGM revision on a consistent belief set $\mathbf{K}$ and C be a set of sentences.*

1. *If C satisfies element consistency and condition $(\mathbf{C} \text{ - } \star)$ holds, then C satisfies single sentence closure.*
2. *If $Cn(\emptyset) \subseteq C$ and condition $(\mathbf{C} \text{ - } \star)$ holds, then C satisfies expansive credibility.*

In the next subsection, we propose a set of postulates to characterize the two level credibility-limited revision operators. At the end of that subsection, we will present explicit definitions for the sets C_H and C_L in terms of a two level credibility-limited revision $\odot$ and investigate the properties that such sets satisfy whenever $\odot$ satisfies some given postulates.

[4] We note that, this version of revision credibility relates two structures (namely $\star$ and C) that are independent of each other, while, on the other hand, the previous version presents a relation between a credibility-limited revision $\circledast$ and its associated set of credible sentences. We note also that, more rigorously *revision credibility* should be named *revision credibility with respect to* $\mathbf{K}$ *and* $\star$ since it relates the set of credible sentences with the outcomes of the revision of $\mathbf{K}$ by means of $\star$. However, we will use the shorter designation for it, since whenever this property is mentioned, it will become clear from the context which is the belief set and the revision operator being considered.

3.1. Two level credibility-limited revision postulates In this subsection we present the postulates that we will use in the representation theorems for the *two level credibility-limited revisions* that we will present further ahead. The first set of postulates were already recalled in Subsection 2.2. and were proposed for revisions.

(Closure) $\mathbf{K} \odot \alpha = Cn(\mathbf{K} \odot \alpha)$ (*i.e.* $\mathbf{K} \odot \alpha$ is a belief set).
(Inclusion) $\mathbf{K} \odot \alpha \subseteq \mathbf{K} + \alpha$.
(Vacuity) If $\neg\alpha \notin \mathbf{K}$, then $\mathbf{K} + \alpha \subseteq \mathbf{K} \odot \alpha$.
(Extensionality) If $\vdash \alpha \leftrightarrow \beta$, then $\mathbf{K} \odot \alpha = \mathbf{K} \odot \beta$.
(Disjunctive Overlap) $\mathbf{K} \odot \alpha \cap \mathbf{K} \odot \beta \subseteq \mathbf{K} \odot (\alpha \vee \beta)$.

The second set of postulates were recalled in Subsection 2.4. and were proposed for credibility-limited revisions:

(Consistency Preservation) If $\mathbf{K}$ is consistent, then $\mathbf{K} \odot \alpha$ is consistent.
(Strict Improvement) If $\alpha \in \mathbf{K} \odot \alpha$ and $\vdash \alpha \rightarrow \beta$, then $\beta \in \mathbf{K} \odot \beta$.
(Regularity) If $\beta \in \mathbf{K} \odot \alpha$, then $\beta \in \mathbf{K} \odot \beta$.
(Disjunctive Distribution) If $\alpha \vee \beta \in \mathbf{K} \odot (\alpha \vee \beta)$, then $\alpha \in \mathbf{K} \odot \alpha$ or $\beta \in \mathbf{K} \odot \beta$.

The next set of postulates are obtained by adapting the *recovery* postulate for contraction and three of the postulates proposed for shielded contractions[5]. These postulates deal with negations.

(N-Recovery) $\mathbf{K} \subseteq \mathbf{K} \odot \alpha + \neg\alpha$.
(N-Relative success) If $\neg\alpha \in \mathbf{K} \odot \alpha$, then $\mathbf{K} \odot \alpha = \mathbf{K}$.
(N-Persistence) If $\neg\beta \in \mathbf{K} \odot \beta$, then $\neg\beta \in \mathbf{K} \odot \alpha$.
(N-Success Propagation) If $\neg\alpha \in \mathbf{K} \odot \alpha$ and $\vdash \beta \rightarrow \alpha$, then $\neg\beta \in \mathbf{K} \odot \beta$.

N-Recovery states that if the result of revising a belief set by a certain sentence is (subsequently) expanded by the negation of that sentence then all the initial sentences are recovered. *N-Relative success* states that when revising by any given sentence either the negation of that sentence is effectively removed, or the belief set is left unchanged. *N-Persistence* intuitively states that if a negation of a sentence is kept when trying to revise a belief set $\mathbf{K}$ by that sentence, then it should also be kept when revising $\mathbf{K}$ by any other sentence. *N-Success propagation* informally states that if the negation of certain sentence is removed when revising a belief set by that sentence, then the same thing happens regarding every logical consequence of that sentence.

The following three postulates are weaker versions of the postulates of *relative success* (for credibility-limited revisions), *vacuity* and *disjunctive inclusion* (for

[5] **Relative success** Rott (1992) $\mathbf{K} \sim \alpha = \mathbf{K}$ or $\mathbf{K} \sim \alpha \nvdash \alpha$.
Persistence Fermé & Hansson (2001) If $\mathbf{K} \sim \beta \vdash \beta$, then $\mathbf{K} \sim \alpha \vdash \beta$.
Success propagation Fermé & Hansson (2001) If $\mathbf{K} \sim \beta \vdash \beta$ and $\vdash \beta \rightarrow \alpha$, then $\mathbf{K} \sim \alpha \vdash \alpha$.

revisions).

(Weak Relative Success) $\alpha \in \mathbf{K} \odot \alpha$ or $\mathbf{K} \odot \alpha \subseteq \mathbf{K}$.
(Weak Vacuity) If $\neg\alpha \notin \mathbf{K}$, then $\mathbf{K} \subseteq \mathbf{K} \odot \alpha$.
(Weak Disjunctive Inclusion) If $\neg\alpha \notin \mathbf{K} \odot (\alpha \vee \beta)$, then $\mathbf{K} \odot (\alpha \vee \beta) + (\alpha \vee \beta) \subseteq \mathbf{K} \odot \alpha + \alpha$.

Weak relative success states that either α is incorporated when revising by it or nothing is added. In both cases, some sentences might be removed from the original belief set. *Weak vacuity* states that if the negation of a sentence is not an element of a given belief set $\mathbf{K}$, then $\mathbf{K}$ is a subset of the revision of $\mathbf{K}$ by that sentence. Thus even if $\neg\alpha \notin \mathbf{K}$ it may happen that $\alpha \notin \mathbf{K} \odot \alpha$. *Weak disjunctive inclusion* ensures that if $\neg\alpha$ does not belong to the revision of a belief set $\mathbf{K}$ by $\alpha \vee \beta$, then everything in the expansion of $\mathbf{K} \odot (\alpha \vee \beta)$ by $\alpha \vee \beta$ must be in the expansion of $\mathbf{K} \odot \alpha$ by α.

We also propose the following postulate:

(Containment) If $\mathbf{K}$ is consistent, then $\mathbf{K} \cap ((\mathbf{K} \odot \alpha) + \alpha) \subseteq \mathbf{K} \odot \alpha$.

Containment informally states that whenever $\mathbf{K}$ is consistent (at least) the sentences added to $\mathbf{K} \odot \alpha$ as a result of an expansion by α are removed when intersecting the outcome of that expansion with $\mathbf{K}$.

The following propositions relate some of the postulates presented above.

Proposition 3.9. *Let* $\mathbf{K}$ *be a consistent and logically closed set and* $\odot$ *be an operator on* $\mathbf{K}$ *that satisfies N-Persistence and closure. Then* $\odot$ *satisfies N-Success propagation.*

Proposition 3.10. *Let* $\mathbf{K}$ *be a consistent and logically closed set and* $\odot$ *be an operator on* $\mathbf{K}$ *that satisfies closure, consistency preservation, weak relative success and N-Recovery. Then* $\odot$ *satisfies N-Relative success.*

The following proposition states that if $\odot$ is an operator, on a consistent belief set $\mathbf{K}$, that satisfies *weak vacuity* and *inclusion*, then it also satisfies the following property that was introduced in Hansson (2017):

(Confirmation) If $\alpha \in \mathbf{K}$, then $\mathbf{K} \odot \alpha = \mathbf{K}$.

Proposition 3.11. *Let* $\mathbf{K}$ *be a consistent and logically closed set and* $\odot$ *be an operator on* $\mathbf{K}$ *that satisfies weak vacuity and inclusion. Then* $\odot$ *also satisfies confirmation.*

In the following proposition explicit definitions of C_H and C_L are given in terms of a two level credibility-limited revision $\odot$. If α is at the highest level of credibility, then it should be an element of the outcome of the revision of the belief set $\mathbf{K}$ by it. Therefore, a natural way to define C_H is by $C_H = \{\alpha : \alpha \in \mathbf{K} \odot \alpha\}$. On the other hand the sentences at the second level of credibility, *i.e.* in C_L, although not being incorporated when revising a belief set by it, should be such that its negation is removed during the revision process. Thus a natural way to define C_L is by

$C_L = \{\alpha : \neg\alpha \notin \mathbf{K} \odot \alpha\} \setminus C_H$. The next proposition illustrates some properties that are satisfied by C_H, C_L and $C_H \cup C_L$ whenever $\odot$ satisfies some of the postulates mentioned in this section.

PROPOSITION 3.12. *Let* $\mathbf{K}$ *be a consistent belief set and* $\odot$ *be an operator on* $\mathbf{K}$. *Let* $C_H = \{\alpha : \alpha \in \mathbf{K} \odot \alpha\}$ *and* $C_L = \{\alpha : \neg\alpha \notin \mathbf{K} \odot \alpha\} \setminus C_H$. *Then:*

1. *If* $\odot$ *satisfies consistency preservation, then it holds that if* $\alpha \in C_H$, *then* $\neg\alpha \notin \mathbf{K} \odot \alpha$.
2. $C_H \cap C_L = \emptyset$.
3. *If* $\odot$ *satisfies N-relative success and weak vacuity, then it holds that if* $\alpha \notin C_H \cup C_L$, *then* $\neg\alpha \in C_H$.[6]
4. *If* $\odot$ *satisfies consistency preservation, then* C_H *satisfy element consistency.*
5. *If* $\odot$ *satisfies closure, then* C_L *satisfies element consistency and* $Cn(\emptyset) \subseteq C_H$.
6. *If* $\odot$ *satisfies consistency preservation and closure, then* $C_H \cup C_L$ *satisfies element consistency.*
7. *If* $\odot$ *satisfies closure and extensionality, then* C_H, C_L *and* $C_H \cup C_L$ *satisfy credibility of logical equivalents.*
8. *If* $\odot$ *satisfies weak vacuity and inclusion, then* C_H *and* $C_H \cup C_L$ *satisfy credibility lower bounding.*
9. *If* $\odot$ *satisfies vacuity, then* C_H *and* $C_H \cup C_L$ *satisfies expansive credibility.*
10. *If* $\odot$ *satisfies N-success propagation, then it holds that if* $\alpha \in C_L$ *and* $\beta \in Cn(\alpha)$, *then* $\beta \in C_H \cup C_L$.
11. *If* $\odot$ *satisfies strict improvement, then* C_H *satisfies single sentence closure.*
12. *If* $\odot$ *satisfies strict improvement and N-success propagation, then* $C_H \cup C_L$ *satisfies single sentence closure.*
13. *If* $\odot$ *satisfies disjunctive distribution, then* C_H *satisfies disjunctive completeness.*
14. *If* $\odot$ *satisfies N-persistence, closure and consistency preservation, then* $C_H \cup C_L$ *satisfies disjunctive completeness.*
15. *If* $\odot$ *satisfies N-persistence, closure and strict improvement, then* C_L *satisfies disjunctive completeness.*

In the next proposition we will construct an operator $\star$ in terms of another operator $\odot$. Being C_H and C_L the sets mentioned above, we will explore some of existing relations between these sets and $\star$ whenever $\odot$ satisfies some of the postulates proposed for two credibility-limited revisions.

PROPOSITION 3.13. *Let* $\mathbf{K}$ *be a consistent belief set and* $\odot$ *be an operator on* $\mathbf{K}$, $C_H = \{\alpha : \alpha \in \mathbf{K} \odot \alpha\}$ *and* $C_L = \{\alpha : \neg\alpha \notin \mathbf{K} \odot \alpha\} \setminus C_H$. *Let* $\star$ *be the operator on* $\mathbf{K}$ *defined (for all* α*) by*

i. If $\neg\alpha \notin \mathbf{K} \odot \alpha$, *then* $\mathbf{K} \star \alpha = \mathbf{K} \odot \alpha + \alpha$;
ii. If $\neg\alpha \in \mathbf{K} \odot \alpha$, *then* $\mathbf{K} \star \alpha = Cn(\alpha)$.

The following statements hold:

1. *If* $\odot$ *satisfies N-persistence and consistency preservation, then condition* ($C_H \cup C_L$ **-** $\star$) *holds.*[7]

[6] Hence $C_H \cup C_L$ satisfies negation completeness.
[7] *I.e.* that the following condition holds:

If $\alpha \notin C_H \cup C_L$ and $\beta \in C_H \cup C_L$, then $\neg\alpha \in \mathbf{K} \star \beta$.

2. *If $\odot$ satisfies regularity and consistent preservation, then C_H satisfies revision credibility.*

3.2. Representation theorems In this subsection we present representation theorems for several classes of two level credibility-limited revisions. More precisely we consider the classes of two level credibility-limited revisions induced by basic AGM and by AGM revision functions and by several alternative types of sets of credible sentences (*i.e.* considering different sets of properties associated to the related sets of credible sentences, C_H and C_L). In the following theorem we present an axiomatic characterization for a two level credibility-limited revision operator induced by a basic AGM revision and sets C_H and C_L satisfying some given properties.

THEOREM 3.14. *Let* $\mathbf{K}$ *be a consistent and logically closed set and* $\odot$ *be an operator on* $\mathbf{K}$. *Then the following conditions are equivalent:*

1. $\odot$ *satisfies weak relative success, closure, inclusion, consistency preservation, weak vacuity, extensionality, N-relative success and containment.*
2. $\odot$ *is a two level credibility-limited revision operator induced by a basic AGM revision operator for* $\mathbf{K}$ *and sets* $C_H, C_L \subseteq \mathcal{L}$ *such that:* $C_H \cap C_L = \emptyset$, C_H *and* C_L *satisfy credibility of logical equivalents and element consistency.*

The next theorem illustrates some one-to-one correspondence between additional two level credibility-limited postulates and additional properties of C_H and $C_H \cup C_L$.

THEOREM 3.15. *Let* $\mathbf{K}$ *be a consistent and logically closed set and* $\odot$ *be an operator on* $\mathbf{K}$. *Then the following pairs of conditions are equivalent:*

1. $\odot$ *satisfies weak relative success, closure, inclusion, consistency preservation, weak vacuity, extensionality, N-relative success, containment and*
   (a) *Vacuity.*
   (b) *Disjunctive distribution.*
   (c) *Strict improvement.*
   (d) *N-success propagation and strict improvement.*
   (e) *N-persistence.*
   (f) *Regularity.*
2. $\odot$ *is a two level credibility-limited revision operator induced by a basic AGM revision operator* $\star$ *for* $\mathbf{K}$ *and sets* $C_H, C_L \subseteq \mathcal{L}$ *such that:* C_H *and* C_L *satisfy credibility of logical equivalents and element consistency,* $C_H \cap C_L = \emptyset$ *and*
   (a) C_H *satisfies expansive credibility.*
   (b) C_H *satisfies credibility lower bounding and disjunctive completeness.*
   (c) C_H *satisfies single sentence closure.*
   (d) C_H *and* $C_H \cup C_L$ *satisfy single sentence closure.*
   (e) *Condition* ($C_H \cup C_L$ - $\star$) *holds.*
   (f) C_H *satisfies revision credibility.*

In the following theorem we present an axiomatic characterization for a two level credibility-limited revision operator induced by an AGM revision (instead of by a basic AGM revision as in the previous theorems) and sets C_H and C_L satisfying some given properties.

THEOREM 3.16. *Let* $\mathbf{K}$ *be a consistent and logically closed set and* $\odot$ *be an operator on* $\mathbf{K}$*. Then the following conditions are equivalent:*

1. $\odot$ *satisfies weak relative success, closure, inclusion, consistency preservation, weak vacuity, extensionality, containment, strict improvement, N-persistence, N-recovery, disjunctive overlap and weak disjunctive inclusion.*
2. $\odot$ *is a two level credibility-limited revision operator induced by an AGM revision operator* $\star$ *for* $\mathbf{K}$ *and sets* $C_H, C_L \subseteq \mathcal{L}$ *such that:* C_L *satisfy credibility of logical equivalents and element consistency,* $C_H \cap C_L = \emptyset$*,* C_H *satisfies element consistency, credibility lower bounding and single sentence closure and condition* $(C_H \cup C_L$ - $\star)$ *holds.*

It follows from Theorems 3.15. and 3.16. that if we also impose, in statement (2) of the Theorem 3.16., that C_H satisfies expansive credibility, then we have to replace *weak vacuity* by *vacuity* in statement (1) of that theorem. Furthermore, if we impose in statement (2) of the Theorem 3.16. that C_H satisfies revision credibility, then we must add *regularity* to the list of postulates presented in statement (1) of that theorem. Having in mind Proposition 3.12. and the constructions used in part (1) to (2) of the proof of Theorem 3.16. it is possible to include other properties of the credible set of sentences in statement (2) without adding any other postulates in the statement (1) (for example, that $C_H \cup C_L$ satisfies credibility lower bounding and that $Cn(\emptyset) \subseteq C_H$).

§4. Conclusion AGM revision operators are always successful, *i.e.* a given belief is always incorporated when revising by it. However, this is not a realistic feature of belief revision, since an agent may be unwilling to incorporate certain potential beliefs into his/her belief set. In this paper we presented a non-prioritized revision operator that we call *two level credibility-limited revision*. The basic idea of two level credibility-limited revision is to define a two-step function. The first step consists of determining which beliefs are credible (and their degree of credibility) and which are not. Then the function should:
- behave as a standard revision when revising by a belief at the highest level of credibility (C_H);
- behave as a standard contraction by the negation of that belief, if that belief is at the second level of credibility (C_L);
- leave the set of beliefs unchanged when the belief to be revised is considered not credible (is not an element of $C_H \cup C_L$).

Two level credibility-limited revisions operators are closely related to credibility-limited revisions. In fact, they can be seen as a generalization of credibility-limited revisions. In the particular case where $C_L = \emptyset$ both types of operators coincide.

In this paper, we proposed several postulates to characterize two level credibility-limited revisions operators and presented axiomatic characterizations for several classes of such operators. We presented some results exposing the relation between the postulates satisfied by two level credibility-limited revision operators and the properties satisfied by the underlying sets of credible sentences.

§5. Acknowledgements The author of this paper was supported by FCT - Fundação para a Ciência e a Tecnologia through projects UID/MAT/04674/2019

(CIMA), PTDC/CCI-COM/30990/2017, and by the bilateral cooperation project FCT/DAAD (Germany) entitled "Advanced belief change operations based on comparisons and conditionals: Towards a general framework".

§6. Appendix: Proofs

Lemma 6.17. *Let* $\mathbf{K}$ *be a belief set. Then:*

1. $(\mathbf{K}+\alpha)\cap(\mathbf{K}+\beta)\subseteq\mathbf{K}+\alpha\vee\beta$.
2. $\mathbf{K}+\alpha\vee\beta\subseteq\mathbf{K}+\alpha$.

Proof.

1. Let $\delta\in(\mathbf{K}+\alpha)\cap(\mathbf{K}+\beta)$. Hence $\delta\in\mathbf{K}+\alpha$ and $\delta\in\mathbf{K}+\beta$. By deduction it follows that $\alpha\to\delta\in\mathbf{K}$ and $\beta\to\delta\in\mathbf{K}$. On the other hand it holds that $\{\alpha\to\delta,\beta\to\delta\}\vdash(\alpha\vee\beta)\to\delta$. Thus $(\alpha\vee\beta)\to\delta\in\mathbf{K}$, from which it follows that $\delta\in\mathbf{K}+\alpha\vee\beta$. Therefore $(\mathbf{K}+\alpha)\cap(\mathbf{K}+\beta)\subseteq\mathbf{K}+\alpha\vee\beta$.
2. Let $\delta\in\mathbf{K}+\alpha\vee\beta$. Hence by deduction $(\alpha\vee\beta)\to\delta\in\mathbf{K}$. It holds that $\{(\alpha\vee\beta)\to\delta\}\vdash\alpha\to\delta$. Thus $\alpha\to\delta\in\mathbf{K}$, from which it follows that $\delta\in\mathbf{K}+\alpha$. Therefore $\mathbf{K}+\alpha\vee\beta\subseteq\mathbf{K}+\alpha$.

□

Proof. Proposition 3.8.

1. Let $\alpha\in C$ and $\beta\in Cn(\alpha)$. Assume, by *reductio ad. absurdum* that $\beta\notin C$. Hence by condition ($\mathbf{C}$ **-** $\star$) it follows that $\neg\beta\in\mathbf{K}\star\alpha$. Thus $\neg\alpha\in\mathbf{K}\star\alpha$. It holds by $\star$ *success* that $\alpha\in\mathbf{K}\star\alpha$. Hence $\mathbf{K}\star\alpha$ is inconsistent. By $\star$ *consistency* it follows that $\alpha\vdash\perp$. Contradiction, since C satisfies element consistency.
2. Let $\beta\in Cn(\emptyset)$ and $\neg\alpha\notin C$. From $Cn(\emptyset)\subseteq C$ it follows that $\beta\in C$. By condition **(**$C_H\cup C_L$ **-** $\star$**)** it follows that $\neg\neg\alpha\in\mathbf{K}\star\beta$. It holds that $\star$ is an AGM revision operator. By $\star$ *closure* it follows that $\alpha\in\mathbf{K}\star\beta$. On the other hand, by hypothesis $\mathbf{K}$ is a consistent belief set. Thus $\beta\in\mathbf{K}$ and $\neg\beta\notin\mathbf{K}$. By $\star$ *vacuity* and *inclusion* it follows that $\mathbf{K}\star\beta=\mathbf{K}+\beta$. Thus $\mathbf{K}\star\beta=\mathbf{K}$, from which it follows that $\alpha\in\mathbf{K}$.

□

Proof. Proposition 3.9.
Let $\neg\alpha\in\mathbf{K}\odot\alpha$ and $\vdash\beta\to\alpha$. Hence $\vdash\neg\alpha\to\neg\beta$. It follows by *N-Persistence* that $\neg\alpha\in\mathbf{K}\odot\beta$. Thus $\mathbf{K}\odot\beta\vdash\neg\beta$. Therefore, by *closure*, it follows that $\neg\beta\in\mathbf{K}\odot\beta$.
□

Proof. Proposition 3.10.
Let $\neg\alpha\in\mathbf{K}\odot\alpha$. By *consistency preservation* it follows that $\alpha\notin\mathbf{K}\odot\alpha$. By *weak relative success* it follows that $\mathbf{K}\odot\alpha\subseteq\mathbf{K}$. Let $\delta\in\mathbf{K}$. By *N-Recovery* it follows that $\delta\in\mathbf{K}\odot\alpha+\neg\alpha$. From $\neg\alpha\in\mathbf{K}\odot\alpha$ it follows, by *closure*, that $\delta\in\mathbf{K}\odot\alpha$. Hence $\mathbf{K}\subseteq\mathbf{K}\odot\alpha$. Therefore $\mathbf{K}\odot\alpha=\mathbf{K}$. □

Proof. Proposition 3.11.
Assume that $\alpha \in \mathbf{K}$. $\mathbf{K}$ is consistent, thus $\neg\alpha \notin \mathbf{K}$. It follows from *weak vacuity* that $\mathbf{K} \subseteq \mathbf{K} \odot \alpha$. On the other hand by inclusion $\mathbf{K} \odot \alpha \subseteq \mathbf{K} + \alpha$. It follows from $\alpha \in \mathbf{K}$ that $\mathbf{K} + \alpha = \mathbf{K}$. Hence $\mathbf{K} \odot \alpha \subseteq \mathbf{K}$. Therefore $\mathbf{K} \odot \alpha = \mathbf{K}$. □

Proof. Proposition 3.12.

1. Let $\alpha \in C_H$. Hence $\alpha \in \mathbf{K} \odot \alpha$. By $\odot$ *consistency preservation* it follows that $\neg\alpha \notin \mathbf{K} \odot \alpha$.
2. Follows trivially by definition of C_H and C_L.
3. Assume that $\alpha \notin C_H \cup C_L$. Hence it holds that $\neg\alpha \in \mathbf{K} \odot \alpha$. Thus, from $\odot$ *N-relative success*, it follows that $\mathbf{K} \odot \alpha = \mathbf{K}$. It holds that $\mathbf{K}$ is consistent, hence $\neg\neg\alpha \notin \mathbf{K}$. It follows, from $\odot$ *weak vacuity* that $\mathbf{K} \subseteq \mathbf{K} \odot \neg\alpha$. Thus $\neg\alpha \in \mathbf{K} \odot \neg\alpha$. Therefore $\neg\alpha \in C_H$.
4. Assume that $\alpha \vdash \bot$. Hence $\mathbf{K} \odot \alpha \vdash \neg\alpha$. By $\odot$ *consistency preservation* it follows that $\alpha \notin \mathbf{K} \odot \alpha$. Thus $\alpha \notin C_H$.
5. Assume that $\alpha \vdash \bot$. Thus, from $\odot$ *closure*, it follows that $\neg\alpha \in \mathbf{K} \odot \alpha$. Therefore, $\alpha \notin C_L$. Hence C_L satisfies element consistency.
   Let $\alpha \in Cn(\emptyset)$. By $\odot$ *closure* it follows that $\alpha \in \mathbf{K} \odot \alpha$. Thus $\alpha \in C_H$. Therefore $Cn(\emptyset) \subseteq C_H$.
6. Assume that $\alpha \vdash \bot$. By (4) and (5) it holds that $\alpha \notin C_H$ and $\alpha \notin C_L$. Thus $\alpha \notin C_H \cup C_L$.
7. Assume that $\vdash \alpha \leftrightarrow \beta$. We will start by showing that C_H satisfies credibility of logical equivalents. Let $\alpha \in C_H$. Hence $\alpha \in \mathbf{K} \odot \alpha$. It follows from $\vdash \alpha \leftrightarrow \beta$ and $\odot$ *closure* that $\beta \in \mathbf{K} \odot \alpha$. On the other hand, it follows from $\odot$ *extensionality* that $\mathbf{K} \odot \alpha = \mathbf{K} \odot \beta$. Thus $\beta \in \mathbf{K} \odot \beta$, from which it follows that $\beta \in C_H$. By symmetry of the case it holds that if $\beta \in C_H$, then $\alpha \in C_H$. Therefore it holds that $\alpha \in C_H$ if and only if $\beta \in C_H$.
   We will now show that C_L satisfies credibility of logical equivalents. Let $\alpha \in C_L$. Then $\neg\alpha \notin \mathbf{K} \odot \alpha$ and $\alpha \notin C_H$. It follows from $\vdash \neg\alpha \leftrightarrow \neg\beta$ that $\neg\beta \notin \mathbf{K} \odot \alpha$. On the other hand it follows from $\odot$ *extensionality* that $\mathbf{K} \odot \alpha = \mathbf{K} \odot \beta$. Thus $\neg\beta \notin \mathbf{K} \odot \beta$. On the other hand, C_H satisfies credibility of logical equivalents thus $\beta \notin C_H$. Therefore $\beta \in C_L$.
   We will now show that $C_H \cup C_L$ satisfies credibility of logical equivalents. Let $\alpha \in C_H \cup C_L$. Thus $\alpha \in C_H$ or $\alpha \in C_L$. Both sets, C_H and C_L, satisfy credibility of logical equivalents. Thus $\beta \in C_H$ or $\beta \in C_L$. In both cases it holds that $\beta \in C_H \cup C_L$.
8. Let $\alpha \in \mathbf{K}$. By Proposition 3.11. it follows that $\alpha \in \mathbf{K} \odot \alpha$. Hence $\alpha \in C_H \subseteq C_H \cup C_L$. Thus C_H and $C_H \cup C_L$ satisfy *credibility lower bounding*.
9. Assume that $\mathbf{K} \nvdash \alpha$. Hence $\mathbf{K} \nvdash \neg\neg\alpha$. It follows from $\odot$ *vacuity* that $\mathbf{K} + \neg\alpha \subseteq \mathbf{K} \odot \neg\alpha$. Thus $\neg\alpha \in \mathbf{K} \odot \neg\alpha$. Therefore $\neg\alpha \in C_H$. Consequently it also holds that $\neg\alpha \in C_H \cup C_L$.
10. Let $\alpha \in C_L$ and $\beta \in Cn(\alpha)$. Then $\neg\alpha \notin \mathbf{K} \odot \alpha$ and $\vdash \alpha \rightarrow \beta$. It follows from $\odot$ *N-success propagation* that $\neg\beta \notin \mathbf{K} \odot \beta$. There are two cases to consider. If $\beta \in C_H$, then $\beta \in C_H \cup C_L$. If $\beta \notin C_H$, then $\beta \in C_L$. Consequently $\beta \in C_H \cup C_L$.
11. Let $\alpha \in C_H$ and $\beta \in Cn(\alpha)$. Hence $\alpha \in \mathbf{K} \odot \alpha$. On the other hand, it follows by $\odot$ *strict improvement* that $\beta \in \mathbf{K} \odot \beta$. Hence $\beta \in C_H$.

12. Let $\alpha \in C_H \cup C_L$ and $\beta \in Cn(\alpha)$. Hence $\alpha \in C_H$ or $\alpha \in C_L$. In the first case it holds by (11) that $\beta \in C_H$. Thus $\beta \in C_H \cup C_L$. In the second case by (10) it holds that $\beta \in C_H \cup C_L$. Therefore, in both cases it holds that $\beta \in C_H \cup C_L$.
13. Assume the $\alpha \notin C_H$ and $\beta \notin C_H$. Hence $\alpha \notin \mathbf{K} \odot \alpha$ and $\beta \notin \mathbf{K} \odot \beta$. It follows from $\odot$ *disjunctive distribution* that $\alpha \vee \beta \notin \mathbf{K} \odot (\alpha \vee \beta)$. Therefore $\alpha \vee \beta \notin C_H$.
14. Assume that $\alpha \notin C_H \cup C_L$ and $\beta \notin C_H \cup C_L$. Hence $\neg\alpha \in \mathbf{K} \odot \alpha$ and $\neg\beta \in \mathbf{K} \odot \beta$. Thus, by $\odot$ *N-persistence*, it holds that $\neg\alpha \in \mathbf{K} \odot (\alpha \vee \beta)$ and $\neg\beta \in \mathbf{K} \odot (\alpha \vee \beta)$. Hence, by $\odot$ *closure*, $\neg(\alpha \vee \beta) \in \mathbf{K} \odot (\alpha \vee \beta)$. Hence $\alpha \vee \beta \notin C_L$. On the other hand by $\odot$ *consistency preservation* it holds that $\alpha \vee \beta \notin \mathbf{K} \odot (\alpha \vee \beta)$. Therefore $\alpha \vee \beta \notin C_H$. Thus $\alpha \vee \beta \notin C_H \cup C_L$.
15. Assume that $\alpha \notin C_L$ and $\beta \notin C_L$. We will consider two cases:
case 1) $\alpha \in C_H$ or $\beta \in C_H$. By (11) it follows that $\alpha \vee \beta \in C_H$. Thus $\alpha \vee \beta \notin C_L$.
case 2) $\alpha \notin C_H$ and $\beta \notin C_H$. Hence $\neg\alpha \in \mathbf{K} \odot \alpha$ and $\neg\beta \in \mathbf{K} \odot \beta$. Thus, by $\odot$ *N-persistence*, it holds that $\neg\alpha \in \mathbf{K} \odot (\alpha \vee \beta)$ and $\neg\beta \in \mathbf{K} \odot (\alpha \vee \beta)$. Hence, by $\odot$ *closure*, $\neg(\alpha \vee \beta) \in \mathbf{K} \odot (\alpha \vee \beta)$. Hence $\alpha \vee \beta \notin C_L$.

□

Proof. Proposition 3.13.

1. Assume that $\alpha \notin C_H \cup C_L$ and $\beta \in C_H \cup C_L$. Hence $\beta \in C_H$ or $\beta \in C_L$. If $\beta \in C_H$, then $\beta \in \mathbf{K} \odot \beta$. By $\odot$ *consistency preservation* it follows that $\neg\beta \notin \mathbf{K} \odot \beta$. Assume now that $\beta \notin C_H$. Then $\beta \in C_L$. By definition of C_L it follows that $\neg\beta \notin \mathbf{K} \odot \beta$. Thus in both cases it holds that $\neg\beta \notin \mathbf{K} \odot \beta$. Hence it holds that $\neg\alpha \in \mathbf{K} \odot \alpha$ and $\neg\beta \notin \mathbf{K} \odot \beta$. By $\odot$ *N-persistence* it follows that $\neg\alpha \in \mathbf{K} \odot \beta$. By definition of $\star$ it holds that $\mathbf{K} \star \beta = \mathbf{K} \odot \beta + \beta$. Hence it holds that $\neg\alpha \in \mathbf{K} \star \beta$.
2. Let $\alpha \in C_H$ and $\beta \in \mathbf{K} \star \alpha$. It holds that $\alpha \in \mathbf{K} \odot \alpha$. Thus, by $\odot$ *consistency preservation*, it follows that $\neg\alpha \notin \mathbf{K} \odot \alpha$. Therefore $\mathbf{K} \star \alpha = \mathbf{K} \odot \alpha + \alpha = \mathbf{K} \odot \alpha$. Hence $\beta \in \mathbf{K} \odot \alpha$. By $\odot$ *regularity* it follows that $\beta \in \mathbf{K} \odot \beta$. Thus $\beta \in C_H$.

□

Proof. Theorem 3.14.
(1) to (2): Let $\star$ be the operation such that:

i. If $\neg\alpha \notin \mathbf{K} \odot \alpha$, then $\mathbf{K} \star \alpha = \mathbf{K} \odot \alpha + \alpha$;
ii. If $\neg\alpha \in \mathbf{K} \odot \alpha$, then $\mathbf{K} \star \alpha = Cn(\alpha)$.

Furthermore let $C_H = \{\alpha : \alpha \in \mathbf{K} \odot \alpha\}$ and $C_L = \{\alpha : \neg\alpha \notin \mathbf{K} \odot \alpha\} \setminus C_H$.
We start this proof by noticing that by $\odot$ *consistency preservation* it follows from $\neg\alpha \in \mathbf{K} \odot \alpha$ that $\alpha \notin \mathbf{K} \odot \alpha$.
We must show that:
(A) $C_H \cap C_L = \emptyset$, C_H and C_L satisfy credibility of logical equivalents and element consistency;
(B) $\star$ is a basic AGM revision operator;
(C) $\odot$ is a two level credibility-limited revision operator induced by $\star$ and the sets C_H and C_L.
(A) Follows trivially by Proposition 3.12..

(B) We must show that $\star$ satisfies the six basic AGM postulates for revision.
It follows trivially by the definition that of $\star$ that it satisfies **success** and **closure**.
Inclusion: It follows by the definition of $\star$ and $\odot$ *inclusion* that $\star$ satisfies *inclusion*.
Vacuity: Assume that $\neg\alpha \notin \mathbf{K}$. We intend to prove that $\mathbf{K} + \alpha \subseteq \mathbf{K} \star \alpha$. Assume by *reductio ad absurdum* that $\neg\alpha \in \mathbf{K} \odot \alpha$. It follows by $\odot$ inclusion that $\mathbf{K} + \alpha \vdash \bot$. Thus $\mathbf{K} \vdash \neg\alpha$. $\mathbf{K}$ is a belief set, thus $\neg\alpha \in \mathbf{K}$. Contradiction. Therefore $\neg\alpha \notin \mathbf{K} \odot \alpha$. Hence it holds that $\mathbf{K} \star \alpha = \mathbf{K} \odot \alpha + \alpha$. It follows by $\odot$ *weak vacuity* that $\mathbf{K} \subseteq \mathbf{K} \odot \alpha$. Thus (by monotony) $\mathbf{K} + \alpha \subseteq \mathbf{K} \odot \alpha + \alpha = \mathbf{K} \star \alpha$.
Consistency: Assume that α is consistent. We intend to prove that $\mathbf{K} \star \alpha$ is consistent.
If $\neg\alpha \notin \mathbf{K} \odot \alpha$, then $\mathbf{K} \star \alpha = \mathbf{K} \odot \alpha + \alpha$. By $\odot$ *closure* it follows that $\mathbf{K} \odot \alpha \nvdash \neg\alpha$. From which it follows that $\mathbf{K} \odot \alpha + \alpha \nvdash \bot$. Hence $\mathbf{K} \star \alpha$ is consistent. If $\neg\alpha \in \mathbf{K} \odot \alpha$, then it follows by the definition of $\star$ that $\mathbf{K} \star \alpha$ is consistent.
Extensionality: Let $\vdash \alpha \leftrightarrow \beta$. By $\odot$ *extensionality* it holds that $\mathbf{K} \odot \alpha = \mathbf{K} \odot \beta$. We will consider two cases:
case 1) $\neg\alpha \notin \mathbf{K} \odot \alpha$. Then $\mathbf{K} \star \alpha = \mathbf{K} \odot \alpha + \alpha$. It holds that $\mathbf{K} \odot \alpha + \alpha = \mathbf{K} \odot \beta + \beta$. On the other hand, it follows by $\odot$ *closure* and *extensionality* that $\neg\beta \notin \mathbf{K} \odot \beta$. Thus $\mathbf{K} \star \beta = \mathbf{K} \odot \beta + \beta$. Therefore $\mathbf{K} \star \alpha = \mathbf{K} \star \beta$.
case 2) $\neg\alpha \in \mathbf{K} \odot \alpha$. Hence it holds, by $\odot$ *closure* and *extensionality* that $\neg\beta \in \mathbf{K} \odot \beta$. Thus $\mathbf{K} \star \alpha = Cn(\alpha) = Cn(\beta) = \mathbf{K} \star \beta$.
(C) We will now prove that $\odot$ is a two level credibility-limited revision operator induced by $\star$ and the sets C_H and C_L. There are three cases to consider:
If $\alpha \in C_H$, then $\alpha \in \mathbf{K} \odot \alpha$. Thus $\mathbf{K} \odot \alpha + \alpha = \mathbf{K} \odot \alpha$. On the other hand, it follows by $\odot$ *consistency preservation* that $\neg\alpha \notin \mathbf{K} \odot \alpha$. Thus, by definition of $\star$, it holds that $\mathbf{K} \odot \alpha = \mathbf{K} \star \alpha$.
Assume now that $\alpha \in C_L$. Hence by $\odot$ *weak relative success* it holds that $\mathbf{K} \odot \alpha \subseteq \mathbf{K}$. On the other hand $\mathbf{K} \star \alpha = \mathbf{K} \odot \alpha + \alpha$. Thus $\mathbf{K} \odot \alpha \subseteq \mathbf{K} \star \alpha$. Therefore $\mathbf{K} \odot \alpha \subseteq \mathbf{K} \cap \mathbf{K} \star \alpha$.
It holds, by the definition of $\star$, that $\mathbf{K} \cap \mathbf{K} \star \alpha = \mathbf{K} \cap (\mathbf{K} \odot \alpha + \alpha)$. On the other hand, from $\odot$ *containment* it holds that $\mathbf{K} \cap (\mathbf{K} \odot \alpha + \alpha) \subseteq \mathbf{K} \odot \alpha$. Thus $\mathbf{K} \cap \mathbf{K} \star \alpha \subseteq \mathbf{K} \odot \alpha$. Therefore $\mathbf{K} \odot \alpha = \mathbf{K} \cap \mathbf{K} \star \alpha$.
Consider now that $\alpha \notin C_H \cup C_L$. Hence $\neg\alpha \in \mathbf{K} \odot \alpha$. It follows from $\odot$ *N-relative success* that $\mathbf{K} \odot \alpha = \mathbf{K}$.

(2) to (1): Let $\mathbf{K}$ be a consistent belief set and $\star$ be a basic AGM revision operator for $\mathbf{K}$. Let $C_H, C_L \subseteq \mathcal{L}$ be such that: $C_H \cap C_L = \emptyset$, C_H and C_L satisfy credibility of logical equivalents and element consistency. Let $\odot$ be the two level credibility-limited revision operator induced by $\star$ and the sets C_H and C_L. Thus

$$\mathbf{K} \odot \alpha = \begin{cases} \mathbf{K} \star \alpha & \text{if } \alpha \in C_H \\ (\mathbf{K} \star \alpha) \cap \mathbf{K} & \text{if } \alpha \in C_L \\ \mathbf{K} & \text{if } \alpha \notin C_L \cup C_L \end{cases}$$

We intend to prove that $\odot$ satisfies *weak relative success, closure, inclusion, consistency preservation, weak vacuity, extensionality, N-relative success* and *containment*. We start by noticing that it follows from $C_H \cap C_L = \emptyset$ that if $\alpha \notin C_H$, then either $\alpha \in C_L$ or $\alpha \notin C_H \cup C_L$.
Weak relative success: If $\alpha \in C_H$, then by $\star$ *success* it follows that $\alpha \in \mathbf{K} \star \alpha = \mathbf{K} \odot \alpha$. If $\alpha \notin C_H$, then either $\alpha \in C_L$ or $\alpha \notin C_H \cup C_L$. In both cases it follows by

definition of $\odot$ that $\mathbf{K} \odot \alpha \subseteq \mathbf{K}$.
Closure: Logical closure is preserved under intersection. $\mathbf{K}$ is a belief set. Thus it follows trivially from the definition of $\odot$ and $\star$ *closure* that $\odot$ satisfies *closure*.
Inclusion: It follows trivially from the definition of $\odot$ and $\star$ *inclusion* that $\odot$ satisfies *inclusion*.
Consistency preservation: Assume that $\alpha \in C_H$. C_H satisfies element consistency. Thus it holds that $\alpha \not\vdash \bot$. From $\star$ *consistency* it follows that $\mathbf{K}\star\alpha$ is consistent. Thus by definition of $\odot$ it follows that $\mathbf{K} \odot \alpha$ is also consistent. Assume now that $\alpha \notin C_H$. It holds that either $\alpha \in C_L$ or $\alpha \notin C_H \cup C_L$. In both cases it follows from the definition of $\odot$ that $\mathbf{K} \odot \alpha \subseteq \mathbf{K}$. On the other hand, by hypothesis, $\mathbf{K}$ is consistent, thus $\mathbf{K} \odot \alpha$ is also consistent.
Weak vacuity: Assume that $\neg\alpha \notin \mathbf{K}$. We intend to prove that $\mathbf{K} \subseteq \mathbf{K} \odot \alpha$. It follows trivially if $\alpha \notin C_H \cup C_L$. Assume now that $\alpha \in C_H \cup C_L$. It holds, by $\star$ *vacuity* and *inclusion*, that $\mathbf{K} \star \alpha = \mathbf{K} + \alpha$. We will consider two cases:
case 1) $\alpha \in C_H$. Hence $\mathbf{K} \odot \alpha = \mathbf{K} \star \alpha$. Thus $\mathbf{K} \subseteq \mathbf{K} + \alpha = \mathbf{K} \odot \alpha$.
case 2) $\alpha \in C_L$. Hence $\mathbf{K} \odot \alpha = (\mathbf{K} \star \alpha) \cap \mathbf{K} = (\mathbf{K} + \alpha) \cap \mathbf{K} = \mathbf{K}$.
Extensionality: Let $\vdash \alpha \leftrightarrow \beta$. It holds that C_H and C_L satisfy credibility of logical equivalents. Thus $\alpha \in C_H$ if and only if $\beta \in C_H$ and $\alpha \in C_L$ if and only if $\beta \in C_L$. We will consider three cases:
case 1) $\alpha \in C_H$. Thus $\beta \in C_H$.
case 2) $\alpha \in C_L$. Then $\beta \in C_L$.
case 3) $\alpha \notin C_H \cup C_L$. Thus $\beta \notin C_H \cup C_L$.
In each case it follows, from the definition of $\odot$ and $\star$ *extensionality* (in the first two cases) that $\mathbf{K} \odot \alpha = \mathbf{K} \odot \beta$.
N-relative success: Assume that $\neg\alpha \in \mathbf{K}\odot\alpha$. We intend to prove that $\mathbf{K}\odot\alpha = \mathbf{K}$. Assume by *reductio ad absurdum* that $\alpha \in C_H \cup C_L$. Hence, either $\alpha \in C_L$ or $\alpha \in C_H$. In both cases it holds, from the definition of $\odot$, that $\neg\alpha \in \mathbf{K} \star \alpha$. Thus, by $\star$ *success* it follows that $\mathbf{K} \star \alpha$ is not consistent. By $\star$ *consistency* it follows that $\alpha \vdash \bot$. Contradiction, since either $\alpha \in C_L$ or $\alpha \in C_H$ and it holds that C_H and C_L satisfy element consistency. Hence $\alpha \notin C_H \cup C_L$. Thus, it follows trivially by $\odot$ definition that $\mathbf{K} \odot \alpha = \mathbf{K}$.
Containment: Above we showed that $\odot$ satisfies *closure, weak vacuity* and *N-relative success*. We will prove by cases:
case 1) $\neg\alpha \notin \mathbf{K}$. Hence by $\odot$ *weak vacuity* it holds that $\mathbf{K} \subseteq \mathbf{K} \odot \alpha$. Thus $\mathbf{K} \cap ((\mathbf{K} \odot \alpha) + \alpha) \subseteq \mathbf{K} \odot \alpha$.
case 2) $\neg\alpha \in \mathbf{K} \odot \alpha$. Thus by $\odot$ *N-relative success* it follows that $\mathbf{K} \odot \alpha = \mathbf{K}$, thus $\mathbf{K} \cap ((\mathbf{K} \odot \alpha) + \alpha) \subseteq \mathbf{K} \odot \alpha$.
case 3) $\neg\alpha \in \mathbf{K}$ and $\neg\alpha \notin \mathbf{K}\odot\alpha$. We intend to prove that $((\mathbf{K}\odot\alpha)+\alpha)\cap\mathbf{K} \subseteq \mathbf{K}\odot\alpha$. It must be the case that $\alpha \in C_H \cup C_L$, otherwise it would follow that $\mathbf{K} \odot \alpha = \mathbf{K}$, which contradicts the hypothesis that $\neg\alpha \in \mathbf{K}$ and $\neg\alpha \notin \mathbf{K} \odot \alpha$. We will consider two cases:
case 3.1) $\alpha \in C_H$. Hence $\mathbf{K} \odot \alpha = \mathbf{K} \star \alpha$. By $\star$ *success* it follows that $\alpha \in \mathbf{K} \odot \alpha$. Hence $(\mathbf{K} \odot \alpha) + \alpha = \mathbf{K} \odot \alpha$. Thus $\mathbf{K} \cap ((\mathbf{K} \odot \alpha) + \alpha) \subseteq \mathbf{K} \odot \alpha$.
case 3.2) $\alpha \in C_L$. Thus $\mathbf{K} \odot \alpha = (\mathbf{K} \star \alpha) \cap \mathbf{K}$. Let $\beta \in ((\mathbf{K} \odot \alpha) + \alpha) \cap \mathbf{K}$. Hence $\beta \in (\mathbf{K} \odot \alpha) + \alpha$ and $\beta \in \mathbf{K}$. From the former it holds by $\odot$ *closure* (and deduction) that $\alpha \to \beta \in \mathbf{K} \odot \alpha$. Hence $\alpha \to \beta \in (\mathbf{K} \star \alpha) \cap \mathbf{K}$. Thus $\alpha \to \beta \in \mathbf{K} \star \alpha$. It follows by $\star$ *success* and *closure* that $\beta \in \mathbf{K} \star \alpha$. Thus $\beta \in (\mathbf{K} \star \alpha) \cap \mathbf{K} = \mathbf{K} \odot \alpha$. Therefore $\mathbf{K} \cap ((\mathbf{K} \odot \alpha) + \alpha) \subseteq \mathbf{K} \odot \alpha$. □

Proof. Theorem 3.15.
(1) to (2): We will use the same constructions as in the corresponding part of Theorem 3.14. Then we need to prove only that C_H, C_L and/or $C_H \cup C_L$ satisfy the properties listed in the corresponding paragraph of item 2. This follows trivially by Propositions 3.12. and 3.13.

(2) to (1): Let $\odot$ be a two level credibility-limited revision operator induced by a basic AGM revision operator $\star$ for $\mathbf{K}$ and sets $C_H, C_L \subseteq \mathcal{L}$ such that: $C_H \cap C_L = \emptyset$, C_H and C_L satisfy credibility of logical equivalents and C_L satisfies element consistency. It follows from Theorem 3.14. that $\odot$ satisfies *weak relative success, closure, inclusion, consistency preservation, weak vacuity, extensionality, N-relative success* and *containment*. It remains to prove that if C_H, C_L and/or $C_H \cup C_L$ satisfy also the property listed in a given paragraph of item 2, then $\odot$ satisfies the postulates listed in the corresponding paragraph of item 1.

(a) Assume that C_H expansive credibility. We must prove that $\odot$ satisfies vacuity. Assume that $\neg\alpha \notin \mathbf{K}$. Hence $\mathbf{K} \nvdash \neg\alpha$. It follows from C_H expansive credibility that $\neg\neg\alpha \in C_H$. It holds that C_H satisfies credibility of logical equivalents thus $\alpha \in C_H$. Therefore $\mathbf{K} \odot \alpha = \mathbf{K} \star \alpha$. It follows from $\star$ *vacuity* that $\mathbf{K} + \alpha \subseteq \mathbf{K} \star \alpha$. Thus $\mathbf{K} + \alpha \subseteq \mathbf{K} \odot \alpha$.

(b) Assume that C_H satisfies credibility lower bounding and disjunctive completeness. We must prove that $\odot$ satisfies *disjunctive distribution.*
Assume that $\alpha \notin \mathbf{K} \odot \alpha$ and $\beta \notin \mathbf{K} \odot \beta$. Hence $\alpha \notin C_H$ and $\beta \notin C_H$ (otherwise it would follow from $\star$ *success* and $\odot$ definition that $\alpha \in \mathbf{K} \odot \alpha$ and $\beta \in \mathbf{K} \odot \beta$). C_H satisfies disjunctive completeness. Thus $\alpha \vee \beta \notin C_H$. It holds that C_H satisfies credibility lower bounding, thus $\alpha \vee \beta \notin \mathbf{K}$. On the other hand, by definition of $\odot$ it follows that $\alpha \vee \beta \notin \mathbf{K} \odot (\alpha \vee \beta)$.

(c) Assume that C_H satisfies single sentence closure. We intend to prove that $\odot$ satisfies *strict improvement.*
Let $\alpha \in \mathbf{K} \odot \alpha$ and $\beta \in Cn(\alpha)$. We will prove by cases:
case 1) $\alpha \in C_H$, then $\beta \in C_H$ (since C_H satisfies single sentence closure). Thus by definition of $\odot$ and $\star$ *success* it follows that $\beta \in \mathbf{K} \odot \beta$.
case 2) $\beta \in C_H$. Thus by definition of $\odot$ and $\star$ *success* it follows that $\beta \in \mathbf{K} \odot \beta$.
case 3) $\alpha \notin C_H$ and $\beta \notin C_H$. From $\alpha \notin C_H$ it follows from the definition of $\odot$ that $\mathbf{K} \odot \alpha \subseteq \mathbf{K}$. Hence $\alpha \in \mathbf{K}$. It follows from $\beta \in Cn(\alpha)$ that $\beta \in \mathbf{K}$. Therefore it follows by the definition of $\odot$ and $\star$ *success* that $\beta \in \mathbf{K} \odot \beta$.

(d) Assume that C_H and $C_H \cup C_L$ satisfy single sentence closure. By (c) $\odot$ satisfies *strict improvement.* It remains to show that $\odot$ satisfies *N-success propagation.*
Let $\neg\alpha \in \mathbf{K} \odot \alpha$ and $\vdash \beta \to \alpha$. It holds that $\vdash \neg\alpha \to \neg\beta$. We will prove by cases:
case 1) $\vdash \neg\alpha$. Hence $\vdash \neg\beta$. Therefore by $\odot$ *closure* it follows that $\neg\beta \in \mathbf{K} \odot \beta$.
case 2) $\nvdash \neg\alpha$. By $\star$ *success* it holds that $\alpha \in \mathbf{K} \star \alpha$. Hence by $\star$ *consistency* $\neg\alpha \notin \mathbf{K} \star \alpha$. Thus from the definition of $\odot$ it follows that $\alpha \notin C_H \cup C_L$. By $C_H \cup C_L$ single sentence closure it follows that $\beta \notin C_H \cup C_L$. Thus $\mathbf{K} \odot \alpha = \mathbf{K} = \mathbf{K} \odot \beta$. Therefore, $\neg\alpha \in \mathbf{K} \odot \beta = \mathbf{K}$. $\mathbf{K}$ is a belief set, thus $\neg\beta \in \mathbf{K} = \mathbf{K} \odot \beta$.

(e) Assume that condition ($C_H \cup C_L$ **-** $\star$) holds. We intend to prove that $\odot$ satisfies *N-persistence.*
Let $\alpha \in \mathcal{L}$ and assume that $\neg\beta \in \mathbf{K} \odot \beta$. We will consider two cases:

case 1) $\vdash \neg\beta$. Hence by $\odot$ *closure* $\neg\beta \in \mathbf{K} \odot \alpha$.
case 2) $\nvdash \neg\beta$. If $\beta \in C_H \cup C_L$, then it holds, by definition of $\odot$ that $\neg\beta \in \mathbf{K}\star\beta$. On the other hand, by $\star$ *success* it follows that $\mathbf{K} \star \beta \vdash \perp$, which contradicts $\star$ *consistency*. Hence $\beta \notin C_H \cup C_L$. Thus $\mathbf{K} \odot \beta = \mathbf{K}$, from which it follows that $\neg\beta \in \mathbf{K}$. We will consider two sub-cases:
2.1) $\alpha \in C_H \cup C_L$. Condition $(C_H \cup C_L$ **-** $\star)$ holds. Hence $\neg\beta \in \mathbf{K} \star \alpha$. Therefore, by the definition of $\odot$, it holds that $\neg\beta \in \mathbf{K} \odot \alpha$.
2.2) $\alpha \notin C_H \cup C_L$. Hence $\mathbf{K} \odot \alpha = \mathbf{K}$. Therefore $\neg\beta \in \mathbf{K} \odot \alpha$.

(f) Let $\beta \in \mathbf{K} \odot \alpha$. We will consider two cases:
case 1) $\alpha \in C_H$. Then $\mathbf{K} \odot \alpha = \mathbf{K} \star \alpha$. Hence $\beta \in \mathbf{K} \star \alpha$. Revision credibility holds, thus $\beta \in C_H$. Therefore $\mathbf{K} \odot \beta = \mathbf{K} \star \beta$. By $\star$ *success* it follows that $\beta \in \mathbf{K} \odot \beta$.
case 2) $\alpha \notin C_H$. Hence $\beta \in \mathbf{K}$. By Proposition 3.11. it holds that $\mathbf{K} \odot \beta = \mathbf{K}$. Thus $\beta \in \mathbf{K} \odot \beta$.

□

Proof. Theorem 3.16.
(2) to (1): Let $\odot$ be a two level credibility-limited revision operator induced by an AGM revision operator $\star$ for $\mathbf{K}$ and sets $C_H, C_L \subseteq \mathcal{L}$ such that: C_L satisfies credibility of logical equivalents and element consistency, $C_H \cap C_L = \emptyset$, C_H satisfies element consistency, credibility lower bounding and single sentence closure and condition $(C_H \cup C_L$ **-** $\star)$ holds.
We note that since C_H satisfies single sentence closure then C_H also satisfies credibility of logical equivalents. We note also that since C_H and C_L satisfy element consistency then $C_H \cup C_L$ also satisfies element consistency. Thus, according to Proposition 3.8., $C_H \cup C_L$ satisfies single sentence closure.
It follows from Theorems 3.14. and 3.15. that $\odot$ satisfies *weak relative success, closure, inclusion, consistency preservation, weak vacuity, extensionality, N-relative success, containment, strict improvement* and *N-persistence.* It remains to prove that $\odot$ satisfies *N-recovery, disjunctive overlap* and *disjunctive inclusion.*
N-recovery: We will consider three cases:
case 1) $\alpha \notin C_H \cup C_L$. Hence $\mathbf{K} \odot \alpha = \mathbf{K}$. Thus $\mathbf{K} \subseteq \mathbf{K} \odot \alpha + \neg\alpha$.
case 2) $\alpha \in C_H$. Hence $\mathbf{K} \odot \alpha = \mathbf{K} \star \alpha$. Then by $\star$ *success* it follows that $\alpha \in \mathbf{K} \odot \alpha$. Hence $\mathbf{K} \odot \alpha + \neg\alpha = \mathcal{L}$. Thus $\mathbf{K} \subseteq \mathbf{K} \odot \alpha + \neg\alpha$.
case 3) $\alpha \in C_L$. Hence $\mathbf{K}\odot\alpha = \mathbf{K}\cap\mathbf{K}\star\alpha$. It holds that $\mathbf{K}\odot\alpha+\neg\alpha = (\mathbf{K}\cap\mathbf{K}\star\alpha)+\neg\alpha$. $\delta \in \mathbf{K} \odot \alpha + \neg\alpha$ iff $\delta \in (\mathbf{K} \cap \mathbf{K} \star \alpha) + \neg\alpha$ iff $\neg\alpha \to \delta \in (\mathbf{K} \cap \mathbf{K} \star \alpha)$ iff $\neg\alpha \to \delta \in \mathbf{K}$ and $\neg\alpha \to \delta \in \mathbf{K} \star \alpha$ iff $\delta \in \mathbf{K} + \neg\alpha$ and $\delta \in (\mathbf{K} \star \alpha) + \neg\alpha$ iff $\delta \in \mathbf{K} + \neg\alpha$ and $\delta \in \mathcal{L}$ iff $\delta \in \mathbf{K} + \neg\alpha$. Thus $\mathbf{K} \odot \alpha + \neg\alpha = \mathbf{K} + \neg\alpha$. Therefore $\mathbf{K} \subseteq \mathbf{K} \odot \alpha + \neg\alpha$.
Disjunctive overlap: We will prove by cases:
case 1) $\alpha \in C_H \cup C_L$ and $\beta \notin C_H \cup C_L$. It holds, by definition of $\odot$, that $\mathbf{K} \odot \beta = \mathbf{K}$ On the other hand $C_H \cup C_L$ satisfies single sentence closure, thus $\alpha \vee \beta \in C_H \cup C_L$. By $\star$ *disjunctive factoring* (Proposition 2.3.) it holds that either: (i) $\mathbf{K}\star(\alpha\vee\beta) = \mathbf{K}\star\alpha$, (ii) $\mathbf{K} \star (\alpha \vee \beta) = \mathbf{K} \star \beta$ or (iii) $\mathbf{K} \star (\alpha \vee \beta) = \mathbf{K} \star \alpha \cap \mathbf{K} \star \beta$.
(i) $\mathbf{K} \odot \alpha \cap \mathbf{K} \odot \beta = \mathbf{K} \odot \alpha \cap \mathbf{K} = \mathbf{K} \star \alpha \cap \mathbf{K} = \mathbf{K} \star (\alpha \vee \beta) \cap \mathbf{K} \subseteq \mathbf{K} \odot (\alpha \vee \beta)$.
(ii) Condition $(C_H \cup C_L$ **-** $\star)$ holds. Hence it follows from $\beta \notin C_H \cup C_L$ and $\alpha \vee \beta \in C_H \cup C_L$ that $\neg\beta \in \mathbf{K}\star(\alpha \vee \beta)$. Therefore $\neg\beta \in \mathbf{K}\star\beta$. It holds by $\star$ *success* that $\mathbf{K} \star \beta = \mathcal{L}$. Thus $\mathbf{K} \star (\alpha \vee \beta) = \mathcal{L}$. Thus $\mathbf{K} \odot \alpha \cap \mathbf{K} \odot \beta = \mathbf{K} \star \alpha \cap \mathbf{K} \subseteq \mathbf{K} = \mathbf{K} \star (\alpha \vee \beta) \cap \mathbf{K} \subseteq \mathbf{K} \odot (\alpha \vee \beta)$.

(iii) As in case (ii), it follows by condition $(C_H \cup C_L$ - $\star)$ that $\mathbf{K} \star \beta = \mathcal{L}$. Thus $\mathbf{K}\star(\alpha\vee\beta) = \mathbf{K}\star\alpha$. Hence $\mathbf{K}\odot\alpha\cap\mathbf{K}\odot\beta = \mathbf{K}\star\alpha\cap\mathbf{K} = \mathbf{K}\star(\alpha\vee\beta)\cap\mathbf{K} \subseteq \mathbf{K}\odot(\alpha\vee\beta)$.
case 2) $\alpha \notin C_H \cup C_L$ and $\beta \in C_H \cup C_L$. This case is symmetric with case 1.
case 3) $\alpha \notin C_H \cup C_L$ and $\beta \notin C_H \cup C_L$. Hence $\mathbf{K} \odot \alpha = \mathbf{K} \odot \beta = \mathbf{K}$. We will consider two sub-cases:
3.1) $\alpha \vee \beta \notin C_H \cup C_L$. Hence $\mathbf{K} \odot (\alpha \vee \beta) = \mathbf{K}$. Thus $\mathbf{K} \odot \alpha \cap \mathbf{K} \odot \beta \subseteq \mathbf{K} \odot (\alpha \vee \beta)$.
3.2) $\alpha \vee \beta \in C_H \cup C_L$. Condition $(C_H \cup C_L$ - $\star)$ holds. Thus it follows by $\star$ *closure* that $(\neg\alpha\wedge\neg\beta) \in \mathbf{K}\star(\alpha\vee\beta)$. Thus, by $\star$ *closure*, $\neg(\alpha\vee\beta) \in \mathbf{K}\star(\alpha\vee\beta)$. By $\star$ *success* it follows that $\mathbf{K}\star(\alpha\vee\beta) = \mathcal{L}$. Thus $\mathbf{K}\odot\alpha\cap\mathbf{K}\odot\beta = \mathbf{K} = \mathbf{K}\star(\alpha\vee\beta)\cap\mathbf{K} \subseteq \mathbf{K}\odot(\alpha\vee\beta)$.
case 4) $\alpha \in C_H \cup C_L$ and $\beta \in C_H \cup C_L$. $C_H \cup C_L$ satisfies single sentence closure. Thus $\alpha \vee \beta \in C_H \cup C_L$. We will consider four sub-cases:
4.1) $\alpha \in C_H$ and $\beta \in C_H$. C_H satisfies single sentence closure. Thus $\alpha \vee \beta \in C_H$. Thus by definition of $\odot$ it holds that $\mathbf{K}\odot\alpha\cap\mathbf{K}\odot\beta = \mathbf{K}\star\alpha\cap\mathbf{K}\star\beta$ and $\mathbf{K}\odot(\alpha\vee\beta) = \mathbf{K}\star(\alpha\vee\beta)$. Hence by $\star$ *disjunctive overlap* it follows that $\mathbf{K}\odot\alpha\cap\mathbf{K}\odot\beta \subseteq \mathbf{K}\odot(\alpha\vee\beta)$.
4.2) $\alpha \in C_H$ and $\beta \in C_L$. C_H satisfies single sentence closure. Thus $\alpha \vee \beta \in C_H$. By definition of $\odot$ it holds that $\mathbf{K} \odot \alpha \cap \mathbf{K} \odot \beta = \mathbf{K} \star \alpha \cap (\mathbf{K} \star \beta \cap \mathbf{K})$. It follows, by $\star$ *disjunctive overlap*, that $\mathbf{K} \star \alpha \cap (\mathbf{K} \star \beta \cap \mathbf{K}) \subseteq \mathbf{K} \star (\alpha \vee \beta) \cap \mathbf{K} \subseteq \mathbf{K} \odot (\alpha \vee \beta)$. Thus $\mathbf{K} \odot \alpha \cap \mathbf{K} \odot \beta \subseteq \mathbf{K} \odot (\alpha \vee \beta)$.
4.3) $\alpha \in C_L$ and $\beta \in C_H$. This case is symmetric with the previous one.
4.4) $\alpha \in C_L$ and $\beta \in C_L$. By definition of $\odot$ it holds that $\mathbf{K} \odot \alpha \cap \mathbf{K} \odot \beta = (\mathbf{K}\star\alpha\cap\mathbf{K})\cap(\mathbf{K}\star\beta\cap\mathbf{K})$. It follows by $\star$ *disjunctive overlap* that $(\mathbf{K}\star\alpha\cap\mathbf{K})\cap(\mathbf{K}\star\beta\cap\mathbf{K}) \subseteq \mathbf{K} \star (\alpha \vee \beta) \cap \mathbf{K}$. It holds that $\alpha \vee \beta \in C_H \cup C_L$. It follows by definition of $\odot$ that $\mathbf{K} \star (\alpha \vee \beta) \cap \mathbf{K} \subseteq \mathbf{K} \odot (\alpha \vee \beta)$. Thus $\mathbf{K} \odot \alpha \cap \mathbf{K} \odot \beta \subseteq \mathbf{K} \odot (\alpha \vee \beta)$.
Weak Disjunctive inclusion: We start by noticing that, according to Lemma 6.17., it holds that $(\mathbf{K} + \alpha) \cap (\mathbf{K} + \beta) \subseteq \mathbf{K} + \alpha \vee \beta$ and $\mathbf{K}' + \alpha \vee \beta \subseteq \mathbf{K}' + \alpha$, for any belief set $\mathbf{K}'$.
Assume that $\neg\alpha \notin \mathbf{K} \odot (\alpha \vee \beta)$. We will prove by cases:
case 1) $\alpha \in C_H$. It holds that C_H satisfies single sentence closure. Thus $\alpha\vee\beta \in C_H$. Thus $\mathbf{K} \odot \alpha = \mathbf{K} \star \alpha$ and $\mathbf{K} \odot (\alpha \vee \beta) = \mathbf{K} \star (\alpha \vee \beta)$. It holds that $\neg\alpha \notin \mathbf{K} \star (\alpha \vee \beta)$. From which it follows by $\star$ *disjunctive inclusion* that $\mathbf{K} \odot (\alpha \vee \beta) \subseteq \mathbf{K} \odot \alpha$. Hence $\mathbf{K} \odot (\alpha \vee \beta) + \alpha \vee \beta \subseteq \mathbf{K} \odot \alpha + \alpha \vee \beta \subseteq \mathbf{K} \odot \alpha + \alpha$.
case 2) $\alpha \notin C_H$. It holds that $C_H \cap C_L = \emptyset$. We will consider two sub-cases:
2.1) $\alpha \in C_L$. It follows by $\odot$ definition that $\mathbf{K} \odot \alpha = \mathbf{K} \star \alpha \cap \mathbf{K}$. It holds that $C_H \cup C_L$ satisfies single sentence closure. Thus $\alpha \vee \beta \in C_H \cup C_L$. We will consider two sub-cases:
2.1.1) $\alpha \vee \beta \in C_H$. It holds that $\mathbf{K} \odot (\alpha \vee \beta) = \mathbf{K} \star (\alpha \vee \beta)$. Thus $\neg\alpha \notin \mathbf{K} \star (\alpha \vee \beta)$. It follows by $\star$ *disjunctive inclusion* that $\mathbf{K} \star (\alpha \vee \beta) \subseteq \mathbf{K} \star \alpha$. On the other hand it holds, by $\star$ *inclusion*, that $\mathbf{K} \star \alpha \subseteq \mathbf{K} + \alpha$.
We will now show that $\mathbf{K} \star \alpha \subseteq (\mathbf{K} \star \alpha \cap \mathbf{K}) + \alpha$. Let $\delta \in \mathbf{K} \star \alpha$. Hence $\delta \in \mathbf{K} \star \alpha$ and $\delta \in \mathbf{K} + \alpha$. It follows by $\star$ *closure* that $\alpha \to \delta \in \mathbf{K} \star \alpha$. It also holds that $\alpha \to \delta \in \mathbf{K}$. Hence $\alpha \to \delta \in \mathbf{K} \star \alpha \cap \mathbf{K}$. Thus $\delta \in (\mathbf{K} \star \alpha \cap \mathbf{K}) + \alpha$. Therefore $\mathbf{K} \star \alpha \subseteq (\mathbf{K} \star \alpha \cap \mathbf{K}) + \alpha$.
Hence it holds that $\mathbf{K} \star (\alpha \vee \beta) \subseteq (\mathbf{K} \star \alpha \cap \mathbf{K}) + \alpha$. On the other hand it holds, by $\star$ *success*, that $\alpha \vee \beta \in \mathbf{K} \star (\alpha \vee \beta)$. Thus $\mathbf{K} \odot (\alpha \vee \beta) + \alpha \vee \beta = \mathbf{K} \odot (\alpha \vee \beta) = \mathbf{K} \star (\alpha \vee \beta) \subseteq (\mathbf{K} \star \alpha \cap \mathbf{K}) + \alpha = \mathbf{K} \odot \alpha + \alpha$.
2.1.2) $\alpha \vee \beta \in C_L$. Hence $\mathbf{K} \odot (\alpha \vee \beta) = \mathbf{K} \star (\alpha \vee \beta) \cap \mathbf{K}$. We will divide this case in two other sub-cases:
2.1.2.1) $\neg\alpha \in \mathbf{K}$. It follows from $\neg\alpha \notin \mathbf{K} \odot (\alpha \vee \beta)$ that $\neg\alpha \notin \mathbf{K} \star (\alpha \vee \beta)$. Therefore

by $\star$ *disjunctive inclusion* $\mathbf{K}\star(\alpha\vee\beta)\subseteq\mathbf{K}\star\alpha$. Hence $\mathbf{K}\odot(\alpha\vee\beta)\subseteq\mathbf{K}\odot\alpha$. Therefore $\mathbf{K}\odot(\alpha\vee\beta)+(\alpha\vee\beta)\subseteq\mathbf{K}\odot\alpha+(\alpha\vee\beta)\subseteq\mathbf{K}\odot\alpha+\alpha$.
2.1.2.2) $\neg\alpha\notin\mathbf{K}$. By $\star$ *vacuity* and *inclusion* it holds that $\mathbf{K}\star\alpha=\mathbf{K}+\alpha$. Thus $\mathbf{K}\odot\alpha=\mathbf{K}$. From which it follows that $\mathbf{K}\odot\alpha+\alpha=\mathbf{K}+\alpha$. It holds by definition of $\odot$ that $\mathbf{K}\odot(\alpha\vee\beta)\subseteq\mathbf{K}$. Thus $\mathbf{K}\odot(\alpha\vee\beta)+\alpha\vee\beta\subseteq\mathbf{K}+\alpha\vee\beta\subseteq\mathbf{K}+\alpha=\mathbf{K}\odot\alpha+\alpha$.
2.2) $\alpha\notin C_L$. Thus $\alpha\notin C_H\cup C_L$. Hence $\mathbf{K}\odot\alpha=\mathbf{K}$. Thus $\mathbf{K}\odot\alpha+\alpha=\mathbf{K}+\alpha$. On the other hand, by $\odot$ *inclusion*, it holds that $\mathbf{K}\odot(\alpha\vee\beta)\subseteq\mathbf{K}+\alpha\vee\beta$. Thus $\mathbf{K}\odot(\alpha\vee\beta)+(\alpha\vee\beta)\subseteq\mathbf{K}+\alpha\vee\beta\subseteq\mathbf{K}+\alpha=\mathbf{K}\odot\alpha+\alpha$.

(1) to (2): We will use the same constructions as in the corresponding parts of Theorems 3.14. and 3.15.. According to Theorems 3.14. and 3.15. it remains to prove that $\star$ satisfies *disjunctive overlap* and *disjunctive inclusion*.
Disjunctive overlap: We start by noticing that according to Observation 1.17 of Hansson (1999) it holds that $Cn(\alpha\vee\beta)=Cn(\alpha)\cap Cn(\beta)$. We will prove by cases:
case 1) $\neg(\alpha\vee\beta)\notin\mathbf{K}$. By $\star$ *vacuity* and *inclusion* it follows that $\mathbf{K}\star(\alpha\vee\beta)=\mathbf{K}+(\alpha\vee\beta)$.
By $\star$ *inclusion* it holds that $\mathbf{K}\star\alpha\subseteq\mathbf{K}+\alpha$ and $\mathbf{K}\star\beta\subseteq\mathbf{K}+\beta$. On the other hand it holds, by Lemma 6.17., that $(\mathbf{K}+\alpha)\cap(\mathbf{K}+\beta)\subseteq\mathbf{K}+\alpha\vee\beta$. Thus $\mathbf{K}\star\alpha\cap\mathbf{K}\star\beta\subseteq\mathbf{K}+\alpha\vee\beta=\mathbf{K}\star(\alpha\vee\beta)$.
case 2) $\neg(\alpha\vee\beta)\in\mathbf{K}$. Hence $\neg\alpha\in\mathbf{K}$ and $\neg\beta\in\mathbf{K}$. We will consider two sub-cases.
2.1) $\neg(\alpha\vee\beta)\in\mathbf{K}\odot(\alpha\vee\beta)$. Thus, by $\odot$ *N-persistence*, $\neg(\alpha\vee\beta)\in\mathbf{K}\odot\alpha$. By $\odot$ *closure* it follows that $\neg\alpha\in\mathbf{K}\odot\alpha$. By symmetry of the case it also holds that $\neg\beta\in\mathbf{K}\odot\beta$. Hence $\mathbf{K}\star\alpha\cap\mathbf{K}\star\beta=Cn(\alpha)\cap Cn(\beta)=Cn(\alpha\vee\beta)=\mathbf{K}\star\alpha\vee\beta$.
2.2) $\neg(\alpha\vee\beta)\notin\mathbf{K}\odot(\alpha\vee\beta)$. Thus $\mathbf{K}\star(\alpha\vee\beta)=\mathbf{K}\odot(\alpha\vee\beta)+\alpha\vee\beta$. If $\neg\alpha\in\mathbf{K}\odot\alpha$ and $\neg\beta\in\mathbf{K}\odot\beta$, then it would follow, by $\odot$ *N-persistence*, that $\neg\alpha\in\mathbf{K}\odot(\alpha\vee\beta)$ and $\neg\beta\in\mathbf{K}\odot(\alpha\vee\beta)$ and, consequently, by $\odot$ *closure* that $\neg(\alpha\vee\beta)\in\mathbf{K}\odot(\alpha\vee\beta)$. Contradiction. Hence, it holds that either $\neg\alpha\notin\mathbf{K}\odot\alpha$ or $\neg\beta\notin\mathbf{K}\odot\beta$. We will subdivide this case:
2.2.1) $\neg\alpha\notin\mathbf{K}\odot\alpha$ and $\neg\beta\notin\mathbf{K}\odot\beta$. Hence $\mathbf{K}\star\alpha\cap\mathbf{K}\star\beta=(\mathbf{K}\odot\alpha)+\alpha\cap(\mathbf{K}\odot\beta)+\beta$. Let $\delta\in\mathbf{K}\odot\alpha+\alpha\cap\mathbf{K}\odot\beta+\beta$. Thus $\delta\in\mathbf{K}\odot\alpha+\alpha$ and $\delta\in\mathbf{K}\odot\beta+\beta$. Therefore $\alpha\to\delta\in\mathbf{K}\odot\alpha$ and $\beta\to\delta\in\mathbf{K}\odot\beta$. On the other hand by $\odot$ *N-recovery* it holds that $\neg\beta\in\mathbf{K}\odot\alpha+\neg\alpha$. Thus, by $\odot$ *closure*, it holds that $\neg\alpha\to\neg\beta\in\mathbf{K}\odot\alpha$. Hence, by $\odot$ *closure*, it holds that $\beta\to\alpha\in\mathbf{K}\odot\alpha$. By symmetry of the case it also holds that $\alpha\to\beta\in\mathbf{K}\odot\beta$. It holds that $\{\alpha\to\delta,\beta\to\alpha\}\vdash(\alpha\vee\beta)\to\delta$. Thus by $\odot$ *closure* it holds that $(\alpha\vee\beta)\to\delta\in\mathbf{K}\odot\alpha$. By symmetry of the case it also holds that $(\alpha\vee\beta)\to\delta\in\mathbf{K}\odot\beta$. Hence it follows by $\odot$ *disjunctive overlap* that $(\alpha\vee\beta)\to\delta\in\mathbf{K}\odot\alpha\vee\beta$. Thus $\delta\in\mathbf{K}\odot(\alpha\vee\beta)+\alpha\vee\beta$. Therefore $\mathbf{K}\star\alpha\cap\mathbf{K}\star\beta=(\mathbf{K}\odot\alpha)+\alpha\cap(\mathbf{K}\odot\beta)+\beta\subseteq\mathbf{K}\odot(\alpha\vee\beta)+(\alpha\vee\beta)=\mathbf{K}\star(\alpha\vee\beta)$.
2.2.2) $\neg\alpha\in\mathbf{K}\odot\alpha$ and $\neg\beta\notin\mathbf{K}\odot\beta$. Hence $\mathbf{K}\star\alpha\cap\mathbf{K}\star\beta=Cn(\alpha)\cap(\mathbf{K}\odot\beta)+\beta\subseteq(\mathbf{K}\odot\alpha)+\alpha\cap(\mathbf{K}\odot\beta)+\beta$. As showed in the previous case it holds that $(\mathbf{K}\odot\alpha)+\alpha\cap(\mathbf{K}\odot\beta)+\beta\subseteq\mathbf{K}\odot(\alpha\vee\beta)+(\alpha\vee\beta)=\mathbf{K}\star(\alpha\vee\beta)$. Therefore $\mathbf{K}\star\alpha\cap\mathbf{K}\star\beta\subseteq\mathbf{K}\star(\alpha\vee\beta)$.
2.2.3) $\neg\alpha\notin\mathbf{K}\odot\alpha$ and $\neg\beta\in\mathbf{K}\odot\beta$. This case is symmetric to the previous one.
Disjunctive inclusion: Assume that $\neg\alpha\notin\mathbf{K}\star(\alpha\vee\beta)$. We will consider two cases:
case 1) $\neg(\alpha\vee\beta)\notin\mathbf{K}\odot(\alpha\vee\beta)$. Hence $\mathbf{K}\star(\alpha\vee\beta)=\mathbf{K}\odot(\alpha\vee\beta)+\alpha\vee\beta$. Thus $\neg\alpha\notin\mathbf{K}\odot(\alpha\vee\beta)$. It follows by $\odot$ *N-persistence* that $\neg\alpha\notin\mathbf{K}\odot\alpha$. Therefore $\mathbf{K}\star\alpha=\mathbf{K}\odot\alpha+\alpha$. It follows by $\odot$ *weak disjunctive inclusion* that $\mathbf{K}\star(\alpha\vee\beta)\subseteq\mathbf{K}\star\alpha$.

case 2) $\neg(\alpha \vee \beta) \in \mathbf{K} \odot (\alpha \vee \beta)$. Hence $\mathbf{K} \star (\alpha \vee \beta) = Cn(\alpha \vee \beta)$. Thus, it holds that $\mathbf{K} \star (\alpha \vee \beta) \subseteq Cn(\alpha) \subseteq \mathbf{K} \star \alpha$. □

BIBLIOGRAPHY

Alchourrón, C., Gärdenfors, P., & Makinson, D. (1985). On the logic of theory change: Partial meet contraction and revision functions. *Journal of Symbolic Logic* **50**, 510–530.

Booth, R., Fermé, E., Konieczny, S., & Pino Pérez, R. (2012). Credibility-limited revision operators in propositional logic. In *13th International Conference on the Principles of Knowledge Representation and Reasoning, KR 2012*, pp. 116–125.

Fermé, E., & Hansson, S. O. (2001). Shielded contraction. In H.Rott & Williams, M.-A., editors, *Frontiers in Belief Revision*, Applied Logic Series, pp. 85–107. Kluwer Academic Publishers.

Fermé, E., & Hansson, S. O. (2011). AGM 25 years: Twenty-five years of research in belief change. *Journal of Philosophical Logic* **40**, 295–331.

Fermé, E., & Hansson, S. O. (2018.). *Belief Change: Introduction and Overview.* Springer Briefs in Computer Science Series. Springer.

Fermé, E., Mikalef, J., & Taboada, J. (2003). Credibility-limited functions for belief bases. *Journal of Logic and Computation* **13:1**, 99–110.

Garapa, M., Fermé, E., & Reis, M. (2018a). Studies in credibility-limited base revision. In *Proceedings of the Sixteenth International Conference on Principles of Knowledge Representation and Reasoning (KR 2018)*, pp. 240–247.

Garapa, M., Fermé, E., & Reis, M. D. (2018b). Shielded base contraction. *Artificial Intelligence* **259**, 186 – 216.

Gärdenfors, P. (1978). Conditionals and changes of belief. *Acta Philosophica Fennica* **30**, 381–404.

Gärdenfors, P. (1982). Rules for rational changes of belief. In Pauli, T., editor, *Philosophical Essays dedicated to Lennart Åqvist on his fiftieth birthday*, Number 34 in Philosophical Studies, pp. 88–101.

Gärdenfors, P. (1988). *Knowledge in Flux: Modeling the Dynamics of Epistemic States.* Cambridge: The MIT Press.

Hansson, S. O. (1999.). *A Textbook of Belief Dynamics. Theory Change and Database Updating.* Applied Logic Series. Dordrecht: Kluwer Academic Publishers.

Hansson, S. O. (2017). *Descriptor Revision*, Volume 46 of *Trends in Logic*. Springer International Publishing.

Hansson, S. O., Fermé, E., Cantwell, J., & Falappa, M. (2001). Credibility-limited revision. *Journal of Symbolic Logic* **66**(4), 1581–1596.

Harper, W. L. (1976). Rational conceptual change. *PSA: Proceedings of the Biennial Meeting of the Philosophy of Science Association* **1976**, 462–494.

Makinson, D. (1997). Screened revision. *Theoria* **63**, 14–23.

Rott, H. (1992). Preferential belief change using generalized epistemic entrenchment. *Journal of Logic, Language and Information* **1**, 45–78.

FACULDADE DE CIÊNCIAS EXACTAS E DA ENGENHARIA
UNIVERSIDADE DA MADEIRA
Campus Universitário da Penteada, 9020-105 Funchal, Portugal
E-mail: marco@uma.pt