Università degli studi di Padova

Dipartimento di Fisica

Corso di Laurea Magistrale in PHYSICS

Improving the understanding of cosmological horizons through the use of interactive animations

Master Thesis by:

Gianmarco Ceccotti

Supervisor(s):

Prof. Ornella Pantano

Dr. Alessio Mattia Leonardi

Year 2021/2022

Contents

List of Figures

Abstract

Animations for educational purposes have been studied for more than 40 years. The results of the research have over time defined some general canons shared by the developers [Adams et al., 2008a, R. E. Mayer, 2009].

The aim of this thesis is to investigate the effectiveness of an interactive didactic graphic software developed ad hoc in a contest of introductory cosmology.

The Physics topic addressed in this teaching unit is that of cosmological horizons. The choice of this topic stems from two main reasons: our willingness to test the use of interactive educational animations in an advanced Physics context and the fact that there are documented difficulties in learning these concepts [Davis and Lineweaver, 2004].

In the development of the interactive software we have followed general guidelines that have been defined on the basis of cognitive load theory[Sweller et al., 1998]. In addition, we have try to include in the work a new paradigm, the so-called ”Perceptual symbol systems” approach [Barsalou, 1999], which has been used in the context of Physics education with good results [Z. Chen and Gladding, 2014].

In the first chapter of this thesis we will make an analysis of the literature on educational animations. After a brief historical overview of the documented use of these media in literature, we will cover the theoretical aspects involved. In our theoretical analysis we distinguish mainly two levels: “understanding”, or the clarity with which information is used, and “learning”, or the mechanisms by which mental models are built in the student. Next we will consider the guidelines for a successful project developing simulation useful for the teaching of Physics: ”PhET” from the University of Colorado.

In the second chapter we analyse the Physics involved in the so called Horizon problem in cosmology. First we outline the necessary prerequisites; then, starting from these concepts, we will set a discussion on the cosmological horizons and which problems are connected to its understanding. Finally, we will conclude by talking about the perspectives that are opened in the cosmological field starting from this problem.

In the third chapter we continue the analysis of the same topic from the teaching perspective. We describe the fundamental steps in learning these concepts and identify the teaching objectives.

In the forth chapter we explain in detail the individual choices in the design of a new proposal of interactive simulations useful for the teaching of the cosmological horizons in the light of what emerged in previous chapters. This discussion addresses not only the design of the simulations but also some technical aspects related to data creation and optimization.

In the fifth chapter we discuss the development of a protocol of questions which have guided the interview with three respondent for a first test of the effectiveness of our animation in favoring the learning of cosmological horizons.

In the sixth chapter we discuss on the outcomes of the interviews in relation to our teaching objectives.

The seventh and last chapter is dedicated to final consideration and suggestion for a possible development of the project.

Chapter 1

The role of animations in Physics education

In the last three decades, increased processing capacities and an increasing use of computers in learning context have resulted in the tremendous development of multimedia instructions. An important subgroup of this multimedia material are animations: according to Bauer- Morrison an animation is "any application, which generates a series of frames, so that each frame appears as an alteration of the previous one, and where the sequence of frames is determined, either by the designer or the user" ([Bauer-Morrison et al., 2000] IVI p. 313). In particular, we talk about *interactive simulations* when different degrees of freedom in the manipulation of the animation are left to the user.

The use of animations in teaching has different efficacy results depending on the area of interest and the strategies used. In the next section, we will present a short chronological history of the scientific research found on this subject. These articles have been extracted mainly from the bibliography of two important meta-reviews [Berney and Bétrancourt, 2016, Höffler and Leutner, 2007] and extended using the buzzwords suggested by these researches [1]; the articles studying animations used in Physics were considered the most.

1.1 An overview of salient aspects of didactic animations

> I believe that the motion picture is destined to revolutionize our educational system and that in a few years it will supplant largely, if not entirely, the use of textbooks.
>
> *Thomas Edison* - The diary and sundry observations of Thomas Alva Edison (ed. Greenwood Pub Group, 1968)

The belief that the use of new technologies leads to an automatic improvement in the quality of learning represents a persistent myth in the educational field and also influenced the first phase of the research on didactic animations.

The research on animations for educational purposes began in the 1980s: the first phase was

---

[1] "didactic", "animation", "simulation", "dynamic picture", "dynamic image", "still picture", "still image", "static picture", "static image", "motion", "steps", etc.

characterized by confidence in the superiority of dynamic representations with respect to the static ones, but nevertheless it failed to show a clear advantage for the animations.

Although some studies found positive effects of animated displays, for example, on students motivation and in implicit learning [Rieber, 1991], there have been few studies showing an advantage of static over animated displays in conceptual learning. Betrancourt in 2002 reviewed over 20 researches that compared learning from static and animated graphics [Tversky et al., 2002]. In the majority of these studies, there was no advantage of animations over static graphics. A few studies showed such an advantage, but in these researches more information was presented in the animated graphics than in the static graphics, i.e., they were not informationally equivalent. These first contradictory results highlighted the need for an expansion of the theoretical background focusing on the individual aspects of the animations, in order to understand the mechanisms that would explain a differential learning effect [Hegarty, 2004].

In subsequent years, the multimedia research shifted towards evaluating the conditions under which and the reasons why dynamic representation may improve or facilitate learning. In particular, the researches investigated the cognitive processes involved in understanding a dynamic visualization, the steps leading to the comprehension of the contents and ultimately to their learning.

Usually in multimedia research, learning refers to the construction of a mental model of the spatial, temporal and functional components of the dynamic content [Narayanan and Hegarty, 2002, Lowe and Boucheix, 2008a].

This analysis made on cognitive processes led to a more practical discussion on the various relevant aspects in the design of animations (e.g., whether these animations should contain text or audio or visual instructions; whether a 3d or 2d animation is better; how much control should be left to the user). This gave general indications on the ideal features of an animation and at the same time defined a set of questions that the developer of the contents should take into account [Höffler and Leutner, 2007].

Now it's clear that the educational effectiveness of dynamic visualizations depends on how well they are designed, used and supported.

The large number of factors that must be taken into account when designing an animation makes it difficult to compare them. For example, some experimental studies failed to make considerations about factors that were subsequently considered relevant. This lack, together with the inaccessibility of the material used in the experiments, reduces the statistical relevance of part of the early studies [Berney and Bétrancourt, 2016].

The large number of factors behind an exhaustive analysis of the didactic animations is not compensated by a sufficient number of studies that allow to have a clear idea. This fragmented landscape indicates the need for more research to understand the effectiveness of this medium. In the following sections, we will try to summarize an overall view of the aspects that will be considered for the creation of a didactic animation.

We will focus on two levels where the student's learning experience takes place: the first is linked to the comprehensibility of the medium, or rather the possibility of extracting information from the animation consistently; the second layer is learning itself, that is, the construction of a mental model that allows the student to recall the acquired knowledge.

The former level will be treated in the section 1.2 "Understanding before learning" while the latter in the section 1.3 "Learning after understanding". Finally, we will analyze the guidelines of the successful interactive animations project called "PhET" of the University of Colorado.

1.2 Understanding before learning

An important distinction emerging from literature is between the comprehensibility and the educational effectiveness of an animation, being the former a necessary but not sufficient requirement for the latter.

While the factors affecting the effect on learning are still unclear, those that determine the ability to understand animation are better identified [Berney and Bétrancourt, 2016]. These can be divided mainly in three categories:

1. *Factors specific to the learners*, such as their prior knowledge level [ChanLin, 1998, Kalyuga, 2008] and visuospatial ability [Hegarty and Sims, 1994, Boucheix and Lowe, 2010].
2. *Factors specific to the instructional material*, such as the type of dynamic changes within the animation [Lowe, 2003], its perceptual salience [Boucheix and Lowe, 2010], the presence of accompanying information [Ginns, 2005, R. Mayer et al., 1999, Tabbers et al., 2001] or the control over the pace of the animation [Fischer et al., 2008, R. Mayer and Chandler, 2001].
3. *Factors specific to the learning context*, such as the type of knowledge and the instructional domain [Bétrancourt and Tversky, 2000], that is how much the subject of study

in general and its concepts are appropriate to be represented as animations.

Among these three categories, we can intervene directly only on the second one. We can use the other two to make ex post observations on the results of a trial. However, the main focus will be understanding what the literature suggests on the specific characteristics of the instructional material.

1.2.1 Multimedia learning theory

Several theoretical models exist in the field of multimedia education: one of the most successful is Mayer's multimedia learning theory (MML) [R. E. Mayer, 2009] which provides a set of "design principles" for creating animated multimedia tutorials.

On a very coarse scale, Mayer's MML theory is based on two main ideas. First, students learn better when more cognitive resources can be used to make sense of the verbal or pictorial material. Second, when auditory and visual signals are presented in coherence, the cognitive resources required to process these signals are minimized. In that case, more cognitive resources can be allocated to making sense of the materials.

The theoretical foundations of MML theory lay in the **cognitive load theory (CLT)** [Sweller, 1994]. CLT refers to the limited capability of humans to process novel information; it asserts that instructional design must be primarily concerned with allocating cognitive resources to those activities which lead to desired changes in long-term memory. A key concept of this theory is the **working memory**; this idea is similar to short-term memory, the ability to store information for a short period of time, with the addition of processing and using data as well as storing them, giving the ability to guide decision-making and reasoning. The working memory resources are used to deal with **Cognitive load**, the mental workload required to perform a task.

Literature identifies three types of cognitive load:

1. **Intrinsic cognitive load** is the inherent level of difficulty associated with a specific instructional topic. The term was first used in the early 90s by Chandler and Sweller. According to them, all instructions have an inherent difficulty associated with them (e.g., the calculation of 2 + 2, versus solving a differential equation). An instructor cannot alter this difficulty. However, many schemas may be broken into individual "sub-schemas"

and taught in isolation, to be later brought back together and described as a combined whole (see section 1.3) [Chandler and Sweller, 1991].

2. **Extraneous cognitive load** is generated by the modality information is presented to learners and it is under the control of instructional designers.
3. **Germane cognitive load**, as it was first described by Sweller and Paas in 1998, "reflects the effort that contributes to the construction of schemas.". Unlike intrinsic and extraneous cognitive load, germane cognitive load does not constitute an independent source of cognitive load. It merely refers to the working memory resources available to deal with the relations between elements associated with intrinsic cognitive load (e.g., creating flowcharts in presentations to explain complex concepts). Differently from intrinsic and extraneous cognitive load that are determined by the characteristics of the material, germane cognitive load is concerned only with learner effort to understand. This can be facilitated in highlighting the connections between the various elements of a graphic. The systematic organization of information makes it easier to learn and remember [Sweller et al., 1998].

Since we cannot intervene directly on the intrinsic cognitive load, we can make design decisions only on the external and germane load. The goal that must be pursuit in good instructional design is to minimize extraneous cognitive load and maximize germane load. [Muller et al., 2008, Sweller, 2010].

Two reasons for the lack of benefit in teaching through animation put on display by this theory include: (a) the imposition on learners of excessive information processing demands ('overwhelming') and (b) a reduction in the extent to which learners engage in valuable processing activities ('underwhelming') [Lowe, 2003]. These cognitive load issues are generated from different aspects of animations and we will briefly summarize them below, exploring their features in the following sections.

The essential instructional characteristic that distinguishes dynamic from static graphics is their capacity to depict temporal change directly. A consideration of the perceptual and cognitive implications of this aspect is that with some animations learners may face higher levels of cognitive load with respect to those expected for static alternatives. [Lowe, 1999]

Also, learners usually focus on more obvious perceptual events rather than on those that are of most conceptual interest; for example, Lowe (1999) found that novices extracting information from a dynamic weather map focused on less important changes rather than the more important but perceptually subtle transformations.

Instructional messages containing redundant information sources inhibit learning in what is called the *redundancy effect* [Sweller et al., 1998].

In the context of MML theory this is called the **coherence principle**:

> Adding extraneous words or pictures to a multimedia message can interfere with [...] cognitive processes [...] by encouraging learners to pay attention to words or images that are not relevant, by disrupting how learners organize words or pictures into a causal chain, and by priming inappropriate schemas to be used to assimilate the incoming words and pictures. [R. Mayer, 2004 IVI, pg. 133]

In short, multimedia messages including non-essential material often distract learners from the cognitive processes required for learning. Extraneous cognitive load, on the other hand, reflects the additional cognitive effort required to learn from poorly designed instruction.

Mayer's principles, therefore, serve as guidelines for coordinating verbal and visual representations, rather than guidelines for designing each individual representation. The most prominent shortcoming of Mayer's model is that it lacks the ability to distinguish the quality between two different visual (or verbal) representations of the same content in terms of learning outcomes [Schnotz, 2002, Reed, 2006]. According to this model, as long as the visual representation is presented following the aforementioned principles, students will understand it. This can be effective in a didactic context where a unique representation of a concept prevails, as for example when the object of the study is a real object with an unequivocal representation; this instead becomes increasingly ambiguous for the representation of abstract concepts, to the point that a symbol that makes perfect sense for the teacher is not understandable for a student. Therefore, although Mayer's model is very useful in guiding certain aspects of multimedia instruction design, it is not exhaustive for our purpose.

In addition to minimizing extraneous cognitive load, what we need to encourage is the Germane cognitive load. This aspect will be taken into greater consideration in paragraph 1.3 "Learning after understanding", where models allowing us to consider the aspects not touched by the MML theory will be discussed. In the next subsections, we will focus on which design choices avoid extraneous cognitive load.

1.2.2 Pacing and time control

The main difference between static and dynamic representation is given by the temporal evolution of the graphics. Being the images of an animation transitory, the salient didactic passages can be soon forgotten due to the perishable short-term memory. Therefore, it seems coherent to question if some temporal control features can be useful to improve user's learning experience.

An important but surprising finding documented in Bétrancourt's meta-study of 2016 state that a positive effect of an animation over a static graphics was found only when learners did not control the pace of the display (system-paced instructional material). Whereas this research reported inconsistent findings with respect to some studies showing a benefit of control [Boucheix and Guignard, 2005; R. Mayer and Chandler, 2001; Schwan and Riempp, 2004] and other studies that found no benefit [for example Adesope and Nesbit, 2012]. The system-paced presentation, together with the transient nature of animation, frequently seen as a drawback, did not impede learners to study the current information and images while integrating the previous learning content [Moreno and Mayer, 2007]. However, it is highly probable that the effect of pacing control interacts with other factors, such as learners' cognitive style [Höffler and Leutner, 2011], prior knowledge, learning objectives, and/or modality of the accompanying information [Schmidt-Weigand et al., 2010; Tabbers, 2002].

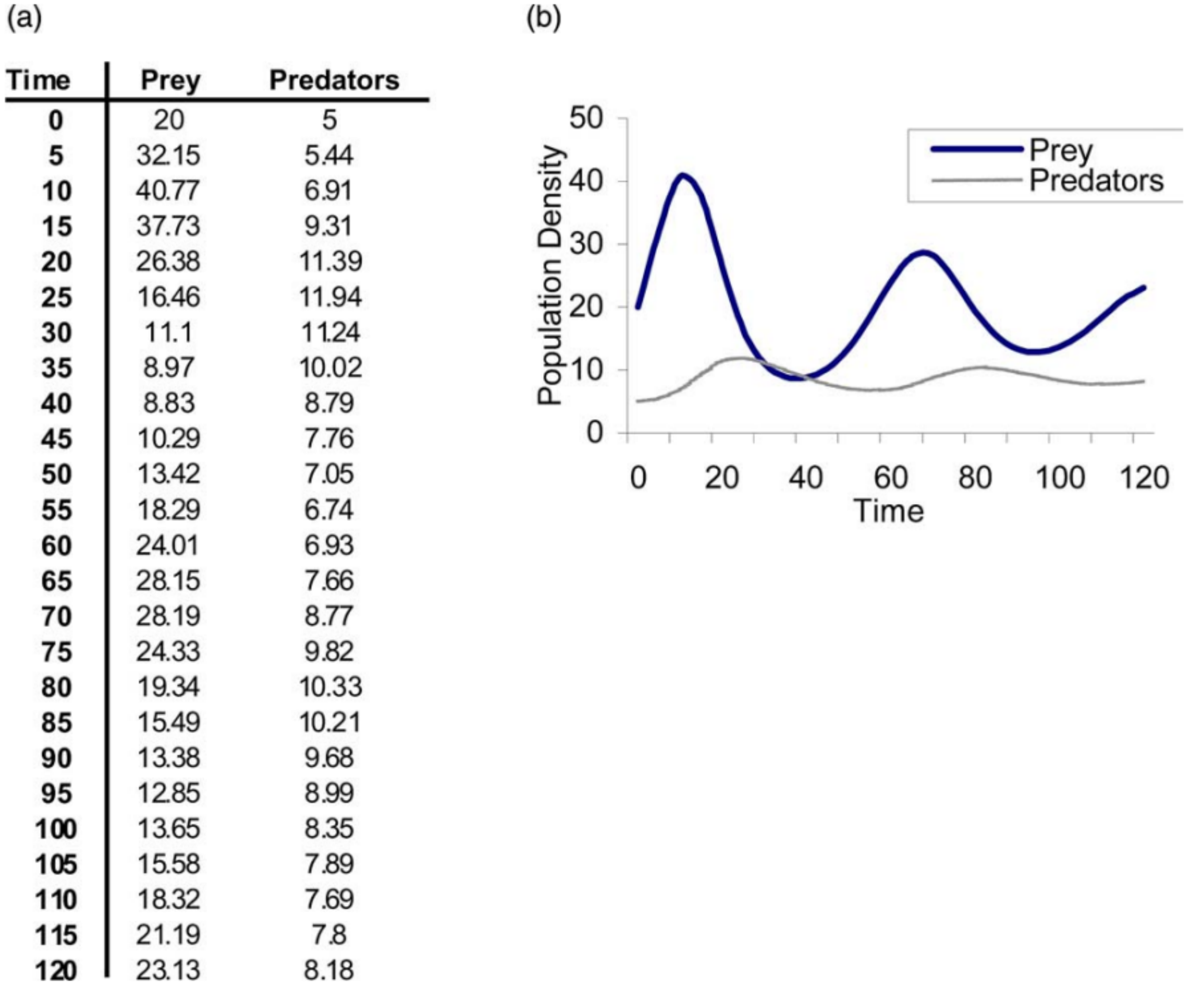
1.2.3 Time representation

In the previous section, we have discussed what the potential benefits of using time controls. A further clarification is needed with respect to the type of time representations that can be used in an animation. Its first conceptualization was carried out by Ainsworth, using the following time categories [Ainsworth and Van Labeke, 2004]:

- Time-persistent representations: in this case the information persists over time, as in figure 1.1 where a graph of two populations as a function of time is displayed.
- Time implicit representation: in this case there is no explicit dependence of the variables on time in the representation, but a curve is represented in the phase space with the 2 populations as axes (figure 1.2).
- Time-singular representation: in this case, single states are presented sequentially, as in

figure 1.3 where a pie chart evolves over time with the percentage of populations.

These multiple representations complement each other, because they differ either in the information each one expresses or in the processes each one supports. By combining representations that differ in these ways, it is likely that learners will benefit from the advantages of each of the individual representations. Multiple representations provide complementary information, while a single representation would be insufficient to carry all the information about the domain.



(a)

Time	Prey	Predators
0	20	5
5	32.15	5.44
10	40.77	6.91
15	37.73	9.31
20	26.38	11.39
25	16.46	11.94
30	11.1	11.24
35	8.97	10.02
40	8.83	8.79
45	10.29	7.76
50	13.42	7.05
55	18.29	6.74
60	24.01	6.93
65	28.15	7.66
70	28.19	8.77
75	24.33	9.82
80	19.34	10.33
85	15.49	10.21
90	13.38	9.68
95	12.85	8.99
100	13.65	8.35
105	15.58	7.89
110	18.32	7.69
115	21.19	7.8
120	23.13	8.18

Figure 1.1: Example of a time persistent representation: predator prey population density in a (a) table, (b) time-series graph.

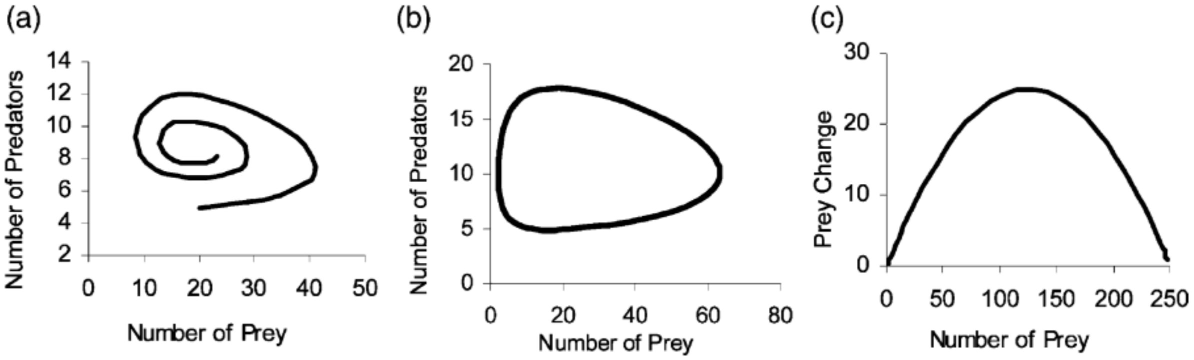


Figure 1.2: Examples of time implicit representation: (a, b) predator prey population density phase-plots, (c) population change vs population density phase-plot.

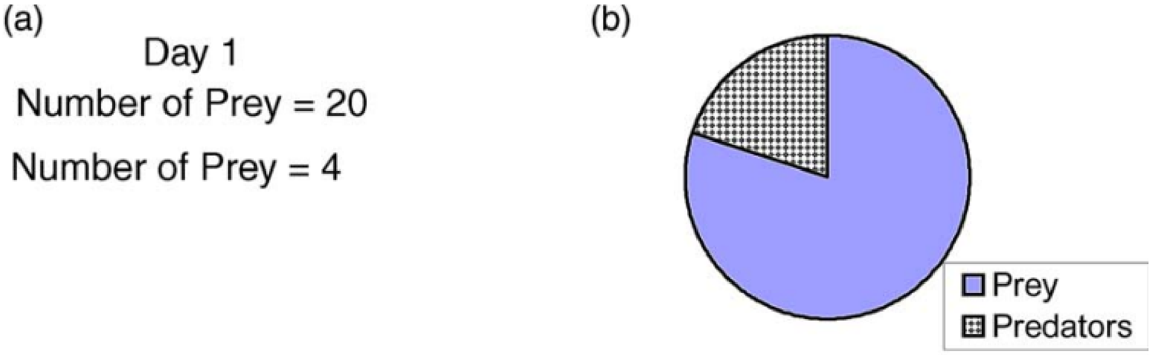


Figure 1.3: A snapshoot of an example of a time singular representation: population density in a (a) dynamic text, (b) pie chart.

1.2.4 Directing attention

As we saw in the previous section, the complexity of animations means that they can be extremely demanding for users to process. Contributions to this complexity range from more obvious characteristics, such as the sheer number of graphic entities shown in the display space, to quite subtle aspects, such as highly transitory temporal relationships occurring between those entities.

Display simplification is a possible way to reduce these demands, even though there are many situations where this is neither feasible nor appropriate. An alternative approach is to retain the animated diagram's complexity but providing users with additional processing support that helps them to cope with its inherent demands. This second approach is addressed here.

Users can cope with a complex dynamic display if they are able to direct attention strategically and assign their limited information processing resources efficiently to extract the most task-appropriate information. For this purpose, users need to direct their attention to the most thematically relevant entities of the display looking in the right place, at the right time, for the right aspects.

For a long time, visual guidance in the form of arrows and highlighting has been used to direct attention in static graphics. This approach tends to offer more precision in their directional function than non-visual alternatives (such as accompanying text). Unfortunately, they can also increase the visual clutter of the display and the arrows may even distract attention from the aspects they are meant to cue by becoming a focus of attention themselves. [Boucheix and Lowe, 2008]

One way of adapting traditional static highlight cueing approaches to the requirements of processing complex animations is to synchronize the operation of the cueing effect with the changes in thematic relevance that occurs as the presentation progresses.

Some examples of cueing can be found in the article of Lowe (2008): a color band, that spreads through the sequence of graphic entities involved in a causal chain, allows viewers to follow the line of action through a mechanism. These spreading color focus on the broad alignment of viewer attention shifts with linear changes in the locus of thematically relevant activity. This method is called *progressive path cueing* [Lowe and Boucheix, 2008b].

In the same paper, we found another cueing technique called *relational intensity*: in this approach, the cue strength is modulated over time to align with the thematic relevance of the relationship between the target graphic entities during the animation's time course. As the relationship becomes more important over time, the cue strength (e.g., color intensity) increases and then decreases as its importance wanes. A relevant aspect of the relational intensity cueing is to give greater visual impact to information with high thematic relevance at the right time.

There are growing evidence of visual cues improving learning in literature [Boucheix and Lowe, 2010; De Koning et al., 2010; Lin and Atkinson, 2011; to name a few]. However, some studies demonstrated that cueing could also counteract spontaneous exploration and add extraneous visual information [Boucheix et al., 2013].

1.2.5 Comparison between 2D and 3D animations

Usually, with respect to two-dimensional, three-dimensional animations are more detailed and more complex ones. Adding a third dimension heightens the complexity of an illustration as the spatial relations between the elements of the animation have to be coded on three axes instead of two. Three-dimensional depictions also typically introduce occlusions and foreshortening (i.e., optical distortions of objects extending along the depth axis). In some circumstances, occlusions may help to understand otherwise unavailable spatial relationships. On the other hand, both occlusions and foreshortening carry the danger of making understanding of the presentation more difficult for the learner [Schwan and Papenmeier, 2017]. In addition, three-dimensional animations allow defining trajectories of point of view that can guide the attention of the viewer on the relevant parts of objects or events. Instead of providing students with a fixed perspective, the designer can flexibly adapt the viewing angle and distance conditions throughout an animation. In addition, the motion of the camera serves different purposes, including completeness of the view, optimizing the points of view, guiding the attention, or simply doing the presentation more appealing.

While some researches consider that viewing dynamic 3D-animations improves mental model construction [Hays, 1996; Wu and Shah, 2004] other studies highlight the danger of cognitive overloading of working memory especially for learners with low spatial ability [Huk, 2006, R. Mayer and Sims, 1994].

The core question the developer must ask himself when he has to decide whether to use the 3D or not is the following : "is it really necessary to represent the phenomenon in 3D, or can I transmit the same information with fewer dimensions?". Given that the cognitive load is most likely greater with an additional dimension, we have to ask ourselves which of the 3 categories of section 1.2.1 this additional cognitive load falls into.

1.3 Learning after understanding

A first essential consideration for designing an educational animation is to have clear what it is hoped to achieve in terms of learning. For many animations, the overarching design goal seems to be largely affective. Their main purpose is to maximize learner interest and provide ongoing motivation.

Even thought these affective aspects are undoubtedly important, they are beyond the scope of this present section. Instead we will focus on how an animation might directly foster deep understanding of the subject addressed. Our criterion for evaluating an animation as educationally effective will be the possibility of encouraging the construction of a high quality mental model of the subject. By high quality, we mean that the mental model is (i) appropriate for the task that the learner will be required to perform, (ii) an accurate internal representation of the referent subject, and (iii) sufficiently comprehensive to encompass all essential aspects of the required task performance. When the subject portrayed by an animation is complex, its understanding requires learners to develop mental models representing this complexity [Lowe et al., 2017].

The question we aim to answer in the following sections is "how can we facilitate the construction of high quality mental models in students?"

1.3.1 The composition approach to animations

The **Animation Processing Model** (APM) by Lowe is a cognitive model that characterizes learning through a set of five interrelated processing phases in order to construct a mental model from such a presentation [Lowe and Boucheix, 2008a].

This theory has a bottom-up educational constructionist approach. "Constructionism" is a theory of learning to postulate the construction of mental models through which the individual organizes his knowledge and can adapt it. The approach is defined as "bottom-up" as it starts decomposing into fundamental concepts the subject of study and establishing foundations that subsequently are collected into increasingly more complex ideas.

Learning mediated by this model is divided into 5 phases. The passage from one phase to the following consists in correlating simple concepts that can be clustered into increasingly complex objects. The expected goal is arriving to a mental model that is gradually more and more flexible and capable of generalization.

The 5 phases, discussed in depth in the aforementioned article, can be summarized as follow:

1. Phase 1: Localized perceptual exploration.

   This initial phase is characterized by the division of our animation into event unit which are, according to Kurby [Kurby and Zacks, 2008] "[...] a segment of time at a given location that is conceived by an observer to have a beginning and an end".

2. Phase 2: Regional structure formation.

Broader scale regional structures emerge due to the formation of relationships between the individual event units. Links between them can be made on the basis of visuospatial characteristics (see in paragraph "Directing attention" (1.2.4)).

3. Phase 3: Global characterization.
   During the third stage of processing, students develop an overview of the animation in terms of operations and components across space and time. The construction of this internal characterization during Phase Three entails the confirmation of the individual causal links identified during the second phase and how these connections are arranged in space and time.

4. Phase 4: Functional differentiation.
   Although a complete description of the animation can be generated during Phase Three, what distinguishes Phase Four is the characterization of individual contributions given by the various events. Events that were identified in the earlier stages of processing are now interpreted in terms of the main purpose and characterized as a set of functionally distinct actions. Phase Four is linked to the understanding of the different subsystems contributing to the overall functioning of the system.

5. Phase 5: Mental model consolidation.
   This last phase is the consolidation of the mental model. The additional ability compared to phase four is understanding the behavior of the system not simply in a single situation, but rather through a wide variety of both usual and unusual circumstances.

In their work Lowe and Boucheix pointed out that APM Phase 1 processing tends to be a major stumbling block for learners [Lowe and Boucheix, 2017]. Even when they are supported by dynamic cues attempting to strongly direct attention to the most relevant aspects of the display, the quality of the mental models learners compose is relatively low. This suggests a question: if decomposition is such an intractable impediment for learners, would it be possible to design educationally effective animations that help relieving learners of this burden?

Phase 1 processing can therefore be regarded essentially as a preliminary enabling activity whose purpose is to provide the raw material for this later and more central composition activity.

The **Composition Approach** [Lowe and Boucheix, 2012] helps learners composing a high quality mental model, easing their difficulties in decomposing an animation in event unit. Instead of expecting learners to decompose a conventional comprehensive animation, it supplies them with parcels of dynamic information that are consistent with the results of an 'ideal' decomposition.

The overall concept is that learners are equipped with a 'kit of parts', each one of which is a small-animated assembly comprising two or possibly more interconnected event units. These assemblies are called *relation sets*. Learners are thus able to progressively evolve simple, individual events into a complex, integrated network of temporal chains and functional relationships that a high quality mental model requires.

1.3.2 Comparison between simultaneous and sequential animations

Ploetzner and Lowe brought out a new question about the effectiveness of animations, namely whether it is better to present sequential episodes of animations or to see more animations at the same time in order to explain a concept [Ploetzner and Lowe, 2017].

In the experiment conducted in the aforementioned study, the authors compared between two groups of students learning from two different versions of a sailing animation. Both versions consisted of four animation episodes portraying the possible maneuvers of a sailboat in relation to the wind direction. While one group of students learned from a sequential presentation of the four episodes (sequential group), the other one learned from a simultaneous presentation of the same episodes (simultaneous group).

This study provides evidence in favor of simultaneity. In detail, the strengths in the simultaneous group are: (1) it resulted in significantly more visual transitions between the episodes; (2) it leads to significantly more bidirectional visual transitions between the episodes; (3) the learning of higher-order relationships was significantly better from the simultaneously presented episodes.

Although the first two results do not directly verify that learners compared the simultaneously presented episodes, the third result suggests that they went beyond a mere mechanistic shifting of their visual attention between episodes. In light of all three results, the article concludes that "[…]it appears plausible that learners actively compared and contrasted the episodes in an effort to identify higher-order relationships of the animated subject. However, it would be pedagogically unwise to conclude from these results that learners should be instructed to simply shift their visual attention as often as possible between simultaneously presented episodes."

The reasons provided by the article refer to the APM model that we considered in the previous section. Presenting component episodes of an animation simultaneously offers affordances considerably more suited to identify and extract high-level cross-episode relationships than sequential presentation. According to the APM, the comparisons and contrasts between co-present episodes in order to establish meaningful relationships are crucial here – the repeated shifts of visual attention are merely perceptual indicators of this deeper processing. Furthermore, the APM suggests that successful learning from simultaneously as well as sequentially presented episodes requires both within-episode and between-episode interrogation. They are both essential for the learner in order to construct the hierarchical knowledge structure that characterizes a well-developed mental model.

Even if we do not have sufficient evidence to unequivocally affirm the effectiveness of the use of simultaneous animations, this choice seems consistent with a development direction oriented toward the APM.

1.3.3 Perceptual symbol systems theory

Until recently, most cognitive psychologists believed that concepts were represented in the brain by the so-called “amodal symbols”, abstract symbols processed by specialized neural circuits of the brain, independently of all the sensory-motor systems [Z. Chen and Gladding, 2014, Reed, 2006].

The symbolic system is characterized by a series of signifiers, each one representing a precise semantic concept. Each signifier is related to other symbols in a rhizome of concepts, in which relations are defined by propositional relations. For example, the amodal symbol “TREE” represents the semantic concept of the word “tree” and is linked to other amodal symbols such as “ROOTS” and “BRANCHES” through the propositional relationship “HAS”.

An unsolved issue of this theory lies in identifying a mechanism of formation of these amodal symbols. Since learning consists in the creation of new amodal symbols (or the development of new connections between them) it is understandable the reason this represents a serious lack for the creation of effective educational theories from this background.

This lack resulted in a growing skepticism towards the system of amodal symbols [Barsalou, 2010, Anderson, 2007], leading some cognitive psychologists to believe that all the mechanisms listed above are somehow “grounded” in the sensory-motor systems of the brain.

The first attempt to create such a theory is the **Perceptual Symbol Systems** [PSS] by Barsa-

lou [Barsalou, 1999]. One of the most significant advantages of a grounded cognition view of knowledge is the direct connection between perception and internal knowledge structures, being they both represented by activation of neural circuits in much the same sensorimotor domains. Therefore, grounded cognition works as an ideal theoretical framework for understanding the impact of visual representational design on learning.
The PSS theory can be summarized in three pivotal ideas:

1. **Perceptual symbols** are records of neural activation in the sensory-motor system as a result of perception. There are a series of neural circuits that are activated in response to perception: thinking of all the characteristics that can be perceived from an object (i.e., shape, color, size, individual vertices, curves, etc.), perception can be both holistic or focused on a specific aspect. Since these activations to be stored in long-term memory, we refer to this record as "Perceptual symbols".
   The PSS theory assumes that even the most abstract concepts, such as "truth" or "justice," are represented solely by the reactivation of various perceptual symbols [Barsalou and Wiemer-Hastings, 2005]. Any concept, whether simple or complex, related to the real or the abstract, is a re-enactment of a series of perceptual symbols. The only difference is that the most complex concepts require the perceptual symbols to possess a number of unique and critical features that differentiates them from holistic perceptions.
   For example while learning of theorems of Euclidean geometry, some perceptive aspects of the figures are selected with respect to others. Surely the presence of a right angle in a triangle will receive more attention than its size.

2. **Simulation** is the act of re-calling a single or a sequence of perceptual symbols [Barsalou, 1999, Barsalou, 2009].
   This can be used to solve tasks, such as calling up the perception of an object to understand if it has certain features. These simulations pass quickly from being conscious to taking place in the subconscious with the habit.
   In many studies, visual representations, particularly animations, are seen as facilitators for creating mental simulations [Z. Chen and Gladding, 2014].

3. **Concepts** are seen as a collection of interrelated perceptual symbols that allow the mind generating many different but related simulations [Barsalou, 1999].

1.3.4 Application of PSS theory in multimedia Physics education

One of the main benefits of using PSS in education is to have a precise definition of "learning". Learning is said to have "acquired a concept" if they are able to generate simulations about an entity or process to a "culturally acceptable degree" [Barsalou, 1999].

This can be achieved either by acquiring new perceptual symbols directly with sensory experience or by combining existing perceptual symbols or smaller scale simulations in a novel way. Since most of the concepts in Physics are not directly observable, their knowledge needs to be built on previously acquired perceptual symbols.

Our goal is therefore to activate the right sequence of perceptual symbols so that the brain is able to generate a new simulation inherent in the argument explained. The question we have to investigate then becomes "how exactly can instructional representations activate perceptual symbols?". In the literature the strategies identified are mainly [Z. Chen and Gladding, 2014] :

1. The **"symbolic method"**: its efficiency relay upon a representation that has already been associated with an existing simulation consisting of multiple perceptual symbols. The proper functioning of this method is based entirely on our previous knowledge of the symbol and the existing convention.
   An example is the circuit diagrams: by means of basic symbolic elements such as those used for resistors and capacitors, it is possible to build the knowledge of a more complex circuit based on previous knowledge called by a symbol.

2. The **"perceptual method"**: in order to understand the meaning of any representation, visual or verbal, the learner must first perceive it using the perceptual domains of the brain. At the same time, according to grounded cognition, the "meaning" behind that representation is also being processed in the same perceptual domains of the brain. Therefore, the perceptual features of the representation must have a significant chance of influencing the activated perceptual symbols. An example from Physics is presented in figure 1.4, where it is shown an electric potential of a charge. In this representation the perceptual model is employed in the use of the gradient color to indicate the intensity of the electric potential.

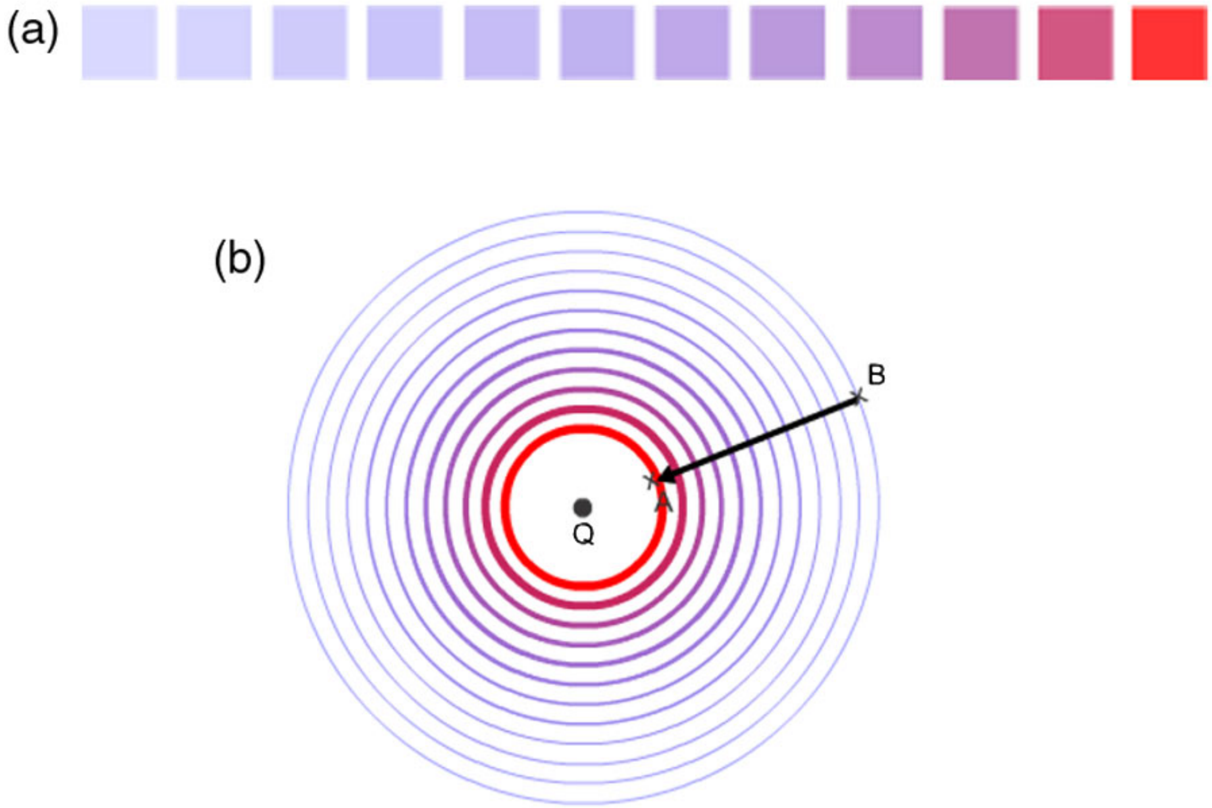


Figure 1.4: Describing using perceptual method of an electric potential by associating a color gradient to field intensity

1.4 PhET interactive simulation, a case study

So far, we have defined the theoretical aspects a developer of educational animations must take into account. Now with this theoretical background, we will analyze a series of successful animations, the “Physics Education Technology” (PhET) Project of the University of Colorado Boulder, founded in 2002 by Nobel Laureate Carl Wieman. PhET simulations are based on extensive education research and engage students through an intuitive, game-like environment where they can learn through exploration and discovery.

This project caught our attention because of the large number of awards won and the large volume of publications.

The project consists of a large number (over 100) of interactive animations freely accessible on the dedicated website or downloadable locally.[2] The target of the project is very diverse: in the dedicated site we find search filters by the level of education ranging from elementary schools to university [Wieman et al., 2010].

The covered topics in the animations belong to different STEM disciplines such as Physics, Chemistry, Mathematics, Biology, and Earth Science.

The most extensive subgroup of animations is the one dedicated to Physics, with several subsections addressing some topics such as: “Movement”; “Sound and Waves” ; “Work, energy and power”; “Heat and Thermos”; “Quantum Phenomena”; “Light and Radiation”; “Electricity, Magnets, and Circuits”.

Thus, the research conducted by this group is very well documented and on the website a lot of information material about the animations. In the next section we will analyze a document containing the general guidelines[3] of the project and try to integrate it with the previous sections.

1.4.1 The guidelines

The guidelines document starts exposing three underlying ideas of this project:

1. Engagement, namely the importance of engaging the students in the exploration of the simulation.

---

[2] PhET site

[3] PhET Look and Feel (Adams, Perkins and Wieman, 2006)

2. The Coherence Principle, which aims to exclude superfluous information for minimizing extraneous cognitive load (section 1.2.1).
3. Consistency, namely the choice to keep identical the graphic aspects of an object in different animations

After presenting these ideas, the document goes into detail about individual aspects of the design, always taking into account what has just been mentioned. The following elements of the animation are considered: **Layout**, **Intuitive Controls**, **Representations**, **Help**, **Encouraged Exploration**.

The developers clarify that the context of this document is not meant to be absolute but that each animation has its own specific needs that can get out of what has been said. **Layout**: it is the graphic representation of a processing system and it is composed of all the elements appearing on the screen to the user. Below we will specify these elements with the respective comments given by the guidelines:

- **Control panel**: it is the portion of the screen addressed to the controls, which makes up much of the possible interactions with the animation. The control panel does not necessary handle all the functions as it will be clarified in the next paragraph.
  Limiting the number of tools/controls and arranging them in small groups, it becomes easier to identify what is available and makes the simulation less intimidating.
  Students only read the text attached to a control. Abbreviations are not understood by most students. Text strings from one to three words work best.

- **Play area**: it is the part of the layout where the animation actually takes place. It must be distinct from the control panel in look and functionality.
  When too many objects are in the play area, the control panel is overlooked. Text is a distraction in the play area.

- **Backgrounds**: backgrounds image should not distract the user from the important features of the simulation. Separation between the features of the simulation and the background is important. It could be used as a clue to give context to the animation.

Intuitive Controls: interviews showed that certain types of controls are intuitive for users. If different controls are used, even with 'help' or tutoring from the interviewer, many students

still can not use the simulation. Indeed their focus is on learning how to manipulate the simulation rather than on the concepts.
It is also pointed out that the functionality of the controls depends on the type of animation:

- **Click and drag interface**: it is the most natural motion for students. It is therefore useful to develop functionalities related to this action.

- **Grabbable objects**: also this type of action is used intuitively by students; it is recommended to implement it whenever is possible and contextualized it must be implemented.

- **Sliders, buttons, checkboxes**: the slider allows controlling a variable in a continuous domain. The buttons and the checkbox instead transmit a single information that is kept in time.
  This control is very common in everyday life: indeed, the interviews conducted by the research team show a good familiarity in the use by students. Sliders are usually used the first time for exploratory purposes, and they are useful tools when student are request to solve a task. The general trend noted by the research group in the use of check-boxing is to activate new functions, but not when it comes to disabling them.

- **Set of tools**: a series of digital instruments (e.g., a meter, a speedometer, a thermometer) can be integrated into the control panel emulating measurements in the simulated environment. The strategy used by PhET researchers agrees with the principle of consistency, namely representing them like their equivalent in the real world. The aim is to make intuitive their purpose.

Representations: a strength of the simulations is providing an explicitly visual mental model. This is not limited to the experience of the individual student but rather to a common picture for discussion. Typically before students begin discussing a concept and using a simulation there can be a fairly long conversation about what the phenomena look like. This step is unnecessary with a functioning simulation and students fell more confident about what they are discussing.
In the following points, we will analyze the aspects we need to consider in order to have a functional visual model for a discussion.

- **Visual cues**: as already said, visual clues are very important for directing the students' attention.

  However, we must be careful not to create an environment full of text because the general tendency of a student who does not understand a concept will be to refer to all the visual clues he finds.

  The role of colors in this context is very important and they must be used consistently not only within the same animation but also between different ones.

- **Consistent representations**: when an object is represented differently from simulation to simulation, students perceive it as two different objects. Otherwise when objects are represented similarly they are perceived as the same, even though they may be completely unrelated.

- **Start up settings**: an interactive simulation should start with minimal use of animations and active objects on the screen. The reason is to induce the student to interact with the software.

Help: the presence of tutorials in a simulation should be reduced to a minimum; ideally, they should not appear as the use of the animation should be intuitive. This can be avoided deciding on a "main object" and giving a clue for its use. In certain contexts of particularly complex animations, one can think of setting up a button that provides help. However even in these cases, one should try to minimize the textual component in favor of visual clues. **Encourage Exploration**: the last aspect taken into account by the guidelines is engagement. There is evidence that a greater level of students involvement is related to a better learning [Rieber et al., 1998, Kearsley and Shneiderman, 1998].

- **Animation and interactivity**: animated features are noticed first by students; however, they ask fewer questions and form significantly fewer new connections when they just observe without interacting. Any additional degree of freedom in controlling a simulation can have a significant learning potential. The developer should try to make the experience as interactive as possible while maintaining an understandable interface.

- **Little puzzles/clues**: a student can encounter an aspect he does not understand but he can still interact with it in this situation is still stimulated to explore how the function changes the simulation until it is possible to create a working definition of the function.

  Using legends and labels on controls allows students to associate a definition to the control behavior.
  Multiple representations of an idea further facilitate its understanding and connections.
  Developers must be careful as exploration is not always productive and thus features that encouraging exploration in an unproductive way must be avoided.

- **Credibility of simulations**: students must believe the simulation in order to engage its exploration.
- **Performance mode**: previous knowledge of a student's subject affects the educational effectiveness of animation. In general, students without any previous knowledge approach animations in a more engaging way than one who is already familiar with the topic.

In the next chapter, we will dissert on the topic of interest of this thesis. The concepts of this chapter will be recovered again in the fourth chapter: there, after having analyzed the subject and defined the teaching objectives, we will try to design a didactic animation on it.

Chapter 2

Dissertation on cosmological horizons

It is not uncommon for a Physics student interested in cosmology to approach for the first time the cosmological horizons without having a solid foundation of general relativity. Indeed, it is possible to address introductory topics of cosmology using simplifications that disregard from a solid background of general relativity. However these simplifications come at the cost of possible misunderstandings, hence the risk of creating an incomplete or incorrect mental model.

We propose to build a teaching unit in the context of the cosmological horizons; in particular, we aim to provide the necessary tools to understand the ”**horizon problem**”.

The **horizon problem** is a discrepancy arising from the difficulty in explaining the observed homogeneity of space regions causally disconnected without a mechanism that establishes the same initial conditions everywhere.

This topic plays a fundamental role in the comprehension of cosmology but it shows different didactic issues related to the apparent contradictions with special relativity, to the ambiguity in the graphic representation, to the use of terminology and other topics that will be explored later [Davis and Lineweaver, 2004, Ellis and Rothman, 1993].

This chapter will begin exposing different approaches to the horizon problem found in literature. Then we will deal with the possible misunderstandings in the interpretation of these concepts and finally we will define the educational objectives for our teaching unit.

2.1 Where to start?

Cosmological horizons and the horizon problem are based on several previous acquaintances in the field of cosmology and therefore we must decide a starting point.

First, the target of this teaching unit will be a student of Physics proficient in special relativity, having also some basic notions of cosmology. With these premises, we will define a series of necessary **prerequisites** to understand the concepts that we aim to explain.

Next we will try to analyze the main topic answering two questions hidden in the name of the problem itself: “**What is a horizon?**” and “**Why is this a problem?**”.

2.2 Prerequisites

In this section, we will recall the concept of the light cone as it is introduced in special relativity and then we will make a brief excursus on the main concepts related to the geometry of the Universe. Starting from the cosmological principle, we will see the implications of the assumption of this strongly symmetrical condition, describing the characteristics and dynamics of the metric of the Universe. Finally, we will talk about the photon geodesics in this metric, setting all the needed tools to understand how light cones are deformed compared to the Minkowski spacetime.

2.2.1 Light cone

In special relativity, a light cone (or null cone) is the surface describing the temporal evolution of a flash of light in Minkowski spacetime.

The function $r_{LC}(t_0, t)$ defining the radius of the light cone has a double dependence on temporal variables: the first t_0 represents the current time of the observer/emitter from whose position the cone is generated; the second variable t is the time at which, given a set t_0 time, the actual radius of the cone is calculated.

If we call the observer **O** with spacetime coordinates (x_0, y_0, z_0, t_0) and **E** the emitter with spacetime coordinates (x_E, y_E, z_E, t_E), then using Minkowski metric we can define the **spacetime interval** as:

$$\Delta s^2 = -(c(t_E - t_0))^2 + (x_E - x_0)^2 + (y_E - y_0)^2 + (z_E - z_0)^2 = -(c\Delta t)^2 + \Delta x^2 + \Delta y^2 + \Delta z^2. \tag{2.1}$$

Using spherical coordinate and symmetry under rotations we can reduce 2.1 to:

$$\Delta s^2 = -(c\Delta t)^2 + \Delta r^2. \tag{2.2}$$

On the base of this definition, we can classify all possible spacetime events in three different categories delimited by the boundary of the light cone:

- If $\Delta s^2 < 0$, then $\Delta r^2 < c^2 \Delta t^2$, namely the spatial separation is less than the distance light travels. The interval is called **timelike** and the event **E** is inside the light cone. This means that if the event **E** is in the past of the observer, it can emit a material particle that could affect observer **O**; otherwise if the event E is in his future, it could be affected by a material particle emitted by **O**.

- If $\Delta s^2 = 0$, then $\Delta r^2 = c^2\Delta t^2$ and the spatial separation is equal to the distance light travels. The interval is called **lightlike** and the event **E** lays on the boundary of the light cone.

  This means that if the event **E** is in the past of **O**, it can emit a light-speed information that could affect him. Otherwise, if the event **E** is in his future, it could be affected by a light-speed information emitted by **O**.

- If $\Delta s^2 > 0$, then $\Delta r^2 > c^2\Delta t^2$ and the spatial separation is greater than the distance light travels. The interval is called **spacelike** and the event **E** is outside the light cone.

  This means that event **E** can never affect the observer **O** nor be affected by him.

This idea can be extended in the context of general relativity for a spacetime with an arbitrary metrics using the appropriate geodetic equations of the photons.

Using different metrics it is common that the typical cone shape is deformed; however the name used to refer to these hypersurfaces remains ”light cone” although it can be misleading.

Those ideas are central to understand cosmological horizons: as we will see, some of them can be thought as particular cases of light cones in the Friedmann Lemaître Robertson Walker (FLRW) metric.

2.2.2 Cosmological principle

The cosmological theories most widely accepted by the scientific community are based on the so-called *Cosmological Principle*:

> [...] is the assertion that, on sufficiently large scales (beyond those traced by the large-scale structure of the distribution of galaxies), the Universe is both homogeneous and isotropic. Homogeneity is the property of being identical everywhere in space, while isotropy is the property of looking the same in every direction. [Coles and Lucchin, 1995 IVI, pg. 3]

There is no necessary relationship between homogeneity and isotropy; a manifold can be isotropic around one point without being homogeneous or vice versa.

Isotropy is supported by our observations of the Universe, such as those concerning the cosmic background radiation (CMB). On the other hand, homogeneity cannot be proved experimentally, but the validity of this hypothesis is support by the correctness of the theories following it.

2.2.3 FLRW metric

The most general spacetime metric describing a homogeneous and isotropic Universe is the **Friedmann-Lemaître-Robertson-Walker (FLRW) metric**:

$$\mathrm{d}s^2 = -c^2\,\mathrm{d}t^2 + a^2(t)\left[\mathrm{d}r^2 + S_k^2(r)\,\mathrm{d}\Omega^2\right]. \tag{2.3}$$

In this expression we are using the **comoving coordinates**, a very popular system of coordinates that "expands" with the Universe itself. This expansion is determined by $a(t)$ a dimensionless function called the "**cosmic scale factor**" or the **"expansion parameter"**. This coordinate system ensures that objects, at rest with respect to the Universe, always remain at the same distance over time. Instead the **proper coordinates** are a coordinate system fixed at a certain time and the distance between two objects at rest and the Universe increases over time with the expansion.

Finally in equation 2.3 r and Ω are spherical polar coordinates and $\mathrm{d}\boldsymbol{\Omega}^2 = \mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2$ (where θ and ϕ are respectively the polar and azimuthal angles); t is the proper (or cosmic) time and $S_k(r)$ represents a function of r depending on the curvature:

$$S_k(r) = \begin{cases} \sqrt{k}^{\,-1}\sin(r\sqrt{k}), & k > 0 \\ r, & k = 0 \\ \sqrt{|k|}^{\,-1}\sinh(r\sqrt{|k|}), & k < 0. \end{cases} \tag{2.4}$$

Here k represents the curvature and it may assume the values $k = (-1, 0, +1)$ for negative, zero, and positive curvature respectively corresponding to a closed, flat and open geometry as shown in figure 2.1. [Coles and Lucchin, 1995 IVI, pg. 9]

We can express the FLRW metric in another interesting form using the so-called **conformal time**, defined as:

$$\mathrm{d}t = a(\tau)\,\mathrm{d}\tau \quad \longrightarrow \mathrm{d}\tau = \frac{\mathrm{d}t}{a(t)}.$$

The term $a(\tau)$ is called the **conformal scale factor**.

Equation 2.3 can be rewritten as:

$$\mathrm{d}s^2 = a^2(\tau)\left[-c^2\,\mathrm{d}\tau^2 + \mathrm{d}r^2 + S_k^2(r)\,\mathrm{d}\Omega^2\right] \tag{2.5}$$

It results that the FLRW metric, as a function of conformal time, is proportional by the factor $a(\tau)$ to the metric of Minkowski in the case of a flat Universe. The transformation that links the two metrics is called conformal transformation, or also **Weyl transformation**.

Hereafter when we refer to the FLRW metric, we will consider only its flat version unless otherwise specified. Indeed the flat Universe is the simplest case and thus it allows to avoid an additional cognitive load on the student. Also, once mastered this geometry, it will be easier to generalize to others.

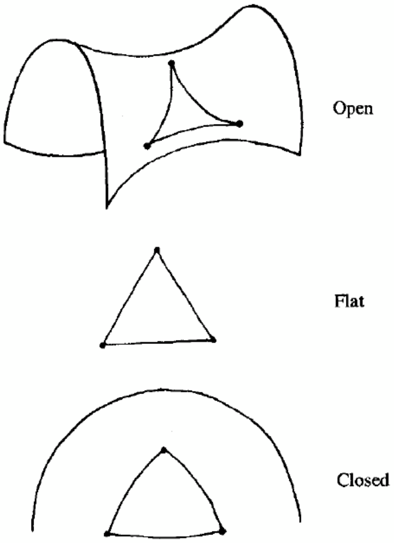


Figure 2.1: Examples of curved spaces in two dimensions: in a space with negative curvature (open), for example, the sum of the internal angles of a triangle is less than 180°, while for a positively curved space (closed) it is greater.

2.2.4 Comoving and proper distance

We consider now a reference frame in the FLRW metric, with an observer in the origin **O** and a light source in the position **E**. If we want to evaluate the distance covered by a light-ray emitted by E at a fixed time t_e, we should impose two conditions in equation 2.3: $ds^2 = 0$, that represents a light-like world line, and $d\Omega^2 = 0$, as light propagates along a straight line and thus the angles (θ, ϕ) remain constant. We than have:

$$c\,\mathrm{d}t = a(t)\,\mathrm{d}r \longrightarrow \mathrm{d}r = \frac{c\,\mathrm{d}t}{a(t)} \longrightarrow \int_0^{r_p(t_0)} dr = c\int_{t_e}^{t_0} \frac{\mathrm{d}t}{a(t)} \tag{2.6}$$

$$r_p(t_0) = c\int_{t_e}^{t_0} \frac{\mathrm{d}t}{a(t)} \tag{2.7}$$

The quantity $r_p(t_0)$ is the **comoving distance** between two points measured along a path defined by the cosmological emission time t_e and the observation time t_0.

The **proper distance** $d_p(t_0)$ between two point is equal to the length of the spatial geodesic between them when the scale factor is fixed at the value of $a(t_0)$.

To evaluate it, we need to use the equation 2.3 at fixed time t_0:

$$\mathrm{d}s^2 = a^2(t_0)\left[\mathrm{d}r^2 + r\,\mathrm{d}\Omega^2\right]$$

Once again along with the geodesics of light-ray the angles (θ, ϕ) remain constant:

$$\mathrm{d}s^2 = a^2(t_0)\mathrm{d}r^2$$

The proper distance d_p is obtained integrating over the radial comoving coordinate r:

$$d_p(t_0) = a(t_0)\int_0^r \mathrm{d}r = a(t_0)r = a(t_0)\;c\int_{t_e}^{t_0} \frac{\mathrm{d}t}{a(t)}. \tag{2.8}$$

We can explicate the relationship between the two distances as:

$$d_p(t_0) = a(t_0)\,r_p(t_0). \tag{2.9}$$

[Ryden, 2003, IVI, page 37]

2.2.5 Friedmann equations

The dynamic features of the spacetime are described by a set of 10 relations known as **Einstein equation**:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}. \tag{2.10}$$

When we characterize the Universe with the FLRW metric, Einstein equations reduce to the **Friedmann equations**

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G\rho + \Lambda c^2}{3}, \tag{2.11}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda c^2}{3}, \tag{2.12}$$

$$\dot{\rho} = -3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right). \tag{2.13}$$

The first two equations are obtained imposing symmetrical conditions used in the description of the Universe on cosmological scale.

The Friedmann equation of energy (eq. 2.11) is derived from the 00 component of the Einstein field equations or, in other words, is a direct consequence of energy conservation. Each of the three space-like components is assumed identical because all energy forms are considered to be spatially isotropic. The Friedmann equation of acceleration (eq. 2.12) is the single remaining space-like component. This equation is more difficult to treat being a second-order differential equation, which remarks its accelerative nature [Nemiroff and Patla, 2008].

Finally, equation 2.13 can be derived from the other two, but also from the first law of thermodynamics with appropriate substitution. This equation is often called "**fluid equation**" and it tells us how the density of Universe changes with time [Liebscher, 2005, IVI, page 53].

2.2.6 Hubble's law and Hubble parameter

To fully understand the evolution of the FLRW metric, we need to find an explicit formulation of the **Hubble parameter**. This it is usually referred with letter H and its most common definition is given by the logarithmic derivative of $a(t)$:

$$H(t) = \frac{1}{a(t)}\frac{da(t)}{dt} \tag{2.14}$$

The most famous expression including this parameter is the **Hubble's law**:

$$v_r \equiv H d_p. \tag{2.15}$$

This simple relation describes the instantaneous velocity v_r of an object at a fixed distance d_p with respect to an inertial observer.

Now our aim is to express $H(t)$ as a function of the proprieties of the Universe, considered as a perfect fluid with density ρ and pressure p. However it can also be thought as composed of separate constituents of density ρ_i and partial pressure p_i, with the condition that $\Sigma_i \rho_i = \rho$ and $\Sigma_i p_i = p$. We also need to specify an additional dimensionless number w, defined as:

$$w = \frac{p}{\rho} \quad \text{or} \quad w_i = \frac{p_i}{\rho_i}. \tag{2.16}$$

The i-th quantities are related by the **linear state equation**:

$$p_i = w_i \rho_i c^2\,. \tag{2.17}$$

Since the redshift z is

$$z = \frac{a(t_0)}{a(t)} - 1, \tag{2.18}$$

using the second Friedmann equation (eq. 2.13), we can express the i-th density ρ_i in terms of z:

$$\rho_i = \rho_{i0}(1+z)^{3(1+w_i)} \tag{2.19}$$

Now we define the following time dependent quantities:

- The **critical density** as
$$\rho_c = \frac{3}{8}\pi H^2, \tag{2.20}$$
- The **dimensionless energy density** as
$$\Omega = \frac{\rho}{\rho_c}, \tag{2.21}$$
- The **dimensionless i-th component of the energy density** as
$$\Omega_i = \frac{\rho_i}{\rho_c}. \tag{2.22}$$

When all these quantities are referred to the current time, we denoted that with a zero subindex. If we now substitute in the definition of Ω_i, the expressions of ρ_c, Ω_{i0} and ρ_i, we obtain:

$$\Omega_i(z) = \Omega_{i0} H_0^2 \frac{(1+z)^{3(1+w_i)}}{H(z)^2} \tag{2.23}$$

Finally, adding all the Ω_i (where the index i ranges over all possible state equations) [Margalef-Bentabol et al., 2013]:

$$H(z) = H_0 \sqrt{\sum_{i=1}^{n} \frac{\Omega_{i0}}{\Omega}(1+z)^{3(1+w_i)}}. \tag{2.24}$$

The time-dependent expression can be obtained substituting the redshift with the scale parameter

$$H(t)^2 = H_0^2 \frac{a_0}{a}\left[\Omega_{0w}\left(\frac{a_0}{a}\right)^{1+3w} + (1-\Omega_{0w})\right]. \tag{2.25}$$

In this equation, Ω_{0w} can be interpreted both as an implicit sum on the various components of the Universe and as an expression of the single component dominating in a certain age of the Universe.

Parameter	Value	Primary Sources
Hubble Constant	$H_0 \simeq 70\mathrm{km} \cdot \mathrm{s}^{-1} \cdot \mathrm{Mpc}^{-1}$	Riess et al., 2022 Aghanim et al., 2020
Cosmological Constant	$\Omega_\Lambda = 0.70$	Riess et al., 1998a Jaffe et al., 2001
Matter	$\Omega_m = 0.30$	Riess et al., 1998a Jaffe et al., 2001
Baryonic matter	$\Omega_b = 0.04$	Jaffe et al., 2001
Dark matter	$\Omega_{\mathrm{CDM}} = \Omega_m - \Omega_b = 0.04$	
Curvature	$\Omega_k = 0$	de Bernardis et al., 2000

Table 2.1: An estimate from some authoritative studies of the cosmological parameters at the present-day Universe and the primary sources we used to obtain them.

The main components of the Universe are: matter (both baryonic and dark matter) with $w_m = 0$, radiation with $w_r = 1/3$ and dark energy $w_\Lambda = -1$. Thus a components-explicit version is:

$$H(t)^2 = H_0^2 \frac{a_0}{a} \left[\Omega_{0m} \left(\frac{a}{a_0} \right) + \Omega_{0r} \left(\frac{a}{a_0} \right)^2 + \Omega_{0\Lambda} \left(\frac{a}{a_0} \right)^{-2} + (1 - \Omega_{0m} - \Omega_{0r} - \Omega_{0\Lambda}) \right]. \tag{2.26}$$

An estimate of the values of each energy density parameter is given in the table 2.1[1]. [Coles and Lucchin, 1995 IVI, pg. 35 and 49]:

2.2.7 Scale factor

Combining equations 2.11 and 2.12 we obtain an explicit formulation of the scale parameter:

$$2 \left(\frac{\ddot{a}}{a} \right) + \left(\frac{\dot{a}}{a} \right)^2 = \Lambda \, c^2. \tag{2.27}$$

The solution is [Grøn, 2002]

$$a(t) = A^{\frac{1}{3}} \sinh^{\frac{2}{3}} \left(\frac{t}{t_\Lambda} \right), \tag{2.28}$$

where $A = \frac{\Omega_{m,0}}{\Omega_{\Lambda,0}} \simeq 0.43$ and $t_\Lambda = (4/3 \ \Lambda \ c^2)^{1/2} \simeq 3.4 \times 10^{17}$ s.

---

[1] The value of H_0 is currently a matter of debate as there are at least two classes of measures that give incompatible results. The value assumed by us is an approximation because as far as we are concerned it does not affect our work. For further information, we recommend the article by Efstathiou, 2021

2.3 What is a horizon?

The word *horizon* derives from the Greek verb ”οριζων” (horizōn), “to divide”, “to separate”; also in a cosmological contest it represents a boundary between two regions of spacetime.
Cosmological horizons set the size and scale of the observable Universe: we can define two kind of cosmological horizons and a third similar concept: the *event horizon*, the *particle horizon* and the *Hubble sphere*.
These three objects are spherical surfaces defined by a spherical symmetry with respect to a central observer and a radius.

2.3.1 Particle horizon

For any observer **O** we can define two regions in the instantaneous three-dimensional space at $t = t_0$. The first one is the region defined by the events that have already been observed by **O**, while the second one is its complement in the three-dimensional space, i.e. the region that cannot be observed by **O** at time t_0. The boundary between these two regions is the *particle horizon* at t_0, which defines the observable Universe for **O**. Notice that the particle horizon takes into account only the past events with respect to **O** [Rindler, 1956].
Then the set of points that could have communicated with **O** must be inside a sphere centered upon **O** with proper radius:

$$R_p(t) = a(t)\; c \int_0^t \frac{\mathrm{d}t'}{a(t')}. \tag{2.29}$$

The definition of this horizon is given by equation 2.8 where the emission time $t_e = 0$. There is the possibility that the integral diverges because $a(t)$ tends to zero for small t. In this case the observer in **O** can, in principle, have received light signals from the whole Universe. If, on the other hand, the integral converges to a finite value with this limit, then the spherical surface with center **A** and radius $R_p(t)$ is called the **particle horizon** at the time t of the observer. Thus the observer cannot have received light signals, at any time in his history, from sources at proper distances greater than $R_p(t)$ from him at the time t.
By combing equation 2.25 and equation 2.29 we obtain:

$$R_p(t) = \frac{c}{H_0}\frac{a(t)}{a_0} \int_0^{a(t)} \frac{da'}{a'\left[\Omega_{0w}\left(\frac{a_0}{a'}\right)^{1+3w} + (1-\Omega_{0w})\right]^{\frac{1}{2}}}. \tag{2.30}$$

The integral in 2.30 can be divergent because of the contributions near to the Big Bang, when $a(t)$ is tending to zero. At such times, the second term in the square brackets is negligible compared with the first and one has:

$$R_p(t) \simeq \frac{c}{H_0 \Omega_{0w}^{1/2}} \frac{2}{3w+1} \left(\frac{a(t)}{a_0} \right)^{3(1+w)/2} . \tag{2.31}$$

$R_p(t)$ has a finite value and which also vanishes as $a(t)$ tends to zero. It can also be shown that:

$$R_p(t) \simeq 3 \frac{1+w}{1+3w} ct. \tag{2.32}$$

The solution 2.32 is valid exactly in any case if $\Omega_w = 1$; interesting special cases are described in eq. 2.33 and 2.34 respectively for the flat dust model for a flat radiative model [Coles and Lucchin, 1995 IVI, pag. 45].

$$R_p(t) = 2ct, \tag{2.33}$$

$$R_p(t) = 3ct. \tag{2.34}$$

2.3.2 Event horizon

Another horizon could be defined taking into account **O**'s future. Qe can introduce the *event horizon*, the hyper-surface in spacetime which divides all events into two classes: those that will be observable by **O** and those that are forever outside **O**'s range of observation. This horizon determines a limit in the future observable Universe [Rindler, 1956]. The future of **O** is considered since a fixed time point t.

The mathematical definition is the same as in 2.29 but with the limits of the integral going from t to a t_{max}, which is either t_f (the time of the Big Crunch) in a closed model or $t = \infty$ in a flat or open model. Thus the radius of the event horizon is given by

$$R_e(t) = a(t)\, c \int_t^{t_{max}} \frac{\mathrm{d}t'}{a(t')} \tag{2.35}$$

The event horizon does not exist in Friedmann models with $-1/3 < w < 1$, but it does exist in a de Sitter model [Coles and Lucchin, 1995 IVI, pg. 47].

2.3.3 Hubble radius

The *Hubble radius* is defined as the distance from **O** of an object moving due to the cosmological expansion at the velocity of light with respect to O. This can derive very easily from the definition 2.15 using c as recession velocity:

$$R_c = c\frac{\dot{a}}{a} = \frac{c}{H}. \tag{2.36}$$

If $p > \frac{1}{3}\rho c^2$, the value of R_c can be used to approximate the value of R_p [Coles and Lucchin, 1995 IVI, pg. 45]:

$$R_c = \frac{3}{2}(1+w)ct = \frac{1}{2}(1+3w)R_p \simeq R_p. \tag{2.37}$$

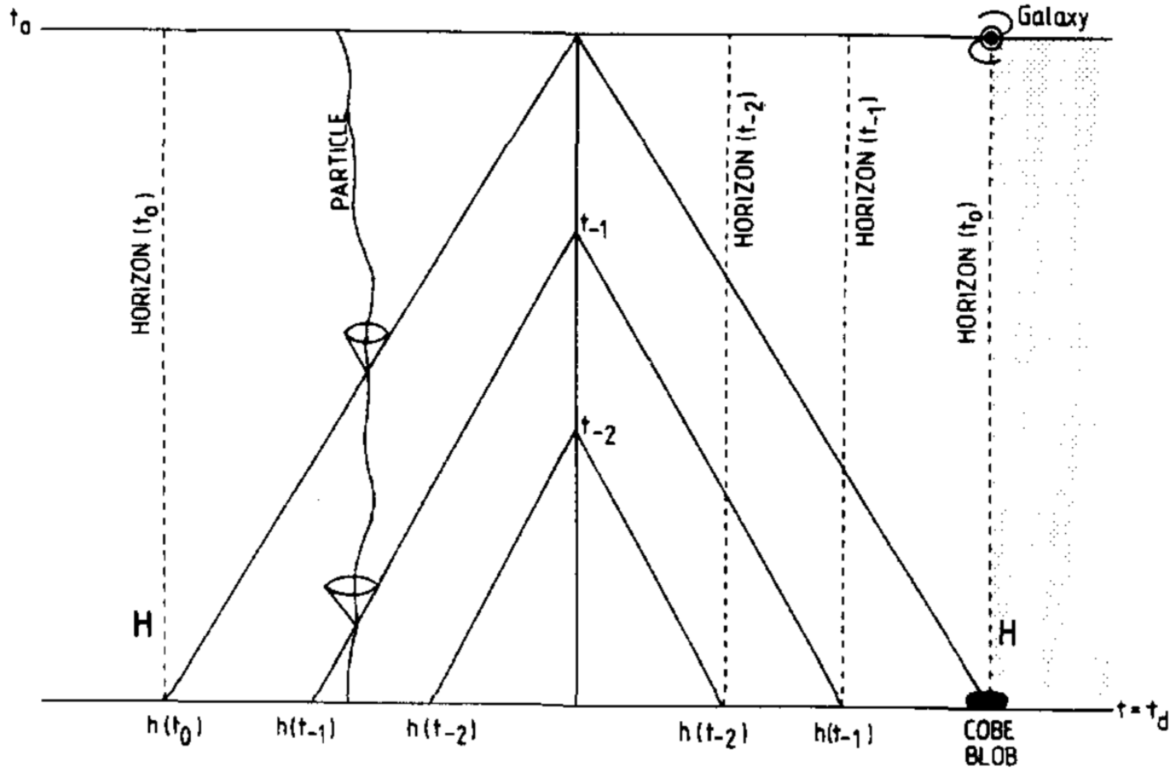


Figure 2.2: "The traditional" depiction. It was a commonly used representation in the past, the value of the particle horizon at time t is obtained from the oldest value of the past light cone of an observer at time t. This type of visualization has a construction of the horizon linked to a concept of the known Physics but at the same time the amount of information transmitted is small because each cone of light can be summarized in a single point.

2.3.4 Graphic representations of the horizons

The last feature of a horizon we have to cover to complete this brief treatise is are the possible depictions through graphics. We propose two kinds of graph, one that could be found in

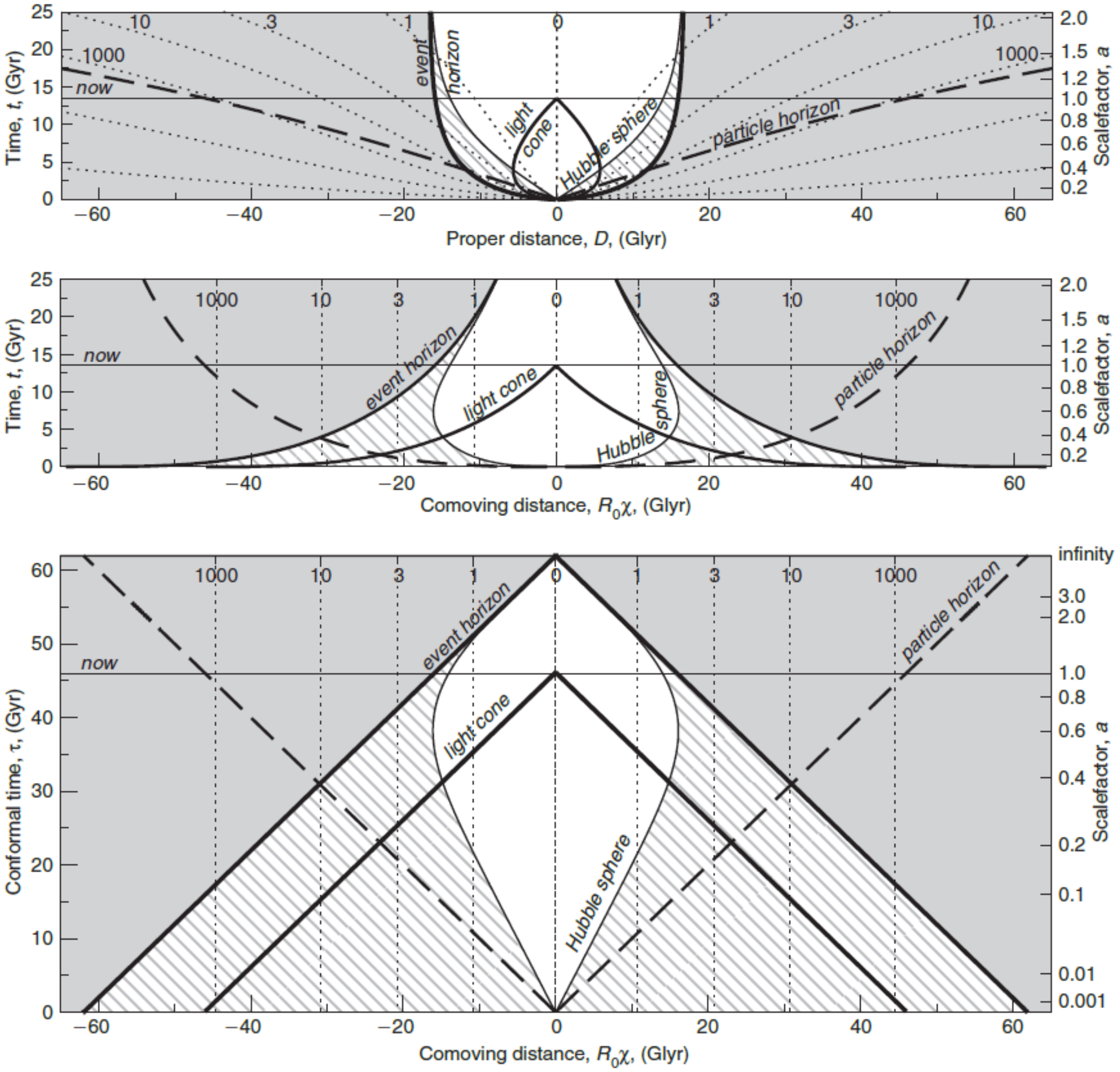


Figure 2.3: ”The alternative” depiction. This modern representation contains more information than that in image 2.2, sacrificing the physical significance of horizons. In this case we have three representations with different coordinate systems: at the top we have proper coordinates on the abscissa and time on the ordinate, at the center comoving coordinates and time, while at the bottom comoving coordinates and time conformal. The different curves shown are mostly labeled on the graph itself and the dotted-lines represent the world-lines of objects at rest with respect to the Universe.

many book of cosmology and an interesting alternative. A third valuable option is the Penrose diagram but, despite being an important theoretical tool, it is not appropriate for an early approach due to the need of prior knowledge in the field of general relativity.

The traditional A graph commonly used to represent horizons and analogous concepts is shown in figure 2.2.

It is a bidimensional graph: space is represented in the abscissa axis (that is reduced to one dimension due to symmetries) using comoving coordinates while the conformal time on the ordinate axis. At the center of the graph there is a vertical line that represents the world-line of an observer: from him past light cones are generated at different times. The length of the semi-base of the light cone triangle corresponds to the value of the particle horizon at the time the light cone is generated. In addition to this, it is possible to distinguish the world-lines of two other objects seen by the observer at different times.

This type of graph gives a ”more physical” meaning of what the particle horizon represents, but at the same time it contains a limited number of information.

The alternative A different approach to depict the same notions but with more information was proposed by Kiang and can be seen here in figure 2.3 [Kiang, 1991].

In fig. 2.3 we have three different spacetime diagrams: in the upper (hence on 2.3.A) proper distance and cosmic time are used, in the middle (2.3.B) we see comoving distance and cosmic time and in the bottom one (2.3.C) comoving distance and conformal time τ defined as:

$$\tau = \int_0^t \frac{dt'}{a(t')}. \tag{2.38}$$

The graph refers to the present time with the horizontal line ”now” and with respect to an observer in a certain position at that time, it shows:

- The **world-lines of comoving objects** represented by dotted lines; these lines are vertical in comoving coordinates (B and C) but this is not true if we consider the proper distance (A) where the recession velocity is not incorporated in the coordinate system.
- The **past light cone** of the observer at time $t =$ now.
- The **particle horizon** represented as the future light cone in the position of the observer at time $t = 0$.

- The **event horizon** represented as the past light cone in the position of the observer at time $t = \text{max}$.
- The **Hubble radius** represented as the ensemble of points reading the speed of light in recession at a given time.

The advantage with respect to the previous graph is that the functions represent the exact value of horizon lengths at a given time, thus containing more information.

2.4 Why is this a problem?

In this section, we will highlight how the founding principles of the modern cosmology lead to contradictions with observations. We will address the problem firstly in a discursive way to frame the main ideas, then in a more formal way.

2.4.1 A loop of apparent contradictions

The theory of a **Hot Big Bang** is the most widely accepted hypothesis for the origin of the Universe: indeed it offers a comprehensive explanation for a broad range of observable phenomena, including the abundance of the light elements, the CMB, the large-scale structure and Hubble's law. The theory depends on two major assumptions: the universality of physical laws and the **Cosmological Principle**.

The most general spacetime metric describing a Universe that obeys to the Cosmological Principle is the **FLRW metric** (equation 2.3) [Weinberg, 1972, sect. 13.1 and 13.5].

Homogeneous and isotropic Universes described by the theory of general relativity possess cosmological horizons under certain conditions. Thus, regions sufficiently distant from each other cannot have been in causal contact ('have never been inside each other's horizon') at any stage since the Big Bang [Coles and Lucchin, 1995 IVI, pg. 5 and 45].

The existence of a cosmological horizon makes it difficult to accept the Cosmological Principle. This principle requires that there should be a correlation (a very strong correlation) between the physical conditions also in regions outside each other's particle horizons which, therefore, have never been in causal contact. For example, the observed isotropy of the microwave background implies that this radiation was homogeneous and isotropic in regions on the last scattering surface. Thus, the problem is how to explain why the Universe appears homogeneous on scales much larger than the one we expect to have been in causal contact up to the present time. In other words, if two regions of the Universe have never been able to communicate with each other by light signals, how can they even know the physical conditions (density, temperature, etc.) of each other? If they cannot know this, why did they evolve in such a way that these conditions are the same in each region? One either has to suppose that causal Physics is not responsible for this homogeneity or that the calculation of the horizon is not correct.

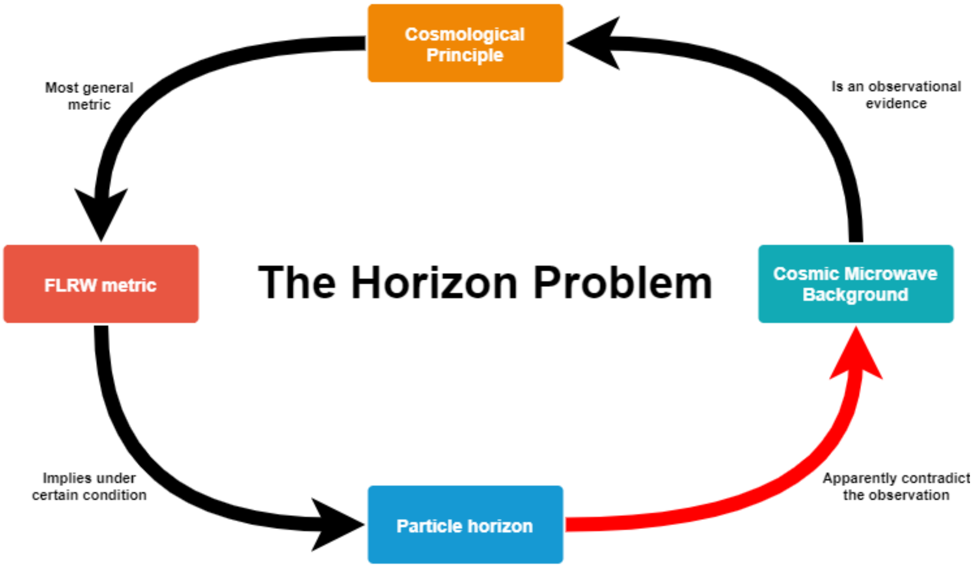


Figure 2.4: This is a summary of the horizon problem: the construction of a metric from the cosmological principle leads to observations that apparently contradict it.

2.4.2 Horizon during last scattering

The distance described in equation 2.29 defines the causally connected regions. Between about 47 000 years and 9.8 billion years after the Big Bang, [Ryden, 2006, eq. 6.33 and 6.41] the energy density of matter exceeded respectively both the energy density of radiation and the vacuum energy density[Zeilik and Gregory, 1998, page 433]. If we want to evaluate the distance of the horizon at the time in which the photons of the cosmic microwave background have decoupled from the matter (and therefore in the epoch of matter), we can approximate the scale parameter as $a(t) = \left(\frac{t}{t_0}\right)^{2/3}$ that leads to the expression of the horizon seen in equation 2.33.

Using equation 2.25, we can make explicit the value of $H(z_{LS})$[2], neglecting all the components excluding matter. We obtain an approximate version of the equation:

$$H(z_{\mathrm{LS}}) = H_0 \Omega_{m_0}^{1/2} (1+z)^{3/2}. \tag{2.39}$$

Substituting the respective numerical values for the constants, we have that $H(Z_{\mathrm{RF}}) = 4,6 \times 10^{-14}\ \mathrm{s}^{-1}$ for $\Omega_{m_0} \simeq 0.3$ and $z_{LS} \simeq 1100$. This leads to a value of $d_{\mathrm{h}}(t_{\mathrm{LS}}) = 0.4\ Mpc$. It is possible to express this distance in terms of angular distance through the relation:

$$d_A = \frac{d_{\mathrm{h}}(t_0)}{1 + z_{\mathrm{LS}}} \simeq 13\ \mathrm{Mpc}. \tag{2.40}$$

Since the angular distance is related to the angle θ via $d_A = \frac{l}{\theta}$:

$$\theta_{\mathrm{LS}} = \frac{d_{\mathrm{h}}(t_{\mathrm{LS}})}{d_{A_{\mathrm{LS}}}} \simeq 0,03\ \mathrm{rad} \tag{2.41}$$

which is equivalent to about 1.7 degree. This means that the only points in the Universe that could have exchanged information at the time of the photon emission were enclosed in an angle of about 1.7°.

2.5 Beyond the horizon problem

The horizon problem together with the flatness problem are issues that have shaped the developments of modern cosmology. The most widely accepted solution to these problems is the theory of inflation [Guth, 1981].

There are currently numerous cosmology articles that consider inflation as a fact and also university courses dedicated to "cosmology of the early Universe" are mainly focused on this model. To learn more about this topic requires a solid knowledge of cosmological horizons (particularly the particle horizon) both to understand the problem that led to the formulation of this model and to understand the behavior of horizons during inflation. In the next chapter we will highlight the delicate steps in the understanding of cosmological horizons.

---

[2] z_{LS} is the redshift value associated to time of last scattering

Chapter 3

Difficulties in learning cosmological horizons

One of the aspects addressed in Physics education research is the nature and development of students scientific ideas.

The better we understand the properties of students' ideas – whether they are coherent or contradictory, context-dependent or independent – the better we may explain and predict students' thinking. The initial non correct conception of students about a particular scientific topic are sometimes referred as misconceptions.[Muller et al., 2008, Scherr, 2007].

As far as the didactic of Cosmology is in concerned, the research is still at its initial states and there are no studies on how students understand the topics covered in the second chapter of this thesis. Nevertheless, there are some works that may be useful. In particular, Davis[1] pointed out several common misconceptions about the expansion of the Universe, showing their presence in the associated literature. Ellis [2] revisited the topic of horizons in a simple language emphasizing the critical points of learning.

Clearly, if these misconceptions stand out in an academic publication as shown by Davis, it is reasonable to assume that students at the first approach with this topic could also be affected by these problems.

Others interesting cues can be found in the articles addressing the misconceptions in special relativity, which share some similarity with the study of horizons.

In the next pages these difficulties will be organized starting from the finding of the articles [Davis and Lineweaver, 2004,Ellis and Rothman, 1993].

Differentials changing over time W. Rindler in his work "Visual Horizons in World Models" (1956) pointed out some ambiguities that can lead to conceptual inconsistencies:

> "…the meaning of many phrases used in discussions of horizons such as, for example, 'all particle of one side of the horizon','crossing the horizon with the speed of light', etc., evidently depend critically on the definitions of time and distance whose diversity is enormous. A statement meaningful and valid on one interpretation can be meaningless or false on another…" [Rindler, 1956]

Here, the highlighted problems result from the use of different spacetime coordinates, in particular the use of proper distance and proper time compared to the comoving distance and cosmic time. This can generate confusion and apparent paradoxes.

---

[1] Davis and Lineweaver, 2004

[2] Ellis and Rothman, 1993

Rindler's quote can be proved with a simple analysis of the FLRW metric, which is studying the behavior of a ray of light. As shown in section 2.2 the light-like trajectory is defined by the relation:

$$ds^2 = 0 \longrightarrow\ c^2 dt^2 = a(t)^2 dr^2 \tag{3.1}$$

A criticism emerges due to the coordinate system. It would seem that in a time interval dt, the traveled distance dr in the spacetime relies exclusively on $a(t)$. Indeed it results that in a matter-dominated Universe $a \propto t^{2/3}$ while in a radiation-dominated one $a \propto t^{1/2}$: thus the distance traveled by light is different.

The point is the choice of the coordinate system in equation 2.3, in this case the comoving coordinates. If in equation 3.1 we consider a fixed dt and an increasing $a(t)$, the dr distance must be decreasing [Ellis and Rothman, 1993].

The understanding of the coordinate systems belong to a wider collection of problems in General Relativity within a didactic context where the student is familiar only with Special Relativity. Indeed General Relativity adopts arbitrary coordinate to formulate the laws of Physics in order to predict motion in not-global inertial reference frame, obviously some choices of coordinates are more "natural" or allow working in a simpler way.

3.1 Super-luminal recession

The most common misconception about the Universe expansion is that it cannot be faster than light [Davis and Lineweaver, 2004, Appendix B 1-8]. It is originated by the limiting description of Special Relativity as Hubble's law predicts superluminal recession at large distances ($D > c/H$). In some texts this problem is solved extending the Hubble's law [Davis and Lineweaver, 2004, appendix B: 6-7]. However it is widely accepted that general relativity (GR), and not special relativity (SR), is necessary to describe cosmological observations.

Phenomena like **the relation between time dilation and redshift for type Ia supernovae** [Goldhaber et al., 2001, Goldhaber et al., 1997, Leibundgut et al., 1996, Riess et al., 1998b] and **the relation between magnitude and redshift for type Ia supernovae** [Riess et al., 1998b, Perlmutter et al., 1999] demonstrate the inadequacy of the SR in explaining the data, in favor of the GR. The superluminal recession does not result in a conflict with the theory of SR as the motion with speeds higher than the speed of light does not happen in an inertial reference frame. In SR a value of redshift is directly associated with a speed. Thus Hubble in

1929 converted the redshift of the "nebulae" in speed, predicting the relation between velocity and distance which now has his name.

The relationships between redshift and speed used in GR and SR are [3]:

$$\textbf{GR} \quad v_{\text{rec}}(t, z) = \frac{c}{R_0}\,\dot{R}(t) \int_0^z \frac{dz'}{R(z')}, \tag{3.2}$$

$$\textbf{SR} \quad v_{\text{rec}}(z) = c\,\frac{(1+z)^2 - 1}{(1+z)^2 + 1} \qquad . \tag{3.3}$$

These speeds are given with respect to a comoving observer looking at an object with redshift z. The description given by GR has an explicit dependence on time, thus allow to specify the speed of recession in different epochs, which is not conceived in SR.

Equation 3.3 has however been used for a long time to convert redshifts into speed, as it closely approximates the trend of equation 3.2 for low values of z.

Super-luminal recession occurred only during inflation

In some text inflation is described as a super-luminary expansion [Davis and Lineweaver, 2004-Appendix B: 22-23] but this is not correct: in fact, as we have seen previously, due to the nature of the law of Hubble, the expansion is always super-luminal for sufficiently large distances. There is therefore no real distinction in the super-luminary recession between the inflationary phase and that post-inflationary. The limit in these phases is determined by the Hubble sphere ($D > \frac{c}{H}$), being the value of the Hubble constant larger during inflation.

3.1.1 Super-luminal recession and observability

Another misconception appears in several texts [Davis and Lineweaver, 2004, Appendix B: 9-13] and can be summarized as: "Recession at speeds greater than that of light implies unobservability". This is not always true as it is possible that photons, in a portion of space with super-luminary recession, subsequently become visible again.

This mechanism is linked to the evolutionary history of the **Hubble sphere** (eq. 2.36) compared to the **Particle horizon**.

In a decelerating Universe, the value of H tends to decrease since $\dot{a}$ is decreasing. In an accelerating Universe, H still decreases as the value of $\dot{a}$ grows slower than a. Thus in both cases

---

[3] Davis and Lineweaver, 2001

the Hubble sphere shrinks. As long as the Hubble sphere recedes faster than the photons immediately external to this region, we will have a passage of photons from a super-luminary to a sub-luminary recession zone. Consequently, they become visible again.

Figure 2.3 show us that the observed photons that were emitted in the first 5 billion years of the Universe were outside the Hubble radius at the time of their emission. Evaluating the super-luminary recession limit in equation 2.15 $v_r = c$ and using the value of H (eq. 2.24) with the conditions $(\Omega_M, \Omega_\Lambda) = (0.3, 0.7)$ (neglecting the others ones) we obtain a redshift value $z = 1.46$. Thousands of galaxies with redshifts higher than this were observed.

The largest observed redshift value is $z = 6.68$ [H. Chen et al., 1999] and Sloan Digital Sky Survey identified 4 galaxies with $z > 6$ [Fan et al., 2004].

Indeed, the observational limit is given by the horizon of the particles and not by the Hubble sphere which is not a horizon.

3.2 The particle horizon in space-time diagrams

As we mentioned in section 2.3.4, the most common representation of the particle horizon is the **past light cones**, a series of events $\mathbf{E}_i$ taking place at different times t_i in the position of the observer at present time. The limits of this kind of graphs are given both by the little information provided on the evolution of the horizon and by the profile of null geodetic of the light cone that can be misunderstood with the evolution of the horizon itself.

The information that this graphic provides is limited and it is hard to interpret the horizon evolution correctly. The boundary of light cone can be misleading especially if the coordinate system does not give the classical 45 degree lines for the light ray. Indeed it *does not* represent a boundary between events we can see and events we cannot see, nor it represents the distance from the horizon of particles at different times.

The alternative representation taken from the article by T. Kaing provides more information on the temporal evolution of the horizon (fig. 2.3): in that graph from each single point of the curve it is possible to build a graph like the one in figure 2.2. [Davis and Lineweaver, 2004]

3.2.1 Other possible misconception

The concepts we have dealt with so far have a history of misunderstandings in the context of academic publications. However, there are other concepts that are considered insidious

[Nemiroff and Patla, 2008, Pössel, 2020] and deserve a mention:

Reference system at rest with respect to the Universe By “rest with respect to the Universe” we mean reference systems that do not measure multipole anisotropies with respect to the CMB. Analysis of the dipole anisotropies made by the Earth show that the Milky Way is moving at approximately 600 km/s with respect to the reference system [Leibundgut et al., 1996].
In special relativity there are no privileged reference systems while in cosmology these often appear in the study. The existence of these reference systems could therefore be counterintuitive [Nemiroff and Patla, 2008].

Energy conservation There is at least one type of known energy that has changing components as the Universe expands: radiation. Individual photons, for example, undergo cosmological redshift and lose energy individually as the Universe expands.
More generally, the speed relative to the cosmic rest frame of any freely moving component of any form of energy will decay as the Universe expands. This decrease in kinetic energy is correlated to a reduced speed. There is no potential energy equivalent for Friedmann’s cosmology. In general, energy is not conserved over large distances and times in general relativity [Nemiroff and Patla, 2008]. This type of concept can be counterintuitive [Pössel, 2020].

3.3 Educational objectives

Now we can try to define teaching objectives for educational project. They were used in the individual design choices of the educational animation we will focus on in the next chapter.

1. **Recognize how to use the light cone.** The light cone is one of the most important topics to capture the causal structure of the spacetime. It specifies the events (that is, which points of space and time) causally connected by trajectories covered by particles slower than light, events that can be connected by trajectories covered by particles traveling at the speed of light and events that cannot be connected by anything traveling at or below light speed. Events in the first group are “timelike” as a physical clock could travel from one event to the other. Events in the second group are “lightlike” as a light

ray can travel from one to the other. Events in the third group are “spacelike”.
We can specify sub-objectives:

   (a) To interpret correctly the spacetime graph in Minkowski metric, knowing how to associate points with events in space-time.
   (b) To distinguish the different elements of the graph and their respective meanings : the observer/emitter, the area of timelike events respect the to observer, the line of lightlike events respect the to observer, the area of space-like inaccessible events to the observer, the past and the future light cone.
   (c) To distinguish the meaning of time variables in the function $d_{lc}(t_0, t)$ that defines the light cone.

2. **Understand the expansion of the Universe from a phenomenological point of view.**
   Student must:

   (a) Differentiate the differences between the Minkowski metric in spherical coordination and that of FLRW (introduction of the scale parameter).
   (b) Explain the Hubble’s law and the distance dependence of Hubble flow.
   (c) Identify Hubble flow behavior at “Hubble radius” distance and why this phenomenon apparently this seems in contradiction with the limit of light speed [4].

3. **Understanding and distinguishing the concepts of comoving distance and proper distance**
   Students have:

   (a) To describe how the comoving coordinate evolve with scale factor. [5]
   (b) To recognize that proper distance is determined at a fixed moment of time as if we could measure distance in frozen time.
   (c) To identify the relation between proper and comoving distance.

---

[4] Davis, Tamara M. and Charles H. Lineweaver (2004). “Expanding Confusion: Common Misconceptions of Cosmological Horizons and the Superluminal Expansion of the Universe”.

[5] Ellis 1993 Lost horizons

4. **Understand how the expansion of the Universe affects the geodesic of light and consequently the light cone.**

   In particular, an emphasis is on the asymmetry between the past light cone and the future light cone.

5. **Understanding of cosmological horizons.**

   These are the fundamental concepts of this teaching unit, in particular the sub-objectives are:

   (a) To describe what the **particle horizon** represents and to frame it as a future light cone obtained at the time $t_0 = 0$.

   (b) To describe what the **event horizon** represents and to frame it as a past light cone obtained at the time $t_0 = \infty$.

   (c) To describe what **Hubble radius** represents.

   (d) To determine that the Hubble radius does not represent a horizon but a surface beyond which the recession becomes super-luminar and its usefulness for approximations in certain contexts.

   (e) To compare the Hubble radius with the particle horizon in order to highlight the misconception[6] related to the observability of objects faster than light.

   (f) To compare the Hubble radius and event horizon.

   (g) To be able to interpret correctly the diagram in figure 2.3 that shows the plots of horizons and other elements of interest over time.

---

[6] Also Davis (2004)

Chapter 4

Animations design

In this chapter, we will illustrate how we developed the simulations of our teaching unit. The approach is a simplified version of the one used by the University of Colorado for the PhET project. This process is divided into several phases [Adams et al., 2008b]: the first of these consist in the development of a trial version for exploratory interviews.
We implemented specific design for the first version of our simulations in order to achieve the set teaching objectives. These choices take into account the guidelines provided by the PhET project itself and some assumptions rely on modern studies (see chapter 1).

The simulations we have built for this teaching unit are: **Hubble flow**, **Light cones comparison** and **Horizons evolution**.
In the next section, we will describe all the graphical aspects and the interaction modalities provided, with reference to the contents of chapter 1. Then we will explain how we developed the simulations starting from the Physics theory presented in chapter 2.

4.1 Description of graphics and interactive elements

Only one of the three simulations is an explicit representation of a physical phenomenon. The other two are interactive graphs: their strength is on the time controls converging more information than their static equivalent.
Although the contents of the animations are different, there are some common strategies.

4.1.1 General strategies

Time control The main difference between our animations and their static equivalent printed on a paper is the chance of seeing a temporal evolution. The available literature does denies the value of the time-pacing strategy (section 1.2.2) [Moreno and Mayer, 2007, Adesope and Nesbit, 2012, Boucheix and Guignard, 2005] and thus we opted for controls similar to those used in video-playback software to increase the interactivity of the applications. This option combined with guided activities allows creating a greater involvement in the student that is related to a higher quality of learning [Rieber et al., 1998, Kearsley and Shneiderman, 1998].
The time commands and their description are shown in figure 4.1.

Figure 4.1: The time controls feature include two buttons, a time bar and an indicator.
The first button from the left is the "PLAY/PAUSE" button: the symbol associate is the typical play triangle and is active when it is blue and off when red. On its right we have the button "RESET" characterized by an "R" inside: pressed, it sets the time to its initial value.
To the right of the button "RESET" we have the time bar: the user can interact clicking in any point of this, thus setting the time to a value proportional to the chosen position.
Above we have an indicator showing the value of the current time of the animation and the value of the associated scale factor (with $a(13.8 \text{ gyr}) = 1$).

Simultaneous display of multiple representations Another common aspect in all animations is the representation of the same concept in different ways or the comparison between similar concepts that benefit from mutual comparison. For instance, in "Hubble radius" and "Horizon Evolution" we can simultaneously see two types of representations for the same concepts, in "Light cone comparison" animation we see the light cone in different metrics: indeed, the comparison between two images leads to more frequent cognitive activations than a sequential presentation (section 1.3.2). Moreover, with respect to the "Horizon Evolution" and "Hubble Flow" animations, we used simultaneous representations differentiating between "explicit time" and "single time" representations (section 1.2.3). This approach appears to have a greater educational effectiveness [Ainsworth and Van Labeke, 2004].

Colors and symbols The use of colors and symbols associated with some concepts must follow the so-called "Coherence principle", mentioned both in PhET guidelines and in MML theory [Adams et al., 2008b, R. Mayer, 2002]. The idea behind is fairly straightforward: the use of an inconsistent representation causes an increase in extraneous cognitive load, thus depriving mental resources to the learning process.
Moreover, bright colors, strongly contrasting with the background, are a useful tool to direct attention to the concepts of interest [Lowe and Boucheix, 2008b].
For this reason the color palette has been chosen to have the main concepts: well distinguishable, consistent between the various animations and standing out from the background. The

coordinate system and didactic tools are represented with softer and less eye-catching colors. Finally we used black for the background, allowing the other colors to stand out. Our preference is consistent with the PhET guidelines expressing that the background must "suggest" the educational context. Thus the black seemed the closet choice to the cosmological content of the animations.

Tools The teaching tools implemented in our simulations are mainly used to measure distances. They give a greater depth to teaching activities, having not only qualitative answer but also numerical ones. Moreover, the PhET guidelines indicate the inclusion of teaching tools as a strategy to increase student involvement.

Option and button interface Each simulation has a series of user-configurable options; they are managed through an interface of buttons with a short text. This is white but it can be colored, thus creating a perceptive association between features with the same color.
We have two types of buttons, one keeps the information on/off (such as those activating are instrument) and the other one are activated only when they are pressed (such as the "reset" button or "set time to today").
The colors that can assume the button are: blue corresponds to ON, red to OFF while a pale yellow indicates that the cursor is over the button. This option that the cursor is in the correct position.
Students are free to manage the amount of objects on the screen and then its cognitive load turning on and off their presence on the screen. Most of these options are set to OFF at the beginning of the animation as suggested by the PhET guidelines being students more motivated to activate new invisible content.

Legend and additional info The last common elements of the animations are legend and other additional information such as formulas. Literature suggest limiting their presence, but in the educational context of our research some used elements as the mathematical background are not intuitive.

4.1.2 Hubble Flow

This first interactive animation aims to intuitively represent how of the expansion of the Universe acts on objects at various distances from an observer, and on the geodesics of photons. The simulation consists of two closely related sketches:

1. The **"time singular simulation" (TSS)** is a 2D representation of the Universe. Whenever the play button is pressed the time of the simulation flows together with the time of the real world, with one seconds lasting 1.5 gry [1]

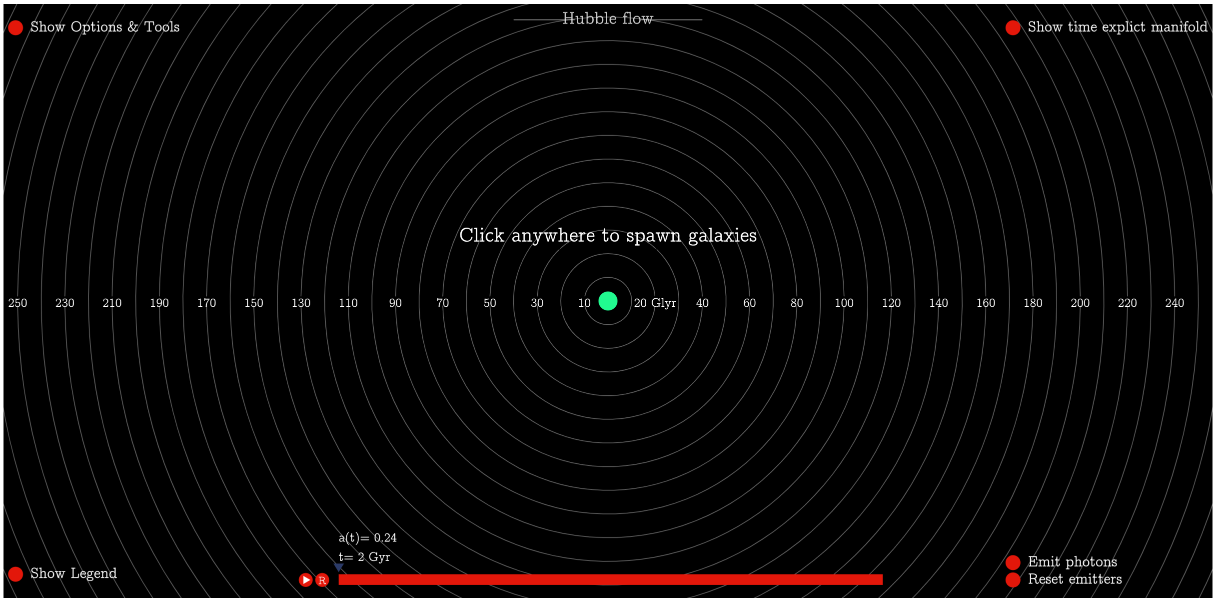
2. The **"time explicit simulation" (TES)** is a 3D representation of a curved surface, showing an alternative description of TSS simulation. Here time is represented along the vertical axis, while the distance between the observer and the emitter are mapped in the azimuth. Finally, the radial coordinate maps the scale factor.

The simulation consists of these two sketches TSS and TES depicting two different views of the same phenomenon. The interface allows to view only the TSS (figure 4.2 to 4.6) or both simulations via the "Show time Explicit manifold" button (figure 4.7). Indeed the TES is meant to be only a support to the main simulation which is TSS.
We will now describe how the simulations work and their main features:

TSS Figure 4.2 shows the simulation at the beginning. It depicts the time singular representation of the expansion of the Universe on cosmic scales. In our window we can see: a circle filled in light green representing our observer, a series of numbered concentric circles representing our coordinate system (by default it is set on comoving), time controls at the bottom center and five different buttons at the corners of the image.
If we click somewhere on the screen (as suggested) it is possible to generate up to 4 galaxies (fig. 4.3). Any additional click would create a new galaxy canceling the oldest one.

---

[1] The framerate it's 30 fps the time interval between two frame it's 0.05 gyr



Click anywhere to spawn galaxies

Figure 4.2: The application at the beginning. In the upper left, there is while the button "Show Options and Tools" at the bottom left, the "Show legend" button. In the upper right, there is the button "Show time explicit manifold" leading the simulation to the state shown in image 4.7. Finally, in the lower right there are the two buttons "Emit photon" whose effects can be seen in the image 4.4 and the button "Reset emitter" which to resets the state of the emitters. At the bottom center is the time bar described in figure 4.1

Figure 4.3: Starting from the situation in Figure 4.2 clicking with the mouse anywhere on the screen, the simulation creates galaxies.

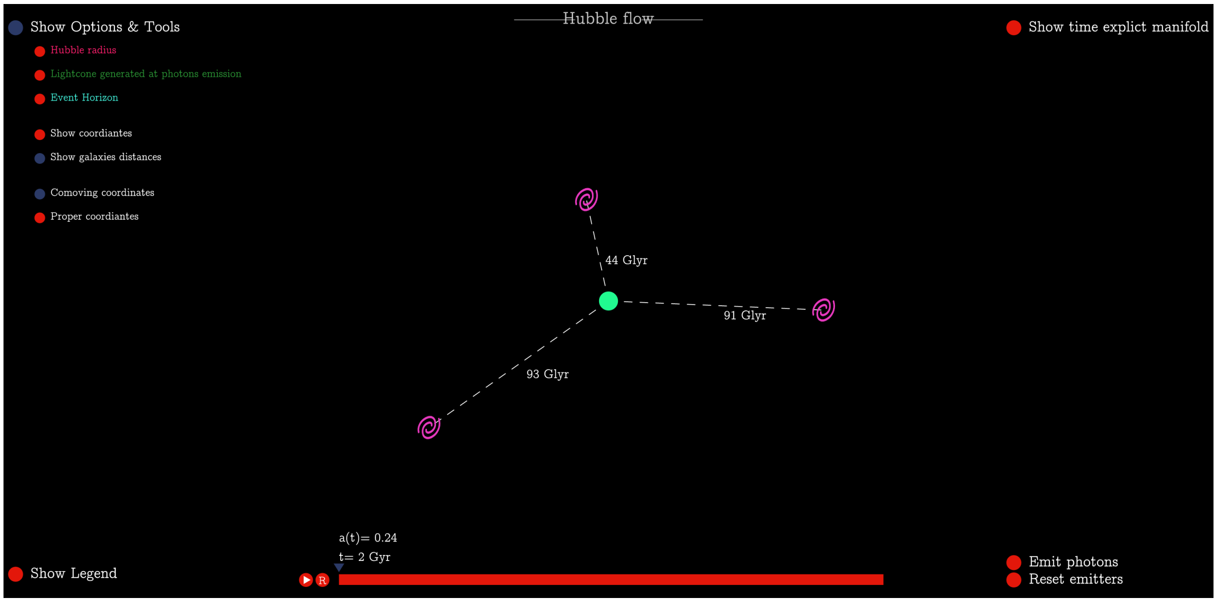


Figure 4.4: In this image we see the open drop-down menu, the coordinate option is disabled while the option showing the precise distance of galaxies is active. These two options are managed by the buttons "Show coordinates" and "Show galaxies distances".

Two purple spiral curves are the symbol used to represent the galaxies, as the legend shows. It is intuitive to anyone familiar with these bodies.

Once the time has started, these galaxies will be subjected to the Hubble flow and will start moving away from the observer with a speed proportional to their distance.

At any time, the user can emit a photon from galaxies in the direction of the observer by clicking on the "Emit photon" button (fig. 4.4). This photon is represented by a yellow curve, referring to the stereotypical wave packet shape. Thus a Physics student who approaches this topic should be familiar with this form. The choice of yellow is made as a matter of contrast with the background. In a subsequent evolution of the animation we could include a color change of the photon to recall the phenomenon of cosmological redshift.

Finally under the button "Emit photons" there is the button "Reset emitters". It allows to delete all the galaxies in the simulation and erase the emission time information from memory. Individual galaxies can be removed simply clicking on them.

As galaxies tend to come out of the screen, a zoom in and out interaction has been mapped to the mouse wheel.

The range in which this zoom is possible increases as the Universe expands to balance this effect.

Options and tools In the upper left corner of the screen there is the button “Show Options and Tools”. It allows to open a drop-down menu showing other buttons.

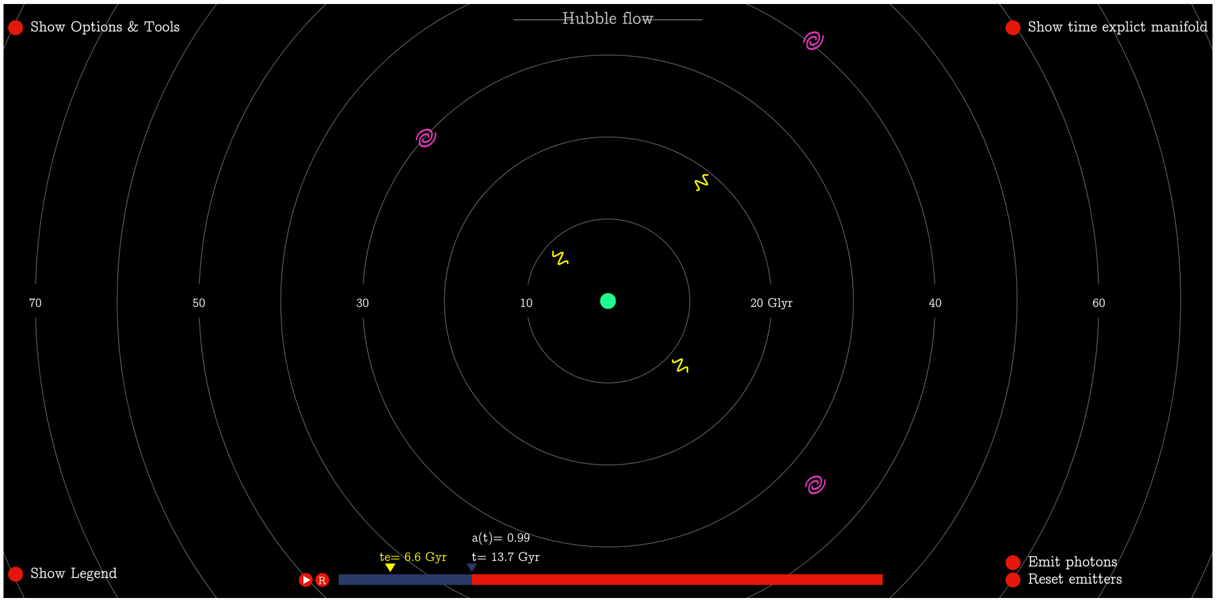


Figure 4.5: Starting from the situation in Figure 4.3 clicking the button “emit photons”, from each of the generated galaxies (purple spirals) a photon (yellow) is emitted in the direction of the observer (light blue).

They all refer to the TSS and are organized in three clusters:

1. The first cluster gather tools showing useful quantities as circles centered in the observer (fig. 4.6); the single elements are:
    (a) “Hubble radius”: this button allows showing a pink circle associated with the exact distance of the Hubble radius at a specific time.
    (b) “Light cone generated at photon emission”: this button allows showing a green circle indicating the evolution of the light cone of the observer generated since the time marked on the time bar when the button “emit photons” was pressed.
    (c) “Event horizon”: this button allows showing a blue circle representing the event horizon at the time indicated in the time bar.

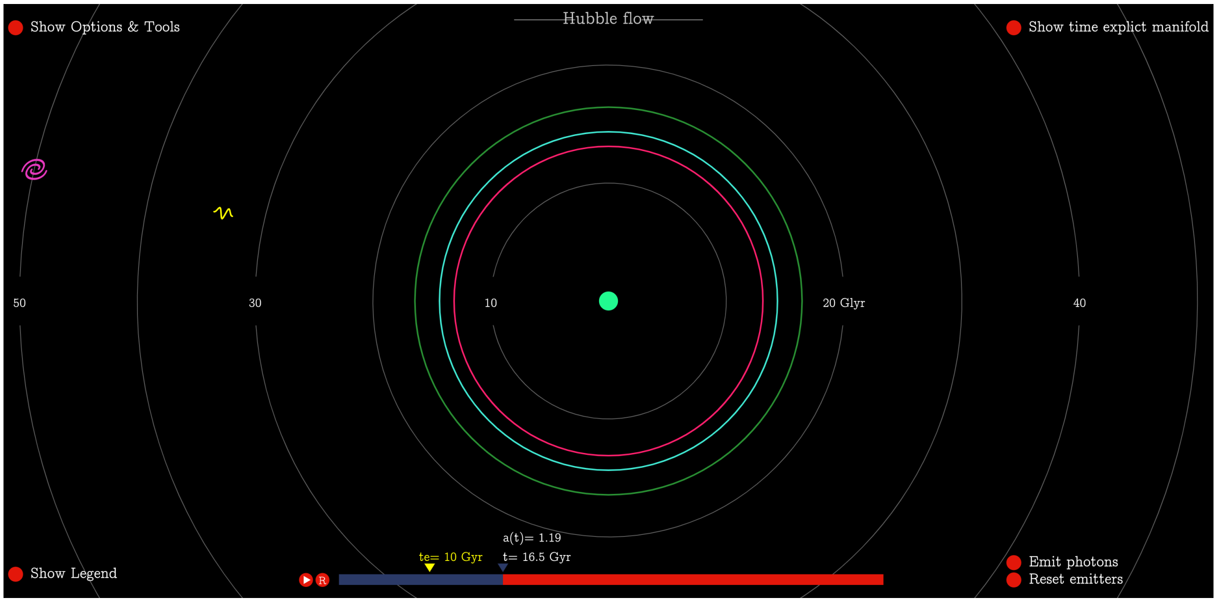


Figure 4.6: In this image we see three circles centered in the observer: the light cone (in green), the Hubble sphere (in purple) and the event horizon (in blue). User can also manage them from the drop-down menu.

2. In the second cluster are enclosed features about on the representation of the reference systems; the single elements are:
    (a) "Show coordinates": this button allows to turn on and off the coordinate system made by concentric circles
    (b) "Show galaxies distances": this button allows to turn on and off a dotted line between observer and galaxy with the numerical value of the distance in the chosen coordinate system (fig.4.5).
3. The third and last cluster allow to switch between comoving and proper coordinates

TES The "show time explicit manifold" button in the upper right corner allows to activate the second simulation (fig 4.7). It shows the temporal evolution of the system in a single image over a surface defined by cylindrical coordinates (ρ,ϕ,z). The z coordinate (or height) maps the time: so the lower part of the pseudo-cone corresponds to the initial time of the simulation while the higher to the final time. The radial coordinate ρ is equal to the scale factor $a(t)$ and finally the azimuthal coordinate ϕ is equal to the physical distance between two objects. In

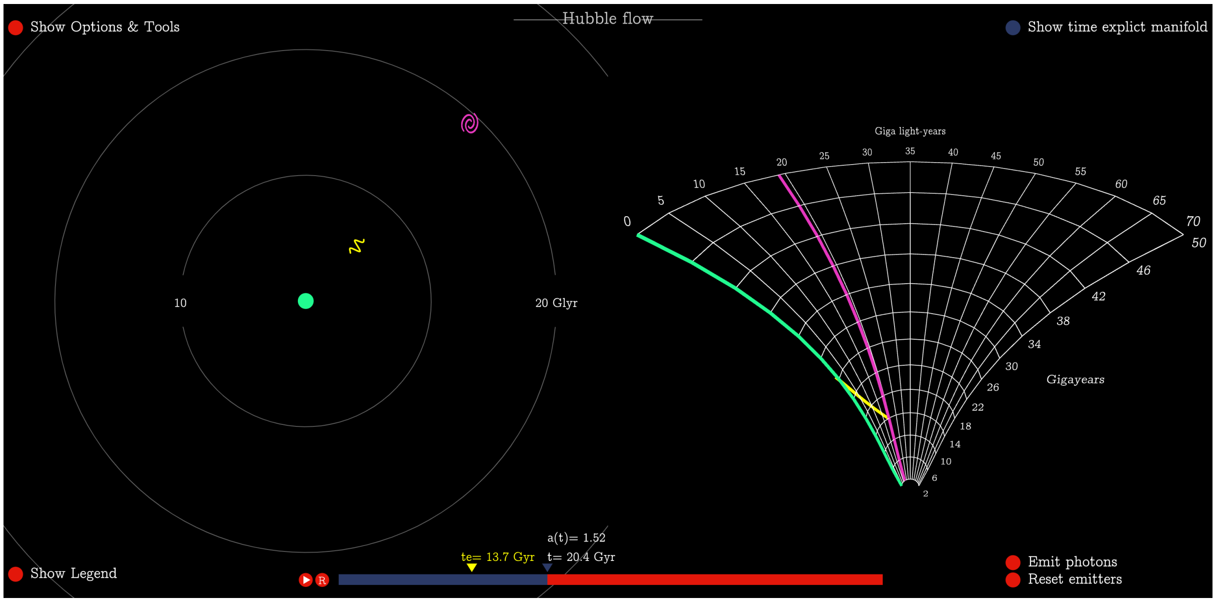


Figure 4.7: View of the simulation with the option “Show time explicit manifold” enabled. In both visualizations the different elements (photon, galaxy and observer) are represented with the same colors.

short,

$$\begin{cases} z \longrightarrow t, \\ \rho \longrightarrow a(t), \\ \phi \longrightarrow r. \end{cases} \tag{4.1}$$

This transformation allows to uniquely map every point of the space-time up to a comoving distance of 70 gigalight-years with time coordinate between 2 and 50 gigayears to another point on the surface.

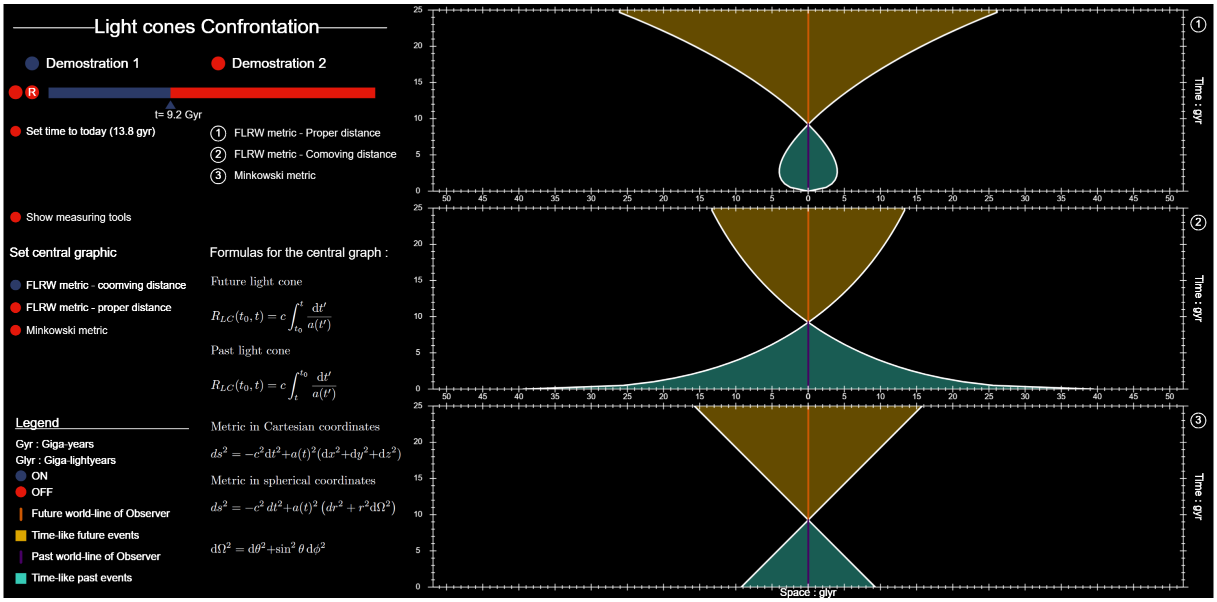
From one representation to the other one, we are denying a spatial coordinate because the system is invariant under rotation.

Finally this transformation converts each trajectory (whether it is a galaxy or a photon) in the TSS representation into an equivalent worldline on the TES surface. Whenever a galaxy or a photon is created in the TSS representation, an equivalent element is created also in the TES one. In Figure 4.7 through the color coding it is possible to recognize the different elements (observer, photon and galaxy) in the two simulations. To avoid confusion, when the TES function is active we have limited the number of galaxies to 1.

The time explicit simulation includes also rotation and zoom controls: the rotation is activated by clicking and dragging the mouse within the space dedicated to the TES, while the zoom in or out is controlled with the mouse wheel.

4.1.3 Light cone comparison

The second interactive animation consists of two demonstrations, showing three different graphs that can evolve through time. The user can switch between them using two buttons "Demonstration 1" and "Demonstration 2" on the top left of the screen, just below the title of the sketch (4.8).



Formulas for the central graph :

Future light cone

$$R_{LC}(t_0,t) = c\int_{t_0}^{t}\frac{dt'}{a(t')}$$

Past light cone

$$R_{LC}(t_0,t) = c\int_{t}^{t_0}\frac{dt'}{a(t')}$$

Metric in Cartesian coordinates

$$ds^2 = -c^2 dt^2 + a(t)^2(dx^2+dy^2+dz^2)$$

Metric in spherical coordinates

$$ds^2 = -c^2\,dt^2 + a(t)^2\left(dr^2 + r^2 d\Omega^2\right)$$

$$d\Omega^2 = d\theta^2 + \sin^2\theta\, d\phi^2$$

Figure 4.8: The interface of the simulation "Light cone comparison" (also called "Demonstration 1"). On the right we see the three light cone ordered as : FLRW metric in proper coordinates, FLRW metric in comoving coordinates and Minkowski metric.

Demonstration 1 The first demonstration is shown in figure 4.8. It is characterized by three light cones, one is represented in the Minkowski metric, the second one in the FLRW metric, using proper coordinates while the last one in the FLRW metric with comoving coordinate. Each one of these graphs evolves using the time commands in the top left corner under the "Demonstration 1" and "Demonstration 2" buttons.

Below the time bar there is the button "Set time to today (13.8 gyr)" which sets the three graphs at the current estimated time of the Universe. Finally the option "Show measuring

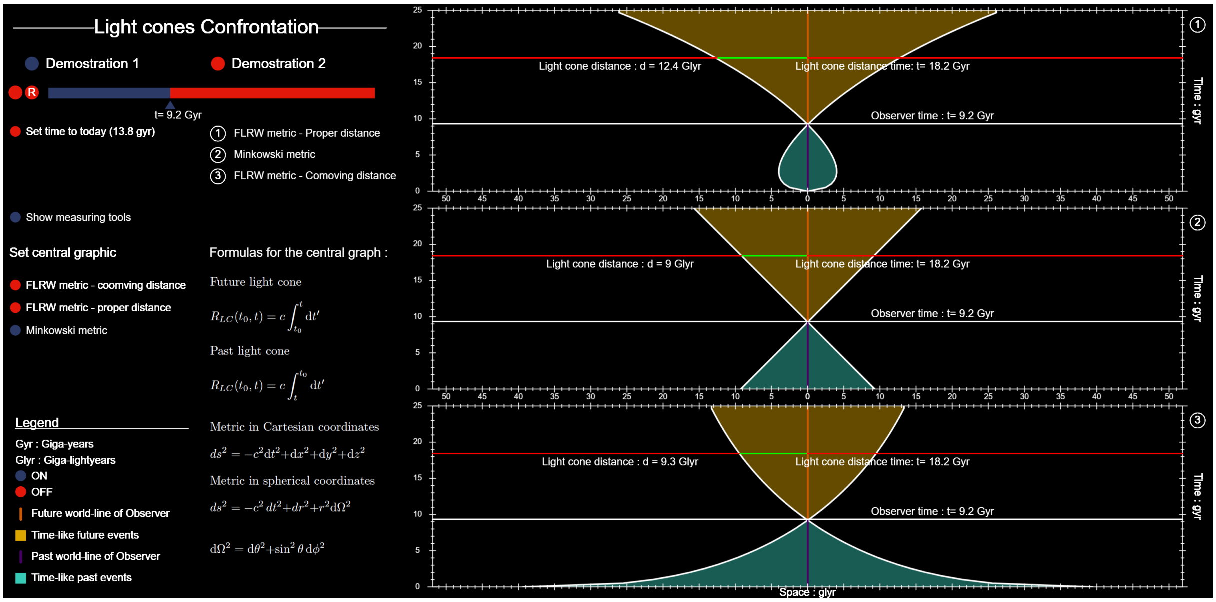


Figure 4.9: Same as shown in figure 4.8 but with the measurement instrument option enabled.

tools" allows to activate the tool shown in figure 4.9. It consists on horizontal and vertical lines on the graphs. They allow us to have the precise value of the time of the observer and, clicking in any point of the three charts, one gets information about the light cone at the time associated to the position of the click.

Below this button there is the cluster of buttons "Set central graphic". This button allows putting any of the three graphs as a central graph. It also displays (at the right of the buttons) the formulas of interest for the understanding of the graph: the definition of future and past light cone, the metric in Cartesian and spherical coordinates.

The last features of this demonstration are purely informative: under the time bar on the right it is displayed the order of the graphs while in the bottom left corner we find a legend.

Demonstration 2 The second demonstration is shown in figure 4.10. There are three graphs: the Event horizon is represented in the top, the light cone in the center and the Particle horizon in the bottom. The graphs of the light cone in the middle evolves with time while the other two graphs are static. The order in the disposition of these diagrams reflects the aim of the animation. Indeed we want to show that the two horizons represent particular cases of the light cone (respectively for an observer placed at time 0 and infinite). Moreover, we want to show that for any observer's time t_0 the value of light cone at $t = 0$ is equal to the value of particle horizon at time t_0.

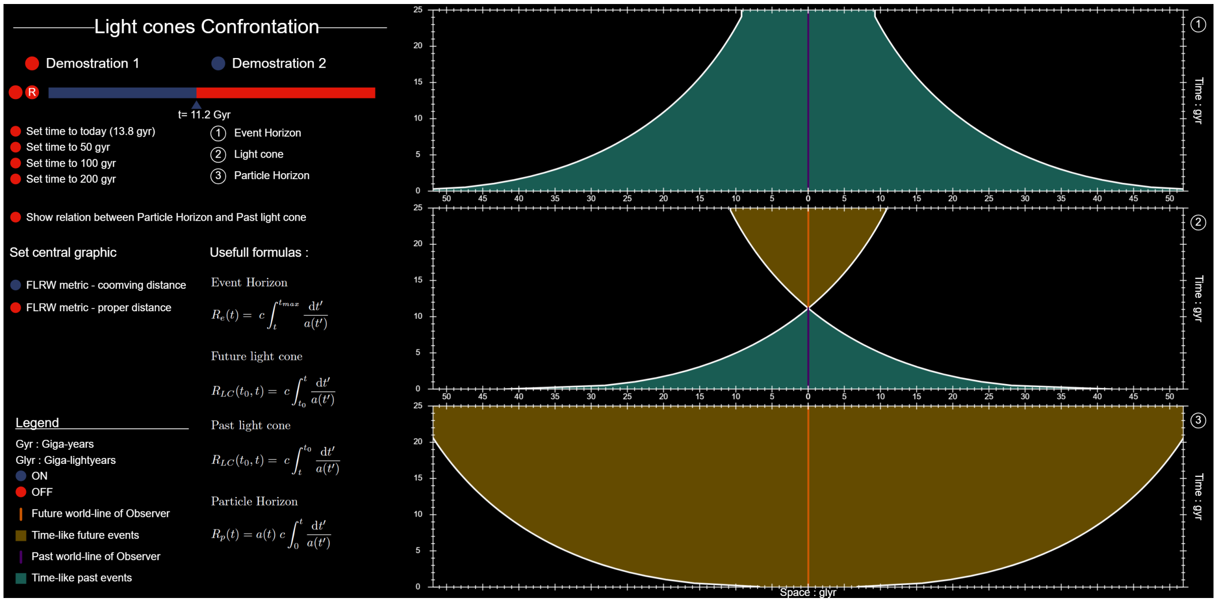


Figure 4.10: The interface of simulation “Light cones comparison” in the modality “Demonstration 2”. On the right, from top to bottom, we see: the event horizon, the light cone and the particle horizon, in coordinate comoving.

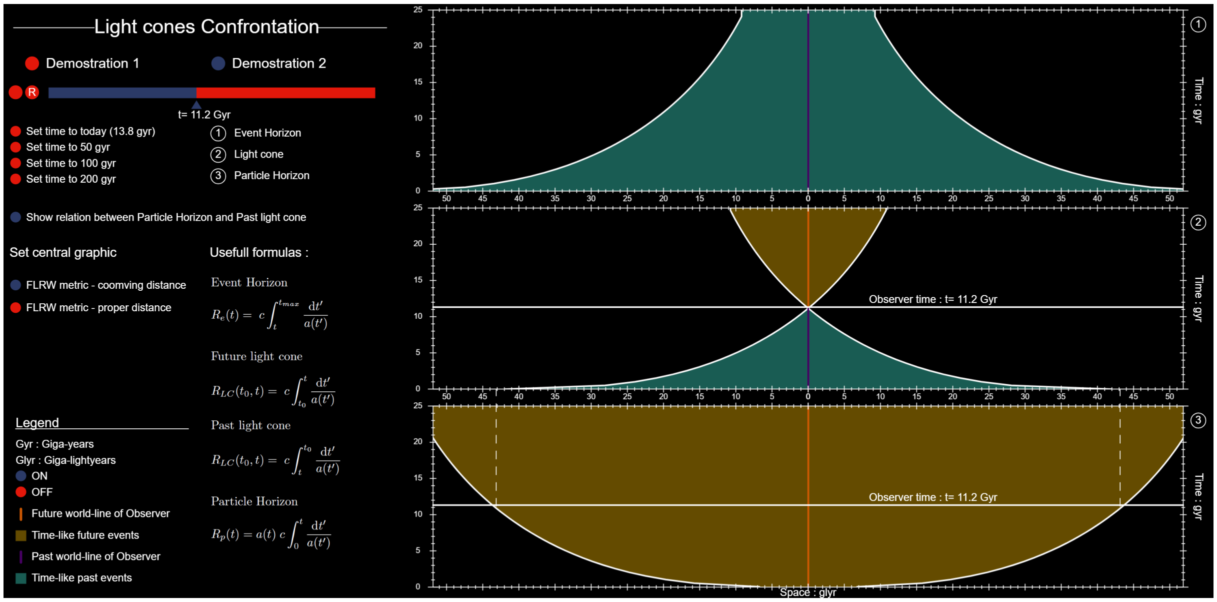


Figure 4.11: Same as shown in figure 4.10 but with the tool “Show relation between Particle horizon and past light cone” active.

In this demonstration the buttons are slightly different: indeed under the time bar on the

left user is able to set the time of the light cone at different ages (respectively 13.8, 50, 100 and 200 gigayears). Some of these ages exceed the length of the time bar but they are necessary to show that for times approaching infinity the cone of light converges to the event horizon.
Below these controls, we find the button "Show relation between Particle horizon and past light cone". This option activates the measuring instrument in figure 4.11. This tool correlates the base of the past light cone with the time value of the observer of the Particle horizon, as shown in the image 2.2.
Finally the last button allow to switch between comoving and proper coordinates.

4.1.4 Horizon Evolution

The last of the three animations is inspired by the classic space-time graph in figure 2.3. The initial configuration of simulation is shown in figure 4.12.
On the left side of the screen there is an interface, in the middle a first rectangular chart (we will call it **"Time explicit graph" (TEG)**) and on the right to the right a second square chart (we will call it **"Time singular graph" (TSG)**). At the beginning these two charts have only

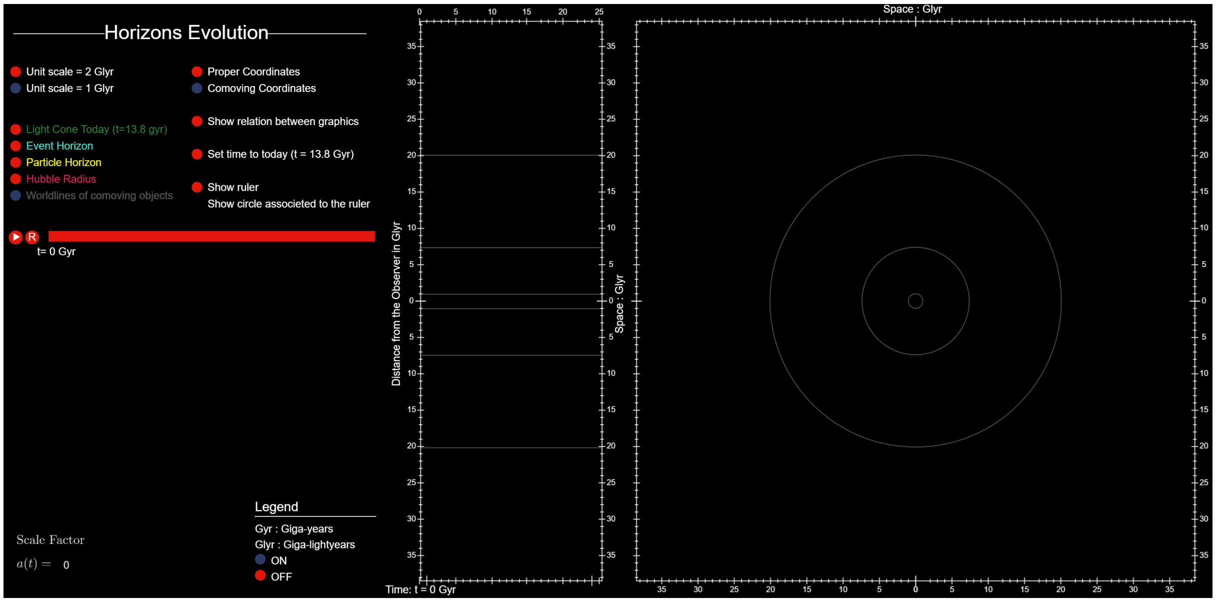
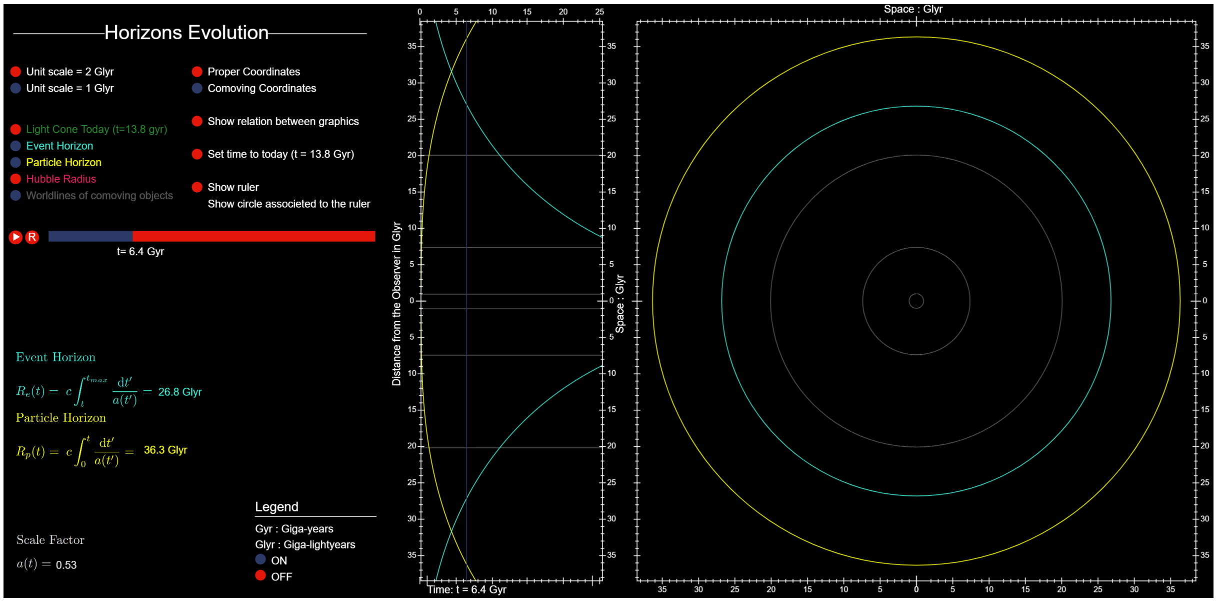


Figure 4.12: The initial aspect of the simulation "Horizon evolution". On the left side of the screen there is the interface while, in the center the graph shown in figure 2.3, Finally, on the right there is the single-time display graph that represents horizons as circles centered in the observer.



Event Horizon

$$R_e(t) = c\int_t^{t_{max}} \frac{dt'}{a(t')} = 26.8 \text{ Glyr}$$

Particle Horizon

$$R_p(t) = c\int_0^{t} \frac{dt'}{a(t')} = 36.3 \text{ Glyr}$$

Scale Factor

$a(t) = 0.53$

Figure 4.13: The same as figure 4.12 with the particle and event horizon shown.

worldline of objects at rest with respect to the Universe which are used as references frames. We will explain below in detail the elements of the simulation.

Interface The elements composing the interface of the animation are: buttons, time controls, mathematical definitions of the physical quantity of interest evaluated at age of the time bar and a legend. We will list below the functions of the individual buttons, starting from the left column from top to bottom, then moving to the right:

- **"Unit scale= 1 Gyr"** and **"Unit scale= 2 Gyr"**: these buttons set the scale of the graphics value at 1 Gigalight-year or 2 Gigalight-year per unit of reference.

- **"Light cone today (t = 13.8 Gyr)**, **Event Horizon**, **Particle Horizon**, **Hubble Radius** and **Worldline of comoving objects**: these buttons turn on or off the singles graphs of the quantity they refer to.

- **"Proper coordinates"** and **"Comoving coordinates"**: they allow choosing between one of the coordinate system.

- **"Show relation between graphics"** show how TEG and TSG graphics relate with each other as shown in fig. 4.14,

- **“Set time to today (t=13.8 Gyr)”**: it sets the time of the simulation at the indicated value.

- **“Show ruler”**: it enables a measuring tool in TSG graph. Clicking anywhere in that graph, the user read the value of the distance associated to the point from the observer. Linked to this button, there is the **“Show circle associated to the ruler”** which generates a circle with radius equal to ruler distance, as shown in figure 4.15.

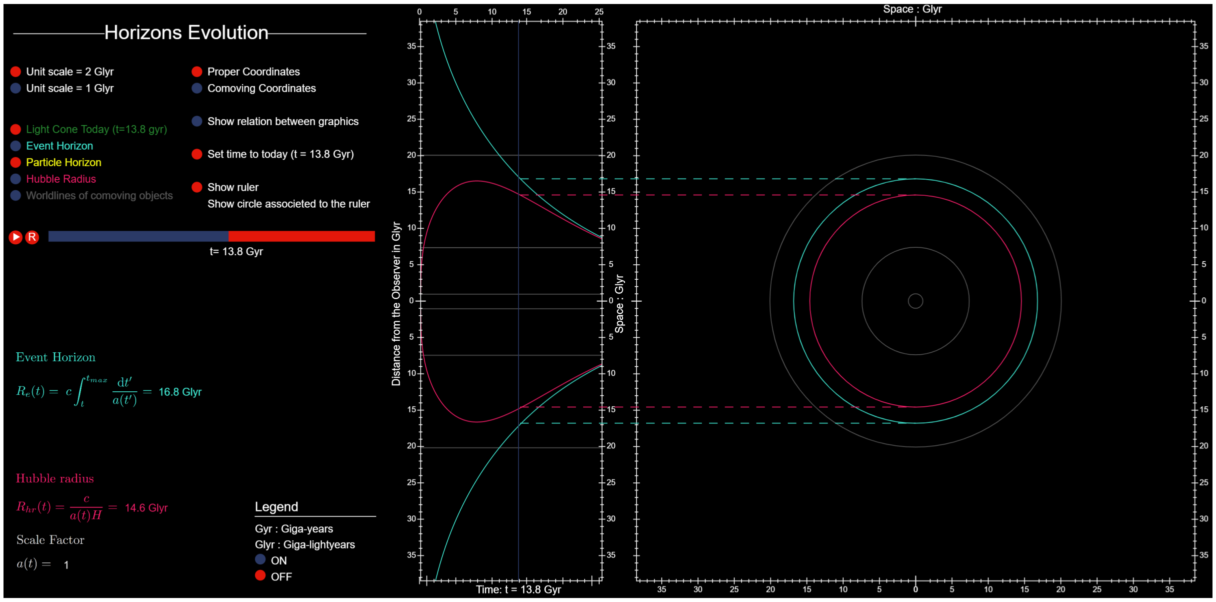


Figure 4.14: The colored dashed lines link the two graphs and are activated by the button “Show relation between two graphics”.

TEG This chart is exactly the one shown in figure 2.3 rotated by 90° degrees. Indeed, there is time on the horizontal axis and space on the vertical axis. This rotation maximizes the information on the screen, as it allows to relate it with the TSG chart as shown in fig. 4.14. This graph has also a blue vertical line that explicitly shows the time of the simulation: this line crosses the plots in two symmetrical points whose distance represents the current size of the selected functions. This distance is equivalent to the radius of the associated circle in the TSG graph. The button “Show relation between graphics” emphasizes this relation.
On the blue line of time there is a cross (Figure 4.15), which represents the position of the measuring instrument used in the TSG.

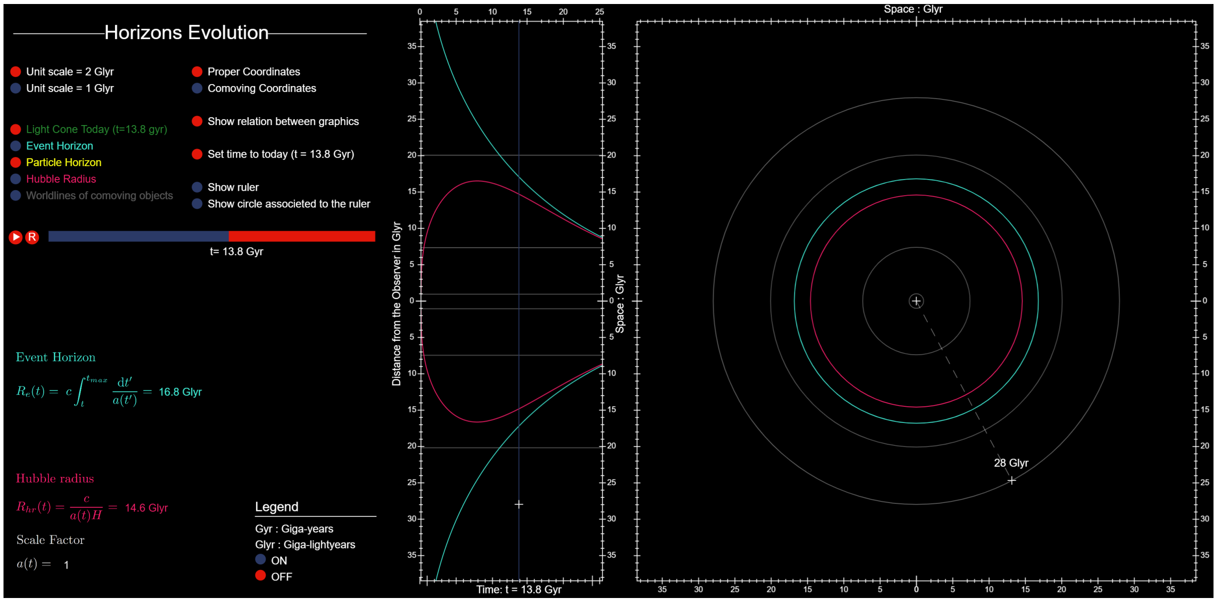


Figure 4.15: The gray dotted lines represent the measuring instrument activated by the homonymous function.

TSG This second graph shows the same physical quantities as the previous one in the form of circles centered in the observer. Their radius is calculated at the time marked by the time bar.

4.2 Technical aspects in the construction of simulations

4.2.1 From theory to data

We used the library **P5js** of the programming language **JavaScript** to create all the simulation we described. Indeed JavaScript is commonly used in client-side Web programming for the creation, of interactive dynamic effects through script functions. Moreover, the access to applications via web browsers eliminates the compatibility issues related to operating systems.
The only downside encountered working with this language was the lack of a solid library for mathematical calculations. This problem was solved generating with **Wolfram Mathematica** the necessary data for the development of the applications.
The data were generated in the format *.csv* on two columns: the former lists all the values of a function used in the simulations. The latter, being these functions time depending, contains the value of the associated time. A full version of the macro used on Mathematica is available

in the appendix.

We have already showed the used formulas in chapter 2 but we gathered all them below:

$$a(t) = \left(\frac{\Omega_{m,0}}{\Omega_{\Lambda,0}}\right)^{\frac{1}{3}} \sinh^{\frac{2}{3}}\left(\frac{t}{(4/3\ \Lambda\ c^2)^{1/2}}\right), \tag{4.2}$$

$$R_c = c\frac{\dot{a}}{a}, \tag{4.3}$$

$$R_p(t) = a(t)\ c\int_0^t \frac{\mathrm{d}t'}{a(t')}, \tag{4.4}$$

$$R_e(t) = a(t)\ c\int_t^{t_{max}} \frac{\mathrm{d}t'}{a(t')}, \tag{4.5}$$

$$R_{LC}(t_0,t) = \ c\int_{t_0}^t \frac{\mathrm{d}t'}{a(t')}. \tag{4.6}$$

Here $a(t)$ is the scale factor, $R_c(t)$ is the Hubble radius, $R_p(t)$ is the particle horizon, $R_e(t)$ is the event horizon and $R_{LC}(t_0,t)$ is a generic light cone with the observer at time t_0. Each of these equations is expressed in comoving coordinates while the equivalent value in proper coordinates can be obtained simply multiplying by the scale factor at the associated time.

For the "Hubble flow" simulation, we used equations 4.2 to 4.5 calculated in a time interval between 2 and 50 gyr with step of 0.05 gyr.

For the "Light cone comparison" simulation, we used equations 4.2.1, 4.5 and 4.6 in a time interval between 0 and 25 gigayears with a step of 0.05 gyr. In particular for equation 4.6 we added data for observers at times t_0 of 50, 100 and 200 gyr.

Finally, for the "Horizon evolution" simulation, we used equations 4.2 to 4.6 above calculated in a time interval between 0 and 25 gigayears with a step of 0.5 gyr. In particular in equation 4.6, we fixed the time of the observer at the present time of the Universe (13.8 gyr).

4.2.2 From data to images

The images of the simulations were created from the data obtained with Mathematica, using different techniques.

Map function "map" is a native JavaScript function. Given a variable with values within a determined range, the function "map" assigns a new value interpolated in a different range to a new variable linearly proportional to the value of the previous variable. For example, a variable x has values in a range between 0 and 10 and we aim to map a variable y to x in a range between 0 and 100: then if the variable x has a value 3.5, the associated value of y is 35. Each time we use the term "to map" we will refer to this operation.

Data plots This technique was used to generate the TEG graph in the simulation "Horizon evolution" and each single frame of the light cone in "Light cone comparison". We decided a suitable scale between our unit of measurement and the pixels on the screen, and we multiplied the data by the scale to get the associated position on the screen. Each plot is made of two symmetrical curves with the origin O of the space axis in the middle. For the temporal evolution of the light cones it is necessary to generate a number of plots equal to all possible time values and show the graph associated with the time of the animation.

Circle graphs The circle graphs were used in the simulations "Horizon evolution" and "Hubble Flow". We obtained them mapping the time of the simulation with the index of the associated table and then using these data to generate a circle with a radius of that value opportunely scaled.

Hubble flow This technique was used in the simulation "Hubble flow", in particular to simulate the recession of galaxies. As we said in the previous sections, galaxies are generated with the user's click at any time in the simulation. At each click, the program saves two information about the galaxy: the time t_g the galaxy was generated and its initial distance d_g from the observer. The distance information of the galaxy is updated following this formula:

$$d(t) = d_g \, \frac{a(t)}{a(t_g)}. \tag{4.7}$$

Photons The photon used in the simulation "Hubble flow" is represented by the graph of the following equation:

$$\gamma(x, t) = S \cos\left(\frac{x}{f}\right) \exp\left(-\frac{(x - \mu(t))^2}{2\sigma^2}\right). \tag{4.8}$$

Here S is a scaling factor useful to give the photon appropriate screen proportions, σ represents the variance of the Gaussian, f the frequency of our packet and $\mu(t)$ the mean of Gaussian as well as the photon position tabulated value.

The mean value is given by the future light cone calculated with the formula 4.6 using as integration extremes the emission time t_{em} and the current animation time t.

To describe the various trajectories of the photon at every possible emission time would require as many light cones tables, this would make the program much slower to operate. It is possible to overcome this problem using a simple property of integrals, *the linearity of the integral* with respect to the extremes:

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx \qquad \text{with } c \in [a,b]. \tag{4.9}$$

By applying this formula to eq 4.6 we obtain:

$$R_{LC}(t_{em},t) = c\int_{t_{em}}^{t} \frac{\mathrm{d}t'}{a(t')} = c\int_0^t \frac{\mathrm{d}t'}{a(t')} - c\int_0^{t_{em}} \frac{\mathrm{d}t'}{a(t')} = R_p(t) - R_p(t_{em}), \tag{4.10}$$

this allows us to use only the Particle horizon data instead of a different dataset for each possible emission time.

Chapter 5

Simulations-based educational activities

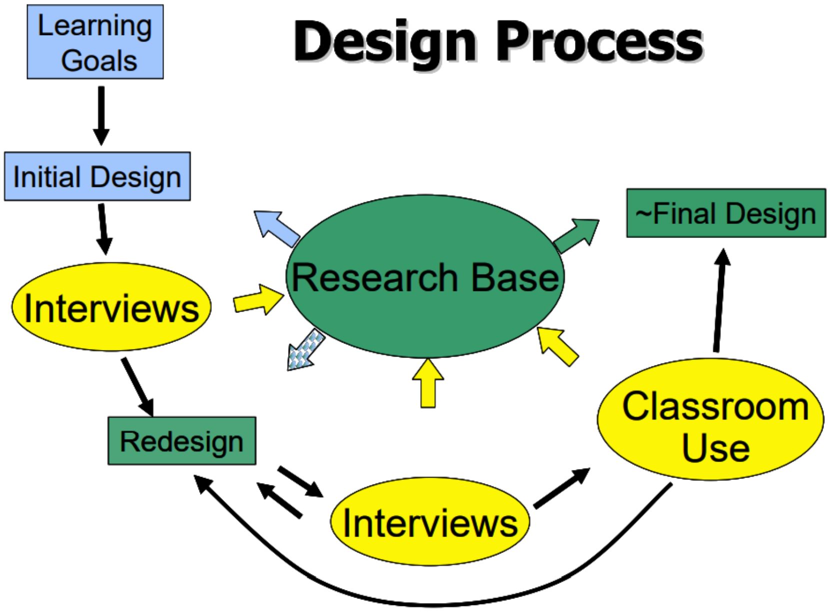


Figure 5.1: The design process flow chart showing the simulation design process [Adams et al., 2008b].

In the first part of the previous chapter, we explained the different design choices adopted in structuring our educational simulations. This action corresponds to the first phase of the protocol used by PhET project for the development of their educational software [Adams et al., 2008a]. A summary of the Simulation Design Process is shown in Figure 5.1.
The following step consists of a series of preliminary interviews. Their main purposes are: reveal the interface weaknesses, resolve ambiguous questions about the interface, to reveal pedagogically undesirable features and subtle programming bugs.
Furthermore, we can start to make preliminary considerations about the educational effectiveness of our simulations in order to subsequently set up an experiment with a larger sample.
These interviews have been carried out through three phases: an initial test on relativistic concepts to assess whether the prerequisites necessary for the understanding of the animations are adequate and a series of simulations-based activities.

Simulation	**Educational objectives**
Hubble flow	2a, 2b, 2c 3a, 3b, 3c 4, 5b, 5c, 5e
Light cone comparison	1a, 1b, 1c 2a, 3c, 4 5a, 5b
Horizon evolution	3c, 5a, 5b 5c, 5d, 5e 5f

Table 5.1: The teaching objectives (section 3.3) addressed by our simulations.

5.1 Purpose of the animations

Before introducing the pre-test and the post-test, we want to point out the didactic purpose of each one of these interactive simulations in order to clarify the link with educational activities. In Table 5.1 we can see the teaching objectives covered by each one of our simulations.

5.1.1 Hubble flow

The fundamental function of this first simulation is to show the behavior of galaxies and photons over times and distances of cosmological interest.

This animation was labelled as introductory because it provides a tangible view of the phenomenon. On the whole, it appears as a problem of elementary kinematics (at least from a “visual” point of view), constituting the foundation for the successive content.

This kind of representation falls under the “perceptual method” (chapter 1): the movement on the screen of photons and galaxies is a perceptive activation that cannot be achieved with a static graph.

This design was also used for the representation of the event horizon, Hubble sphere, light cone and comoving coordinates. All these concepts are represented as one or more circles centered in the observer with a variable radius in time. The light cone in particular is characterized by an interesting occurrence in terms of perceptions: the synchrony of the arrival of the photon to the observer with the light cone reading the emitter.

Another interesting feature for our teaching purpose is the so-called “time explicit manifold”, emerging in a comparison between two representations. It allows us to see if the student has managed to deconstruct the individual concepts in the correct way and can associate them in a different context.

5.1.2 Light cone comparison

This animation allows students to become familiar with the light cones in a cosmological context and it shows how they compare them with the cosmological horizons. Firstly, the temporal evolution of the light cones is explicitly shown as time passes.

Observer time in SR is always considered zero, because the Minkowski light cone is invariant under temporal translation. This is not true for light cones in the FLRW metric, as there is a privileged reference time (the Big Bang) and the shape of the light cone is affected by the difference between the time of the observer and the time when the big bang occurred. The comparison of different types of light cones allows the user to grasp useful differences from an educational point of view and makes it possible to stimulate him by asking his interpretation of these differences.

5.1.3 Horizon Evolution

This animation shows the temporal evolution of cosmological horizons. In addition to the well-known spacetime graph, we show horizons as a series of concentric circles with time-dependent radius. This type of representation allows a more direct comparison from a perceptual point of view than the search for information on a graph.

5.2 Special relativity pre-test

Before the interview, we decided to verify whether the student has adequate bases of special relativity. We used a short pre-test mode of fine questions; we used for this purpose four items of the ”Relativity Concept Inventory” (RCI) [Aslanides and Savage, 2013]. The RCI refers to the basic concepts of Special Relativity and assesses its understanding with conceptual questions. The questions that we have chosen for our pre-test concern two concepts: the second postulate and the velocity addition.

We finally added a question on the light cone and on different concepts associated with it.

5.2.1 Second postulate

The questions related to this topic are:

1. True or false: "In principle, it is possible for an observer following a pulse of light at a constant high speed to observe the light to be almost stationary."

   A True

   B False

2. Consider a spaceship travelling from Earth towards a distant star at a constant high velocity v relative to Earth. The spaceship sends a light pulse back to Earth. On Earth, the speed of this pulse is measured to be:

   (a) c

   (b) $c + v$

   (c) $c - v$

These first two questions (number 3 and 4 of the RCI) portray two situations that can be solved simply appealing to the invariance of the speed of light in the different reference systems. The second question in the original experiment was answered correctly by more people than the second, although the request is similar. This is probably due to the more "extreme" conditions of the first problem than of the second. We were interested in these questions as in the study of cosmological horizons there are apparent aspects which are often confused with violations of the second postulate. Thus we want the interviewee to have well internalized this concept.

5.2.2 Velocity addition

The questions related to this topic are:

> In the following two questions, the scenario is as follows: Alex and his friend Bianca decide to set off on separate voyages in identical spaceships. They each speed away from Earth in opposite directions – Alex at v = 0.75c to the left, and Bianca at v = 0.75c to the right, relative to an observer on Earth.

3. If Alex measures the rate at which his distance to Bianca is increasing, he will obtain a value that is:
    - (a) Equal to 1.5c
    - (b) Greater than c but less than 1.5c
    - (c) Equal to c
    - (d) Greater than 0.75c but less than c
    - (e) Equal to 0.75c
4. If Cameron, an observer on Earth, measures the rate at which the distance between Alex and Bianca is increasing, he will obtain a value that is:
    - (a) Equal to 1.5c
    - (b) Greater than c but less than 1.5c
    - (c) Equal to c
    - (d) Greater than 0.75c but less than c
    - (e) Equal to 0.75c

These questions (number 9 and 10 of the RCI) in the original article have much lower success rates than the previous ones (67% for question 3 and 49% for question 4). Indeed sometimes the motion of the photon in a context of expanding Universe is interpreted as a composition of velocity. Knowing speed composition is important to highlight how Hubble flow is a different phenomenon. The second question has the highest failure rate of the entire test, probably because the correct answer seems to violate the speed of light limit (the answer is $1.5\,c$). This has strong similarities with the misconception that is highlighted by Davis [Davis et al., 2003], that recession can happen at speeds faster than light. In both cases, motion faster than light occurs outside the observer's inertial frame.

5.2.3 Light cone

The last question asked is:

5. Describe the various elements of the light cone graphic in figure 5.2
   (a) What the yellow area refers to?
   (b) what the blue area refers to?
   (c) what the black area refers to?
   (d) what the orange line refers to?
   (e) What the purple line refers to?
   (f) What the white lines refer to?

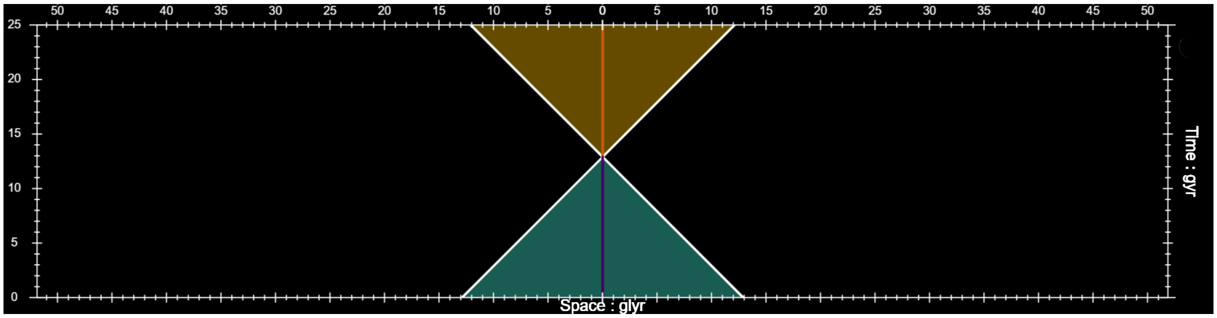


Figure 5.2: Light cone in Minkowski's metric

The intent of this group of questions is very direct: to understand if the interviewee is able to recognize the individual elements of the light cone.

5.3 Activities and questions simulations-based

The core of our interview consists of a series of activities aimed to explore our simulations in all their aspects. This interview protocol is divided into 12 main steps, each one addressed to one of the different teaching objectives we described. We ordered the activities in order to favor a progressive learning of the functionalities of the simulations. Table 5.2 summarizes the proposed activities, the animations involved and the new controls each time are introduced.

#	**Activity name**	**Simulations**	**Initial setting**	**New controls**
1	Hubble flow experiment	Hubble flow	Default setting	Time controls Spawn galaxies
2	Coordinates comparison	Hubble flow	Show Options and Tools: ON	Show coordiantes Show galaxies distances Coordinates A Coordinates B
		Horizon evolution	Default setting	Time controls Comoving coordinates Proper coordiantes
3	Photons and light cones	Hubble flow	Show Options and Tools: ON Light cone genereted at photons emission: ON	Emit photons Reset emitter
		Light cones comparison	Default setting	Time controls Show measuring tools
4	Hubble radius explanation	Horizons evolution	Hubble radius: ON	
5	Hubble radius and photons	Hubble flow	Hubble radius: ON	
6	Event horizon demonstration	Hubble flow	Event horizon: ON	
7	Crossing the Event horizon	Hubble flow	Event horizon: ON	
8	Event and Particle horizons explanation	Horizon evolution	Particle horizon: ON Event horizon: ON	
9	Light cones and horizons	Light cones comparison	Demonstration 2: ON	Set time to X gyr Show relation [...]
10	Time explicit Manifold	Hubble flow	Show time explicit manifold: ON	Camera rotation
11	“Cosmic lighthouse” exercise	Hubble flow	Event horizon: ON	
12	Free interaction	All		

Table 5.2: Summary table of activities proposed to the student, in terms of: the number of the task, the name, the simulations used, how to configure the settings and the new buttons that can be used for each new task.

5.3.1 Hubble flow experiment

Setting This activity uses the simulation “Hubble Flow” in its default condition (active options are: “proper coordinates” and “show distance galaxies”). The interviewee is introduced to the use of time controls and how galaxies are generated.

Content Hubble’s law, because of its extreme simplicity and historical value, is an excellent starting point to talk about the expansion of the Universe from a phenomenological point of view.
In this first activity, no information about this law is provided. Rather the student is asked to obtain a formula of the recessional speed of galaxies. Following the PhET guidelines, we provide as little as possible help of the student in the research process [Ariel et al., 2013]. The things we expect to be noticed are:

- The speed depends on the distance between galaxy and observer,
- The relationship between speed and distance is linear.

We therefore expect the student to initially create several galaxies at different distances and to notice the difference in speed. Then to observe the time evolution of the distance of these galaxies to draw a linear plot. Or more simply, by putting two galaxies at different distances and noting that the doubling time of these distances is the same.

5.3.2 Coordinates comparison

Setting This task uses the “Hubble Flow” simulation with the “Show options and tools” drop-down menu open and “Horizon evolution” in its default condition. The interviewee is introduced to the use of all the buttons in the drop-down menu.

Content In this task, we show two different simulations where the coordinate system can be changed. In the simulation “Hubble flow” the coordinates are not explicated with their name but with the letters A and B. The request is to understand which coordinate system the letters refer to motivating the choice with a convincing argument.

5.3.3 Photons and light cones

Setting This task uses the “Hubble Flow” simulation with the light cone enabled and “Light cone comparison” in its default condition. The interviewee is introduced to the button “Emit Photons” in the simulation and the basic interface commands in the later one former.

Content Similarly to the previous point, we ask the student to associate the light cones presented in the simulation “Light cone comparison” to the associated coordinate system.

5.3.4 Hubble radius explanation

Setting This task uses the “Horizon evolution” simulation with “Hubble radius” option enabled.

Content This step consists in an explanation of the physical meaning of the Hubble radius using its formula and phenomenological description. The purpose is to introduce the next activity.

5.3.5 Hubble radius and photons

Setting This task uses the “Hubble flow” simulation with “Hubble radius” option enabled.

Content In this passage we deal with a problem proposed by Davis’ [Davis et al., 2003]. It was asked the student if it is possible to observe a photon emitted outside the Hubble sphere. Regardless of the answer we then go on with a demonstration and we will ask if there is a limit beyond which it is never possible to observe the photon and the origin of this limit.

5.3.6 Event horizon demonstration

Setting This task uses the “Hubble flow” simulation with “Event horizon” option enabled.

Content After the demonstration with the Hubble sphere it is interesting to show the behavior of the photons emitted inside and outside the event horizon. The aim is to clarify that the observability of a photon emitted at a given time determined only by the position of the emitter with respect to the event horizon.

5.3.7 Crossing the Event horizon

Setting This task uses the “Hubble flow” simulation with “Event horizon” option enabled.

Content Once we have showed that the event horizon is the limit for undetectable photons, it is possible to examine the observability of objects exceeding this limit. The asked question to the student is: “If a source that emits photons continuously passes from within to outside the event horizon, does it continue, to be visible? How long?”.
Actually it continues to be visible forever, but an observer will not have information after the object exits the horizon.

5.3.8 Event and Particle horizons explanation

Setting This task uses the “Horizon evolution” simulation with “Event horizon” and “Particle horizon” options enabled.

Content At this point of the interview the concept of particle horizon is introduced. It is highlighted that the observability of an object is not determined by its position with respect to the event horizon but rather to the horizon particle. Indeed the former particle horizon quickly exceeds the event horizon. Another interesting observation is to point out that (in comoving coordinates) the value of the event horizon at time 0 is very close to that of the particle horizon at maximum time.

5.3.9 Light cones and horizons

Setting This task uses the “Light cones comparison” simulation with “Demonstration 2” option enabled.

Content Through the second demonstration of the simulation “Light cones comparison” it is possible to notice how the particle horizon and the event horizon are particular cases of the light cones [1].

5.3.10 Time explicit Manifold

Setting This task uses the “Hubble flow” simulation with “Show time explicit manifold” option enabled.

Content This activity has been conceived as an evaluation of the knowledge acquired up to that moment by the student. Here we resume the demonstration of the path of a photon. However, we represent this phenomenon in a three-dimensional manifold with the time shown on an axis. The student is asked to link the various elements on the screen to what has been seen so far. Among the various recognizable entities, we consider most valuable in didactic terms the recognition of the scale factor as a variable radius of the manifold described in cylindrical coordinates.

---

[1] The light cones with observer time equals to zero coincides with particle horizon and with observer time equal infinity coincides with event horizon

This step can represent a possible hint for a "high quality mental model" as the student applies the knowledge he has acquired in a new context.

5.3.11 "Cosmic lighthouse" exercise

Setting This task uses the "Hubble flow" simulation with "Event horizon" option enabled.

Content The last guided task consists in the following exercise:

> "A lighthouse is located in a galaxy within the event horizon. The lighthouse emits at regular intervals of one second. At a certain time the galaxy goes out of the horizon. What does an observer see as time flows? Is the measured frequency the same or does it change?"

As the previous activity, this task is a test of acquired knowledge. The simulation "Hubble flow" allows recreating the scenario of this exercise, thus encouraging the qualitative reasoning.

5.3.12 Free interaction

After this series of addressed activities the interviewee is allowed to interact autonomously with the animations. In this context the interviewee is also asked to comment his experience.

Chapter 6

Interview results

In this final chapter, we will analyze three exploratory interviews [Adams et al., 2008a] based on the protocol developed in the previous chapter. The interviewees have different Physics-related backgrounds: a bachelor student, a master student and a worker with a master's degree in Physics. Each one of them have no academic knowledge in the topic of interest.

After that we will make a comparison between the three interviews trying to highlight the similarities and differences emerged during the interviews.

Finally, we will make some concluding remarks about: the possible developments in technical aspects, expansions in didactical contents and application for experimental purpose.

6.1 Interview #1

This interview was conducted on a 29-year-old male with a master's degree in Physics. He is currently working in a technological field related to photonics.

Pre-test The respondent answered correctly all the questions declaring maximum confidence in each answer.

Activity 1 - Hubble flow experiment At the beginning of the activity, the interviewee stated that he did not know exactly the Hubble's law. However he had a vague idea as "something related to the redshift".

With some interaction with the simulation he understood that the speed of recession depended on distances.

The question "what kind of dependence there is between speed and distance?" initially led to some hesitation but he was able to solve it correctly by means of an experimentation on the simulation. He placed two galaxies at different distances and he verified if they could reach twice distances in the same time interval. The interviewee understood that the relationship was linear seeing that his prediction was correct.

Activity 2 - Coordinates confrontation At the beginning of this activity, the interviewee was confused observing the two coordinate systems in the “Hubble Flow” animation (figure 4.2) because the comparison took place when time was not flowing.
To solve this problem, we showed the other graph in “Horizon Evolution” (figure 4.12); depicting the worldlines of comoving objects in the space-time graph. Then we asked to switch between the two coordinates system.
After this demonstration, he realized that the difference could be in a dynamic component of the coordinates and that in one of the two cases “the coordinates expanded with the Universe”. Subsequently, he looked for confirmation in the other simulation, with a positive result. After some considerations aloud, he also managed to correctly identify each one of the coordinate systems.

Activity 3 - Photons and light cones The task in this activity was to associate light cone graphs to the respective coordinate systems. In this activity the interviewee failed to resolve the request as the comoving coordinates were associated with the light cone of the Minkowski metric. At this point we showed the formulas associated with light cones but he still failed to associate them. After a brief explanation, he “understood but not internalized the concept to the point of being able to recognize it from a graph”.

Activity 4 - Hubble radius explanation The interviewee was able to understand the physical meaning of Hubble radius reading its mathematical definition without further explanation.

Activity 5 - Hubble radius and photons Initially the interviewee was confident that a photon from outside Hubble radius could never reach the observer. After the demonstration, he initially doubted whether he really understood the meaning of the Hubble radius. Thus he proposed an alternative definition for it as "some sort of asymptotic time limit" and therefore the superluminar recession did not take place at the present time. Once we remarked that this occurred at the present time of simulation, he said that this was "a very strange phenomenon due to the complexity of General Relativity". Unfortunately the lack of a more accurate explanation limited him to accept this as a fact without understanding the mechanisms.

Activity 6 - Event horizon demonstration The interviewee understood without further explanation that the photons emitted outside the event horizon do not reach the observer while the inner ones do. This outcome was deduced noticing that both the inner and outer photons move away from the event horizon (inward and outward, respectively). He concluded that this represented the true limit of observability of the Universe.

Activity 7 - Crossing the Event horizon and "Cosmic lighthouse" exercise To the question "What happens to a galaxy that goes out of the event horizon?" interviewee replied "The galaxy goes out and we can no longer observe it". Then he was asked "When do we stop seeing the galaxy?". He immediately gave the correct explanation, saying "According to me the image of the passage would have lasted infinite time".

Finally, we proposed the exercise of the "cosmic lighthouse" (section 5.3.11). Even in this case the interviewee answered correctly saying "The frequency tends to 0 approaching the horizon"

Activity 8 - Event and Particle horizons confrontation After a brief explanation of the physical meaning of the particle horizon, the task was explaining the trend of the graphs (figure 4.11). In particular we asked the interviewee "Why does the particle horizon soon exceed the size of the event horizon?" The answer was "Because the event horizon is the limit of the Universe tied to us while the particle horizon is bound to a particle and a particle goes further. I guess the particle horizon is the limit of a particle that we shoot from the observer".
Before our explanation the interviewee observed the behavior of the worldlines with respect to horizons. This allowed him to correctly conclude that "The event horizon determines the farthest point at time t that will be visible in the future. While the particle horizon is the farthest point visible right now."

Activity 9 - Light cones and horizons The interviewee was autonomously able to find the similarities between light cone and cosmological horizons. He also stated that the colors helped him.

Activity 10 - Time explicit manifold In this last demonstration the interviewee was able to quickly recognize all the elements present on the manifold (coordinates, photon, observer and galaxy). We pointed out that the surface was defined in cylindrical coordinates. Then was able to understand that: time was represented in the vertical axis, the angular distance represented the comoving distance, the size of the arc associated with the angular distance represented the proper distance and finally the radius represented the scale factor.

6.2 Interview #2

This interview was conducted on an undergraduate 23-year-old male. He is currently enrolled in a bachelor's degree course in Physics. His previous knowledge about astrophysics and cosmology was almost absent.

Activity 1 - Hubble flow experiment The interviewee never heard of Hubble's law but he was aware that the Universe is expanding. He was also able to correctly recognize the units of measure involved.
The interviewee found difficult to understand what the speed of recession depended on. Firstly he associated it with the total mass of the system as he noticed a change in speed while adding more galaxies. Then he gave up answering the request through some guided observations. After we pointed out the speed dependence on distance, we asked to discover the explicit relationship between them. He proposed a promising procedure based on acquiring position data at different times. Despite this proposal, he suggested a quadratic dependence, as "the motion he observed was accelerated".
Finally, through a guided demonstration (observing the evolution of two galaxies at different distances over an equal time interval) he was able to understand that the relation was linear.

Activity 2 - Coordinates confrontation We started asking the interviewee to describe the difference between the two coordinate systems in the "Hubble flow" simulation. His first impression was that in the comoving coordinates the space expanded while in the proper coordinates did not. This conclusion was probably inferred as the simulation was started without galaxies. Later, including galaxies, he was able to correctly identify the differences between the two coordinate systems, although in his definition he confused the word "space" with "coordinate system". During the task of linking coordinate systems in different graphs, he was able to answer correctly using compelling reasons.

Activity 3 - Photons and light cones The interviewee was able to correctly identify the light cones associated with the coordinate systems. The method he used consisted in observing the profiles of the light cones: he understood that the one widening more later in time was the light cone associated with proper coordinates. However his exercise raised concerns related with the speed of light which did not seem constant over time.

Activity 4 - Hubble radius explanation There are not relevant observations.

Activity 5 - Hubble radius and photons The interviewee was confident that the photon could not exceed the Hubble sphere. After attending the demonstration he gave us no other feedback but wonder.

Activity 6 - Event horizon demonstration The interviewee understood that both photons were moving away from the event horizon, the inner one towards the observer while the outer one away from it. He also asked if this horizon represented our limit of observability.

Activity 7 - Crossing the Event horizon and "Cosmic lighthouse" exercise The interviewee initially claimed that an object crossing the event horizon stopped being visible. Then he questioned his statement, noticing that the light emitted just before the crossing of the horizon reach the observer much later.
In the exercise "Cosmic lighthouse" he answered correctly, claiming that the frequency tended asymptotically to 0. He was not able to relate it to the previous question.

Activity 8 - Event and Particle horizons confrontation We firstly explained that the event horizon did not represent the limit of observability which instead is determinate by the particle horizon. We also clarified their physical meaning, also comparing them.

Activity 9 - Light cones and horizons The interviewee associated horizons with light cones at a particular time.

Activity 10 - Time explicit manifold The analogy between explicit and single time representation was partly grasped by the interviewee. Critical aspects emerged determining the coordinate systems and associating the radial coordinate with scale factor. The other parts of the activity were carried out correctly.

6.3 Interview #3

This interview was conducted on a 24-year-old male with a bachelor's degree. He is currently enrolled in a master degree course in Physics of Complex Systems. Previous knowledge about astrophysics and cosmology was almost absent.

Activity 1 - Hubble flow experiment The interviewee heard about Hubble law in the context of Universe expansion but he did not know its mathematical formulation.
When we asked about dependence of velocity, he placed two galaxies at different distances from the observer and started the simulation. He noticed that there was a relation between the distance and the speed of recession.
We asked him to find out the formal relation of the Hubble law. He proposed to use one galaxy: in a table he wrote down distances and times acquired from the simulation and then he plotted on a graph the data.
Finally we asked him to find a more direct way: he answered proposing the same strategy used in interview 1.

Activity 2 - Coordinates confrontation The interviewee was able to recognize coordinate systems easily. He understood that the straight worldlines in an expanding Universe are determined by the expansion of the coordinates themselves.

Activity 3 - Photons and light cones The interviewee was able to recognize coordinate systems with more hesitation with respect to the previous activity. The shape of the light cone profile helped him to identify them correctly.
The case of comoving coordinates remembered him a ”light slowdown” that he correctly attributed to the expansion of the coordinates themselves. The time he needed to answer was longer and the interviewee himself noticed that this question was the most challenging. He also used extensively the light cone measuring instrument.

Activity 4 - Hubble radius explanation There are not relevant observations.

Activity 5 - Hubble radius and photons The interviewee understood that the answer was not obvious because the speed of recession is “diluted” over all the space between the observer and the emitter. His final answer was that the light reaches the observer but he did not have enough knowledge to justify it.

Activity 6 - Event horizon demonstration Before starting the demonstration, the interviewee said that the word “horizon” presaged that external photons would not have reached the observer.

Activity 7 - Crossing the Event horizon and “Cosmic lighthouse” exercise The interviewee noticed that the time tend to infinity as the emitter approaches the horizon. He then correctly assumed that the event horizon represented an asymptote for these reception times.

Based on the previous reasoning, the “Cosmic lighthouse” exercise was easily solved as a direct consequence.

Activity 8 - Event and Particle horizons confrontation Although the interviewee answered the questions on the event horizon, he initially found difficult to understand the concept of the particle horizon. The doubt is analogous to the one reported during the same activity in interview 1, that it does not seem possible that the particle horizon exceeds the event horizon. He felt satisfied with the explanation given using the same with the example worldlines made in the first interview.

Activity 9 - Light cones and horizons This activity was different from those carried out in the other interview as we asked different question.

It was “Why are light cones in the cosmological sphere defined also by the time of the observer unlike those in special relativity?” The answer was corrected and he mentioned two fundamental aspects: the fact that time 0 is defined by the Big Bang and the lack of symmetry under temporal translations of the light cones themselves. Once this aspect has been clarified we asked about the comparison between the light cone at different times and the horizons. The interviewee did not notice the relation between horizons and light cones until we pointed it out.

Activity 10 - Time explicit manifold Each element of the explicit time representation was easily recognized with appropriate argument.

6.4 Assessment of interviews

To evaluate the performance of the interviewees, we will rely on the standard developed by Etkina in 2006. For each request expecting an answer from the interviewee, we will give an assessment on a scale from 0 to 3 [Etkina et al., 2006]. The values of this scale correspond to the following outcomes:

- 0 is equivalent to "not replying or giving a completely wrong answer". By "completely wrong" we mean that it demonstrates serious conceptual gaps on which it is impossible to build new knowledge.
- 1 is a response with correct information but they are not contextualized in a general framework. This information creates knowledge that has little versatility and more exposed to doubt.
- 2 is an answer with correct information contextualized in a more general framework but with limits of versatility and still doubtful. The reasons given for the answers are unclear and contain some conceptual leaps.
- 3 is equivalent to modelling concepts in a versatile and doubtless framework. The reasons given for answering a question are solid and follow a logical and comprehensive development.

The evaluation of our interviews is summarized in table 6.1. From these results it emerges a clear difference between the interviewees with a solid foundation of special relativity (I1, I3) and the one without (I2). It is noteworthy that despite the gaps in I2 special relativity he managed to reach a score of 2 in some questions. In general, the scores of I1 and I3 are greater than 70The most difficult question is definitely number 4, which is directly inspired by one of the misconceptions in Davis' article [Davis et al., 2003]. The question arises with the purpose of pointing out the apparent paradox but the simulation leaks in giving a phenomenological explanation to this fact.

It is also interesting to note that questions 6 and 7, the ones mostly designed to test the interviewees, were answered correctly by all of them, even by the one without solid foundation in special relativity.

#	Questions	Educational objectives	I1	I2	I3
1	Velocity distance relation	2b	3	1	3
2	Coordinate confrontation	3a,3b	2	1	3
3	Photons and light cones	4	1	2	2
4	Hubble radius and photons	2c,5c	0	0	1
5	Crossing the Event horizon	5b	3	1	3
6	Cosmic lighthouse	5b	3	2	3
7	Time explicit Manifold	2a,3c,4	3	2	3
	Final score		15/21	9/21	18/21

Table 6.1: Evaluation table, the last three columns each correspond to one respondent

Chapter 7

Final thoughts and future development

The encouraging preliminary results presented in the previous chapter need to be extended and verified with a large sample of students, however we can consider optimistically some of the aspects that have emerged. In all the interviews there was a high level of involvement denoted by the quantity and quality of interactions with the animations. The answers were never given from the simple observation of the images but they emerged through the interaction. This aspect is positively related to learning [Adams, 2010].

Visual transitions between various images have also been multiple. These are usually measured with an eye tracker, but, as we don't have this equipment, we could partially deduce it from the comments of the interviewees. These transitions are considered positive as they are a symptom of student relating contents and thus building new knowledge [Ploetzner and Lowe, 2017].

Based on what was observed during the interviews and our subsequent analysis, we can imagine different possible developments for this project on three levels: the **technical development**, the improvement of the **didactic contents** and the possible uses for **experimental purposes** of the simulation in its final version. By "**technical development**" we refer to possible improvements on usability and on the way our simulation are used. For example the ambiguity of colors is one of the relevant aspects. The colors we used in the interview were called with different names by the students. This did not create any problems during the interviews but it could be a source of ambiguity in possible future activities. Thus we should use colors that are uniquely identifiable by a name (such as green, red), avoiding shades. Another critical point to underline is related to the graphical rendering of the expansion. One of the interviewees initially confused the expansion for a zoom. So he did not interpret the simulation as if the distances were increasing but rather as if we were "approaching" the system. We should therefore think of a strategy to limit this perception. A possible solution could be the introduction of a scale segment showing the size of a gigalight year at the time of the simulation in the selected coordinates.

During the interviews, the different activities required different initial configurations depending on the request. On the other hand, at the opening our simulations always appear in the same way. This led to two main problems: we used some time at the beginning of each teaching activity to correctly set up the simulation and to provide some information useful to solve a task. These problems can be solved simply creating a "**test mode**". This mode should be used during interviews and would consist in presenting our simulations in an orderly sequence with different settings depending on the teaching activity. Also the development of a tutorial might be useful to encourage autonomous use. We have thought to some possible solution for this problem: the preparation of an illustrative document and a series of video tutorials or a "**?**" button inside the simulations that, when activated, allows showing a pop-up element the cursor stands above.

Regarding the **educational content**, some aspects deserve to be considered and could be implemented in a future version of the animations. First of all, at present it is absent a direct reference to the redshift. Although this topic is important in cosmology it is not fundamental for the understanding of cosmological horizons. Actually, this phenomenon has been sketched out in the simulation "Hubble flow" where it is possible to observe the photon wavelength increasing over time. A possible way of developing further this feature are: to put the value of z together with the one of the scale factor in the different simulations or to show explicitly a change of color in the photon to suggest "perceptively" this phenomenon. The cosmological redshift is also present in one of the interview question: the teaching activity "cosmic lighthouse" shows that the phenomenon of cosmological redshift is not exclusively linked to the photon frequency. It is also possible to generalize it for a different signal. This could be pointed out on a perceptual level if we developed in the simulation "Hubble flow" the possibility of emitting multiple photons with regular frequency from a galaxy. This would be also a visual solution to the exercise of the “cosmic lighthouse”.

Another interesting extension is to include in the simulation ’Hubble flow’ the vector field of velocities associated with the various positions in space. Some graphic solutions for this purpose are: a series of arrows at regular distances with a modulus equal to the value of the recession velocity, a gradient of color similar to that used in the figure 1.4.

Once the aspects discussed earlier have been widened and implemented, it may be appropriate a second round of interviews with a sample of students with a more uniform background and using a more standardized procedure. The ideal scenario would be the development of a interface which allows the interviewee to use the animation without any external intervention. When this second phase of preliminary interviews will be concluded, it will be possible to move on to actual experiments.

The possible objectives of a possible future research can be multiple. We can inquiry if the teaching unit is more or less effective with respect to a more traditional lesson. It might be interesting to test and compare some of the paradigms used in the development of the design. For example the use of the **perceptual method** rather than the symbolic method. This technique of conceiving animations has give some evidence of being effective in the teaching of Physics [Z. Chen and Gladding, 2014] but currently the literature is lacking. These simulations could be an appropriate tool to test this paradigm through an experiment comparing them with another hypothetical animation where the symbolic method would be adopted.

Another feature that could be tested is how the simultaneous presentation of several animation compare with sequential animations. Although the first choice has some evidence of better performance compared with the second, this claim needs to be supported by more research [Ploetzner and Lowe, 2017].

The use of simultaneous animations is present in every simulation of our project. Through small changes to the code, simulations can be presented sequentially and not simultaneously. Then it would be possible through an experiment to compare these two modes and see if differences in the learning profile emerge.

Ultimately but not least, it is possible to imagine purposes outside of the research interest. The main use for which this software has been conceived is in the context of university teaching, specifically in an introductory course in cosmology. However we can think to a wider use for information purposes. For example, with appropriate precautions, it might be appropriate for a high school class as an introduction to cosmology or a deepening of special relativity. In addition, these simulations could also be used in more informal educational contexts (such as science museums, conferences, videos, etc.). Given the pervasiveness and importance of computer media in today's context, we would be happy to contribute to the modernization and usability of the contents of this type.

Bibliography

Adams, W. (2010). Student engagement and learning with phet interactive simulations. *Il nuovo cimento C*, *33*(3), 21–32.

Adams, W., Reid, S., LeMaster, R., McKagan, S., Perkins, K., Dubson, M., & Wieman, C. (2008a). A study of educational simulations part ii–interface design. *Journal of Interactive Learning Research*, *19*(4), 551–577.

Adams, W., Reid, S., LeMaster, R., McKagan, S., Perkins, K., Dubson, M., & Wieman, C. (2008b). A study of educational simulations part i-engagement and learning. *Journal of Interactive Learning Research*, *19*(3), 397–419.

Adesope, O., & Nesbit, J. (2012). Verbal redundancy in multimedia learning environments: A meta-analysis. *Journal of Educational Psychology*, *104*(1), 250.

Aghanim, N., Akrami, Y., Ashdown, M., Aumont, J., Baccigalupi, C., Ballardini, M., Banday, A., Barreiro, R., Bartolo, N., Basak, S., & et al. (2020). Planck 2018 results-vi. cosmological parameters. *Astronomy & Astrophysics*, *641*, A6.

Ainsworth, S., & Van Labeke, N. (2004). Multiple forms of dynamic representation. *Learning and Instruction*, *14*, 241–255.

Anderson, M. (2007). The massive redeployment hypothesis and the functional topography of the brain. *Philosophical Psychology*, *20*(2), 143–174.

Ariel, P., Podolefsky, N., & Perkins, K. (2013). Guiding without feeling guided: Implicit scaffolding through interactive simulation design. *AIP Conference Proceedings*, *1513*(1), 302–305.

Aslanides, J., & Savage, C. (2013). Relativity concept inventory: Development, analysis, and results. *Phys. Rev. ST Phys. Educ. Res.*, *9*, 010118. https://doi.org/10.1103/PhysRevSTPER.9.010118

Barsalou, L. (1999). Perceptual symbol systems. *Behavioral and brain sciences*, *22*(4), 577–660.

Barsalou, L. (2009). Simulation, situated conceptualization, and prediction. *Philosophical Transactions of the Royal Society B: Biological Sciences*, *364*(1521), 1281–1289.

Barsalou, L. (2010). Grounded cognition: Past, present, and future. *Topics in cognitive science*, *2*(4), 716–724.

Barsalou, L., & Wiemer-Hastings, K. (2005). Situating abstract concepts. *Grounding cognition: The role of perception and action in memory, language, and thought*, 129–163.

Bauer-Morrison, J., Tversky, B., & Betrancourst, M. (2000). Animation: Does it facilitate learning. *International Journal of Human Computer Studies*, *57*(4), 279–315.

Berney, S., & Bétrancourt, M. (2016). Does animation enhance learning? a meta-analysis. *Computers & Education*, *101*, 150–167.

Bétrancourt, M., & Tversky, B. (2000). Effect of computer animation on users' performance: A review/(effet de l'animation sur les performances des utilisateurs: Une sythèse). *Le travail humain*, *63*(4), 311.

Boucheix, J., & Guignard, H. (2005). What animated illustrations conditions can improve technical document comprehension in young students? format, signaling and control of the presentation. *European Journal of Psychology of Education*, *20*(4), 369–388.

Boucheix, J., & Lowe, R. (2008). Eye tracking as a basis for improving animation design. *Livre/Conférence International Journal of Psychology*, *43*, 747–748.

Boucheix, J., & Lowe, R. (2010). An eye tracking comparison of external pointing cues and internal continuous cues in learning with complex animations. *Learning and instruction*, *20*(2), 123–135.

Boucheix, J., Lowe, R., Putri, D., & Groff, J. (2013). Cueing animations: Dynamic signaling aids information extraction and comprehension. *Learning and Instruction*, *25*, 71–84.

Chandler, P., & Sweller, J. (1991). Cognitive load theory and the format of instruction. *Cognition and instruction*, *8*(4), 293–332.

ChanLin, L. (1998). Animation to teach students of different knowledge levels. *Journal of Instructional Psychology*, *25*(3), 166.

Chen, H., Lanzetta, K., & Pascarelle, S. (1999). A spectroscopically identified galaxy of probable redshift z = 6.68.

Chen, Z., & Gladding, G. (2014). How to make a good animation: A grounded cognition model of how visual representation design affects the construction of abstract physics knowledge. *Physical Review Special Topics-Physics Education Research*, *10*(1), 010111.

Coles, P., & Lucchin, F. (1995). Cosmology, the origin and evolution of cosmic structure. *Chichester: Wiley, —c1995*, *-1*.

Davis, T., & Lineweaver, C. (2001). Superluminal recession velocities. *AIP Conference Proceedings*, *555*(1), 348–351.

Davis, T., & Lineweaver, C. (2004). Expanding confusion: Common misconceptions of cosmological horizons and the superluminal expansion of the universe. *Publications of the Astronomical Society of Australia*, *21*(1), 97–109. https://doi.org/10.1071/as03040

Davis, T., Lineweaver, C., & Webb, J. (2003). Solutions to the tethered galaxy problem in an expanding universe and the observation of receding blueshifted objects. *American Journal of Physics*, *71*(4), 358–364. https://doi.org/10.1119/1.1528916

De Koning, B., Tabbers, H., Rikers, R., & Paas, F. (2010). Attention guidance in learning from a complex animation: Seeing is understanding? *Learning and instruction*, *20*(2), 111–122.

de Bernardis, P., Ade, P., Bock, J., Bond, J., Borrill, J., Boscaleri, A., Coble, K., Crill, B., De Gasperis, G., Farese, P., Ferreira, P., Ganga, K., Giacometti, M., Hivon, E., Hristov, V., Iacoangeli, A., Jaffe, A., A.H.and Lange, Martinis, L., S., M., … Vittorio, N. (2000). A flat universe from high-resolution maps of the cosmic microwave background radiation. *Nature*, *404*(6781), 955–959. https://doi.org/10.1038/35010035

Efstathiou, G. (2021). To ih/i0 or not to ih/i0? *Monthly Notices of the Royal Astronomical Society*, *505*(3), 3866–3872. https://doi.org/10.1093/mnras/stab1588

Ellis, G., & Rothman, T. (1993). Lost horizons. *American Journal of Physics*, *61*(10), 883–893.

Etkina, E., Van Heuvelen, A., White-Brahmia, S., Brookes, D., Gentile, M., Murthy, S., Rosengrant, D., & Warren, A. (2006). Scientific abilities and their assessment. *Phys. Rev. ST Phys. Educ. Res.*, *2*, 020103. https://doi.org/10.1103/PhysRevSTPER.2.020103

Fan, X., Hennawi, J., Richards, G., Strauss, M., Schneider, D., Donley, J., Young, J., Annis, J., Lin, H., Lampeitl, H., & et al. (2004). A survey ofz¿5.7 quasars in the sloan digital sky survey. iii. discovery of five additional quasars. *The Astronomical Journal*, *128*(2), 515–522. https://doi.org/10.1086/422434

Fischer, S., Lowe, R., & Schwan, S. (2008). Effects of presentation speed of a dynamic visualization on the understanding of a mechanical system. *Applied Cognitive Psychology: The Official Journal of the Society for Applied Research in Memory and Cognition*, *22*(8), 1126–1141.

Ginns, P. (2005). Meta-analysis of the modality effect. *Learning and instruction*, *15*(4), 313–331.

Goldhaber, G., Deustua, S., Gabi, S., Groom, D., Hook, I., Kim, A., Kim, M., Lee, J., Pain, C., R.and Pennypacker, Perlmutter, S., Small, I., Goobar, A., Ellis, R., Mcmahon, R., Boyle, B., Bunclark, P., Carter, D., Glazebrook, K., Irwin, M., … Couch, W. (1997). Observation of cosmological time dilation using type ia supernovae as clocks. In P. Ruiz-Lapuente, R. Canal, & J. Isern (Eds.), *Thermonuclear supernovae* (pp. 777–784). Springer Netherlands. https://doi.org/10.1007/978-94-011-5710-0_48

Goldhaber, G., Groom, D., Kim, A., Aldering, G., Astier, P., Conley, A., Deustua, S., Ellis, R., Fabbro, S., Fruchter, A., & et al. (2001). Timescale stretch parameterization of type ia supernovab-band light curves. *The Astrophysical Journal*, *558*(1), 359–368. https://doi.org/10.1086/322460

Grøn, Ø. (2002). A new standard model of the universe. *European journal of physics*, *23*(2), 135.

Guth, A. (1981). Inflationary universe: A possible solution to the horizon and flatness problems. *Physical Review D*, *23*(2), 347.

Hays, T. (1996). Spatial abilities and the effects of computer animation on short-term and long-term comprehension. *Journal of educational computing research*, *14*(2), 139–155.

Hegarty, M. (2004). Mechanical reasoning by mental simulation. *Trends in cognitive sciences*, *8*(6), 280–285.

Hegarty, M., & Sims, V. (1994). Individual differences in mental animation during mechanical reasoning. *Memory & Cognition*, *22*(4), 411–430.

Höffler, T., & Leutner, D. (2007). Instructional animation versus static pictures: A meta-analysis. *Learning and instruction*, *17*(6), 722–738.

Höffler, T., & Leutner, D. (2011). The role of spatial ability in learning from instructional animations – evidence for an ability-as-compensator hypothesis [Current Research Topics in Cognitive Load Theory]. *Computers in Human Behavior*, *27*(1), 209–216. https://doi.org/https://doi.org/10.1016/j.chb.2010.07.042

Huk, T. (2006). Who benefits from learning with 3d models? the case of spatial ability. *Journal of computer assisted learning*, *22*(6), 392–404.

Jaffe, A., Ade, P., Balbi, A., Bock, J., Bond, J., Borrill, J., Boscaleri, A., Coble, K., Crill, B., de Bernardis, P., Farese, P., Ferreira, P. G., Ganga, K., Giacometti, M., Hanany, S., Hivon, E., Hristov, V., Iacoangeli, A., Lange, A., … Wu, J. (2001). Cosmology from MAXIMA-1, BOOMERANG, and COBE DMR cosmic microwave background observations. *Physical Review Letters*, *86*(16), 3475–3479. https://doi.org/10.1103/physrevlett.86.3475

Kalyuga, S. (2008). Relative effectiveness of animated and static diagrams: An effect of learner prior knowledge. *Computers in Human Behavior*, *24*(3), 852–861.

Kearsley, G., & Shneiderman, B. (1998). Engagement theory: A framework for technology-based teaching and learning. *Educational technology*, *38*(5), 20–23.

Kiang, T. (1991). A new diagram for illustrating the horizon problem. *Acta Astrophysica Sinica*, *11*(3), 197–212.

Kurby, C., & Zacks, J. (2008). Segmentation in the perception and memory of events. *Trends in cognitive sciences*, *12*(2), 72–79.

Leibundgut, B., Schommer, R., Phillips, M., Riess, A., Schmidt, J., B.and Spyromilio, Walsh, J., Suntzeff, N., Hamuy, M., Maza, J., & et al. (1996). Time dilation in the light curve of the distant type ia supernova sn 1995k. *The Astrophysical Journal*, *466*(1), L21–L24. https://doi.org/10.1086/310164

Liebscher, D. (2005). *Cosmology* (Vol. 210). Springer Science & Business Media.

Lin, L., & Atkinson, R. (2011). Using animations and visual cueing to support learning of scientific concepts and processes. *Computers & Education*, *56*(3), 650–658.

Lowe, R., & Boucheix, J. (2008a). Learning from animated diagrams: How are mental models built? *International conference on theory and application of diagrams*, 266–281.

Lowe, R., & Boucheix, J. (2008b). Supporting relational processing in complex animated diagrams. *International Conference on Theory and Application of Diagrams*, 391–394.

Lowe, R., & Boucheix, J. (2012). Dynamic diagrams: A composition alternative. *International Conference on Theory and Application of Diagrams*, 233–240.

Lowe, R., & Boucheix, J. (2017). A composition approach to design of educational animations. In *Learning from dynamic visualization* (pp. 5–30). Springer.

Lowe, R., Boucheix, J., & Fillisch, B. (2017). Demonstration tasks for assessment. In *Learning from dynamic visualization* (pp. 177–201). Springer.

Lowe, R. (1999). Extracting information from an animation during complex visual learning. *European journal of psychology of education*, *14*(2), 225–244.

Lowe, R. (2003). Animation and learning: Selective processing of information in dynamic graphics [External and Internal Representations in Multimedia Learning]. *Learning and Instruction*, *13*(2), 157–176. https://doi.org/https://doi.org/10.1016/S0959-4752(02)00018-X

Margalef-Bentabol, B., Margalef-Bentabol, J., & Cepa, J. (2013). Evolution of the cosmological horizons in a universe with countably infinitely many state equations. *Journal of Cosmology and Astroparticle Physics*, *2013*(02), 015–015. https://doi.org/10.1088/1475-7516/2013/02/015

Mayer, R. (2002). Multimedia learning. In *Psychology of learning and motivation* (pp. 85–139). Elsevier.

Mayer, R. (2004). Teaching of subject matter [PMID: 14744232]. *Annual Review of Psychology*, *55*(1), 715–744. https://doi.org/10.1146/annurev.psych.55.082602.133124

Mayer, R., & Chandler, P. (2001). When learning is just a click away: Does simple user interaction foster deeper understanding of multimedia messages? *Journal of educational psychology*, *93*(2), 390.

Mayer, R., Moreno, R., Boire, M., & Vagge, S. (1999). Maximizing constructivist learning from multimedia communications by minimizing cognitive load. *Journal of educational psychology*, *91*(4), 638.

Mayer, R., & Sims, V. (1994). For whom is a picture worth a thousand words? extensions of a dual-coding theory of multimedia learning. *Journal of educational psychology*, *86*(3), 389.

Mayer, R. E. (2009). *Multimedia learning* (2nd ed.). Cambridge University Press. https://doi.org/10.1017/CBO9780511811678

Moreno, R., & Mayer, R. (2007). Interactive multimodal learning environments. *Educational psychology review*, *19*(3), 309–326.

Muller, D., Bewes, J., Sharma, M., & Reimann, P. (2008). Saying the wrong thing: Improving learning with multimedia by including misconceptions. *Journal of Computer Assisted Learning*, *24*(2), 144–155. https://doi.org/https://doi.org/10.1111/j.1365-2729.2007.00248.x

Narayanan, N., & Hegarty, M. (2002). Multimedia design for communication of dynamic information. *International journal of human-computer studies*, *57*(4), 279–315.

Nemiroff, R., & Patla, B. (2008). Adventures in friedmann cosmology: A detailed expansion of the cosmological friedmann equations. *American Journal of Physics*, *76*(3), 265–276. https://doi.org/10.1119/1.2830536

Perlmutter, S., Aldering, G., Goldhaber, G., Knop, R., Nugent, P., Castro, P., Deustua, S., Fabbro, S., Goobar, A., Groom, D., & et al.

(1999). Measurements of ω and λ from 42 high-redshift supernovae. *The Astrophysical Journal*, *517*(2), 565–586. https://doi.org/10.1086/307221

Ploetzner, R., & Lowe, R. (2017). Looking across instead of back and forth: How the simultaneous presentation of multiple animation episodes facilitates learning. In *Learning from dynamic visualization* (pp. 51–68). Springer.

Pössel, M. (2020). Interpretations of cosmic expansion: Anchoring conceptions and misconceptions. *Physics Education*, *55*(6), 065006. https://doi.org/10.1088/1361-6552/aba3b1

Reed, S. (2006). Cognitive architectures for multimedia learning. *Educational psychologist*, *41*(2), 87–98.

Rieber, L. (1991). Effects of visual grouping strategies of computer-animated presentations on selective attention in science. *Educational Technology Research and Development*, *39*(4), 5–15.

Rieber, L., Smith, L., & Noah, D. (1998). The value of serious play. *Educational technology*, *38*(6), 29–37.

Riess, A., Filippenko, A., Challis, P., Clocchiatti, A., Diercks, A., Garnavich, P., Gilliland, R., Hogan, C., Jha, S., Kirshner, R., & et al. (1998a). Observational evidence from supernovae for an accelerating universe and a cosmological constant. *The Astronomical Journal*, *116*(3), 1009.

Riess, A., Filippenko, A., Challis, P., Clocchiatti, A., Diercks, A., Garnavich, P., Gilliland, R., Hogan, C., Jha, S., Kirshner, R., & et al. (1998b). Observational evidence from supernovae for an accelerating universe and a cosmological constant. *The Astronomical Journal*, *116*(3), 1009–1038. https://doi.org/10.1086/300499

Riess, A., Yuan, W., Macri, L., Scolnic, D., Brout, D., Casertano, S., Jones, D., Murakami, Y., Anand, L., G.S.and Breuval, & et al. (2022). A comprehensive measurement of the local value of the hubble constant with 1 km s- 1 mpc- 1 uncertainty from the hubble space telescope and the sh0es team. *The Astrophysical Journal Letters*, *934*(1), L7.

Rindler, W. (1956). Visual horizons in world models. *Monthly Notices of the Royal Astronomical Society*, *116*(6), 662–677.

Ryden, B. (2003). *Introduction to cosmology*. Addison-Wesley. http://gen.lib.rus.ec/book/index.php?md5=8738a0bfa6e4a2977fdc19e67e128d45

Ryden, B. (2006). *Introduction to cosmology*. http://carina.fcaglp.unlp.edu.ar/extragalactica/Bibliografia/Ryden_IntroCosmo.pdf

Scherr, R. (2007). Modeling student thinking: An example from special relativity. *American Journal of Physics*, *75*(3), 272–280. https://doi.org/10.1119/1.2410013

Schmidt-Weigand, F., Kohnert, A., & Glowalla, U. (2010). A closer look at split visual attention in system-and self-paced instruction in multimedia learning. *Learning and instruction*, *20*(2), 100–110.

Schnotz, W. (2002). Commentary: Towards an integrated view of learning from text and visual displays. *Educational psychology review*, *14*(1), 101–120.

Schwan, S., & Papenmeier, F. (2017). Learning from animations: From 2d to 3d? In *Learning from dynamic visualization* (pp. 31–49). Springer.

Schwan, S., & Riempp, R. (2004). The cognitive benefits of interactive videos: Learning to tie nautical knots. *Learning and instruction*, *14*(3), 293–305.

Sweller, J. (1994). Cognitive load theory, learning difficulty, and instructional design. *Learning and Instruction*, *4*(4), 295–312. https://doi.org/https://doi.org/10.1016/0959-4752(94)90003-5

Sweller, J. (2010). Element interactivity and intrinsic, extraneous, and germane cognitive load. *Educational psychology review*, *22*(2), 123–138.

Sweller, J., Van Merrienboer, J., & Paas, F. (1998). Cognitive architecture and instructional design. *Educational psychology review*, *10*(3), 251–296.

Tabbers, H. (2002). The modality of text in multimedia instructions: Refining the design guidelines.

Tabbers, H., Martens, R., & Van Merrienboer, J. (2001). The modality effect in multimedia instructions. *Proceedings of the Annual Meeting of the Cognitive Science Society*, *23*(23).

Tversky, B., Morrison, J., & Betrancourt, M. (2002). Animation: Can it facilitate? *International journal of human-computer studies*, *57*(4), 247–262.

Weinberg, S. (1972). *Gravitation and cosmology*. John Wiley; Sons.

Wieman, C., Adams, W., Loeblein, P., & Perkins, K. (2010). Teaching physics using phet simulations. *The Physics Teacher*, *48*(4), 225–227.

Wu, H., & Shah, P. (2004). Exploring visuospatial thinking in chemistry learning. *Science education*, *88*(3), 465–492.

Zeilik, M., & Gregory, S. (1998). *Introductory astronomy and astrophysics* (4th). Cengage Learning. http://gen.lib.rus.ec/book/index.php?md5=ebf1b960aedfd90792dcf7feb2b1cef6

Appendix A

Links to Web applications of the simulations

Hubble Flow Simulation Code repository

Light cones comparison Simulation Code repository

Horizon evolution Simulation Code repository