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CRYPTOCURRENCIES TO ENHANCE PORTFOLIO PERFORMANCE: IS IT WORTH IT? AN ANALYSIS FOLLOWING MARKOWITZ AND RISK-BUDGETING APPROACHES.

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Table of Contents

1. **Chapter 1:** An introduction to the Cryptocurrency's world 7
1.1. Introduction 7
1.2. History of cryptocurrencies: E-money and Virtual Currencies 11
1.3. Cryptocurrencies and Real Money 15
1.4. Bitcoin Innovations 21
2. **Chapter 2 -** Dataset and Descriptive Analysis of Returns 27
2.1. The Base Portfolio Composition. 27
2.2. Base Portfolio - Descriptive analysis of returns 29
2.2.1. Government Bonds 29
2.2.2. Corporate Bonds 30
2.2.3. Equities 31
2.2.4. Commodities 32
2.3. The Cryptocurrency's Portfolio 33
2.4. Cryptocurrencies - Descriptive Analysis of Returns 35
2.4.1. Bitcoin 35
2.4.2. Namecoin, Litecoin, Peercoin, Feathercoin 38
2.4.3. Primecoin 39
2.4.4. Ripple 40
2.4.5. Doge 40
2.4.6. Ethereum 41
2.5. Assessing the Correlation 42
2.6. Final considerations: is there a bubble going on? 44
3. **Chapter 3**: Markowitz analysis, portfolios and constraints 47
3.1. Returns and Covariances 48
3.1.1. Rolling Sample Moments 48
3.1.2. EWMA 49
3.2. The Efficient Frontier 51
3.3. Maximum Sharpe and Global Minimum Variance Portfolios 53
3.4. Portfolio with intermediate degrees of risk aversion 54
3.5. Portfolio Constraints 57
3.5.1. No short selling constraint 59
3.5.2. Upper and Lower Bounds: 60

3.5.3. Group constraints 1 (Crypto at 10%) - Group constraint 2 (Crypto at 20%) 61
3.5.4. Turnover Constraint 64
4. **Chapter 4:** The Risk Budgeting analysis 67
5. **Chapter 5.** Results: Return Analysis..73
5.1. Markowitz strategy returns..74
5.1.1. Static approach Portfolios 75
5.1.2. Dynamic approach Portfolios...79
5.2. Risk Budgeting Results...81
5.2.1. Static Approach Portfolios 82
5.2.2. Dynamic Approach Portfolios..86
6. **Chapter 6:** Performance Indicators..89
6.1. Sharpe Ratio - Sh...90
6.2. Sortino Ratio - So 90
6.3. Treynor Ratio - Tr 91
6.4. Value at Risk - VaR 92
6.5. Expected Shortfall - Es...93
6.6. Drawdown Sequence - DD...93
6.7. Farinelli-Tibiletti Ratio 94
6.8. Performance Indicators Analysis 95

List of Tables

Table 2.1: Traditional Assets 27
Table 2.2: Cryptocurrencies dataset 34
Table 3.1: GMV portfolio attributes 53
Table 3.2: MS Portfolio Attributes 54
Table 3.3: Portfolios included in the analysis with the associated degree of Risk Adversion 56
Table 3.4: GMV and MS under Positivity of weights 59
Table 3.5: determination of the Efficient Frontier under upper and lower bounds. 61
Table 3.6: determination of the Efficient Frontier under upper and lower bounds. 61
Table 3.7: Group Constraint 1 (Crypto at 10%) 62
Table 3.8: Group Constraint 2 (Crypto at 20%) 62
Table 3.9: GMV and MS under Group Constraint 63
Table 3.10: GMV and MS under Turnover Constraints 65
Table A - Appendix: Markowitz Portfolios Summary Statistics - Rolling inputs 112
Table B - Appendix: Markowitz Portfolios Summary Statistics – Rolling Inputs 113
Table C - Appendix: Risk Budgeting Portfolios Summary Statistics – Rolling Inputs 114
Table D - Appendix: Risk Budgeting Portfolios Summary Statistics – EWMA Inputs 114
Table 1 - Appendix: Performance Indicators Rank Markowitz Strategies – Rolling inputs 115
Table 2 - Appendix: Performance Indicators Rank Markowitz Strategies – EWMA inputs 116
Table 3 - Appendix: Performance Indicators Ranking Risk Contribution Strategies – Rolling inputs 117
Table 4 - Appendix: Performance Indicators Ranking Risk-Contribution Strategies – EWMA inputs 117

List of Figures

Figure 2.1: Government Bond Index trend 29
Figure 2.2: Government Bond Cumulated returns 29
Figure 2.3: Corporate Bond trends 30
Figure 2.4: Corporate Bond Cumulated returns 30
Figure 2.5: Equities trends 31
Figure 2.6: Equities Cumulated returns 31
Figure 2.7: Commodities trends.. 32
Figure 2.8: Commodities trends.. 32
Figure 2.9 - Figure 2.10: Bitcoin Trend and Cumulated Return 35
Figure 2.11: Namecoin-Litecoin-Peercoin-Feathercoin Trends 38
Figure 2.12: Namecoin-Litecoin-Peercoin-Feathercoin Cumulated Returns 38
Figure 2.14-2.15: Primecoin Trend and Cumulated Return.. 39
Figure 2.16-2.17: Ripple Trend and Cumulated Return 40
Figure 2.18-2.19: DOGE Trend and Cumulated Return.. 40
Figure 2.20-2.21: Ethereum Trend and Cumulated Return 41
Figure 2.21: Portfolio Correlations.. 43
Figure 2.22: Cryptocurrencies Correlations 44
Figure 3.1: Efficient Frontier with GMV and MS 54
Figure 3.2: Efficient Frontier with GMV, MS, Intermediate risk aversion portfolios 57
Figure 3.3: Efficient Frontier Unconstrained vs Positivity of weights 58
Figure 5.1: Rolling MaxSharpe Portfolios with Group Constraints, Cumulated Returns 76
Figure 5.2: EWMA MaxSharpe Portfolios with Group Constraints, Cumulated Returns 76
Figure 5.3: Weights Area Plots GMV Portfolios - Rolling 78
Figure 5.4: Weights Area Plots GMV Portfolios - EWMA 78
Figure 5.5: MS Portfolios with Crypto at 20% of weight: Static strategy vs Dynamic strategy 80
Figure 5.6: Area Plots for Relative Risk Contribution, Static Strategy: Rolling 84
Figure 5.7: Area Plots for Weights, Static Strategy: Rolling 85
Figure 5.8: Area Plots for Relative Risk Contribution, Static Strategy: Rolling 87
Figure 1 - Appendix: Markowitz portfolios with Rolling inputs – Static approach 106
Figure 2 - Appendix: Markowitz portfolios with EWMA inputs – Static approach 107
Figure 3 - Appendix: Markowitz portfolios with Rolling inputs – Dynamic approach 108
Figure 4 - Appendix: Markowitz portfolios with EWMA inputs – Dynamic Approach 109
Figure 5 - Appendix: Risk-Budgeting Portfolios Rolling and EWMA– Static approach 110
Figure 6 - Appendix: Risk-Budgeting Portfolios Rolling and EWMA–Dynamic approach 111

Introduction

My master thesis project deals with Financial Technology, also known as "Fintech", a topic assuming more and more relevance over the last few years in the economic field. In particular, inside the wide area of Fintech, the main topic of my work are Cryptocurrencies.

If we adopt strictly a speculative point of view, the new instruments born as a result of Fintech activities have brought plenty of new opportunities to the worldwide investors, both in terms of the possibility to obtain higher returns than what an instrument belonging to a traditional asset class can offer, both in terms of portfolio diversification, which is probably the main concern of every market operator. If the advantages brought by financial innovations seem to be quite consistent, also their drawbacks are worth to be considered in order to have a more precise view about this kind of instruments. First of all, almost all the assets born in the "Fintech" environment are instruments whose nature and whose mechanics are difficult to be understood: for example, if you consider a stock, this is simply a fraction of a company share capital, a bond is instead a fraction of a company debt, a derivative is an option contract with another asset as underlying. However, what does a Cryptocurrency is? Put it differently, is this only an innovative instrument of payment or a new transaction method as it was originally meant to be? I can go further: going back to the stock, its value should reflect the dynamics of a company, its growth prospects and so on. What does instead determine the value of a Cryptocurrency? Actually there is not a precise answer to these questions, I will try to deal with them in the following pages.

Then, another characteristic of this kind of instruments one may be worried about is their high volatility, leading to huge intraday fluctuations, which are responsible of very attractive gains when things go well but also of dangerous losses when things go poorly. The latter may be a serious problem if this kind of instruments are approached mainly by uninformed investors whose choices are driven only by speculative aims: this is precisely what happened in the last months for what concern Cryptocurrencies. Unexperienced investors put their money in Cryptos because they were attracted by the high potential gains made by people who had already invested before them: this dynamic has been even more strengthened when also the media started to spread this sort of "easy-money" opportunity, facilitating their knowledge to that kind of people who only wants to make money without being aware of the huge amount of risk he is taking by investing in such instruments. This is why this kind of assets is very prone to bubbles. The fact that all the people can invest in such instruments with extreme facility is mainly a matter of lack of regulation around this topic.

Following this, what we can conclude is that financial innovations can be quite useful even if they are used as speculative instruments, provided that investors are aware of what they are handling, given the strong risk-return trade-off: “all that glitters is not gold”.

Why Cryptocurrencies?

Personally, I have been interested in this topic from its early stages, when the Bitcoin was the only known Cryptocurrency around. Even if Bitcoin still remains the most important Cryptocurrency, also in terms of volume traded, plenty of new assets have been introduced in the market over time being. There are many interesting and innovative aspects surrounding Cryptocurrencies, for example the virtual process through which they are generated: this is called “mining”, and it is completely virtual. Every person through it can “mine” (generate) fractions of Bitcoin by its own through the mean of adequately powered working stations. Consider for a while the traditional “fiat” currency: this is generated through a legal tender regulated by a central bank, a completely different process with respect to the one through which a Cryptocurrency is generated.

However, in my opinion, the most interesting aspect in absolute related to this environment is the concept of “Blockchain”: basically this is the public ledged, based on a peer to peer technology, where all the transaction in Bitcoin are recorded: one can think of it as a list of “blocks”, linked together and put in chronological order, where each block represents an encrypted transaction. This allows market participants to keep track of the transactions made via that currency without the need of a central authority that guarantees the process. That’s why we refer to crypto as “decentralized currencies”. Blockchain technology has many advantages that makes it suitable to be applied in many fields beyond Fintech: first, as the majority of the decentralized systems, Blockchain is safe, no one until now has succeeded in hacking it. Second, if it is used to record transaction, as it is originally meant to, it almost completely eliminates human error, at the same time protecting the data. For example, one of the most recent development of this system outside the financial field concern the process of voting in government or local elections, significantly diminishing the risk of electoral fraud. We will discuss about these aspects more in deep later, giving also some examples of real world implementation of this technology.

Finally, what we can conclude is that Bitcoin and all the other Cryptocurrencies are meant to become very interesting payment systems in case of a future regulation toward this direction, supported by an innovative and safe technology, simplifying many aspects of the transaction processes. I will make in the following pages a comparison between Cryptocurrencies and Real Money to verify to what extent the former could take the place of the latter.

As explained earlier, things are different when this kind of instruments are used to accomplish speculative aims, and it is precisely for speculative aims that these instruments are known to the public. While at the early stages Cryptocurrencies were handled only by a small fraction of people, it was during 2017 (in particular the last months of the year) that the common people started to be aware of their existence: during this time Bitcoins, but also other Cryptos, experienced a shocking price increase. Newspapers and TVs started to give them notoriety and many investors have been attracted to invest: basically, they have been convinced by the daily positive news that the end-2017 shocking value increase would have continued for the time being. At the same time, many trading platforms, which were originally focused only on assets belonging to the traditional classes, began to allow for the trading of Bitcoins and other Cryptos. Many new exchanges were even created for this purpose. It was precisely at this point that many experts started to talk about a "Crypto Bubble": Cryptocurrencies reached their peak between December 2017 and January 2018, as we will see more in deep later. As a byproduct of this increasing in demand, a lot of new cryptocurrencies have been introduced in the market.

During this period of "irrational exuberance" affecting Cryptocurrencies, as a finance student I asked myself: should this sort of instruments be considered as a traditional asset class, such as stock, bond, commodities? What would have happened if an investor some years ago decided to extend a diversified portfolio made up of traditional assets including also Cryptocurrencies? What about its performance? This has been the starting point of my work.

In the following Chapter 1 I will make a discussion more in deep about the Cryptocurrency environment, starting from the analysis of the historical context from which they come from, concluding with the analysis of the main components of the Bitcoin's technology, trying to extract from them the main sources of innovation that will be useful for many other real worlds applications in the future.

Chapter 1:

An introduction to the Cryptocurrency's world.

1.1. History of cryptocurrencies: E-money and Virtual Currencies

In order to better understand the main focus of my work I think it is worth to do a short brief concerning Cryptocurrency phenomenon, taking a look at the historical context from which they come from.

The one of Cryptocurrency is a relative short history: it has been 20 years since the famous Bill Gates sentence: "Banking is essential, banks are not". These words, even if quite strong, seemed to perfectly reflect the revolution arising in the financial environment during those years, whose main effect was the proliferation in the market of FinTech firms, providing plenty of new financial services: the main objective of the most part of them was the simplification of the payment system trying also to reduce the frictions concerning international transactions, eliminating transaction costs as a byproduct. These frictions were mainly due to the limitation of the so called "Fiat Currency": according to the ECB, Fiat Currency is defined as "any legal tender designated and issued by a central authority that people are willing to accept in exchange for goods and services because it is backed by regulation, and because they trust this central authority"[1]. Put it simply, Fiat currency is the traditional way we are used to think about money, it is the physical money. In order to bypass the limitation generated by the fiat currency, limitations that were mainly related to the spreading of the internet technology and the ability to make online payments, these new firms started to develop alternative systems of payments that exploited innovations such as the "e-Money" or the "Virtual Currencies": the two can be considered to all extent the Cryptocurrency's ancestors. If we take a look at e-Money and Virtual Currencies, we can identify many characteristics that are visible also in Cryptocurrencies. Moreover, by looking at them, we can also realize to what extent they differentiate.

According to Gareth W. Peters et al. in "Trends in cryptocurrencies and blockchain technologies: a monetary theory and regulation perspective" (2015), e-Money can be defined as it follows:

[1] European Central Bank – Eurosystem, "Virtual Currencies Schemes", Chapter 2 pp. 13-18, available at https://www.ecb.europa.eu/pub/pdf/other/virtualcurrencyschemes201210en.pdf

"Electronic Money (e-money) is electronically (including magnetically) stored monetary value, represented by a claim on the issuer, which is issued on receipt of funds for the purpose of making payment transactions, and which is accepted by a person other than the electronic money issuer". Early forms of e-money date to the early 1980s, when David Chaum introduced the concept of some sort of electronic Money feasible to set up transactions in the real world economy: two aspects had to be satisfied: first, the e-Money must emulate physical currency and second, it must respect the privacy feature. Based on these two starting points Chaum developed Digicash, which is known as the first form of e-money. After Digicash many other e-money systems were developed by small venture capital firms. The 1994 EU Report by the Working Group on EU payment systems has been a turning point, since it was the first initial regulatory response to this phenomenon, limiting mainly the privacy feature among the others: after the release of the report three e-Money operators seem to become the leaders in this sector: these were *E-gold*, *Liberty Reserve* and *Paypal*. The first problems arose when authorities discovered that E-gold and Liberty Reserve were mainly used for money laundering and other criminal purposes (which is unfortunately one of the critical points affecting also Cryptocurrencies), leading to the shutdown of these platforms. For what concern the third, the adopted solution was the integration of the Paypal platform into the monetary system without breaking the directives of the central authorities: Paypal has become the leading platform to perform online payments. To sum up, we can say that e-money is not a new form of money, but it is simply a virtual representation of the fiat money.

However, there are also other systems leading in turn to the creation of new currencies, which is nearer to the concept of Cryptocurrency: this is the case of those instruments that fall under the umbrella of "Virtual Currencies". According to the European Central Bank Opinion of October 12, 2016[2], Virtual Currency can be defined as "a digital representation of value that is neither issued by a central bank or a public authority, nor necessarily attached to a fiat currency, but is accepted by natural or legal persons as a mean of payment and can be transferred, stored or traded electronically". According to Peters et al. (2015), virtual currencies are typically a 1990s phenomenon, and they were mainly used as a mean of payment for online messaging platform and virtual gaming environments: their usage was indeed very limited. One of the first virtual currencies introduced was the *Q-coin*, functional to the Chinese *Tencent* online messaging

[2] OPINION OF THE EUROPEAN CENTRAL BANK of 12 October 2016
"on a proposal for a directive of the European Parliament and of the Council amending Directive (EU) 2015/849 on the prevention of the use of the financial system for the purposes of money laundering or terrorist financing and amending Directive 2009/101/EC"

platform. Limited to their virtual scope, Virtual Currencies show a lot of similarities with the Fiat Money: first, both of them are used to purchase goods and services (virtual in case of virtual currencies), moreover, Fiat and Virtual Currencies have in common the presence of a central authority, which is the Central Bank for Fiat and the issuing virtual platform for Virtual Currencies: in both cases the main role of the central authority is money supply management and inflation control. Making now a comparison with e-Money, one aspect that differentiate Virtual Currency from them is the scarce presence in the real economy of the former relative to the latter: according to Peters et Al. (2015) the scarce presence of Virtual Currencies in the real economy seems to be due to the fact that the flow between Fiat and Virtual Currency is unidirectional: you can only use Fiat Currency to purchase Virtual Currency, but you can't use Virtual to purchase Fiat currency. However, the most important distinction between e-Money and Virtual Currencies is that the former can be fully considered as a substitute of Fiat, while Virtual Currency can't, since the latter is only valid for purchases made in the hosting platform and not widespread across the real economy. Finally, while there it exists a precise conversion link (established by the law) regulating the conversion of e-Money in Fiat currency, there is any kind of conversion link governing the Virtual Currency-Fiat Currency swap, this is freely established by the issuing entity.

Now that I have given light to the historical context and to the instruments that can be considered to all extent their ancestors, we can now switch to the analysis of the concept of Cryptocurrencies. Following with the Peters et Al. (2015) analysis, Bitcoin but in general all the Cryptocurrencies are defined as a "Decentralized ledger of transactions"[3]. The key word to distinguish Cryptocurrencies from the other two categories descripted above is "Decentralized": decentralized in the sense that there is no need of a financial intermediary to perform a transaction involving Cryptocurrency, but also in the sense that there is no need of a central authority performing monetary policy. This is the main distinction but also the main innovation with respect to e-Money and Virtual Currency counterparties. Considering for example Virtual Currencies, these are clearly centralized systems: the role of the central authority is performed by the issuing platform, which establishes the monetary policy and the transaction rules, but it also verifies the correctness of the transactions themselves, taking the role of a "verifying third party". The latter role for what concern Cryptocurrencies is taken instead by all the network participants who have a stake in the correct functioning of this business. So Cryptocurrencies can be considered as some sort of *decentralized*

[3] Peters G.W., Panayi E., Chapelle A., "Trends in Cryptocurrencies and blockchain technologies: a monetary and regulation perspective", p.10.

Virtual Currencies. Bringing the distinction further, Peters et al. sum up the breakpoints differentiating Virtual Currencies from Cryptos as it follows:

1) Virtual Currencies characteristics can be modified by the issuing company, while Crypto specifications are agreed by "cryptographic consensus", so there is not a central entity that can alter their properties.
2) Virtual Currency usage is limited to the online platform that originate them, while Crypto are intended for a widespread usage: the main aim is to set up a payment platform for the purchase of real goods and services which is valid worldwide.
3) As already said, virtual currencies are subject to the monetary policy and the control of a central authority, while cryptocurrency's management is decentralized: users control the generating process proportionally to the generating power they give to the system.
4) The link between Virtual currencies and Fiat currency is unidirectional, while the link between Crypto and Fiat is intended to be bi-directional.
5) Different value generation mechanisms.

While it is clear now what are the main elements distinguishing Cryptocurrencies from Virtual Currencies, we can draw also a map of the distinction between Cryptos and their second (more distant) ancestor, which is e-Money.

1) Cryptos generating process is autonomous and it only responds to computational algorithms regulating their supply. On the opposite, e-money generating process is strictly dependent on the underlying fiat currency.
2) E-money business relies on the principle of "know your customer": as already said e-Money firms are subject to a very strict regulation to stay in the market without any issue, and one of the main point of this regulation aim at avoiding that these instruments are used to accomplish criminal purposes. One of the best ways to discourage this kind of behavior is by forcing customers to reveal their identity when operating with these instruments. The same is not true for what concern Cryptos, where one can enter the business without any obligation to reveal his identity, in such a way that the principle of anonymity which is common also to the Virtual currencies is guaranteed.
3) Crypto business (but also the Virtual currency one) is opened only to those people who can dispose of an internet connection, precluding its diffusion mainly in the third world countries. e-Money business relies also on mobile technology systems that make it more

accessible to everyone. Recent developments in the Cryptocurrency industry go in the direction of closing this gap.

As said the original purpose leading to the creation of Cryptocurrencies was the setting up of a new payment system that should coexist in parallel to the traditional payment system which is served by the Fiat Currency as the dominating mean of exchange. Is this possible? What are the main difficulties for the implementation of Cryptocurrencies as a new payment system? What kind of risk such an integration would involve? I will try to deal with this question in the following paragraph, where I will focus on the interaction between Real Money and Cryptocurrencies.

1.2. Cryptocurrencies and Real Money

Even if nowadays Cryptocurrencies are known mainly as speculative instruments, originally they were supposed to accomplish the role of a mean of payment parallel to the traditional one, with the advantage of facilitating cross border online payments and reducing transaction costs. For this reason I would like to examine now the interconnection between the Cryptocurrency environment and the real money world: in particular, I am trying to identify firstly to what extent Cryptocurrencies can be considered as a substitute of the Fiat Money, fulfilling its role in the actual payment systems, and secondly what are the main risks that could arise from this interaction.

For what concern the first point, it is useful to analyze an extract of the Q3 2014 Quarterly Bulletin by the Bank of England titled "The Economies of Digital Currencies": here the comparison between Cryptocurrency and Fiat Currency is assessed following two different guidelines: first, looking at the different process through which these types of asset are generated and spread in the market, second, starting from the identification of the main three functions of the fiat currency, checking if these can be fulfilled also by the Cryptocurrencies.

For what concern the generating process, according to McLeay, Radia and Thomas (2014) the Money may simply be considered as a series of claims, or IOUs. The vast majority of Money is held as bank deposit and the most common way through which new Money is spread in the market is by originating new loans: each time a bank originates a new loan it simultaneously creates a matching deposit in the borrower's bank account, creating in this way new money. On the opposite, if we consider Cryptocurrencies, they cannot be considered properly a claim. If we use their generating process as the discriminating factor to identify one asset class they could belong to, Cryptocurrencies should be included into the commodity family. Obviously we are not talking about a physical commodity asset class, such as gold, oil, copper, since Cryptocurrencies are intangible assets: using the definition by the Bank of England, we can talk about "Digital Commodities"[4]. The most important characteristic all the assets under the name of Digital Commodities have in common is that they do not have any meaning per se, the only meaning they have is the one that all the business participants (including the owners) agree to give them. So the central point is an agreement, which defines not only the characteristics that the asset must satisfy, but also the correct process through which that asset is generated: for what concern Cryptocurrencies the latter is called "Mining", and as we will see more in deep in the following paragraph, it is a completely different process with respect to the legal tender process through which a new fiat currency is issued. Typically, the mining process is a continuous process, but it has a precise deadline, it is time bounded. This is because the vast majority of Cryptocurrencies is meant to exist in limited supply: when the maximum quantity of a specific Crypto is "mined", the process ends automatically. If we consider the Bitcoin, its supply is meant to be limited at 21 million of Bitcoins; according to the forecasts, this will be reached within 2040.

Following the analysis by the Bank of England (2014), we will now try to assess if Cryptocurrency can fulfill the traditional functions accomplished by the Fiat Currency. Traditional economic theory identifies three main functions that are proper of the Fiat:

1. Currency as a **store of value,** through which one can transfer purchasing power from the present to the future (Saving).
2. Money as a **medium of exchange,** through which one can obtain other goods in exchange of it.
3. Money as a **unit of account,** through which one can make measure to value of a given item.

It is not clear to what extent Cryptocurrencies can absorb these attributes. For what concerns the first, the capacity of the Cryptos to store value relies mainly on the people beliefs about future related demand and supply. While the latter should not constitute a problem to be forecasted, given the limited-supply nature of many Cryptos, the story is different if we consider the demand, since at the currently state of things it seems impossible to forecast Crypto's demand trend. Despite these

[4] Robleh A., Barrdear J., Clews R., Southgate J., "The Economics of Digital Currencies", BoE Q3 2014 Bulletin, Definition included in "Digital Currencies versus Fiat Money, how are they created?", p.278.

uncertainties, it is excluded a priori their ability to be a store of value in the short term, and this is clearly due to the extreme volatility affecting these assets, proving that actually Cryptocurrencies are mainly intended as speculative instruments rather than instruments of payment. On the opposite, their capacity to become a store of value in the medium-long term solely depends on the sustainability of their future demand: the main determinants of it will be people beliefs about the future developments of this business. For example, in the case of a worldwide regulation banning Cryptocurrencies from the market, their demand will be immediately nullified.

Moving on to the second function, on a very simplistic way one can measure the capacity of a specific asset to fulfill the role of a medium of exchange by simply measuring the number of sellers in the market who are willing to accept that asset in exchange of their goods: for what concerns Cryptocurrencies, the quantity of retailers who are willing to accept them has been quite consistent until now. The problem with this approach is that the willingness of retailers to accept Cryptocurrencies to be paid does not mean that Cryptocurrencies are indeed widely used. A more accurate approach measures instead the overall quantity of transactions involving the mean of payment under examination over a given period of time. Following this approach, results show that very few transactions have been conducted through the mean of Cryptocurrencies (in the case under analysis Bitcoins), and this seems to reject their ability to fulfill the role of a medium of exchange. This result seems to be counterintuitive if considered together with the evidence that the volume of Bitcoin traded over the same time period have increased significantly. One possible explanation of this phenomenon could be that people purchase Bitcoin not to use them for day by day transactions, but simply to hold them, hoping to sell them later in the future at a higher price or to obtain some sort of diversification benefit: even this consideration seems to confirm that what matters now is mainly the speculative trait. Finally, similarly to the two previous cases, there is little evidence about the ability of Cryptocurrencies to be used as a unit of account. This result stems from the fact that it is very unlikely that two parties agree a price in Bitcoin terms during a transaction: they typically agree it in Euro, or in Dollars, and they will eventually make the transaction in Bitcoin. This evidence seems to be related to the huge volatility affecting these instruments, which would lead to very frequent and annoying price adjustments.

The considerations made in the bulletin by the Bank of England (2014) seem to reject the possibility for Cryptocurrencies to substitute Fiat Currency in the traditional payment system, given the difficulties to fulfill the three main roles that are proper of the Fiat currency itself.

Moreover, in case of an eventual implementation as real world currency, a monetary model for Cryptos would be needed: there is plenty of theoretical solutions proposed by the experts until now, but none of them have been satisfactory and an agreed consensus has not been reached yet. For example, one of the main problems is related to the limited supply nature of many Cryptocurrencies: what does this mean in practice? This simply means the risk of deflation: a satisfactory way to deal with it has not been found yet. Then, monetary models concerning Cryptos should take into account not only of those assets taken per se, but also of their interaction with the established monetary system. Interaction that can be fruitful to some extent but which can also involve different kind of risks: in order to better understand this step, I think it is useful to classify the risks that could arise from this interaction in four different areas according to the analysis by Sauer, "Virtual Currencies, the Money Market, and Monetary Policy", 2016, in compliance with studies made by the ECB and the Bank of England. As a result of their studies, they found out four main sources of risk that could arise from the interaction between an eventual Cryptocurrency inclusion in the monetary system and the monetary system itself. These are:

1) **Risks to price stability**

An entrance of the Cryptocurrencies into the traditional monetary system could endanger price stability for different reasons: First, it affects the *quantity of money* in circulation: once introduced into the monetary supply Cryptos crowd out partially the demand of national currency, so the Central Bank should adapt traditional money supply to the reduced demand. However, the supply of Cryptocurrencies is expected to continue its increasing, and consequently demand of national currency will be even more under pressure in the future. Secondly, according to Sauer, the introduction of Cryptocurrencies may affect the *velocity of money*: as the name suggests, the ability to influence velocity of money depends on how quickly central bank reacts to the changes in cash demand. With additional currencies in circulation other than the established fiat currency, money demand becomes less predictable and decisions by the Central Bank become more complicated. Moreover, the reduced use of the cash for sure will generate a contraction in the Central Bank balance sheet, which in turn will diminish the ability of the Central Bank to influence short term interest rates. Finally, Cryptos could add an unpredictable component to the real economic indicators. However, at the current state of things their expansion is too limited to affect them, but if we consider that Cryptos' environment for sure is going to expand in the future, we could end up in a situation where they will indeed be able to generate real economic effects.

2) Risk to Financial Stability and Payment System Stability

Results of the studies conducted by the ECB and the Bank of England show that Cryptocurrencies could constitute a serious threat to financial stability, even more if in the future these will reach the ability to influence people expectations. But why they could affect financial stability? If we consider that the exchange rate regime linking Cryptos to the traditional Fiat currencies is flexible, we should observe a low level of volatility and relatively stable currencies, given that flexible exchange rate regimes are known as an optimal dampening or absorption for external shocks, and this would be good for the economy as a whole. The problem is that this is not the case for Cryptocurrencies, because these instruments are affected by a very high level of volatility: the reason behind this volatility is always the same, speculation. Every time the market is affected by high volatility also financial stability is in danger, so that's why Cryptocurrencies could become a serious threat: nowadays they are one of the biggest source of volatility among all the other financial instruments in the market. While concerns are limited at the actual state of things, since Cryptos still represent a relatively small phenomenon, if we consider that probably the expansion of this business will continue over time being, the menace to the stability will be more and more visible if the speculation trait won't be restrained.

Moreover, another important aspect that could contribute to instability is that, differently from the fiat currency, Cryptocurrencies do not have a trustworthy central authority that control for them. As already said, Cryptocurrencies are decentralized systems where the control function is exerted by the other network participants, so we can say that the trust resides in the efficiency and the safety of this network and of the technology behind it. If this aspect could be positive to some extent, the drawback is that such a system is prone to fraud, because the participants have an incentive to intervene and cheat, for example by redirecting transactions occurred in Bitcoins toward their accounts. Even if this possibility is remote, due to the high level of encryption of the Blockchain technology and to the huge amount of computational power an eventual hack would require, this does not automatically exclude the possibility that someone could find a way to cheat the system in the future. In such circumstances a crash would occur.

3) Lack of Regulation

While the traditional Fiat Currency is subject to a very strict and well defined regulation, limiting many aspects of its existence, we cannot say the same for what concern Cryptocurrencies: in this case both supervision authorities and financial market regulators have not been able not only to regulate this phenomenon, but also to precisely state what is the essence of it. Srokosz W. and Kopysciansky T., and in "Legal and economic analysis of the Cryptocurrencies impact on the financial system stability" (2015), states that according to the civil law Cryptocurrencies can be defined as a "measure of value other than money"[5]. According to Srokosz and Kopysciansky analysis, this definition corresponds to the "perception of Cryptocurrencies as an abstract measure of value, which is the monetary unit"[6]. But this is not sufficient, because the introduction of the Cryptos in the monetary system would require also a new concept of "property right", in fact we wouldn't be wrong if we considered Cryptocurrencies themselves as a new type of property. Given the digital nature of this kind of assets, the property right concerns something that is not tangible: it is simply a record in the public ledger of transactions (Blockchain) where all the transactions in terms of a given Cryptocurrency are recorded. In case of an integration into the established monetary system, a new concept of property right would be needed in order to avoid misunderstandings in case for example of judicial controversies.

4) Reputational Risk

The reference authority for what concern the monetary environment is for sure the Central Bank. If you consider the Central Banking activity, the reputational aspect is fundamental, provided that only a credible authority is able to influence people expectation, which is in turn the main requirement for the effectiveness of a given policy. Without any doubt we can say that the reputation and the credibility of the central authority lie on the basis of the correct functioning and the stability of the financial system. Suppose that the integration process occurs and Cryptocurrencies become to all extent a monetary phenomenon: given the monetary nature of this business, people would be induced to think that Cryptocurrency business fall under the responsibility of the Central Bank, even if this is not the case. This implies that in case of problems such as frauds or crashes affecting the Cryptocurrency systems, people would blame the central

[5] Srokosz W., Kopysciansky T., "Legal and Economic Analysis of the Cryptocurrencies Impact on the Financial System Stability", p.2.

[6] Srokosz W., Kopysciansky T., "Legal and Economic Analysis of the Cryptocurrencies Impact on the Financial System Stability", p.2.

bank as responsible of these facts, and this would affect negatively the Central Bank reputational aspect: put it simply, people would lose confidence towards the Central Bank due to something that is not related to the Central Bank itself. However, as already mentioned, trust towards the Central Bank plays a fundamental role for the correct functioning of its policy measures. So what we can conclude is that problems in the Cryptocurrency system could indirectly affect negatively the role of the Central Bank in the financial system, and this would be true even if the hypothetical share of Cryptocurrencies among the entire money market remains substantially small.

What we can conclude from this examination is that the road to the integration of Cryptocurrencies into the monetary system is long and full of handicaps, and at the actual state of thing it is very unlikely that authorities will be able to set up a transaction system where Cryptocurrencies and Fiat Money coexists. Maybe a solution will be found in the middle, allowing the usage of Cryptocurrencies in the real world economy but restricting the same usage only to some specific scopes, in such a way to limit the interference with the established payment system. In this way Cryptos could be seen as a complement rather than a substitute of the fiat currency.

While it seems that they have failed to reach the original aim of constituting a payment system simplification, with an associate reduction of the frictions concerning international transactions and also transaction costs, nowadays Cryptocurrencies seems to fit very well the role of speculative instruments, and as speculative instruments I will treat them for the rest of my work. But before starting with the analysis I would like to spend a few words about the Bitcoin, which is the most important cryptocurrency introduced in the market and also the oldest one. The fact is that when we talk about Cryptocurrencies the innovation aspect is not only limited to the concept of Cryptocurrency itself, or to the functions for which this kind of instruments have been thought, but there is plenty of other developments whose common denominator is the technology linked to the Cryptocurrency's environment. The point is that this technology is supposed to find in the future many others uses that have little to do with the Cryptocurrencies world, so from this point of view these instruments can be seen as a good starting point for many future innovations which originate from the same technology linked to their implementation.

1.3. Bitcoin Innovations

In order to understand where the main innovations concerning Bitcoin technology resides, I think it is useful to decouple Bitcoin in its three main technical components, according to the analysis by

J.Bonneau et al. "SoK: Research Perspectives and Challenges for Bitcoin and Cryptocurrencies", (2015), which is focused precisely on Bitcoins. According to it, Bitcoin system has three main components:

1) **Transaction and scripts**: currently, if one asks what a Bitcoin is, the most accurate answer would be that it is a series of messages called transactions. A transaction contains an arrays of inputs and an arrays of outputs, where the latter includes an integer value representing the amount of Bitcoin exchanged through that transaction. The further one unit of currency can be divided into smaller units, the more precise that value will be. For what concerns Bitcoins, the smallest unit is called "Satoshi", and it corresponds to 10^{-8} Bitcoins. Besides the number representing the value of the transaction, each output includes also a code named "ScriptPubKey": this simply represents the code which is necessary to redeem that transaction (it is the "key" necessary to unlock the transaction). In other words, a given transaction will end successfully only if the following transaction contains the same ScriptPubKey as an input, constituting some sort of chain. Transaction inputs in turn include a code named "ScriptSig" which redeem the transaction output. Considering this, Bonneau et al. (2015) drew two conditions to be satisfied for the correct execution of a Bitcoin transaction: first, every input must match a previous transaction output, second, the two scripts "ScriptPubKey" and "ScriptSig" must execute successfully. The way this system works leads to a more precise concept of *ownership* with respect to the very abstract one based on the findings by Srokosz and Kopyscianski (2015) about which I have written before[7]. Here *Ownership* can be defined as the property of the private key which is necessary to redeem a given transaction: an individual owns as many Bitcoins as its private key is able to redeem. To conclude, the innovation we can keep from this first component of the Bitcoin technology is this new concept of ownership arising from the payment system.
2) **Consensus and Mining**: following with the analysis by Bonneau et al. (2015), this is the second relevant component of the Cryptocurrency technology. One of the main problems that could affect the system as depicted in point 1) is that there is nothing to prevent that transactions can be redeemed multiple times: in other words, one user could use the same transaction input twice to unlock two different transactions, each of them appearing to be

[7] See "1.3 Cryptocurrencies and Real Money", p.15.

valid in isolation. If this was true the system would be prone to the so called *"double spending attacks"*. For this reason, all the transactions in Bitcoin are recorded in a "global, permanent transaction log and any individual transaction output may only be redeemed in one subsequent transaction"[8]. Following this, another requirement is added to ensure the validity of the transaction, besides the two written above: to be considered valid, the transaction must have been successfully published in this ledger. The latter is called "Blockchain", and it can be considered to all extent the main innovation that the Cryptocurrency environment has brought. Blockchain can be simply seen as a list of blocks, where each block represents a transaction which contains the necessary script to redeem the previous transaction, following the input-output mechanism as written in the previous point. One assumption necessary for its correct functioning is that all the users trust the content of this ledger, so that a global consensus is built around the existing Blockchain, preventing the birth of new Blockchains that are not trusted. How to ensure people trust towards the original Blockchain? One way to do it would be the institution of a central authority that guarantee the consensus. However, this solution would not be desirable since the decentralization is one of the strength points of the Cryptocurrency business. In order to solve this problem the concept of *Nakamoto consensus* has been introduced. How does Nakamoto consensus work? The main ingredient is the commonly called *"proof of work"*, which is simply a computational puzzle: its main function is to determine the following valid block to continue the Blockchain. How is this next block determined? According to Bonneau et al. (2015), the first valid block produced by the network participants containing the solution of the computational puzzle is considered correct. If the solution provided is invalid all the other network participants have to reject it (decentralization) and apply to find a new one. What does it happen instead if two valid solutions are found at the same time? This is what we call *temporary fork:* the existing ledger divides into two separate branches: one of them is supposed to be discarded as soon as all the network participants converge on the other (the longer one), so that the system follows along a unique path. Elsewhere, a double spending attack would occur because one could use the same script to redeem transactions in both branches.

[8] Bonneau J., Miller A., Clark J., Narayanan A., Kroll J.A., Felten E.W., "SoK: Research Perspectives and Challenges for Bitcoin and Cryptocurrencies, p.106.

Why this second component of the Bitcoin technology is so important? As said before, because Blockchain is suitable for many applications outside the cryptocurrency environment, bringing innovation to many other processes that before its introduction were done following different procedures. In order to provide concrete examples on how this new technology is penetrating the real world I will follow in the next rows the analysis by Bocek T. and Stiller B., "Smart Contracts – Blockchains in the wings" (2017) which provides many real applications for the Blockchain. For what concern the financial area they provide the example of *CargoChain*™, a new procedure based on the Blockchain technology whose main aim is the reduction in the usage of paperwork, such as invoices, bills, custom documentation, and authenticity certifications. Besides the financial sector, Bocek and Stiller report many other areas where the Blockchain technology is finding new implementations: among them Fraud Detection, with *Everledger*™, *Blockverify*™, *Verisart*™, Global Rights Databases, with *Mediachain*™ and *Monegraph*™, Identity Management with *Blockstack*™, *UniquID*™ and *ShoCard*™, Ridesharing with *LaZooz*™ and *Arcade*™, Document Verification with *Tierion*™ and *Factom*™. These are the main fields where the Blockchain found application until now, but there are many others where the work is still in progress, expected to make their appareance in the following years.

3) **Peer to Peer Communcation Network**: according to Bonneau et al. (2015) this is the third and final component of the Bitcoin's technology worth to be considered, and probably the less innovative among the three. It is simply a decentralized peer to peer communication network used to announce the new transactions and the proposed blocks to be validated in order to continue the ledger. The characteristics that the system must satisfy are firstly the speed of execution, because any crash, even if small and temporary, increases significantly the probability of a temporary fork. Secondly, as before, decentralization, because if one participant was able to control the majority of the network, he could increase the probability of his own blocks winning an existing computational puzzle, enhancing in this way his mining rewards.

After having done this brief discussion about the cryptocurrencies environment, it is now the time to go back to the original scope of my work: as I already mentioned in the previous rows, despite the many interesting innovation brought by this new technology, the main role in the economy of

Cryptocurrencies at the actual state of things is the one of speculative instruments. For this reason, in my work you will find Cryptos included in a diversified portfolio together with different kind of traditional assets. Put it in a simplistic way, what I did in my analysis was tracking the performance first of portfolios without cryptocurrencies, and then of the same portfolios including also Cryptocurrencies. Finally, I compared the two things to assess if Cryptocurrency could have brought some sort of benefits during the period under analysis. Clearly, the analysis of the performances have been conducted following different methodologies, but I will discuss about this later. However, I remark that this has nothing to do with Cryptocurrencies intended as instrument of payment, since I focused only on their speculative trait.

In the first section of the following chapter I will discuss about my dataset, reporting the portfolios subject of the analysis and their composition, giving also light to my investment choices. Then, in the second section, I will show the result of the descriptive analysis of returns performed for each asset class included in the portfolios, trying to identify what are the main events driving the assets' performances.

Chapter 2

Dataset and Descriptive Analysis of Returns

2.1. The Base Portfolio Composition.

My work started from a portfolio of traditional assets (indexes), beginning from those with the lowest level of risk (Government Bonds), arriving to those with the highest (Equity Indexes). My aim was to construct a solid portfolio with the highest level of diversification achievable, both in terms of the type of asset classes included, but also into the asset classes themselves: for this reason I included four different asset classes, each of them in turn comprehending at least three indexes: Government Bonds, Corporate Bonds, Equities, Commodities. The final result has been an 18 assets portfolio. I choose the provider of the indexes according to the availability of the data, and selecting the ones that better fit my interests: in particular, I used data by Citigroup for what concern Bonds and data by S&P for equities and commodities. You can see the final portfolio composition in Table 2.1:

	Index Name	Asset Class	Provider	Starting point
1	**Citi US Government Bond all maturities**	Government Bonds	Citigroup	05/05/2008
2	**Citi US Government Bond Index 5-7 year**	Government Bonds	Citigroup	05/05/2008
3	**Citi US Government Bond Index 15+ year**	Government Bonds	Citigroup	05/05/2008
4	**Citi US Banking**	Corporate Bonds	Citigroup	05/05/2008
5	**Citi US Utility**	Corporate Bonds	Citigroup	05/05/2008
6	**Citi US Industrial Energy**	Corporate Bonds	Citigroup	05/05/2008
7	**Citi US Health Care**	Corporate Bonds	Citigroup	05/05/2008
8	**Citi US Info Tech**	Corporate Bonds	Citigroup	05/05/2008
9	**S&P500 Banks**	Equity	S&P	05/05/2008
10	**S&P500 Energy**	Equity	S&P	05/05/2008
11	**S&P500 Utilities**	Equity	S&P	05/05/2008
12	**S&P500 ConsDur**	Equity	S&P	05/05/2008
13	**S&P500 Semicond**	Equity	S&P	05/05/2008
14	**S&P GSCI Energy**	Commodities	S&P	05/05/2008
15	**S&P GSCI Precious Metal**	Commodities	S&P	05/05/2008
16	**S&P GSCI Ind Metals**	Commodities	S&P	05/05/2008
17	**S&P GSCI Gold**	Commodities	S&P	05/05/2008
18	**S&P GSCI Crude Oil**	Commodities	S&P	05/05/2008

Table 2.1: *Traditional Assets*

For what concern the traditional assets, all the data have been downloaded from the Thomson Reuters Eikon platform.

An important aspect to be remarked is that this is a US based portfolio with dollar denominated assets: the choice has been made mainly for two reasons: first, I wanted to limit the geographical risk only to events affecting the US market, which is the most important worldwide financial market. Second, I wanted my analysis to abstract from currency risk. The latter would have entered if I had included in my portfolio indexes concerning for example the Euro or the Japanese economy. Last but not least, Cryptocurrencies Exchange rates are commonly expressed versus US dollars.

I would like now to spend a few words about the assets chosen. For what concern Government bonds, I considered the best choice to include an index for the overall maturities, then one specific for the medium term (5-7 years) and finally one for the long term (15+ years), covering in this way all the time horizon. Moving to the next risk level we have Corporate Bonds: I tried to select indexes based on completely different sectors of the economy, from Banks to Info Tech passing through Health care: this has been clearly done in order to reduce as much as possible the correlation inside the asset class. Moving up to Equities, I choose sectorial indexes constructed based on the S&P500 index. Even in this case I tried to reach the highest possible degree of diversification. Moreover, even if the provider in this case is different than the one for Corporate Bond indexes (Citigroup), I tried to look for equity indexes which replicate the sectors included in the Corporate Bond asset class: as you can see, Banking, Utilities and Energy are sectors in common between the two asset classes. Finally, for what concerns commodities, I selected initially three general indexes (Energy, Precious Metals, Industrial Metals), then I decided to go more in deep including in the portfolio even the Crude Oil index (that should show some correlation with the energy index) and Gold Index (which should correlate in turn with Precious Metals). All the commodity indexes are GSCI (Goldman Sachs Commodities Index), which are the Standard and Poor reference indexes for what concern the commodity market (as the name suggest GSCI indexes were originally set up by Goldman Sachs but after some time they were purchased by Standard and Poors in 2007). I will now move to the Descriptive analysis of returns for what concern the base portfolio.

2.2. Base Portfolio - Descriptive analysis of returns.

2.2.1. Government Bonds.

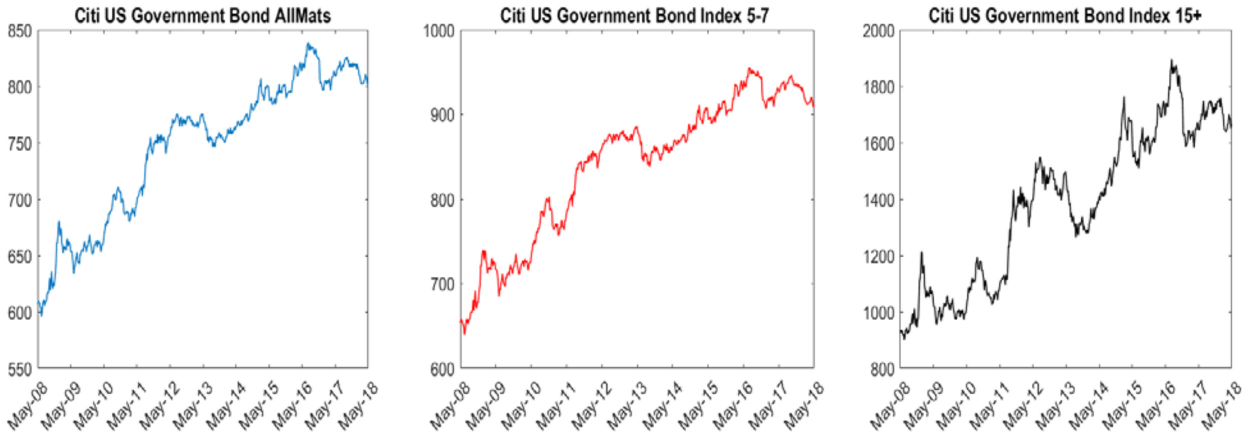

***Figure 2.1**: Government Bond Index trend*

As one could expect, Government Bonds shows a small level of volatility: the smoother among the three seems to be the All Maturities index: this is obvious since it is the most diversified one, given that it is constructed combining Government Bond index for all maturities, and small maturities indexes have a very low level of volatility.

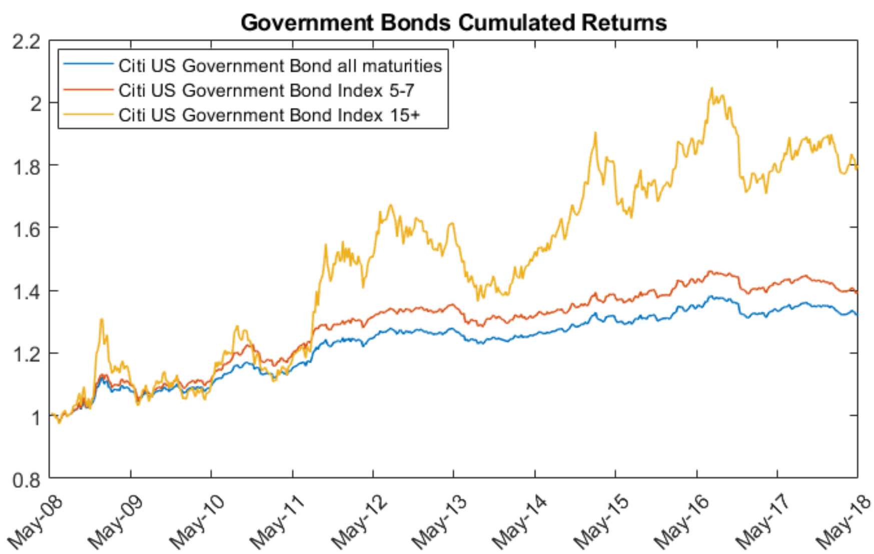

***Figure 2.2**: Government Bond Cumulated returns*

Figure 2.2 shows the cumulated returns of the three Government bond indices. Basically, this tells us how much your wealth would be at the end of the period (May-18) if you invested 1$ worth of wealth at the beginning of the period (May 2008) for every index. Clearly wealth is much more volatile if one invest in the longer maturity index (graphically spikes are much more pronounced).

2.2.2. Corporate Bonds.

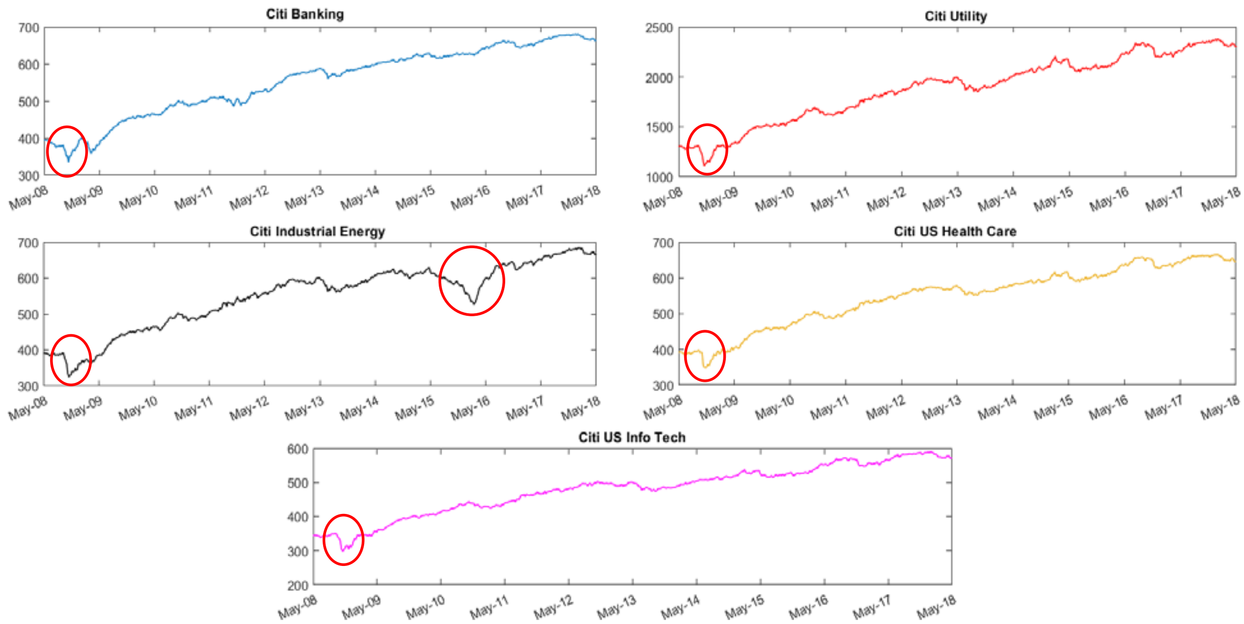

Figure 2.3: *Corporate Bond trends*

All the Corporate Bond indexes show a clear positive trend over the last ten years. One thing that could be noticed is a downward spike between May 2008 and May 2009 (red circle). The rest of the trend is comparable among these assets, a part for a strong downward spike affecting Industrial Energy at the beginning of 2016 (blue circle).

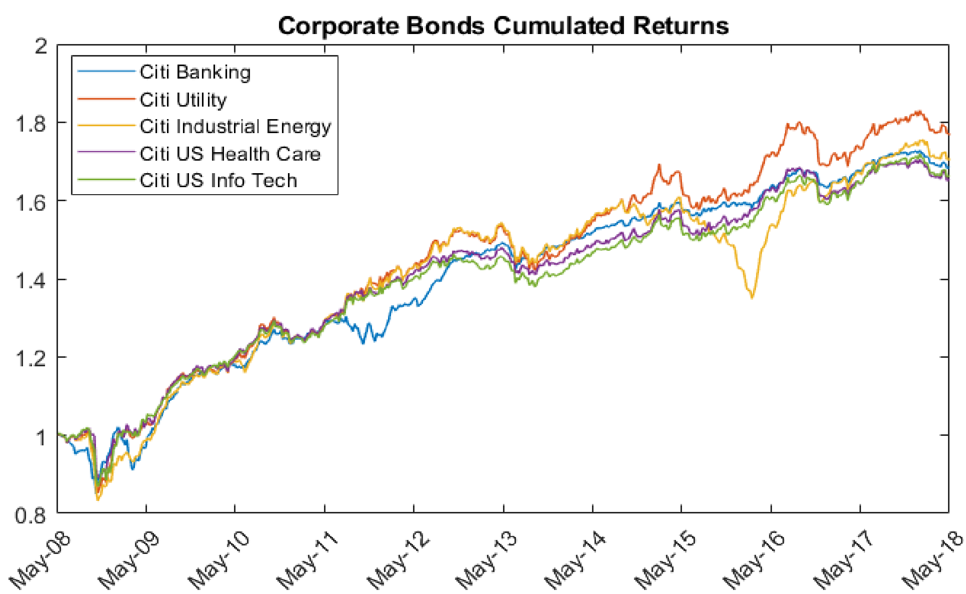

Figure 2.4*: Corporate Bond Cumulated returns*

Figure 2.4 shows us the comparison between corporate bond indexes' cumulated returns: these are very similar, for some periods of time they are almost completely overlying.

2.2.3. Equities.

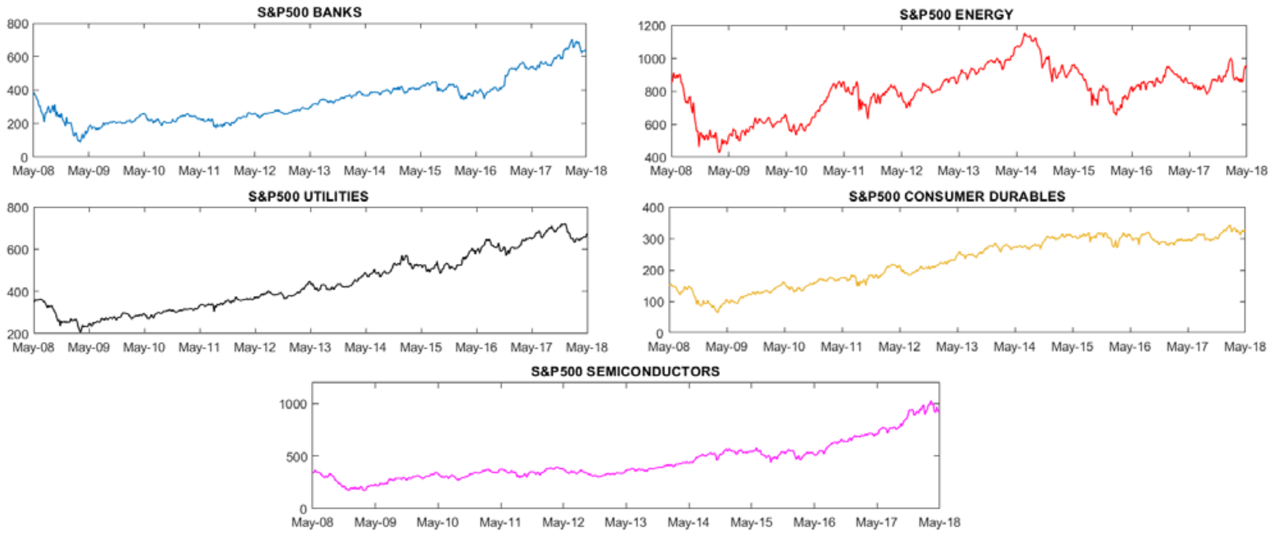

Figure 2.5: *Equities trends*

From Figure 2.5 we can see that the trend is positive in all the sectors, but volatility is slightly different among them: Consumer Durables is almost flat compared to the others, ranging from $100 to $400. Even Banks and Utilities are quite stable, ranging from $200 to $800. These are indeed "Value Sectors". Evidence is a little different for what concern Semiconductors and Energy. The first one ranges from $400 to $1100 and the trend is clearly positive, meaning that this sector experienced very strong growth over the last decade. Indeed this is a "growth" sector. Energy ranges from $400 to $1200 but shows much more volatility than semiconductors, even if the trend seems to be positive also in this case.

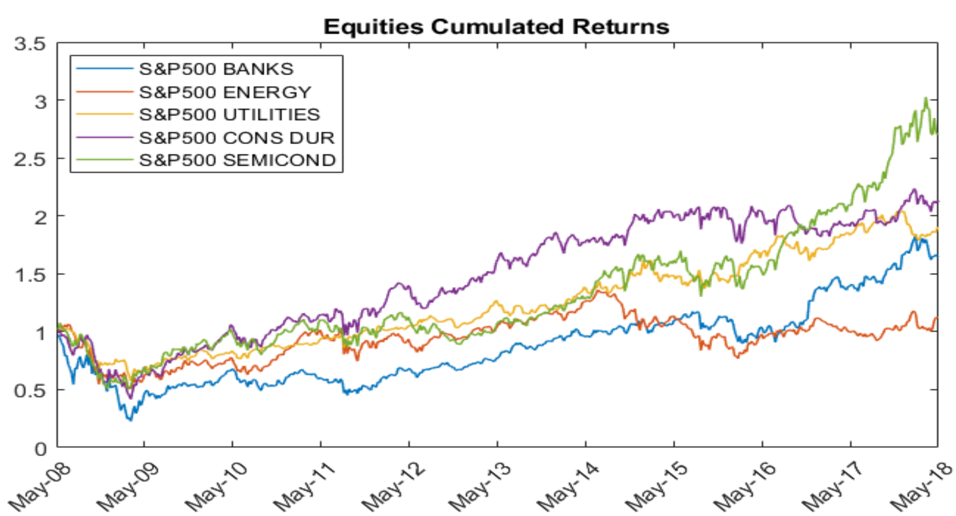

Figure 2.6: Equities Cumulated returns

Energy index cumulated return is almost flat, meaning if you had invested in this index your wealth would not have increased. All the others increased, with substantial difference among each other.

2.2.4. Commodities.

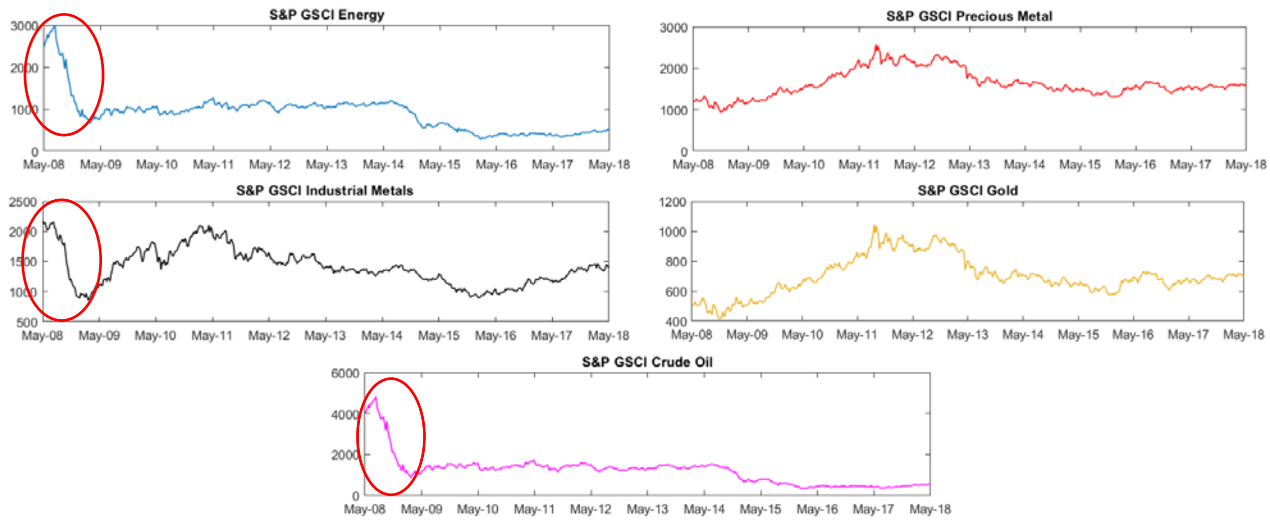

Figure 2.7*: Commodities trends*

Commodities assets can be dividend in two groups according to their trends: on one side there are Precious Metals and Gold, whose trend is very similar. On the other side there are Energy, Industrial Metals and Crude Oil, all of them showing a similar path characterized by a huge declining in the first year (red circle) followed by a very volatile but flat trend.

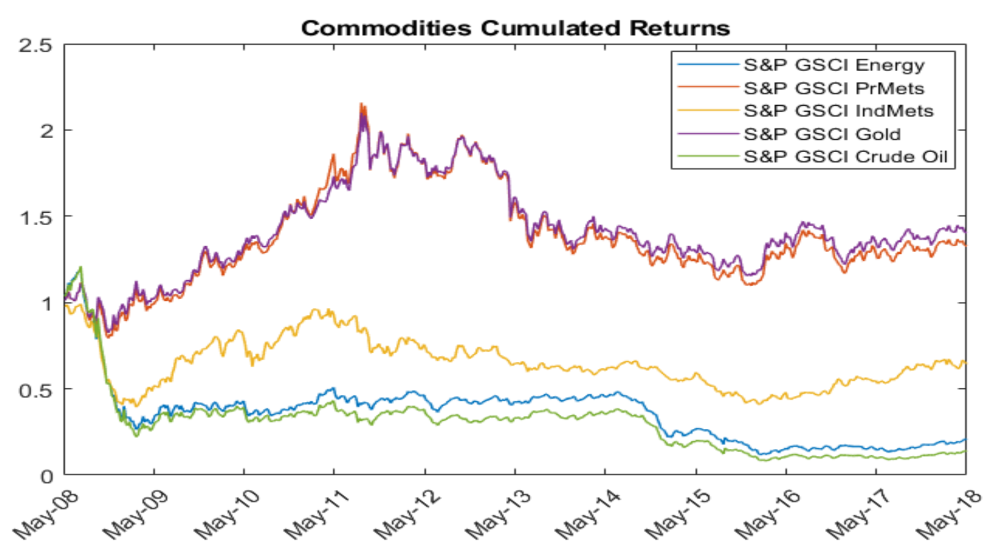

Figure 2.8: *Commodities Cumulated Returns*

Even Cumulated Returns (Figure 2.8) confirm the scenario depicted from Figure 2.7: Gold and Precious metals cumulated returns are almost equal for all the time being. Then, there is similarity between Crude Oil and Energy indexes Cumulated Returns, as one could expect, leading to a loss at the end of the period. Industrial metals index lies between the two groups, leading to a smaller loss at the end of the period than an investment in Crude Oil or Energy index. However, Industrail

Metals Cumulated Return is more similar to Oil and Energy index rather than to Gold and precious metals, particularly in the first year of negative returns. The situation in terms of returns is quite negative for what concern commodities. However, I decided to include them anyway for the diversification benefit they could bring to my portfolio.

2.3. The Cryptocurrency's Portfolios

If the starting point of my analysis is the base portfolio made of traditional assets as descripted in the previous pages, the additional part of the portfolio is made up of cryptocurrencies, which are the focus of my work. In particular, I wanted to assess how the performance of the base portfolio changes if at a certain point in time this kind of new asset enter the portfolio itself, both in terms of returns and volatility. As I will discuss more in deep later, I conducted this analysis following two different approaches: the easiest one, call it the "Static approach", involves only five cryptocurrencies, which are the ones with the longest available historical time series of prices (5 years). The second, most sophisticated one, is called "Dynamic approach", and it involves nine cryptocurrencies. I had to make this distinction because some cryptocurrencies were introduced only very recently in the market (for example Ethereum), and if I followed only the first approach they would have been excluded from the analysis, even if they definitely deserve to enter it.

When dealing with an activity of portfolio construction, the first thing you may do is selecting the assets that are better suitable to take part of your basket. As already told, when constructing the base part of the portfolio made up of traditional assets my main principle to be followed was diversification, both in terms of asset classes included (four) both within the asset classes (five assets for each class). For what concern Cryptocurrencies, it does not make sense to speak about diversification: these are assets whose fundamentals (if they exist) are not clear, and I cannot exclude the possibility that their level of correlation is high. For this reason, I followed different principles other than diversification to select the cryptocurrencies to be included.

First, I picked up only those assets that have a minimum degree of knowledge by the financial environment: almost every day over the last period a new type of Crypto has been spread in the market, but finally only a few of these will survive for a reasonable number of years, while all the others are destined to expire. For this reason, I tried to select only those that in my opinion will have more chances to survive, looking to parameters such as their history, their provider, or even the quality of information available in the internet about their regard.

Second, I looked at the volume traded: the amount of Cryptocurrencies in circulation is huge, but plenty of them are traded in very small quantities or even not traded at all. In other words, a market for them does not exist. It does not make sense to include such assets on the board, their prices don't have a meaning: for this reason, I decided to include only those cryptocurrencies whose volume traded is reasonable, assuring that a market for that asset exists. The price of all the crypto included in my analysis is not just a number, but it is meaningful according to their demand and supply.

Finally, I considered the availability of the data: constructing the cryptocurrencies dataset has been quite a hard challenge for me: if I exclude Bitcoin, whose time series is provided by Blockchain.org, which is a trusted source as the issuer of this instrument, for what concern all the other Crypto there is not a unique trusted data provider as it happens for traditional assets classes. In particular, historical time series of prices for the traditional asset classes indexes can be found on platforms such as Thomson Reuters Eikon, Bloomberg or Factset, which are all trusted sources (personally I used Eikon as data provider for the traditional assets). However, for what concern cryptocurrencies, there it exists a plenty of private web sources which make their time series available. My data provider of reference was the website "coinmarketcap.net", because I considered it the most reliable and complete source among all the others. The problem was that using a unique data provider for Cryptocurrencies has not been sufficient. This was due to the fact that the daily time series downloaded from these web sources suffered for some problems such as missing data on specific days along the series. This made it necessary to take into consideration also other data sources to replace missing data and finally check if the integration was consistent.

The construction of the Crypto dataset required much time: however, I can say that it was worth it, because the result was a very satisfactory and reliable Cryptocurrency dataset. The latter has been structured as shown in Table 2.2, according to the two different approaches outlined below.

	Cryptocurrency	Starting Date
1	Bitcoin BTC	19/07/2010
2	Namecoin NMC	06/05/2013
3	Litecoin LTC	06/05/2013
4	Peercoin PPC	06/05/2013
5	Feathercoin FTC	06/05/2013
6	Primecoin XPM	15/07/2013
7	Ripple XRP	05/08/2013
8	Dogecoin DOGE	16/12/2013
9	Ethereum ETH	10/08/2015

***Table 2.2**: Cryptocurrencies dataset*

The Static approach involves only the first five Cryptocurrencies of Table 2.2 beyond the base portfolio assets. Here Cryptocurrencies enter the dataset all together from Monday 6, May 2013 (even if Bitcoin time series is available from 2010). The second (more sophisticated) "Dynamic" approach involves all the nine Cryptocurrencies, each of them entering the dataset from the date from which they are available. Bitcoin, for example, entered the portfolio on Monday 19, July 2010, Namecoins, Litecoins, Peercoins, Feathercoins from Monday 6, May 2013, Primecoin from Monday 15, July 2013 and so on. This dynamic approach, besides allowing the inclusion of more Crypto than the first one, allows also for more flexibility, because none of the time series has been cut to conduct the analysis.

I will now move to the descriptive analysis of returns of the Cryptocurrency asset class.

2.4. Cryptocurrencies - Descriptive Analysis of Returns

For what concern Cryptocurrencies, descriptive analysis of returns has been conducted sorting them in different groups. First, I isolated Bitcoin, since this the most important and known Crypto among all the others, but also because this is the asset whose time series of prices start before all the others. I can anticipate that Cryptocurrency exchange rates, as we will see from the charts below, are very similar to each other, differing only for the magnitude of the movements, suggesting a high level of correlation among them. I will test for correlation at the end of this chapter after the descriptive analysis.

2.4.1. Bitcoin

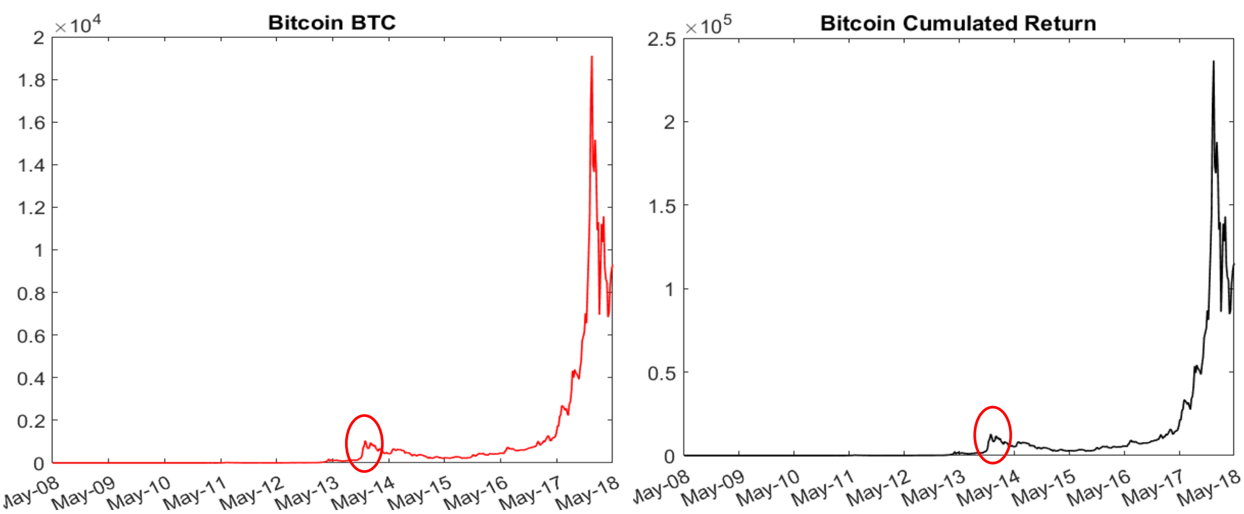

Figure 2.9 - Figure 2.10: *Bitcoin Trend and Cumulated Return*

The first thing that should be noticed from Figure 2.9 and 2.10 is the scale of the Y axis: this is huge. Bitcoin/USD exchange rate have reached levels that are probably unthinkable for even the riskiest traditional asset class in the market: if one had invested a dollar in Bitcoin as soon as they entered the market in 2010, at the end of the year 2017 he would have seen his investment worth almost 230.000$ (computed when the asset reached its peak on December 2017). The second thing we can notice is that the pattern of the exchange rate can be splitted in two parts along the X axis (that represents time). The first one goes from the starting date July 19, 2010 (which in the graphs appear to be zero because during the first years of its existence Bitcoin's value was indeed near zero $) to the first half of the year 2017. This period of time is characterized by an initial jump between May 2013 and May 2014 (red circle): according to Gandal N., Hamrick JT., Moore T., Oberman T., in *"Price Manipulation in the Bitcoin Ecosystem"* (2017), this jump is probably linked with the suspicious trading activity happening in the Mt.Gox cryptocurrency exchange[9]. Basically, the suspicious activity was mostly performed by two bots, called Markus and Willy, which used Bitcoins that they didn't own to perform what they appeared to be valid trades. These fraudulent transactions were included in the overall trading volume by the Mt.Gox exchange, showing a higher than normal amount of trading activity and thus prompting the public to begin trading legitimately. This demonstrates us how much these unregulated Cryptocurrency markets are prone to manipulation. After this jump, BTC/USD seems to follow a decreasing trend until the first half of 2015, becoming then slightly positive until 2017. The second part starts from the second half of 2017: this is probably the most interesting part characterized by a huge increasing that lasts until the end of the year 2017, when Bitcoin reached their highest peak. This is followed by a sharp declining over the beginning of 2018, then a rapid and strong recovering occurred soon followed by a new decreasing. Besides the huge amount of volatility characterizing Bitcoin over the last year, another thing that can be noticed is that this pattern refers very closely to the one of a bubble. Is this indeed a bubble? I will try to clarify this aspect later on through a very interesting article by the International Monetary Fund. However, in this section my main interest is to concentrate on what are the main determinants of this pattern. Even under these circumstances, manipulation plays an important role as it already happened in 2013, but it is not sufficient alone to explain what happened between 2017 and 2018. In the latter case there is also the "Euphoria" component that must be taken into account. For what concern the manipulation aspect, the most known attempt to

[9] Gandal N., Hamrick JT., Moore T., Oberman T., "Price Manipulation in the Bitcoin Ecosystem", Workshop of Economics of Information Security (WEIS) 2017.

manipulate Bitcoin price in 2017 was made by an anonymous actor called "Spoofy" that manipulated the Bitcoin markets by "spoofing" (bidding or offering with the intent to cancel the trades before the execution) large amounts of Bitcoins. Through this practice, Spoofy was able to successfully manipulate prices mainly on the Bitfinex exchange, which in turn has influenced the market of Cryptocurrencies as a whole. However, the main determinant of the huge price increasing of the 2017 was the rising interest by the regular market participants toward this class of financial instruments, fueled also by a surprising increasing in media coverage that clearly contributed to spread the knowledge also to people who were previously not aware of the existence of such instrumets. The down-jump occurred in the following months was mainly due to the attempts by the authorities of some countries to regulate and limit the phenomenon, among which the US Commodity Futures Trading Commission (CFTC) request on June 9, 2018, of trading data from several cryptocurrency exchanges, with the goal of investigating market manipulation. However, the down-jump was also due to the decreasing interest of the public toward such instruments, leading to a sharp decrease in the volume traded, and consequently in the price.

I decided to analyze in deep the dynamics of Bitcoin because this is far the most important among all the other Crypto assets, but also because his movements are many times explicative also of the other cryptocurrencies price dynamics. I will be more synthetic for what concern the rest of the Crypto portfolio.

2.4.2. Namecoin, Litecoin, Peercoin, Feathercoin ("2013 Crypto")

I decided to group these four assets together under the name of "2013 Crypto" for two reasons: all their time series start at the same day, which is May 6, 2013. Moreover, their exchange rates present a very similar pattern.

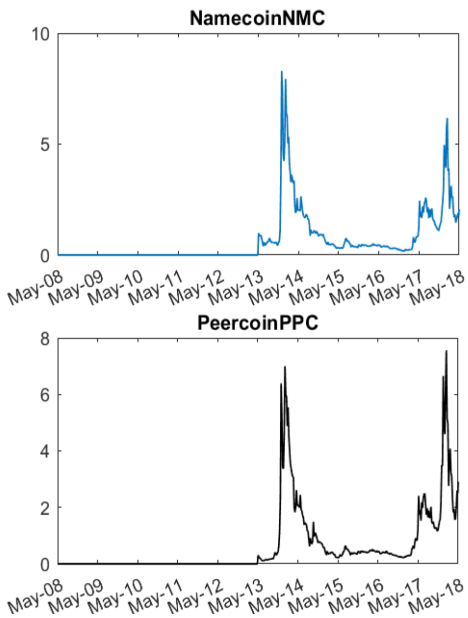

***Figure 2.11**: Namecoin-Litecoin-Peercoin-Feathercoin Trends*

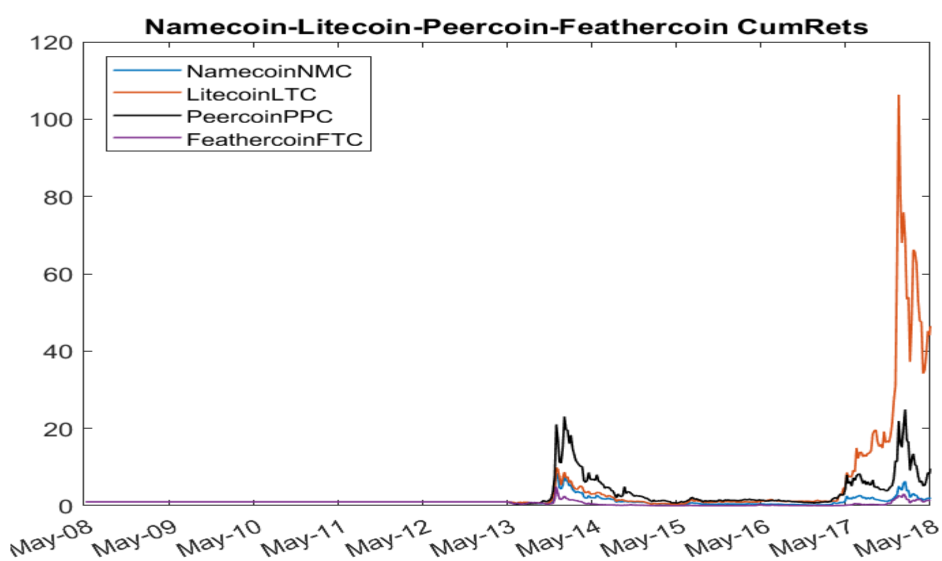

***Figure 2.12:** Namecoin-Litecoin-Peercoin-Feathercoin Cumulated Returns*

The first thing we can notice is that none of them is able to reach the level touched by the Bitcoin. For what concern Namecoin, Peercoin and Feathercoin the 2013 jump and the 2017 jump are almost of equal magnitude (to be more precise, Namecoin's and Feathercoin's 2017 jumps are quite smaller than the 2013 one). This means that over the last year these three cryptocurrencies have been less affected by the Euphoria effect that overwhelmed Crypto market, probably because they were less known by the public than the Bitcoin was. Regarding Litecoin, the situation is different: in this case there is a huge peak reached in 2017, in concurrence with the rapid increase of the Bitcoin, meaning that also the Litecoins were strongly swept up by the wave of Enthusiasm affecting Cryptocurrencies over the last year. The huge increasing experienced in 2017 by Litecoins makes the 2013 peak appearing very small, even if it was not in absolute value. Litecoin exchange rate seems to replicate very closely the Bitcoin's one, even if in a smaller magnitude.

2.4.3. Primecoin

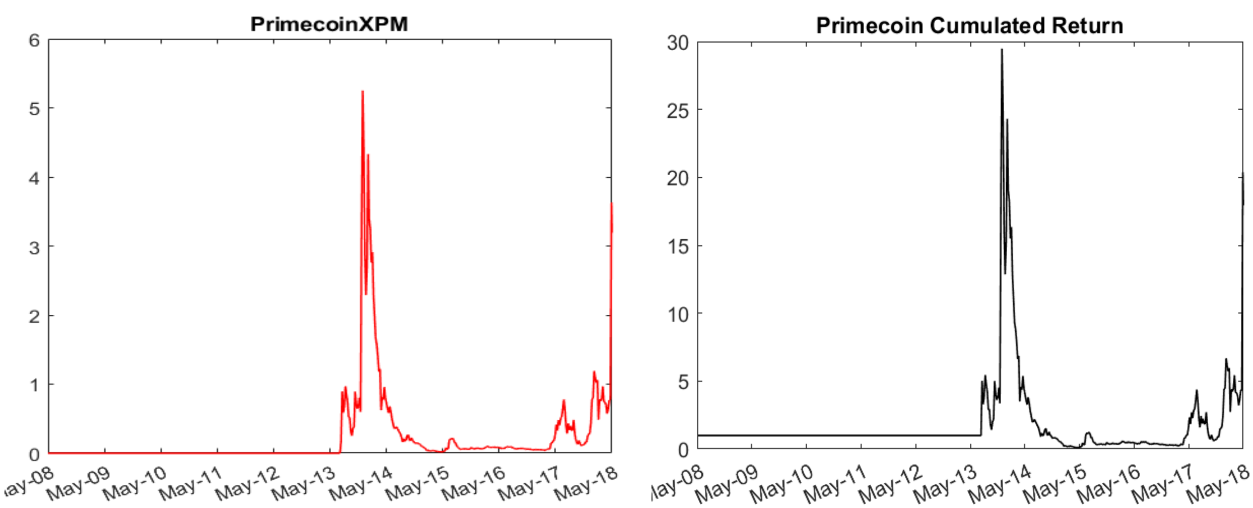

Figure 2.14-2.15: *Primecoin Trend and Cumulated Return*

Primecoin started to be available in the market on July 2013, and it is the first Cryptocurrency being excluded from the Static approach and entering only the Dynamic one. As you can see by comparing Figure 2.14-2.15 (Primecoin) with Figure 2.11-2.12 (2013 Crypto), Primecoin exchange rate pattern is very similar to the one of Namecoin-Peercoin-Feathercoin: the peak reached in 2013 is higher (in this case much higher) relative to the one reached in 2017. In this case the situation could be misleading, so in order to understand the point we must reason in absolute values: the 2013 jump seems huge compared to the one of 2017, but in fact it is in the same order of magnitude of the first 2013 jump of the other Crypto. The problem in this case is that the jump experienced in 2017 was very small in absolute value, meaning that people investing in Cryptocurrencies following the wave of enthusiasm of the last year did not consider Primecoin very attractive.

2.4.4. Ripple

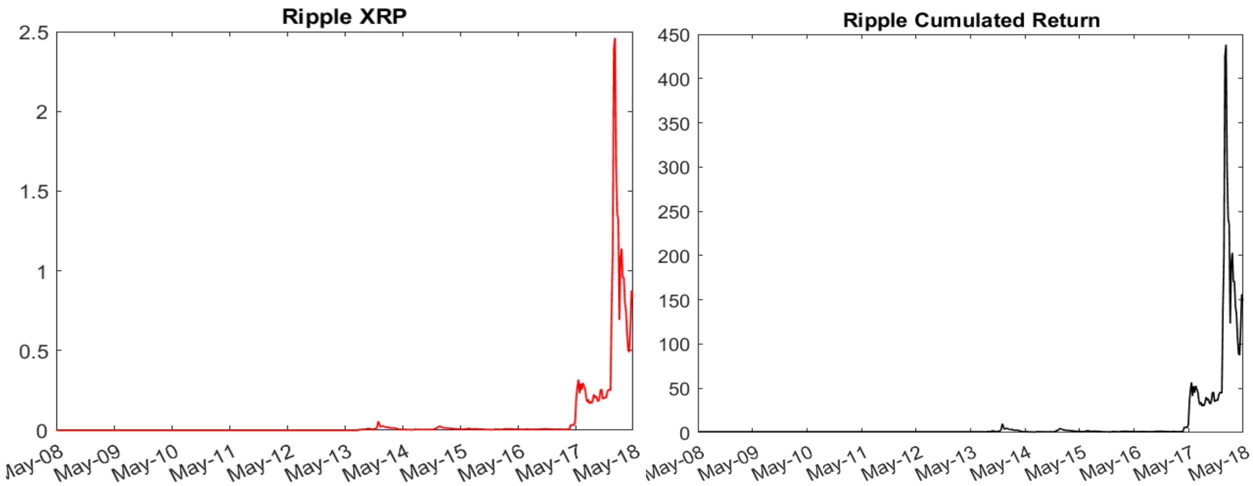

Figure 2.16-2.17: *Ripple Trend and Cumulated Return*

Ripple has been available in the market starting from August 2013 and it is the second Crypto entering the dynamic portfolio set up for my analysis. Ripple is probably the most famous cryptocurrency in circulation together with Bitcoin and Ethereum, and you can see this from the magnitude of the jump experienced in 2017: Ripple is the asset that mostly benefitted from the Crypto euphory: one Ripple was worth almost zero until May 2017, but its value exploded in the next months: if one had invested 1$ in before May 2017, at the end of the year he would have seen his wealth worth in Ripple amounting to 450$. On the opposite, the 2013 jump has been negligible, even if compared relative to the one experienced by the other Cryptocurrencies. This is mainly due to the fact that Ripple at that time had just overcome the ICO (Initial Coin Offering) phase, so probably investors preferred to put their money into more mature Cryptocurrencies at that time.

2.4.5. Doge

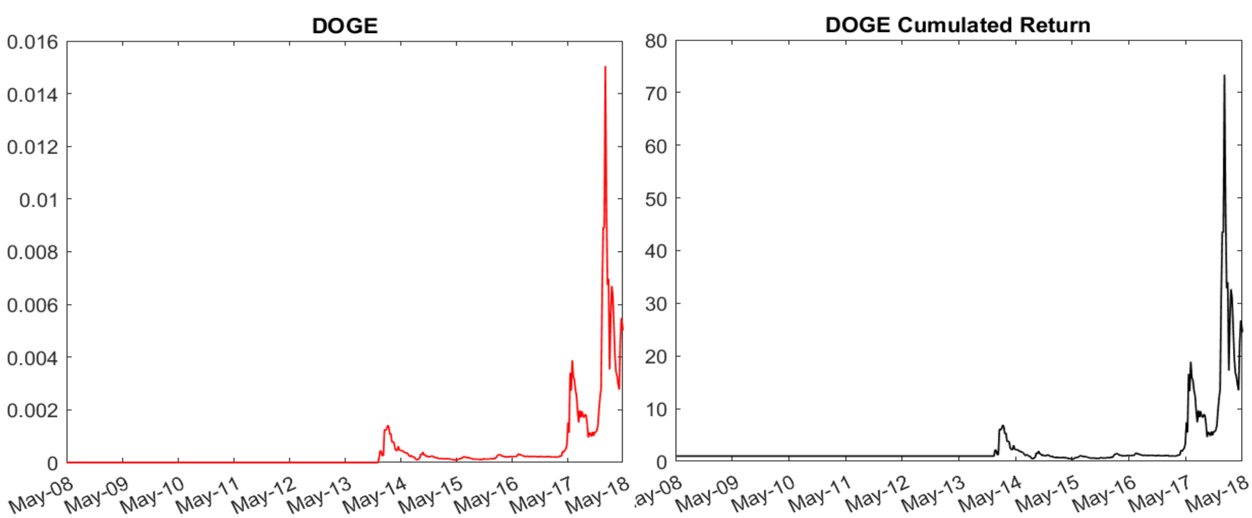

Figure 2.18-2.19: *DOGE Trend and Cumulated Return*

Dogecoin was originally introduced in the market on December 6 2013, and it enters my portfolio starting from Monday 16 December 2013. Its inventors, the programmers Billy Marcus and Jackson Palmer from Adobe, based Dogecoin on an existing Cryptocurrency, Luckycoin, which features a randomized reward that is received for “mining” a block. Luckycoin is in turn based on Litecoin, one of the Cryptocurrencies already included in my portfolio, so the programmers did not create Doge by zero. Even in this case we can notice a small 2013 spike relative to the one of 2017: in particular, at the end of December 2013 Doge raised by 300% in 3 days, and the volume traded amounted to Billions of dollars. In the next few days the value of Doge felt by 80% as a result of the China Government prohibition toward the national banks to invest in the Crypto’s market, but also of a hack to the Cryptocurrency wallet “Dogewallet” that resulted in a huge amount of Doge stolen. For what concern 2017, Doge experienced a huge increasing, and it was one of the cryptocurrencies benefitting the most of the new investments in Cryptocurrency market. In January 2018, Dogecoin reached an amount of two billion market capitalization.

An important thing I have to underline concerning this asset is that I have been forced to exclude it from a fraction of my analysis: this is because at a certain point my optimal “Dynamic” portfolios (DOGE entered indeed only the dynamic approach, not the static), presented volatility beyond reasonable levels. After some testing, I was able to figure out that the problem was precisely DOGE: it has been quite a hard challenge to integrate DOGE in the portfolio without having abnormal results. The problem was that for some periods of time the correlation with another crypto in the portfolio was too high, giving some difficulties to the execution of the optimization algorithm

2.4.6. Ethereum

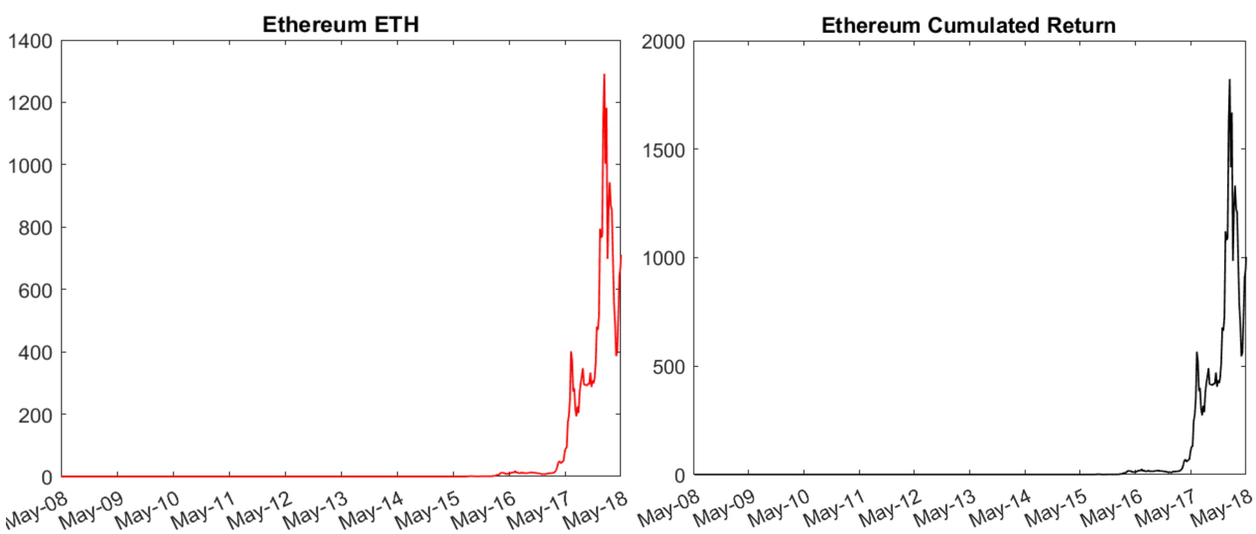

Figure 2.20-2.21: *Ethereum Trend and Cumulated Return*

According to the “Ethereum Foundation” Ethereum is “a decentralized platform that runs smart contracts: applications that run exactly as programmed without any possibility of downtime, censorship, fraud or third-party interference”[10]. If we adopt the speculative point of view of my analysis, Ethereum is the youngest but probably one of the most important Cryptocurrencies included in my portfolio. It was officially introduced in the market starting from July 30, 2015, so it has been available in my dynamic-approach portfolio for almost three years. From the moment of its introduction in the market, Ethereum experienced huge growth: As you can see from Figure 20, Ethereum currency grew almost 13000% over the year 2017. The total supply of Ethereum was around 100 Million dollars as of June 2018, and together with Bitcoin it is one of the most commonly traded Cryptocurrencies on many online wallets (and even online exchanges that are used tipically to trade stocks or bonds now allow for the trading of Bitcoin and Ethereum).

2.5. Assessing the Correlation

As the last thing I wanted to measure the level of correlation within the portfolio, in order to assess if there are assets showing a high level of correlation (near +1) or assets that correlate negatively. In the latter case we would have a benefit in terms of diversification. However, the main focus of this correlation test are Cryptocurrencies, for this reason at the end of this paragraph I decided to isolate them. In particular, given the fact that we are not sure about the fundamentals of such instruments, I wanted to identify if they correlate with one of the other asset classes included in my portfolio so as to explain at least partially their patterns. Moreover, I wanted also to identify the correlation among them, given the strong doubt that they correlate strongly mainly with the Bitcoin. Correlation has been measured from the week 380 of my dataset, which is July 30, 2015. Why I decided not to measure correlaton from the beginning? Because my datasets starts with only 18 assets, indeed Cryptocurrencies enter them later. It wouldn’t have made sense to assess the correlation of only 18 assets while my complete portfolio includes 27. The choice has been July 30, 2015 because this is the day when the last Cryptocurrencies (Ethereum) enters the dataset, so this is the first day from which my portfolio includes all the assets under analysis. Results are shown in the following surface plot (Figure 2.21).

[10] Definition taken from the website “www.ethereum.org”

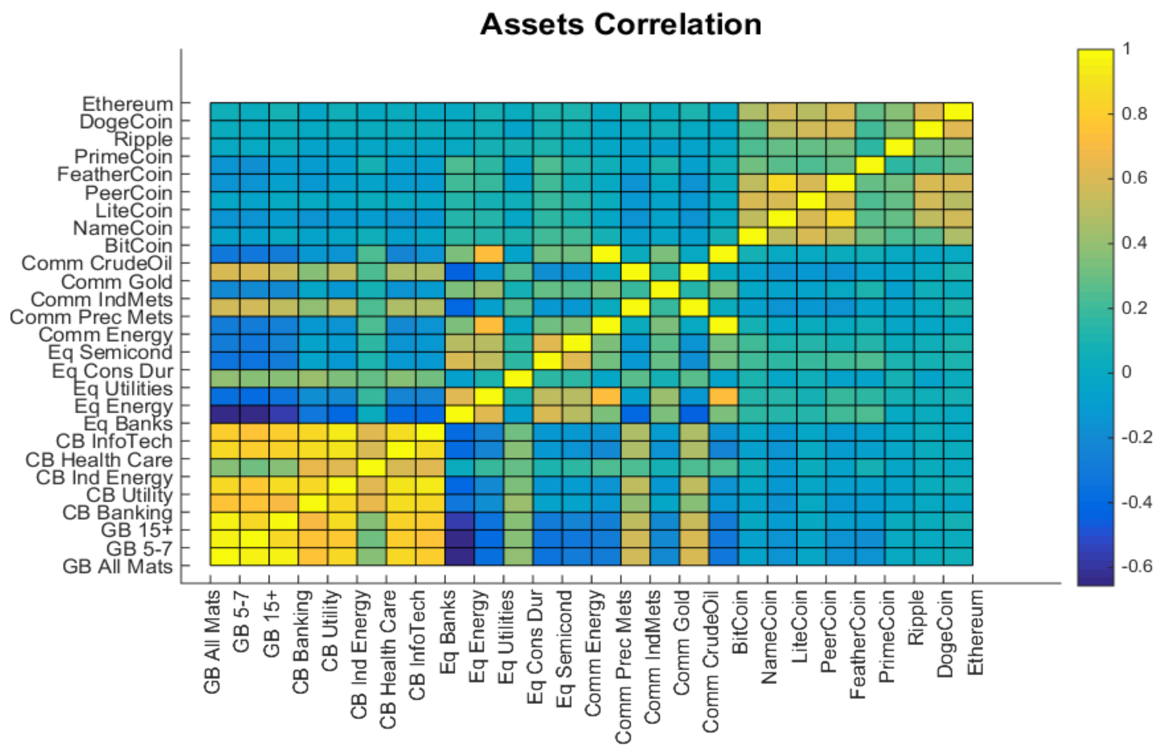

Figure 2.21: *Portfolio Correlations*

From this surface plot we can see that the correlation is quite strong mainly among the Bond asset classes, both Government and Corporate (bottom left corner), ranging from 0.6 to 0.8. In addition, bonds seem to correlate with some commodity indexes, but this correlation is weak. Moving up to the Equity asset class, the level of correlation within the asset class seems to be quite limited. Moreover, Equities do not correlate with Bonds (in some cases correlation is even strongly negative), but they show weak correlation with some Commodity indexes. However, here the situation is very heterogeneous, since in some cases the correlation becomes even negative (for example in case of Eq. Banks and Comm Crude Oil). Switching to Commodities, we can identify some correlation between Precious Metals and Gold with the Bond indexes. This is probably explained by the fact that Precious Metals are "defensive" instruments, precisely as Bonds. Moreover, we can also see that there is a strong correlation between Gold Index and Precious Metal index, but as I have already written, we could expect this. Finally, we can now move to Cryptocurrencies: the first thing one can notice is that Cryptocurrencies do not correlate with any of the other asset classes included in the portfolio (ρ ranges from 0 to 0.1), meaning that cryptocurrency pattern is not explained by any other traditional asset class. What we can observe instead is that they correlate quite strongly with each other (upper-right corner of the surface plot). If we focus on this part of the correlation area plot we obtain the output included in Figure 2.22.

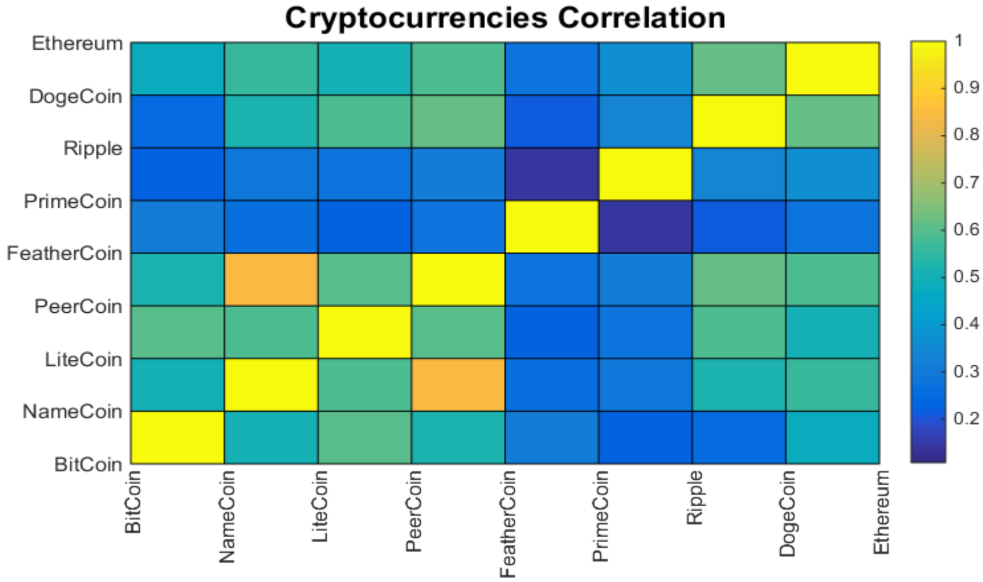

Figure 2.22*: Cryptocurrencies Correlations*

Correlation among Cryptocurrencies ranges from 0.2 to 0.8. Moreover, it seems to be stronger for what concern Namecoins, Litecoins, Peercoins and Fearhercoins, while it is weaker for the more recent Cryptocurrencies.

2.6. Final considerations: is there a bubble going on?

Looking at the pattern of all Cryptocurrencies involved in my analysis, it clearly brings to our mind a financial bubble. Many experts claim that we are fully in the middle of a financial bubble for what concern Cryptocurrencies, others claim instead that this kind of assets cannot be considered a bubble, because they lack of some important characteristics that financial bubbles do indeed have. Even if this is beyond the scope of my work, I think that it is important to discuss about this aspect, since this can also give some predictability about the assets' future returns. In order to clarify the ideas I will discuss briefly a short article by the International Monetary Fund, named "a Short History of Crypto Euphoria" (2018) by Andreas Adriano. The article is based on the literature of the Harvard Professor John Kenneth Galbraith, authors of milestones such as "The Great Crash, 1929" or "A Short History of Financial Euphoria". Galbraith is claimed as one of the maximum experts of financial bubbles worldwide.

Adriano (2018) considers first that, according to Galbraith, financial disasters are quickly forgotten, so even if it is only a decade from the 2008 financial crisis and two decade from the "dotcom" bubble, two of the most dramatic events affecting worldwide economy recently, the "irrational exhuberance of those two period has largely faded from memory": this implies that even if

Cryprocurrencies could be a possible menace to replicate what happened in those periods, people are not able currently to avoid the risk because they tipically don't learn from past errors. Even more if you consider that Cryptocurrencies are very often traded among the youngest people, who did not experienced the past. Another aspect that characterizes a financial bubble according to Galbraith is the speculative trait: as the bubble grows speculation becomes more and more important, hiding the fundamentals of the instruments traded. And this is precisely what is happening now in the Crypto market: think for example to the Initial Coin Offering (ICO), which is the process through which people buy tokens reedimable in the new Cryptocurrency when this will be introduced in the market (if all the process goes well). Tokens do not give any right of ownership to the people who purchase them, so there is the risk of losing it all if the ICO does not succeed. So put in this way, why does one investor purchase tokens? Is it really because he is interested in the final redemption? Clearly not, one typically buy tokens simply to resell them to other investors eager to join the race at a higher price, making some gain in this way. This is pure speculation.

If all the elements described above seem to confirm that cryptocurrencies are without doubts a financial bubble, according to Adriano (2018) there is one important detail which is still not clear: how much debt is involved in the Crypocurrency business? Galbraith (1929) in his literature claims that financial bubbles always involve large amount of debt, just think about the dotcom bubble. This happens because those people who see other people gaining a lot of money, as it happens during the increaing phase of the bubble, are induced to borrow money in order to "join the race" themselves. One thing is sure: debt is involved in the Crypto business, and this is simply explained by the fact that many exchanges allow to buy these assets with leverage, even using 100 times the cash balance in the investor's portfolio. The problem is that nowadays it is impossible to know precisely how many people are borrowing to buy cryptocurrencies.

Summing up, there are some elements such as the tendency of the human nature to repeat the same error of the past or the speculative feature becoming more and more important, which seem to confirm that Cryptocurrencies are indeed a financial bubble. Other aspects, such as the amount of leverage involved, are still not clear.

And what will happen when/if the bubble will burst? The consequences of the bursting of a financial bubble are the more dramatic the higher is the number of investors that own in their portfolio the assets involved in the bubble. For what concern Crypto, their level of diffusion is still far from the level reached for example by the dotcom stocks at the peak of the dotcom bubble. Moreover, Adriano (2018) reports that, as said by the governor of the Bank of England Mark

Carney, "even at their peak, all crypto assets combined are worth less than 1% of the world GDP": to give a comparison, Dotcom stocks were worth one third of the world GDP when the bubble burst. So, what we can say is that even if the bubble will burst, consequences are expected to be limited and likely to affect only a small fraction of the market participants.

The main aim of my work is to try to figure out what would have happened to a balanced-diversified portfolio made up of traditional assets if at a certain point in time Cryptocurrencies had entered it. Clearly, what I expect to find is that the performances of the portfolio including Cryptocurrencies improve substantially with respect to the correspondent base portfolio without this kind of assets, and this is because all the Cryptocurrencies involved in the analysis experienced huge growth during the period. But in the light of what has just been said, you have to consider where does this growth come from, and above all you have to consider that this growth is not linked to the fundamentals at all. So, if it is reasonable for a private investor to try to invest part of his wealth in Crypto following a speculative aim, it is more difficult that professionals, such as mutual fund managers, pension fund managers, or even small private bankers decide to invest wealth in cryptocurrencies to improve the performances of their portfolios. This means that my analysis is valid, but with some reserves.

Chapter 3

Markowitz analysis, portfolios and constraints

Starting from the dataset as illustrated above I developed a portfolio analysis following Markowitz strategy: the final objective of this analysis is the determination of the weights of the optimal portfolios, determined according to the investor degree of risk aversion. After having performed Markowitz analysis, I adopted also a risk contribution approach in order to have a wider spectrum of results: the difference with respect to the traditional Markowitz analysis is that in this case I am not working with weights anymore, since I am imposing budgets on the risk contribution of each asset to the total risk of the portfolio. I will deal with it in the following Chapter 4.

I started with the analysis of the Base portfolio composed of asset by the traditional classes, which is the starting point of my work: this includes ten years of weekly data, starting from May 6, 2008 to May 6, 2018 (522 data). The next step has been making the same analysis including in the dataset also the Cryptocurrency part (following the Static and the Dynamic approach), in order to assess to what extent the return and the risk of the portfolio including Cryptocurrencies differs compared to the one without the cryptocurrency component.

As I already mentioned in the previous lines the integration of the cryptocurrency component has been done following two different approaches, so that I ended up with two kind of portfolios, an easier one involving only five Cryptocurrencies (Static) and a more sophisticated one including also all the other cryptocurrencies (Dynamic).

In the Static approach I took Bitcoin, Namecoin, Litecoin, Peercoin, Feathercoin, all of them starting from May 6, 2013, and I added them to the Base portfolio starting from that date. The final result has been a portfolio composed by two distinct parts of almost equal time lenght: the first one goes from May 5, 2008 to April 29, 2013 and it includes only traditional assets, the second one goes from May 6, 2013 to May 7, 2018 and it includes both traditional assets and cryptocurrencies.

The Dynamic approach starts with the 18 assets of the base portfolios as well: beyond the fact that here all the cryptocurrencies are included in the analysis, the main difference with respect to the Static approach is that here cryptocurrencies time series do not begin all at the same point in time, since each of them enters the analysis from the date on which it is available in the market. So, for example, in the Dynamic portfolios Bitcoins enter from July 19, 2010, which is the day on

which they started to be exchanged in the market, while in the Static approach they enter later (May 6 2013), in order to start together with the next four cryptocurencies (2013Crypto).
If the Static approach portfolios can be ideally split in two blocks, the Dynamic ones can be splitted in much more (one block for every crypto's new entry). This resulted in a more difficult analysis, but the latter approach is more precise and does not require to eliminate data in excess.
After having defined how the portfolios are structured, the next step is to explain what methodologies have been used to conduct the analysis.

3.1. Returns and Covariances

Markowitz analysis require two basic inputs: assets returns (**r**) and asset Covariances (**Σ**). There are different estimation methods through which these inputs can be retrieved: one can simply estimate sample moments of observed returns over the entire time period T.

$$Return = r = \frac{1}{N}\sum_{j=1}^{T} r_t \tag{3.1}$$

$$Covariance = \sum = \frac{1}{N}\sum_{j=1}^{T}(r_t - r)(r_t - r)' \tag{3.2}$$

In this way the best one step ahead forecast is given by sample moments of observed returns. However, this approach requires that the asset returns are independently and identically distributed (IID) along all the time period, and this is quite a strong assumption for what concern historical time series of asset returns. For this reason in my analysis I decided to use two alternative standard approaches for the evaluation of the inputs in order to mitigate this problem: these are the **Rolling** approach for sample moments and the Esponential Weighted Moving Average (**EWMA**). So for both the Static and the Dynamic portfolios Returns and Covariances have been estimated using both methods.

3.1.1. Rolling Sample Moments

This method implies the usage of a recent observation window than the overall sample. You simply compute sample moments of assets' returns over a smaller time window *(w)* than the overall time length of the analysis *(T)*. First, you must decide the length of the time window: the size of the window depends on the length of the sample size (T) and on the periodicity of data. Tipically the window is short for data collected in short intervals, longer for data in longer intervals: another thing that must be considered is that longer rolling windows yield smoother rolling estimates than shorter windows. In my case the sample size is about 10 years (T=522 weekly returns) which is quite short. For this reason I decided to use a short window size of 1 year (w=52 weeks). After this first step the sample mean of the first *w* observations of the original sample must be computed and the result will be the first element of the Rolling return sample. Then, you move one period head and you take the sample mean from the second return of the original sample to return w+1. This will be the second element of the rolling return sample. You repeat this procedure until you reach the last observation.

The final result will be a Rolling sample of *T-w* observations, in my case 470 weeks equivalent to a 9 year period. The same procedure is applied to all the assets in the portfolio.

The final result is the variable called "**ErS"**, the rolling return sample, structured as a two dimensional matrix where the number of raws is *T-w = 470*, equivalent to the number of observation in the rolling sample (9 years of data) and the number of column is *c* (taking the notation of my Matlab code), equivalent to the number of assets in the portfolio.

For what concern covariances the procedure is the same, but instead of estimating the sample mean over the time window you estimate the sample covariance.

The final result is **EvS**, the rolling covariances sample, a three dimensional matrix sized *470xCxC*, where *C* is still the number of assets in the portfolio.

I finally obtained ErS and EvS, the 2 inputs needed to run the Markovitz analysis with rolling input.

3.1.2. EWMA

The main restriction of the rolling approach is that it is a simple moving average (SMA) (the unweighted mean of the previous n data), so by applying the rolling approach one gives the same weight to the past and to the recent observations. Sometimes, considering the accuracy of the final result, it is more convenient to give more weight to recent observation than to older ones, and for this purpose it is useful to implement a dynamic moving average model, such as the Esponential Weighted model: the latter assigns weights that decrease exponentially across the sample size. The final result will be an average where the recent observation will weight more than less recent ones, which is precisely what we are looking for.

For what concern returns, the general form of the estimator is the following (3.3):

$$r = \frac{1}{\bar{\lambda}} \sum_{j=1}^{T} \lambda_j \, r_{T-j} \quad \text{, where } \bar{\lambda} = \sum_{j=1}^{T} \lambda_j \tag{3.3}$$

Where r are the sample returns of each asset, λ is the smoothing factor and T is the sample size. There are many different options of smoothing factors among which one can choose: in my analysis I exploited the most commonly used, which is defined as in (3.4):

$$\lambda_j = (1-\lambda)\lambda^{j-1}, \text{ where } \lambda \in [0.9, 0.99] \tag{3.4}$$

Substituting the equation (3.4) on equation (3.3), Returns at time *t (3.5)* and *t-1 (3.6)* become:

$$\boldsymbol{r_t} = \sum_{j=1}^{t} (1-\lambda)\lambda^{j-1}\, r_{t-j} \tag{3.5}$$

$$\boldsymbol{r_{t-1}} = \sum_{j=1}^{t} (1-\lambda)\lambda^{j-1}\, r_{t-1-j} \tag{3.6}$$

Where *t* is the present EWMA return and *t-1* the yesterday EWMA return.
If we isolate the yesterday return (r_{t-1}) from the $\boldsymbol{r_t}$ formula, and if we adopt a recursive evaluation approach, what we obtain is:

$$\boldsymbol{r_t} = (1-\lambda)r_{t-1} + \lambda \sum_{j=1}^{t-1} (1-\lambda)\lambda^{j-1}\, r_{t-1-j} \tag{3.7}$$

Notice that the second addendum of the RHS of the previous equation is simply the yesterday EWMA return as defined above (3.6) multiplied by the smoothing factor λ, so we can write:

$$\boldsymbol{r_t} = (1-\lambda)r_{t-1} + \lambda \boldsymbol{r_{t-1}} \tag{3.8}$$

As you can see from the last formula the current EWMA return is a combination between the previous smoothed value ($\boldsymbol{r_{t-1}}$) and the current observation (r_{t-1}), where λ controls the closeness of the interpolated value to the most recent observation[11]. The importance given to the present and to the past depends on the magnitude of the smoothing factor λ. For example, if λ=0 we have that $\boldsymbol{r_t} = r_{t-1}$, so present return is exactly equal to the most recent observation. This means that you are giving more weight to the recent period. If instead λ=1 we get $\boldsymbol{r_t} = \boldsymbol{r_{t-1}}$, so past returns acquire more importance than the most recent ones. For this reason tipically λ is set into the interval $\lambda \in [0.9, 0.99]$. In my research I chose $\lambda = 0.95$.
The first EWMA return (ErE(1,:) in Matlab) is simply computed as the average of the 52 returns of the time window *w*, as it happens in the rolling case. The difference from the Rolling case is that

[11] Moving Average and Exponential Smoothing Models, https://people.duke.edu/~rnau/411avg.htm

the following EWMA returns are not simply the average of the one-step-ahead 52 returns. The following EWMA returns are computed giving different weights to present and past returns. The final result is the Matrix of returns ErE, which is a double dimension matrix of (470 x c) (c is the number of assets in the portfolio).

For what concern covariances, the strategy is similar to the recursive one adopted to compute returns: in particular, assuming assets have zero mean, we can define assets covariances as it follows:

$$\boldsymbol{\Sigma_t} = \sum_{j=1}^{T} (1-\lambda)\lambda^{j-1}\, r_{t-j} r'_{t-j} \tag{3.9}$$

$$\boldsymbol{\Sigma_{t-1}} = \sum_{j=1}^{T} (1-\lambda)\lambda^{j-1}\, r_{t-1-j} r'_{t-1-j} \tag{3.10}$$

$$\boldsymbol{\Sigma_t} = (1-\lambda) r_{t-1} r'_{t-1} + \lambda \boldsymbol{\Sigma_{t-1}} \tag{3.11}$$

Even one step ahead EWMA covariances are defined as a combination of Present Covariances Σ_{t-1} and Past Covariances $r_{t-j} r'_{t-j}$. The final result is EvW, a Matrix sized 470xCxC.

3.2. The Efficient Frontier

Markowitz analysis requires two inputs: Assets Mean and Covariances. In the previous paragraph I have just described two methods in order to obtain them. The next step of the work is the computation of the efficient frontier and of the optimal portfolios. In my analysis I did not include the risk free asset, so we are talking about an efficient frontier *without risk free.*

The efficient frontier is defined as the the set of efficient portfolios, which constitute a subset of all the admissible portfolio set satisfying the two following properties:

1) It does not exist a Portfolio with the same return as the efficient portfolio but with a lower level of risk.
2) It does not exist a Portfolio with a higher return than the efficient portfolio and with the same level of risk.

So, the next step is the determination of the efficient portfolio subset. This problem has a dual represantion because an efficient portfolio could be determined both fixing the risk and maximizing the return for that level of risk, both fixing the return and minimizing the risk associated with that return level.

In the **first** case the problem is set up as it follows (3.12):

$$\begin{aligned} & min_{\omega} \quad \omega' \Sigma \omega \\ \text{s.t:} \quad & \mu_p = \quad \omega' r \\ \text{s.t:} \quad & \omega' 1_n = 1 \end{aligned} \tag{3.12}$$

Where ω is the vector of weights of the assets in the portfolio, Σ is the covariance matrix, r are the returns and 1_n is a vector of ones with length "n" = number of assets in the portfolio. Clearly $\omega'\Sigma\omega$ is the variance σ_p of the portfolio P while μ_p represents its return.

As we can see, the minimization problem is subject to two different constraints: first, the portfolio return $\mu_p$is fixed to a given level because we are minimizing the risk for a given level of return. Second, the sum of asset weights equal to 1. After having solved this problem we obtain the Portfolio with the lowest risk for that level of return: it does not exist another portfolio in the investment universe with the same return and lower volatility.

In order to solve this minimization problem the following Lagrangian function is set up:

$$min_{\omega}\ L(\omega) = \frac{1}{2}\ \omega'\Sigma\omega - \lambda_1(\omega' 1_n - 1) - \lambda_2\left(\omega' r - \mu_p\right) \tag{3.13}$$

The solution to the Lagrangean (3.13) is the following relation between μ_p and σ_p^2:

$$\sigma_p^2 = \frac{C}{\Delta}\mu_p^2 - \frac{2B}{\Delta}\mu_p + \frac{A}{\Delta} \tag{3.14}$$

Where:

$$A = r'\,\Sigma^{-1} r \qquad B = 1'_n \Sigma^{-1} r \qquad C = 1'_n \Sigma^{-1} 1_n$$

Equation (3.14) is the equation of the Efficient Frontier: we can define this as the set of portfolios with the highest expected return for each level of risk.

The Efficient frontier is commonly defined on a $(\sigma_p\,, \mu_p)$ plane, which is a plane where the X axis is the portfolio volatility and the Y axis is the portfolio return: Efficient Frontier takes the form of a hyperbola with vertex V (3.15):

$$V = \left(\frac{1}{\sqrt{C}}\,, \frac{B}{C}\right) \tag{3.15}$$

Clearly, we consider only the branch of the hyperbola lying above the vertex: the branch lying below the vertex does not have an economic meaning, but it is only a result of computations.

The problem could be set up also by fixing the risk and maximizing the return for that level of risk. In this **second** case the problem is defined as in (3.16):

$$
\begin{aligned}
& max_{\omega} \quad \omega' r / \sqrt{\omega' \Sigma \omega} \\
\text{s.t:} \quad & \sigma_p^2 = \omega' \Sigma \omega \\
\text{s.t:} \quad & \omega' 1_n = 1
\end{aligned}
\tag{3.16}
$$

Here we are maximizing the returns for a given level of risk. For this reason the maximization problem is subject to two constraints: there is the *given risk level* σ_p^2 and the usual *sum of weights equal to 1*. Solving this maximization problem we obtain the portfolio having the highest return for that level of volatility.

3.3. Maximum Sharpe and Global Minimum Variance Portfolios

Among all the efficient portfolios, we can identify two of them playing a peculiar role: these are the Max Sharpe portfolio (MS) and the Global Minimum variance portfolio (GMV).

- The **GMV portfolio** is the solution to the minimization problem (3.17):

$$
\begin{aligned}
& min_{\omega} \quad \omega' \Sigma \omega \\
\text{s.t:} \quad & \omega' 1_n = 1
\end{aligned}
\tag{3.17}
$$

Its characteristics are summarized in the following table (Table 3.1):

Weights	ω_{GMV}	$\frac{\Sigma^{-1} 1_n}{1_n' \Sigma^{-1} 1_n}$
Return	r_T	$\frac{r' \Sigma^{-1} 1_n}{1_n' \Sigma^{-1} 1_n}$
Volatility	σ_T	$\frac{1}{\sqrt{1'_n \Sigma^{-1} 1_n}}$

Table 3.1: GMV portfolio attributes

The Global Minimum Variance portfolio is the lowest vertex of the Efficient Frontier, and it represents the Portfolio with the lowest risk among the Efficient Portfolios universe.

- The **Max Sharpe portfolio** is obtained by solving the following maximization problem:

$$
\begin{aligned}
& max_{\omega} \quad \omega' r / \sqrt{\omega' \Sigma \omega} \\
\text{s.t:} \quad & \omega' 1_n = 1
\end{aligned}
\tag{3.18}
$$

Where $\omega' r / \sqrt{\omega' \Sigma \omega}$ is the so called Max Sharpe index.

Its characteristics are summarized in the table below (Table 3.2):

Weights	ω_{MS}	$\frac{\Sigma^{-1} r}{1_n' \Sigma^{-1} r}$
Return	r_T	$\frac{r' \Sigma^{-1} r}{1_n' \Sigma^{-1} r}$
Volatility	σ_T	$\frac{\sqrt{r' \Sigma^{-1} r}}{\lvert 1'_n \Sigma^{-1} r \rvert}$

Table 3.2: MS Portfolio Attributes

In the following plot (Figure 3.1) you can see the location of the Global Minimum Variance portfolio and of the Max Sharpe Portfolio along the efficient frontier:

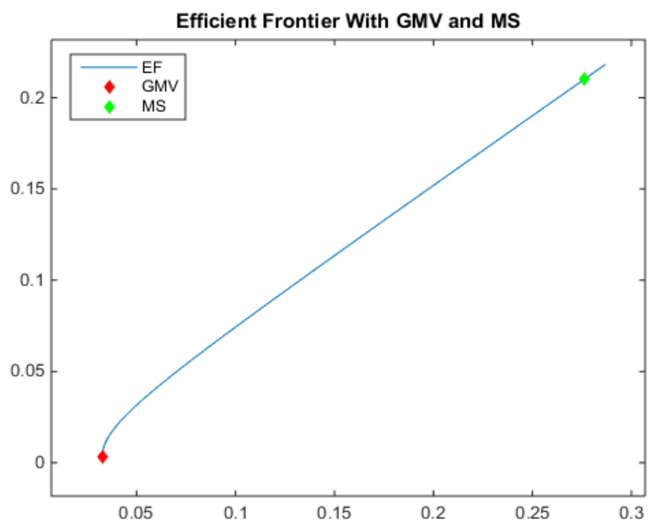

***Figure 3.1**: Efficient Frontier with GMV and MS*

3.4. Portfolios with intermediate degrees of Risk Adversion

Global Minimum Variance and Max Sharpe portfolios can be considered the two extreme choices of a rational investor. The GMV is the portfolio, among all the efficient ones, with the lowest level of risk, so its composition should reflect the needs of an investor with the maximum degree of risk adversion. On the opposite side, the Max Sharpe, which maximize the returns, is the optimal portfolio choosen by the investor with the minimum level of risk aversion.

Between these two extremes there are all the other efficient portfolios composing the frontier. As already said, even these portfolios are optimal, and they are suitable for investors with different degrees of risk aversion. I have decided to consider in my analysis, besides GMV and MS, three additional portfolios with different degrees of risk adversion that are located in the efficient frontier bewteen the two "extreme" portfolios. In particular, I constructed a portfolio suitable for an investor with a high level of risk aversion **(Risk Averse, RA)**, one for an investor with a medium degree of risk aversion **(Moderate Risk Averse MRA)** and finally one suitable for an investor with a very small degree of risk aversion **(Risk Lover RL)**. Empirically I found out that the higher the degree of risk adversion, the more the portfolio composition becomes similar to the GMV in terms of weights, while the smaller the degree of risk adversion the more the portfolio composition is similar to the MS. This is coherent with our expectations.

How to set up these kind of portfolios?

They are the result of a Utility maximization problem.

Consider an economic agent with the folowing mean variance utility function (3.19):

$$U = \mu_p - \frac{\gamma}{2}\sigma_p^2 \tag{3.19}$$

$$\text{With} \begin{cases} \mu_p = \omega' r = Portfolio\ Return \\ \sigma_p^2 = \omega'\Sigma\omega = Portfolio\ Variance \\ \gamma = Agent\ Degree\ of\ Risk\ Adversion \end{cases}$$

In order to define the optimal portfolio we solve the Maximization problem (3.20) subject to the usual constraint of sum of asset weights equal to 100%:

$$\begin{aligned} & max_\omega U = \mu_p - \tfrac{\gamma}{2}\sigma_p^2 \\ \text{s.t:} \quad & \omega' 1_n = 1 \end{aligned} \tag{3.20}$$

Solving the problem, it results that the optimal weights of the portfolio with a specific degree of risk adversion γ are determined according to the formula in (3.21):

$$\omega = \frac{1'_n \Sigma^{-1} r}{\gamma}\,\widehat{\omega}_{MS} - \frac{1'_n \Sigma^{-1} r - \gamma}{\gamma}\,\widehat{\omega}_{GMV} \tag{3.21}$$

As we can see, the weights of an Intermediate Risk Averse investor portfolio are determined as some sort of weighted average between the weights of the Max Sharpe portfolio and the weights of the Global Minimum Variance portfolio. Clearly, the final result will depend on the investor's degree of risk aversion, which is represented in the formula by the Parameter "γ": graphically, the optimal solution is the point of tangency between the efficient frontier and the agent utility function, whose shape depends precisely on "γ". A high value of γ represents a high degree of risk aversion, while a small level of γ represents a small degree of risk aversion: considering the two extreme cases ($\gamma \to \infty$ and $\gamma \to 0$) we obtain:

- For $\boldsymbol{\gamma} \rightarrow \infty$ (Infinitely Risk Averse Agent) the formula above gives $\omega = \widehat{\omega}_{GMV}$, so the solution converge to the Global Minimum Variance portfolio, and the two portfolios will have the same compositions. We could expect this, since the GMV is the portfolio with the lowest possible standard deviation, so it is the favorite portfolio for an agent who is extremely risk averse.
- For $\boldsymbol{\gamma} \rightarrow \mathbf{0}$ (Infinitely Risk Lover Agent) the solution provides weights that are a result of an extremely long position in the Max Sharpe portfolio and an extremely short position in the GMV portfolio. The situation is different from the previous extreme case: in the latter the resulting portfolio was equal to the GMV one, in this case one could expect to find a portfolio whose composition is equal to the Max Sharpe: however, this is not the case. The fact is that the resulting portfolio could be even riskier than the MS one, so we may have a portfolio that will be positioned in the efficient frontier on the right side of the MS.

In my analysis I do not consider a degree of risk aversion such that the resulting optimal portfolio is riskier than the Max Sharpe one, since I decided to keep the MS as the right limit of the efficient frontier.

As already mentioned, my work involves three portfolios with intermediate degree of risk aversion besides the GMV and the MS ones, as summarized in Table 3.3. Here, all the portfolios included in my analysis are listed based on their degree of risk aversion (from the highest to the lowest).

Portfolio Name	**Code**	**Degree of Risk Aversion**
Global Minimum Variance	GMV	$\gamma \rightarrow \infty$
Risk Averse Agent	RA	$\gamma = 40$
Moderate Risk Averse Agent	MRA	$\gamma = 10$
Risk Lover Agent	RL	$\gamma = 3$
Max Sharpe	MS	/

Table 3.3: Portfolios included in the analysis with the associated degree of Risk Adversion

In the following plot (Figure 3.2) you can observe the efficient frontier together with MS, GMV but also with the Risk Lover (RL), Moderate Risk Adverse (MRA) and Risk Adverse (RA) portfolios, which are those portfolios with an intermediate degree of risk aversion, resulting from the optimization problem as defined in equation (3.20), associated to parameters of risk adversion as in Table 3.3.

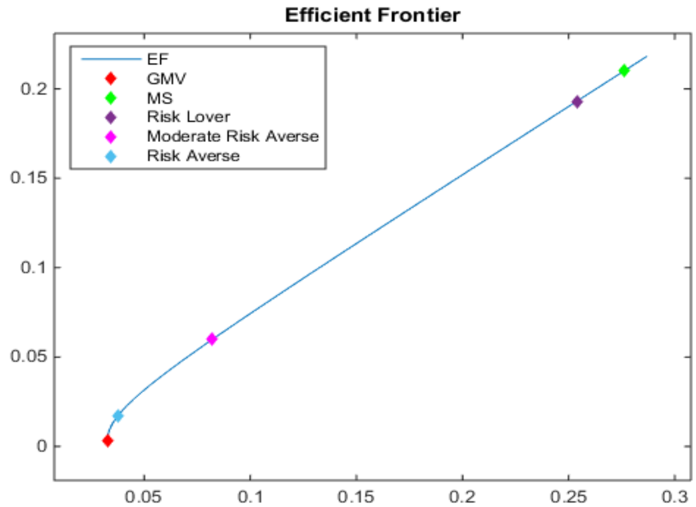

***Figure 3.2**: Efficient Frontier with GMV, MS, Intermediate risk aversion portfolios*

As you can expect, the portfolio suitable to the investor with a high degree of risk aversion (Risk Adverse) lies near the GMV, while the portfolio with a low degree of risk aversion (Risk Lover) lies near the Max Sharpe portfolio. The one with an intermediate degree of risk aversion (Moderate Risk Averse) lies in the middle, nearer by construction to the GMV one.

3.5. Portfolio Constraints

Until now I have spoken about the Markowitz approach in his basic form, considering as the only constraint to the optimization problem the summation of asset weights equal to 100% ($\omega' 1_n = 1$). This is equivalent to say that in each period under analysis the 100% of the investor wealth is invested in the portfolio, I did not allow for the possibility of entering leverage in the portfolio ($\omega' 1_n > 1$), nor for the possibility of keeping liquidity available instead of investing it ($\omega' 1_n < 1$). One of the main drawbacks of the Markowitz model in his basic form, call it the "unconstrained" approach, is that the general solution to the optimization problem often lead to portfolios which are characterized by the presence of extreme long or short position. For example, one of the possible results could be that the 100% of the investor wealth is concentrated on a single asset. This means that there is a problem of lack of diversification, because the performance of the portfolio is linked only to the performance of one single asset. Moreover, short positions could be a problem because if you consider a typical financial market operator, such as a retail investor, a private banker, or even a pension fund manager, typically these operators aren't allowed to open short positions or even if they are allowed to, they cannot maintain them opened for long periods of time.

What we can conclude is that many times the results of the Unconstrained Markowitz approach are portfolios that in the real world are not feasible. In order to deal with this problem what I did in my analysis was trying to create portfolios which meet the real investors' needs or the constraints set up by the existing regulation. How to implement this? Simply imposing constraint on weights that the optimal portfolio must satisfy.

I conducted the analysis considering five different set of constraints on weights, so five different strategies for every portfolio: each new constraint is simply a new "subject to" added to the basic unconstrained optimization problems as described in the previous pages, modeled according to what kind of characteristics the weights of the final portfolio are expected to satisfy.

What does it happen to the Efficient Frontier? By putting constraints on weights, you are reducing the set of efficient portfolios only to those whose weights satisfy certain characteristics, so the investment universe reduces. This means that the efficient frontier shifts bottom-right with respect to the unconstrained one, as you can see from Figure 3.3, where an unconstrained efficient frontier is represented together with a constrained one (Positivity of Weights).

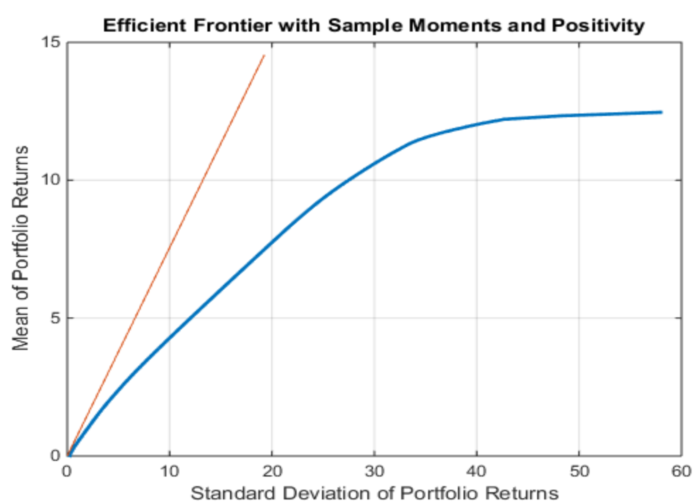

Figure 3.3: *Efficient Frontier Unconstrained vs Positivity of weights*

As I written before, I adopted 5 set of constraints:

1) No short selling
2) Upper and Lower Bounds
3) Group constraints 1 (Crypto at 10%)
4) Group constraint 2 (Crypto at 20%)
5) Turnover Constraint.

Now I will spend a few word on each of them.

3.5.1. No short selling constraint

This constraint allows for the possibility of eliminating from the portfolio every short position: in my case it is reasonable to add this kind of constraint, beyond the regulation concerns, because there is no way to open a short position on a Cryptocurrency asset. Maybe one could do this Over the Counter, but I decided to exclude this case.

In order to allow for the positivity of weights you must start from the usual unconstrained Minimization problem and simply add to it the constraint of positive weights (3.22):

$$\begin{aligned} &min_{\omega} \quad \omega'\Sigma\omega \\ \text{s.t:} \quad & \mu_p = \omega' r \\ \text{s.t:} \quad & \omega' 1_n = 1 \\ \text{s.t:} \quad & \omega' \geq 0 \end{aligned} \tag{3.22}$$

The new added constraint with respect to the base Markowitz approach is the one marked in red. An important thing that must be considered is that this problem does not have an analytical solution, for this reason we must resort to numerical methods: by construction all the resulting weights will be included in the interval [0, 1].

For what concern the Efficient Frontier, in this case we have that it will be both Upward and Downward limited. In particular, the upward limit is given by the portfolio where the 100% of the investor's wealth is allocated on the asset with the highest return. On the opposite, the Downward Limit is the maximum between two different portfolios: the GMV under no short selling constraint or the Portfolio where 100% of the wealth is allocated on the minimum expected return asset.

Speaking about the Global Minimum Variance and Max Sharpe portfolios, these are the result of optimization problems very similar to the ones for their determination in the unconstrained case, with the only difference that the constraint for the positive weights must be added (Table 3.4).

GMV	MS
$min_{\omega} \quad \omega'\Sigma\omega$ s.t: $\quad \omega' 1_n = 1$ s.t: $\quad \omega' \geq 0$	$max_{\omega} \quad \dfrac{\omega' r}{\sqrt{\omega'\Sigma\omega}}$ s.t: $\quad \omega' 1_n = 1$ s.t: $\quad \omega' \geq 0$

***Table 3.4**: GMV and MS under Positivity of weights*

What I have obtained by setting this constraint is the possibility of getting rid of short positions, but the problem of lack of diversification keeps unsolved, since there is still the possibility that all the wealth is allocated in one or a few assets.

3.5.2. Upper and Lower Bounds:

After forbidding short selling, I considered a portfolio with upper and lower bounds. As just said, the positivity of weight constraint does not solve the problem of a possible lack of diversification inside the portfolio. Indeed, chances are that the wealth keeps concentrated on a small number of assets, and this is not optimal, because the link between the performance of the portfolio and the performance of those assets would be too strong in such a case. One way to deal with this problem is by setting Upper and Lower bounds on the weight of the single assets.

For what concern bounds, in my analysis I decided to handle differently traditional assets and Cryptocurrencies. Every single traditional asset has an Upper Bound of 0.3 and a Lower Bound of -0.3. On the opposite, every single Cryptocurrency has the same Upper Bound set at 0.3 and a Lower Bound of 0. The difference between Traditional Asset classes and Cryptocurrencies is that while short selling is allowed for the former (on the limit of 30% for each asset), the same practice is still forbidden for Cryptocurrencies. This choice is related to the impossibility to go short on this kind of assets in the real market.

The optimization problem in the case of upper and lower bounds is set up as it follows (3.23):

$$\begin{aligned} & min_{\omega} \quad \omega'\Sigma\omega \\ \text{s.t:} \quad & \mu_p = \quad \omega' r \\ \text{s.t:} \quad & \omega' 1_n = 1 \\ \text{s.t:} \quad & l \le \omega_i \le u \end{aligned} \tag{3.23}$$

Where l is the lower bound (set at -30% for traditional assets and at 0 for Cryptocurrencies) and u is the upper bound, set up at 30% for every asset in the portfolio. By imposing bounds we are imposing limits on the cardinality of the portfolio: by construction all the resulting weights will be included in the interval [-0.3, +0.3].

For what concern the Efficient Frontier, even in this case we have that it will be both upward and downward limited. Under these circumstances, the identification of the minimum and the maximum returns limits require the solution of two different optimization problems, as in Table 3.5:

Minimum Return (lower frontier bound)	**Maximum Return (Upper frontier bound)**
$min_{\omega} \quad \omega' r$ s.t: $\omega' 1_n = 1$ s.t: $l \leq \omega_i \leq u$	$max_{\omega} \quad \omega' r$ s.t: $\omega' 1_n = 1$ s.t: $l \leq \omega_i \leq u$

Table 3.5*: determination of the Efficient Frontier under upper and lower bounds.*

By construction, the efficient frontier is narrower than the one under no short selling, and it lies on its right side: in this case more restriction have been added, so the investment universe reduces.

Finally, for what concern Global Minimum Variance and Max Sharpe portfolios under upper and lower bounds, the optimization problems for their determination are the following (Table 3.6):

GMV	**MS**
$min_{\omega} \quad \omega' \Sigma \omega$ s.t: $\omega' 1_n = 1$ s.t: $l \leq \omega_i \leq u$	$max_{\omega} \quad \dfrac{\omega' r}{\sqrt{\omega' \Sigma \omega}}$ s.t: $\omega' 1_n = 1$ s.t: $l \leq \omega_i \leq u$

Table 3.6*: determination of the Efficient Frontier under upper and lower bounds.*

Through the inclusion of upper and lower bounds in the portfolio, we are able to reach a higher level of diversification. The smaller is the upper bound, the higher is the level of diversification: in my portfolio I decided to maintain an upper bound (30%) which is quite high considering the number of assets in the portfolio. I made this choice in order not to impose artificially diversification through the setting of too small Bounds.

3.5.3. Group constraints 1 (Crypto 10%) – Group constraint 2 (Crypto 20%)

As the name suggests, this kind of constraint does not concern the single asset position, but group of assets. For example, you can group different assets having some common characteristics and impose that their total weights in the portfolio must be less than a given percentage. In my analysis, I grouped assets according to the class they belong to: Government Bonds with Government Bonds, Equities with Equities, Crypto with Crypto and so on. Then, through group constraint, I put a limit (both upper and lower) on the weight of the group on the total portfolio weight. In the first case the constraints are set up as it follows (Table 3.7):

Group Constraints 1 (GC1)		
Asset Class	Lower Bound	Upper Bound
Government Bonds - Group 1	-0.1	+0.4
Corporate Bonds – Group 2	-0.1	+0.4
Equities – Group 3	-0.1	+0.4
Commodities – Group 4	-0.1	+0.4
Cryptocurrencies –Group 5	+0.1	+0.1

Table 3.7: *Group Constraint 1 (Crypto at 10%)*

In the first group constraint, I forced the asset class "Cryptocurrencies" at the 10% of the total portfolio weight by setting an upper and lower bound to the Cryptocurrency group both equal to 0.10. Technically, this means that for all the periods under analysis the weight of the Cryptocurrency Asset Class does not go below or above the 10% of the total portfolio weight.

For what concern the second Group Constraint (GC2, Table 3.8), Bounds on the traditional asset classes are the same as the previous case (Table 3.7).The only difference is that in this case Crypto are forced at the 20% of the total portfolio weight.

Group Constraints 2 (GC2)		
Asset Class	Lower Bound	Upper Bound
Government Bonds - Group 1	-0.1	+0.4
Corporate Bonds - Group 2	-0.1	+0.4
Equities - Group 3	-0.1	+0.4
Commodities - Group 4	-0.1	+0.4
Cryptocurrencies - Group 5	+0.2	+0.2

Table 3.8: *Group Constraint 2 (Crypto at 20%)*

Technically, in order to set up Group Constraints, one should start from the usual unconstrained optimization problem and add the new constraints, exactly the same I have done in the two previous cases.

Under Group Constraints the optimization problem takes the form specified in (3.24).

$$
\begin{aligned}
& min_{\omega} \quad \omega' \Sigma \omega \\
\text{s.t:} \quad & \mu_p = \quad \omega' r \\
\text{s.t:} \quad & \omega' 1_n = 1 \\
\text{s.t:} \quad & H\omega' \geq \ p \quad and \quad H\omega' \leq h
\end{aligned}
\qquad (3.24)
$$

H is a double dimension Matrix $(q\ x\ n)$, where the first dimension q is the number of groups you want to set up and n is the number of assets in the portfolio. This is a [0, 1] matrix: the first raw defines the first group: assets belonging to the first group are marked with 1, all the others by 0. The second raw is the second group: assets belonging to the second group are marked by 1, all the others by 0. And this until the last raw, which is the last group. By multiplying this Matrix for the weight vector ω' you divide your sample in groups (the sum of weights of the asset marked by 1 in the first raw is the first group, the sum of weights of the asset marked by 1 in the first raw is the first group, and so on). p and h have instead size $(q\ x\ 1)$, and they are the vectors of upper and lower bound each group must satisfy. The equation states that each group resulting from the product $\mathrm{H}\omega'$ must be above its lower bound and below its upper bound.

To give you a practical idea, if you consider my Static dataset including 23 assets from week 260 (18 traditional +5 Cryptocurrencies), the constraint $H\omega' \leq h$ analytically takes the following form:

$$\begin{vmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{vmatrix} \begin{vmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \cdots \\ \omega_{23} \end{vmatrix} \leq \begin{vmatrix} 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.1 \end{vmatrix}$$

The first matrix of the product is the matrix H: the first row defines the Government Bound group, the second defines the Corporate Bond group, the third the Equity group, the fourth Commodities and finally the fifth stands for Cryptocurrencies. The second Matrix is the transposed of the weight vector, comprehending the weight of the 23 assets included in the portfolio, and finally the vertical vector h defines the upper bounds that the groups must satisfy.

Even in this case the problem may not have an analytical solution, for this reason it is typically solved through numerical methods.

Looking at the determination of GMV and MS portfolios we have that their determination follows the two problems included in Table 3.9:

GMV	MS
$min_\omega \quad \omega'\Sigma\omega$ s.t: $\omega' 1_n = 1$ s.t: $H\omega' \geq p \quad and \quad H\omega' \leq h$	$max_\omega \quad \dfrac{\omega' r}{\sqrt{\omega'\Sigma\omega}}$ s.t: $\omega' 1_n = 1$ s.t: $H\omega' \geq p \quad and \quad H\omega' \leq h$

Table 3.9: *GMV and MS under Group Constraint*

3.5.4. Turnover Constraint

Turnover is defined as a measure of how many positions the portfolio changes between two different points in time: mathematically it is the sum of the absolute values of the weight difference between period t and t+1:

$$\text{Turnover} = \sum|\omega - \widetilde{\omega}| \tag{3.24}$$

Where $\widetilde{\omega}$ is the actual portfolio composition.

Why does it make sense to implement a turnover? Because portfolio changes are not free of charge: every trade implies some costs. One of the main problems that Portfolio Managers have are the Transaction Costs, which are those costs that arise when one wants to replace an existing position in his portfolio with a new one. In the Markowitz approach, transaction costs are not taken into account. Typically, Transaction Costs are computed as the sum of the penalties for the closing of the existing position and the opening of the new one: they include commissions paid to the broker, spreads, taxation fees (such as the taxation on the capital gain if the position that the investor is going to close is gaining). So, what we can say is that the higher the portfolio turnover, the higher the transaction costs associated to it: considered this, it is reasonable to assume that every portfolio Manager aims at reducing this kind of costs which can be considered as some sort of sunk costs.

There are two different ways through which this aim can be reached, and these two ways are linked to some extent together: first, one could reduce the frequency of the trades. Second, one could reduce the quantity of trades. The two are linked together because one implies the other: if you reduce the frequency of the trades you are also reducing the amount of trades over the period and vice versa.

Personally, I originally constructed my dataset based on daily data, but I soon concluded that this wouldn't have been a reasonable choice, precisely because of the huge amount of costs that I would have incurred by changing daily the composition of my portfolio. Costs that would not have appeared in the analysis since Markowitz approach does not take into account transaction costs. In such a case, results would not have been truthful. This is the reason why I decided to reduce the frequency of data and I switched from daily to weekly. Then, in order to mitigate the problem further, I implemented among the others constraints a portfolio with a turnover constraint: a turnover constraint put some sort of limit to the trading activity, for example limiting the amount of the portfolio that could be changed from period to period.

The optimization problem now becomes:

$$\begin{aligned} & min_{\omega} \quad \omega'\Sigma\omega \\ \text{s.t:} \quad & \mu_p = \omega' r \\ \text{s.t:} \quad & \omega' 1_n = 1 \\ \text{s.t:} \quad & |\omega - \tilde{\omega}|'^{1_n} \leq \tau \end{aligned} \tag{3.25}$$

Where τ is the limit on the turnover expressed as the fraction of the portfolio that changes: for example in my analysis I imposed a 30% turnover, meaning that no more than 30% of the portfolio composition is allowed to change from week t to week $t+1$.

Even under the turnover constraint, the optimization problem cannot be solved analytically, so it is solved through the help of numerical methods.

We can identify the Global Minimum Variance and the Max Sharpe portfolios:

GMV	**MS**
$min_{\omega} \quad \omega'\Sigma\omega$ s.t: $\omega' 1_n = 1$ s.t: $\lvert\omega - \tilde{\omega}\rvert'^{1_n} \leq \tau$	$max_{\omega} \quad \frac{\omega' r}{\sqrt{\omega'\Sigma\omega}}$ s.t: $\omega' 1_n = 1$ s.t: $\lvert\omega - \tilde{\omega}\rvert'^{1_n} \leq \tau$

Table 3.10: *GMV and MS under Turnover Constraints*

The reason that lead me to implement this kind of constraint in my model is clearly to limit the impact of transaction costs to the final result: it is worthless to construct a portfolio with a satisfying return if the amount of transaction costs is so huge that it erodes all the gain.

Chapter 4:

The Risk Budgeting analysis

Until now we have followed the approach by Markowitz, whose main scope is the determination of the optimal portfolio weights. I will now switch to a different kind of analysis called Risk Budgeting, introduced by Roncalli T. in "Introduction to risk parity and budgeting" (2013): while the main focus of the approach by Markowitz was the determination of the optimal portfolio weights, on which I also imposed different kind of constraints according to the characteristics I wanted the resulting optimal portfolio to satisfy (weight budgeting), now the main subject of the analysis becomes Risk. In particular, Risk Budgeting approach imposes constraints on the risk contribution of each assets to the total portfolio risk.

First of all, we need to define a generic measure of the portfolio risk, call it $\mathcal{R}(\omega)$, defined by the portfolio weight vector ω. Looking at the single asset, the risk contribution $\mathcal{R}C_i$ of the asset i to the portfolio P is defined as the first derivative of the risk contribution measure $\mathcal{R}(\omega)$ with respect to the asset weight ω_i, multiplied by the weight ω_i (Equation 4.1):

$$\mathcal{R}C_i = \omega_i \frac{d\mathcal{R}(\omega)}{d\omega_i} \tag{4.1}$$

Until now I have talked about a "generic Risk Measure" $\mathcal{R}(\omega)$: in order to be more precise, one of the most important properties this generic risk measure must satisfy is the so called Euler Decomposition Rule, implying that the total Portfolio Risk Contribution $\mathcal{R}(\omega)$ must be a linear combination of the Risk Contributions of the single assets $\mathcal{R}C_i$:

$$\mathcal{R}(\omega) = \sum_{i=1}^{n} \omega_i \frac{d\mathcal{R}(\omega)}{d\omega_i} = \sum_{i=1}^{n} \mathcal{R}C_i \tag{4.2}$$

If we consider our formula for the standard deviation ($\sigma(\omega) = \sqrt{\omega'\Sigma\omega}$) as the measure of risk of the Portfolio, we will end up with a Risk Contribution which takes the form as in (4.3):

$$\mathcal{R}C_i = \omega_i \frac{d\mathcal{R}(\omega)}{d\omega_i} = \omega_i \frac{d\sigma(\omega)}{d\omega} = \omega_i \frac{(\Sigma\omega_i)}{\sqrt{\omega\prime\Sigma\omega}} \tag{4.3}$$

Risk Contribution can be computed both ex-ante both ex-post. In the ex-post case one is simply verifying the realized risk contributions given the optimal portfolio weights resulting for example from the Markowitz approach. In the ex-ante case, one is firstly setting limit on the risk contributions, and then computing the optimal weights based on them. Clearly, I followed the ex-ante approach to develop the risk-budgeting analysis.

The first step of the risk budgeting approach is the determination of the risk budgets: risk budgets are monetary amounts representing the risk in a given asset class: in our analysis we can represents risk budgets as percentages of the overall portfolio risk. For example, if one set a given asset class to contribute for the 30% of the total portfolio risk, the risk budget b_i associated to that asset class will be $b_i = 30\%$. Under these circumstances, risk budgets will be defined as it follows:

$$\begin{cases} \mathcal{R}C_1 = b_1\mathcal{R}(\omega) \\ \mathcal{R}C_2 = b_2\mathcal{R}(\omega) \\ \mathcal{R}C_3 = b_3\mathcal{R}(\omega) \\ \quad\quad\;\; \dots \\ \mathcal{R}C_n = b_n\mathcal{R}(\omega) \end{cases} \tag{4.4}$$

Where $\mathcal{R}C_i$ is the risk contribution of asset i to the total portfolio risk, b_i is the risk budget assigned to the asset i expressed as a percentage of the total risk of the portfolio, and $\mathcal{R}(\omega)$ is the total risk of the portfolio. n is the number of assets in the portfolio, so in the Static approach portfolios will end up with $n = 23$, while in the Dynamic approach $n = 27$ at the end of the analysis. What I am doing here is simply a decomposition of the total portfolio risk into the risks associated to the various assets.

For what concern the solution to this problem, the main difference with respect to the Markowitz approach is that here we have n non-linear equations in n unknowns, and we do not have a criterion function to be minimized/maximized. This means that we can’t solve the usual optimization problem such as the one characterizing the approach by Markowitz. In my analysis, I have applied risk budgeting together with the no-short selling constraint, in order to ensure that all the asset in the portfolio have positive weights despite their level of risk contribution. In such circumstances, the problem is defined as it follows:

$$\begin{aligned} &\mathcal{R}C_i = b_i\mathcal{R}(\omega) \\ s.t \quad & b_i \geq 0, \quad i = 1 \dots n \\ s.t \quad & \omega_i \geq 0, \quad i = 1 \dots n \\ s.t \quad & \sum_{i=1}^{n} b_i = 1, \quad i = 1 \dots n \\ s.t \quad & \sum_{i=1}^{n} \omega_i = 1, \quad i = 1 \dots n \end{aligned} \tag{4.5}$$

The first and the second constraints impose the positivity of weights and the positivity of the risk budgets (it does not make any sense from an economic perspective to put a negative risk contribution, in this case this simply would be 0). The third and the fourth constraints impose the summation of budgets and weights equal to 1.

It is important to underline that the problem as defined in (4.5) does not have an analytical solution. For this reason, an alternative approach must be used in order to determine the optimal portfolios. One of the possible solutions could be the definition of an objective function to be optimized. In my analysis the role of the objective function is taken by (4.6), which represents the summation of the squared difference between the actual Risk Contribution of each asset and the Risk Budget assigned to that asset. Analytically the equation takes the following form:

$$f(\omega, b) = \sum_{i=1}^{n} (\omega_i \frac{d\mathcal{R}(\omega)}{d\omega_i} - b_i \, \mathcal{R}(\omega))^2 \tag{4.6}$$

The minimization of this squared difference should ensure that the risk contribution of each asset will be equal to the target risk contribution assigned to that asset, which is precisely what we are looking for.

So the risk budgeting approach could be reduced to a minimization problem which takes the following form:

$$\begin{aligned} &min_{\omega} \; f(\omega, b) \\ &s.t. \quad \omega' 1 = 1 \\ &s.t. \quad \omega' \geq 0 \end{aligned} \tag{4.7}$$

This is very similar to the usual Markowitz optimization problem except for the form of the objective function. The optimal solution provides the weights corresponding to the risk budgets imposed.

Following this approach, one could set whatever budget on risk he deserves. For example in my analysis, I set up various portfolios with risk budgets: firstly, an Equally-Risk-Contribution portfolio, where each asset contributes equally to the total portfolio risk. So, dealing with the static approach, risk budgets will be $b_i = 1/18$ for the first period, when only the traditional assets are available, and finally $b_i = 1/23$ from the entrance of Cryptocurrencies. On the opposite, in the Dynamic approach, the risk budgets will range from $b_i = 1/18$ in the first period to arrive to $b_i =$

$1/27$ when all the Crypto are available. However, I will explain better this mechanism in Chapter 5 through the help of some risk contribution area plots. What I expect to find in this case is that, given the same risk budgets, those assets that are less risky than the others (such as Government Bonds and Corporate Bonds) will have more weight on the portfolio than those which are riskier (Equities, Cryptocurrencies). I will verify this assumption in Chapter 5. Then, I set up two other Generalized-Risk-Contributions portfolios (GRC1 and GRC2): differently from the ERC case, in the first one (GRC1) I forced Cryptos to contribute for the 10% of the total portfolio risk, while in the second one (GRC2) I forced Cryptos at the 30% of the portfolio risk. Even in these cases, considering the high level of volatility (which is our measure of risk) of Cryptocurrencies, I expect that their weights won't exceed reasonable levels even if their risk contribution is quite high. Finally, I included the usual GMV, MS, RL, MRA, RA portfolios, constructed via the risk budgeting optimization problem.

In order to implement this model in Matlab, two different inputs are needed: first, we need a script containing the criterion function $f(\omega, b)$ to be minimized, second, we need a Matlab function performing numerical optimization. For what concern the Equally Risk Contribution case I used the script called ERCfun.m. For the two Generalized Risk Contribution portfolios I used the function GRCfun.m. What is the difference between the two? Simply the construction of the objective function: the ERC script it is constructed exploiting the fact that all the risk budgets are equal, while the GRC script is different. Analytically, the two cases take the following form:

$$\textbf{ERC:}\ f(\omega, b) = \Big(repmat\big(RC, 1, lenght(RC)\big) - repmat(RC', lenght(RC), 1)\Big)\hat{}2 \tag{4.8}$$

Where "repmat" is a Matlab function which repeats copies of arrays: for what concern the first element of the difference it copies the vector RC (risk contr.) along one raw and a number of column equal to the length of the vector RC, creating a matrix of dimension (1, length(RC)). For the second element it copies the transpose of the vector originating a matrix (length(RC),1).

$$\textbf{GRC:}\ f(\omega, b) = \sum (RC - bR).* (RC - bR) \tag{4.9}$$

Where R is our risk measure, so it is the portfolio volatility computed in the usual way.

The main difference between the two functions is that the former takes only two inputs (weights and covariances), the second instead takes three inputs: (weights, covariances and the vector of risk budgets b). The problem is that in both cases the function must be minimized changing only weights, leaving the other inputs unaffected. In order to deal with this, an anonymous function for portfolio weights is set up as it follows (4.10 and 4.11):

$$fERC = @(\omega)ERCfun(\omega, Sigma) \tag{4.10}$$

$$fGRC = @(\omega)GRCfun(\omega,, budgets) \tag{4.11}$$

Where $Sigma$ is the variance covariance matrix, ω is the weight vector, $budgets$ is the vector of risk budgets.

For what concern the second element needed, which is the Matlab function performing the numerical optimization problem, I used the “fmincon” function, which uses as target the anonymous functions just defined and it provides the optimal portfolio weights as output. The fmincon function simply attempts through numerical methods to find a minimizer of the objective function (first input of the fmincon) starting from a given point, in my case initial portfolio weights (second input of the fmincon), subject to the linear constraints (for example no short selling) specified as the following inputs in the fmincon function.

The solution of this problem, if it exists, is the vector of optimal portfolio weights according to the risk budgets I have imposed to the assets included into the portfolio itself. From the optimal weights, one can simply compute the realized returns of the optimal portfolios over the period under analysis.

Chapter 5

Results: Return Analysis

As I have already mentioned in the previous stages of my work, the solution to the optimization problems as defined in Chapter 3 (Markowitz) and in Chapter 4 (Risk-Budgeting) provides us the vector of optimal weights for a given portfolio over the period under analysis. Taken per se, optimal weights do not tell us very much about the realized performance of the correspondent portfolio, neither weights alone allow us to compare the performance of the portfolios relative to each other. However, by multiplying the vector of optimal weights ω_t of a given strategy by the vector of asset returns r_t we can easily obtain Realized Returns of that strategy, call it "r_i". Unfortunately, neither realized returns are very helpful to assess a proper comparison among the various approaches: in order to compare the various portfolios relative to each other, the basic element we need are the Cumulated Returns R_t. Cumulated Return of a given portfolio can be easily computed starting from the Realized Return according to the formula included into the following Equation (5.1):

$$R_t .=[\ \prod_{i=w+1}^{t}(1+\ r_i)\ ]-1 \tag{5.1}$$

All the Comparisons you will find for the rest of this Chapter are based on Cumulated Returns, and also all the plots include Cumulated returns. After having done this necessary premise, we can now move to the core of this chapter, which is the Return Analysis. In order to provide an initial answer to my research question, which is to what extent Cryptocurrencies can be useful to boost the performance of the portfolios, I will firstly focus on returns of the optimal portfolios resulting from Markowitz problem (Chapter 3). In particular, I will try to assess if the Portfolios including Cryptocurrencies perform better or worse in terms of returns than those without Cryptocurrencies. The return analysis has been conducted by plotting the cumulated returns of a given portfolio without Cryptocurrencies versus its counterparty (same risk adversion and same constraints) including Crypto, trying to assess if the Portfolios including Cryptocurrencies perform better or worse in terms of returns than those without Cryptocurrencies. I am aware that focusing only on returns may be reductive, because the portfolios with Cryptocurrencies are likely to show very high volatility with respect to those without, worsening in this case the risk-return trade off. However, a return analysis can be considered a good starting point. Then, I will perform the Return Analysis in the same manner for those portfolios resulting from the Risk Budgeting analysis (Chapter 4).

5.1. Markowitz strategy returns

In this section I will present the output of the analysis by Markowitz.

Given the considerable amount of portfolios involved in my Markowitz analysis (50 Rolling + 50 EWMA portfolios), I will show in this section only the most relevant cases, including all the others into the appendix. Where does these 50 portfolios come from? You have to consider that both Rolling and EWMA include 2 approaches, so we will have 25 Static and 25 Dynamic portfolios. Why 25? Because each approach involves 5 different portfolios (GMV, MS, Risk Lover, Moderate Risk Adverse, Risk Adverse) each of them in turn with a set of 5 constraints (NS, Bounds, GC1, GC2, Turnover). So we will have 5 GMV portfolios, 5 MS portfolios and so on.

The identification of the portfolios could be problematic, so in the following lines I will show the logic behind the nomenclature process. Every portfolio is identified through a set of four attributes: the first attribute identifies the type of Returns and Covariances included (EWMA or Rolling), the Second attribute identifies the Static or the Dynamic approach. The third attribute identifies the kind of portfolio (GMV, MS, Risk Lover, Moderate Risk Adverse, Risk Adverse), and finally the fourth the weight constraint adopted for its construction (NS, Bounds, GC1, GC2, Turnover). So, for example, we will have a Rolling-Static-GMV-NS portfolio to identify a Static Global Minimum Variance with rolling inputs and positivity of weight constraint. Or a Dynamic-EWMA-MRA-GC1 to identify a Moderate Risk Averse Dynamic portfolio with EWMA returns and covariances and Crypto at 10% of the total weight (Group Constraint 1). In this way, we can identify all the 100 portfolios via these four attributes.

After having performed the return analysis, I will switch to a deeper analysis focusing not only on returns but also on risk. In particular, I will conduct this analysis through the mean of different performance indicators that take into account not only the return, but also the risk. This will allow me to rank all the spectrum of the portfolios relative to each other, in order to assess not only if the portfolios with Crypto are better than those without (giving in this case a positive answer to my research question), but also, among those portfolios including Crypto, if the Dynamic approach is better than the other or viceversa.

Structure of the Analysis.

In this section, I will focus on the returns of the optimal portfolios resulting from my analysis. As already mentioned, I will present in this paragraph only the most relevant ones, while I will include all the cases together in the Appendix. Into the Appendix you can find four figures (Figure 1-Appendix, Figure 2-Appendix, Figure 3-Appendix, Figure 4-Appendix), each of them including 25 plots where returns of the portfolio without Crypto are plotted against the corresponding portfolio including Crypto, where "corresponding" means with the same risk adversion and the same set of constraints. The first two figures are respectively the Rolling Static and Dynamic approaches portfolios, while the third and fourth include the EWMA Static and Dynamic portfolios respectively. In each of the four figures the plots are distributed following a $5\ x\ 5$ grid, where the horizontal dimension is the kind of portfolio (GMV, MS, RL, MRA, RA) while the vertical dimension is the Constraint adopted (NS, Bounds, GC1, GC2, Turnover). To give an idea, you have to treat each figure as a matrix, structured as it follows:

GMV-NS	GMV-Bounds	GMV-GC1	GMV-GC2	GMV-Turnover
MS-NS	MS-Bounds	MS-GC1	MS-GC2	MS-Turnover
RL-NS	RL-Bounds	RL-GC1	RL-GC2	RL-Turnover
MRA-NS	MRA-Bounds	MRA-GC1	MRA-GC2	MRA-Turnover
RA-NS	RA-Bounds	RA-GC1	RA-GC2	RA-Turnover

The first thing we can notice by looking at Figure 1-Appendix and Figure 2-Appendix is that always the returns of the base portfolios without Crypto and the returns of the corresponding portfolios with Crypto coincide for the beginning weeks under analysis. This is because you have to consider that Cryptos do not enter the portfolio from the beginning of the analysis, which is 2008, but they enter later, in particular in May 2013 (week 260) for what concern the Static approach and in July 2010 (week 115) for the Dynamic approach. For this reason, the two returns coincide for the first period and then they start to differentiate when Cryptos enter into the game. The blue line represents the return of the portfolio including Crypto, while the red line represents the return of the portfolio without Crypto.

5.1.1. Static approach Portfolios

I will now focus on the static portfolios (figure 1 and figure 2 Appendix). What we can observe is that for almost all the strategies the return of the portfolio including Cryptocurrencies exceed the return of the portfolios without Cryptocurrencies. We could expect this, since in the period under analysis Crypto have grown a lot in terms of value, so all the portfolios including them have exploited this growth, enhancing the returns. There are only four portfolios including Crypto out of one hundred in total whose return seems to suffer at least initially, going below the corresponding portfolios without Crypto. In particular, I am referring to the Static-MaxSharpe portfolios with Group Constraints. This problem concerns both the Rolling and the EWMA approaches. I report these two cases below (Figure 5.1 and Figure 5.2).

ROLLING

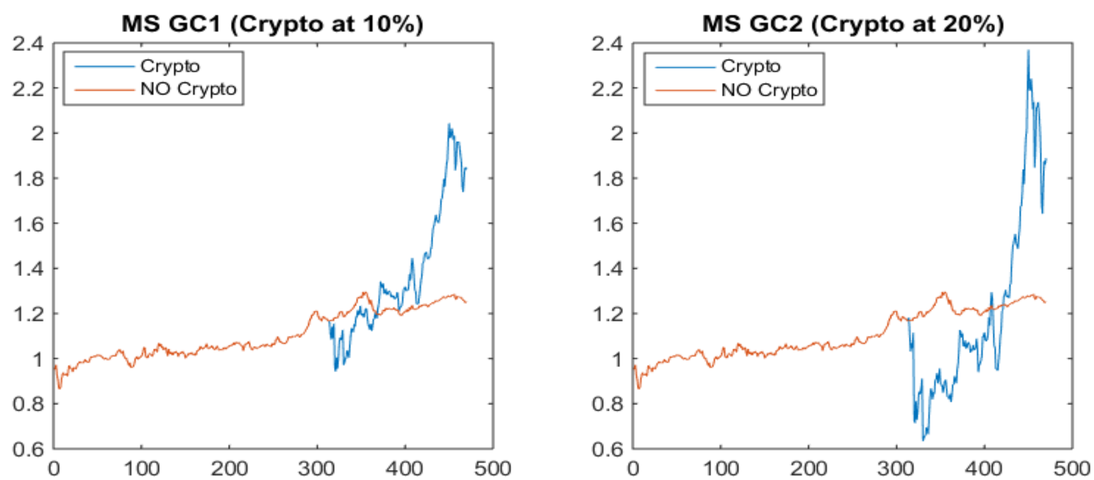

Figure 5.1*: Rolling MaxSharpe Portfolios with Group Constraints, Cumulated Returns*

EWMA

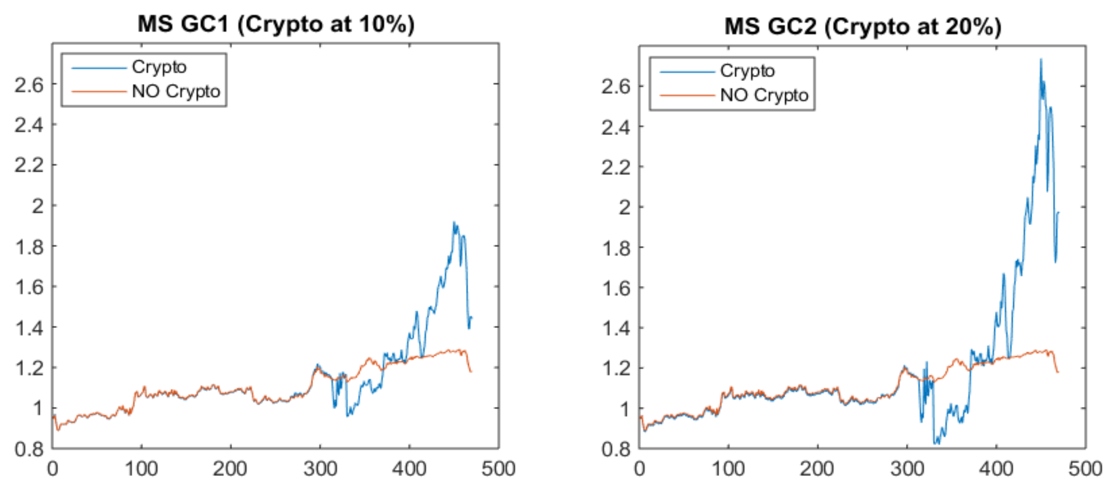

Figure 5.2*: EWMA MaxSharpe Portfolios with Group Constraints, Cumulated Returns*

As you can see from Figure 5.1 and Figure 5.2, the problem is even more pronounced in both cases for the Group Constraint 2 portfolios, where the Crypto are forced at the 20% of the total portfolio weight. What we can say about this evidence is that Max Sharpe portfolios plus Group Constraint are probably the riskiest portfolios among my investment universe, showing the highest level of volatility (and you can see this also from the return plots themselves). By imposing a constraint that forces Crypto assets to remain at the 10%/20% of the total portfolio weight respectively, for sure the portfolio will replicate at least partially their return. In fact, the downside pattern of these portfolios coincides with the first descending Cryptocurrencies phase, happening after the 2013 peak (see Chapter 2). This descending phase is then followed by a rapid and strong recovery that make these portfolios the ones with the highest return at the end of the period under analysis. So finally, returns for these strategies seem to be very high but at a price of very high volatility. Is the trade-off worth it? We will verify this later through the help of the performance indicators.

Going on with the Static approach, another interesting thing we can notice concerns the GMV portfolios, both Rolling and EWMA (First raw of plots from Figure 1-Appendix and Figure 2-Appendix). Here the interesting thing is that if we exclude the portfolios with Group Constraints that force Crypto inside the portfolio, returns of the portfolios without cryptocurrencies (red) and of the portfolio with cryptocurrencies (blue) do not differ so much relative to each other. This is also true (but to a smaller extent) for the other portfolios characterized by a high degree of risk aversion (RA, fifth raw, and MRA, fourth raw). Why does this happen? The answer relies precisely on the high degree of risk aversion that characterizes these portfolios, which are mainly composed by assets with a low level of volatility such as Government Bonds or Corporate Bonds (indeed the cumulated return of these strategies at the end of the period is lower than the other strategies). Typically, the returns of the portfolios including Cryptos differentiate from the relative base portfolio without crypto starting from the period when Cryptos enter the dataset of the portfolio including them. The fact is that for these very risk adverse portfolios, despite the availability of Cryptocurrencies from a certain point in time, these portfolios continue to be invested in those safe assets without benefitting of cryptocurrencies availability. In other words, the Markowitz optimization problem for these portfolios sets the weights of the Crypto assets at zero (Crypto are too risky for the parameters of the problem), so there is no differentiation with respect to the base portfolio, and the two returns continue to be equal. To provide further evidence to confirm the absence of the Crypto, I include below the area plots of weights (figure 5.3 and 5.4) for the GMV portfolios I pointed out before (excluding the Group Constraints where Cryptos are artificially forced inside the portfolio, so that there is indeed a differentiation in returns). This is to show that these portfolios indeed do not include Crypto. The X axis of these area plots represents time (number of weeks=470), Y represents the total Portfolio weights. What you can observe from these area plots is the evolution of assets weights over time for a given portfolio.

Rolling

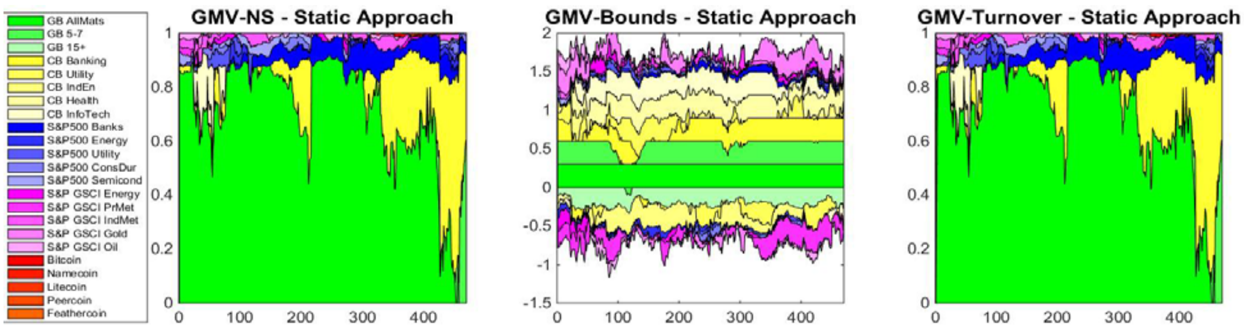

Figure 5.3*: Weights Area Plots GMV Portfolios - Rolling*

EWMA

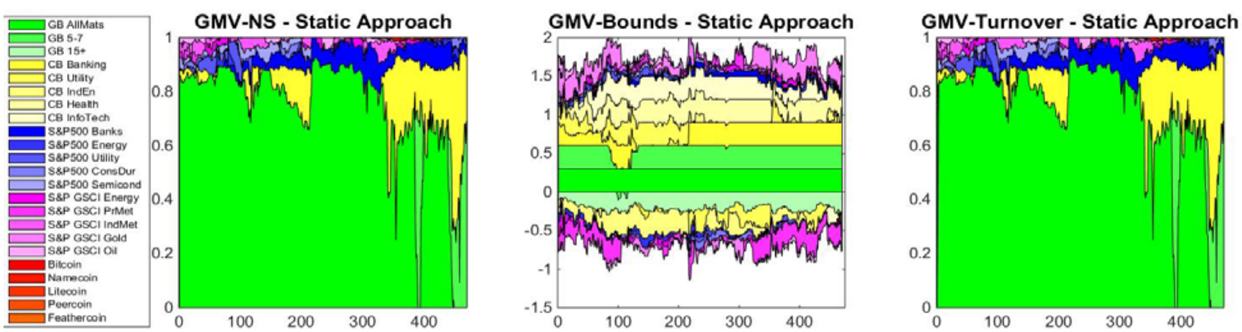

Figure 5.4*: Weights Area Plots GMV Portfolios - EWMA*

What it can be noticed from the area plots included in Figures 5.3 and Figure 5.4 is the almost absolute absence of Cryptocurrencies (whose weights would be marked in red/orange) for all the time being in these portfolios. This confirms my hypothesis made before concerning the reason why the returns of these crypto-portfolios do not differentiate much from their corresponding base portfolios: Cryptocurrencies are too risky to enter them.

Going on with the analysis, the Static approach strategies whose returns differ the most from the corresponding base portfolios are the riskiest ones, such as MaxSharpe portfolios (second raw of plots) or the Risk Lover ones (third raw), even more if associated to a Group Constraint. Not by chance the strategies with the highest cumulated return are the Group Constraint 2 ones (fourth column for both the Rolling and EWMA figures) where crypto are forced at the 20% of the portfolio weight. However, at a price of more volatility.

To conclude with the Static approach, the final thing we can observe is that returns of the portfolios with turnover constraint (fifth column) are very similar to the returns of no short selling portfolios (first column). This is true for both the Rolling and the EWMA cases. This simply means that the turnover constraint applied to the no short selling case is not effective, because even without this constraint the weights of the portfolio do not change more than 30% (the maximum turnover I have imposed) from one period to the next one. So, the returns seems to be equal. You can realize this also looking at the weights area plots in the previous page: the third area plot in Figure 5.3 and Figure 5.4, which is the one of No short selling plus turnover constraints is very similar to the first one, which is just no short selling.

The final thing we can observe is that none of the portfolios including Cryptos has a return at the end of the period that lies below the corresponding portfolio with Crypto, suggesting that cryptocurrencies effectively improve portfolio performances, at least in terms of returns.

5.1.2. Dynamic approach Portfolios

In this paragraph, we will concentrate on Figure 3-Appendix and figure 4-Appendix, which include the Rolling Dynamic approach portfolios and the EWMA Dynamic approach portfolios respectively. Remember that the Dynamic approach differentiates from the Static to the extent that it includes 9 crypto versus 5 and that in the Dynamic every Crypto is included from the date on which it is available. The structure of the figure is the same as for the Static approach: the plots are distributed following a $5x5$ grid, where the rows are the kind of portfolio and the columns are the constraint adopted. There is one plot for every strategy, and each plot includes the return of the portfolio including Crypto (blue) against the return of the corresponding base portfolio without crypto (red).

For what concern the comparison within the Dynamic approach, I can make similar considerations to the ones I have done for the Static approach. For this reason, I will now concentrate on the differences between the two approaches to highlight the peculiarities of the Dynamic approach.

The first thing we can notice for what concern the Dynamic portfolios is that the severance between the portfolio including Crypto from the correspondent base portfolio starts earlier than what happens in the Static approach. You have to consider that in the Static Approach Cryptocurrencies enters the portfolios all together starting from May 2013, while in the Dynamic approach the Bitcoin, which is the first available Crypto, starts from July 2010. That’s why the differentiation starts earlier in the Dynamic approaches, because the assets that differentiate the portfolios from their correspondent base portfolio are included earlier than what occurs in the Static portfolios.

Another fact I have pointed out in the Static approach was that the cumulated returns of some strategies, at least initially, were below the return of the correspondent base portfolio. Here it is not the case: the problem in the Static approach was that Cryptos started to be included in concomitance to the descendant phase of their first peak (see Chapter 2), so by forcing them inside the portfolio (as it happens in the Group Constraint MaxSharpe cases) the return of the optimal portfolio for sure would suffer. Things are different for what concern the Dynamic approach, and now I will explain why. As I have just mentioned the Bitcoin is included earlier in the Dynamic than in the Static approach, so the Dynamic portfolios do not only suffer the descendant phase of the 2013 peak, but they also benefit their ascendant phase that is not available for the Static portfolios, simply because Crypto had been included later. For this reason, even for those portfolios whose return seem to suffer in the Static case (and I am referring in particular to the MaxSharpe portfolios with Group constraints reported in Figure 5.3 and 5.4), in the Dynamic approach the Cumulated returns of these portfolios lie for all the time being above the return of base portfolios. You can realize this from the plots below (Figure 5.5) where I compare the MaxSharpe with Crypto at 20% strategy Static (first plot) versus its Dynamic counterparty (second plot).

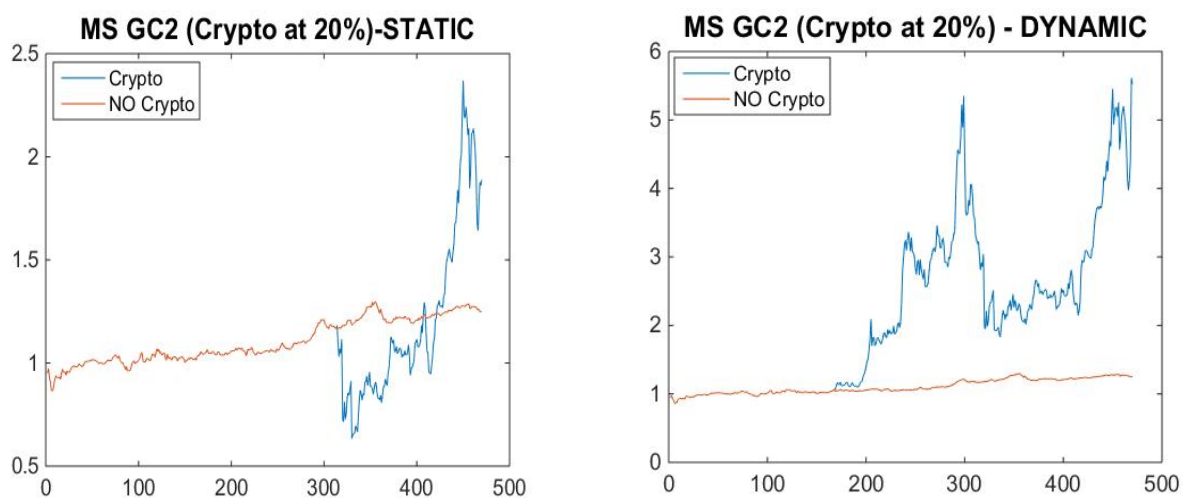

Figure 5.5: *MS Portfolios with Crypto at 20% of weight: Static strategy vs Dynamic strategy*

We can say that for what concern the Dynamic approach, in all the 50 strategies portfolios-with-crypto returns exceed returns of the correspondent base portfolio without crypto. Going on with the analysis, another aspect that differentiates Dynamic returns from the Static ones stems from the fact that in some Static strategies returns of the portfolios with Crypto is equal to the return of the correspondent portfolio without cryptocurrencies. In particular, I am referring to the cases highlighted in the previous paragraph, such as the GMV-Static-Ns, the GMV-Static-Bounds or the GMV-Static-Turnover, both for Rolling and EWMA. Previously, I have explained this evidence starting from the fact that these portfolios have a very high degree of risk aversion, so the optimization problem sets the weights of the risky assets such as Crypto at 0. For what concern the Dynamic approach, we can say that the situation seems to improve a bit for the Rolling strategies, indeed here GMV returns exceed those without Crypto. Considering the EWMA instead, here the situation does not improve and the returns of these high-risk-adverse strategies do not differentiate from the relative base portfolios without Cryptocurrencies.

Finally, probably the most important thing, we can say that all the Dynamic strategies seem to have a higher cumulated return in absolute value if compared to the correspondent Static strategies (it is sufficient to look at the previous comparison between the Group-Constraint-2 Static and the Group-Constraint-2 Dynamic, Figure 5.5). This evidence is related to two different facts: firstly, the Bitcoin, which is the cryptocurrency experiencing the stronger growth among all the others, persists for more time in the Dynamic than in the Static approach. This has for sure contributed to enhance the return of the Dynamic strategies more than what happens for the Static strategies, where the Bitcoin enters only in week 260 (May 2014). Secondly, Dynamic portfolios are characterized by the presence of new cryptocurrencies, such as the Ethereum or the Ripple, which have experienced strong growth over the last months. These new cryptocurrencies are not available in the Static approach, so Static portfolios did not benefit from their growth.

I will assess in the following chapter if these higher returns happen at the price of a higher volatility for the dynamic strategies with respect to the static ones or if effectively dynamic strategies are better than the static ones. I will do this through the mean of the performance indicators.

5.2. Risk Budgeting Returns

In this section, I will present the return analysis for the Risk-Budgeting portfolios. The structure of the analysis is similar to the one followed to perform Markowitz analysis, but the number of portfolios on which I performed Risk-Budgeting is smaller than in Markowitz. This is because I decided to apply this kind of analysis only to the No-Short-Selling (NS) case: it does not make sense to apply risk budgeting also to portfolios subject to alternative strict weights constraints such as Bounds or Group Constraints. In fact, chances are that an optimal solution to such a problem results in a portfolio which is not feasible due to the overlapping of the weights constraints and of the Risk Budgeting itself. For this reason, I applied risk budgeting analysis to the less restrictive among the five constraints, which is No Short Selling, letting the risk-budgeting to make his job. Moreover, in the risk budgeting there are two Generalized Risk Contribution cases (GC1 and GC2), where I forced Cryptocurrencies respectively at the 10% and 30% of the total portfolio risk, could play the role of some sort of Markowitz Group Constraint on weights. The only difference is that here I am not working directly on weights but I am working on them indirectly by imposing risk budgets, following the methodology shown in Chapter 4. Even in this case, I present the results of this approach by plotting the cumulated return of each portfolio including Cryptocurrency against its without-Cryptocurrencies counterparty (same risk budgets), in order to identify to what extent Crypto are able to enhance portfolio performance, at least in terms of returns (for now). In chapter 6 instead I will focus not only on returns but also on risk, through the help of the already mentioned performance indicators.

In order to sum up, my risk budgeting analysis is structured as it follows: two different mean-covariance inputs (Rolling and EWMA, as in Markowitz), two different approaches (Static, with five cryptocurrencies entering the dataset all together from 2013, and Dynamic, with nine crypto starting from 2010, as in Markowitz), only one constraint (which is No Short Selling, differently from Markowitz characterized by five constraints), and finally three different sets of risk budgets beside the usual set of 5 portfolios (GMV, MS, RL, MRA, RA), so 8 portfolios. The three portfolios with Risk Budgets are: Equally-Risk-Contribution (ERC), where each asset contributes equally to the total portfolio risk, Generalized-Risk-Contribution-1 (GRC1), where cryptocurrencies are forced at the 10% of the portfolio risk and Generalized-Risk-Contribution-2 (GRC2), where Crypto contribute for the 30% of the total portfolio risk. Clearly, the most interesting portfolios under this analysis are the last three portfolios. This analysis has resulted in a total of 32 portfolios, 16 Rolling and 16 EWMA, (8 Static + 8 Dynamic), and similarly to what I have done for Markowitz, each strategy including Crypto is plotted against its corresponding base case without Crypto. The plots are included into the appendix: in Figure 5-Appendix I have included the Static strategies: the first and the second columns of plots include portfolios with Rolling inputs, while the third and the fourth one portfolios with EWMA inputs. The same is true for Figure 6-Appendix, with the only difference that here I included Dynamic portfolios.

5.3.1. Static Approach Portfolios

For what concern the risk contribution for the Static portfolios (Figure 5-Appendix), the first thing we can notice is that the No Short Selling GMV portfolios show the same problem pointed out for their Markowitz counterparties. In particular, the return of the GMV portfolios with Crypto is almost equal to their relative portfolios without Crypto. This means that also in the risk budgeting the optimization problem seems to set Crypto weights at zero due to the high degree of risk aversion for the GMV portfolios. I will check this out in the following rows.

For what concern the other strategies, the one showing the highest cumulated return is the MaxSharpe, both for the Rolling and the EWMA cases. However, MaxSharpe portfolios seem also the most volatile among the others, while the RL, MRA and RA are some sort of intermediate case between the 2. Switching to the three portfolios with artificial risk budgets, the peculiarity of the Equally Risk Contribution (ERC) portfolios is that, as the name suggests, each asset contributes equally in terms of risk to the total portfolio risk. This means that in the Static approach we will have a total amount of 23 assets from week 260 when Crypto enters the game, so the risk contribution for each asset is $1/23$ in this period. Provided that the number of Crypto included in this approach is 5, the total risk contribution for the Crypto asset class is $5/23 \approx 22\%$. For this reason, we can consider ERC in the Static approach as some sort of middle way between the GRC1 case (Crypto at 10% of the total risk) and GRC2 case (crypto at the 30% of the total risk).

In order to better understand the results, I think it is useful to include the area plots showing the assets risk contributions during the period under analysis (Figure 5.6). In the following area plots the X axis represents time (in my case 470 weeks), while the Y axis each asset risk contributions. The result is shown below (for sake of space I have included only the Rolling portfolios, omitting also the RL, MRA and RA cases). The output is very similar for the EWMA-Static portfolios.

In order to facilitate the consultation of these area plots, I have assigned a different color for different asset classes. In particular, Government Bonds risk contributions are marked in green, Corporate Bonds risk contributions in yellow, Equities risk contributions in blue, Commodities in violet and finally the Crypto risk contribution is marked in Red/Orange. In this way, if for example you are looking at the total risk contribution of the Cryptocurrencies' asset class it is sufficient to look at the portion of the area plot that takes the color red/orange. If you are trying to assess the risk contribution of the Government Bond asset class, you have to look to the portion of the area plot that is marked in green. The same is true for all the other assets classes included in my portfolios.

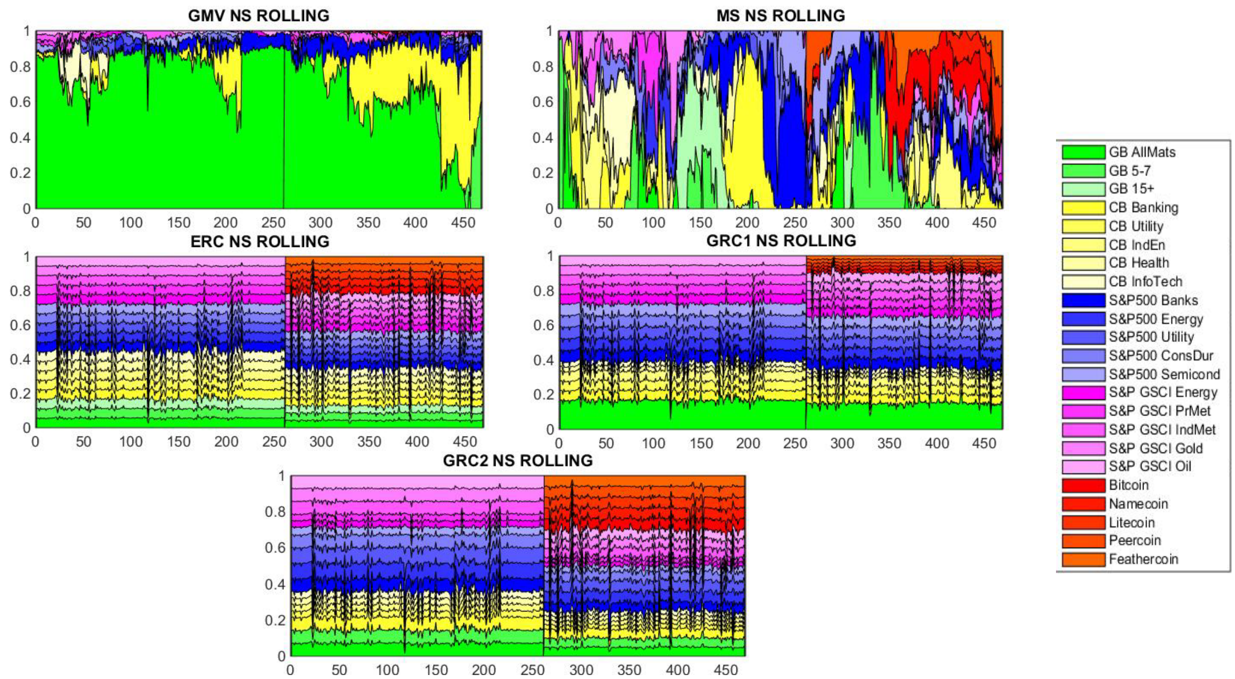

Figure 5.6: *Area Plots for Relative Risk Contribution, Static Strategy: Rolling*

These area plots show the risk contribution of each asset to the total portfolio risk: in the Static Approach, each plot can be ideally split in two parts, representing two different periods: the first one goes from week 1 to week 260 and it is characterized by the presence of only the 18 traditional assets. The second fraction goes from week 260 to 470 and it is characterized by the presence of 23 assets (18 traditional assets plus 5 Cryptocurrencies). This separation is mainly visible in the ERC/GRC1/GRC2 subplots. Focusing on the ERC portfolio, the first part is made up of 18 bands of equal width (because each asset contributes equally to the portfolio risk, and before crypto entering the portfolio is indeed composed by 18 assets). The second part is composed instead by 23 bands of equal width because here also Crypto are included in the portfolio: the red/orange area appearing in the second part of the subplots stands precisely for the total Cryptocurrency Risk Contribution.

Going on with the analysis, another thing we can observe concerns the GMV portfolio: here the portfolio risk mainly replicates the risk of the Government bond asset class (huge green band), while Crypto are included in the second period only to a very limited extent. This explains why the return of the GMV portfolios including crypto is almost equal to their counterparties GMV without Crypto even under Risk Budgeting. Things are different for what concerns the other portfolios. In particular, looking at the MaxSharpe, we can observe that the risk contribution of the Crypto in the second period is quite large (red/orange area), meaning that cryptocurrencies indeed enter the MaxSharpe portfolio. RL, MRA, RA, even if not included in Figure 5.6, are middle ways between the GMV and the MS. Finally, taking a look at the ERC, GRC1, GRC2 area plots, these provide an intuitive idea on how risk budgeting method works. In the second part of the GRC1 plot, the orange/red bands fill the 10% of the total area, because risk budgets have been set in such a way to force Crypto at the 10% of the total portfolio risk. In the GRC2 plot, the orange/red band is much larger than in the GRC1 case, and this is because Crytpo are forced at the 30% of the portfolio risk. For what concern the ERC case, as I already mentioned, Crypto Risk Contribution ($\approx 22\%$) is a middle way between the two GRC cases.

To conclude the analysis, I would like to deepen one supposition made in the previous stages of the work (Chapter 4, The Risk Budgeting Methodology). In particular, I wrote that for a given level of risk contribution, riskier assets are expected to have lower weight in the portfolio than safer assets. This means that, taking the ERC case as an example, a safe asset like a Government Bond is expected to have much more weight than a Cryptocurrency given the same risk contribution. In order to check this, I include in the next page (Figure 5.7) the weight area plots of the same portfolios whose risk contribution has been shown in the previous area plots (Rolling Static, Figure 5.6).

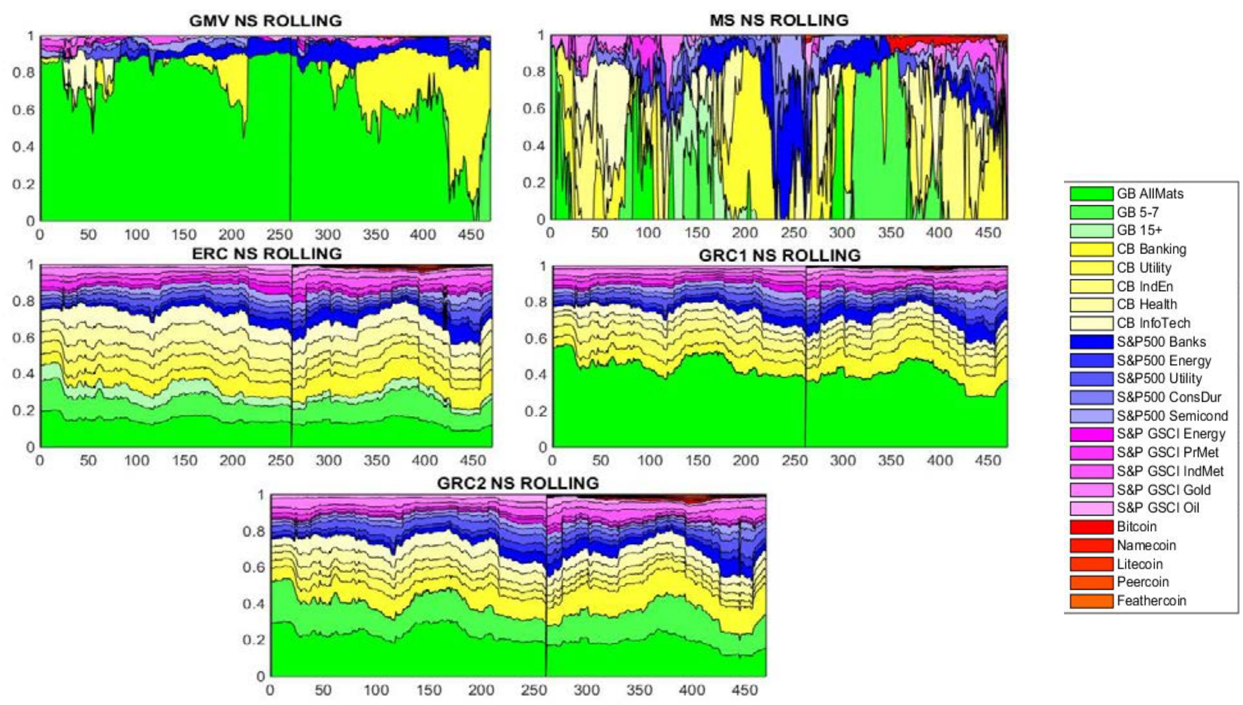

Figure 5.7: *Area Plots for Weights, Static Strategy: Rolling*

My supposition is confirmed. Given the same risk contribution, safer assets (such as Government or Corporate bonds) have much more weight than riskier ones (Equities-Crypto). This is true also for all the other portfolios. The GRC2 case is emblematic: despite Cryptocurrencies contribute for the 30% of the total risk, their contribution in terms of weight (red/orange band) is minimal.

5.3.2. Dynamic Approach Portfolios

Dynamic strategies are shown in the Figure 6 of the appendix. First, we can observe that returns of the portfolio including crypto departs from the return of the base portfolio earlier than what happens in the Static approach. The reason is the same I have explained for Markowitz: Bitcoin is included earlier in the Dynamic than in the Static approach. Moreover, evidence shows that differently from the Static approach, here even the GMV portfolios seem to over-perform their relative base portfolio at least in the last weeks under observation. Going on with the analysis, we can notice that the MaxSharpe is the portfolio which seems to reach the highest return at the end of the period, as it happens in the Static approach, but at a price of a higher volatility with respect to all the other strategies. Middle way between the GMV and the MS we have all the other portfolios.

Focusing on the three portfolios with risk budgets, in the previous paragraph I wrote that the ERC case could be considered as a middle way between the GRC1 and the GRC2 portfolios. This is true only for the Static approach, and here I will explain why. While the Static portfolios can be ideally split in two time-parts, the first one with 18 assets and the second one with 23 assets, Dynamic portfolios can be split in much more parts, because in this approach Crypto do not enter all together at the same point in time (which in Static Approach is week 260 = May 2013). So, taking the ERC case as an example, in the Dynamic approach we will have a first part without Crypto where each asset contributes for 1/18 of the total risk (as in the Static approach), a second part starting when Bitcoin enters alone around week 115 (July 2010), where each asset contributes for 1/19 of the risk, and so on, until we arrive to the last fraction starting with the entrance of the Ethereum, where each asset contributes for the 1/27 of the total portfolio risk. This means that in the second period the total Crypto risk contribution will be 1/19, since there is only one crypto (Bitcoin). Then Crypto risk contribution becomes 5/23 when also the four "2013 Cryptos" enter the portfolio, then 6/24 with the Primecoin and finally 9/27 in the last period when all the Crypto are available. This is a different story with respect to the ERC Static portfolios: here there is only a second part characterized by the presence of 5 cryptocurrencies, so the risk contribution keeps fixed at almost 20% from the time on which Cryptocurrencies enter until the end of the analysis. In the Dynamic ERC approach instead, Crypto total risk contribution ranges from 5.3% (1/19) to arrive at 33.3% (9/27). This means that the ERC Dynamic portfolios cannot be considered a middle way between the two GRC cases as it happens for the Static approach, but Crypto Risk Contribution increases progressively over time. I attach below the risk contributions area plots for the Rolling Dynamic portfolios (Figure 5.8): you can see that in the ERC portfolio the Cryptocurrency risk contribution increases over time being. The results are similar for the EWMA portfolios.

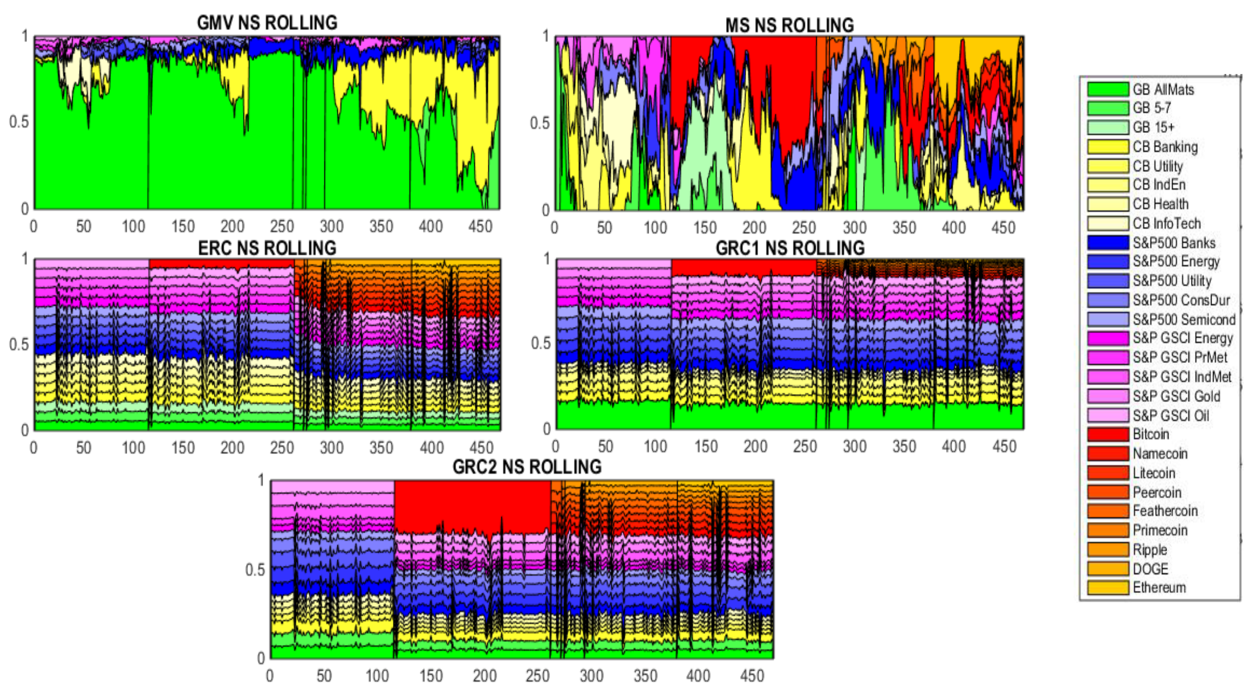

***Figure 5.8**: Area Plots for Relative Risk Contribution, Static Strategy: Rolling*

As I figured out for the static case, even for what concern the Dynamic approach GMV portfolios are dominated by assets characterized by a very low level of volatility (in particular Government Bonds), while the Cryptocurrencies risk contribution seems minimal, explaining the low return. For what concern the MS strategy instead, cryptocurrency risk contribution seems to be quite significant, enhancing the return of the portfolio. In the ERC portfolio the risk contribution of cryptocurrencies is minimal at the beginning and it becomes larger with time being as long as all the Cryptocurrencies enter. As in the Static approach, Cryptocurrencies' risk contribution is fixed at 10% for GRC1 and at 30% for GRC2 for all the time being (from the date on which they are available).

To conclude, even for the Dynamic case it is valid the consideration concerning weights that given the same risk contribution, risky assets will have smaller weights than safe assets in the portfolio. If you consider for example the MS or the GRC2 case, even if the risk contribution of cryptocurrencies seems to be quite strong, their weights will not be as significant.

Chapter 6

Performance Indicators

From the previous paragraph we have seen that return analysis seems to provide a positive answer to my research question: at least in terms of returns, it is worth it to include cryptocurrencies into a balanced portfolio made up of traditional assets. But as I have already stressed out, returns are not sufficient alone to provide a complete answer to the question: the aim of my analysis is to assess if the inclusion of cryptocurrencies could lead to a performance improvement not only in terms of returns, but also in terms of risk-return trade-off. For this reason, I evaluated the performance of the portfolios through many different performance indicators, taking into account not only the risk, but also the volatility. In the following lines I will present these indicators together providing a brief description of their main characteristics: for the most part, they take the form of financial ratios. Typically, these ratios require portfolio returns, variances or both as inputs.

Before starting with the performance indicator analysis, in order to provide a deepen view on the attributes of my portfolios, I have included in the appendix the summary statistics tables (Table A, Table B, Table C, Table D). Table A stays for Markowitz portfolios with Rolling inputs (Rolling=CASE 1), Table B for Markowitz portfolios with EWMA inputs (EWMA=CASE 2), Table C for Risk-Budgeting with Rolling inputs and Table D for Risk Budgeting with EWMA inputs. Each Table is split in three sections: in the first section there are the Dynamic portfolios, in the second section Static portfolios, in the third base portfolios without crypto (No-Crypto).

What you can find in this tables are the basic attributes, such as Mean, Variance, Minimum, Maximum (column 1, column 2, column 3, column 4), but I included also the quantiles at 5%, 50% and 95% (column 5, column 6, column 7). Taken alone, these data do not tell us very much about the performance of the portfolios: they become useful if combined together through the help of the performance indicators, so you can think about them as the inputs of the performance indicator analysis. Finally, the last three columns include the Tracking Error (TE), Tracking Error Volatility (TEV) and the Information Ratio (IR). The last three indicators measure the deviation of the return of a given strategy from the return of a benchmark: in my case the benchmark has been computed as an equally weighted of a Government Bond Index all maturities, a Corporate Bond Index all sectors, the S&P500 equity index and a General Commodity index. These indexes replicate the same asset classes included in the Base Portfolios, but in a more general manner. The higher the TE, the more the strategy has over-performed the benchmark. The TEV is instead the volatility of the Tracking Error. It is useful to compute the IR, which is included in the last column: the Information Ratio is simply the ratio between the TE and the TEV. Analytically, Tracking Error, Tracking Error Volatility and Information ratio take the following form:

$$TE = E[R_t - R_t^B] \tag{6.1}$$

$$TEV = V[[R_t - R_t^B] \tag{6.2}$$

$$IR = \frac{TE}{TEV} \tag{6.3}$$

What we can observe is that the TE seems to be higher for the Dynamic portfolios, but I will reserve to confirm this evidence through the help of the following performance indicators.

6.1. Sharpe Ratio - Sh

Sharpe ratio is simply a measure of the portfolio return per unit of risk, where the return is typically computed in excess to the risk free rate. Provided that I did not include risk free asset in my analysis, I computed Sharpe ratio using simply Expected Return at the numerator. The measure of risk at the denominator is Portfolio Volatility.

$$Sharpe = \frac{E[R_t]}{\sigma[R_t]} \tag{6.4}$$

Sharpe is probably one of the most commonly used performance indicators, given the easiness of use and of interpretation. However, it suffers for some drawbacks that make it inappropriate under certain circumstances. For example, the fact that Sharpe ratio uses Standard Deviation σ at the denominator as a measure of portfolio risk is equivalent to assume that asset returns are normally distributed. This seems to be a quite strong assumption, since evidence shows that asset returns are not normally distributed: in particular, the vast majority of times return distributions are skewed. This could make the results from the Sharpe Ratio misleading.

More accurate alternatives to the Sharpe ratio could be Sortino Ratio and Treynor Ratio.

6.2. Sortino Ratio - So

Sortino Ratio is computed as the ratio of portfolio expected returns (or excess returns in case of risk free) at the numerator and portfolio downside risk at the denominator.

$$Sortino = \frac{E[R_t]}{\sigma[R_t I(R_t < 0)]} \tag{6.5}$$

Sortino ratio could be considered an improvement with respect to the Sharpe ratio in the sense that it isolates downside volatility from the total volatility: this allows us to take into account only of the so-called "harmful volatility", which is the volatility associated to the negative asset returns. This is good in terms of performance evaluation because upside volatility is good for the investors, so it should not be included in the ratio as it happens in the Sharpe case.

6.3. Treynor Ratio - Tr

Treynor Ratio is computed as the ratio between portfolio expected returns and systematic risk, expressed by the portfolio $beta$. So, the Treynor ratio provides a measure of return per unit of systematic risk.

$$Treynor = \frac{E[R_t]}{\beta} \tag{6.6}$$

Treynor ratio is very similar to the Sharpe ratio, and the logic behind its functioning is similar as well. The only difference is that Treynor uses the β as a measure of the portfolio risk, while Sharpe uses standard deviation. The β is a measure of the sensitivity of the portfolio to market movements, so it is a measure of systematic risk, which is that kind of risk which cannot be eliminated via diversification. Differently from the Sharpe ratio, Treynor lies on the assumption that an investor should be compensated only for the amount of systematic risk he bears, not for the idiosyncratic component that can be eliminated simply via diversification.

The estimation of the β requires also a benchmark. One of the main limitation of the Treynor ratio is common also to the other two ratios described before: in particular, I am referring to their backward-looking nature: they track performance on the basis of past returns, but the past is not always useful to predict the future.

I will now switch to a different kind of performance measures, which, differently from the three ratios just illustrated, are not based on a risk/return trade off.

6.4. Value at Risk - VaR

Value at risk $VaR(\alpha)$ can be defined as a threshold, and VaR represents is the probability to observe returns below that threshold (α) within a time horizon equal to the frequency of the data (in my case weekly) (6.7).

$$VaR(\alpha) = \int_{-\infty}^{VaR(\alpha)} R_t\, f(R_t) dR_t = \alpha \tag{6.7}$$

Put it differently, we can define $VaR(\alpha)$ as the maximum potential loss the portfolio can suffer in a given time horizon with a probability $(1 - \alpha)$. From an analytical point of view, it is simply computed as a quantile of the return density.

Value at Risk has been probably the most commonly used risk performance measure in the financial environment over the last years. However, the attribute "most commonly used" does not mean that VaR is the most reliable performance measure. Acerbi C., Tasche D, in "Expected Shortfall: a natural coherent alternative to Value at Risk" (2001), affirm that VaR cannot be considered as a "Coherent Risk Measure". In order to understand why, it is worth to take a look at their definition of Risk Measure:

Definition 1 (Risk Measure): Consider a set V of real-valued random variables. A function $\rho: V \to \mathbb{R}$ is called a risk measure if it is:

(i) *Monotonous*: X ∈ V, X ≥ 0 ⇒ $\rho(x) \leq 0$

(ii) *Sub-Additive*: X, Y, X+Y ∈ V ⇒ $\rho(X + Y) \leq \rho(X) + V(Y)$

(iii) *Positively Homogeneous*: X ∈ V, h>0, hX ϵ V ⇒ $\rho(hX) = h\rho(X)$,

(iv) *Transition Invariant*: X ∈ V, a ∈ ℝ ⇒ $\rho(X + a) = \rho(X) - a$

The main problem related to the VaR implementation is that it does not satisfy the second property, which is *sub-additivity*. Suppose you are dealing with a 23-assets portfolio, similar to the ones of my Static Approach. Sub-Additivity property requires that the total portfolio risk is smaller or at maximum equal to the sum of the risk of the 23 assets taken separately. At the actual state of things portfolio diversification, which is obtained including in the portfolio assets that correlate negatively each others, always lead to a reduction in the level of risk. Acerbi and Tasche demonstrated that this evidence is not true for those risk measures which violate the sub-additivity axiom, such as VaR. Under certain circumstances it could happen also that "diversification may produce an increase in the level of risk even when partial risks are triggered by mutually exclusive events". This is exactly the opposite effect of the diversification benefit.

One of the possible ways to deal with this problem is adopting a different risk performance measure that satisfies all the four properties. This could be for example the Expected Shortfall.

6.5. Expected Shortfall - Es

As just explained, Value at Risk is not a sub-additive measure. Moreover, another limitation concerning the VaR as a performance measure is that it is nothing else than a threshold of the possible α% losses, but it does not take into account how serious losses above this threshold effectively are. The main strength point of Expected Shortfall performance measure (also known as CVaR, or Conditional-VaR, because it is a VaR-based performance measure) is that it overcomes these two limitations affecting VaR: ES is sub-additive and it allows also to quantify how serious losses above the threshold effectively are. It is computed as it follows:

$$ES(R_t, \alpha) = E[R_t \mid R_t \leq VaR(\alpha)] \tag{6.8}$$

In particular, this formula simply quantifies the loss if the VaR threshold is ever crossed.

6.6. Drawdown Sequence – DD

Drawdown measures the largest losses and the time taken to recover from that loss. It is computed recursively. The first Drawdown (at time 0) is set at 0 while the following periods drawdowns are computed according to the following formula:

$$Drawdown\ D_t = min(0, (1 + D_{t-1})(1 + R_t) - 1) \tag{6.9}$$

If the product between the previous period Drawdown $(1 + D_{t-1})$ and the actual period return minus 1 $(1 + R_t)$ is negative, D_t is negative and it will take the value of this product. If the second term of the minimum function is instead positive, the formula provides $D_t = 0$, meaning that the loss has been recovered. By measuring the number of negative drawdowns preceding the zero one (recovery) we can have a measure of the time taken by a given portfolio to recover the loss. Clearly, the lower the drawdown associated to a given strategy, the better that strategy is, because this means that it recovers quickly from losses.

Drawdown is used also as an input for the so called **Sterling Ratio,** which is computed as the ratio between the expected returns and the average of the k largest drawdowns: in other words, it can be considered as some sort of return per unit of extreme downside risk.

$$Sterling = \frac{E[R_t]}{D_t} \tag{6.10}$$

6.7. Farinelli-Tibiletti Ratio

According to Cuizhen N. et al. in "Farinelli and Tibiletti ratio and stochastic dominance" (2017), Farinelli and Tibiletti ratio has been introduced as a performance evaluator alternative to the Max Sharpe ratio, which was one of the most commonly used ratio to assess portfolio performance. As I have already said, one of the main limitations of Sharpe index, even according to Cuizhen et al., is that the standard deviation cannot be considered a good measure of risk because it penalizes upside deviations as well as downside deviations. However, this is not consistent with the view of many investors, which consider risk only the return below a given threshold without worrying about upside movements. For this reason, upside deviations should not be penalized as Sharpe implicitly does, but they should be accounted positively. Farinelli-Tibiletti in its basic form is computed as it follows:

$$FT(\tau, p, q) = \frac{E[\max(0, R_t - \tau)^p]^{1-p}}{E[\max(0, \tau - R_t)^q]^{1-q}} \tag{6.11}$$

If computed in this way, The FT ratio is "essentially a ratio of average above-benchmark returns (gains) to average below-benchmark returns (losses), each raised by some power index, p and q (for upside and downside respectively), to proxy for the investor's degree of risk aversion". We can notice that when $p = 1$ and $q = 2$, FT ratio is just equal to the Sortino Ratio.

One of the main issues related to the implementation of the performance measures is that they are not concordant each other. For example, it could happen that a strategy, which is the best one in terms of VaR, may be the worst in terms of Sharpe, simply because these indicators are computed in different ways using different inputs. One thing is sure: there is not a better performance indicator among all the others, it depends on what one is looking for. In order to deal with this problem, I implemented a composite index that is computed as the sum of the different indicators, but I will explain the process more in deep later. In this way, I ended up with a unique reliable performance indicator. This allowed me to rank the various strategies following this unique and reliable performance indicator.

6.8. Performance Indicators Interpretation

As I said in the previous paragraph, conducting a return analysis may be a good starting point to provide an initial idea to see if Cryptocurrencies could improve portfolios performance. However, we need to go more in deep to verify if it is really worth it: this is because if we limit ourselves to analyze and compare only the strategies' returns, we are not taking into account of the other fundamental component of the trade off, which is the risk, expressed in terms of portfolio volatility. Clearly, if Cryptocurrencies lead to an improvement in terms of return, but at the same time they increase portfolio volatility beyond reasonable levels, this evidence could mine the optimism arising from the return analysis results. Honestly, this is precisely what I thought before starting with my analysis, because it is true that almost every Cryptocurrency experienced a huge growth in terms of value from its introduction in the market, but you have to consider that Cryptos are also the riskiest instruments available in the market due to their huge intraday fluctuations. For this reason, I expected a deterioration in the risk-return trade off in such a way that strategies without Cryptocurrencies (Base Portfolios) still performed better than portfolios including Cryptocurrencies (both Static and Dynamic), even if the formers are worse in terms of returns than the latters. In order to deepen my analysis I evaluated all the portfolios through the mean of the performance indicators I have described in the previous paragraphs. I anticipate that my initial beliefs have been denied. In particular, Cryptocurrencies seem to improve portfolios performance not only in terms of returns, but also in terms of the risk-return trade off. This is true mainly for what concern the Dynamic potrtfolios: according to the ratios, Dynamic portfolios seem to be the best among all the others strategies. I will demonstrate this in a while.

The output of each performance indicator is simply a number, for example in the case of the Sharpe ratio the output is the ratio between portfolio expected return and volatility. Typically, the higher the ratio the better a given strategy is: this is true also for the other eight performance indicators besides Sharpe used to conduct the analysis on each portfolio strategy. The main problem related to the performance indicators analysis relies precisely on the fact that the output is simply a number, so results are difficult to interprete. In particular, it is difficult to establish if one strategy is better than another by simply looking at the resulting indicators outputs. For this reason, I acted as it follows: firstly, I computed the various performance indicators for each portfolio included in my analysis (first Markowitz and then Risk-Budgeting), secondly, instead of reporting for each indicator the associated absolute value, I computed a ranking which sorts the strategies assigning to every indicator output a number. This number is nothing else than the position within the ranking of a given strategy with respect to the others according to a given performance indicator. I repeated this process for all the 9 performance indicators included in my analysis. So, for example, if you consider the Markowitz Rolling strategies, we will have a total amount of 70 portfolios plus 1 benchmark: according to my procedure, the portfolio with the highest Sharpe Ratio will rank 1, meaning that this portfolio is the best among the others in terms of Sharpe. The one with the lowest Sharpe Ratio will rank 71, meaning that it is the worst one. The same I have done for all the other indicators, each of them ranging from 1 to 71 in case of Markowitz and from 1 to 32 for the Risk Contribution approach. One of the main problems related to this kind of analysis stems from the fact that results among the various performance indicators may be discordant, so for example a strategy which ranks first in terms of Sharpe may not rank first in terms of VaR or Expected Shortfall. This is related to the fact that these indicators are computed in different ways and take different inputs. As I have already mentioned before, I tried to deal with this problem by computing for each strategy a Composite Index (CI) made up simply as the sum of the rankings of all the 9 performance indicators. For example, if a given portfolio ranks first according to all the evaluators, its Composite Index will be 9. Clearly, the lower the Composite Index the better a given strategy is, because a lower Composite Index means that the portfolio has a good ranking in all the performance indicators. Following this reasoning, I assume that the best strategy is the one with the lowest Composite Index. You can see the results of this analysis in Table 1-Appendix, Table 2-Appendix, Table 3-Appenidx, Table 4-Appendix. In Table 1 you can find the ranking of the Markowitz strategies with Rolling inputs (case 1 means Rolling, case 1.1 means Rolling no short, case 1.2 Rolling bounds and so on). In Table 2, I included the analysis of Markowitz portfolios with EWMA inputs (case 2 means EWMA), while in Table 3 and Table 4 there is the analysis of the Risk Budgeting portfolios, with Rolling and EWMA inputs respectively. The reason why you find distinct tables for Rolling and EWMA is that I decided to compare Rolling and EWMA startegies separately, so Rolling against Rolling and EWMA against EWMA. The reason why I made this choice was that I wanted to concentrate on the comparison between the Static, Dynamic and NoCrypto strategies, since the main aim of my work is to determine if Cryptocurrencies could improve portfolio performances. On the opposite, I was not interested in a comparison between Rolling and EWMA: this goes beyond the scope of my analysis. However, if the results of the performance indicators analysis go in the same direction for both the Rolling and EWMA portfolios, this must be intended as a further confirmation of the goodness of my work.

For what concern the interpretation of the results, I will start from Table 1, so from Markowitz portfolios with Rolling inputs. Look at the structure of the table: this is splitted in three sections. In the first section I included the 25 Dynamic portfolios, in the second section the 25 Static portfolios and finally in the third one the 20 Base Portfolios without Crypto (only 20 because Base portfolios do not make the distinction between GC1 with Crypto at 10% and GC2 with Crypto at 20%, simply because they do not include Crypto).

The first evidence I would like to figure out is that No-Crypto portfolios generally seems to be the worst among the others according to the performance indicators analysis. You can realize this by looking at the Column 1 of the Table 1-Appendix: the Composite Indexes associated to the No-Crypto strategies (ranging from 250 to 500) are far higher than the CI of the Static and Dynamic strategies, meaning that No-Crypto portfolios performed poorly with respect to the portfolios including Crypto. This is a first confirmation of the result found out in the return analysis: Crypto portfolios, no matter if static or dynamic, perform better than portfolios without Crypto. Following this evidence, I can add "not only in terms of returns". If we limit instead our focus to the Static and to the Dynamic portfolios, evidence shows that the best performing ones between the two are for sure the Dynamic ones. The CI associated to the dynamic portfolios (ranging from 40 to 500) are substantially lower than the one associated to the static portfolios (ranging from 120 to 500), meaning that in general Dynamic portfolios rank better than Static portfolios.

To sum up, what this analysis is telling us is that portfolios including Crypto perform better than those without crypto. Among the portfolios including Crypto, Dynamic strategy seems to do better than Static strategy. This could be due to the same two facts already highlighted in the previous stages of my work: the more persistent presence of Bitcoin in the Dynamic than in the Static approach, the higher variety of Cryptocurrencies available in the Dynamic than in the Static approach.

However, this kind of analysis allows us to give a step further: from the performance indicators table we can try to identify what are the portfolios which are the best in absolute among all the others. These are the ones with the lowest Composite Index. For what concern Markowitz-Rolling strategies, the two portfolios with the lowest Composite Index are both Dynamic portfolios, as we could expect. In particular, I am referring to the **MRA Dynamic CASE 1.4** (Moderate Risk Adverse portfolio with Turnover Constraint), with a Composite Index of 46, and to the **RA Dynamic CASE 1.3.1**, (Risk Adverse Portfolio with Crypto forced at 10% of the weight), with a

CI of 44. Even if the CI is a little bit smaller for the latter than for the former, from the Table 1 we can see that the Moderate Risk Adverse Portfolio ranks first in almost all the performance indicators, but performs quite poorly according to the Sortino ratio ("So", fourth column, 27/71). Provided that Sortino mesures the expected return per unit of downside risk, probably this strategy suffers a bit a problem of downside volatility. However, even the Sortino rank, which is 27/71, is not so bad if considered in absolute value. For what concern the Risk Adverse portfolio, the ranking is good for all the indicators, even if it does not rank first in any of the strategies.

The main thing I would like to underling following this evidence is that according to the performance indicator analysis the best portfolios among the others are both characterized by a **medium-high degree of risk adversion**. This result is surprising, since we are used to think at Cryptocurrencies as extreme speculative instruments suitable mainly for risk lover investors. Now, this result allows us to say that Cryptocurrencies are suitable also for an investor with a high degree of risk adversion, provided that some measures are adopted to limit the volatility they bring to the portfolio. These could be for example a Turnover constraint, or a limit to Cryptocurrencies' weight on the portfolio. This is a very useful insight.

Besides these main results, the analysis shows some other evidences that are worth to be considered: first, there are many other Dynamic Portfolios performing very well (Composite Index substantially below 100), enforcing the idea that Dynamic is the best strategy. Secondly, in the same way I have identified the best portfolios, I can also figure out the worst ones. In particular, I am referring to those portfolios whose Composite Index lies around the 500 threshold: besides the benchmark we are not interest in now, this is the case of 8 portfolios: these are the same for the Static and the Dynamic strategies, and they are the MS and RL portfolios CASE 1.3.1 and CASE 1.3.2 (MaxSharpe and RiskLover portfolios with group constraints). Provided that MaxSharpe and Risk Lover are the riskiest portfolios among the others, we can say that Cryptocurrencies seem not to perform well for investors with a low degree of risk adversion, particularly if associated to group constraints. This seems to deny in part the output of the return analysis, where we found that precisely these portfolios were the best in terms of returns reaching the highest cumulated return at the end of the period. However, we have also noticed that they were also the most volatile, and at this point, through the help of the performance indicator analysis, I can say that the trade off is definitely not worth it for them.

The result I have found out is quite surprising: Cryptocurrencies seem to fit better the needs of an investor with a moderate level of risk aversion than those of a risk lover investor. This may be explained from the fact that, due to their very volatile nature, a limited usage of Cryptocurrencies performs better than an intensive one.

I will now take a look at Table 2-Appendix, where I have included the results of the Markowitz performance indicator analysis with EWMA inputs. Before starting with this analysis, I asked myself if Rolling and EWMA output would have gone in the same direction. The answer seems to be affirmative, and all the evidences found out in Table 1 are confirmed also for the EWMA. For what concern Rolling, we said first that strategies performing worse are those without Crypto. This is true also for EWMA, with the CI of these strategies ranges in the same interval as in No-Crypto Rolling strategies (250-500). So, even for EWMA, we can say that portfolios including Crypto performs better than those without. Secondly, among the portfolios including Crypto, I pointed out that Dynamic portfolios performed better than Static. This is true also for EWMA, with a lot of Dynamic portfolios with an associated CI consistently below 100.

Finally, I tried to identify the single best and worst positions among all the strategies. Even in this case, EWMA portfolios replicate (at least partially) the evidence from the Rolling approach, and the best portfolios are both Dynamic. The portfolio with the lowest CI is the same we sorted out before for the Rolling inputs: I am referring to the Moderate Risk Adverse **MRA Dynamic CASE 2.4** (Turnover constraint), with a CI of 35. But I can tell more: even in this case this portfolio ranks first or second for all the indicators, performing weakly only according to Sortino. Even its Rolling counterparty highlighted before performed weakly only for Sortino, suggesting a problem of downside volatility. The other portfolio I sorted out in Table 2 was the **RL Dynamic CASE 2.4** (Risk Lover portfolio with turnover constraint), with an excellent ranking in all the performance indicators except some weakness in Sharpe and Sortino. We can think about the latter as a “quite risky” portfolio associated to a “safe” constraint.

Looking at the worst portfolios, these are exactly the same I have sorted out for the Rolling strategies. In particular, I am referring to the MS and RL portfolios with group constraints, both Static and Dynamic, with a Composite Index lying around 500, as in table 1.

Finally, they are valid the same considerations I have made for the Rolling inputs. What we have to keep is that Rolling and EWMA methods seem to go towards the same direction, and this is great.

Switching now to the Risk Budgeting, we must look at **Table 3** (Rolling) and **Table 4** (EWMA) of the Appendix. Even in this case, as in Markowitz, portfolios including Crypto seem to perform better than base portfolios without cryptocurrencies (CI from 110 to 180). Among those including Crypto, Dynamic seem to perform much better than Static. You can realize this by looking at the Composite Index, which is consistently smaller for Dynamic (20-90) than for Static portfolios (60-160). This is true both for the Rolling (Table 3-Appendix) and for the EWMA (Table 4-Appendix) cases.

Focusing now on Rolling, the best performing portfolio according to the indicators used is the **Dynamic GRC2**, that is the generalized risk contribution with crypto at the 30% of the total portfolio risk. This strategy gets an excellent ranking in all the measures a part from Sortino (17/25), suggesting as before a problem of downside volatility. The other two Rolling portfolios performing well are the Equally Risk Contribution one (ERC), ranking second in all the strategies but suffering of the same weakness in Sortino (16/25), and the Moderate Risk Adverse (MRA) , which ranks generally third but it does not suffer any Sortino weakness (3/25). Worst portfolios are in general those without Crypto.

Switching to **EWMA** (Table 4-Appendix), here the best performing portfolio seems to be with any doubt the **Moderate Risk Adverse** one with a Composite Index of 21. The Risk Contribution portfolios on the opposite seems to perform worse than what we observed in the Rolling case: GRC2 portfolio, with a CI of 52, still remains a good portfolio but the ranking of the Calmar and Sterling ratios deteriorates with respect to the GRC2 Rolling Counterparty (13 Calmar and 14 Sterling). For what concern the worst portfolios, even in this case they are in general those without Cryptocurrencies.

Both Markowitz and Risk Budgeting approaches are almost coherennt with the result found in the return analysis that Cryptocurrencies have improved portfolio performance (except for MaxSharpe and some Risk Lover portfolios). This is true mainly if we consider the Dynamic ones: Dynamic portfolios in all the cases not only overperform portfolios without Crypto, but also the Static portfolios including Crypto.

Conclusions

In order to conclude my work, I would like to briefly re-examine what I have done in my analysis. Firstly, I started with a description of the Cryptocurrencies environment at the actual state of thing: I explained their history, the main risks related to their implementation as a new and alternative payment system to the one dominated by the fiat currency, and the most important innovations these instruments have brought to the real world economy. The main thing I pointed out is that, despite Cryptocurrencies were originally meant to fulfill the role of payment instruments, at the actual state of thing they seem to fulfill better the role of speculative assets. This is true if you consider what happened at the end of year 2017 when a wave of enthusiasm ran over Cryptocurrencies, pulling their exchange rates toward very high levels. Is this only an instance of "irrational exuberance"? Put it differently, taking the point of view of a rational investor with a diversified portfolio made up of traditional assets: would have been a good choice for him to enlarge its existing portfolio to cryptocurrencies? My analysis tries to give an answer precisely to this question: starting from a diversified portfolio of traditional assets (Government Bonds, Corporate Bonds, Equities and Commodities), I included in it Cryptocurrencies following two different approaches, one Static and one Dynamic, corresponding to two different cryptocurrencies datasets. So, I ended up with three different datasets: the first is the dataset of the base portfolios, which includes only traditional assets, the second is the dataset of the "Static Approach", including traditional assets plus five Cryptocurrencies all together entering in 2013, and the third is the "Dynamic Approach" dataset, including nine cryptocurrencies from 2010 beyond the traditional assets. Then, I conducted the analysis following the Markowitz Optimization approach and the Roncalli's Risk Budgeting. I set up many different portfolio taking into account of different degrees of risk adversion and different constraints on weights (Markowitz) and on risk (Roncalli). Both these methods provide as a result the optimal composition of the portfolio (weights) according to the parameters adopted, from which you can track the performance to see how the optimal portfolios would have performed over the period.

Firstly, I conducted the analysis on the portfolios without crypto, then on the Static portfolios and finally on the Dynamic portfolios.

The result section has been structured in two parts (Chapter 5 and Chapter 6 respectively). In Chapter 5 I focused only on returns, in particular I wanted to investigate if the portfolios with Cryptocurrencies would have performed better than those without at least in terms of returns. To assess this, I first computed the cumulated returns of every portfolio resulting from my analysis. Then I plotted the cumulated returns of every strategy without Cryptocurrencies against its counterparty (same Risk Adversion and same Constraint) strategy including crypto, first Static and then Dynamic. The result of this "return analysis" seem to provide a positive answer to my research question: all the portfolios including crypto perform better than the corresponding portfolio without, and this is true for both the Static portfolios and the Dynamic ones. Even if this result is satisfactory, it is not very surprising considering the huge growth in terms of returns experienced by Cryptocurrencies over the period under analysis. If you had included in your portfolio an asset experiencing very high growth, the performances of your portfolios are very likely to improve at the end of the period.

However, the main problem concerning cryptocurrencies is volatility, because if the upside of these instruments has been quite consistent, the downside has been strong as well, particularly in some market phases. So, if I had limited my conclusions only to returns, ignoring the volatility component, this would have constituted a serious bias. Probably, the portfolios with crypto had performed better than those without in terms of returns, but at the price of worsening the risk return trade off. This would have mined the positive answer given to the research question following the return analysis. In order to avoid this bias and measure more precisely the consistency of my results I gave a step further, and I evaluated the portfolios using performance indicators taking into account also volatility beyond the return component (Chapter 6).

And this time the result has been quite surprising. Evidence strongly confirms that portfolios with cryptocurrencies have perform better than their counterparty without cryptocurrencies according to all the performance indicators taking into account of different parameters. This is true mainly for the Dynamic approach. Another important result I found out thanks to this analysis has been that the best portfolios are those with an intermediate degree of risk adversion, performing better than those with a low degree of risk adversion. This has been surprising given the fact that we are used to think at cryptocurrencies as speculative instruments. This also suggests that cryptocurrencies indeed have brought some benefit, but they are more useful in a limited weight (intermediate risk adversion) than in a more consistent one (low risk adversion).

Performance indicators result provide a further positive answer to my initial research question.

One natural criticism that could be addressed to my work is that Cryptocurrencies are included in the dataset until May 2018, and during the following months all of them experienced a sharp decreasing in value. One can think that this could be a serious threat, leading to a deterioration in the performances of the portfolios and invalidating the results of my analysis. I don't think that things would go in this direction. What I expect is that from the moment on which cryptocurrencies start to perform poorly, the optimization problems will set crypto weights at zero, switching the composition of the portfolio toward other asset classes such as bonds, equities or commodities, which do not correlate with cryptocurrencies. The returns of the optimal portfolios won't be as high as before, but I do not expect them to fall in such circumstances. On the opposite, I expect a deterioration in the performances of those portfolios where Crypto are artificially forced inside (and I am referring in particular to the Markowitz portfolios with Group Constraints or to the portfolios with Equal or Generalized Risk Contribution).

APPENDIX

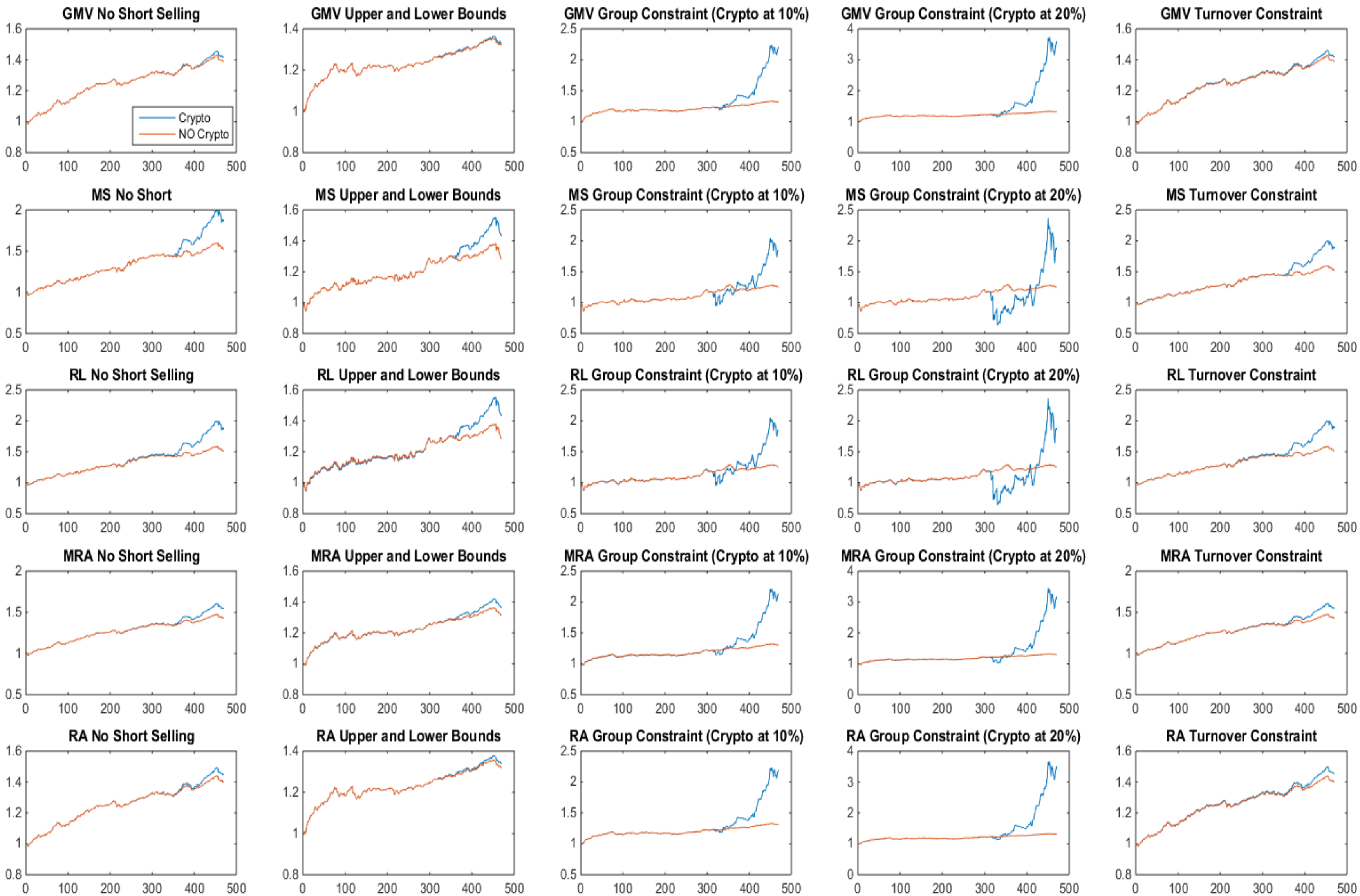

Figure 1 - Appendix: *Markowitz portfolios with Rolling inputs – Static approach*

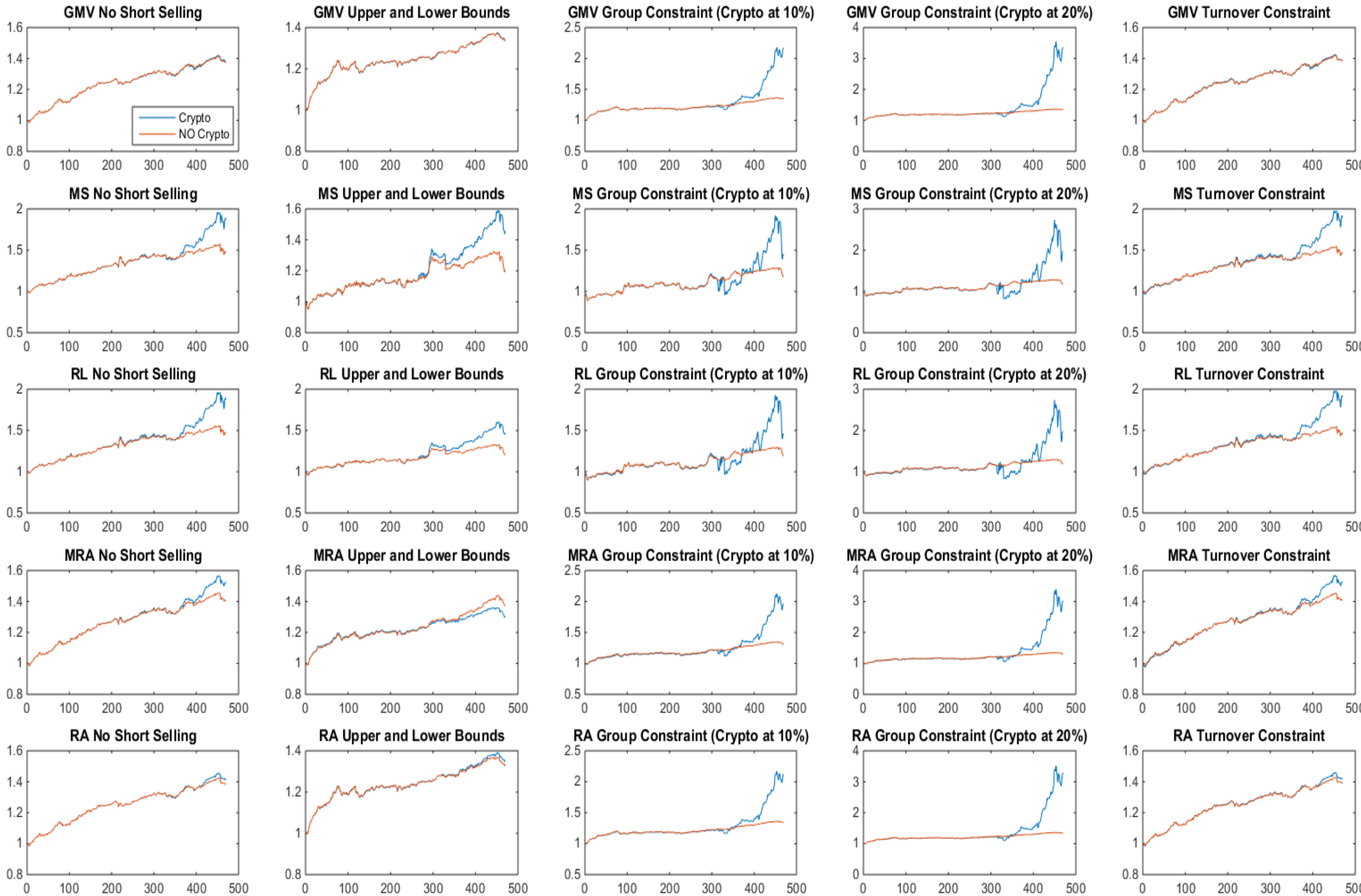

Figure 2 - Appendix: *Markowitz portfolios with EWMA inputs – Static approach*

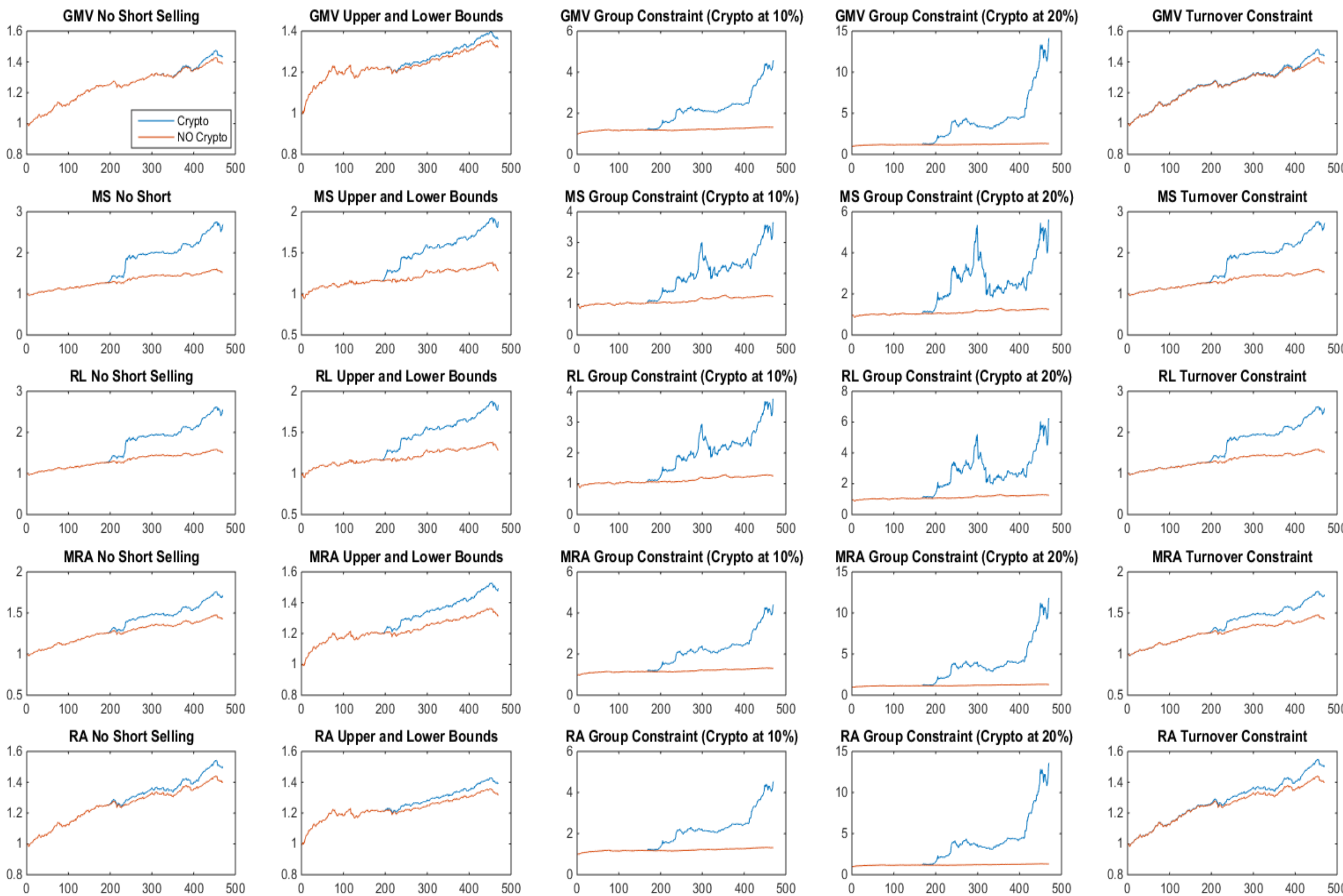

Figure 3 - Appendix*: Markowitz portfolios with Rolling inputs – Dynamic approach*

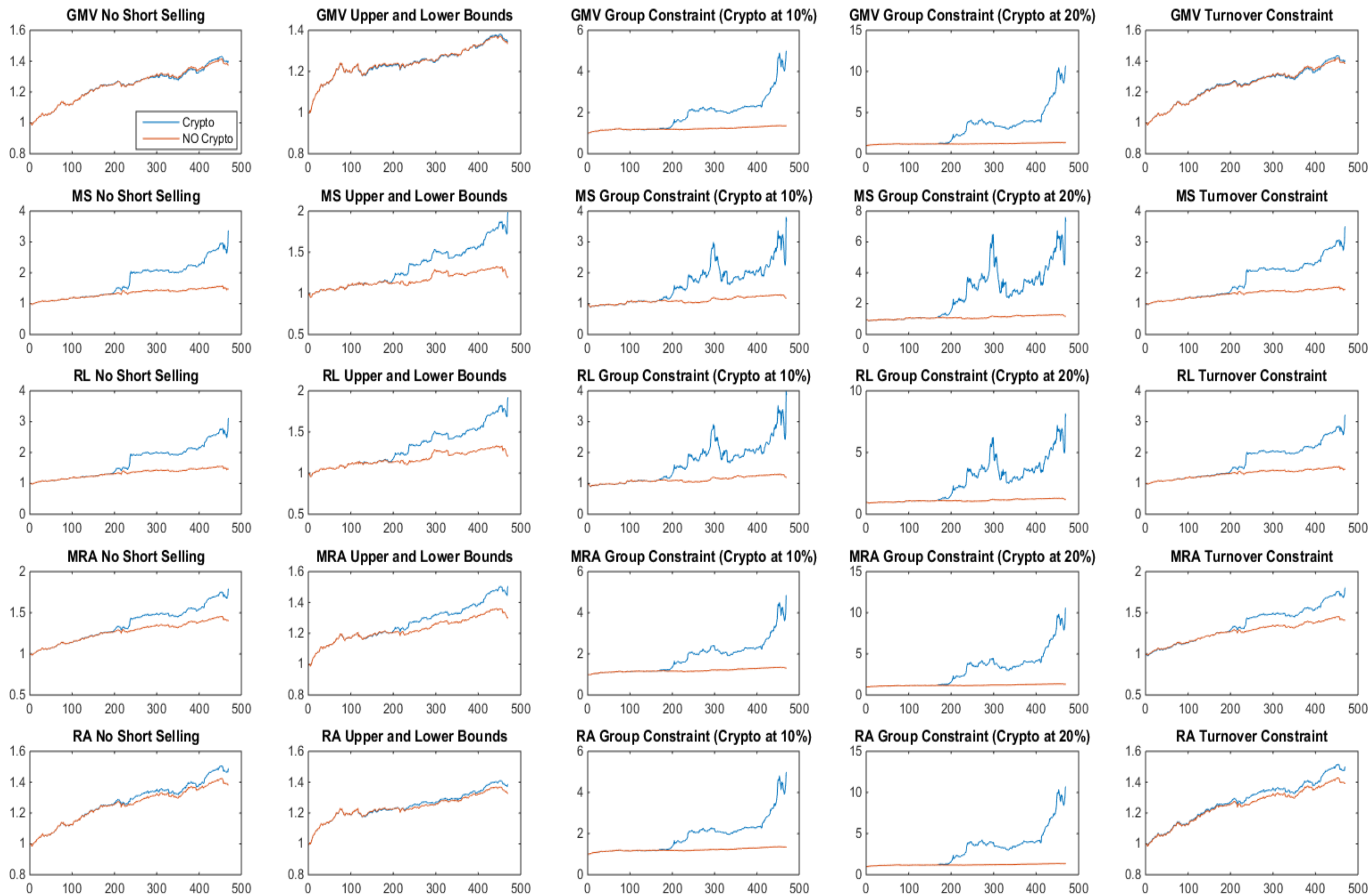

Figure 4 - Appendix*: Markowitz portfolios with EWMA inputs – Dynamic Approach*

Rolling **EWMA**

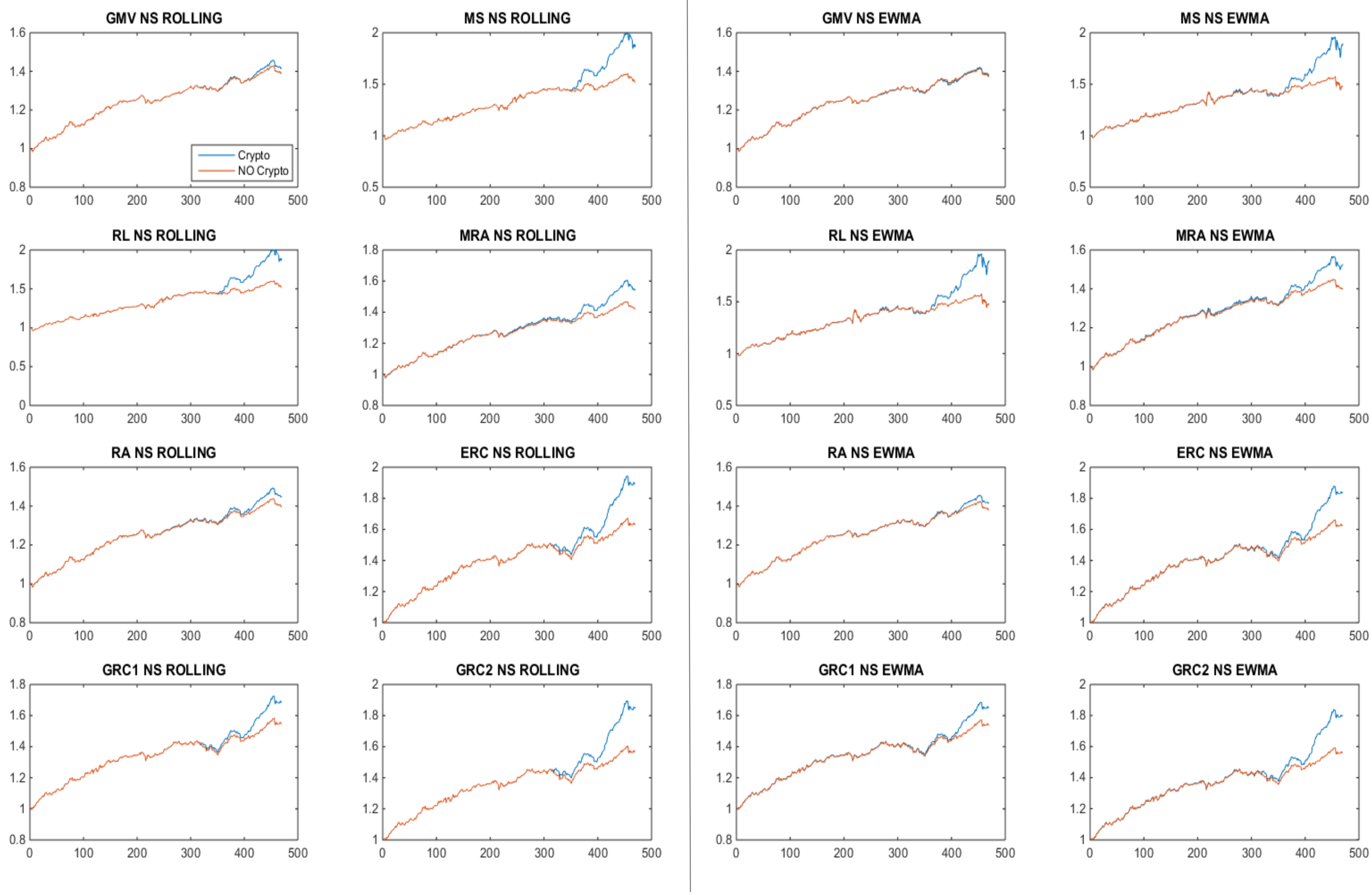

***Figure 5 - Appendix**: Risk-Budgeting Portfolios Rolling and EWMA– Static approach*

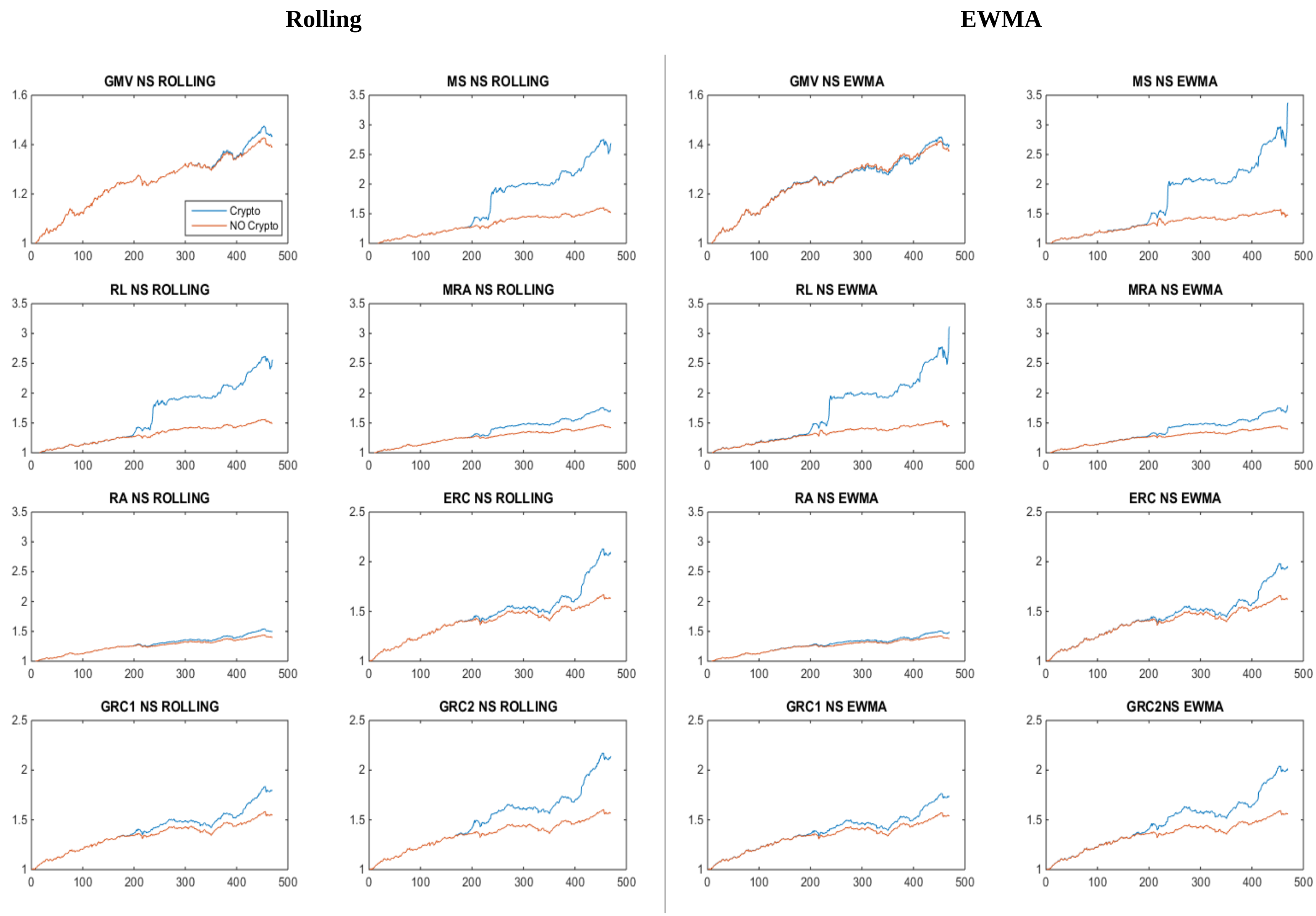

Figure 6 - Appendix*: Risk-Budgeting Portfolios Rolling and EWMA–Dynamic approach*

Strategy	Mean	StDev	Min	Max	q5	q50	q95	TE	TEV	IR
Benchmark	0.1045	1.8646	-8.9489	7.1412	-2.9434	0.3042	3.0307	/	/	/
GMV Dynamic CASE 1.1	0.0778	0.4003	-1.9545	1.3309	-0.5325	0.0792	0.6436	-0.0267	3.2310	-0.0083
GMV Dynamic CASE 1.2	0.0665	0.4048	-2.9038	1.6194	-0.4869	0.0812	0.7610	-0.0380	3.2993	-0.0115
GMV Dynamic CASE 1.3.1	0.3360	1.5694	-9.2797	10.5958	-1.7843	0.1580	2.7604	0.2315	5.1299	0.0451
GMV Dynamic CASE 1.3.2	0.6068	2.9336	-16.3265	20.5981	-2.9260	0.1890	4.9053	0.5023	10.8903	0.0461
GMV Dynamic CASE 1.4	0.0803	0.4026	-1.9545	1.3309	-0.5293	0.0810	0.6497	-0.0242	3.2154	-0.0075
MS Dynamic CASE 1.1	0.2165	1.0827	-4.6360	10.6920	-1.2409	0.1608	1.7020	0.1120	3.8591	0.0290
MS Dynamic CASE 1.2	0.1378	0.8166	-2.7852	4.4520	-1.1973	0.1399	1.2716	0.0333	3.9256	0.0085
MS Dynamic CASE 1.3.1	0.3106	2.6773	-17.0539	13.1208	-3.5672	0.2734	4.1173	0.2061	11.4210	0.0180
MS Dynamic CASE 1.3.2	0.4808	4.7556	-34.8918	27.2726	-6.2453	0.2490	7.0676	0.3763	27.4851	0.0137
MS Dynamic CASE 1.4	0.2187	1.0739	-3.7305	10.6920	-1.2405	0.1671	1.7020	0.1142	3.8258	0.0298
RL Dynamic CASE 1.1	0.2051	1.0095	-4.2185	9.8444	-1.1577	0.1579	1.5819	0.1005	3.7376	0.0269
RL Dynamic CASE 1.2	0.1319	0.7655	-2.6180	4.1262	-1.1253	0.1398	1.1805	0.0274	3.8345	0.0071
RL Dynamic CASE 1.3.1	0.3127	2.5226	-15.5234	12.4390	-3.3121	0.2563	3.8717	0.2082	10.4856	0.0199
RL Dynamic CASE 1.3.2	0.4913	4.5017	-31.8771	25.8277	-5.7571	0.2571	6.7514	0.3867	24.9212	0.0155
RL Dynamic CASE 1.4	0.2072	1.0018	-3.3878	9.8444	-1.1577	0.1613	1.5891	0.1027	3.7074	0.0277
MRA Dynamic CASE 1.1	0.1160	0.5056	-2.1096	3.2592	-0.6248	0.1243	0.8543	0.0115	3.2207	0.0036
MRA Dynamic CASE 1.2	0.0861	0.4460	-2.8027	1.6276	-0.5774	0.1019	0.7952	-0.0184	3.3683	-0.0055
MRA Dynamic CASE 1.3.1	0.3290	1.6324	-9.3225	10.6454	-1.8091	0.1646	2.9861	0.2245	5.7681	0.0389
MRA Dynamic CASE 1.3.2	0.5721	3.0537	-17.0087	20.6651	-3.1333	0.2280	5.2487	0.4676	12.3203	0.0380
MRA Dynamic CASE 1.4	0.1183	0.5062	-2.1096	3.2592	-0.6320	0.1253	0.8646	0.0138	3.2047	0.0043
RA Dynamic CASE 1.1	0.0874	0.4136	-1.9933	1.3449	-0.5453	0.0921	0.6885	-0.0171	3.2154	-0.0053
RA Dynamic CASE 1.2	0.0714	0.4065	-2.8785	1.5120	-0.4926	0.0833	0.7260	-0.0331	3.3092	-0.0100
RA Dynamic CASE 1.3.1	0.3343	1.5606	-9.2904	10.6082	-1.6126	0.1883	2.8008	0.2298	5.2116	0.0441
RA Dynamic CASE 1.3.2	0.5981	2.9262	-16.4971	20.6149	-2.8694	0.1921	4.8815	0.4936	11.0244	0.0448
RA Dynamic CASE 1.4	0.0898	0.4159	-1.9933	1.3449	-0.5436	0.0943	0.6899	-0.0147	3.2000	-0.0046
GMV Static CASE 1.1	0.0750	0.3964	-1.9545	1.3309	-0.5325	0.0792	0.6436	-0.0295	3.2323	-0.0091
GMV Static CASE 1.2	0.0616	0.4016	-2.9038	1.6194	-0.4869	0.0716	0.7281	-0.0429	3.2984	-0.0130
GMV Static CASE 1.3.1	0.1731	0.8935	-3.9757	6.2773	-0.9609	0.0816	1.7662	0.0686	3.5626	0.0193
GMV Static CASE 1.3.2	0.2860	1.6356	-7.2498	12.3129	-1.4381	0.0933	3.1883	0.1815	5.1620	0.0352
GMV Static CASE 1.4	0.0774	0.3988	-1.9545	1.3309	-0.5293	0.0802	0.6497	-0.0271	3.2167	-0.0084
MS Static CASE 1.1	0.1383	0.8118	-4.4599	3.2486	-1.1332	0.1337	1.4061	0.0338	3.3308	0.0101
MS Static CASE 1.2	0.0804	0.7578	-3.0768	3.1023	-1.2012	0.1064	1.1954	-0.0241	3.7955	-0.0064
MS Static CASE 1.3.1	0.1506	1.9689	-17.0539	7.6833	-2.1304	0.1535	3.1705	0.0461	7.7669	0.0059
MS Static CASE 1.3.2	0.2049	3.5508	-34.8918	13.5318	-3.7531	0.1441	5.3232	0.1004	16.9276	0.0059
MS Static CASE 1.4	0.1405	0.8060	-3.8071	3.2486	-1.1052	0.1380	1.4061	0.0360	3.3053	0.0109
RL Static CASE 1.1	0.1387	0.8155	-4.4921	3.2692	-1.1418	0.1335	1.4119	0.0342	3.3343	0.0103
RL Static CASE 1.2	0.0805	0.7615	-3.0921	3.1217	-1.2063	0.1056	1.1998	-0.0240	3.8018	-0.0063
RL Static CASE 1.3.1	0.1505	1.9803	-17.1763	7.7340	-2.1480	0.1515	3.2003	0.0460	7.8193	0.0059
RL Static CASE 1.3.2	0.2043	3.5713	-35.1329	13.6355	-3.7490	0.1508	5.3379	0.0998	17.0851	0.0058
RL Static CASE 1.4	0.1409	0.8097	-3.8350	3.2692	-1.1136	0.1373	1.4119	0.0364	3.3086	0.0110
MRA Static CASE 1.1	0.0941	0.4681	-2.1379	1.2865	-0.6061	0.1135	0.8143	-0.0104	3.1725	-0.0033
MRA Static CASE 1.2	0.0673	0.4390	-2.7928	1.6674	-0.5967	0.0801	0.7314	-0.0372	3.3553	-0.0111
MRA Static CASE 1.3.1	0.1663	0.9747	-4.6295	5.2288	-1.1212	0.0804	1.9875	0.0618	4.0542	0.0152
MRA Static CASE 1.3.2	0.2615	1.7745	-9.4298	10.4404	-1.7781	0.0813	3.9460	0.1570	6.1889	0.0254
MRA Static CASE 1.4	0.0965	0.4695	-2.1379	1.2865	-0.6114	0.1153	0.8177	-0.0080	3.1567	-0.0025
RA Static CASE 1.1	0.0798	0.4067	-2.0004	1.3079	-0.5473	0.0900	0.6799	-0.0247	3.2101	-0.0077
RA Static CASE 1.2	0.0630	0.4020	-2.8760	1.5016	-0.4854	0.0760	0.6815	-0.0415	3.3051	-0.0126
RA Static CASE 1.3.1	0.1714	0.8793	-3.9873	5.4825	-0.9348	0.0921	1.7789	0.0669	3.6224	0.0185
RA Static CASE 1.3.2	0.2798	1.6089	-7.3941	10.6944	-1.3009	0.0841	3.3611	0.1753	5.2138	0.0336
RA Static CASE 1.4	0.0822	0.4090	-2.0004	1.3099	-0.5473	0.0924	0.6864	-0.0223	3.1947	-0.0070
GMV No Crypto CASE 1.1	0.0711	0.3925	-1.9545	1.3309	-0.5325	0.0834	0.6266	-0.0334	3.2346	-0.0103
GMV No Crypto CASE 1.2	0.0601	0.4002	-2.9038	1.6194	-0.4869	0.0637	0.7281	-0.0444	3.3043	-0.0134
GMV No Crypto CASE 1.3	0.0598	0.3626	-2.1537	1.3238	-0.4537	0.0402	0.6540	-0.0447	3.1630	-0.0141
GMV No Crypto CASE 1.4	0.0711	0.3925	-1.9545	1.3309	-0.5325	0.0834	0.6266	-0.0334	3.2346	-0.0103
MS No Crypto CASE 1.1	0.0922	0.7221	-2.5619	2.5621	-1.1251	0.1375	1.1530	-0.0123	3.2949	-0.0037
MS No Crypto CASE 1.2	0.0562	0.7254	-2.5930	3.1023	-1.2012	0.0896	1.1336	-0.0483	3.7902	-0.0128
MS No Crypto CASE 1.3	0.0518	0.8973	-4.6481	3.8852	-1.3656	0.1036	1.4319	-0.0527	4.2814	-0.0123
MS No Crypto CASE 1.4	0.0926	0.7231	-2.5619	2.5621	-1.1251	0.1391	1.1530	-0.0119	3.2962	-0.0036
RL No Crypto CASE 1.1	0.0905	0.6845	-2.5117	2.3555	-1.0258	0.1284	1.0818	-0.0140	3.2673	-0.0043
RL No Crypto CASE 1.2	0.0565	0.6826	-2.5668	2.8593	-1.1581	0.0973	1.0627	-0.0480	3.7200	-0.0129
RL No Crypto CASE 1.3	0.0525	0.8347	-4.2301	3.6579	-1.2742	0.1072	1.3255	-0.0520	4.1361	-0.0126
RL No Crypto CASE 1.4	0.0908	0.6854	-2.5117	2.3555	-1.0258	0.1304	1.0818	-0.0137	3.2685	-0.0042
MRA No Crypto CASE 1.1	0.0769	0.4424	-2.1217	1.2737	-0.5989	0.1022	0.7429	-0.0276	3.1918	-0.0086
MRA No Crypto CASE 1.2	0.0590	0.4264	-2.8027	1.6276	-0.5903	0.0768	0.6891	-0.0455	3.3589	-0.0135
MRA No Crypto CASE 1.3	0.0576	0.4216	-1.8554	1.8925	-0.5887	0.0566	0.6847	-0.0469	3.3317	-0.0141
MRA No Crypto CASE 1.4	0.0770	0.4428	-2.1217	1.2737	-0.5989	0.1022	0.7429	-0.0275	3.1921	-0.0086
RL No Crypto CASE 1.1	0.0726	0.4003	-1.9963	1.3100	-0.5436	0.0914	0.6640	-0.0319	3.2197	-0.0099
RL No Crypto CASE 1.2	0.0598	0.3999	-2.8785	1.5120	-0.4848	0.0686	0.6789	-0.0447	3.3123	-0.0135
RL No Crypto CASE 1.3	0.0592	0.3649	-2.0791	1.3692	-0.4823	0.0521	0.6445	-0.0453	3.1953	-0.0142
RL No Crypto CASE 1.4	0.0726	0.4004	-1.9963	1.3118	-0.5436	0.0914	0.6640	-0.0319	3.2197	-0.0099

Table A - Appendix: *Markowitz Portfolios Summary Statistics - Rolling inputs*

Strategy	Mean	StDev	Min	Max	q5	q50	q95	TE	TEV	IR
Benchmark	0.1045	1.8646	-8.9489	7.1412	-2.9434	0.3042	3.0307	/	/	/
GMV Dynamic CASE 2.1	0.0724	0.3952	-1.7259	1.4398	-0.5372	0.0798	0.6805	-0.0321	3.2337	-0.0099
GMV Dynamic CASE 2.2	0.0643	0.3871	-2.7223	1.5598	-0.4644	0.0593	0.7280	-0.0402	3.2647	-0.0123
GMV Dynamic CASE 2.3.1	0.3594	1.8165	-10.1093	12.6691	-1.8692	0.1755	3.2253	0.2549	5.9349	0.0430
GMV Dynamic CASE 2.3.2	0.5443	2.8282	-19.6517	20.7364	-2.8278	0.1934	4.8044	0.4398	10.4545	0.0421
GMV Dynamic CASE 2.4	0.0746	0.3970	-1.7259	1.4398	-0.5340	0.0826	0.7134	-0.0299	3.2193	-0.0093
MS Dynamic CASE 2.1	0.2700	1.5197	-5.1692	16.4158	-1.3508	0.1519	1.7645	0.1654	4.8488	0.0341
MS Dynamic CASE 2.2	0.1520	1.0875	-4.5476	14.0741	-1.1896	0.1229	1.3930	0.0474	4.7734	0.0099
MS Dynamic CASE 2.3.1	0.3512	4.0454	-19.7220	54.6509	-4.5964	0.1915	4.8941	0.2466	20.7303	0.0119
MS Dynamic CASE 2.3.2	0.5631	5.4024	-27.6639	54.6509	-7.0120	0.1729	7.8456	0.4585	34.4137	0.0133
MS Dynamic CASE 2.4	0.2819	1.5222	-5.1692	16.4158	-1.3221	0.1540	1.8244	0.1774	4.7612	0.0373
RL Dynamic CASE 2.1	0.2511	1.3892	-4.7732	14.9032	-1.2623	0.1401	1.6206	0.1466	4.5207	0.0324
RL Dynamic CASE 2.2	0.1436	0.9937	-4.1661	12.7367	-1.0848	0.1207	1.2712	0.0391	4.5328	0.0086
RL Dynamic CASE 2.3.1	0.3519	3.7516	-17.8515	50.2987	-4.2090	0.1774	4.5046	0.2474	18.2739	0.0135
RL Dynamic CASE 2.3.2	0.5613	5.0350	-25.0044	50.2987	-6.2092	0.1912	7.4568	0.4568	30.3146	0.0151
RL Dynamic CASE 2.4	0.2621	1.3919	-4.7732	14.9032	-1.2541	0.1572	1.6836	0.1576	4.4405	0.0355
MRA Dynamic CASE 2.1	0.1260	0.5850	-2.1470	4.8723	-0.6340	0.1047	0.8755	0.0215	3.2737	0.0066
MRA Dynamic CASE 2.2	0.0881	0.4573	-2.4367	3.8679	-0.5689	0.0946	0.7840	-0.0164	3.4532	-0.0048
MRA Dynamic CASE 2.3.1	0.3572	2.1109	-9.7433	21.4379	-1.8013	0.1710	3.4579	0.2527	7.5604	0.0334
MRA Dynamic CASE 2.3.2	0.5494	3.0985	-19.0002	21.4379	-3.0032	0.1945	5.2948	0.4449	12.8090	0.0347
MRA Dynamic CASE 2.4	0.1308	0.5887	-2.1470	4.8723	-0.6340	0.1073	0.8826	0.0263	3.2406	0.0081
RA Dynamic CASE 2.1	0.0858	0.4181	-1.7920	1.8580	-0.5154	0.0872	0.7026	-0.0187	3.2159	-0.0058
RA Dynamic CASE 2.2	0.0702	0.3863	-2.6509	1.4295	-0.4464	0.0706	0.7059	-0.0343	3.2964	-0.0104
RA Dynamic CASE 2.3.1	0.3589	1.8498	-10.0178	12.6076	-1.7213	0.2061	3.2930	0.2544	6.1744	0.0412
RA Dynamic CASE 2.3.2	0.5456	2.8477	-19.4888	19.8161	-2.7358	0.2100	4.8007	0.4411	10.7535	0.0410
RA Dynamic CASE 2.4	0.0886	0.4209	-1.7920	1.8580	-0.5084	0.0886	0.7144	-0.0159	3.1969	-0.0050
GMV Static CASE 2.1	0.0698	0.3880	-1.7259	1.3590	-0.5146	0.0845	0.6776	-0.0347	3.2380	-0.0107
GMV Static CASE 2.2	0.0632	0.3862	-2.7008	1.5598	-0.4593	0.0565	0.7319	-0.0413	3.2641	-0.0127
GMV Static CASE 2.3.1	0.1690	0.8879	-4.8882	6.6866	-0.8692	0.0728	1.6607	0.0645	3.6041	0.0179
GMV Static CASE 2.3.2	0.2723	1.6530	-8.7393	13.0653	-1.4844	0.0689	3.2666	0.1678	5.2734	0.0318
GMV Static CASE 2.4	0.0720	0.3899	-1.7259	1.3590	-0.5108	0.0859	0.6816	-0.0325	3.2236	-0.0101
MS Static CASE 2.1	0.1405	0.9308	-4.8722	5.3890	-1.1751	0.1199	1.4380	0.0360	3.4208	0.0105
MS Static CASE 2.2	0.0835	0.8357	-3.9803	3.4701	-1.2496	0.1064	1.3072	-0.0210	4.3062	-0.0049
MS Static CASE 2.3.1	0.0995	2.1139	-17.9604	13.6169	-2.7721	0.0996	2.9310	-0.0050	8.5405	-0.0006
MS Static CASE 2.3.2	0.2050	3.4650	-27.6175	25.8027	-3.5913	0.1114	4.9980	0.1005	16.2209	0.0062
MS Static CASE 2.4	0.1507	0.9365	-4.8722	5.3890	-1.1556	0.1375	1.4863	0.0462	3.3483	0.0138
RL Static CASE 2.1	0.1410	0.9357	-4.9075	5.4247	-1.1801	0.1224	1.4485	0.0365	3.4264	0.0107
RL Static CASE 2.2	0.0836	0.8405	-4.0029	3.4933	-1.2572	0.1072	1.3154	-0.0209	4.3175	-0.0048
RL Static CASE 2.3.1	0.0991	2.1261	-18.0810	13.7023	-2.7978	0.0998	2.9234	-0.0054	8.6007	-0.0006
RL Static CASE 2.3.2	0.2046	3.4841	-27.8061	25.9655	-3.5794	0.1132	5.0114	0.1001	16.3642	0.0061
RL Static CASE 2.4	0.1512	0.9414	-4.9075	5.4247	-1.1645	0.1377	1.4921	0.0467	3.3535	0.0139
MRA Static CASE 2.1	0.0912	0.4774	-2.1056	1.8181	-0.6449	0.0899	0.8380	-0.0133	3.1544	-0.0042
MRA Static CASE 2.2	0.0693	0.4189	-2.4213	1.1722	-0.5486	0.0841	0.7674	-0.0352	3.4393	-0.0102
MRA Static CASE 2.3.1	0.1480	1.0130	-5.2154	5.8260	-1.1498	0.0957	1.9681	0.0435	4.2213	0.0103
MRA Static CASE 2.3.2	0.2520	1.8122	-9.2050	11.5583	-1.9757	0.1097	3.4839	0.1475	6.3305	0.0233
MRA Static CASE 2.4	0.0958	0.4816	-2.1056	1.9747	-0.6449	0.0949	0.8418	-0.0087	3.1222	-0.0028
RA Static CASE 2.1	0.0752	0.3983	-1.7532	1.3325	-0.5294	0.0832	0.6948	-0.0293	3.2058	-0.0092
RA Static CASE 2.2	0.0647	0.3800	-2.6309	1.4148	-0.4646	0.0599	0.6817	-0.0398	3.2966	-0.0121
RA Static CASE 2.3.1	0.1637	0.8814	-4.8893	5.8756	-0.9689	0.0728	1.6761	0.0592	3.6875	0.0161
RA Static CASE 2.3.2	0.2672	1.6395	-8.7595	11.5738	-1.5107	0.0715	3.2335	0.1627	5.3553	0.0304
RA Static CASE 2.4	0.0780	0.4009	-1.7532	1.3319	-0.5108	0.0859	0.6951	-0.0265	3.1870	-0.0083
GMV No Crypto CASE 2.1	0.0688	0.3876	-1.7259	1.3618	-0.5568	0.0759	0.6777	-0.0357	3.2332	-0.0110
GMV No Crypto CASE 2.2	0.0627	0.3853	-2.6991	1.5525	-0.4578	0.0516	0.7379	-0.0418	3.2608	-0.0128
GMV No Crypto CASE 2.3	0.0652	0.3579	-1.9117	1.5882	-0.4104	0.0427	0.6372	-0.0393	3.1263	-0.0126
GMV No Crypto CASE 2.4	0.0704	0.3868	-1.7259	1.3618	-0.4914	0.0756	0.6777	-0.0341	3.2326	-0.0105
MS No Crypto CASE 2.1	0.0874	0.7986	-3.9969	4.2265	-1.1008	0.1083	1.2715	-0.0171	3.2908	-0.0052
MS No Crypto CASE 2.2	0.0423	0.7578	-3.6506	2.5631	-1.2609	0.0958	1.1752	-0.0622	4.1880	-0.0149
MS No Crypto CASE 2.3	0.0396	0.9142	-4.3536	3.7356	-1.5731	0.0730	1.4680	-0.0649	4.4025	-0.0147
MS No Crypto CASE 2.4	0.0858	0.7811	-3.9969	3.3931	-1.0993	0.1070	1.2803	-0.0188	3.2699	-0.0057
RL No Crypto CASE 2.1	0.0858	0.7512	-3.6278	3.9413	-1.0607	0.1032	1.2206	-0.0187	3.2528	-0.0057
RL No Crypto CASE 2.2	0.0440	0.7062	-3.3239	2.3733	-1.1789	0.0923	1.0651	-0.0605	4.0711	-0.0149
RL No Crypto CASE 2.3	0.0417	0.8455	-4.0080	3.4418	-1.4538	0.0640	1.3631	-0.0628	4.2347	-0.0148
RL No Crypto CASE 2.4	0.0857	0.7811	-3.9969	3.3931	-1.0993	0.1070	1.2803	-0.0188	3.2701	-0.0057
MRA No Crypto CASE 2.1	0.0739	0.4438	-1.8252	1.7257	-0.6121	0.0780	0.8006	-0.0306	3.1616	-0.0097
MRA No Crypto CASE 2.2	0.0571	0.3994	-2.4443	1.1367	-0.5556	0.0735	0.7202	-0.0474	3.4099	-0.0139
MRA No Crypto CASE 2.3	0.0581	0.3986	-1.5790	1.3125	-0.5952	0.0596	0.7175	-0.0464	3.3135	-0.0140
MRA No Crypto CASE 2.4	0.0746	0.4401	-1.8337	1.2865	-0.6020	0.0804	0.7823	-0.0299	3.1602	-0.0095
RA No Crypto CASE 2.1	0.0701	0.3945	-1.7508	1.3376	-0.5669	0.0857	0.6934	-0.0344	3.2091	-0.0107
RA No Crypto CASE 2.2	0.0613	0.3791	-2.6354	1.4271	-0.4669	0.0567	0.6872	-0.0432	3.2905	-0.0131
RA No Crypto CASE 2.3	0.0634	0.3523	-1.8285	1.4624	-0.4599	0.0495	0.6233	-0.0411	3.1614	-0.0130
RA No Crypto CASE 2.4	0.0715	0.3934	-1.7508	1.3368	-0.5225	0.0857	0.6934	-0.0330	3.2086	-0.0103

***Table B - Appendix**: Markowitz Portfolios Summary Statistics – Rolling Inputs*

Strategy	Mean	StDev	Min	Max	q5	q50	q95	TE	TEV	IR
Benchmark	0.1045	1.8646	-8.9489	7.1412	-2.9434	0.3042	3.0307	0.0000	0.0000	0.0000
GMV Dynamic Rolling	0.0778	0.4003	-1.9545	1.3309	-0.5325	0.0792	0.6436	-0.0267	3.2310	-0.0083
MS Dynamic Rolling	0.2165	1.0827	-4.6360	10.6920	-1.2409	0.1608	1.7020	0.1120	3.8591	0.0290
RL Dynamic Rolling	0.2051	1.0095	-4.2185	9.8444	-1.1577	0.1579	1.5819	0.1005	3.7376	0.0269
MRA Dynamic Rolling	0.1160	0.5056	-2.1096	3.2592	-0.6248	0.1243	0.8543	0.0115	3.2207	0.0036
RA Dynamic Rolling	0.0874	0.4136	-1.9933	1.3449	-0.5453	0.0921	0.6885	-0.0171	3.2154	-0.0053
ERC Dynamic Rolling	0.1596	0.6569	-2.8666	4.8121	-0.7409	0.1391	1.0821	0.0551	2.6212	0.0210
GRC1 Dynamic Rolling	0.1265	0.5528	-2.5633	2.2634	-0.6707	0.1288	1.0034	0.0220	2.5606	0.0086
GRC2 Dynamic Rolling	0.1637	0.6513	-2.7686	4.4660	-0.7420	0.1491	1.0564	0.0592	2.5474	0.0232
GMV Static Rolling	0.0750	0.3964	-1.9545	1.3309	-0.5325	0.0792	0.6436	-0.0295	3.2323	-0.0091
MS Static Rolling	0.1383	0.8118	-4.4599	3.2486	-1.1332	0.1337	1.4061	0.0338	3.3308	0.0101
RL Static Rolling	0.1387	0.8155	-4.4921	3.2692	-1.1418	0.1335	1.4119	0.0342	3.3343	0.0103
MRA Static Rolling	0.0941	0.4681	-2.1379	1.2865	-0.6061	0.1135	0.8143	-0.0104	3.1725	-0.0033
RA Static Rolling	0.0798	0.4067	-2.0004	1.3079	-0.5473	0.0900	0.6799	-0.0247	3.2101	-0.0077
ERC Static Rolling	0.1383	0.6085	-2.8801	1.8609	-0.7686	0.1466	1.0707	0.0338	2.5705	0.0131
GRC1 Static Rolling	0.1129	0.5366	-2.5516	1.5077	-0.7004	0.1326	0.9107	0.0084	2.5521	0.0033
GRC2 Static Rolling	0.1332	0.5903	-2.7403	2.0429	-0.6967	0.1377	0.9834	0.0287	2.4839	0.0116
GMV No Crypto Rolling	0.0711	0.3925	-1.9545	1.3309	-0.5325	0.0834	0.6266	-0.0334	3.2346	-0.0103
MS No Crypto Rolling	0.0922	0.7221	-2.5619	2.5621	-1.1251	0.1375	1.1530	-0.0123	3.2949	-0.0037
RL No Crypto Rolling	0.0873	0.6179	-2.4195	1.9762	-0.9654	0.1272	1.0082	-0.0172	3.2273	-0.0053
MRA No Crypto Rolling	0.0760	0.4310	-2.0940	1.2610	-0.5810	0.1015	0.7169	-0.0286	3.1958	-0.0089
RA No Crypto Rolling	0.0723	0.3988	-1.9894	1.3134	-0.5436	0.0947	0.6568	-0.0322	3.2219	-0.0100
ERC No Crypto Rolling	0.1062	0.5805	-2.8801	1.6726	-0.7888	0.1264	1.0111	0.0017	2.5571	0.0007
GRC1 No Crypto Rolling	0.0947	0.5246	-2.5516	1.5077	-0.7159	0.1131	0.8865	-0.0098	2.5516	-0.0038
GRC2 No Crypto Rolling	0.0976	0.5533	-2.7155	1.7125	-0.7557	0.1171	0.9274	-0.0070	2.4666	-0.0028

***Table C - Appendix**: Risk Budgeting Portfolios Summary Statistics – Rolling Inputs*

Strategy	Mean	StDev	Min	Max	q5	q50	q95	TE	TEV	IR
Benchmark	0.1045	1.8646	-8.9489	7.1412	-2.9434	0.3042	3.0307	0.0000	0.0000	0.0000
GMV Dynamic EWMA	0.0724	0.3952	-1.7259	1.4398	-0.5372	0.0798	0.6805	-0.0321	3.2337	-0.0099
MS Dynamic EWMA	0.2700	1.5197	-5.1692	16.4158	-1.3508	0.1519	1.7645	0.1654	4.8488	0.0341
RL Dynamic EWMA	0.2511	1.3892	-4.7732	14.9032	-1.2623	0.1401	1.6206	0.1466	4.5207	0.0324
MRA Dynamic EWMA	0.1260	0.5850	-2.1470	4.8723	-0.6340	0.1047	0.8755	0.0215	3.2737	0.0066
RA Dynamic EWMA	0.0858	0.4181	-1.7920	1.8580	-0.5154	0.0872	0.7026	-0.0187	3.2159	-0.0058
ERC Dynamic EWMA	0.1446	0.6407	-2.8453	4.6351	-0.7777	0.1428	1.0461	0.0401	2.6003	0.0154
GRC1 Dynamic EWMA	0.1193	0.5492	-2.4826	2.8499	-0.6701	0.1221	0.9075	0.0148	2.5656	0.0058
GRC2 Dynamic EWMA	0.1512	0.6327	-2.6526	4.3861	-0.7495	0.1492	1.0453	0.0467	2.5313	0.0184
GMV Static EWMA	0.0698	0.3880	-1.7259	1.3590	-0.5146	0.0845	0.6776	-0.0347	3.2380	-0.0107
MS Static EWMA	0.1405	0.9308	-4.8722	5.3890	-1.1751	0.1199	1.4380	0.0360	3.4208	0.0105
RL Static EWMA	0.1410	0.9357	-4.9075	5.4247	-1.1801	0.1224	1.4485	0.0365	3.4264	0.0107
MRA Static EWMA	0.0912	0.4774	-2.1056	1.8181	-0.6449	0.0899	0.8380	-0.0133	3.1544	-0.0042
RA Static EWMA	0.0752	0.3983	-1.7532	1.3325	-0.5294	0.0832	0.6948	-0.0293	3.2058	-0.0092
ERC Static EWMA	0.1315	0.6032	-2.8525	2.2291	-0.7872	0.1501	1.0344	0.0270	2.5575	0.0106
GRC1 Static EWMA	0.1085	0.5303	-2.4931	1.5179	-0.6831	0.1114	0.8967	0.0040	2.5482	0.0016
GRC2 Static EWMA	0.1272	0.5840	-2.6793	2.4246	-0.7143	0.1397	0.9760	0.0227	2.4734	0.0092
GMV No Crypto EWMA	0.0688	0.3876	-1.7259	1.3618	-0.5568	0.0759	0.6777	-0.0357	3.2332	-0.0110
MS No Crypto EWMA	0.0874	0.7986	-3.9969	4.2265	-1.1008	0.1083	1.2715	-0.0171	3.2908	-0.0052
RL No Crypto EWMA	0.0830	0.6669	-2.9499	3.4175	-1.0038	0.0933	1.0931	-0.0215	3.1986	-0.0067
MRA No Crypto EWMA	0.0731	0.4297	-1.8088	1.5686	-0.5997	0.0723	0.7653	-0.0314	3.1689	-0.0099
RA No Crypto EWMA	0.0699	0.3930	-1.7466	1.3416	-0.5654	0.0842	0.6912	-0.0346	3.2128	-0.0108
ERC No Crypto EWMA	0.1052	0.5764	-2.8525	1.6692	-0.7937	0.1318	1.0010	0.0007	2.5647	0.0003
GRC1 No Crypto EWMA	0.0935	0.5181	-2.4931	1.4718	-0.7120	0.1128	0.8514	-0.0110	2.5640	-0.0043
GRC2 No Crypto EWMA	0.0965	0.5468	-2.6793	1.5665	-0.7157	0.1177	0.8923	-0.0080	2.4799	-0.0032

***Table D - Appendix**: Risk Budgeting Portfolios Summary Statistics – EWMA Inputs*

Strategy	CI	Sh	Tr	So	VaR	ES	Cal	Ste	FT
Benchmark	546	71	68	63	71	68	71	71	63
GMV Dynamic CASE 1.1	191	20	24	34	27	23	17	22	24
GMV Dynamic CASE 1.2	338	43	44	33	31	40	51	51	45
GMV Dynamic CASE 1.3.1	50	5	5	12	6	2	6	9	5
GMV Dynamic CASE 1.3.2	96	8	4	9	3	1	31	33	7
GMV Dynamic CASE 1.4	164	17	19	35	21	20	15	17	20
MS Dynamic CASE 1.1	97	16	10	18	16	11	11	6	9
MS Dynamic CASE 1.2	208	41	23	5	49	29	21	21	19
MS Dynamic CASE 1.3.1	486	58	60	69	56	58	62	62	61
MS Dynamic CASE 1.3.2	507	62	62	66	60	62	64	64	67
MS Dynamic CASE 1.4	71	11	6	20	15	9	5	1	4
RL Dynamic CASE 1.1	89	12	9	21	14	10	10	5	8
RL Dynamic CASE 1.2	194	37	21	8	48	28	19	16	17
RL Dynamic CASE 1.3.1	468	57	57	71	53	53	59	60	58
RL Dynamic CASE 1.3.2	494	59	61	70	57	59	61	63	64
RL Dynamic CASE 1.4	60	7	2	22	13	8	4	2	2
MRA Dynamic CASE 1.1	54	2	3	26	8	6	2	4	3
MRA Dynamic CASE 1.2	222	23	28	17	23	25	37	42	27
MRA Dynamic CASE 1.3.1	101	13	11	4	11	7	20	24	11
MRA Dynamic CASE 1.3.2	160	25	15	3	10	12	34	40	21
MRA Dynamic CASE 1.4	46	1	1	27	7	5	1	3	1
RA Dynamic CASE 1.1	120	6	16	30	18	16	9	11	14
RA Dynamic CASE 1.2	295	30	41	28	29	35	47	47	38
RA Dynamic CASE 1.3.1	44	4	7	10	4	3	3	7	6
RA Dynamic CASE 1.3.2	103	10	8	7	2	4	26	36	10
RA Dynamic CASE 1.4	107	3	14	31	17	15	7	8	12
GMV Static CASE 1.1	221	24	29	37	30	27	23	25	26
GMV Static CASE 1.2	377	46	46	42	42	46	55	54	46
GMV Static CASE 1.3.1	122	22	12	25	12	13	13	10	15
GMV Static CASE 1.3.2	212	31	22	16	5	17	42	37	42
GMV Static CASE 1.4	200	21	25	38	26	26	18	23	23
MS Static CASE 1.1	333	39	42	51	46	43	39	34	39
MS Static CASE 1.2	426	60	58	15	63	60	57	57	56
MS Static CASE 1.3.1	522	65	65	68	61	65	65	65	68
MS Static CASE 1.3.2	544	69	70	65	65	70	67	68	70
MS Static CASE 1.4	285	32	36	53	40	41	29	18	36
RL Static CASE 1.1	341	40	43	52	47	44	40	35	40
RL Static CASE 1.2	432	61	59	14	64	61	58	58	57
RL Static CASE 1.3.1	529	66	67	67	62	66	66	66	69
RL Static CASE 1.3.2	552	70	71	64	66	71	69	70	71
RL Static CASE 1.4	294	33	38	54	41	42	30	19	37
MRA Static CASE 1.1	160	14	18	44	20	22	12	14	16
MRA Static CASE 1.2	361	47	48	29	50	49	45	45	48
MRA Static CASE 1.3.1	249	38	35	11	24	32	36	32	41
MRA Static CASE 1.3.2	336	50	51	6	25	47	49	46	62
MRA Static CASE 1.4	144	9	17	47	19	19	8	12	13
RA Static CASE 1.1	196	18	26	39	28	24	16	20	25
RA Static CASE 1.2	360	45	45	36	37	45	52	53	47
RA Static CASE 1.3.1	127	19	13	19	9	14	22	13	18
RA Static CASE 1.3.2	219	35	27	13	1	18	44	38	43
RA Static CASE 1.4	170	15	20	41	22	21	14	15	22
GMV No Crypto CASE 1.1	256	29	31	46	32	31	28	31	28
GMV No Crypto CASE 1.2	392	48	49	43	43	48	56	56	49
GMV No Crypto CASE 1.3	335	42	34	58	36	36	48	49	32
GMV No Crypto CASE 1.4	252	28	30	45	33	30	27	30	29
MS No Crypto CASE 1.1	431	56	56	62	59	57	43	44	54
MS No Crypto CASE 1.2	467	64	64	23	68	64	63	61	60
MS No Crypto CASE 1.3	482	68	69	1	70	69	70	69	66
MS No Crypto CASE 1.4	422	55	55	61	58	56	41	43	53
RL No Crypto CASE 1.1	407	54	53	60	55	55	38	41	51
RL No Crypto CASE 1.2	458	63	63	24	67	63	60	59	59
RL No Crypto CASE 1.3	471	67	66	2	69	67	68	67	65
RL No Crypto CASE 1.4	396	53	52	59	54	54	35	39	50
MRA No Crypto CASE 1.1	302	36	39	56	39	38	33	27	34
MRA No Crypto CASE 1.2	397	51	54	32	51	52	50	52	55
MRA No Crypto CASE 1.3	397	52	47	48	52	51	53	50	44
MRA No Crypto CASE 1.4	292	34 1	37	55	38	37	32	26	33
RA No Crypto CASE 1.1	264	27	33	50	35	34	25	29	31
RA No Crypto CASE 1.2	394	49	50	40	44	50	54	55	52
RA No Crypto CASE 1.3	354	44	40	57	45	39	46	48	35
RA No Crypto CASE 1.4	256	26	32	49	34	33	24	28	30

***Table 1 - Appendix**: Performance Indicators Rank Markowitz Strategies – Rolling inputs*

Strategy	CI	Sh	Tr	So	VaR	ES	Cal	Ste	FT
Benchmark	523	66	64	55	67	66	71	71	63
GMV Dynamic CASE 2.1	182	20	26	27	36	30	10	15	18
GMV Dynamic CASE 2.2	314	36	40	33	30	36	47	52	40
GMV Dynamic CASE 2.3.1	104	6	9	8	12	7	25	25	12
GMV Dynamic CASE 2.3.2	131	10	12	5	11	10	29	31	23
GMV Dynamic CASE 2.4	153	16	21	29	28	24	8	13	14
MS Dynamic CASE 2.1	91	30	6	14	5	5	18	7	6
MS Dynamic CASE 2.2	315	51	30	70	42	34	31	29	28
MS Dynamic CASE 2.3.1	488	62	60	63	60	57	62	62	62
MS Dynamic CASE 2.3.2	460	58	54	61	56	54	58	60	59
MS Dynamic CASE 2.4	52	18	3	16	1	2	6	3	3
RL Dynamic CASE 2.1	81	25	4	15	7	4	15	6	5
RL Dynamic CASE 2.2	288	49	27	71	37	29	30	28	17
RL Dynamic CASE 2.3.1	473	61	55	66	54	55	61	61	60
RL Dynamic CASE 2.3.2	445	54	53	62	53	53	57	59	54
RL Dynamic CASE 2.4	44	15	1	17	2	1	5	2	1
MRA Dynamic CASE 2.1	52	2	5	20	8	6	3	4	4
MRA Dynamic CASE 2.2	122	9	13	9	21	15	22	23	10
MRA Dynamic CASE 2.3.1	164	34	16	2	9	13	32	34	24
MRA Dynamic CASE 2.3.2	174	31	17	1	14	11	33	35	32
MRA Dynamic CASE 2.4	35	1	2	21	4	3	1	1	2
RA Dynamic CASE 2.1	92	4	10	24	18	16	4	8	8
RA Dynamic CASE 2.2	232	23	33	22	20	23	36	41	34
RA Dynamic CASE 2.3.1	97	8	8	7	3	8	26	26	11
RA Dynamic CASE 2.3.2	118	11	11	3	6	9	27	30	21
RA Dynamic CASE 2.4	77	3	7	25	16	12	2	5	7
GMV Static CASE 2.1	213	26	29	30	35	33	14	20	26
GMV Static CASE 2.2	326	39	42	36	32	38	48	50	41
GMV Static CASE 2.3.1	127	13	15	18	10	14	20	12	25
GMV Static CASE 2.3.2	247	38	35	12	13	25	38	37	49
GMV Static CASE 2.4	173	19	25	32	27	27	11	16	16
MS Static CASE 2.1	360	45	48	43	49	48	43	38	46
MS Static CASE 2.2	478	59	61	68	61	61	59	57	52
MS Static CASE 2.3.1	536	69	70	57	66	69	68	69	68
MS Static CASE 2.3.2	513	64	66	59	64	64	63	63	70
MS Static CASE 2.4	324	43	44	45	39	44	34	32	43
RL Static CASE 2.1	368	46	49	44	50	49	44	39	47
RL Static CASE 2.2	484	60	62	67	62	62	60	58	53
RL Static CASE 2.3.1	544	70	71	56	68	70	70	70	69
RL Static CASE 2.3.2	517	65	67	58	63	65	64	64	71
RL Static CASE 2.4	332	44	45	46	40	45	35	33	44
MRA Static CASE 2.1	177	12	20	40	26	21	24	19	15
MRA Static CASE 2.2	287	37	41	11	44	43	37	36	38
MRA Static CASE 2.3.1	339	47	50	6	41	47	51	46	51
MRA Static CASE 2.3.2	362	52	52	4	43	51	52	47	61
MRA Static CASE 2.4	140	5	14	42	23	18	19	10	9
RA Static CASE 2.1	162	14	24	34	25	26	9	11	19
RA Static CASE 2.2	281	32	38	23	29	35	41	44	39
RA Static CASE 2.3.1	165	17	18	13	17	17	28	24	31
RA Static CASE 2.3.2	262	40	37	10	15	28	42	40	50
RA Static CASE 2.4	134	7	19	37	22	20	7	9	13
GMV No Crypto CASE 2.1	246	29	32	35	47	37	16	21	29
GMV No Crypto CASE 2.2	338	41	43	38	33	40	50	51	42
GMV No Crypto CASE 2.3	246	21	22	50	19	19	46	49	20
GMV No Crypto CASE 2.4	192	22	28	31	24	31	12	17	27
MS No Crypto CASE 2.1	453	57	59	52	57	58	56	56	58
MS No Crypto CASE 2.2	527	67	65	60	69	67	67	67	65
MS No Crypto CASE 2.3	550	71	69	64	71	71	69	68	67
MS No Crypto CASE 2.4	446	55	57	53	58	59	54	54	56
RL No Crypto CASE 2.1	432	53	56	51	55	56	53	53	55
RL No Crypto CASE 2.2	513	63	63	65	65	63	65	65	64
RL No Crypto CASE 2.3	541	68	68	69	70	68	66	66	66
RL No Crypto CASE 2.4	454	56	58	54	59	60	55	55	57
MRA No Crypto CASE 2.1	302	35	39	48	48	46	23	27	36
MRA No Crypto CASE 2.2	368	50	51	19	51	52	49	48	48
MRA No Crypto CASE 2.3	343	48	46	28	52	50	40	42	37
MRA No Crypto CASE 2.4	281	33	36	47	45	42	21	22	35
RA No Crypto CASE 2.1	256	28	34	41	46	39	17	18	33
RA No Crypto CASE 2.2	329	42	47	26	38	41	45	45	45
RA No Crypto CASE 2.3	256	27	23	49	31	22	39	43	22
RA No Crypto CASE 2.4	217	24	31	39	34	32	13	14	30

***Table 2 - Appendix**: Performance Indicators Rank Markowitz Strategies – EWMA inputs*

Strategy	CI	Sh	Tr	So	VaR	ES	Cal	Ste	FT
Benchmark	200	25	25	25	25	25	25	25	25
GMV Dynamic Rolling	90	13	12	5	12	12	11	13	12
MS Dynamic Rolling	49	11	5	1	8	5	8	6	5
RL Dynamic Rolling	42	9	4	2	7	4	7	5	4
MRA Dynamic Rolling	27	3	3	3	5	3	4	4	2
RA Dynamic Rolling	62	7	9	4	10	9	6	8	9
ERC Dynamic Rolling	31	2	2	16	2	2	2	2	3
GRC1 Dynamic Rolling	71	4	7	19	4	7	12	11	7
GRC2 Dynamic Rolling	28	1	1	17	1	1	3	3	1
GMV Static Rolling	104	14	14	6	14	14	14	14	14
MS Static Rolling	153	21	21	11	21	21	19	18	21
RL Static Rolling	161	22	22	12	22	22	20	19	22
MRA Static Rolling	78	10	10	8	11	11	9	9	10
RA Static Rolling	93	12	13	7	13	13	10	12	13
ERC Static Rolling	59	5	6	18	6	6	5	7	6
GRC1 Static Rolling	93	8	11	21	9	10	13	10	11
GRC2 Static Rolling	55	6	8	20	3	8	1	1	8
GMV No Crypto Rolling	123	17	16	9	16	16	16	17	16
MS No Crypto Rolling	177	24	24	15	24	24	21	21	24
RL No Crypto Rolling	167	23	23	14	23	23	18	20	23
MRA No Crypto Rolling	139	20	18	13	19	19	17	15	18
RA No Crypto Rolling	125	16	17	10	17	17	15	16	17
ERC No Crypto Rolling	143	15	15	22	15	15	23	23	15
GRC1 No Crypto Rolling	163	18	19	23	18	18	24	24	19
GRC2 No Crypto Rolling	167	19	20	24	20	20	22	22	20

***Table 3 - Appendix**: Performance Indicators Ranking Risk Contribution Strategies – Rolling inputs*

Strategy	CI	Sh	Tr	So	VaR	ES	Cal	Ste	FT
Benchmark	200	25	25	25	25	25	25	25	25
GMV Dynamic EWMA	80	11	13	5	15	13	4	7	12
MS Dynamic EWMA	42	18	3	1	2	2	10	3	3
RL Dynamic EWMA	31	13	1	2	3	1	7	2	2
MRA Dynamic EWMA	21	6	2	3	4	3	1	1	1
RA Dynamic EWMA	49	7	8	4	9	9	2	4	6
ERC Dynamic EWMA	67	2	5	15	5	5	15	15	5
GRC1 Dynamic EWMA	87	5	7	19	7	7	17	17	8
GRC2 Dynamic EWMA	57	1	4	16	1	4	13	14	4
GMV Static EWMA	92	15	14	6	13	15	5	10	14
MS Static EWMA	155	21	21	11	21	21	21	18	21
RL Static EWMA	163	22	22	12	22	22	22	19	22
MRA Static EWMA	87	9	11	10	12	11	14	9	11
RA Static EWMA	73	10	12	7	11	12	3	5	13
ERC Static EWMA	72	3	6	18	8	6	12	12	7
GRC1 Static EWMA	101	8	10	21	10	10	16	16	10
GRC2 Static EWMA	68	4	9	20	6	8	6	6	9
GMV No Crypto EWMA	110	17	16	8	18	16	9	11	15
MS No Crypto EWMA	185	24	24	17	24	24	24	24	24
RL No Crypto EWMA	175	23	23	14	23	23	23	23	23
MRA No Crypto EWMA	136	20	20	13	20	20	11	13	19
RA No Crypto EWMA	112	16	17	9	19	18	8	8	17
ERC No Crypto EWMA	133	12	15	22	16	14	18	20	16
GRC1 No Crypto EWMA	149	14	18	23	17	17	20	22	18
GRC2 No Crypto EWMA	155	19	19	24	14	19	19	21	20

***Table 4 - Appendix**: Performance Indicators Ranking Risk-Contribution Strategies – EWMA inputs*

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