Università degli Studi di Padova

FACOLTÀ DI INGEGNERIA
Corso di Laurea Magistrale in Ingegneria dell'Automazione

Tesi di laurea magistrale

Interactive Motion Prediction using Game Theory

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Anno Accademico 2012–2013

This thesis has been developed at the Institute of Automatic Control Engineering (LSR) of the Technische Universität München under the supervision of the Dipl.-Ing. Daniel Althoff, under an exchange program from the 1.5.2012 to 1.3.2013. It has been a great opportunity for me and I personally thank him, my italian supervisor Prof. Luca Schenato and my Erasmus coordinator Prof. Silverio Bolognani for having made this possible.

Abstract

Prediction of human behaviour is a crucial argument in the integration of robots with people in everyday's life, especially for path planning's purposes. In this thesis we consider two specific scenarios where people interact between themselves: the first one is a pedestrian scenario where humans walk towards different destination in a open hall; the second is a congested highway scenario, where several cars move searching their best trajectories, taking in consideration possibles interaction with other drivers around them.

This analysis is developed adopting the game theory to the different subjects: in these scenarios we assume that when planning trajectories in an interactive area each behaviour is influenced by the other participants. We convert this problem into a game, where each driver or pedestrian is a player with his correspondent set of actions. Typical solutions of these games will be then configured as explanation of the motion in the different scenarios.

Contents

1 Introduction **5**
1.1 Motivation 5
1.2 Related Works 7

2 Game Theory Fundamentals **9**
2.1 Normal Form Games 9
2.2 Extensive Form Games 11
2.3 Nash Equilibria 13
2.4 Multistage Games 15

3 Problem Formulation and Implementation **17**

4 Pedestrian Scenario **23**
4.1 Data filtering and cost function choice 26
4.1.1 Recalls on B-Spline 28
4.1.2 Implementation of Kalman filter 30
4.1.3 Game formulation and setup 34
4.2 Analysis with 2-players games 37
4.3 Analysis with N-players games 47
4.4 Discussion 58

5 Automotive Application **61**
5.1 Description of the simulated scenario 62
5.2 Game setup and choice of cost function 65
5.3 Simulations 67
5.4 Discussion 84

6 Conclusions and Future Works **87**
6.1 Conclusions 87
6.2 Future Works 88

Bibliography **91**

Chapter 1

Introduction

1.1 Motivation

Robotics can be considered by now, under every point of view, integral part of human's everyday life. Although in the past the principal aim of research was to obtain robots that could replace a person in the daily actions, new applications and improvements are particularly challenging in the integration between human and machine. Motion planning and navigation can be surely considered one of these: while in the past the approach to the problem consisted essentially of creating privileged workspaces or areas unaccessible to humans, so that inside them the robot could be programmed without any particular considerations on its motion in the open space, but rather on the efficiency of the motors on its kinematic chain and joints, in the last years we have noticed a quick change on the motion conception, making possible the existence of a mutual interaction between robot and human, where both of them move in the same space without necessarily bringing to critical situations like collisions or danger towards people.

This problem is very interesting when robots move in close areas (for example, a narrow corridor or a room with many furnitures), where walls or any sort of obstacles can be relevant problems when facing an human in its path. Usually a solution can be found ensuring that the robot put itself on a lower priority level respect to humans, so that it could calculate its own trajectory under some constraints, that brings to a constrained optimum problem. In case of a presence of an interaction between a high number of robotic entities (as well as humans) this could bring to movement problems, since that it would be difficult to assign a hierarchical ranking between the moving robots.

Our research set as a goal to find a possible solution that could predict the motion of several agents in a dynamic scenario, so that all the agents can move following a path that minimize their effort and maximize a certain gain index, according to some criteria (basically kinematic like speed and direction variation, travel time). This is possible only if all the trajectories of humans in different scenarios (walking, driving..) can be predicted with reliable accuracy, so that robots could be integrated following the same motion schemes. The basis of our research will be then focused on trajectories where robots

- eliminate or minimize the collision risk between them and between humans
- move with maximum comfort, which means that their trajectories maximize certain indexes depending on path' smoothness or jerk
- everyone follows a unique model of motion, likely to those usually executed by a human walking

In order to do this, we will solve these problems through the Games Theory. In fact, this is a theory, born and developed since 1944 as a study over economic factor trying to obtain a possible reliable prediction of some data, that lend itself to our situation: every human interacting will be considered as a player and every action as an available executable trajectories. The interaction between the different users will lead to some situations where each user obtains the maximum possible benefit resulting from the combination of all the players' choices. These situations will be our predictions on the motion of the different players.

We will therefore assign to the studied scenario a function that will initially define the corresponding costs associated to the trajectories: this function will be crucial because it will represent the model through which a robot will interface when choosing which trajectory to execute in the free motion space; from this cost we will obtain through another function the corresponding game payoffs that we will use, considering also possibles collisions in the scenario (which will be assumed always to be the minimum possible payoff); once obtained the game description, we will search the equilibrium points in our system, those that according to our mathematical model will be optimal in navigation; we will verify through a dataset of real moving people how these points will be prediction with satisfactory performances of human motion.

In the second part we will focus on a second application of the multiagent navigation, based on game theory: we will describe indeed how also in a scenario populated by car in a high speed traffic situation (in our case, a highway), using a suitable model that explains which could be the possible actions of a driver, it's possible to make a prediction on the urban motion the be followed in order to minimize both kinetic costs and possible collisions.

1.2 Related Works

Several different methodologies are investigated in literature in order to find out a reliable solution to this problem. Let's make then a schematic overview about the different concepts for predicting human motion. One typical approach consists of using the HMM theory for computing most probables trajectories executed by people; although generally accurate and reliable for static scenarios, HMM's approaches lack in flexibility when modifying the scenario: in fact, the trajectories are obtained after refining with a research over all the possibles trajectories for the most probable one. But when modifying even lightly the space of action (like moving furniture, creating walls and so on), the results could be not desirable since that they could cross these new obstacles. In [1] it's presented a novel approach that tries to minimize these characteristics, making the changes on the scenario less effective than the normal approach. In this case the model focus on the dynamic properties of an agent's interaction with its environment. Differently from the typical approaches, based on static observable parameters, such as position, their new approach consists of considering also visible variables (change in position and angles) with dynamic properties to insert in the motion model

Other methods are learning based algorithms: these algorithms require a preliminary phase where typically in the motion patterns used for the prediction some variable parameters are set, basing on the experimental collected data. In [2] it's described a method that uses the "Expectation-Maximization Algorithm" (EM): initially it's described an algorithm that in input considers a set of N trajectories $d = \{d_1, ..., d_n\}$ and a set of M outputs corresponds to the M possible motion patterns $\theta = \{\theta_1, ..., \theta_M\}$ which will be performed with the highest probability by a pedestrian in the assigned space. In this case to each trajectory d_i corresponds a set of of time samples describing the current position; each position is approximate through assigning it to a box on a grid that divides the motion space: under the assumption that each pattern could be represented with a set of probability density functions $p(x|\theta_m^t)$ that describe the probability that the person is at location x after t steps given that that he or she is engaged in this motion pattern, the likelihood of a trajectory d_i under the m-th motion model θ_m as

$$p(d_i|\theta_m)) = \Pi_{t=1}^{T} p(x_i^t|\theta_m^t)$$

. This probability function, that is assumed to be Gaussian with deviation σ, is maximized through the EM-algorithm. This method unfortunately reveals some intrinsic problems: First of all because the convergence of such algorithm is not always guaranteed; moreover, learned algorithms result to be a few flexible, since the modification of the scenario with the introduction of possible obstacles could decrease clearly the performances.

Another method is that one depicted in [3]: in this work the motion of a pedestrian is modeled through a summation of concurrent forces on the subject, consequence of his desired direction, but also of different social and environmental interactions that appears in the space. This summation is decomposed through linearity as a summation of the individual contributions due to the j-th person or k-th obstacle. In details, it's assumed at the beginning that, without interactions, the pedestrian motion p_i with mass m_i desires to move with velocity $\hat{v}_i$ along the direction $\hat{e}_i$, adapting his actual velocity v_i during time τ_i (considered as the necessary time for obtaining this deviation): this motion is a consequence of the total force

$$F_i^{pers} = m_i \frac{\hat{v}_i \hat{e}_i - v_i}{\tau_i}$$

which is the only one present, without interactions from other entities. Afterwards it's introduced the presences of the obstacles, which is divided into terms $f_{i,j}^{soc}$, coming from the presence of other pedestrians nearby, and terms $f_{i,o}^{soc}$ corresponding to the presence of any fixed obstacle, like chairs, benches etc.. In total, the total force contribution is defined as the summation of the individual components, so that

$$F_i^{soc} = f_{i,j}^{soc} + f_{i,o}^{soc}$$

. Differently from the other approaches, this method also considers possible environmental constraints, like wall: in this case another force $f_{i,k}^{phys}$ it's added, depending as well on the distance between pedestrian and wall. The general expression of the motion is finally ruled by the total force

$$F_i = F_i^{pers} + F_i^{soc} + F_i^{phys}$$

. Using the general law of the motion, the authors obtain a model of the motion of the pedestrian, where his position are estimated though a Kalman filter. This approach is efficient particularly because it overcomes the limits of the previous studies, which means especially modeling possible behaviour constraint and mobile obstacles. Unfortunately, in different behaviours it needs to be tuned because in limit cases it could happen that those forces could result too strong and therefore effecting on a low performances estimation.

Chapter 2

Game Theory Fundamentals

We want to make some recalls on the game theory, focusing on the concepts that will be crucial for our methods and aims, like for example the Nash equilibria and all the various representation forms for the games[1]. The game theory is the study of the different interactions between agents, where everyone of them can obtain a certain outcome depending on which choices it makes, but this income will also depend on other agents' choices. In order to be more rigorous, every agent that interacts must be considered as an entity that can express preferences on a set of choices: depending on the particular combination of choices taken from all the agents, it's assigned to them a value that describes, according to a certain ranking of varying nature (economical, social, psychological..) an utility. This utility assures that they could be considered as self interested agents, which means that in every particular situation of study (the game) they will try to maximize this utility (the income or payoff) could varying on their set of preferences. It's important to underline that the definition of "self-interested" doesn't mean that they try to damage other agents, but rather that their choices will be determined in order to maximize their payoff and to come into a situation that give them the best possible state situation.

2.1 Normal Form Games

Normal Form Games (NFG) is the simplest and most common way of describing a game. There are many reasons about this, first of all because its matrix form is very clear and intuitive when the number of players and choices is reduced, so that sometimes even important properties like equi-

[1] For theoretical rigour and clarity the following definitions and theorems are taken from [4]. Further and more detailed explanations about games theory can be found in this book

libria, dominances and so on could be identified quickly from the graphical representation; there are also other properties that make it the most fundamental representation, that is the property of most of the games in other game representations to be reduced to a normal form.

Definition 2.1 (Normal-form game). *A (finite, n-person) normal-form game is a tuple (N,A,u), where:*

- *N is a finite set of n players, indexed by i;*
- *$a = (a_1, ..., a_n) \in A$ is called an action profile;*
- *$u = (u_1, ..., u_n)$ where $u_i : A \mapsto R$ is a real-valued utility (or payoff) function for player i;*

The typical way to represent graphically a NFG is through a n-dimensional matrix. In this matrix, every dimension has a length l_i depending on the number of possibles action in a_i and in every cell are arranged the incomes related to those choices.

Table 2.1: Example of a 2-players game in NFG

	Player2 - choice 1	Player2 - choice 2
Player1 - choice 1	(a_{11}, b_{11})	(a_{12}, b_{12})
Player1 - choice 2	(a_{21}, b_{21})	(a_{22}, b_{22})
Player1 - choice 3	(a_{31}, b_{31})	(a_{32}, b_{32})

Once defined the players and the sets of actions, we need to describe how a player chooses its action to play. The most immediate solution is to select one single action in the set a_i and play it. So in this case we will have a income equal to the value in the cell corresponding to the selected actions. This strategy of choosing one single action to play is called *pure strategy* and if every player uses a pure-strategy, this situation will be called a *pure-strategy profile*.

Another typical strategy profile is to randomize with a certain probability distribution over the available s choices, that's the case called *mixed strategy*

Definition 2.2 (Mixed strategy). *Let (N, A, u) be a NFG, and for any set X let $\Pi(X)$ be the set of all probability distribution over X. Then the set of mixed strategies for player i is $S_i = \Pi(A_i)$.*

Definition 2.3 (Mixed-strategy profile). *The set of mixed-strategy profiles is simply the Cartesian product of the individual mixed-strategy sets, $S_1 x ... x S_n$.*

By $s_i(a_i)$ we denote the probability that an action a_i will be played under mixed strategy s_i. The subset of actions that are assigned positive probability by the mixed strategy s_i is called the *support support* of s_i.

Definition 2.4 (Support). *The support of a mixed strategy s_i for a player i is the set of pure strategies $a_i|s_i(a_i) > 0$.*

Particular mixed strategies are those where all the possible actions have non-zero probabilities, in this case we call them *fully mixed strategies*; in case that only one action has positive probability we are in the previous situation of pure strategy. With a mixed-strategy profile, the calculation of the expected payoff is not straightforward as in a pure-strategy profile, but we have to execute the sum of the payoffs, where all the possible choices are weighted through their assigned probability. Formally, the definition is the following:

Definition 2.5 (Expected utility of a mixed strategy). *Given a NFG (N, A, u), the expected utility u_i for player i of the mixed-strategy profile $s = (s_1, ..., s_n)$ is defined as*

$$u_i(s) = \sum_{a \in A} u_i(a)\Pi_{j=1}^{n} s_j(a_j)$$

2.2 Extensive Form Games

We go now through another common way of describing a game. We look now at (Perfect information) extensive-form games (EFG), where the sequence of the choices is assumed not to be simultaneous, but rather in a temporal alternance between the players. This alternance is a news respect to the NFG and requires a different graphic representation, that this a tree, where an action corresponds to every branch and the payoff incoming from those choices corresponds to every terminal leaf: since we will consider only a finite set of possible actions, there will be only finite trees. Speaking about solving games and properties, EFG can be rearranged in a NFG just eliminating the temporal property, so when we will introduce the notion of Nash Equilibria, all the theory about NFG could be used also for EFG.

Definition 2.6 (Perfect-information game). *A (finite) perfect-information game (in extensive form) is a tuple $G = N, A, H, Z, \chi, \rho, \sigma, u)$, where:*

- *N is a set of n players;*
- *A is a)single) set of actions;*

- *H is a set of nonterminal choice nodes;*

- *Z is a set of terminal nodes, disjoint from H;*

- *$\chi : H \mapsto 2^A$ is the action function, which assigns to each choice node a set of possible actions;*

- *$\rho : H \mapsto N$ is the player function, which assigns to each nonterminal node a player $i \in N$ who chooses an action at that node;*

- *$\sigma : H \times A \mapsto H \cup Z$ is the successor function, which maps a choice node and an action to a new choice node or terminal node such that for all h_1, $h_2 \in H$ and a_1 $a_2 \in A$, if $\sigma(h_1, a_1) = \sigma(h_2, a_2)$ then $h_1 = h_2$ and $a_1 = a_2$; and*

- *$u = (u_1, ..., u_n)$ where $u_i : Z \mapsto \Re$ is a real-valued utility function for player i on the terminal nodes Z*

A pure-strategy profile here is less intuitive respect to a NFG, in fact let's consider a game with 2 players: when the first player has made his choice, we will be on a part of the tree that exclude some possible choices for player 2. In this case, a pure strategy for this player is not only to indicate which action is going to play at that node, but also which other actions he would play in the other (not reachable) nodes. Formally speaking, a pure strategy is defined as follows:

Definition 2.7 (Pure strategies)**.** *Let $G = (N, A, H, Z, \chi, \rho, \sigma, u)$ be a perfect-information extensive-form game. Then the pure strategies of player i consist of the Cartesian product $\Pi_{h \in H, \rho(h)=i} \chi(h)$.*

Since that we will compute equilibria through NFG we need to introduce this procedure: for every pure strategy we assign a correspondent payoff that will be inserted in the NFG matrix. Doing this we can see that this redundancy cause several pure strategies in the normal form to have the same income, which make the size of the related NFG increased when compared with the extensive one.

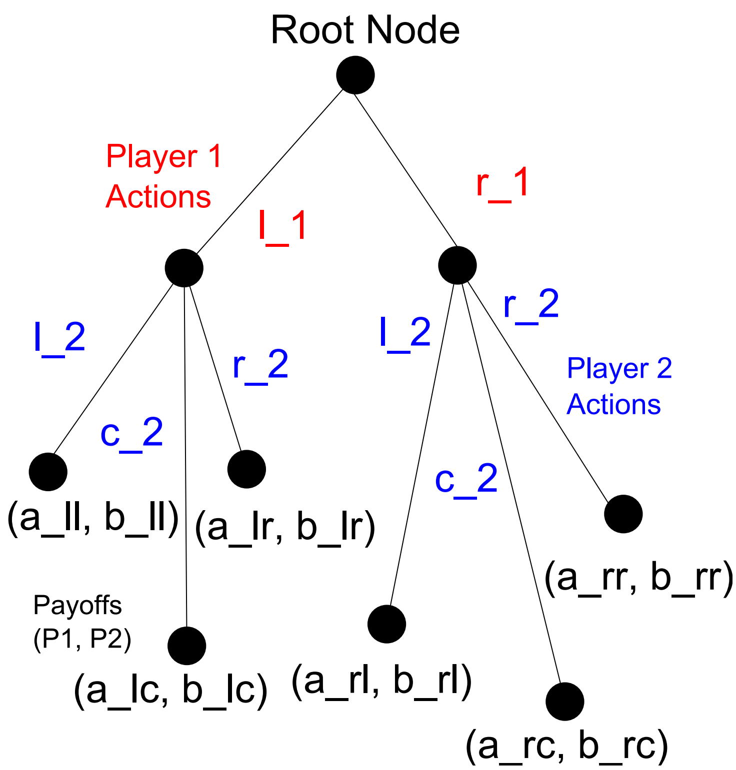

Figure 2.1: Typical representation for EFG with 2 players of respectively 2 and 3 actions

2.3 Nash Equilibria

One of the most important goals when studying a game is certainly the *Nash Equilibrium*. Intuitively, once that N-1 players has selected their pure/mixed strategy to play, the remaining one will select its strategy that will let him gain the highest possible income. If we apply this reasoning to all the players, we will be in a situation where every agent wouldn't desire to change their strategy because they would get a lower payoff. Before defining rigorously what a Nash Equilibrium is, we introduce the concept of *domination*, which is central in the algorithms when searching for equilibria.

Definition 2.8 (Domination)**.** *Let s_i' and s_i be two strategies of player i, and S_{-i} the set of all strategy profiles of the remaining players. Then s_i (strictly) dominates s_i' if for all $s_{-i} \in S_{-i}$, it is the case that $u_i(s_i, s_{-i}) > u_i(s_i', s_{-i})$.*

If one strategy dominates all others, we say that is *strongly dominant*.

Definition 2.9 (Dominant strategy)**.** *A strategy is strictly dominant for an agent if it strictly dominates any other strategy for that agent*

Domination is central in the study of a game, because if we manage to prove that a strategy (which could be pure or even mixed between 2 or more actions) dominates another one, than in any case we won't consider the dominated strategy to be played, since it could give a worse income to the player. Let's focus now on the Nash Equilibria (NEs), which is one of the most important solution concepts in Games Theory. In fact, the research of NEs will be our goal in the real scenarios. In NFG, with the previous definition of expected payoff, NE and best response come straightforward:

Definition 2.10 (Best response)**.** *Player i's best response to the strategy profile s_i is a mixed strategy $s_i^* \in S_i$ such that $u_i(s_i^*, s_{-i}) \geq u_i(s_i, s_{-i})$ for all strategies $s_i \in S_i$.*

Except for special cases where there is a unique pure strategy that represents the best response, usually there are more than one. In fact, it can be proved that if we have 2 different strategies that which are both best responses, any mixture of those 2 is itself a best response (otherwise we would prefer one strategy instead of the other). When all the players go for their possible responses we come to a equilibrium point between the players

Definition 2.11 (Nash equilibrium)**.** *A strategy profile $s = (s_1, ..., s_n)$ is a Nash equilibrium if, for all agents i, s_i is a best response to s_{-i}.*

The research of NEs in a game have sense since a fundamental theorem assure us the existence of at least one NE, considering mixed-strategy profiles.

Theorem 1 (Nash, 1951)**.** *Every game with a finite number of players and action profiles has at least one Nash equilibrium.*

In real scenarios, due to approximations and noises, the notion of NE could be too restrictive: in fact, our expected calculated NEs won't be the real chosen actions, but they won't be too far from them with regard of payoff. This means that up to an additive little positive constant value ϵ, they will satisfy all the properties of the best response. Let's formalize better this concept, including it into a definition:

Definition 2.12 (ϵ-Nash)**.** *Fix $\epsilon > 0$. A strategy profile $s = (s_1, ..., s_n)$ is an ϵ-Nash equilibrium if, for all agents i and for all strategies $s_i' \neq s_i, u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i}) - \epsilon$*

Of course, ϵ-Nash equilibria always exist and also, once founded a NE this is surrounded by an entire set of ϵ-NE, for a certain ϵ.

2.4 Multistage Games

Multistage (finite) games are games where the game is played multiple times. Usually the set of actions and players is not modified, but the table of the correspondent payoffs may change, so that each time that a game is played (single stage game) a different NE could be found. The values in the payoffs' table could be different according to the strategies played in the previous stage: there could be therefore a different sequence of Nash equilibria. There are several methods for analyzing a multistage game, but the most performed is to select the sequence of NE so that the total payoff is the highest possible.

Definition 2.13 (Game payoff)**.** *Given a sequence of payoffs* $\{p_1, p_2, ..., p_N\}$ *corresponding to the strategies* $\{s_1, s_2, ..., s_N\}$*, the game payoff is the sum of all the* N *payoff values* $GP = \Sigma_{k=1}^{N} p_k$

Definition 2.14 (Multistage payoff)**.** *Given a sequence of* M *stages with* M *game payoffs corresponding to* M *strategies* $\{s_{1_M}, s_{2_M}, ..., s_{N_M}\}$*, the multistage payoff is defined as the sum of all the* M *game payoffs* $\Sigma_{k=1}^{M} GP_k$

Chapter 3

Problem Formulation and Implementation

Analyzing a problem through the game theory means basically being able to associate to it a scheme that could describe completely its characteristics using the features of a game. There are several type of games that can be associated, depending on the typology of the players and on how their relationships modify the results of the strategies: in this case, when we speak about the individual motion of each subject, we suppose always that each pedestrian moves in the space following an individual optimality criterion, which means that when it moves, either in conditions of interaction with other pedestrians or not, it will perform a trajectory that is the most "natural" possible for it. In order to characterize better this concept, we focus therefore on the so-called "Non cooperative games", that are those that express in the best way these behaviours. In fact, in these games similarly to the concept of maximum comfort for the pedestrian trajectory corresponds the concept of "maximum payoff" for the player, that is what it earns from the final NE of the game. While describing now the implementation of the problem, we will consider the pedestrian scenario (taken into account in Chapter 4), with people walking and performing different trajectories towards different goals. Same considerations will be valid afterwards also for the following automotive scenario.

First of all, following the definition of NFG, we need to define a set of N players, where each player corresponds to a set of possible actions $(a_1, a_2, ..., a_N)$: this correspondence is clearly straightforward, since that we can fix that to each player corresponds a pedestrian and to each action corresponds a trajectory. As regards the choice of the trajectories to introduce in the set $a_i = \{a_{i_1}, a_{i_2}, ..., a_{i_{M_i}}\}$ of possible actions for player i, it will depend on which kind of trajectories will be considered in the relative game, so that they could be valid for our study. For example, in case of a pedestrian, all the possible trajectories with the same starting point and same final destination will be selected (in this case, in order to avoid possible problems and motion inequalities, we will choose only trajectories having an initial speed v_0 comparable to that one corresponding to the case that we are studying); in the automotive scenario will be instead followed other criterions that we'll explain directly in the relative chapter.

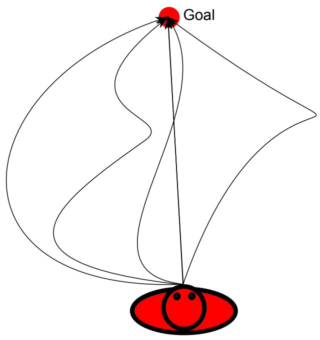

Figure 3.1: Scheme of the selected trajectories for the pedestrian scenario

Obtained now a set of players and a set of actions, we need to describe now how to assign the payoffs in the table relative to the normal form: in this case we choose to divide this phase in 2 parts, where the first one is free from possible interactions, introduced in the second one. In the first part we assign to each combination of strategies a set of payoffs that depend only on the strategy player by the relative player. This payoff is a value that describes numerically how much this trajectory is "liked" from the pedestrian, therefore it will depend directly on kinematic factors, typical of the motion. In particular, since that in the motion of the pedestrian (same considerations count for a car driver) more present are elements such as accelerations, direction changes etc.. less feasible is the correspondent trajectory, therefore we will have assigned initially to each trajectory a cost function (where to a high cost corresponds a trajectory little "liked"), while afterwards this cost value will became, through a linear transformation, a value describing it's appeal (that is, to each high payoff corresponds a highly "liked" trajectory). The second part of the construction of the payoff matrix consists basically in the introduction of the interactive component between the different players, which means that the payoffs obtained from a player are not only function of his choices, but also of strategies played by the all the other players in the game. In this case, we decided that the only one situation that could modify substantially the player's payoff is the collision with another player, that is when the 2 trajectories are actually not compatible between them. It's also performed a check on each combinations of strategies, so that they could be compatible or not between them: in negative case, the correspondent payoff is modified and it's assigned to it a different value, correspondent to a lower payoff (it's assumed lower than all the possible not colliding payoffs, since that on dominance hypothesis, one strategy leading to collision won't be chosen). This collision check is performed verifying that the 2 player are not on the same position (even through a small overlapping) for a time instant. In Figure 3.2 it's reported the scheme describing the algorithm of creation of assignment of the payoff corresponding to a combination of strategies.

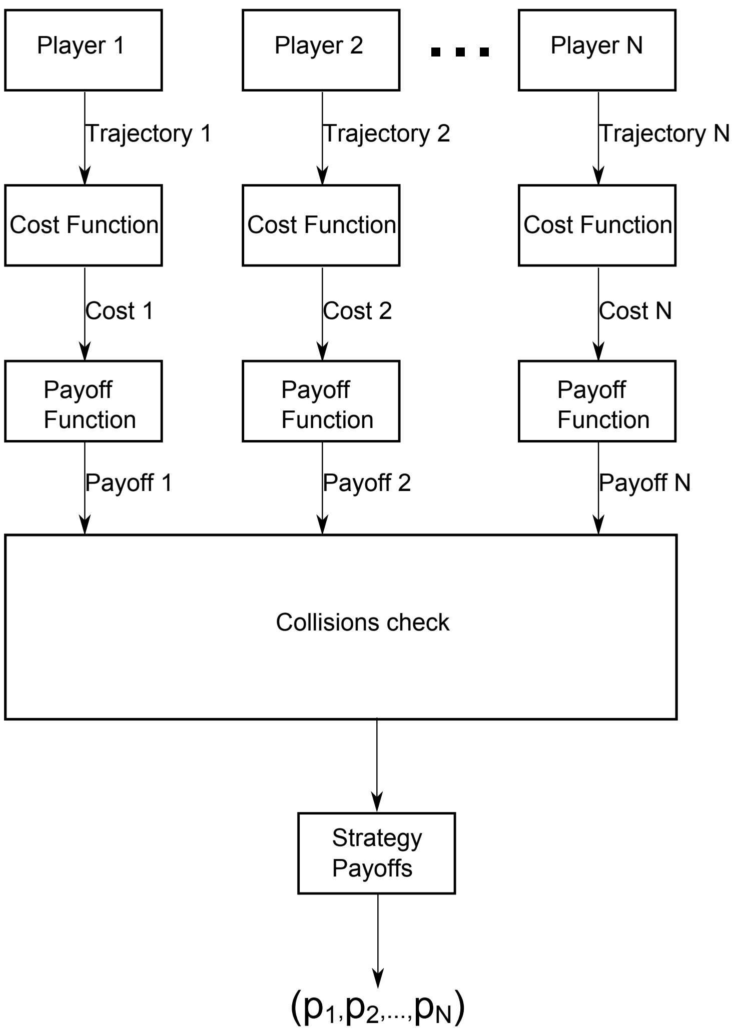

Figure 3.2: Scheme of the payoff assignment algorithm

Table 3.1: Description of the Feasibility Algorithm

Input: $S = (S_1, ..., S_n)$
Output: NE p, if there exists both a strategy profile $p = (p_1, ..., p_n)$ and a value profile $v = (v_1, ..., v_n)$ such that: $\forall i \in N, a_i \in S_i : \Sigma_{a_{-i} \in S_{-i}} p(a_{-i} u_i(a_i, a_{-i}) = v_i$ $\forall i \notin N, a_i \in S_i : \Sigma_{a_{-i} \in S_{-i}} p(a_{-i} u_i(a_i, a_{-i}) \leq v_i$ $\forall i \in N : \Sigma_{a_i \in S_i} p_i(a_i) = 1$ $\forall \in N, a_i \in S_i : p_i(a_i) \geq 0$ $\forall \in N, a_i \notin S_i : p_i(a_i) = 0$

Considerations over the computational cost when searching for Nash Equilibria

As we have seen, NEs are strategies operated by the agents that benefit of particular properties. In the research of them the algorithm is based essentially on checking these properties that can be put in synthesis through a feasibility check of some conditions(see Table 3.1).

As we can notice, this problem results to be linear only in the special case for $N = 2$ (so that lead to a quick resolution). Vice versa, when the number of agents increases (we recall here that we suppose that this study could be applied in a contest where the number of human and robotic users can be even high, in the order of dozens) this problem is no more linear and the complexity increases. It can be proved that it raises with an exponential trend in the size of the problem.

The resolution of these problems is a problem often debated in literature and several theoretical methods exist in order to accelerate this process (like for example Lemke-Howson Algorithm for 2 players or Govindan-Wilson method [5] for N generic players) but those refer almost uniquely to the research of a single NE. This research is object of study on its computational complexity: it has been proved to be NP-Hard [6].

The study of dynamic traffic situations (for example, the car scenario introduced later) could require the calculation of all the possibles NEs present in the system: this force to a research that analyze all the possibles strategy combinations and so is not able to improve the performances of a brute-force algorithm, therefore we can underline how in some situations is necessary to introduce some assumptions about the possibles action sets, in order to reduce the computational load of the problem, where there are many players and the program needs to satisfy any predetermined frequency requirements.

During our simulations, we will execute our research of NEs through the software Gambit (available on www.gambit-project.org). Since it implements also several alternative algorithms like the previous quoted ones, it configures indeed itself as a faster alternative compared to a MatLab routine that solve the feasibility check with the normal polynomial method, letting us to study cases even with several players and actions.

Chapter 4

Pedestrian Scenario

As first scenario that we want to investigate using our game theory-based approach, we consider one where may exists an interaction between people that walks individually or in group moving into a crosswalk area when they face obstacles or other humans while walking on their paths: our target will be to underline how in some peculiar pedestrian situations different persons choose to modify, sometimes even significantly, their path or their speed trajectory in order to avoid possible collisions. These modifications could be introduced by different reasons, but the most recurrings and significatives are those connected to unknown persons which cross others path, or the coupled trajectory of persons in group that for compactness and social interaction reasons decide to walk side by side placed. This could change the individual behaviour as well as the single ones of the contingent pedestrians that may overcome on the scene and may be forced to circumvent this group of people, seen as an obstacle to elude: for example when 2 people are walking side by side for social reasons like friends talking, it could be formulated a constraint where each person has to walk with the same speed and inside a range from the other partner; at the same time, another pedestrian walking in the opposite direction could consider a wider obstacle the approaching of two people walking in a close distance, rather then 2 singular ones.

Vice versa, there could be other reasons that could modify the natural development of a trajectory, especially of a interrelationship nature (think for istance about a men that stops for receiving a phone call or about 2 people that meet in the room and decide to stand chatting) or, simpler, of a target change (in this case our considered subject could decide while walking to change his destinations or could have decided to go back to the start place where he chould have forgotten something) but since these reasons are not connected to possible robotic motions (or in any case extremely hard to investigate matematically, seen the impossibility of understand when actually a subject has modified its destination) we will esclude social features and constraints from our analysis.

We decided then to use a database that satisfies the following requirements such as:

- A high number of trajectories, in order to have lots of possibles interactions (which we remember, could be more or less clear depending on different factors that we will examine) and also many other no-interacting trajectories, that could be useful to compare them with the previous ones for underlining possible changes of speed or direction introduced by other people

- A high resolution of the sampled trajectories, in order to obtain a reliable estimation of the people's motions and that won't be too much filtered or smoothed

- A study environment as much as possible not connected to external factors that could introduce a not natural motion also in the singular person (think for istance about temporary placed obstacles: they could modify human trajectories without being considered in the tracking system)

Our choice goes then on a dataset collected the in the facilities of University of Edinburgh (which results to be that one that satisfies more than the others our requirements).

Dataset description

This dataset is located in the Informatics Forum, the main building of the School of Informatics at the University of Edinburgh. Since July 4, 2010, one camera recorded and tracked the trajectories of all the people walking through the hallway for 121 days; approximately 1000 trajectories were observed everyday, so the total number of real trajectories detected was higher than $92,000$. Here's a brief description of the area:

The main entry/exit points (marked, see Figure 4.1) are at the bottom left (front door), top left (cafe), top center (stairs), top right (elevator and night exit), bottom right (labs). The camera is fixed overhead (although it might drift and vibrate a little over time) approximately 23 m above the floor. The distance between the 9 white dots on the floor is 297 cm vertically and 485 cm horizontally. The images are 640×480, where each pixel (horizontally and vertically) corresponds to 24.7 mm on the ground. The capture rate is about 9 frames per second depending on the local ethernet and capture host machine loads. Unfortunately, the sample rate can vary over short periods. More detailed informations regarding the detecting system can be found in [7].

Figure 4.1: Hallway where trajectories take place

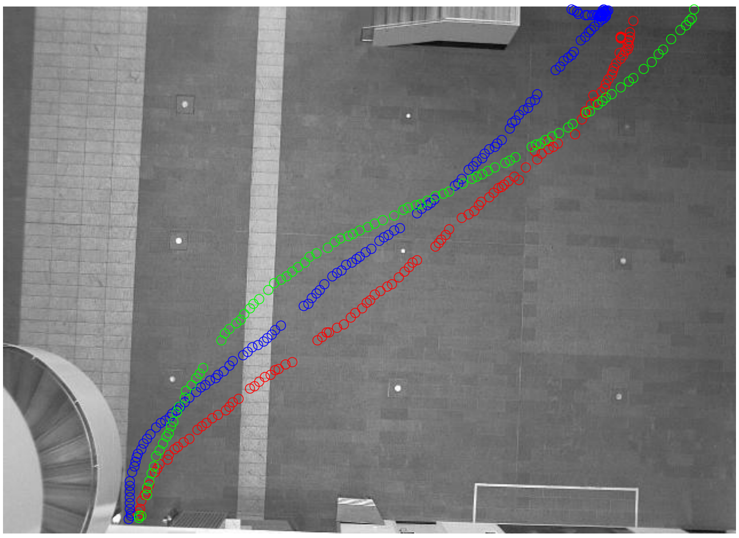

Figure 4.2: Example of tracking results with three pedestrians

4.1 Data filtering and cost function choice

Once that we have described how our data are obtained, we assign to each considered trajectory a cost. In fact, for every of these we would like to find a model that could represent on a quality level the effort accomplished by the considered subject while moving. The aim of this rank is first of all to find the optimum trajectory which could be executed, once given a starting point and a goal: optimum means that it describes how a human pedestrian would move in case of absence of interaction with the environment, so that his only aim is to arrive at the destination avoiding unspontaneous motions: this tool then will manage, at least theoretically, to underline possibles deviations or speed variations introduced in a dynamical scenario when a moving pedestrian face a dynamic obstacle (which in this case we will always consider to be another walking pedestrian); this will be also the starting point in order to obtain the payoffs when applying game theory.

In literature exist several studies concerning the optimum analysis of human walking that obtained a mathematical model that could describe in a consistent way how a pedestrian walks in a space. Since our data are noisy and we have only position data, we will take into account only simple functions, especially when working on non-position data, since most of the times we will obtain them through discrete derivatives that could indeed decrease sensibly the performances.

Before introducing the two cost functions, we make a small summary about the data and the symbols used for represent them:

$x(t)$ position at time t in the coordinate x;

$y(t)$ position at time t in the coordinate y;

$\theta(t)$ angular direction of the tangent at time t;

$v_x(t)$ speed at time t in the coordinate x;

$v_y(t)$ speed at time t in the coordinate y;

$a_x(t)$ acceleration at time t in the coordinate x;

$a_y(t)$ acceleration at time t in the coordinate y;

$v_{\text{long}}(t)$ norm of the longitudinal speed computed from $v_x(t)$ and $v_y(t)$ at time t;

$v_{\text{ang}}(t)$ angular speed at time t;

$a_{\text{long}}(t)$ norm of the longitudinal acceleration computed from $a_x(t)$ and $a_y(t)$ at time t;

$\text{curv}(t)$ curvature of the trajectory computed at time t;

The two functions that we want to consider are then those discussed in [8] and [9]:

The first function is less complex than the latter one: it sums two parameters like the longitudinal accelleration of the pedestrian and the derivative of the curvature of the path. In fact, as seen in [8], these are the two most relevant factors to be taken into account when assigning a cost to a curve. The mathematical expression of this function is

$$c_1(a_{\text{long}}(t), k(t)) = \int_0^T [a_{\text{long}}(t)^2 + k(t)^2]dt \tag{4.1}$$

where $k(t) = \frac{\partial \,\text{curv}(t)}{\partial t}$

The second considered function is a sum of terms that take into account angular and longitudinal acceleration (which are approximately similar to the parameters considered in the previous cost function), but also the time duration of the trajectory (in this way possibles path with high curvatures or frequent stops won't be considered. In particular we want to underline that in many cases a pedestrian prefers to get a strong deceleration for a short time instead of a large deviation that may get the path much longer) and difference between the walk-direction of the human and the goal (we assume that usually a pedestrian tries to walk as straight as possible towards the destination). Mathematically speaking, the model is described as:

$$c_2(T, x(t), y(t), \theta(t), a_{\text{long}}(t), a_{\text{ang}}(t)) =$$

$$\int_0^T [\alpha_0 + \alpha_1 a_{\text{long}}(t)^2 + \alpha_2 a_{\text{ang}}(t)^2 + \alpha_4 \Psi(x(t), y(t))^2)]dt \tag{4.2}$$

where α_i are constant and value

$$\alpha_0 = 1 \quad \alpha_1 = 1.2 \quad \alpha_2 = 1.7 \quad \alpha_3 = 5.2$$

and the function Ψ is defined as

$$\Psi(x(t), y(t)) = \arctan\left(\frac{y_e - y(t)}{x_e - x(t)}\right) - \theta(t)$$

with $-\pi \leq \Psi(x(t), y(t)) \leq \pi$ and (x_e, y_e) are the coordinates of the destination point. Since the second cost function results to be more accurate, we decide to implement it to get our costs.

The accuracy of our data is obviously worsened by several kinds of noises and errors introduced in the tracking/data capture system: for example environmental factors (wind or fog that could disturb the camera) or simply the walk in group of some targets, so that the position of a target could be swapped with the position of another one. In literature this is called "correspondence problem": there are several ways of solving, at least partially, these problems (like those considered in [10]), but we decided for our purposes to solve them with a manual check directly on the tracking data. For other error causes, like noised measures is necessary to accomplish a filtering and an estimation of our data, using suitable algorithms. The first method that we want to use in order to obtain data that, although filtered, could still preserve well-defined speed and directions profiles (paths too smoothed couldn't underline the differences, sometimes even only step-by-step,between the various behaviours, especially sudden direction's changes) is B-splining: we want then, starting from a raw data set, obtain a B-spline that could represent the real path of a pedestrian

4.1.1 Recalls on B-Spline

A spline is a smooth polynomial function piecewise defined, and is C^2 at the places where the polynomial pieces connect (which are known as knots). These curves are often used in mathematic and informatic applications, which requires simple representations of a curve that link several points. Basically, they are obtained assigning a polynomial function that links two close points to every couple of points in the original set. The curve obtained linking these functions is defined as a spline.

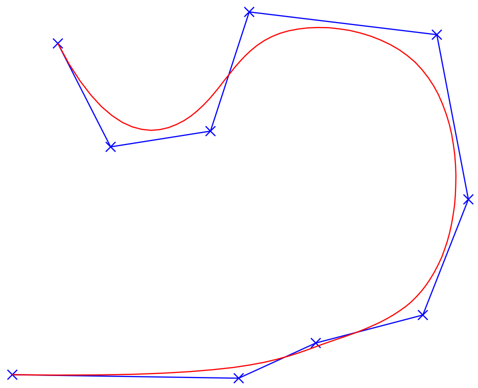

Figure 4.3: Example of B-spline

Special kinds of spline curves are the *Bèzier curves*, which are particular smooth ones that don't pass necessarily for each point. Finally, we define a *B-Spline* as the curve that links all the *Bèzier curves* between every couple of points (see an example in Figure 4.3). Mathematically speaking, we can formally define them (for more details see [11]) in the following way:

Definition 4.1 (B-spline)**.** *Given m real valued t_i, called knots, with*

$$t_0 \leq t_1 \leq ... \leq t_{m-1}$$

a B-spline of degree n is a parametric curve

$$S : [t_n, t_{m-n-1}] \mapsto R^d$$

composed of a linear combination of basis B-splines $b_{i,n}$ of degree n

$$S(t) = \Sigma_{i=0}^{m-n-2} P_i b_{i,n}(t)$$

with $t \in [t_n, t_{m-n-1}]$

The points $P_i \in R^d$ are called control points or *de Boor points*. There are $m - n - 1$ control points, and the convex hull of the control points is a bounding volume of the curve. When the knots are equidistant the B-spline is said to be *uniform*, otherwise *non-uniform*

As we can see, typical results of B-splining consist of smoothed curves (a focus is showed in Figure 4.4), where depending on the number of control points for the representation of the spline.

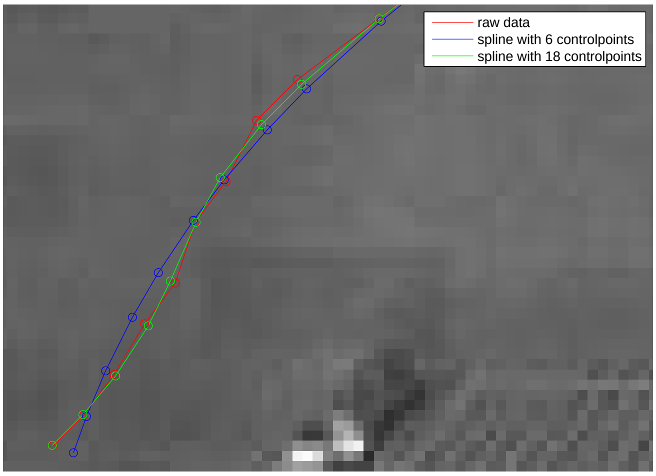

Figure 4.4: Particular of 2 different B-splines compared with raw data

4.1.2 Implementation of Kalman filter

In literature the formulas that describe the evolution of Kalman filter are well known (see [12]), we quickly report them here:

$$\hat{\xi}_{k+1|k+1} = A\hat{\xi}_{k|k} + K_{k+1}(y_{k+1} - CA\hat{\xi}_{k|k})$$

$$P_{k+1|k} = AP_{k|k}A^T + Q$$

$$P_{k+1|k+1} = P_{k+1|k} - P_{k+1|k}C^T(CP_{k+1|k}C^T + R)^{-1}CP_{k+1|k}$$

$$K_{k+1} = P_{k+1|k}C^T(CP_{k+1|k}C^T + R)^-1$$

In this case, the noisy inputs that we have at our disposal are the target positions at each sample timestep T. Since that, as seen previously, there is the risk through discrete derivative of obtaining high peaks not particularly reliable when derivative are iterated more times, we decide to introduce directly in the system state also speed and acceleration, so that they could be estimate keeping a continuous profile sufficiently robust and without peaks.

We can then write a state model that results to be linear in inputs and outputs, described by the following equations (we'll assume that the system evolves at discrete time, since the nature of our observations are clearly discrete):

$$\vec{\xi}(k+1) = \begin{bmatrix} x(k+1) \\ y(k+1) \\ v_x(k+1) \\ v_y(k+1) \\ a_x(k+1) \\ a_y(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 0 & T & 0 & 0 & 0 \\ 0 & 1 & 0 & T & 0 & 0 \\ 0 & 0 & 1 & 0 & T & 0 \\ 0 & 0 & 0 & 1 & 0 & T \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \vec{\xi}(k) + P\vec{u};$$

$$\vec{y}(k+1) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \vec{\xi}(k) + P\vec{v}$$

where the variance matrixes are set with values that fit the noise variances introduced by the system.

There are several factors that induce us to prefer Kalman filtering over B-splines, some of which are concretely meaningful:

- The curve representation changes critically with the number of control-points, so that varying slightly this number may correspond in having a different ranking of costs assigned to the different trajectories, which is one of the first factors to be avoided

- We still need to proceed with a discrete derivative of the data in order to get the accelerations along the axis, which may cause undesiderable effects

- We don't have time references, since splining in the positional dimensions (i.e. x and y) removes connections with the time scale of our dataset

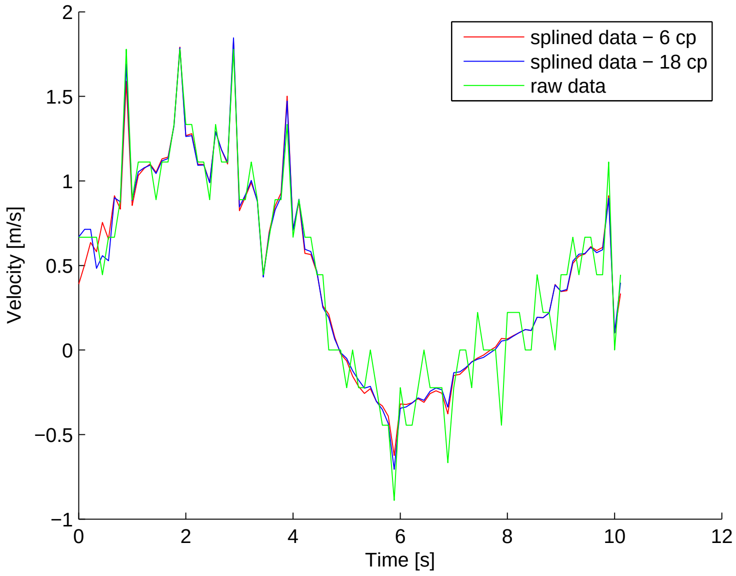

Figure 4.5: Comparison between velocity profiles using B-splines

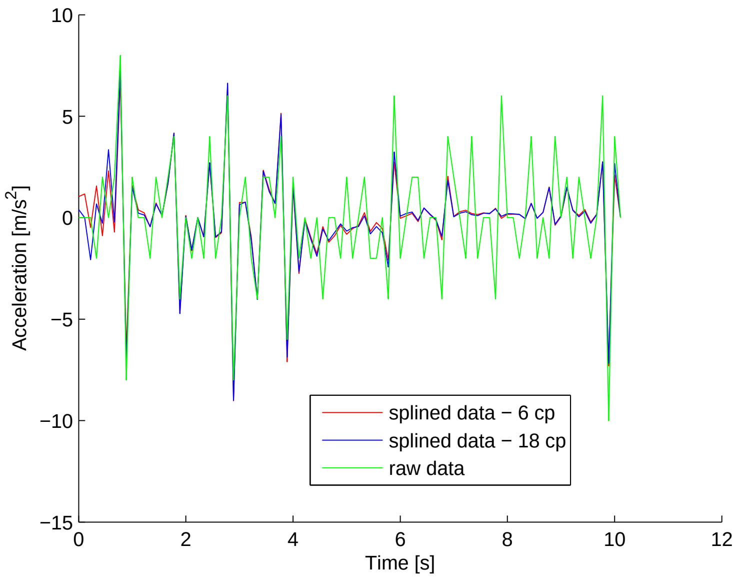

Figure 4.6: Comparison between acceleration profiles using B-splines

In figures 4.5 and 4.6 we can see an example of the first 2 points: in fact, varying from 6 (high smoothness) to 18 (medium smoothness) controlpoints we obtain 2 profiles which show a substancial difference. This variation is accentuated and becomes crucial in the acceleration, where we can obtain ranges relevant in norm. In these figures we notice also how noisy is the profile elaborated by a discrete algorithm, infact speed graphs show point-to-point variations around the general trend: this factor induces some peaks which are absolutely undesiderable and which bring to distort the computation of the cost related to the corresponding trajectory. We also notice that adopting techniques of discrete filtering (like relaxing the derivative over a wider time window, for example 5 or 7 timesteps) could bring improvements (sometimes even noticeable) to the previously introduced problems, but nevertheless still not sufficient to consider them solved and therefore they are not accounted (see Figure 4.7).

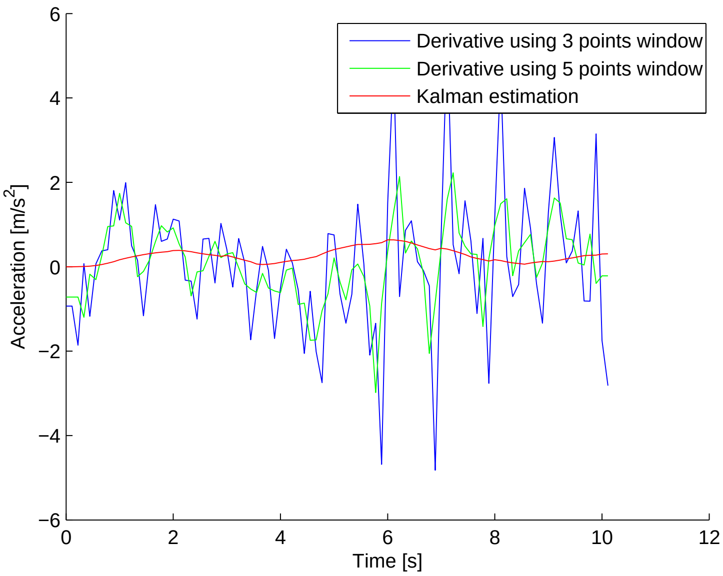

Figure 4.7: Comparison between acceleration profiles using different windows data when deriving

Let's clear better now the third point: in fact when getting a B-spline of a 2-D curve, the output is only a geometrical description of it that depends only on a parameter u which can be considered as a progress parameter of the curve, $u \in [0, 1]$.

In order to avoid this obstacle, we may think about performing a B-spline in 3 dimensions which includes then also the time as input dimension, running at a later stage an evaluation of it on a linear scale respect to the third dimension (so that we could obtain a complete temporal description according to our requirements). Unfortunately this method results extremely unefficient for several reasons, first of all the obtaining of negative timesteps (since steps are really short is possible that splining them the temporal curve goes "back" in the time, characteristic absolutely to be avoided), besides a lower stability according the previously introduced factors.

We consider then not suitable the use of B-splines as data filter so we decide to follow the Kalman concept.

As we can notice, if we perform an estimation of positions, speed and acceleration profiles of the target, we obtain a shape which is surely more reliable than the previous one obtained through B-splines.

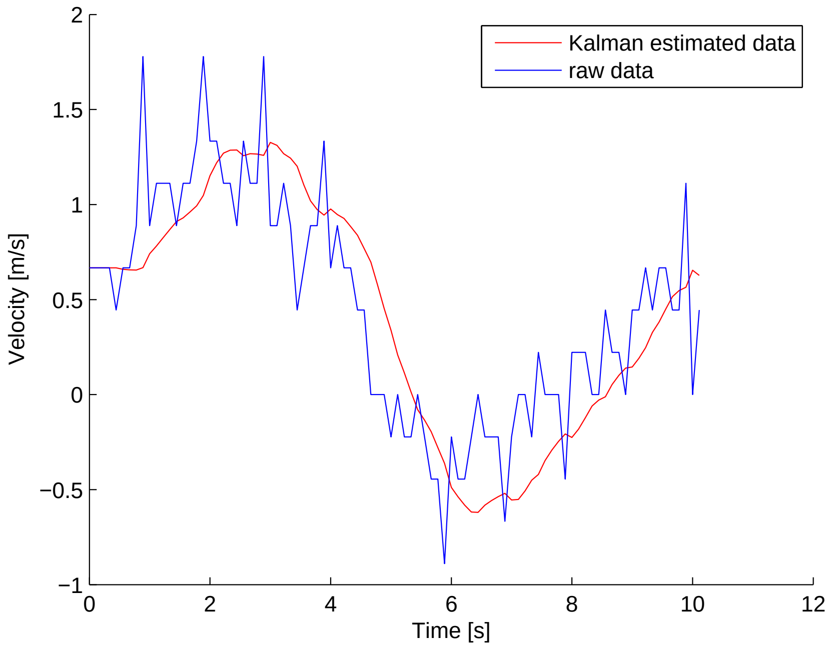

Figure 4.8: Comparison between velocity profiles using Kalman filter

4.1.3 Game formulation and setup

In our scenarios we will assume that each pedestrian walks with a trajectory that is planned independently from the others and that possible interactions may lead only to deviations or variations of speed on it. Our assumption is that each trajectory is goal oriented, so that the human plans to go to destination with the best (regarding comfort but also time) possible way. So each player has a payoff associated to their actions, that could be worsened if these planned trajectories bring to undesirable situations (i.e. collisions).

This is represented as a non cooperative game, where each player has a number of action corresponding to each trajectory that was found in the database with the same goal and the same destinations. Expected solutions of these scenarios (i.e. the predicted motions) are the Nash equilibria of the system, which could be singular or multiple.

Before showing the results obtained through simulations, we report here a brief description about how is created the game corresponding to an interactive pedestrian situation: After having conveniently assigned the relative cost to each trajectory of each player, we create a N-dimensional matrix of dimensions $l_1 \times l_2 \times ... \times l_N$, where l_i is the cardinality of the set with all the possible actions of i-th player.

At this point we execute a linear normalization that, starting from a cost value, assigns a certain income to each trajectory: therefore we'll have a function with costs as input and payoffs as output. We point out that the criteria that we adopted when defining the game creation are the followings:

- payoffs can vary over a fixed interval from 0 to 10
- collisions between players must have the worst possible payoff, corresponding to a strictly lower value respect to any possible non-colliding trajectory
- if the value assigned from the cost function is high (therefore bigger is the effort made by the pedestrian), then the income gained by the player will be low and vice versa

The normalization function chosen is the following one:

$$\Phi(\mathrm{pay_i}) = 10 - 10 \times (\frac{\mathrm{pay_i}}{\mathrm{pay_{max}}})$$

where $\mathrm{pay_{max}}$ is the maximum value between all the possibles costs in the game matrix and $\mathrm{pay_i}$ is the payoff associated to each to trajectory i (in symbol: $\mathrm{traj_i}$). We want to underline that this normalization is implemented giving the same weight scale to each possible trajectory, since that for our data we assumed that all the pedestrian have equal importance in the scenario. Another possible variation would be to realize different normalizations, personalized for each player, where different payoffs are assigned, with higher scale to a more relevant player (for example a pedestrian moving urgently respect to another one).

This statement may result a bit unclear: in fact, the structure of the Nash equilibrium (NE) is not modified, since it's costant respect to linear changes of values, but rather in some situations where there are multiple NEs the choice of which one to select (which usually is decided through the sum of all the different payoffs corresponding to each player in the considered NE) may vary.

Once introduced then the matrix that describes the different game payoffs we need finally to set the interactive component between the players: we will assume that every time two players (i and j) following trajectories $\mathrm{traj}_{\mathrm{i_h}}$ and $\mathrm{traj}_{\mathrm{j_k}}$ are located into a common space with width radius not enough large to avoid collisions (which means that simulating it we would assist to a body intersecting anotherone), their resulting payoff will be negative (fixed to -100) for each group strategy which includes actions $\mathrm{action}_{\mathrm{i_h}}$ and $\mathrm{action}_{\mathrm{j_k}}$ together. In this contest generally we fixed as collision distance 35 cm. This value is obtained considering the minimum distance detected in the simulations maintained by the pedestrians when walking together or side-by-side in the same direction. We assume that this behaviour is the social one that keep the minimum acceptable distance between people, in order the prevent possibles missed collisions. It's obvious that this distance is relatively short and may decrease our performances in the simulations, considering admissible even trajectories generally not optimal (think about 2 pedestrian getting closer one facing the other, in this situation staying 35 centimeters far is surely not comfortable and pratically consists in a collision), nevertheless being difficult to consider a model for each dynamic situations when target approaching, we considered the worst valid case: for example considering 2 friends walking alongside, thay can stay without problems inside a close distance (around $40 - 50cm$) without noticing any particulars problems of comfort; we decide to set this value even smaller because of the noisy many times some real paths appeared to be in this distance, so that every couple of paths using the same dimension had to be considered itself a collision-free one.

4.2 Analysis with 2-players games

We consider now the simplest case of traffic scenario, which means a situation where 2 people interact. We present then some of the most relevant scenarios that could be faced in these situations, such as frontal or lateral intersections, with different initial velocities.

The first case we want to analyze is quite simple (Figure 4.9): we have 2 pedestrians where both of them go through a long corridor in an open space. More precisely, the first person (blue path) starts from a side of the atrium and goes straightly following a corridor towards another exit; while it walks across the space, another human (red path) starts from the target of the other person and, moving on the same path but in an opposite direction, goes towards the start point of the other person, which is faced oppositely.

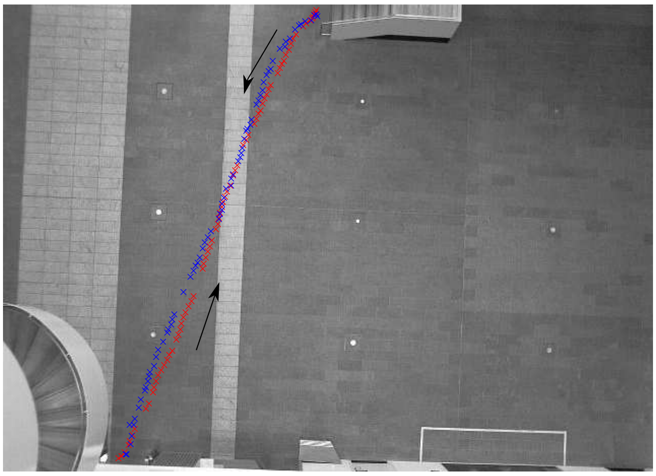

Figure 4.9: Tracking of the Scenario 1

Since that the time duration where both of the pedestrians stand in an interaction area is definitely limited, considering also that both of them before facing execute a trajectory which is perfectly straight, they don't feel uncomfortable in their encounter and therefore they continue forward staying on a side, modifing basicly their path with a deviation that is probably smaller than the noise (so that it can't be measured) and not relevant in order to assign a cost (Figure 4.10);

We can therefore conclude that in these situations is very hard to highlight and model possibles interactions or collisions, because the risk of a contact is very reduced, when people go in different directions and stay close only for a

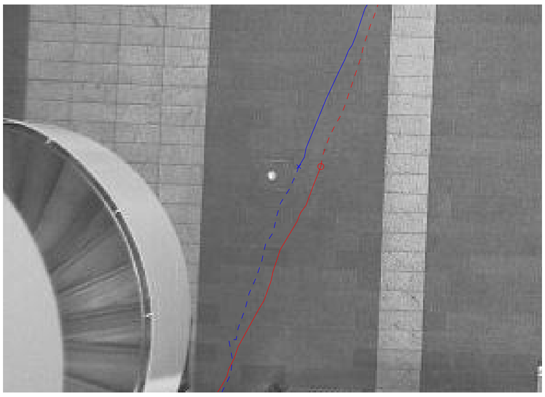

Figure 4.10: Scenario 1: particular when 2 pedestrians are facing

short time duration. In Figure 4.11 we can see that the typical shape of the alternative trajectories for player blue follows the real path, changing only with small deviations or in the time evolution. This induces us to conclude that in the previous case no significant deviation was introduced from any pedestrian.

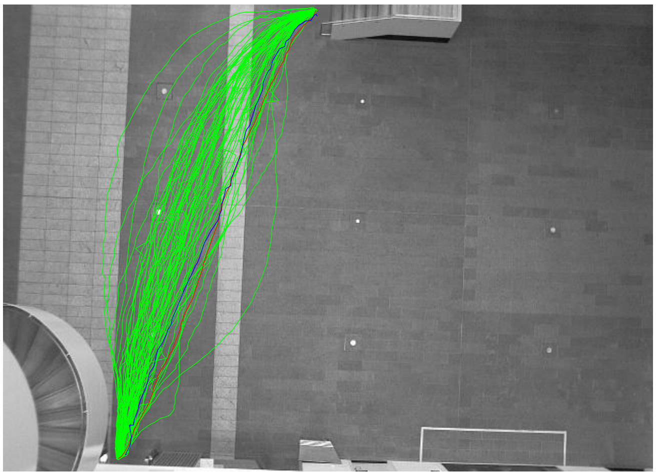

Figure 4.11: Scenario 1: Alternative trajectories for blue pedestrian

Our previous conclusion on possibles interaction is also showed by the numerical results, in fact over 1560 we can notice that 1037 of them are still valid, so that 66.5 of the combinations are still valid; as we will see later, for two players interactions, the collision aspect can be not so selective as in N-player scenarios, where this ratio will increase. The real point results to be a ϵ-NE, with $\epsilon = 0.7059$ (on a scale where not-colliding trajectories can bring to payoffs varying from 0 to 10): with this value we can find a set of equivalent combinations with size 144 over 1560, which means that the real path stays inside the 9.2% of the best solutions, which is a good performance that will be our target when facing more complex scenarios, i.e. for N players. We report here in Figure 4.12 the plot of the computed Nash equilibrium: in this case the obtained payoffs are the highest possible of the scenario and correspond to $[9.12; 9.77]$. In this case we can notice that the two players perform trajectories in a path very similar to the real situation, so that we can conclude that the eventual benefit introduced by the computed Nash-equilibrium is associated only to a more constant speed and a reduced time of motion.

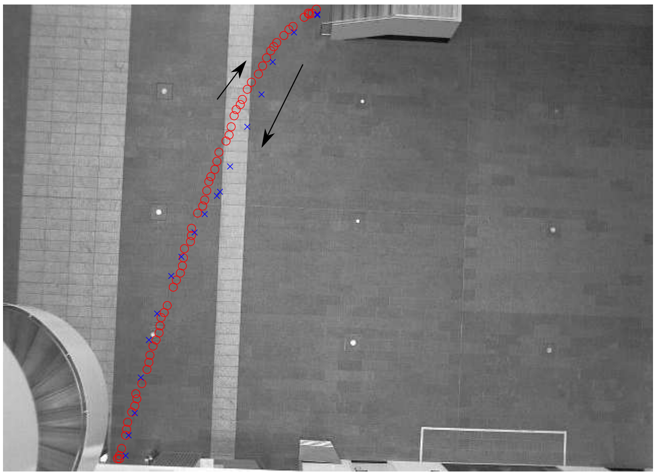

Figure 4.12: Scenario 1: computed Nash equilibrium

We notice then one of the characteristics of the 2-players scenarios: the real point rarely coincide with the best available choice, but the computed NE with the highest benefit in terms of payoffs is not so different as regard the path, where the differences may be introduced by absence of acceleration or reduced time for reaching the goal. The set of possibles combination with equivalent payoffs (which means an ϵ with comparable size with the real situation) is very large, because most of the humans tend to walk in the same way when moving on a long straight path. In Figure 4.13 we can notice that the percentuage of the valid combinations stay not farer than $\epsilon = 2$ from the best solution.

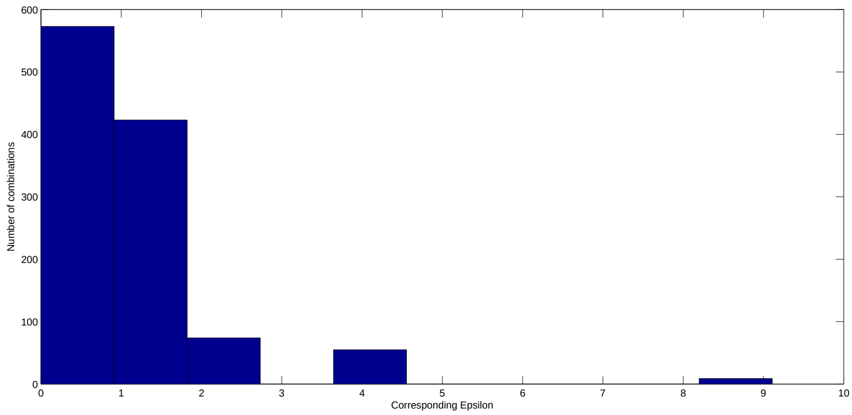

Figure 4.13: Scenario 1: distribution of the valid possible solutions in terms of ϵ

We consider now another scenario (Figure 4.14), where all the features of two player game expressed above can be found: As before, two different pedestrians face themselves when walking in a almost straight line: in this case the red one intersect the path of the red one with a lateral angulation and not directly facing as before. Both of them don't need to modify sensibly their direction, since they can walk in a wide open area without relevant constraint.

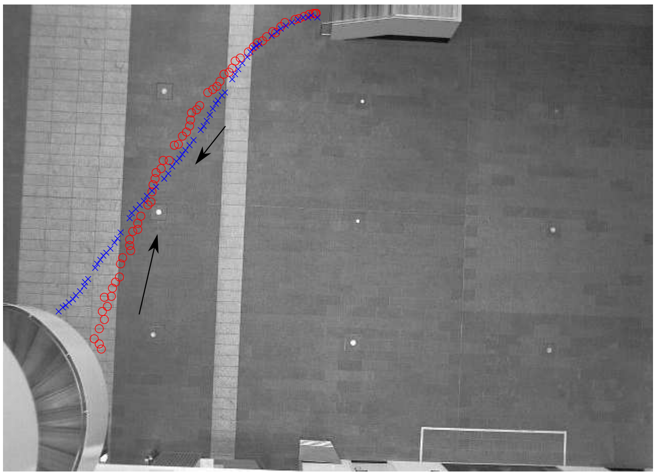

Figure 4.14: Overview of paths with lateral approach in Scenario 2

In this case the possibility we can notice (Figure 4.15) that the red pedestrian crosses the path of the blue pedestrian, but this don't bring to any modification.

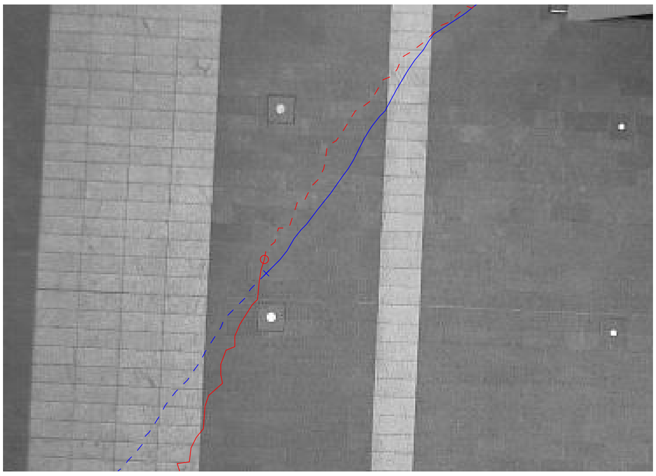

Figure 4.15: Scenario 2: particular when 2 pedestrians are facing

This conclusion is another time supported by the datas, that show that over 400 possibles combinations, only 44 of them bring into a collision so that

356 are still valid choices (this corresponds to the 89% of the total). This brings to conclude that in two players game corresponding to pedestrian situations with low collision rate the general strategy selected by the two players is to maximize their path as they would be alone in the scenario, because even if their paths are partially overlapping, typical trajectories don't bring into collisions, especially in situations with possible lateral intersections. This can be show for example plotting the alternative set of paths for player red (Figure 4.16) and blue (Figure 4.17) noticing that they don't share basically the same motion space, so they can plan the motion independently from each other.

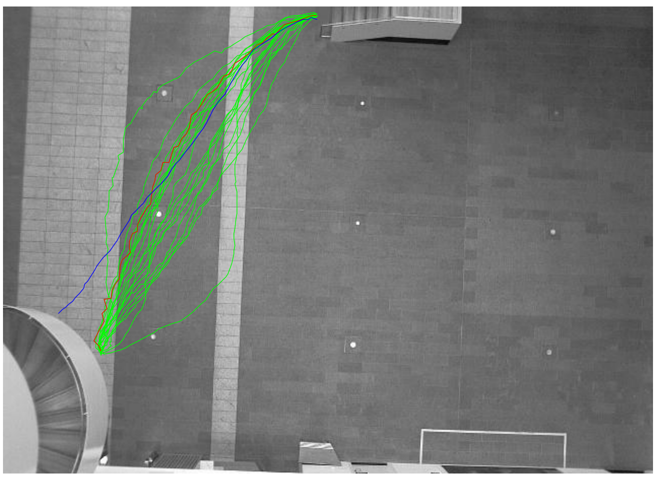

Figure 4.16: Scenario 2: alternative paths for red pedestrian

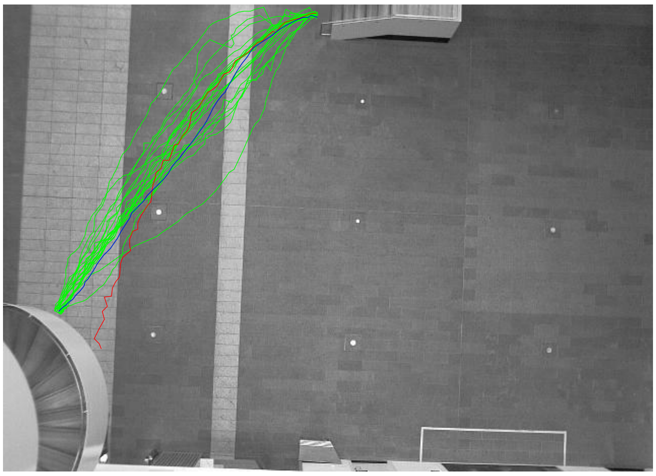

Figure 4.17: Scenario 2: alternative paths for blue pedestrian

In this case the real situation results to be an ϵ Nash equilibrium, with $\epsilon = 0.24$, with a set of valid alternatives of 157 combinations, which corresponds to the 39% of the total. This value is pretty high, but the low difference value between the real situation and the computed NE, that is the previous ϵ, is relatively small so that there are not significant improvements that could be brought to the real situation. We report finally the plot of the computed Nash equilibrium in Figure 4.18, where the two paths confirm the indepence property introduced before:

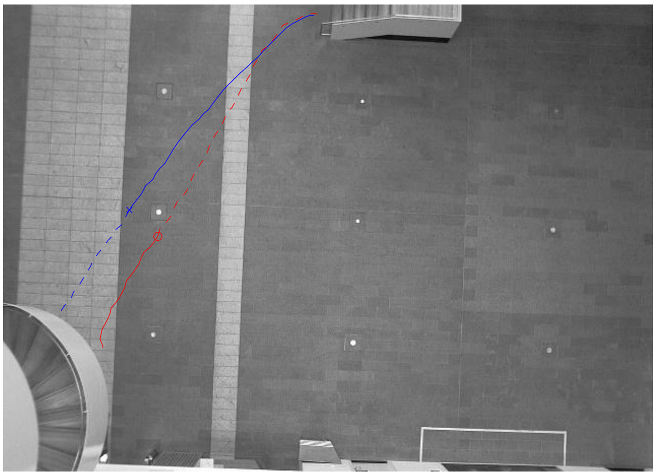

Figure 4.18: Scenario 2: computed Nash equilibrium

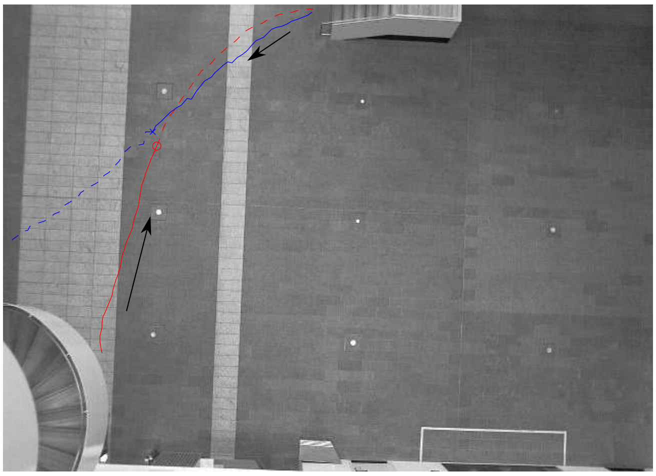

Figure 4.19: Scenario 3: example of path with not optimal actions

Let's focus now on the scenario of Figure 4.19: in this situation we can find a new characteristic that is important to underline when predicting motion using game theory, which is the goal direction. In this scenario we have a typical situation of the red human moving towards the stairs, facing from the lateral side the blue one that comes from his destination going into the atrium. Although similar to the previously analyzed scenario, here we can notice that the red human moves differently: in fact, we notice that his change of direction is sudden, like if he changed goal during the trajectory. In Figure 4.20 we can notice like his direction vector changes, because at a certain time he decides to move towards the stairs.

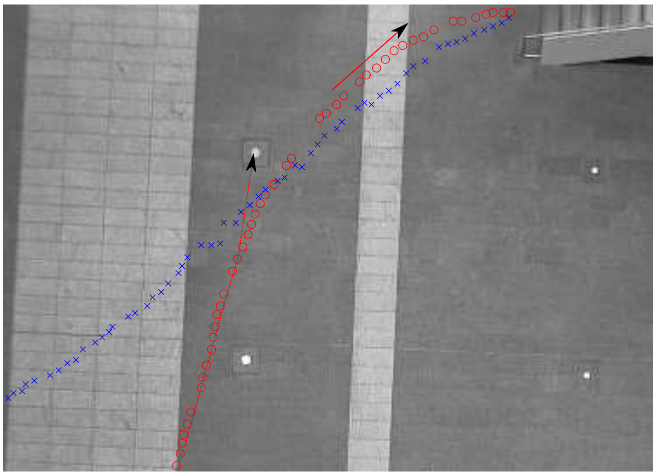

Figure 4.20: Scenario 3: focus on change of direction for red pedestrian

This consideration can be underlined showing (Figure 4.21) that the red path moves standing external respect to all the possible alternatives, that follow the typical goal-directed trajectories. This means that the red human didn't follow a goal-directed trajectories towards the stairs already from the beginning, but rather that he changed the idea and need to replan the trajectory: this brings to an unoptimality of the motion, because the same results can be achieved by other trajectories that go directly to the goal from the beginning.

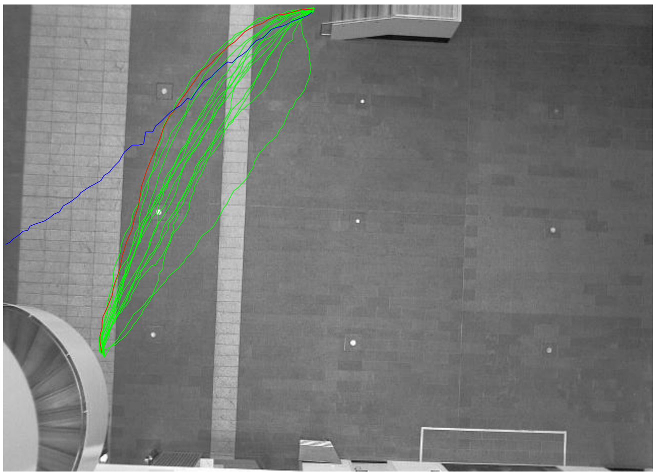

Figure 4.21: Scenario 3: alternative trajectories for red pedestrian

If we report the results obtained for this scenario, we can notice that the ϵ is increased to $\epsilon = 2.81$, that is clearly higher than before. This is a consequence of not acting in a optimal way that shows one important feature and limitation of the game theory: in fact, it works with good accuracy and reliability only if all the pedestrian move with optimality using goal directed trajectories; in this case the scenario satisfies our principal assumption that the humans move towards destinations using an optimality criterion and therefore their motion can be predicted. The real scenario in a set of equivalent valid combination of 91 over 210 that is the 43%, higher than the previous situations in any case. We report finally the computed Nash equilibrium for this situation showing that the best solution is when both of the humans move in a straight line directing always to the goal.

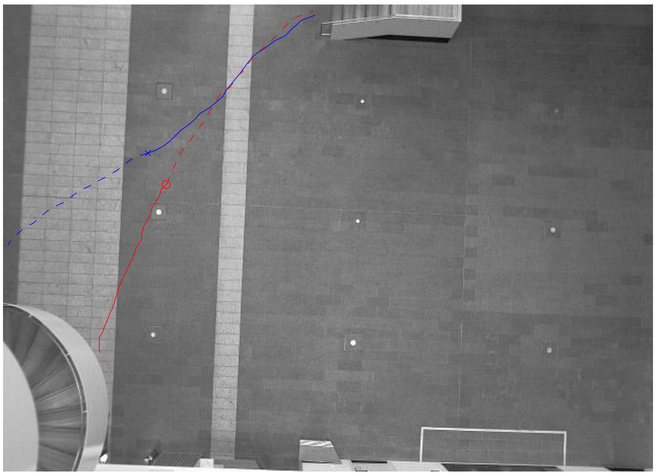

Figure 4.22: Scenario 3: computed Nash equilibrium

4.3 Analysis with N-players games

We move now on situations where more than 2 players are interacting in their motion. In these cases we will notice how the collision possibilities increase considerably when more than 2 players are approaching at the same time. This can introduce more effective changes in the pedestrian motion, so that real studied trajectories can be not optimal when facing an empty way, but become interesting and meaningful when having these kind of obstacles.

In the first scenario in Figure 4.23, we have an interaction between 3 players: at the beginning, two pedestrian start moving from the upper-left side of the hallway, coming out from the lifts and walking side-by-side crossing all the area towards the atrium. When they are approaching the entrance of the atrium, another human comes out from it going outside moving straight and passing just between them. In this case, we can notice that the blue pedestrian accentuates his curve leaving enough free space for the black one to pass through them.

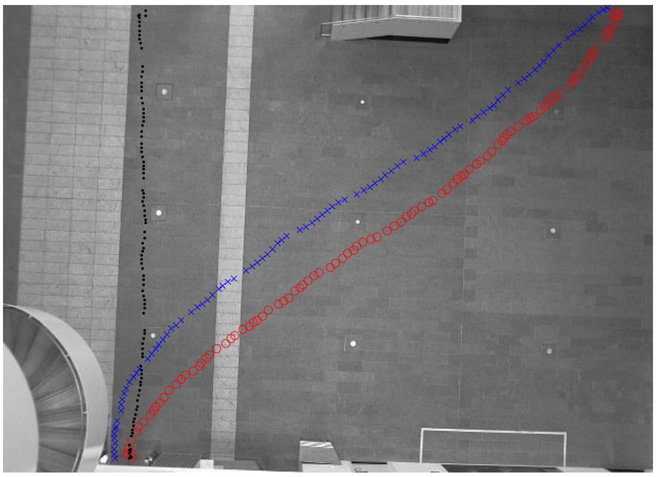

Figure 4.23: Scenario 4: Overview of interaction with three players

The two situations where we can find some possible collisions are first of all in the side-by-side walking of the blue and red pedestrians: in fact, if the blue one would decide to move more directed to the goal as if he was moving alone, he could collide into the red one and vice versa, the same could happen if the red one would get more straight his trajectory he could intersect the blue motion (Figure 4.24).

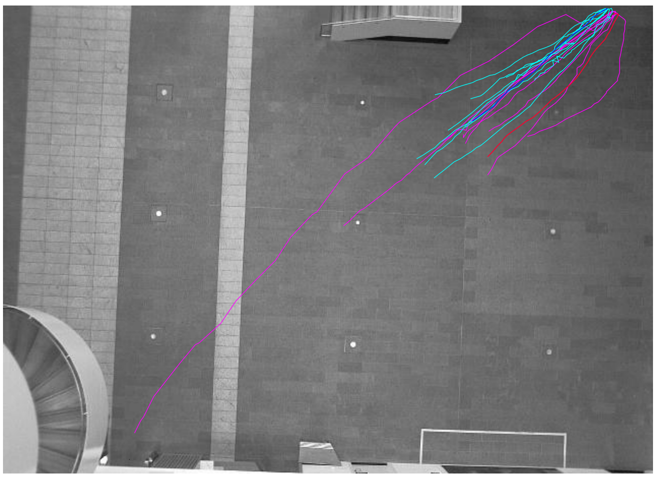

Figure 4.24: Scenario 4: alternative paths for red and blue pedestrian

The second possible interaction is when the three players are close to them, which means when the black is moving out from the destination of the couple: in this case possible alternatives for the blue pedestrian could collide directly with the black human or even with the red one if he was smoothing his trajectory. Same happens with possible alternatives of the black and red trajectories.

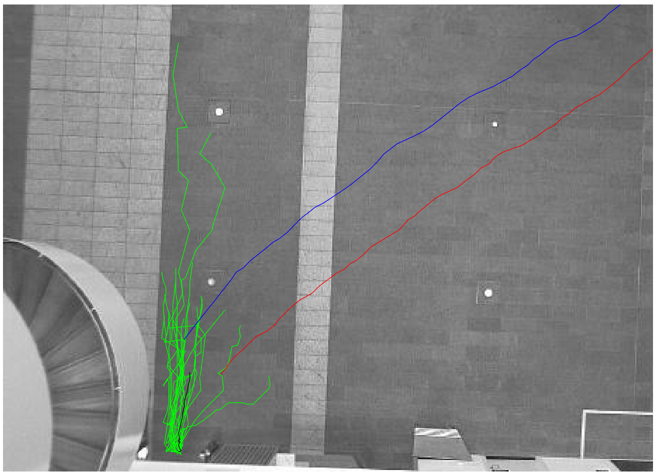

Figure 4.25: Scenario 4: alternative paths for black pedestrian

Differently from the 2 player game studied in the previous section, we can notice that here the collision rate increases and is relevant when computing equilibria: in fact, from a starting number of combinations of 1782 we get only 817 valid solutions, which corresponds to the 46%. In this case, the real situation is an ϵ-Nash equilibrium of 0.89, which is higher compared with the values founded in the previous section (corresponding to values of $[5.42, 4.49, 5.96]$. This value brings to a set of equivalent valid combinations of 80, that is the 4.4% of the total, so it decreased because even with bigger differences of payoffs, a big number of colllisions lead to small number of possibles valid equilibria. This is also show in Figure 4.26, where we can notice that the distribution of the different ϵ-Nash equilibria is more homogeneous.

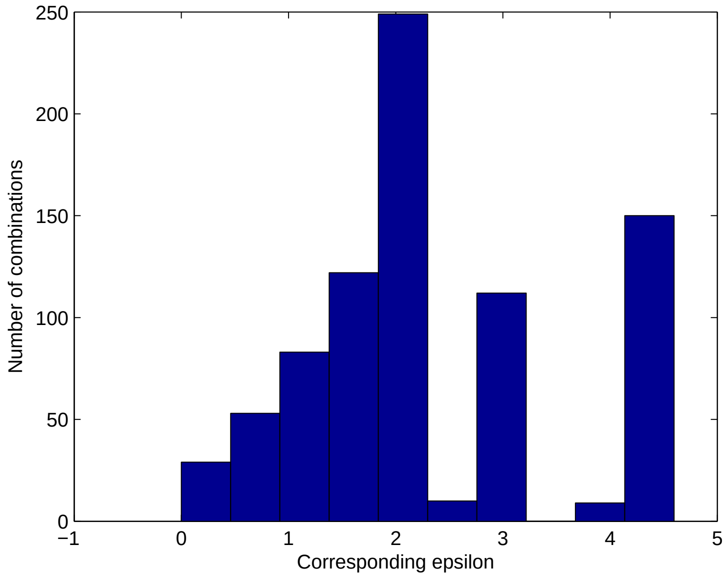

Figure 4.26: Scenario 4: distribution of the different epsilon-Nash equilibria

We finally report the trajectories with the computed Nash equilibria: In the computed one with highest payoffs, as we can notice, the behaviour is quite different from the original situation, because the red pedestrian tends to be faster than the blue, so that they can stay in similar optimal path with similar trajectories. This also brings the black human to avoid completely possibles collisions because in this case the other pedestrian approach the exit leaving enough motion space for the black human. The correspondent payoff is $[5.59, 4.6, 6.99]$.

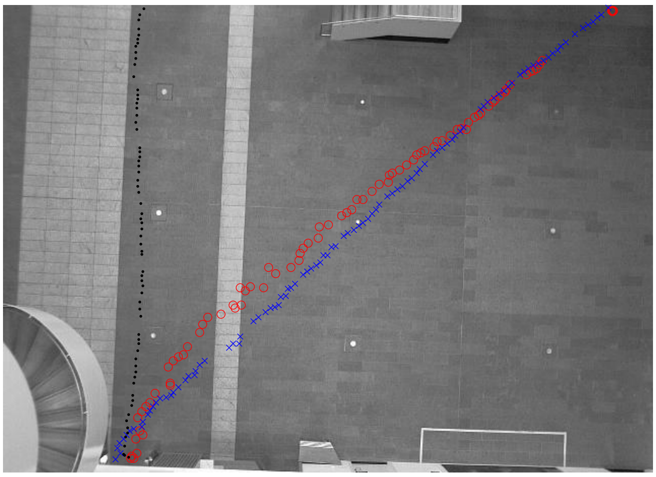

Figure 4.27: Scenario 4: computed Nash equilibrium

We also report (Figure, 4.28) the representation of an alternative Nash-equilibrium with lower payoffs, where the motion of the players is very similar to the previous one, where the difference is only that the red human doesn't overtake the blue one from the start, but rather with a small acceleration during the trajectory. The related payoffs are $[5.42, 4.6, 6.86]$.

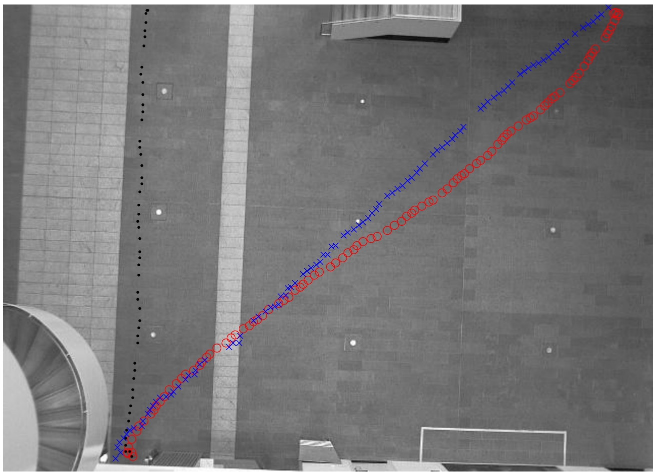

Figure 4.28: Scenario 4: alternative computed Nash equilibrium

In the second scenario of Figure 4.29 we see another interaction between three pedestrians: at first the black human is moving in the same direction of the red-blue in the previous scenario, when two other humans come from the atrium exit and approach him moving in opposite direction towards the lifts: In this case all the three player needs to interact, because while the black one avoid them with a arched path, the blue one makes a similar deviation from the same reason and this induces the red human to move a bit external in order to avoid the blue.

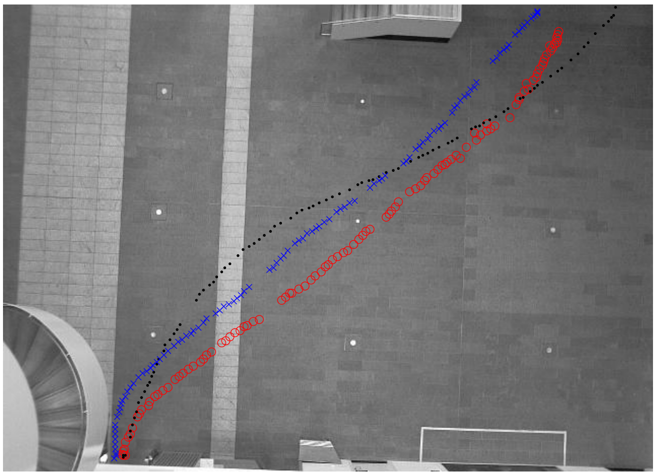

Figure 4.29: Scenario 5: Overview of interaction with three players

As previously considered, the two typical situations of possible collisions are connected to the side-by-side walking and to the facing of the humans all together, because alternative paths may not take in consideration these deviations (since they are obtained generally from free space situations): as we can notice, from Figure 4.30 alternatives of these paths are generally overlapping with those of other pedestrians, especially in this case where we can notice that the black human decided to make a large deviation from the beginning (in order to avoid sudden deviation changes or acceleration) in order to avoid the couple of pedestrians.

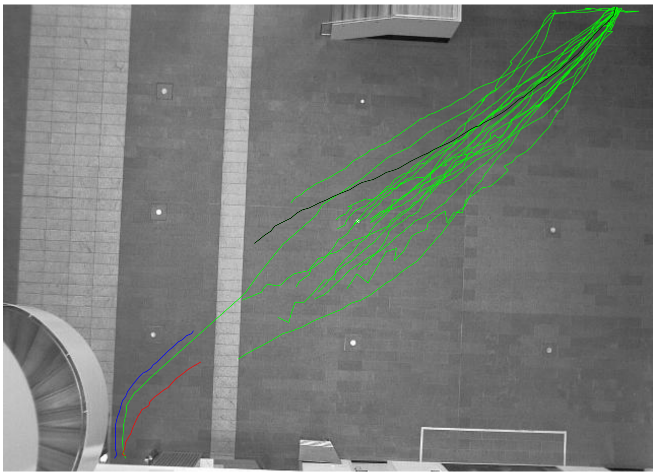

Figure 4.30: Scenario 5: alternative paths for black pedestrian

In this situation there are two computed Nash equilibria: in both of the situations the black pedestrian prefers to move more on the left, leaving free space to the couple of pedestrian; vice versa in the dynamic of Figure 4.31 they move one after the other (and no more side-by-side), where the blue human prefers to accelerate and stay beside the red. In the second Nash equilibrium the situation is the same, but is the red one that overtakes the blue.

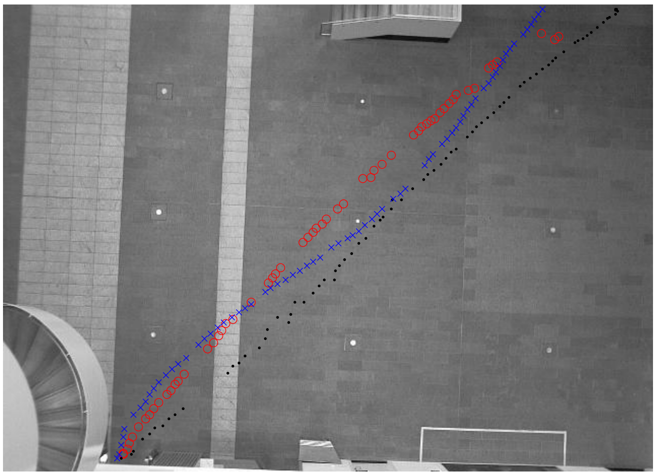

Figure 4.31: Scenario 5: computed Nash equilibrium

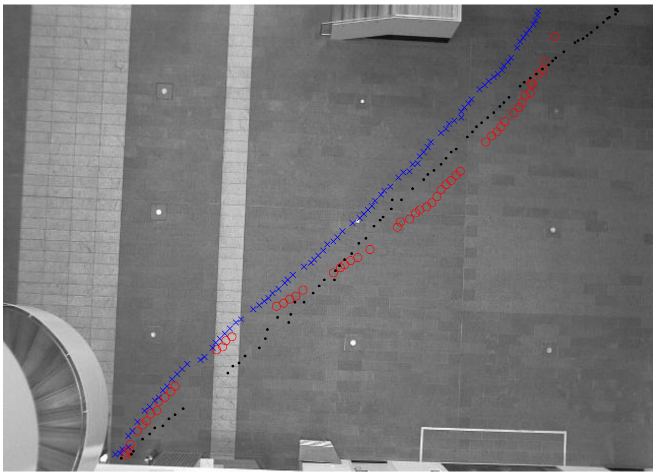

Figure 4.32: Scenario 5: alternative computed Nash equilibrium

Computed payoffs for the NEs are respectively $[8.56, 8.60, 8.65]$ and $[8.45, 8.60, 8.62]$. The real situation has values corresponding to $[8.04, 7.62, 7.8]$; real situation is a ϵ-NE, where $\epsilon = 0.98$, that has a set of equivalent valid solutions of 346 combinations over 3380 (10.24%). In this case the valid solutions once filtered from collisions were 782 (23%), so we can see another time the typical characteristic of the N-player interaction, i.e. that the possible collisions are more frequent than 2-player situations (which is quite obvious, since the probabilities of collision increase esponentially).

As we have seen these two scenarios, one important feature is underlined by game theory: in fact, the optimal strategies chosen by the player when computing Nash equilibria lead to motion situations where people prefer to walk alone on a single path, in order to minimize their costs associated to their trajectories. This is quite obvious, but in the reality we have seen that normally people could decide to sacrifice their comfort in order to walk side-by-side with other people, for social reasons. Of course this lead to real trajectories that are not optimally and decrease the performances of the game theory, since the assumption is that each player moves according to his own possibility of comfort. Possible methods for improving these performances could be related to insert reductions of payoffs when people moving in group stay in a distance bigger than a maximum bound that assures social interactions.

We finally move on a more complex situation (Figure 4.33): in this case we have an interaction with 5 pedestrian. Here we can notice that four human start walking all together in a group, moving towards the stairs; while they're approaching the destination, a fifth human (the blue one) comes out from the stairs and face them; since four people walking together in a group can be seen as an obstacle from the blue pedestrian, he decides to modify and curve his trajectory so that he avoids possibles collisions.

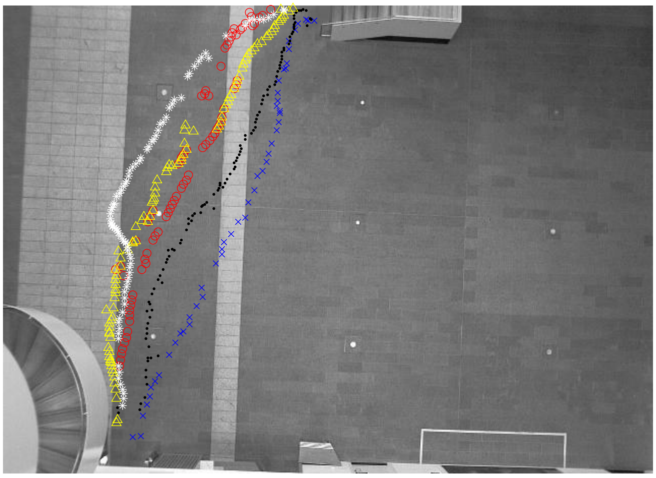

Figure 4.33: Scenario 6: Overview of interaction with five players

As in the previous situations, two are the situations with possibles collisions: the most important is when the blue pedestrian starts, because he has to modify his trajectory in order to avoid the others. This can be shown in Figure 4.34, where many of the alternatives for the blue trajectory may intersect colliding with the others humans.

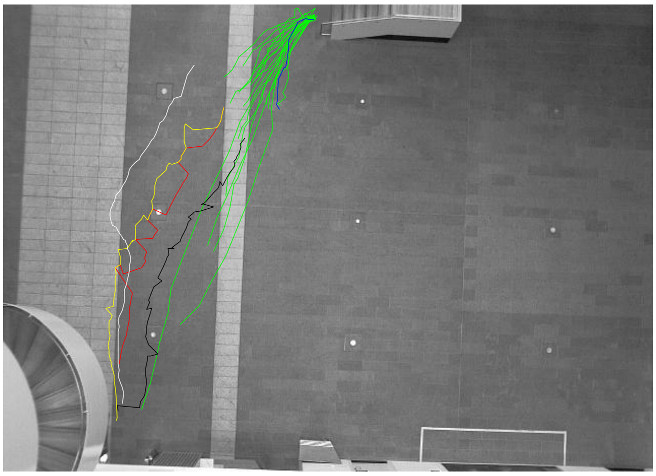

Figure 4.34: Scenario 6: alternative paths for blue pedestrian

The other typical situation where the pedestrian can collide is when they walk in group, because in the original scenario they were mantaining a safe distance between them, but applying optimal trajectories could induce to collide with others people. This is shown in Figure 4.35 for black pedestrian.

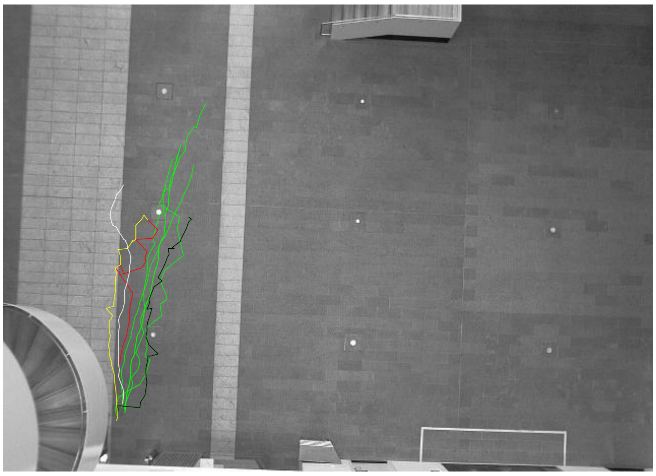

Figure 4.35: Scenario 6: alternative paths for black pedestrian

If we report the computed results, we can notice how the collision rate increases noticeably, which means that more people are interacting in a real scenario modifying the motions of the pedestrian around them, bigger is the possibility that optimal solutions computed in free space situations lead to collisions. In fact, from a total combination of 10560, we obtain 1807 valid ones, which corresponds to 17.1%. The real situation results to be a ϵ-Nash equilibrium, with $\epsilon = 7.43$, which have a set of valid equivalent solutions of size 111, that is the 1.05%. As we have seen, the size of the possible solutions is quickly decreased, even if the ϵ is still relevant. If we take a look at the payoffs, we notice that is caused by the payoffs of the white player, that makes an unspontaneous curve not standing in a straight line: $[5.35, 7.94, 3.82, 5.38, 0]$. From the motion of the player we can't say if the trajectory was goal oriented (which means that he wanted to go to another destination and then changed because he changed his ideas, or maybe because he wanted to let other people pass before him) or not. We can verify that if we are in a not goal oriented situation and we remove this action, considering the scenario as a four player game, we see that the ϵ associated modifies drastically: the combinations become 2640, with only 221 valid; the ϵ associated is 2.97 and corresponds to only 19 equivalent valid points, which is a very high performance (0.72%). We report now the the description of the computed Nash equilibrium: in this case another time the collision is avoided by speeding up the trajectory of the four pedestrian in group. Specifically, the white one goes faster alone, where the three pedestrian remain close in group. This allows the blue pedestrian to face a smaller obstacle and basically to perform an optimal solution without curving. The payoffs obtained with these strategies are $[8.85, 9.77, 7.07, 3.96, 7.43]$.

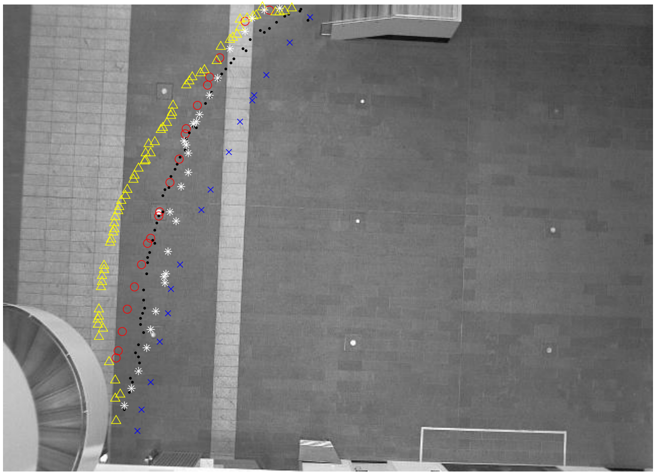

Figure 4.36: Scenario 6: computed Nash equilibrium

4.4 Discussion

We have analyzed the main features of the pedestrian motion prediction in 2-player games: normally the chances of interaction are very reduced if we consider people not moving together side by side (which is nevertheless a singular situation that leads to a suboptimality of results, since it doesn't satisfy completely our assumptions on goal oriented trajectories), so that also collision ratio is usually not relevant; as we have seen in the previous section, the collision ratio stands in percentuages that may vary from 10% with peaks up to 33% (that is the highest value measured for these scenarios), leaving a wide set of remaining valid combination of trajectories. As regards the performances of the predictions, we have seen that the real situation results to have a correspondent ϵ that is very reduced (lower than 0.75), which reflect the fact that computed and real situation don't show relevant differences in performances, where a better payoff could be obtained even only from a more constant speed profile. If we consider non optimal situations, where for example the goal of a pedestrian is modified during the trajectory, then we notice that the performances tend to decrease very fast, as in the presented scenario 3. The set of alternative solutions to the real one remains very wide, because most of the trajectories coming from 1-pedestrian scenarios are optimal trajectories for these scenarios, since that the collision rate don't filter out many of them. What we can underline is that these trajectories don't show particular differences from those studied in a 2-pedestrian scenarios, so we can conclude that the presence of a single pedestrian not involved in social relationships with him doesn't modify substantially the trajectory of the other one. The problem of predicting those trajectories could be therefore modified into a single pedestrian motion problem.

The second part of the study has taken into account the N-players games, that are the most interesting to consider: in this case we notice immediately that the trajectories performed by the pedestrian can be clearly different from the correspondent trajectories computed with the same starting and ending point, but in a situation of no interaction or even in a 2-players game: in this case it's the interaction introduced by the collision rate that eliminates most of these alternatives. In fact, we have seen that in these situations this rate increases up to 83% of all the valid combinations. In these games the performances are still high, since that the set of valid alternatives it's clearly reduced: the real situations appear to be ϵ-NE with still higher values than the previous cases (up to 1.1), but with equivalent solutions that cover only a percentuage of the total number from 1% up to 10%. These performances could be still increased if we would model the social relationships: in fact, the computed NE of the different scenarios present trajectories where most of the times the pedestrian prefer to walk in queue one after the other in an optimal way, rather than walking in group as in the real situations. These real situations present therefore significant differences in the motion strategies performed by the pedestrians: therefore is fundamental for future works to model a social constraint on the motion of the pedestrians in order to get a reliable prediction of the group motion. Nevertheless, we could clearly notice that the trajectory of a single pedestrian facing a group of other humans approaching him on the same direction was predicted with a good approximation, showing also in the correspondent NE a trajectory similar to the real one

Chapter 5

Automotive Application

We analyze a second possible application related to motion prediction, which means one traffic scenario: in fact nowadays, as a consequence of a increasingly massive presence of street vehicles and multilane roads, the danger of collisions and accidents increases. This may lead to damages of even serious entity. Therefore, the problem of motion prediction consists of obtaining a model that could manage with an acceptable accuracy to predict which could be the typical behavior of the various drivers when an interaction is present. We also remark that usually the typical human driving behaviour is based on maintaining the maximum comfort and the maximum safety when driving, so in general we will assume that risk situations due to collisions or even only to sudden stops are consequences of wrong driving behaviors and therefore they aren't part of the ideal behaviour of a person. This study will therefore have as target first of all a check that in simple cases of traffic, the model works and respects the typical base choices that a driver executes; in addition, it suggests to solve in a centralized way some very complex traffic situations, where even a little mistake caused by the human driver could lead to an accident. These results could be used in order to introduce vehicles that, operated automatically by a control system, would manage to interact with optimal performances with vehicles operated by humans, taking advantage of the autonomous navigation. As in the previous chapter, also this one is a valid application for game theory: in fact we assume that each driver moves towards his target with the safest and most comfortable trajectory for him, adapting his choices to the possibilities offered by the world around him; his motion plan is modified from the presence of other agents (the other drivers) that are looking for their goals at the same time, so once given the others' strategies he will maximize his comfort and safety, that is the same goal obtained with the research of the Nash Equilibrium. This attempt of combining all the possible best trajectories with the constraints introduced by the other players while moving is in fact the typical application for the game theory.

5.1 Description of the simulated scenario

One typical approach of game theory with a traffic scenario is when cars are driving through a lane and face other cars performing trajectories that may lead to possible collisions. These collisions can be a consequence of invading other's lane without predicting carefully other's car motion, but also the presence of several car in the same lane with different speed. Interaction between cars can occur especially if cars are going following the same traffic direction, but not only: in fact, in narrow double-lane roads some possibilities of collision may occur between car coming from opposite directions, especially in situations where overtakes are possible. In some congested situations is very meaningful to solve these problems, because the presence of many cars modify clearly the trajectories computed a single one when moving in absence of traffic. The prediction of the motion is very important in order to avoid collisions especially when speeds can be relevant and eventual accident may lead to dangerous consequences. Therefore, we decide to focus our attention on a typical scene of automotive traffic scenarios, that is a unidirectional highway with more than one lane[1]. We consider a carriageway where there are 3 possibles driving lanes: in our case, we will assume that the time duration of the trajectories is that one corresponding to the space sufficient for providing a complete motion for different maneuvers of acceleration, deceleration or overtaking, while the width of our lanes is standard so that is enough wide to allow a standard size car to drive trough, but not enough for more than one car at the same time. The standard size for the cars will be $5 \times 2\,\mathrm{m}$, since that considering cars of different size doesn't lead to relevant modifications. Typical initial speeds for the cars are in the range 40 to 130 km/h, with possible final variations in the range 10 to 30 km/h. We also assume that this scenario could allow the cars to drive in parallel lines, which means that it is possible to drive in any lane and that it is possible to overtake in the right lane. When analyzing the obtained results we will try always to make some considerations about the actual correspondence, especially when some traffic rules are added (think for example a highway where is forbidden to overtake a car from the right side).

[1]All the rights of the cars silouhettes in the following image belong to Andrey Kokidko

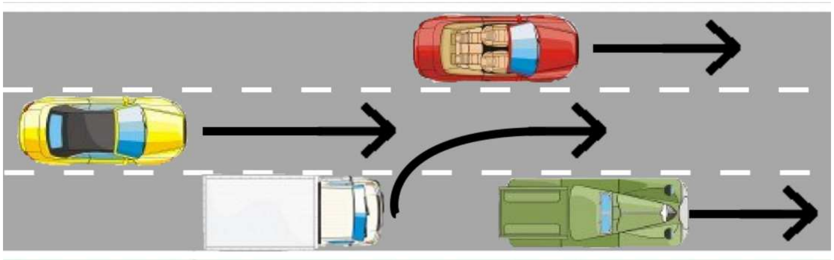

Figure 5.1: Designed highway scenario

Generation of the Trajectories

We consider now the problem of the generation of trajectories, also deciding one modality through which only some trajectories are considered; in fact, for a moving car the possibles executable trajectories are infinite (for example, think about a normal straight path, which could present different trajectories according to a more or less intense acceleration or deceleration), therefore we are forced to introduce some other assumption and simplification in order to make this scenario solvable in finite time: we assume that every driver could move in a single maneuver in a lateral deviation that bring him to move at most to an adjacent maneuver. With this assumption, each car can maintain his straight direction, move to the adjacent left lane or to the right lane. For each of these maneuver we introduce the possibility for the driver to modify the speed of the car respect to the initial one or to keep it constant: so the driver can deviate and move in the left lane, maintaining his initial velocity or even accelerate or decelerate, the same happens with the other trajectories. This difference between initial and final speed may vary and we won't considered it as fixed for all the cars (in fact, different driver and different models of cars could be more or less physically able to accelerate or decelerate). In total, each car i has a set M_i of 9 different possible trajectories $M_i = \{l_1, l_2, l_3, c_1, c_2, c_3, r_1, r_2, r_3\}$, where respectively l_k are the 3 trajectories moving into the adjacent left lane, c_k are the 3 trajectories staying on the initial lane and r_k are the 3 trajectories moving into the adjacent right lane.

Let's focus on the generation of the optimal trajectories, that describe through a mathematical model the possible choices executed by the human user in order to maximize his comfort when driving: we will assume that the goal of each user won't be to reach one specific destination point or the time duration of a maneuver, but rather the general comfort perceived by the driver all along the movement, starting from the defined initial position and speed. This comfort essentially consists of speed profile as constant as possible, avoiding unnecessary changes of directions; it's proved in literature

[13] that the class of the possible trajectories which satisfy this requirement are the so called "minimum jerk trajectories", i.e. those that during the period of movement T minimize a cost function J that is quadratic in the jerk (defined as the third time derivative of the position coordinates) associated to them:

$$J = \int_0^T f(\dddot{d}(t), \dddot{s}(t))^2 dt$$

where $d(t)$ and $s(t)$ are the Frenet coordinates ([14]). Trajectories can be described in several ways, but we prefer to use the Frenet coordinates since they introduce some useful features, like the independent computation of the acceleration components of the lane direction and those introduced when curving. To specify better the implementation, we will call $s(t)$ the component that describes the advancing of the car over the central axis of the lane, while $d(t)$ will be the component of the vector corresponding to the lateral motion. In this system the 2 vectors $s(t)$ and $d(t)$ are perpendicular, so that the corresponding accelerations a_{long} and a_{lat} result to be perpendicular themselves. This choice also allows to simplify the algorithm that computes possible collisions, once made some assumptions: in fact, we assume that:

- the timestep is enough reduced to prevent possible hidden collisions, so that if 2 trajectories collides, there will be at least one timestep where this collision is present

- once defined respectively l_i and w_i the length and width of the car i, 2 cars collide when their distance is smaller than l and w, with $l = \frac{l_i}{2} + \frac{l_j}{2}$ and $w = \frac{w_i}{2} + \frac{w_j}{2}$

With these assumptions it's sufficient to check if the distance between the vehicles is over the minimum bound described by the sizes of the vehicles at any time step. A possible disadvantage could be the harder description for imposing eventual physical constraints in the movement of the vehicles, but in our assumption of simple trajectories these won't be relevant factors for us.

Before introducing the formulas through which the algorithm computes the trajectories, let's make first a schematic recap defining symbols used:

s(t) position of the car at time t along the lane central axis in longitudinal direction

d(t) position of the car at time t along the lateral direction

$\mathrm{a_{long}(t)}$ acceleration of the car at time t along the component s(t)

$\mathrm{a_{lat}(t)}$ acceleration of the car at time t along the component d(t)

Following the approach used in [13], we briefly introduce how the trajectories are calculated: the first important property is that the coordinates can be computed independently one from the other. The curve is represented mathematically as a quintic polynomial, where the coefficients of it can be derived from

$$\begin{aligned} c_{012} &= M_1^{-1}(0)\xi_i(0) \\ c_{345} &= M_2^{-1}(\tau)\left[\xi_i(\tau) - M_1(\tau)c_{012}\right] \end{aligned}$$

with

$$M_1(t) = \begin{pmatrix} 1 & t & t^2 \\ 0 & 1 & 2t \\ 0 & 0 & 2 \end{pmatrix} \quad M_2(t) = \begin{pmatrix} t^3 & t^4 & t^5 \\ 3t^2 & 4t^3 & 5t^4 \\ 6t & 12t^2 & 20t^3 \end{pmatrix}$$

and

$$c_{012} = \begin{bmatrix} c_0 & c_1 & c_2 \end{bmatrix}^T \quad c_{345} = \begin{bmatrix} c_3 & c_4 & c_5 \end{bmatrix}^{\mathrm{T}}$$

where $\xi_i(t)$ is the state vector $\xi_1(t) = \begin{bmatrix} d(t) & \dot{d}(t) & \ddot{d}(t) \end{bmatrix}^T$ or respectively $\xi_2(t) = \begin{bmatrix} s(t) & \dot{s}(t) & \ddot{s}(t) \end{bmatrix}^T$. For the computation of $d(t)$, the final state position is fixed, since it's related to the final lane chosen, so this representation can be reduced to a quartic polynomial. For what concerns the $s(t)$, the final point is not fixed since we will only fixed the eventual speed variation, that will univocally determine the final position.

5.2 Game setup and choice of cost function

We describe now the process of implementation for this scenario, that follows the scheme illustrated in Figure 3.2. In the carriageway each car is seen as a player which can choose strategies based on his actions; for each player we assume that the number of actions is fixed to 9, corresponding to each maneuver of a driver (moving to the left side, moving to the right side, going straight) multiplied for the possible speed variations (accelerating, decelerating, maintaining constant longitudinal speed). To each strategy is assigned a payoff, depending from which strategies the other players chose to act. These payoffs are strictly connected to their progresses on the road, so that each player will try to maximize its income: the interaction between the players is represented by the possible collisions, which means that in some situations it's not possible to maximize their own payoff without considering other' strategies (or at least make predictions on it). We assume that each car cannot talk and decide directly with other players so that they act their strategies independently from the others: this bring to consider this scenario as a non cooperative game.

Let's focus on the choice of the cost function, that will allow us to convert each scenario in a game associated to it. As in the previous scenario, the pedestrian one, also here we assume that the elements that influence the cost are associated to the accelerations. Following the considerations express in [15], we define the cost function as an integral over the time that sums the various components of linear and lateral acceleration. Mathematically speaking we define the cost function as

$$\phi(d(t), s(t)) = \int_0^T a_{\mathrm{long}}(t)^2 \lambda_1 + a_{\mathrm{lat}}^2 \lambda_2 dt$$

where $\lambda_1 = 1/T/a_f^2$ and $\lambda_2 = 74/T/\phi_{\max}$, with $\phi_{\max} = 0.5\,\mathrm{rad}$ and $a_f = 9.1$ m/s^2. In these formulas T is the horizon time considered in a single stage of a game, a_f and $\phi_{\max}$ are respectively the maximum longitudinal acceleration and the maximum steering angle for a typical car. Since the cost function was previously implemented in scenarios that could have even huge differences of parameters with our simulated one, especially the period time T, we decided to remove the term based on the longitudinal speed (differently, considered in the previous paper) because it could lead to undesired results, where the longitudinal speed overweighted and suboptimal trajectories such as going ahead with longitudinal constant velocity varying continuously the lateral direction could be considered optimal.

This cost is assigned to every action that doesn't lead to a collision. The following step is to introduce the interactive part, related to the possible collisions between the trajectories corresponding to the performed actions: the algorithm that checks for possible collisions consists of verifing at any timesteps that the following inequality holds:

$$||d_1(t) - d_2(t)|| > h \quad ||s_1(t) - s_2(t)|| > l$$

where d_i and s_i are the positions of the vehicle's barycenter and h and l are the previously introduced dimensions for a typical car.

We apply then a linear normalization, where 0 to 10 is the scale of the possibles payoffs (logically if a trajectory has low cost, then its related payoff will be high). If 2 trajectories collide even for only 1 timestep a value of -10 is assigned to this situation. We also set that in case that a car is on an external lane, it's be possible for the driver only to go ahead in the same lane or move to an internal one, since that moving out of the road is not allowed. With this assumption, the set of possible actions for the player on an external lane is reduced to 6, because the three trajectories corresponding of moving into a forbidden lane are removed.

5.3 Simulations

We go now for simulating the common traffic situations, where we want to show that the solutions of the games reveal the typical behaviours of the driver in the real situations. With this, we mean that in an ideal scenario game theory provides reliable predictions of the possible future motions. We point out that for us the solutions will be always an unique point (i.e. a unique combination of strategies), so that in the case we should find more than one NE, we will focus on that one that assures all the positive payoffs for all the player (i.e. no collisions) or at least the biggest average sum of payoffs between them, but making some considerations even on the other possible solutions. This assumption follows from the idea that although NEs with lower payoffs could be realistic solutions too, that one selected will be the effective prediction of the strategies followed by the drivers. The parameter that will determine which NE is the most reliable one is the game payoff, which consists of the average payoff obtained by all the players in the game.

The first situation (figure 5.2) we investigate is first of all a simple validation that the model works fine, so we consider 3 cars in 3 different lanes going forward with different initial velocities:

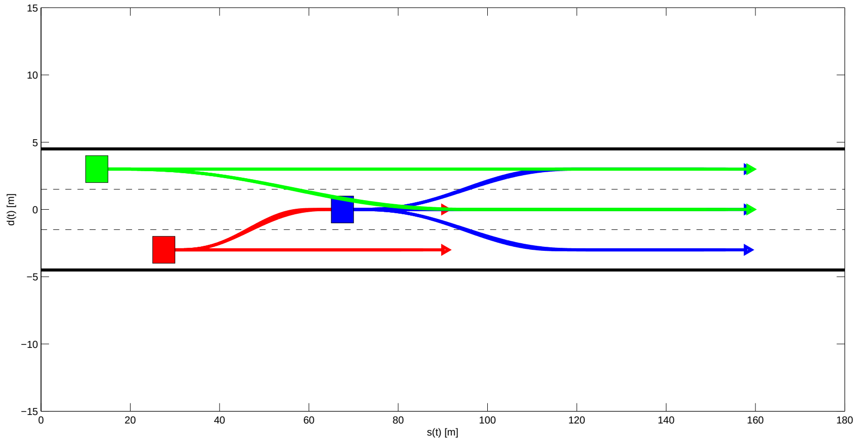

Figure 5.2: Basic scenario with 3 cars running through different lanes (initial speeds for the cars: red = 40 km/h, blue = 60 km/h, green = 100 km/h)

As we can expect, for each driver the maximum comfort is obtained just continuing going forward without any changes, since neither linear nor lateral accelerations are provided. In fact the cost function is 0 for each straight path with constant velocity and so they gain the maximum profit without invading other's lanes (see Figure 5.3). All the speeds of the cars remain constant since there is no reason for them to move from the desired one (assumed equal to the initial one).

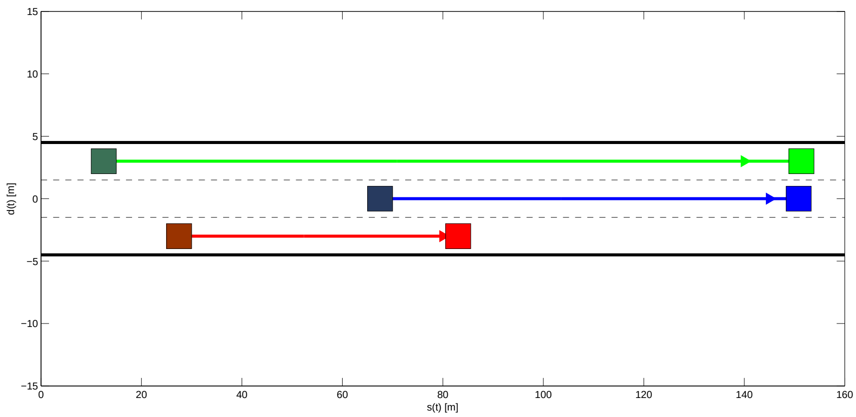

Figure 5.3: NE computed with the highest game payoff (corresponding to NE 1 in table 5.1)

Another NE is computed, but the game payoff is definitely lower, since that two cars need to switch lanes between them (see 5.4): this confirms the expectations on the model, if no interaction is required to improve payoff, each player will get his best payoff just maximizing the payoff function without constraints (in this case, correspondent to moving on a trajectory that maximize the cost function).

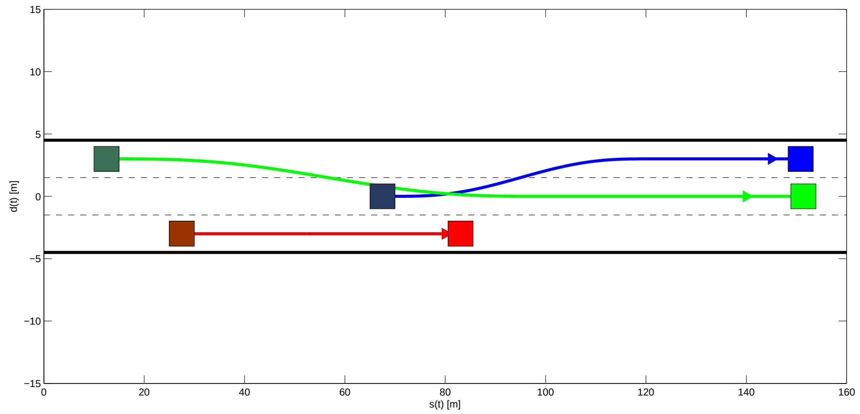

Figure 5.4: Alternative NE where the average payoff is not maximum (corresponding to NE 2 in table 5.1)

In table 5.1 we report the results of the Nash equilibria: We notice a huge difference of payoff for the blue and green cars choosing the strategies according to the first and the second equilibrium. In this situation the cars have initial speeds of 40, 60 and 100 km/h, a different selection of initial speeds wouldn't result in different NEs, but could have eliminated the second one, since that a collision could occur between the blue and the green car.

	NE 1	NE 2
Red	10	10
Blue	10	$1.08 * 10^{-4}$
Green	10	$1.08 * 10^{-4}$

Table 5.1: Scenario 1: payoffs computed for the Nash equilibria

So we can conclude that the trajectories of the drivers are verified, therefore we can move on other common situations.

In Figure 5.5 we have 2 cars standing in the same lane, the right one. In this case, the blue car is going faster than the red one (respectively 65 and

50 km/h, standing one 40 meters far from the other) therefore without any change a collision will take place between them. Running the game theory, we get the following solutions: if the initial speed difference between the two cars is not big, the adjustment is set just forcing the blue car to decelerate until it reach the same speed of the red one (figure 5.6). In fact, the comfort variation introduced by the deceleration is definitely lower than that one introduced by an eventual lateral move towards another lane.

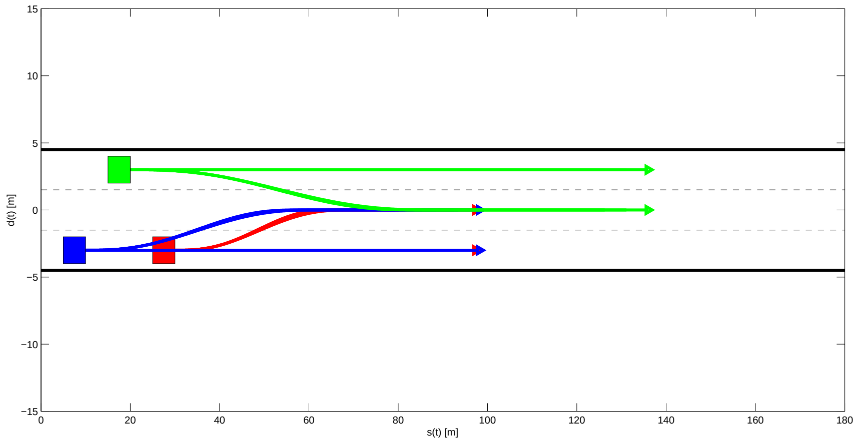

Figure 5.5: Scenario 2: blue car approaching to the red car with a reduced higher speed, while green car is on the left lane (initial speeds: red = 50 km/h, blue = 65 km/h, green = 100 km/h)

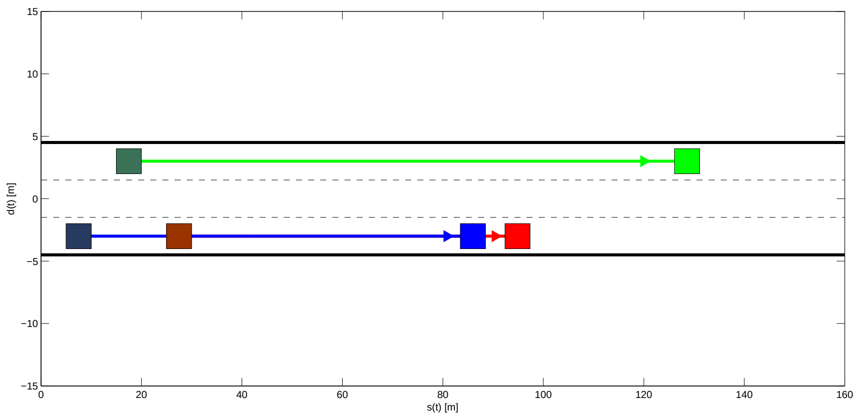

Figure 5.6: Scenario 2: Nash equilibrium computed (corresponding to NE 1 in table 5.2)

We clarify that this is not the only solution with the best game payoff, since that another equilibrium (corresponding payoffs are reported in table 5.2 as NE 2) is computed corresponding to trajectories where the red car accelerate while the blue maintains constant speed. This alternative equilibrium gives the same overall result, but in the real world usually is more plausible the first one since that is the faster car that needs to adapt itself to the other and not vice versa. We remark that this possible equilibrium stands only if the approaching car has a small speed velocity respect to the other one (otherwise it would prefer to switch lane and overtake the other car), or enough distance between it and the other car (otherwise it couldn't decelerate on time). In this case the relative payoffs are:

	NE 1	NE 2
Red	10	9.9
Blue	9.9	10
Green	10	10

Table 5.2: Scenario 2: Computed payoffs for computed equilibria

A similar scenario is suggestive if we consider that the car are moved by a centralized controller that prevents possible accidents: for even bigger speed differences, instead of forcing an overtake the controller could set speed variations for both the cars, so that they come (one accelerating, the other decelerating) to a common velocity, getting better payoffs than the overtake situations (see in the following scenario). The evolution of the scenario in this case is basically the same as in Figure 5.6, as seen before. The payoffs gained are respectively 9.9, 9.9 and 10 and these strategies to a Nash equilibrium similar to those in the previous scenario, but in this case both red and blue cars change speed (respectively blue decelerating and red accelerating). For the third situation (Figure 5.7), we just make a simple modification to the previous one, assuming that the blue and the red cars are in the same lane, but closer and with a bigger speed difference between them. We set as initial speed values 40 and 60 km/h, while the green car's data initial velocity unchanged, since it is not interacting with the others. We also reduce the initial distance between the blue and red car to 20 m.

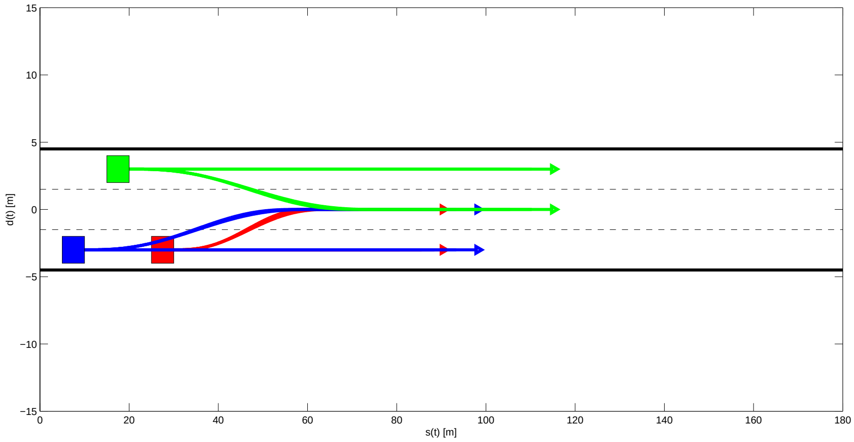

Figure 5.7: Scenario 3: blue car approaching the red car in the right lane with much higher speed, while green car is in the left lane (initial speeds for the cars: red = 40 km/h, blue = 60 km/h, green = 100 km/h)

In this case the 2 cars approach really fast and if there is no possibility to avoid a collision without that one car changes the lane, the only solution is for one of the 2 cars to go in the middle lane. This situation is typical when a car (in this case, the blue one) wants to proceed with its desired velocity and faces another one clearly slower: this lead to a strategy where it overtakes the red car (figure 5.8). As seen before, it's present (Figure 5.9) also another equivalent NE, which is that one where is the red car changing the lane going to the center: this could be an unspontaneous behaviour, since typically the car approaching faster from behind tends to prefer to overtake rather than expecting the other to switch lane. This assertion is also supported by the fact that usually a driver tends to pay attention with different intensity depending on which direction is coming the thread, so if a faster car is approaching somebody from behind, the driver usually put less intention on avoiding this collisions, since it will be the other driver's responsibility to do it (see [15]). We also want to underline that even if possibly unnatural, it could be preferred in a centralized path planning, since it results in trajectories with lower costs.

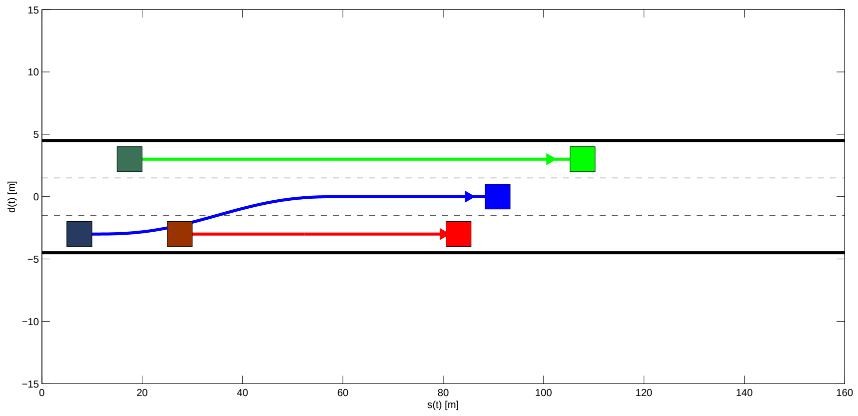

Figure 5.8: Scenario 3: Nash equilibrium where blue car overtakes the red car switching the lane (corresponding to NE 1 in table 5.3)

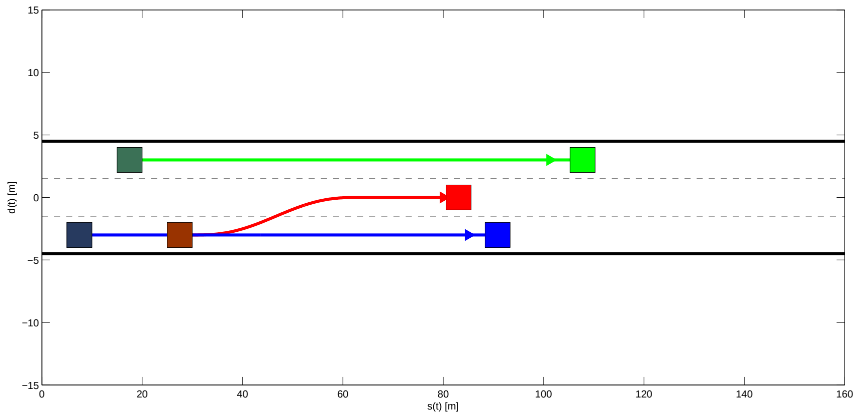

Figure 5.9: Scenario 3: Nash equilibrium where red car changes lane (corresponding to NE 2 in table 5.3)

As before, we report a table with the values of the payoffs obtained by the drivers: as we can notice, these solutions are equivalent.

	NE 1	NE 2
Red	10	$1.08 * 10^{-4}$
Blue	$1.08 * 10^{-4}$	10
Green	10	10

Table 5.3: Scenario 3: Computed payoffs for Nash equilibria when one car is forced to change lane

Let's move now on a more complicated scenario (figure 5.10): we introduce a fourth car (in magenta) with a reduced velocity, comparable with the red one (we set 40 km/h, as the red car, while blue and green have respectively 70 and 130 km/h). This scenario forces the green car (by now considered in the scenario but not interacting with the other cars) to interact with the traffic situation.

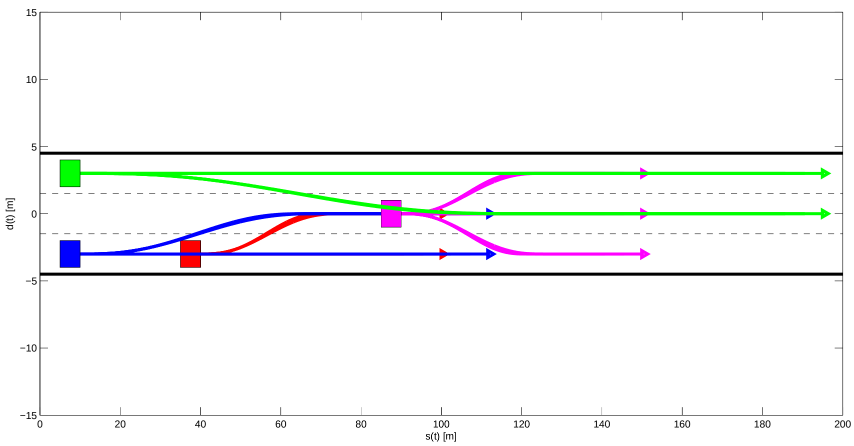

Figure 5.10: Scenario 4: Blue car approaching the red car with higher speed in the right lane, while magenta and green car are respectively in the middle and left lane (initial speeds for the cars: red = 40 km/h, blue = 70 km/h, green = 130 km/h), magenta = 40 km/h)

Until now, our traffic situation were solved in only one step, since after deciding which strategy could maximize their payoffs, the driver could continue on the selected lanes with constant speed without facing further constraints. In this case we want to analyze a situation of complex traffic, where we need to play 2 times the game (2-stage game). The solution of this game will be than the sequence of Nash Equilibria with the highest sum of the 2 game payoffs.

If we compute the Nash equilibria of this scenario we find a solution with typical strategies executed by the drivers, which means a situation where the blue car overtakes the red one, continuing in the middle lanes and approaching to the magenta one, while the others continue in a straight line with constant speed: this results to be as expected the equilibrium with the highest average payoff (Figure 5.11).

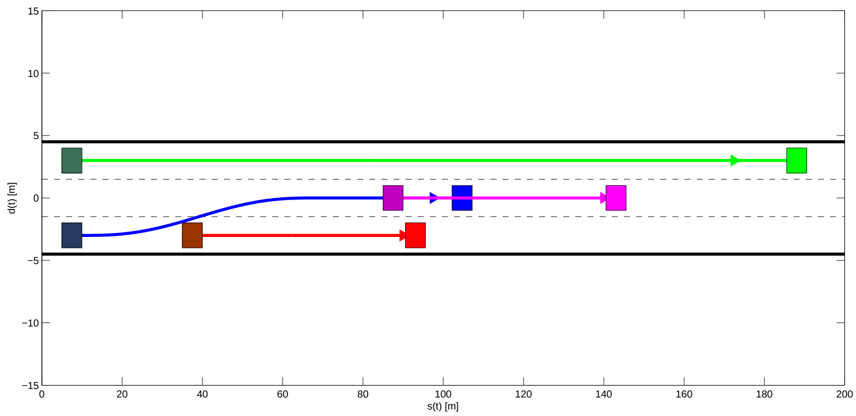

Figure 5.11: Scenario 4: computed Nash equilibrium with highest average payoff (corresponding to NE 1 in table 5.4)

This strategy doesn't bring to an equilibrium that avoids possible future collisions, since we will see later that the blue car approaches the magenta car with a huge speed difference, so that it will need to overtake this car.

As seen also in the previous, it's present another equivalent equilibria (shown in Figure 5.12), where this time is the red car that decide to change lane moving into the center, leaving free space maneuver for the blue car:

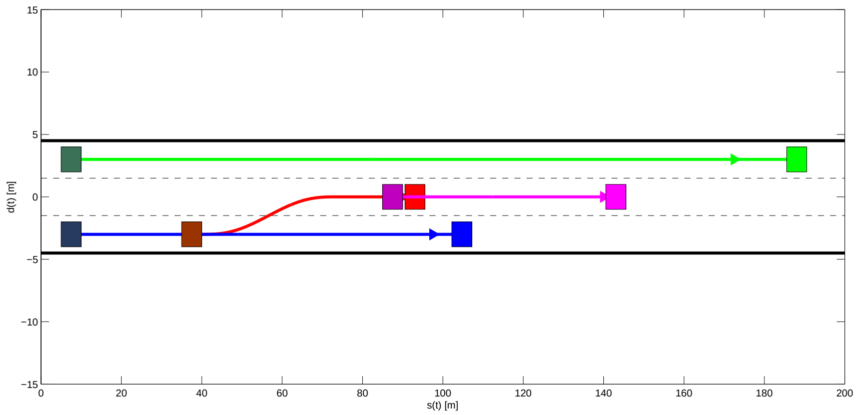

Figure 5.12: Scenario 4: alternative computed Nash equilibrium with highest average payoff (corresponding to NE 2 in table 5.4)

We want to show that with lower game payoff, are present 2 other equilibria: in this case the green car, that is the fastest in the carriageway moves to the center, forcing the magenta one to switch to the left one. The blue car may change to the center lane, since the huge velocity difference with the green car allows them to maintain a safe distance while approaching, increasing during the time (Figure 5.13). Similarly, the last computed Nash equilibrium is symmetrical, but with the red car moving to the center (Figure 5.14):

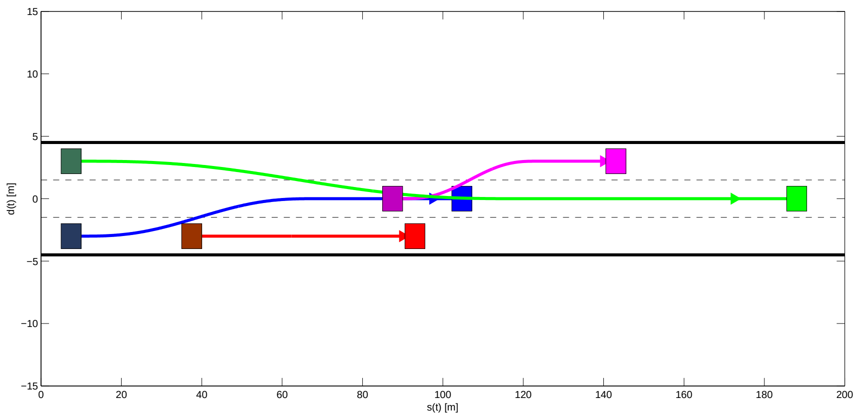

Figure 5.13: Scenario 4: alternative computed Nash equilibrium with lower average payoff - blue car deviating (corresponding to NE 3 in table 5.4)

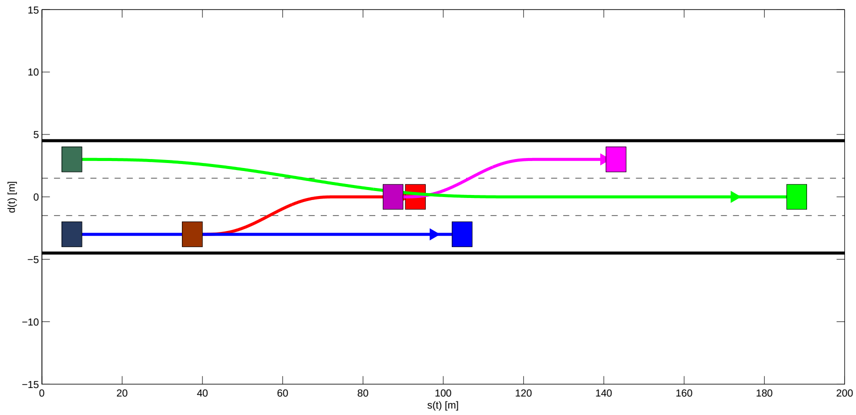

Figure 5.14: Scenario 4: alternative computed Nash equilibrium with lower average payoff - red car deviating (corresponding to NE 4 in table 5.4)

We report finally the partial table relative to the first part of the scenario, with the relative payoffs:

	NE 1	NE 2	NE 3	NE 4
Red	10	1.0810^{-4}	10	1.0810^{-4}
Blue	1.0810^{-4}	10	1.0810^{-4}	10
Green	10	10	1.0810^{-4}	1.0810^{-4}
Magenta	10	10	1.0810^{-4}	1.0810^{-4}

Table 5.4: Scenario 4: Computed payoffs for Nash equilibria for different equilibria

Let's analyze now the second part of the situation, starting from the previous 4 equilibria in order to find out which could be the most probable traffic solution for this scenario. From 5.11 we have now the blue car in the middle lane, approaching very fast the magenta car. Since we assumed that the two cars have a big initial speed difference, they can't stay in the same lane and therefore one has to move to another one. Several equivalent equilibria come out from this situation, for example the magenta car moves to the right, in a compatible speed with the red one (Figure 5.15: this happens if for instance the magenta car accelerated to get a slow overtaking over the red car and later comes back to its slow lane, leaving free lane for the blue car.

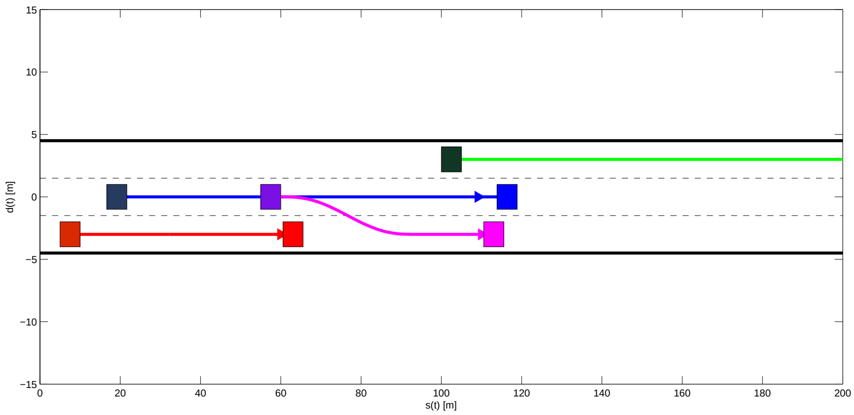

Figure 5.15: Scenario 4 - Future motion according to NE1: magenta car moving right (corresponding to NE 1a in table 5.5)

The other alternative option (Figure 5.16) is to force the blue one in moving right, so that it comes back to the initial lane, but after having overtaken the red car. This scenario is very frequent when most of the cars are occupying all the lanes with more or less the same speed and a new car approaches with a much higher velocity, so that it will try to pass this group of driver with many maneuvers, without stressing other drivers to make any special deviation or speed variation.

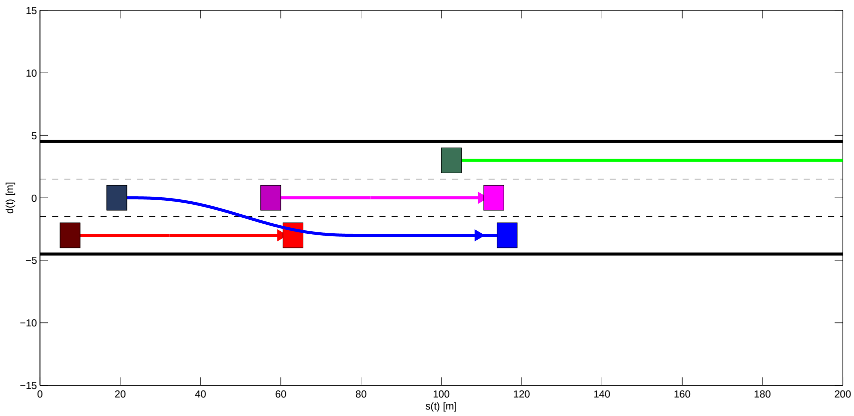

Figure 5.16: Scenario 4 - Future motion according to NE1: blue car moving right (corresponding to NE 1b in table 5.5)

Another safe solution (Figure 5.17) that could be adopted from the blue car (or the magenta in Figure 5.18) is to shift into the left lane, left free from the green car if its speed was enough high to go out from the interaction interest's range.

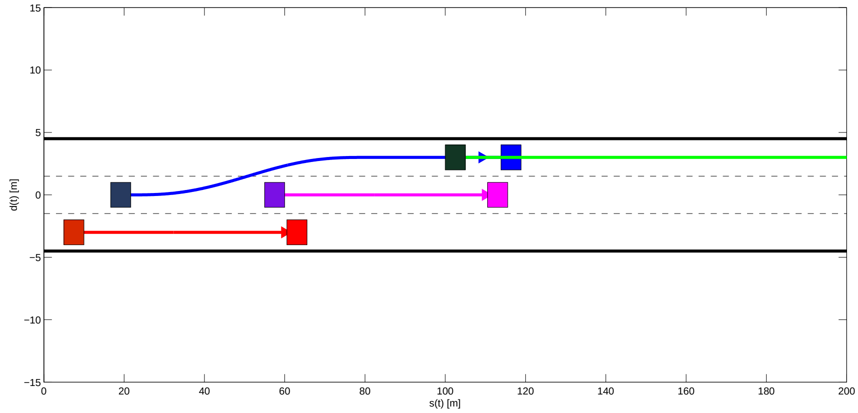

Figure 5.17: Scenario 4 - Future motion according to NE1: blue car moving left (corresponding to NE 1c in table 5.5)

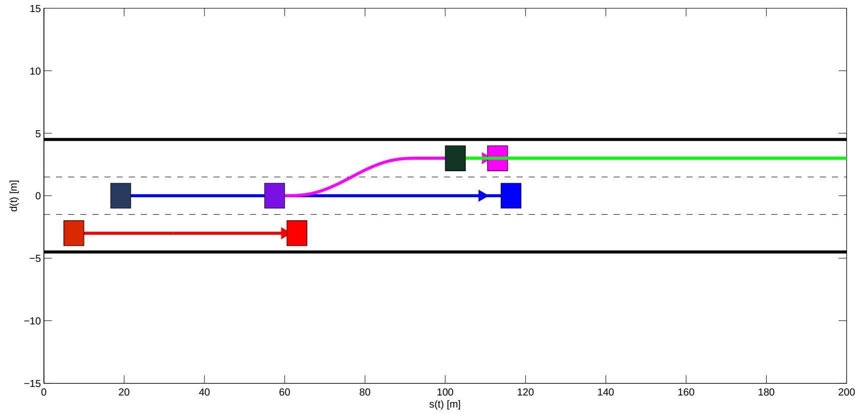

Figure 5.18: Scenario 4 - Future motion according to NE1: magenta car moving left (corresponding to NE 1c in table 5.5)

So in this 2 game, adopting the first equilibrium as intermediate stage for the prediction of the trajectories, we can notice that in any case the safety will be reached after 2 maneuvers, no matter from which car. We analyze the evolutions of the situation when equilibrium 2 is applied: In this case only one dominant equilibrium is present, since no cars are approaching, is obvious that without any risk of collision the solution is already solved and the cars continue on their trajectories following the lanes (Figure 5.19). This situation is therefore that one using the smallest number of total maneuvers, so that in a centralized control this could be the solution chosen by the control system in order to assign the highest average payoff to each driver. We claim that this situation is not so unnatural as it could be appear at the first sight: in fact is a typical situation where an emergency vehicle is present and need free space in order to reach with the highest comfort and the smallest time a goal; we can notice it because the driver car maintains its initial lane and the unique change is introduced when the red car moves to let free way for the other car.

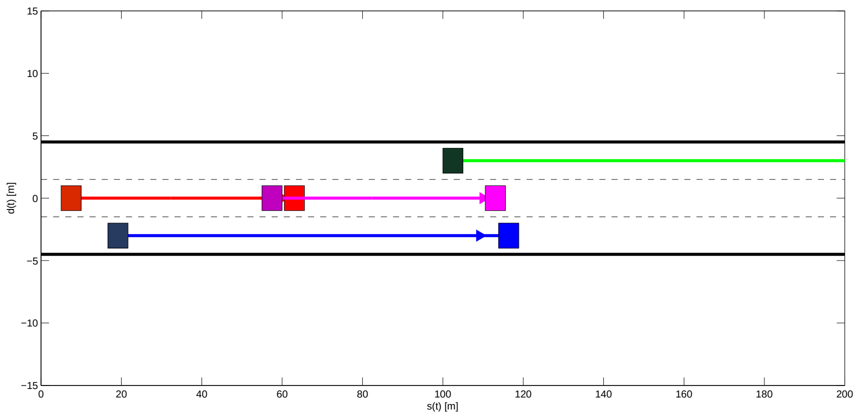

Figure 5.19: Scenario 4 - Future motion according to NE2: All cars continuing in their straight trajectories (corresponding to NE 2a in table 5.5)

Another solution considered as equilibrium is that one described in Figure 5.20, where blue and magenta car swap lanes. This mentioned solution result to in a low game payoff (besides unnatural) and therefore is not generally adopted

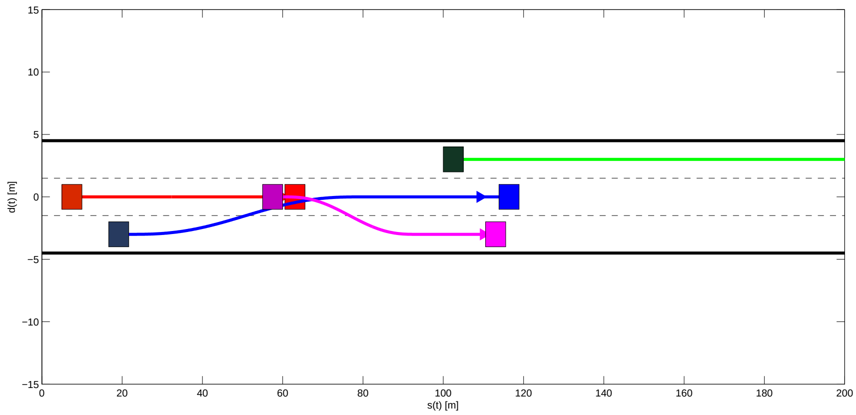

Figure 5.20: Scenario 4 - Future motion according to NE2: Blue and magenta cars swapping lanes (corresponding to NE 2b in table 5.5)

Let's make some considerations on the final trajectories coming from situation of Figures 5.13 and 5.14. We claim that this solutions are not probable in the real situation, since in the first part of the scenario they present lower payoffs in general so that the players generally tend to prefer other strategies. In both the situations, all the cars can continue driving on their lanes without risks of collision: both magenta and blue (or red in Figure 5.21 have a free lane to move through and the green car is clearly faster than the red (or blue) one, so they stay in the same lane maintaining a safe distance

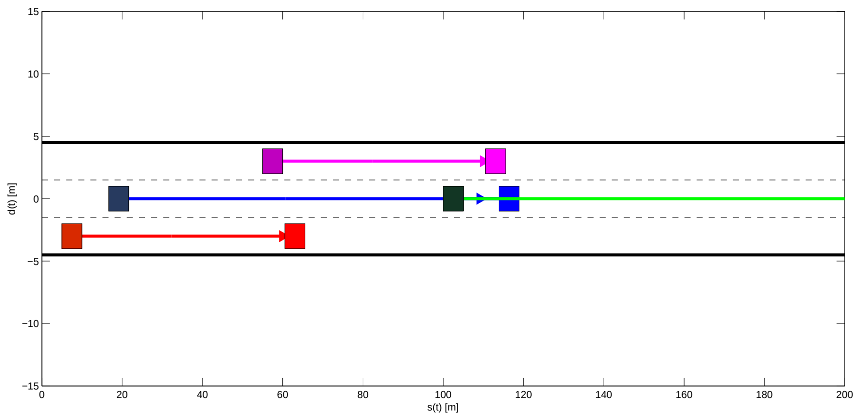

Figure 5.21: Scenario 4 - Future motion according to NE3: all cars continuing in their straight trajectories (corresponding to NE 3a in table 5.6)

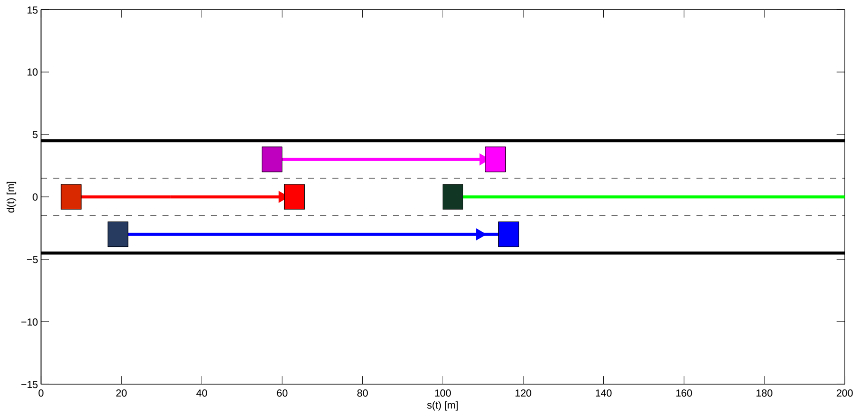

Figure 5.22: Scenario 4 - Future motion according to NE4: all cars continuing in their straight trajectories (corresponding to NE 4a in table 5.6)

Another further equilibrium is that one in Figure 5.23: in this case blue and red car swap lanes. We won't investigate in deep this situation since is neither with a high game payoff nor plausible in the real scenario (with 4 lane changes it's by far the worst solution between all the possible ones).

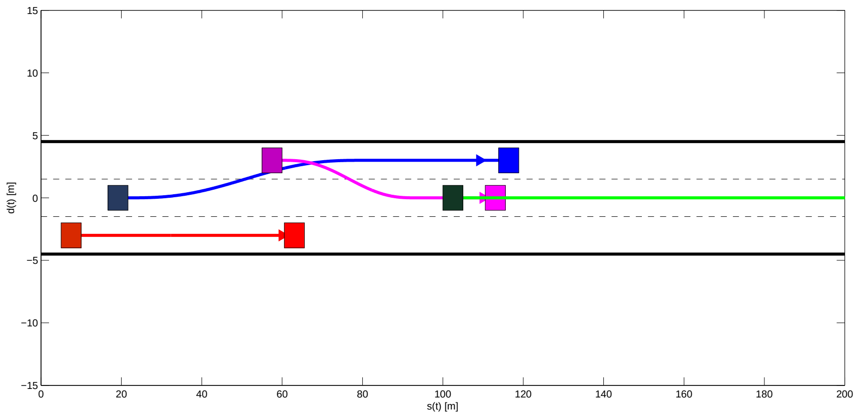

Figure 5.23: Scenario 4 - Future motion according to NE3: blue and magenta cars swapping lanes (corresponding to NE 3b in table 5.6)

So finally we report the payoffs tables corresponding to the played strategies: in Table 5.5 and 5.6 we present the payoffs corresponding to the second part scenario, while in Table 5.7 and 5.8 we sum the payoffs of the previous 2 tables and obtain the final payoffs. As previously considered, the strategies that bring to situation of Figures 5.12 and then 5.19 result to get the better average payoffs and configures themselves as the best possible solution. We remark that usually the players tend to play the action that bring to a highest income since from the first step, so also situation of Figure 5.11 and followings are possible to be chosen. We remark also that in roads where overtaking by right is not allowed, this is the only solution that can be achieved without lowering the original payoffs introducing decelerations or accelerations.

	NE 1a	NE 1b	NE 1c	NE 1d	NE 2a	NE 2b
Red	10	10	10	10	10	10
Blue	10	1.0810^{-4}	$1.08 * 10^{-4}$	10	10	1.0810^{-4}
Green	10	10	10	10	10	10
Magenta	1.0810^{-4}	10	10	1.0810^{-4}	10	1.0810^{-4}

Table 5.5: Scenario 4: Computed payoffs for Nash equilibria starting from more convenient equilibria 1 and 2 previously considered

	NE 3a	NE 3b	NE 4a
Red	10	10	10
Blue	10	10	1.0810^{-4}
Green	10	10	10
Magenta	10	10	1.0810^{-4}

Table 5.6: Scenario 4: Computed payoffs for Nash equilibria starting from less convenient equilibria 3 and 4 previously considered

	NE 1a	NE 1b	NE 1c	NE 1d	NE 2a	NE 2b
Red	20	20	20	20	$10+\epsilon$	$10+\epsilon$
Blue	$10+\epsilon$	2ϵ	2ϵ	$10+\epsilon$	20	$10+\epsilon$
Green	20	20	20	20	20	20
Magenta	$10+\epsilon$	20	20	$10+\epsilon$	20	$10+\epsilon$

Table 5.7: Scenario 4: Total Computed payoffs for Nash equilibria trajectories passing through equilibria 1 and 2

	NE 3a	NE 3b	NE 4a
Red	20	20	$10+\epsilon$
Blue	$10+\epsilon$	$10+\epsilon$	$10+\epsilon$
Green	$10+\epsilon$	$10+\epsilon$	$10+\epsilon$
Magenta	$10+\epsilon$	$10+\epsilon$	2ϵ

Table 5.8: Scenario 4: Total Computed payoffs for Nash equilibria trajectories passing through equilibria 3 and 4
where $\epsilon = 1.0810^{-4}$

5.4 Discussion

In the previous chapter we have seen some typical situations of traffic in a highway. First of all we have considered a basic situation where the effective risk of collision consists basically of 2 cars moving on the same lane with different longitudinal velocities. In this situations the related Nash equilibrium consisted alternatively on a reduction or an increase of speed from one car or even both. When this collision avoidance was not anymore possible, the Nash equilibrium suggested to one of the car to switch lane and eventually overtake the other one. This result is credible and reflects the reality until the longitudinal speed variation is limited and performing a strong braking is not uncomfortable. In fact, the typical driver could prefer to change lane instead of performing an evident decreasing of speed: in this case unfortunately the cost function (and the following payoffs) is not enough tuned in order to show these aspects. Another aspect concerns the fact that in some situations, with 2 cars in the same lane with the one in the back approaching faster the other one, some alternative solutions where showing the car in the head accelerating or moving lane. These solutions are possible scenarios in special cases (as suggested, an emergency and so on), but not typical. One solution in order to prefer the other more credible equilibria could be to modify the cost function so that possible longitudinal accelerations are more weighted than the correspondent longitudinal decelerations; another solution could be to weight more lane changes when the lane ahead is empty instead of doing it when it's the only alternative to collide. As discussed in [15], the introduction of a probability distribution over the possible directions, implemented through Bayesian games could improve these situations.

Same considerations are valid for the multistage games, where this assume a less important role: in fact with the presence of several cars the first goal is always to predict safe trajectories avoiding collision. In our situation we saw how the predicted solutions were enough reliable to be considered valid, in fact all the cars were performing trajectories with the maximum comfort, avoiding collisions and useless switches of lanes when not strictly necessary. We can therefore conclude that our target of predicting trajectories is reached, even in more complex situations such as the last example. The research of Nash equilibria can bring to several alternative solutions that in this case were not filtered out by the algorithm, that could be less credible respect to the reality. For these solutions a future review could solve it working on the cost function in order to make them still solutions, but with a lower game payoff.

Chapter 6

Conclusions and Future Works

6.1 Conclusions

We have introduced a new approach able to predict the motion in some dynamic situations. In the first scenario, we have noticed that people tend to walk following some optimality criteria according to cost indexes depending on accelerations, time and goal direction. From this assumption we have seen, reducing these situations to N-player games that the solution of these representations can predict with small approximations the real trajectories executed by humans while walking. In the first section we have analyzed games with 2 players only, noticing that the trajectories of the 2 pedestrians are not sensibly modified by the presence of another pedestrian in the neighbourhood: the result of this is that the trajectories performed in the reality are not very different from the alternative ones performed when there is absence of interaction. This was shown also from the fact that, even if the real situations was not a Nash equilibrium but rather an ϵ-NE, the differences between computed solutions and real trajectories where qualitatively very close, where the difference is only based on a better trajectory planning. In the second part of the analysis, with N player, we have noticed that the interactive part is more relevant when computing payoffs: in fact the trajectories performed from the pedestrians where clearly different from those computed by a pedestrian when he's alone. This was expected because especially when a single pedestrian moves facing a group of persons, he feels as having a moving obstacle approaching him, so he will modify clearly even the path of his motion. Although the performances were as expected high and the computed solutions were in a very reduced set of possible alternatives, we saw a clear differences sometimes between the expected trajectories and the real ones: this is related to the limitations of our database, where sometimes our assumption that a human moves in a goal directed trajectory not influenced from other factors except for other outsiders pedestrians was not completely valid. We have in fact noticed social behaviours that made suboptimal these trajectories and therefore hard to be predicted. However, the reliable prediction of the individual motion of one pedestrian facing a group of people walking towards him was successfully reached, so we can state that the algorithm works fine.

In the second scenario we have simulated the typical situations faced by humans when driving in congested highways: Even if the action scheme of each player was reduced in order to simulate also problems with more players, we could notice that the predictions obtained through the Nash equilibria of the different examples were reflecting the typical behaviours followed by people while driving. In particular, we have noticed that in any case the possible computed solutions were based on strategies where each player was not colliding and was not performing a variation from the initial data (that is a change of lane or a speed variation) if it was not leading to an direct improvement of the traffic status: this reflect the assumption that every human driver normally acts in order to maximize his comfort and to avoid in any case any collision.

6.2 Future Works

There are still some open problems. As we have seen, motion prediction in these studied situations is sufficiently reliable even only if the cost associated to each trajectory depends only on the motion profiles. However, performances could still increase, using some advanced features of game theory, like for example the introduction of Bayesian games: Indeed previously was always assumpted that the motion of each subject was performed in a deterministic way as a solution of a NFG, where the NE is the strategy that maximize his payoff respect to the other's strategies. This method produces reliable prediction of the real motion and is correct until that is valid the assumption that each player knows totally all the possible trajectories and destinations of the other players. Practically, when an individual (for example, a pedestrian) is moving, he doesn't have immediately all the informations connected to the possible intentions of the other pedestrians, therefore is necessary to introduce an uncertainty on those actions. This uncertainty can model several other aspects of the motion, like for example the lack of accuracy of a driver when detecting the speed of other vehicles around him. This has been shown in the previous simulations, underlining that a probability distribution could be implemented in order to model better how a driver pays attention on the road while driving. More in general, a specific tuning of the cost functions making them perfectly suitable for each scenario could refine more the results.

Another typical feature of game theory to implement are the multistage games: as we have seen (although only in 2-stages games) in the automotive scenario, the motion performed by a driver is a collection of trajectories that are optimal respect to partial time windows, so that the total sum of these maneuvers result to be optimal in general. As we have seen, in traffic situations where cars move with quick maneuvers and spaces relatively close, the optimal prediction of the total motion generated by all the vehicles is that one that consists of a sequence of Nash equilibria. This method can also be applied in pedestrian scenarios, where the areas are narrow or where the presence of many people induce each pedestrian to replan optimally each situation of interaction with the others.

Finally, another factor is introduced by the so called "social constraints": in fact people moving in group can break some optimality criteria when moving, in particular they could choose some parallel path or avoid some obstacles in a way that they wouldn't do if they were moving alone. The exact prediction of the trajectories of human in groups could improve noticeably the integration of robots motions with people in real scenarios.

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List of Figures

2.1 Typical representation for EFG with 2 players of respectively 2 and 3 actions . . . 13

3.1 Scheme of the selected trajectories for the pedestrian scenario 18
3.2 Scheme of the payoff assignment algorithm . . . 19

4.1 Hallway where trajectories take place . . . 25
4.2 Example of tracking results with three pedestrians . . . 25
4.3 Example of B-spline . . . 28
4.4 Particular of 2 different B-splines compared with raw data . . 30
4.5 Comparison between velocity profiles using B-splines . . . 32
4.6 Comparison between acceleration profiles using B-splines . . . 32
4.7 Comparison between acceleration profiles using different windows data when deriving . . . 33
4.8 Comparison between velocity profiles using Kalman filter . . . 34
4.9 Tracking of the Scenario 1 . . . 37
4.10 Scenario 1: particular when 2 pedestrians are facing . . . 38
4.11 Scenario 1: Alternative trajectories for blue pedestrian . . . 38
4.12 Scenario 1: computed Nash equilibrium . . . 39
4.13 Scenario 1: distribution of the valid possible solutions in terms of ϵ . . . 40
4.14 Overview of paths with lateral approach in Scenario 2 . . . 41
4.15 Scenario 2: particular when 2 pedestrians are facing . . . 41
4.16 Scenario 2: alternative paths for red pedestrian . . . 42
4.17 Scenario 2: alternative paths for blue pedestrian . . . 43
4.18 Scenario 2: computed Nash equilibrium . . . 44
4.19 Scenario 3: example of path with not optimal actions . . . 44
4.20 Scenario 3: focus on change of direction for red pedestrian . . 45
4.21 Scenario 3: alternative trajectories for red pedestrian . . . 46
4.22 Scenario 3: computed Nash equilibrium . . . 47
4.23 Scenario 4: Overview of interaction with three players . . . 48

4.24 Scenario 4: alternative paths for red and blue pedestrian . . . 48
4.25 Scenario 4: alternative paths for black pedestrian 49
4.26 Scenario 4: distribution of the different epsilon-Nash equilibria 50
4.27 Scenario 4: computed Nash equilibrium 51
4.28 Scenario 4: alternative computed Nash equilibrium 51
4.29 Scenario 5: Overview of interaction with three players 52
4.30 Scenario 5: alternative paths for black pedestrian 53
4.31 Scenario 5: computed Nash equilibrium 54
4.32 Scenario 5: alternative computed Nash equilibrium 54
4.33 Scenario 6: Overview of interaction with five players 55
4.34 Scenario 6: alternative paths for blue pedestrian 56
4.35 Scenario 6: alternative paths for black pedestrian 57
4.36 Scenario 6: computed Nash equilibrium 58

5.1 Designed highway scenario 63
5.2 Basic scenario with 3 cars running through different lanes (initial speeds for the cars: red = 40 km/h, blue = 60 km/h, green = 100 km/h) 68
5.3 NE computed with the highest game payoff (corresponding to NE 1 in table 5.1) 68
5.4 Alternative NE where the average payoff is not maximum (corresponding to NE 2 in table 5.1) 69
5.5 Scenario 2: blue car approaching to the red car with a reduced higher speed, while green car is on the left lane (initial speeds: red = 50 km/h, blue = 65 km/h, green = 100 km/h) 70
5.6 Scenario 2: Nash equilibrium computed (corresponding to NE 1 in table 5.2) 70
5.7 Scenario 3: blue car approaching the red car in the right lane with much higher speed, while green car is in the left lane (initial speeds for the cars: red = 40 km/h, blue = 60 km/h, green = 100 km/h) 72
5.8 Scenario 3: Nash equilibrium where blue car overtakes the red car switching the lane (corresponding to NE 1 in table 5.3) . . 73
5.9 Scenario 3: Nash equilibrium where red car changes lane (corresponding to NE 2 in table 5.3) 73
5.10 Scenario 4: Blue car approaching the red car with higher speed in the right lane, while magenta and green car are respectively in the middle and left lane (initial speeds for the cars: red = 40 km/h, blue = 70 km/h, green = 130 km/h), magenta = 40 km/h) 74

5.11 Scenario 4: computed Nash equilibrium with highest average payoff (corresponding to NE 1 in table 5.4) 75
5.12 Scenario 4: alternative computed Nash equilibrium with highest average payoff (corresponding to NE 2 in table 5.4) 76
5.13 Scenario 4: alternative computed Nash equilibrium with lower average payoff - blue car deviating (corresponding to NE 3 in table 5.4) 76
5.14 Scenario 4: alternative computed Nash equilibrium with lower average payoff - red car deviating (corresponding to NE 4 in table 5.4) 77
5.15 Scenario 4 - Future motion according to NE1: magenta car moving right (corresponding to NE 1a in table 5.5) 78
5.16 Scenario 4 - Future motion according to NE1: blue car moving right (corresponding to NE 1b in table 5.5) 78
5.17 Scenario 4 - Future motion according to NE1: blue car moving left (corresponding to NE 1c in table 5.5) 79
5.18 Scenario 4 - Future motion according to NE1: magenta car moving left (corresponding to NE 1c in table 5.5) 79
5.19 Scenario 4 - Future motion according to NE2: All cars continuing in their straight trajectories (corresponding to NE 2a in table 5.5) 80
5.20 Scenario 4 - Future motion according to NE2: Blue and magenta cars swapping lanes (corresponding to NE 2b in table 5.5) 81
5.21 Scenario 4 - Future motion according to NE3: all cars continuing in their straight trajectories (corresponding to NE 3a in table 5.6) 81
5.22 Scenario 4 - Future motion according to NE4: all cars continuing in their straight trajectories (corresponding to NE 4a in table 5.6) 82
5.23 Scenario 4 - Future motion according to NE3: blue and magenta cars swapping lanes (corresponding to NE 3b in table 5.6) 82

Acknowledgements

I am grateful to my supervisor at TUM Daniel Althoff for his precious and constant help during my study period in Germany. He has been for me a valid advisor and leading figure, always present, especially when my work seemed not to give the hoped results.

Moreover, I want to thank all the people with whom I have lived during these years and shared the most part of my time: my first thought goes certainly to Francesco, Stefano and all the people from ADJ. As regards my colleagues at university, I can't forget to cite my biggest supporters at DEI, that made me proud of beeing an example for them and still daily don't miss to praise me: Basilio, Clara, Matteo and Ale (and those who used to comment and still will comment on my wall). Furthermore, if I managed to finish triumphant this course, it's also because I lived in a pleasant and "goliardic" surrounding location: therefore my thought goes to all the guys of Via Proust.

I also want to thank personally all the people that I met during my Erasmus in Munich: if this year spent in Germany has been the best one of my life, as well as a unique experience, I owe you. I'm happy especially because now I know that wherever I go, I know that I have a brother or a sister waiting for me with open arms, from Sweden to Chile, from Greece to Spain, via Italy but also through Turkey, Mexico, Argentina, Denmark, India etc.. A special gratitude goes to the people of Ludwigskolleg, with whom I have a special bond: I came there alone and feared from an unknown city and a difficult language, but you made me familiarize it in the best way. If today I'm madly in love with this city full of dindlr, maß and "Bitte, zurück bleiben!", this is due to all the people that daily filled my heart with Guacamole and Sangria, so "Danke und Toma ya!".

Finally, last but not least, I want to give my biggest gratitude to my family, withouth whom I would have never managed to achieve this goal: during these years you have always supported and encouraged me in the difficult moments, inciting me not to give up. I thank you also because you gave me the opportunity and incitement to make new experiences, that each time were converted in my personal rich of satisfactions growth. For these reasons, I give you my biggest hug, thank you Mamma, Papà, Federico and the rest of you, I love you.