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TESI DI LAUREA MAGISTRALE

Identification of the electrical parameters of the capacitors' discharging circuit in the Morgan-Botti Lightning Laboratory (Cardiff)

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SUMMARY

Preface

This thesis work has been accomplished during my experience Erasums in the Laboratory "Morgan-Botti Lightning" in Cardiff. It is based on a reporter of the fundamental experiences preatiche it addressed, the measurements and the conclusions drawn from these. Basically reporte resumes the results obtained from the work of previous students (Kasim Sabir in particular) in the evaluation of the electrical parameters inherent in the circuit of the Bank "D" currently operating in the laboratory. These results are thus repeated in this reporter as a starting point and reference for subsequent measurement. This experience is based on three ways of calculating the same electrical parameters (inductance and resistance). The three pathways discussed here are distinguished by type and each of them leads to the evaluation of these parameters, starting from different initial data. The end result of this experience can be considered positive because 2 of these pathways lead to values close to those previously estimated $R = 0.34\ \Omega$ and $L = 4.4\ \mu H$. The last way can be considered only partially Satisfactory, as it brings good results, close to the previous ones, only for the value of resistance and is very different to previously estimated values of inductance. The final values obtained from 1th method are:

$R = 0{,}35\ \Omega$

$L = 4{,}4\ \mu H$

The final inductance value obtained by 2nd Method is:

$L = 4{,}8\ \mu H$

The final resistance value obtained by the measurement is:

$R = 0{,}38\ \Omega$

At the end of this reporter were also taken of personal considerations in view of the results obtained, based on what came into my knowledge while studying Electrical Engineering. They can be useful to have a rough idea of some electrical phenomena that occur during the operation of the Laboratory.

Chapter 1

1.1 INTRODUCTION

Until the destruction of a glider in 1999, to which the case was assigned to a lightning strike, the danger coming from a bolt of lightning had been underestimated (others are cases of planes crashed, it has been suggested as the cause of the discharge of a lightning). So it becomes essential to understand how dangerous lightning strikes can be. First of all it must be eliminated the idea that they are harmless because very rare. 'An average lightning strike can generate a current of 32 kilo-Amperes (kA) with temperatures being as high as up to 30,000 Degrees Celsius [31]'. Such lightning strikes have the potential to cause destruction. In the early and mid-1900s many deaths occurred as a result. Transportation systems such as automotive vehicles, ships and aircraft were more prone to being struck by lightning. Since countless people used aircraft as transport there was a major concern. If a lightning strike hit the wing of an aircraft (where the fuel tank usually is) it could possibly ignite the fuel tank and therefore cause the tank to explode. This in turn would result in a plane crash (death figures of over hundred people from one plane crash). The only safety measures put into place in the early 1900s were to predict the weather. New measures since the early 1900s have been developed for protection against lightning. Protection such as shielding and usage of metal materials for aircraft structures has been put into place to sustain lightning strikes. 'Nowadays, an aircraft is built to be able to withstand current magnitudes of 250 kA [1]'. Although metal materials have been a good source of protection against lightning, there has been a need to use newer materials today such as carbon composites. There are many reasons to why aircraft structures are changing from metals to composites – one of the reasons being that they are lightweight in comparison to metals. However, since carbon composite materials are not very good conductors they need to be further examined for use in aerospace applications.

1.2 REVOLUTION OF THE MAIN MATERIALS USED IN AIRLINE

As was said at the beginning of 900 aircraft were not designed to withstand lightning strikes (given the lack of knowledge of this type of electrical discharges and their effects). For this reason the materials used for aircraft construction have changed a number of times since the 1900s to present. One of the first aircraft, in 1903, was constructed primarily using spruce and ash wood with muslin covering the wings. Other materials such as steel and aluminum were also used. Wood (spruce) was used in the wings as it is light, flexible and easy to shape into parts. It is a material which was cheap and readily available. Metals such as steel were used to hold the parts together since steel is easy to shape, very hard and suited for making bolts. Muslin was used to cover the wings as it is extremely light and easy to shape, and aluminum was used to make the engine as it could be heated and moulded easily.

The disadvantages of the materials used were that wood was fragile. Steel was good to hold the parts together but it would rust easily and muslin was very easy to tear.

Subsequent to the first metal aircraft, in 1915, metal materials have played a big role in aircraft construction. The most used metal in an aircraft has been aluminum since it has good mechanical properties such as strength, stiffness, ductility and formability and is a very good conductor of electricity.

Composite materials made entry into aerospace applications in the 1960s. Composite materials (such as Graphite Reinforced Plastic/Carbon Reinforced Plastic) have been used for the construction in the wing and tail of aircraft manufactured by Boeing and Airbus. Some advantages of composites over metals are that they are lighter in weight, more corrosion resistant and are easier to shape into complex parts. The first ever large scale aircraft to deploy predominantly composite materials is the Boeing 787 Dreamliner (produced in 2006). Due to the performance of aircraft, using composite materials, it is believed that the era of aluminum, which has been the standard material used for aircraft construction, has come to an end. Composites will be the future material in the aircraft industry.

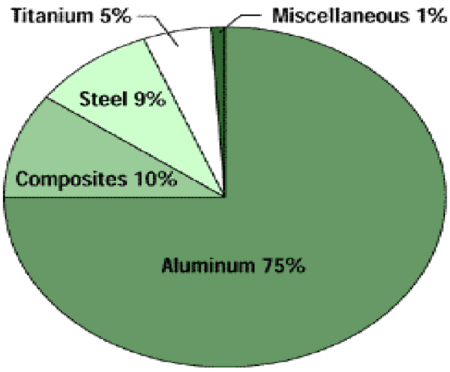


Figure 1.1 – Metals and composites used in earlier Boeing aircraft models

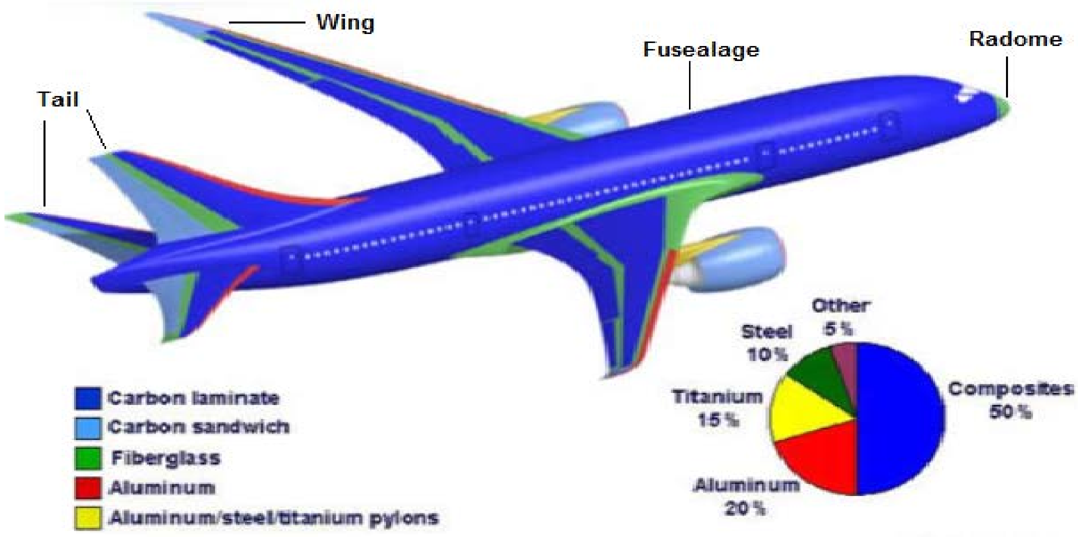


Figure 1.2 – Metals and composite materials used in the Boeing 787 Dreamliner

1.3 COMPOSITE MATERIALS

Currently you are passing more and more use of composite materials which are made, in basic terms, of two or more materials, with different properties, combined together to form a new material with greater properties. Composite materials consist of a reinforcement and matrix material [8]. An example is shown in Figure 1.3.

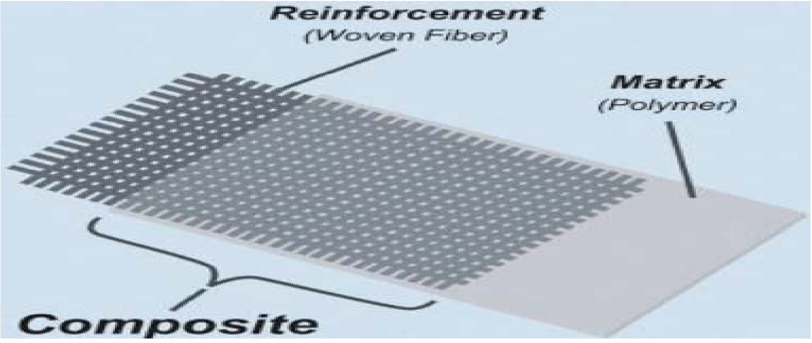


Figure 1.3 - Composite material

Figure 1.3 illustrates a composite material where a, woven, fiber fabric represents the reinforcement, whilst polymer is representing the matrix material. Examples of reinforcements are glass and carbon fibers which provide strength and stiffness. Matrix materials such as ceramic, metal, cement or polymer holds the fibers together, transfers loads between the fibers and protects them from damage.

1.4 COMPOSITE MATERIALS' MANUFACTURE

To create a handmade composite material the fiber reinforcement and matrix material are joined together by a layup procedure. In the layup procedure the fiber reinforcements are impregnated with resin by means of injecting, rolling, pressing and spraying, amongst various other methods. There are two kinds of well-known layup procedures known as a wet and dry layup. In a wet layup the fabricator of the composite material has to handle and spread the liquid matrix resin in an open mould. With a dry type of layup, the fabricator only makes contact with the fiber reinforcement and does not directly handle the liquid resin as such. The dry type of layup requires careful handling and certain equipment for the procedure.

1.5 COMPOSITE MATERIALS USED IN AEROSPACE

Is crucial in the aerospace the use of composite materials prepreg. These materials, a sheet pre-impregnated (also known as a ply), are prepared by impregnating fibers with a controlled volume of resin and until needed for use they can be stored in a partially cured state. Various forms of prepregs are available to purchase such as woven (bi-directional fibers) fabrics and unidirectional tapes.
Advantages of using prepregs include good strength, greater ease of forming complex shapes and a consistent fiber and resin combination. Prepregs are also easy to handle (unlike the messy handmade layup procedure) and are readily available on rolls. There are no excess resin problems with prepregs and hence there are fewer probabilities of encountering manufacturing defects such as voids (voids reduce strength and can also cause partial discharge in the material). Disadvantages of prepregs include expensive curing equipment (autoclave ovens used for curing), added cost in making prepregs and a limited shelf life (this can be extended, however, by storing them in freezers).

1.6 LIGHTNING PHENOMENA

The lightning is an electrical discharge large that occurs in the atmosphere and that is established between two bodies with a high electric potential difference. Therefore, for the generation of a lightning, is required a charge separation in atmospheric clouds that are called cumulonimbus, characterized by large accumulations of static electricity. The main causes of their formation are the atmospheric perturbations (wind, humidity, freezing and melting of rain droplets and of ice particles, friction, and atmospheric pressure). These perturbations involve collisions in the air's particles. Usually, by these collisions, it's generated a positive charge, built up on the upper region, and one negative, on the lower region of the cloud. It is also possible to have a positive pocket of charge at the lower base of the cloud. As collisions continue within the cloud the charge build up becomes greater. Whenever there is a charge separation there is an electric field, which is proportional to charge, and hence the electric field strength increases as the charge build up in the cloud increases. When the electric field of the cloud exceeds the electric field intensity of the air (30 kV/cm) the air will breakdown and become ionized. This results in a current path being created by the ionized air. It is important to note that an ionized current path is not created instantaneously. A lightning flash begins with a leader which advances in steps downwards to ground. Since air is ionized in different directions there are many leaders trying to make their way to ground. As the stepped leaders approach their way to ground ionization from ground creates an electric field in the opposite direction which sends streamers back up towards the cloud. Eventually a streamer meets with one of the stepped leaders and a full path is created for the lightning current to flow from cloud to ground.

Figure 1.4 shows the formation of lightning for a typical cloud to ground strike to make clear to the reader of the process just described. In this section only the cloud to ground strike has been made apparent. The next section will give a brief insight to other common types of lightning strikes.

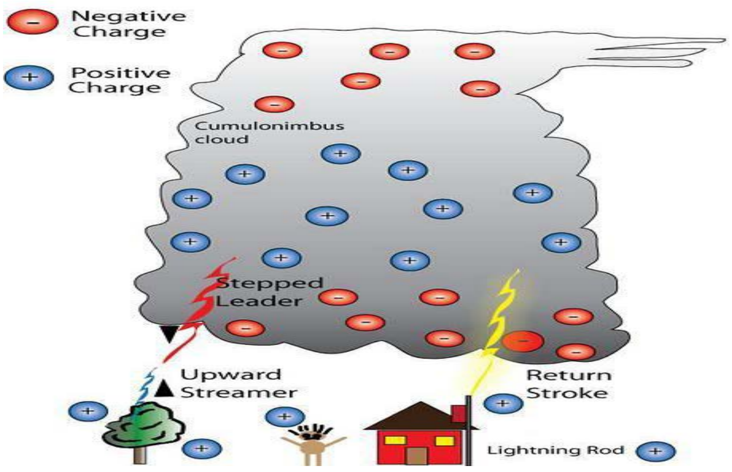


Figure 1.4 – Lightning formation

A cloud to ground strike can be of a positive or negative polarity depending on the charge polarity in the base of the cumulonimbus cloud. The strike can also be of ascending or descending type. Negative flashes normally occur more frequently than positive flashes (approximately 95% of flashes are negative). However, although positive flashes occur rarely (around 5% of the time) they are said to be more severe than negative flashes since they can travel longer distances, carry significant charge and last a longer duration. Positive flashes generally consist of only one stroke whilst a negative flash normally has between 1 and 11 of strokes. A maximum of 24 strokes have been recorded in a negative flash. Following some strokes in a negative flash there is sometimes a low level intermediate and continuing current.

1.7 TYPES OF LIGHTNING

Many are the types of lightings, which differ from one another because they are formed between the cloud itself (intra-cloud), between cloud to air and between cloud to cloud (inter-cloud). The most frequent occurrence of lightning is intra-cloud. This type of lightning occurs as a result of the charge separation (of opposite polarities) in the cumulonimbus cloud. Inter-cloud lightning occurs between two clouds due to attraction of unlike charges of each cloud. Figure 1.5 illustrates the types of lightning phenomenon, described above, which naturally occur.

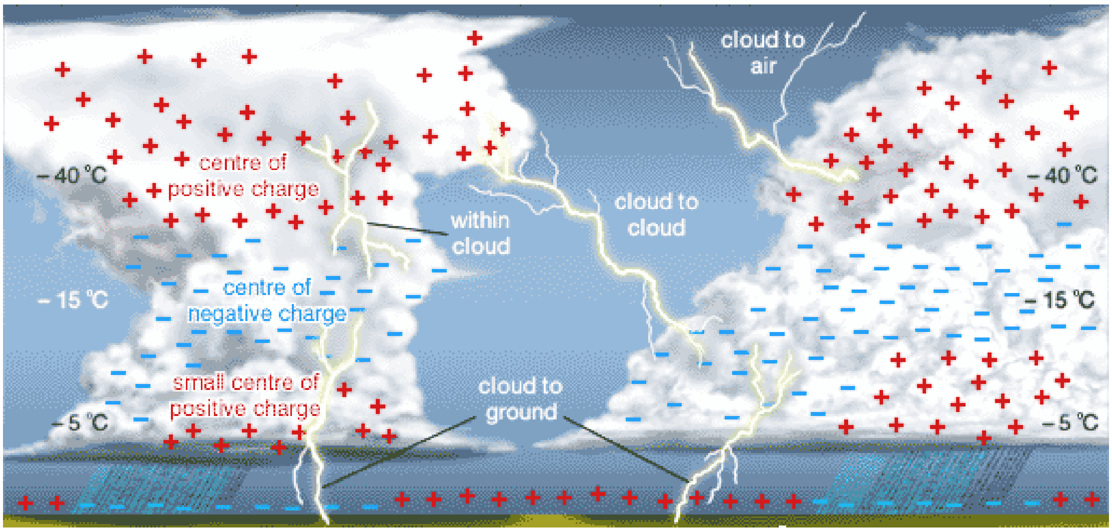


Figure 1.5 – Types of lightning strikes

Chapter 2

2.1 DESCRIPTION OF THE MORGAN-BOTTI LIGHTNING LABORATORY

Figure 2.1 – Picture of the Morgan-Botti Lightning Laboratory

The Morgan-Botti Lightning Laboratory is a building (figure 2.1) whose fundamental use is for producing a kind of lightning (different type of electric arc that goes from 100÷200 kA to 20kA of peak, as it will be showed next) for tasting the aircrafts material. This happens in a test room where is putting the test carbon material (used in the aircrafts) under the output of an electric circuit that generates an electrical discharge. This electrical discharge has an impulsive waveform near to the electric waveform of a lightning. For obtain a waveform more near as possible to the real lightning's waveform is need an high level of current in a very short time (the current in the arc could arrive at a level of 100-200 kA, the level reached by the current waveform's peak coming out from "A/D-bank's circuit"). This high level of current arrive, at the end of the circuit, after eight/six cables, to an electric arc (a kind of spark gap inside the test cabin). It produced at the end of the cables (at the upper end of the spark gap) a corresponding voltage level such as to overcome the dielectric of the air, bringing it into conduction. In this way the electric discharge is generated, very similar to that of the lightning. On the other side of the spark gap is placed a sheet of material to be tested, which will therefore be covered by this discharge current. This level of current, necessary to generate the discharge, is produced through the transient of an RLC (the only way to obtained from an electric circuit an high peak of current in a small time). This RLC transitory circuit is in particular the circuit of discharging of a capacitor's bank that just before has been charged by DC-generator. The capacitor's bank discharging transitory is obtained in closing a system of switch in a dissipative circuit, just after a charging period in which the capacitor's bank is kept open by the rest of the circuit through the switches and is maintained in charge by a DC generator. In the discharging transitory the capacitor's bank is closed in an electrical circuit, basically a dissipative circuit, in which can be considered an overall resistance and inductance (made by many different kind of electrical components) that, together with the capacitor's bank, produces the performance of a typical current transient RLC. So by closing of the previous charged capacitor's bank in a circuit made of inductance and resistance you arrive to a transitory periodic exponential dumped course of current. Down here are shows the fundamental parameters of the characteristic Lightning waveform:

Table 2.1 - Lightning's parameters

	Component A	Component B	Component C	Component D
Io (A)	218810	11300	400	109405
α (s^{-1})	11354	700	N/A	22708
β (s^{-1})	647265	2000	N/A	1294530
Peak Current (kA)	200 ($\pm$10%) *	4.173 ($\pm$10%) *	400	100 ($\pm$10%) *
Time to Peak (μs)	6.4	808	750	3.18
Rise Time (μs)	$\leq$ 50 *	-	-	$\leq$ 25 *
Charge Transfer (C)	N/A	10 C	200 C	N/A
di/dt $_{max}$ (A/s)	1.4×10^{11}A/s	N/A	N/A	1.4×10^{11}
Action Integral (A^2s)	2×10^6	N/A	N/A	0.25×10^6
Note: * - Indicates the parameter tolerance of a particular current waveform to conform to the ED84 standard. For more information regarding the waveform parameters shown see the ED84 standard [11].				

Lightning current waveform:

$I(t) = I_0\left(e^{-\propto t} - e^{-\beta t}\right)$

This finally shape come out from the RLC transitory theory, that can be shortly describe here.

Chapter 3

3.1 RLC TRANSITORY THEORY

First of all we have to introduce some parameters and constant, characteristic of an RLC transitory:

- **Resonance**

An important property of the RLC circuit is its ability to resonate at a specific frequency, the resonance frequency, f_0 (hertz). Dually we could call an angular frequency, ω_0 (radians), is used which is more mathematically convenient. Related by the equation:

$$\omega_0 = 2\pi f_0$$

Resonance occurs because energy is stored in two different ways: in an electric field as the capacitor is charged and in a magnetic field as current flows through the inductor. Energy can be transferred from one to the other within the circuit and this can be oscillatory.

{A mechanical analogy is a weight suspended on a spring which will oscillate up and down when released. A weight on a spring is described by exactly the same second order differential equation as an RLC circuit and for all the properties of the one system there will be found an analogous property of the other. The mechanical property answering to the resistor in the circuit is friction in the spring/weight system. Friction will slowly bring any oscillation to a halt if there is no external force driving it. Likewise, the resistance in an RLC circuit will "damp" the oscillation, diminishing it with time if there is no driving AC power source in the circuit.}

The resonance frequency is defined as the frequency at which the impedance of the circuit is at a minimum. Equivalently, it can be defined as the frequency at which the impedance is purely real (that is, purely resistive). This occurs because the impedance of the inductor and capacitor at resonance are equal but of opposite sign and cancel out. Circuits where L and C are in parallel rather than series actually have a maximum impedance rather than a minimum impedance. For this reason they are often described as anti resonators, it is still usual, however, to name the frequency at which this occurs as the resonance frequency.

- **Natural frequency**

The resonance frequency is defined in terms of the impedance presented to a driving source. It is still possible for the circuit to carry on oscillating (for a time) after the driving source has been removed or it is subjected to a step in voltage (including a step down to zero). This is similar to the way that a tuning fork will carry on ringing after it has been struck, and the effect is often called ringing. This effect is the peak natural resonance frequency of the circuit and in general is not exactly the same as the driven resonance frequency, although the two will usually be quite close to each other. Various terms are used by different authors to distinguish the two, but resonance frequency unqualified usually means the driven resonance frequency. The driven frequency may be called the un-damped resonance frequency or un-damped natural frequency and the peak frequency may be called the damped resonance frequency or the damped natural frequency. The reason for this terminology is that the driven resonance frequency in a series or parallel resonant circuit has the value:

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

This is exactly the same as the resonance frequency of an LC circuit, that is, one with no resistor present, that is, it is the same as a circuit in which there is no damping, hence un-damped resonance frequency. The peak resonance frequency, on the other hand, depends on the value of the resistor and is described as the damped resonance frequency. A highly damped circuit will fail to resonate at all when not driven. A circuit with a value of resistor that causes it to be just on the edge of ringing is called critically damped. Either side of critically damped are described as underdamped (ringing happens) and over-damped (ringing is suppressed).

Circuits with topologies more complex than straightforward series or parallel (some examples described later in the article) have a driven resonance frequency that deviates from and for those the un-damped resonance frequency, damped resonance frequency and driven resonance frequency can all be different.

- **Damping**

Damping is caused by the resistance in the circuit. It determines whether or not the circuit will resonate naturally (that is, without a driving source). Circuits which will resonate in this way are described as underdamped and those that will not are over-damped. Damping attenuation (symbol α) is measured in nepers per second. However, the unitless damping factor (symbol ζ, zeta) is often a more useful measure, which is related to α by

$$\zeta = \frac{\alpha}{\omega_0}$$

The special case of $\zeta = 1$ is called critical damping and represents the case of a circuit that is just on the border of oscillation. It is the minimum damping that can be applied without causing oscillation.

- **Bandwidth**

The resonance effect can be used for filtering, the rapid change in impedance near resonance can be used to pass or block signals close to the resonance frequency. Both band-pass and band-stop filters can be constructed as a filter circuits. A key parameter in filter design is bandwidth. The bandwidth is measured between the 3dB-points, that is, the frequencies at which the power passed through the circuit has fallen to half the value passed at resonance. There are two of these half-power frequencies, one above, and one below the resonance frequency

$$\Delta\omega = \omega_2 - \omega_1$$

where $\Delta\omega$ is the bandwidth, ω_1 is the lower half-power frequency and ω_2 is the upper half-power frequency. The bandwidth is related to attenuation by

$$\Delta\omega = 2\alpha$$

when the units are radians per second and nepers per second respectively. Other units may require a conversion factor. A more general measure of bandwidth is the fractional bandwidth, which expresses the bandwidth as a fraction of the resonance frequency and is given by

$$F_b = \frac{\Delta\omega}{\omega_0}$$

The fractional bandwidth is also often stated as a percentage. The damping of filter circuits is adjusted to result in the required bandwidth. A narrow band filter, such as a notch filter, requires low damping. A wide band filter requires high damping.

- **Q factor**

The Q factor is a widespread measure used to characterise resonators. It is defined as the peak energy stored in the circuit divided by the average energy dissipated in it per cycle at resonance. Low Q circuits are therefore damped and high Q circuits are under-damped. Q is related to bandwidth; low Q circuits are wide band and high Q circuits are narrow band. In fact, it happens that Q is the inverse of fractional bandwidth

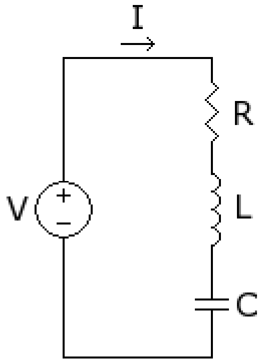
$$Q = \frac{1}{F_b} = \frac{\omega_0}{\Delta\omega}$$

Q factor is directly proportional to selectivity, as Q factor depends inversely on bandwidth.

- **Scaled parameters**

The parameters ζ, F_b, and Q are all scaled to ω_0. This means that circuits which have similar parameters share similar characteristics regardless of whether or not they are operating in the same frequency band. Next are given the analysis for the series RLC circuit in detail. Other configurations are not described in such detail, but the key differences from the series case are given. The general form of the differential equations given in the series circuit section are applicable to all second order circuits and can be used to describe the voltage or current in any element of each circuit.

3.2 SERIES RLC CIRCUIT



V – the voltage of the power source

I – the current in the circuit

R – the resistance of the resistor

L – the inductance of the inductor

C – the capacitance of the capacitor

Figure 3.1 - RLC series circuit

In this circuit, the three components are all in series with the voltage source. The governing differential equation can be found by substituting into Kirchhoff's voltage law (KVL) the constitutive equation for each of the three elements. From KVL,

$$v_R + v_L + v_C = v(t)$$

Where v_R, v_L, v_C are the voltages across *R*, *L* and *C* respectively and *v(t)* is the time varying voltage from the source. Substituting in the constitutive equations,

$$Ri(t) + L\frac{di}{dt} + \frac{1}{C}\int_{-\infty}^{\tau=t} i(\tau)d\tau = v(t)$$

For the case where the source is an unchanging voltage, differentiating and dividing by L leads to the second order differential equation:

$$\frac{d^2i(t)}{dt^2} + \frac{R}{L}\frac{di(t)}{dt} + \frac{1}{LC}i(t) = 0$$

This can usefully be expressed in a more generally applicable form:

$$\frac{d^2i(t)}{dt^2} + 2\alpha\frac{di(t)}{dt} + \omega_0^2 i(t) = 0$$

α and ω_0 are both in units of angular frequency. α is called the *neper frequency*, or *attenuation*, and is a measure of how fast the transient response of the circuit will die away after the stimulus has been removed. Neper occurs in the name because the units can also be considered to be nepers per second, neper being a unit of attenuation. ω_0 is the angular resonance frequency.
For the case of the series RLC circuit these two parameters are given by:

$$\alpha = \frac{R}{2L}$$

and

$$\omega_0 \frac{1}{\sqrt{LC}}$$

A useful parameter is the *damping factor*, ζ which is defined as the ratio of these two,

$$\zeta = \frac{\alpha}{\omega_0}$$

In the case of the series RLC circuit, the damping factor is given by,

$$\zeta = \frac{R}{2}\sqrt{\frac{C}{L}}$$

The value of the damping factor determines the type of transient that the circuit will exhibit. Some authors do not use ζ and call α the damping factor.

3.3 TRANSIENT RESPONSE

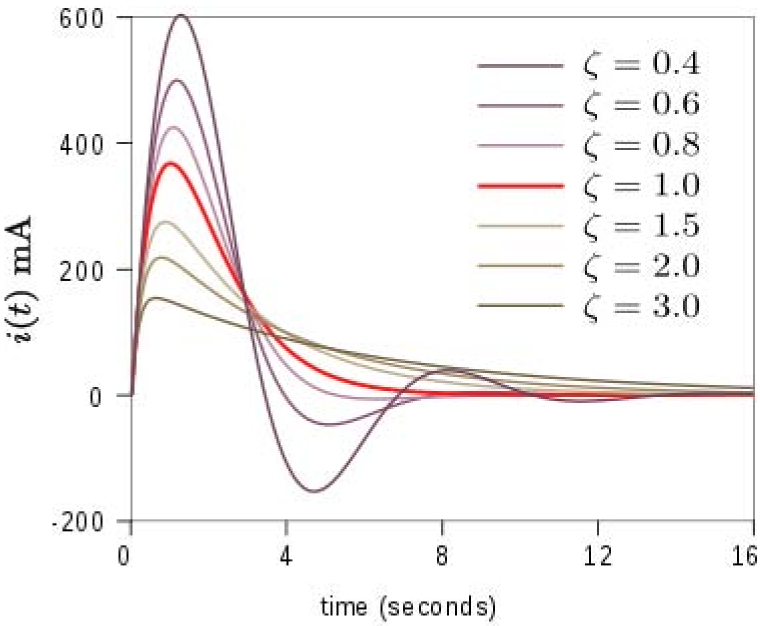


Figure 3-2 - Shape of current *i(t)* for different ζ values

Plot showing under-damped and over-damped responses of a series RLC circuit. The critical damping plot is the bold red curve. The plots are normalized for $L = 1$, $C = 1$ and $\omega_0=1$

The differential equation for the circuit solves in three different ways depending on the value of ζ. These are underdamped ($\zeta < 1$), over-damped ($\zeta > 1$) and critically damped ($\zeta = 1$). The differential equation has the characteristic equation,

$$s^2 + 2\alpha s + \omega_0^2 = 0$$

The roots of the equation in *s* are,

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

The general solution of the differential equation is an exponential in either root or a linear superposition of both,

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

The coefficients A_1 and A_2 are determined by the boundary conditions of the specific problem being analyzed. That is, they are set by the values of the currents and voltages in the circuit at the onset of the transient and the presumed value they will settle to after infinite time.

3.4 OVER-DAMPED RESPONSE

The over-damped response ($\zeta > 1$) is,

$$i(t) = A_1 e^{-\omega_0\left(\zeta+\sqrt{\zeta^2-1}\right)t} + A_2 e^{-\omega_0\left(\zeta-\sqrt{\zeta^2-1}\right)t}$$

The over-damped response is a decay of the transient current without oscillation.

3.5 UNDER-DAMPED RESPONSE

The under-damped response ($\zeta < 1$) is,

$$i(t) = B_1 e^{-\alpha t}\cos(\omega_d t) + B_2 e^{-\alpha t}\sin(\omega_d t)$$

By applying standard trigonometric identities the two trigonometric functions may be expressed as a single sinusoid with phase shift,

$$i(t) = B_3 e^{-\alpha t}\sin(\omega_d t + \varphi)$$

The underdamped response is a decaying oscillation at frequency ω_d. The oscillation decays at a rate determined by the attenuation α. The exponential in α describes the envelope of the oscillation. B_1 and B_2 (or B_3 and the phase shift φ in the second form) are arbitrary constants determined by boundary conditions. The frequency ω_d is given by,

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \omega_0\sqrt{1-\zeta^2}$$

This is called the damped resonance frequency or the damped natural frequency. It is the frequency the circuit will naturally oscillate at if not driven by an external source. The resonance frequency, ω_0 which is the frequency at which the circuit will resonate when driven by an external oscillation, may often be referred to as the un-damped resonance frequency to distinguish it.

3.6 CRITICALLY DAMPED RESPONSE

The critically damped response ($\zeta = 1$) is,

$$i(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$$

The critically damped response represents the circuit response that decays in the fastest possible time without going into oscillation. This consideration is important in control systems where it is required to reach the desired state as quickly as possible without overshooting. D_1 and D_2 are arbitrary constants determined by boundary conditions.

3.7 LAPLACE DOMAIN

The series RLC can be analyzed for both transient and steady AC state behavior using the Laplace transform. If the voltage source above produces a waveform with Laplace-transformed $V(s)$ (where s is the complex frequency ($s=\sigma+i\omega$), KVL can be applied in the Laplace domain:

$$V(s) = I(s)\left(R + Ls + \frac{1}{Cs}\right)$$

where $I(s)$ is the Laplace-transformed current through all components. Solving for $I(s)$:

$$I(s) = \frac{1}{R + Ls + \frac{1}{Cs}} V(s)$$

And rearranging, we have that

$$I(s) = \frac{s}{L\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)} V(s)$$

3.8 LAPLACE ADMITTANCE

Solving for the Laplace admittance $Y(s)$:

$$Y(s) = \frac{I(s)}{V(S)} = \frac{s}{L\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)}$$

Simplifying using parameters α and ω_o defined in the previous section, we have

$$Y(s) = \frac{I(s)}{V(s)} = \frac{s}{L(s^2 + 2\alpha s + \omega_0^2)}$$

3.9 POLES AND ZEROS

The zeros of $Y(s)$ are those values of s such that $Y(s) = 0$:

$$s = 0 \; and \; |s| = 0$$

The poles of $Y(s)$ are those values of s such that $Y(s) \rightarrow \infty$. By the quadratic formula, we find

$$s = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

The poles of $Y(s)$ are identical to the roots s_1 and s_2 of the characteristic polynomial of the differential equation in the section above.

3.10 GENERAL SOLUTION

For an arbitrary $E(t)$, the solution obtained by inverse transform of $I(s)$ is:

$I(t) = \frac{1}{L}\int_0^t E(t-\tau)e^{-\alpha\tau}\left(\cos\omega_d\tau - \frac{\alpha}{\omega_d}\sin\omega_d\tau\right)d\tau$ in the underdamped case $(\omega_0 > \alpha)$

$I(t) = \frac{1}{L}\int_0^t E(t-\tau)e^{-\alpha\tau}(1-\alpha\tau)d\tau$ in the critically damped case $(\omega_0 = \alpha)$

$I(t) = \frac{1}{L}\int_0^t E(t-\tau)e^{-\alpha\tau}\left(cosh\,\omega_r\tau - \frac{\alpha}{\omega_r}sinh\,\omega_r\tau\right)d\tau$ in the overdamped case $(\omega_0 < \alpha)$

where $\omega_r = \sqrt{\alpha^2 - \omega^2}$, and *cosh* and *sinh* are the usual hyperbolic functions.

3.11 PULSE DISCHARGE CIRCUIT

An overdamped series RLC circuit can be used as a pulse discharge circuit. Often it is useful to know the values of components that could be used to produce a waveform that is described by the expression:

$$I(t) = I_0\left(e^{-\alpha t} - e^{-\beta t}\right)$$

Such a circuit could consist of an energy storage capacitor, a load in the form of a resistance, some circuit inductance and a switch – all in series. The initial conditions are that the capacitor is at voltage V_0 and there is no current flowing in the inductor. If the inductance L is known, then the remaining parameters are given by the following – capacitance:

$$C = \frac{1}{L\alpha\beta}$$

Resistance (total of circuit and load):

$$R = L(\alpha + \beta)$$

Initial terminal voltage of capacitor:

$$V_0 = -I_0 L\alpha\beta\left(\frac{1}{\beta} - \frac{1}{\alpha}\right)$$

Rearranging for the case where R is known – Capacitance:

$$C = \frac{\alpha + \beta}{R\alpha\beta}$$

Inductance (total of circuit and load):

$$L = \frac{R}{\alpha + \beta}$$

Initial terminal voltage of capacitor:

$$V_0 = \frac{-I_0 R\alpha\beta}{\alpha + \beta}\left(\frac{1}{\beta} - \frac{1}{\alpha}\right)$$

Chapter 4

4.1 ELECTRIC COMPOSITION AND DESCRIPTION OF THE CIRCUITS

From now, for simplification, the circuits in which are placed the capacitor's banks will be called as "bank circuit" with the indication A, D, B/C for the corresponding A, D, B/C currents' waveform that come out from those circuit. Will be showed the physical real composition of each circuit (A, D, B/C bank circuit), the correspondent description of their main component and their disposition. The circuits are fundamentally:

- **A-bank's circuit**
- **D-bank's circuit**
- **B/C-bank's circuit**

The A-bank circuit is not working for the high level of capacity that is required in the capacitor (and the laboratory don't have those kind of capacitor yet). The circuit is incomplete and is fundamentally the same of the D-bank. Therefore, the description will start from the D-bank circuit.

4.2 D-BANK CIRCUIT

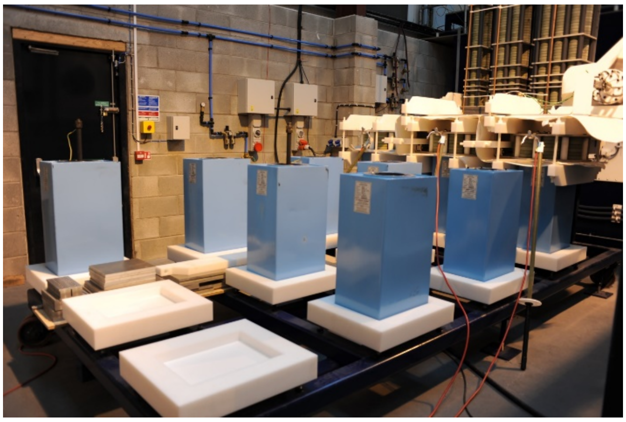


Figure 4.1 - Picture of the D-bank circuit

As it has been said, this circuit is the same one used for the A-Bank. The circuit gives a peak of current of 100 KA and is the biggest peak that for the moment can be obtained by the laboratory. Most of the considerations here taken and of the estimations are refered to this circuit.
The fundamental scheme is showed down:

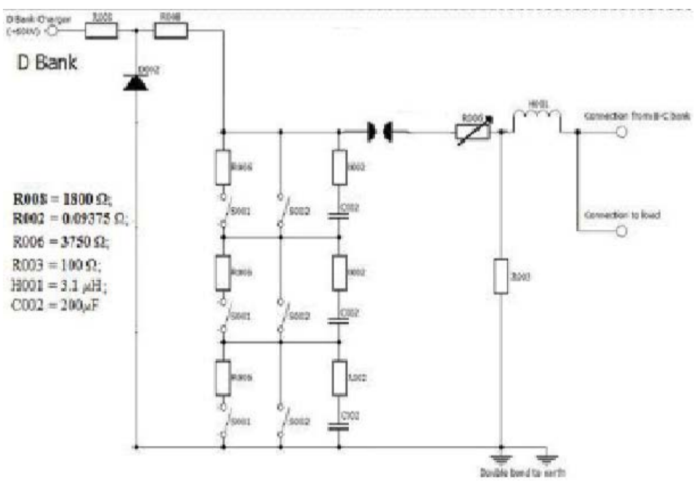


Figure 4.2 - D-bank circuit scheme

This scheme is described as following:

Starting from a charger of 54 KV, follow by two resistance of 1800Ω each one and a diode for the come-back current. After there are the series of three D-Capacitor (each one with capacity of 197µF and voltage supply of 18 KV). Up to each of this capacitor there is a resistance of 0,12875Ω and two switch that close the current in this capacitor to the ground, the first to be close is with a resistance of 1800**Ω** used for dissipated (joule effect) the energy stored in the capacitor and so to prevent the broken of the switch for too much high current in the direct short-circuit. Therefore, after this little time necessary for the dissipation of energy in the first switch, the second one is closed (is this the way in which each capacitor is grounded). There are also three manual switches (one for each capacitor) that are wires applied up each capacitor, after that the other two switches are closed, for push the current of the capacitor directly to the ground and give to the personal a better safety. After the series of capacitors there is a different kind of switch (which open directly all those capacitors from the other part of the circuit and from the test cabin by an dielectric separation). After there are three variables resistances in parallel. These are made by a column of block-resistance and, by closing the resistances' parallel up or down along this column, are included more or less blocks so to obtained a bigger or shorter value of overall resistance. After there is a very big resistance of 100 Ω that puts to the ground (for his big value is considered as an open in the next considerations) and eight wires that go inside the test cabin and eight wires of come-back. These wires could be considered the total value of the fundamental inductance of the D-circuit, and so they will be essentials for the next considerations and estimations.

4.3 D-BANK AND B/C-BANK CONNECTION

In the future (when also A and C bank will work) A and B/C bank will work together in only one circuit. Infact the two circuit A and B/C are concerned to work together for obtained the final complete waveform typical of a lightning.

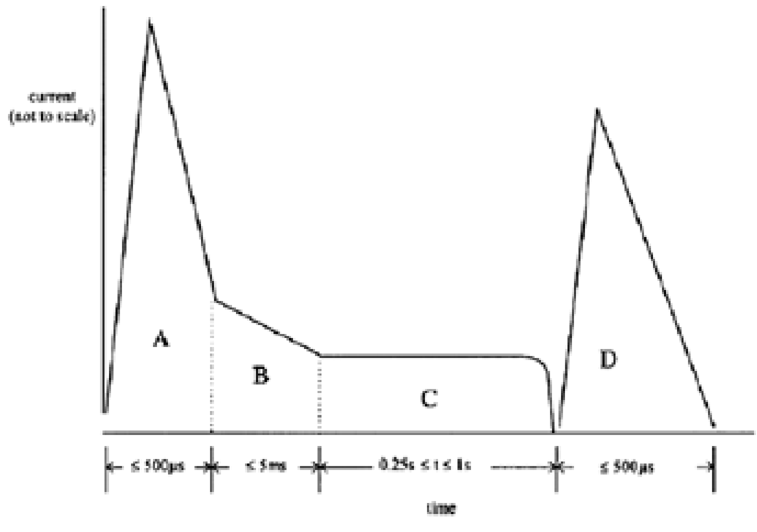


Figure 4.3 - Complete waveform of the 4 bank of capacitor

So for the moment are considered separately the two circuit. A switch (figure 4.4) is put to keep together the two bank's circuit or maintain them open from each other:

Figure 4.4 - Switch between the D-bank and the B/C-bank

4.4 B/C-BANK CIRCUIT

In figure 4.5 is written the fundamental scheme. It's divided in more region for simplify its description as is showed down:

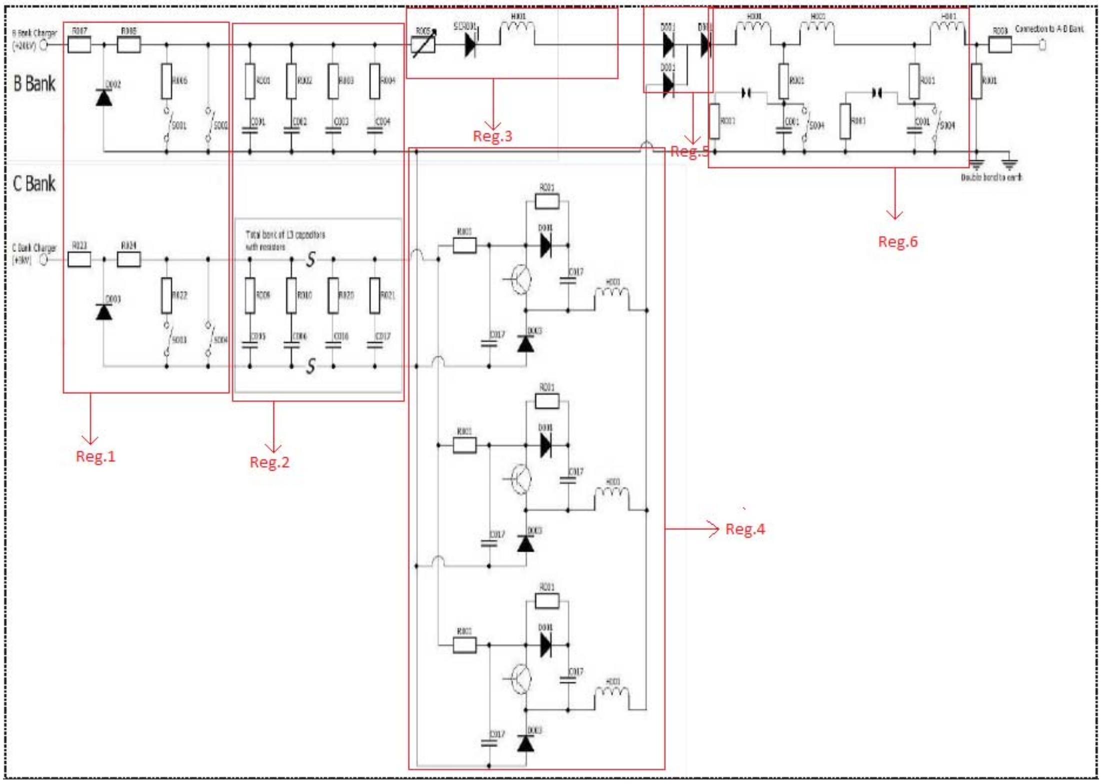


Figure 4.5 - Complete electric scheme of C/D-bank

Here are analyzed each parts of the circuit by a division in six region:

First of all there are two type of DC Charger, one of 20kV for the B-bank and another of 3kV for the C-bank

Region 1

There are two resistance of 1800Ω, between them a diode, used for protect the charger of the C and B bank from the feedback of high voltage in the circuit that is coming back from the test cabin, during charging and discharging. Before arrive to the C/B banks capacitor, two switch in parallel, one with up a resistance of 1000Ω and one other without any resistance, are put for grounded the DC Charger in the discharging.

This resistance should not be considered in the simplified circuit that connects the test cabin, after charging the capacitor, to create lightning.

These above the switch are called "dump resistance" and are used to protect the other switch by the passage of current fed from the condenser (he same type of dump circuit used in the D-bank).

The switches under these resistances are shut during the discharging of the circuit and are open during the charging of the circuit.

The same is for the C and B bank (“dump resistance” and grounding switches) the only difference is that in the C-bank there is also in parallel to the switch a trigger used to protect the other switch of the C-bank. This trigger/switch is not necessary in the A-B-D bank because of the current’s waveform coming out from the A-B-C discharging circuit that has an exponential decrease like (figure 4.6):

$$i(t) = A_1 e^{-at} + A_2 e^{-bt}$$

and is so already arrived at a low value of current when the switches are been shut

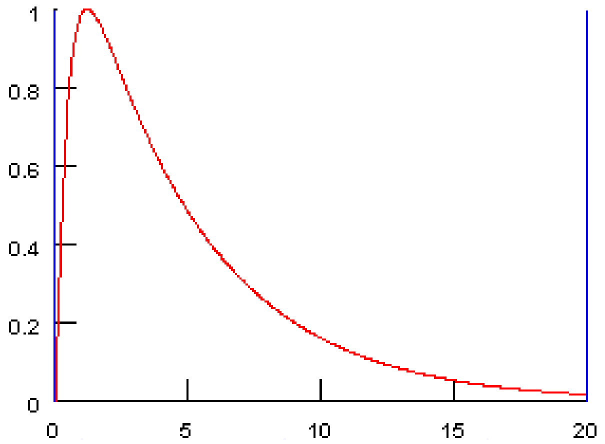


Figure 4.6 - Shape of switching current in the A-B-D Bank

As in D-bank’s circuit, there is another manual switch used just for personal safe.

The commands of the switch are controlled in this way: the switch with the resistance above is shut for first (in it the resistance protect the same switch from the abrupt change in the shape of the current during the switching brought by the C/B-bank’s capacitor in the discharging), the other switch is shut just after, when all the current in C/B is gone. After those for personal safe we can put also a manual switch.

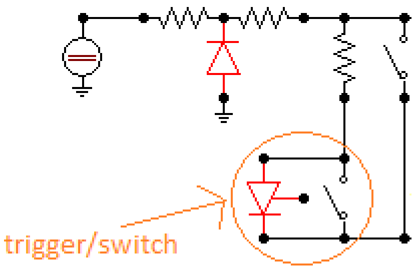


Figure 4.7 - Trigger switch in the C-Bank

Figure 4.8 - Switch for charging/discharging of the C/D-bank

Region 2
For the B-bank we have three capacitor of 250µF that could support a voltage of 20kV (the charger voltage) with above each one a resistance of 3Ω used just for protection.
For the C-bank we have a total of thirteen capacitors' bank (16,700µF each one) that could support a voltage of 3kV (the charging voltage) with above each one a resistance of 3Ω used just for protection.

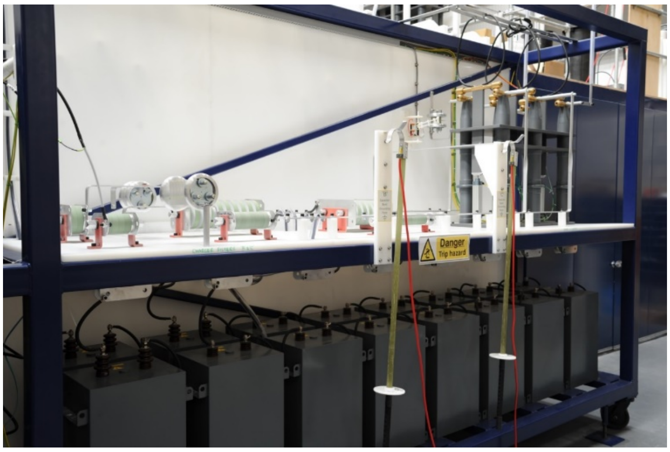


Figure 4.9 - Picture of the B/C-bank

Region 3
There is a variable resistance used just for have the right resistance needed for the correct current flow. After there is a trigger shut during discharging open during charging and after there is an inductance of 260µH (the same of the other three that come after in series) used just for current flow.

Region 4
Here there are three IGBT used for the C-bank circuit that control the current. These IGBT are open during discharging and shut during the charging of the circuit. At the end from these IGBT is obtained a controlled current waveform like this:

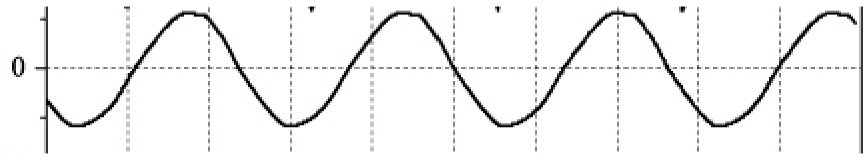


Figure 4.10 - Current waveform that come out from the by the IGBT.

As could be seen the result isn't an exponential waveform, like for the A-B-D bank (figure 4.6), but a ramp near to the square waveform that we want (the ideal waveform for C-bank is a square of 600A) like in figure 4.11:

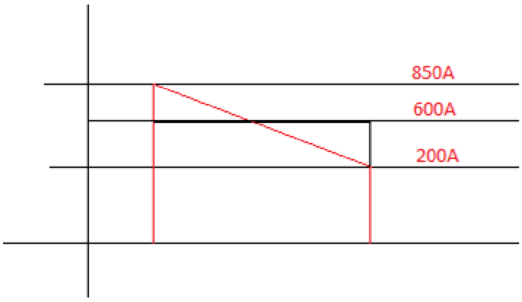


Figure 4.11 - Shape of switching current in C-bank

Each of those IGBT has a capacitor behind (of 15µF and voltage 20kV) used just for current flow and after a SNUBBER that give a protection to the IGBT.
The IGBT are close all together and open all together (actually there is a little distance of time in the closing of each IGBT but is neglectable). After the SNUBBER there are three inductance used just for current flow. In the future just after the SNUBBER it will be put a "TA" (current transformer) for control perfectly the IGBT in a separate time.

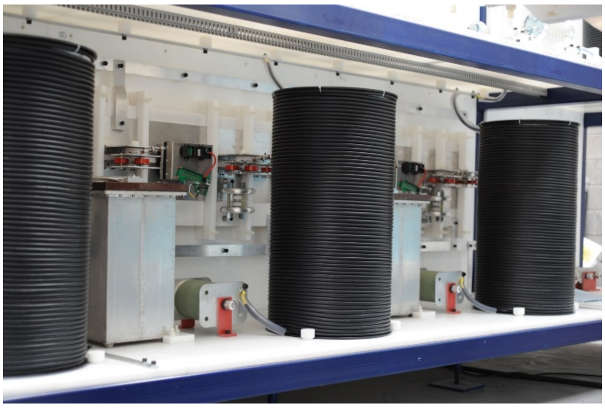


Figure 4.12 - Picture of the IGBT system

Region 5
There is this disposition of three diode used just for protect the charger from the feedbeck of high voltage (coming from the test cabin) just during the discharging and from the current or from the high voltage that are coming back from the "D-bank". (As we will see at the end of this script, the feedbeck's current has the same level of peak of the current in the electric arc in the test cabin). This current should be stopped by these particular diode. For be supported the voltage of break-down (V_{bd}) by each diode they are put in series so that the total voltage at their leaders is divided through each diode). A picture of these diode is in figure 4.13:

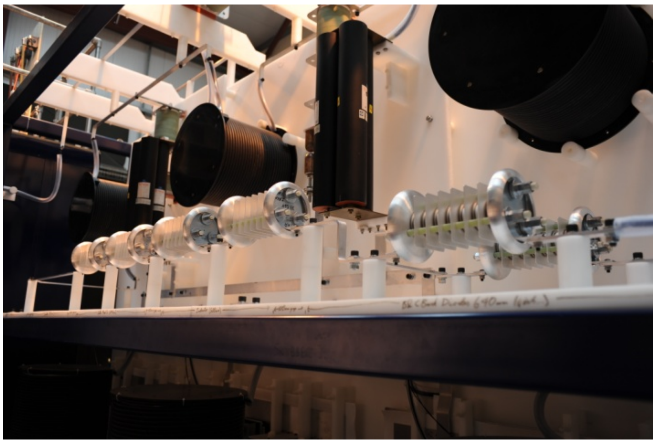


Figure 4.13 - Pictures of the series of diode and of the singular diode.

Region 6

These are two transit filter used just for make a filter for the transitory wave. Before, in between and after those two transit filter there is an inductance of 260µH used for current flow, so totally there are four inductances of 260µH in series along the same wire.

Figure 4.14 - Transit filter in the B/C-bank

At the end of this circuit there is a resistance of 2 Ω in the reality there is a parallel of two resistance of 4 Ω.

Inside the test cabin is all black and dark for have a good picture of the light and for don't have too much reflection (so is very dark). It's used a little wire for help to have the dump of the current (after the "light" this wire will burn and will disappear). The figure 4.15 show better this little wire:

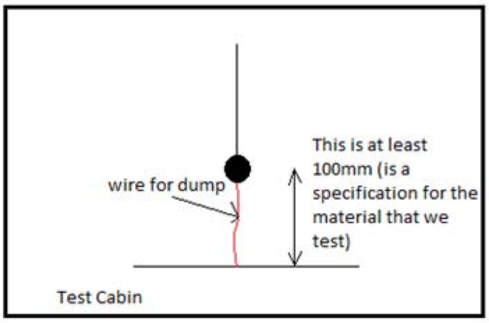


Figure 4.15 - Picture of the test cabin with the wire for the dump

So for more than 100 mm distance we need a wire for have a direct short circuit of the current (60000 Voltage of the D-bank is not enough to do the jump of 100mm for this is needed an help, this is the reason to used the wire).

The wires that bring the current from the outside to the inside of the test cabin are eight wires outside that become six inside with a particular geometrical connection with a platform as show in figure 4.16.

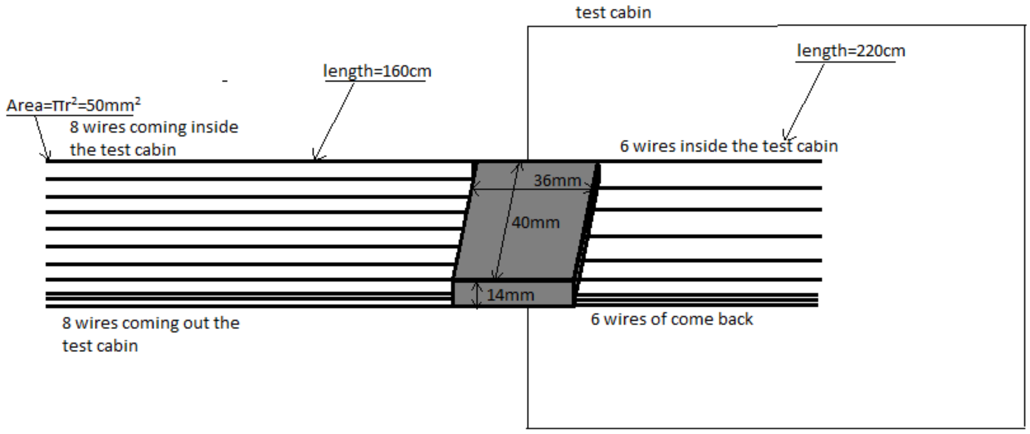


Figure 4.16 - Wires of connection to the test cabin from the bank-circuit

For the future the geometrical connection will include four more conductor inside (two up and two down) so that is more reduced the overall inductance of this last part of the circuit (the part that influence the inductance more than all the other D-bank circuit)
All the other component that appear in the circuit (resistances, capacitors, inductances) are used with their specify levels just for current flow's balance.
This division in eight wire at the end of the D-bank instead of only a big one wire is made for reduce the inductance by maintain them separate.

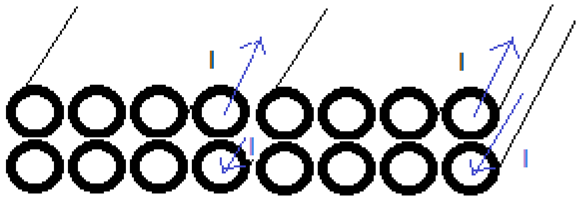


Figure 4.17 - Representation of the eight wires

Less inductance with those eight wires than the inductance of only one big wire (that has the same area obtained by putting together those eight wires).
Figure 4.18 is a picture of the test cabin and of photogram in which we capture the lightning of one experiment. This picture, because the ark appears at around 2μs, is made by a photo machine with a kind of stroboscope inside (a flash that with a variable flash frequency could light up the exact instant of the lightning):

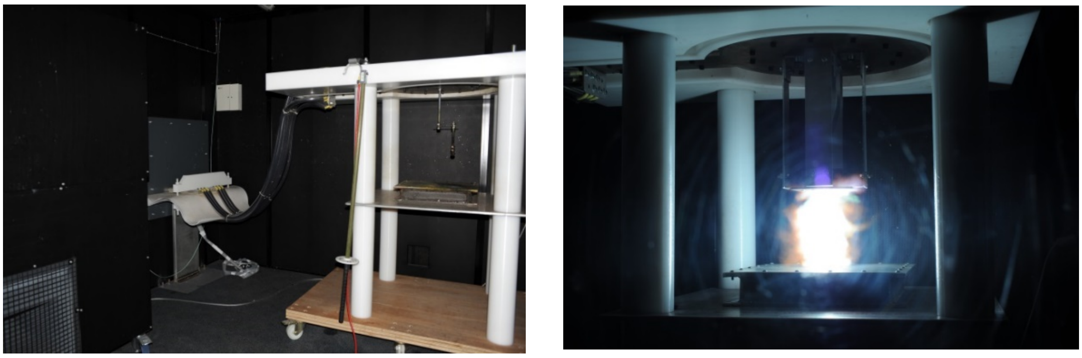
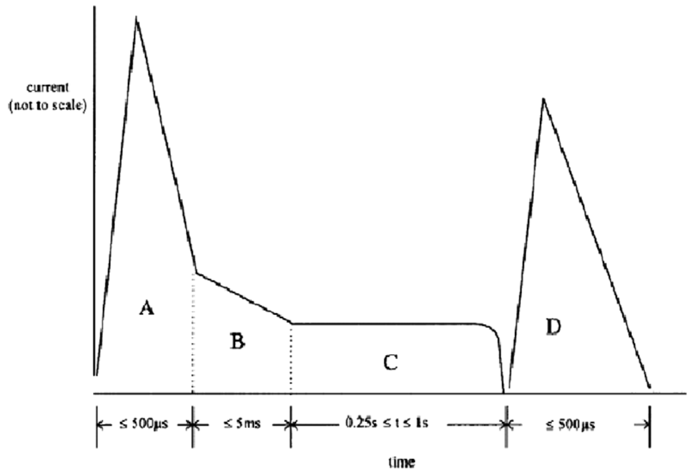


Figure 4.18 - Finally electric arc used for testing material inside test cabin. This picture is made with a particular photo machine that has inside, a kind of stroboscope, necessary for capture the electric arc's moment

Chapter 5

5.1 CIRCUIT WAVEFORM AND SIMULATION

The ideal waveform that is obtained as output of the circuit is something like this:



COMPONENT A (First Return Stroke)

Peak Amplitude	:	200kA (± 10%)
Action Integral	:	$2 \times 10^6 A^2s$ (± 20%) (in 500µs)
Time Duration	:	≤ 500µs

COMPONENT B (Intermediate Current)

Max. Charge Transfer	:	10 Coulombs (± 10%)
Average Amplitude	:	2kA (± 20%)
Time Duration	:	≤ 5ms

COMPONENT C (Continuing Current)

Amplitude	:	200 - 800A
Charge Transfer	:	200 Coulombs (± 20%)
Time Duration	:	0.25 to 1 s

COMPONENT D (Subsequent Return Stroke)

Peak Amplitude	:	100kA (± 10%)
Action Integral	:	$0.25 \times 10^6 A^2s$ (± 20%) (in 500µs)
Time Duration	:	≤ 500µs

Figure 5.1 - Complete waveform of A,B,C,D-bank and their main parameters

With the “A”, “B”, “C” and “D” waveform's nomenclature are indicated the different kind of current obtained by the different circuits that there are in the lab. These circuits differ specially for the capacitors' banks (A = 9·197 µF, B = 3·250 µF, C = 13·16,7 µF, D = 3·197 µF). Actually the "A" and the “C” banks doesn’t work: for the “A” bank there isn't enough capacity of 197 µF and for the "C" bank there is a problem of charging the capacitors' bank . The circuits are described with the use of a simulation program called ATP. The complete circuit is shown next (figure 5.2)

The “A”-bank's circuit is not working because of the lack of two more capacitors (in total must be a bank of nine capacitors of value 197 µF each and each one rated at 18000 kV, the same capacitors used in the B-bank). For this reason the “A”-bank's electric components are considered and mounted only for future use.

As well the “C”-bank's circuit is not working because of a problem with the charging of the capacitors that it will be described after (paragraph 5.3).

For these reason the concentration in the description of the circuit is on the “D”-bank and “B”-bank (the two banks that already operating).

The attention will be focused on the “D”-bank and specifically for the calculation, with different kind of methods, of the electrical parameters' values (R and L) hidden in the wires, in the cables and in the other components of the circuit (much of the measurement methods used in this circuit could be reused in the “B”-bank, for a same kind of L/R values' estimation).

So first of all it will be analyzed accurately the “D”-bank.

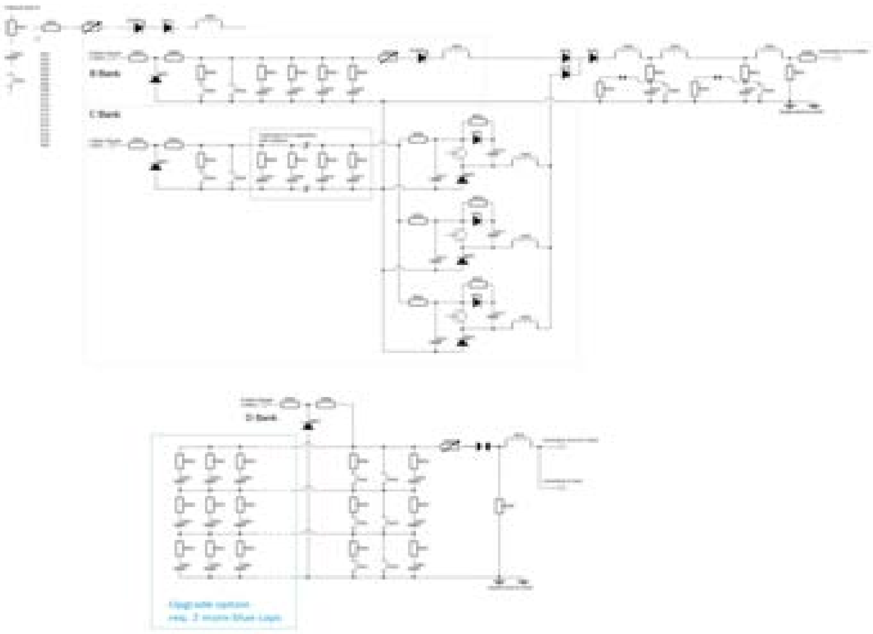


Figure 5.2 - Scheme of the complete circuit for B/C-bank and “D”-bank

5.2 D-BANK’S SIMULATION

The description of the simplified circuit, with “ATP” simulator program, is:

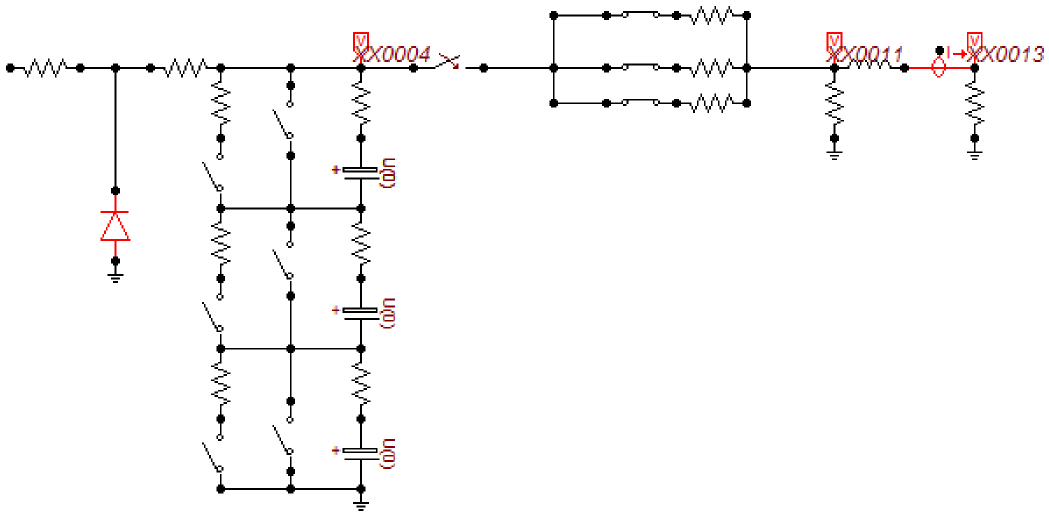


Figure 5.3 - ATP simulator scheme

In this circuit each component is an ideal electric element of the real one, used in the circuit. As result for the simulation is obtained the following waveform of the voltage or the current for the different highlight point of the circuit (XX0004, XX0011, XX0013):

Voltage XX0011

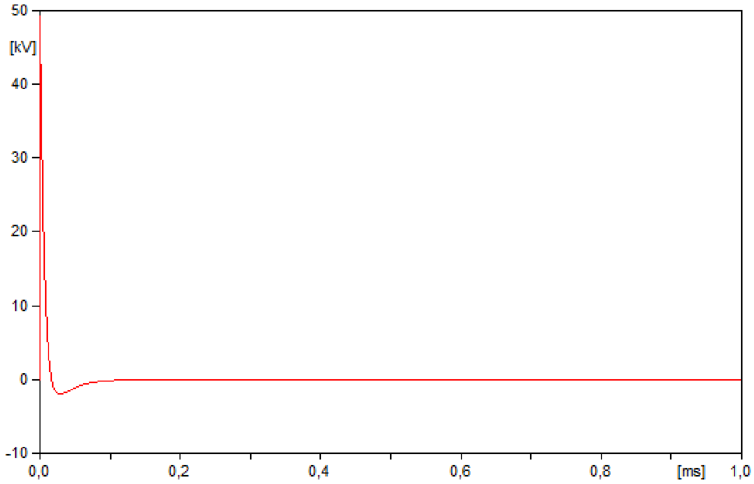


Figure 5.4 - Plot of the Voltage XX0011 that coma from the ATP simulation

Voltage XX0013

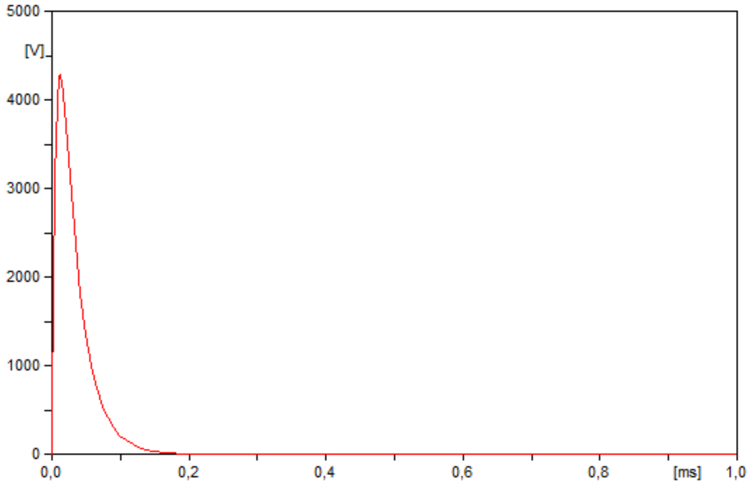


Figure 5.5 - Plot of the Voltage XX0013 that coma from the ATP simulation

Voltage XX0004

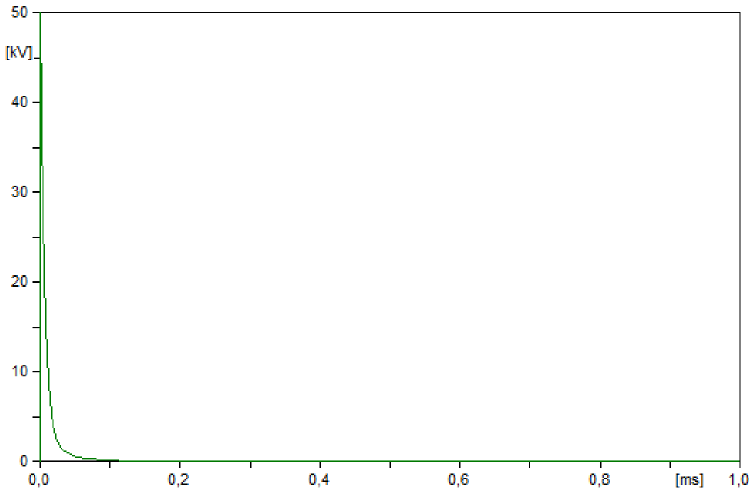


Figure 5.6 - Plot of the Voltage XX0004 that coma from the ATP simulation

Current XX0012-XX0013

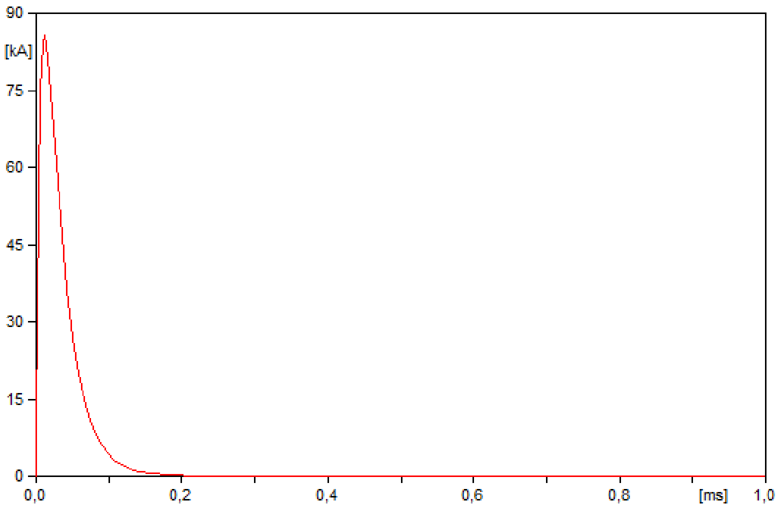


Figure 5.7 - Plot of the current XX0012-XX0013 that coma from the ATP simulation

These waveform could be obtained also with Matlab program. The code used (1.1) is a function of the current "*i(t)*" obtained by the solution of a differential equation drawn by a simplify circuit that contains the complete value of R, L and C each one in only one component. This circuit will be shown and studied better after. The code of the program used in Matlab is the "code 1.1" in appendix I. By applying this code is obtained the following shape of current:

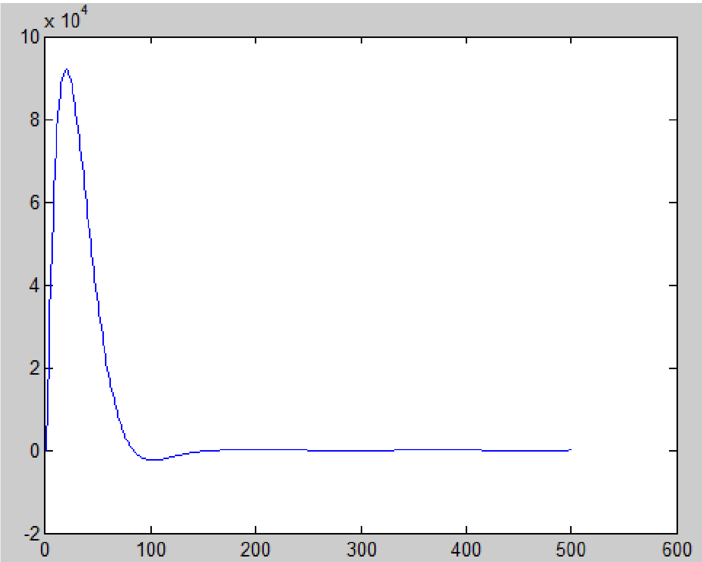


Figure 5.8 - Matlab's simulation of the Shape of *i(t)* generate at the end of the D-bank's circuit (Current "A" on vertical axis, time "µs" on horizontal axis)

In this graphic in vertical axis there is time in "µs" and in horizontal axis the current in "kA".
This function "*i(t)*" and his waveform come from a circuit where is put already the correct value of R and C, while the value of L is obtained by a process in which its value is changed in the ATP circuit until the waveform drawn is coincident with the waveform that comes from the direct measurement of the current. This require a lot of tentative in which is used excel for made the graphic comparison. The direct measurement is made by an electric probe and, with fiber optic wire, its measurement arrive directly to the computer as a number of ten thousand time steps and of ten thousand current's values corresponding.
The result values of the measurement are put in a excel file shown here:

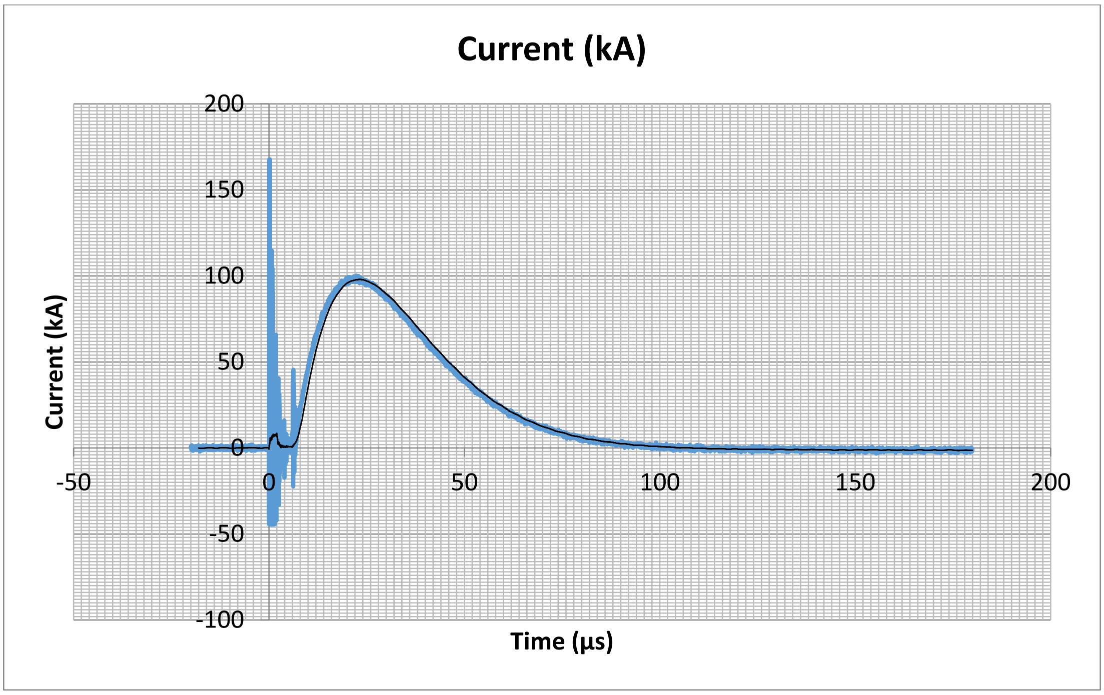


Figure 5.9 – *i(t)*-Current's shape obtained by plotting, in excel, the sampling values measured by the oscilloscope during the test of aluminum plate with D-bank's circuit.

The ATP simplify circuit used for an easier comparison is the follow:

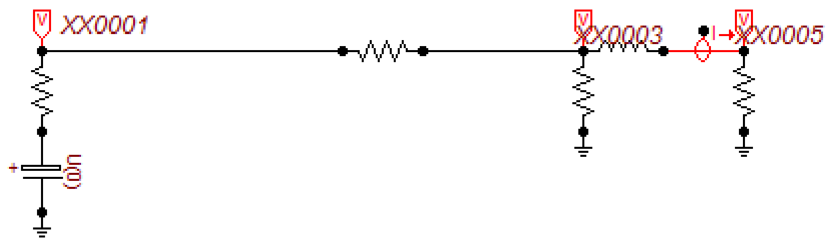


Figure 5.10 – ATP simplify scheme for the D-bank's circuit

In this circuit is considered a total capacitor and a total resistance, composed by the series of the three capacitors and of the three resistances.

$$\frac{1}{Ctot} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} = \frac{3}{C} = \frac{3}{197}\ {}^{1}/_{\mu F}$$

$$Ctot = \frac{197}{3} = 65{,}6667\ \mu F \tag{5.1}$$

$$R = 0.12875\ \Omega \qquad R_{tot} = R + R + R = 3R = 3 * 0.12875 = 0.38625\ \Omega$$

The other resistance is made by the parallel of three resistances

$$\frac{1}{R_{tot}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = 16\ S$$

$$R_{tot} = \frac{1}{16} = 0.0625\ \Omega$$

The components behind the parallel capacitors are neglected. In fact their use is necessary only for the charging of the capacitor.

The "L" component is $L = 4{,}4\ \mu H$ as resulted after a lot of test. These tests have been previously performed in the following way:
has been reported in a single Excel file the graphs of the current values obtained by direct measurement with an oscilloscope (Figure 5.9) and the current values at the end of the circuit simulated in ATP. With the change in the value of inductance in the circuit simulated "ATP" has finally come to an equality of the two graphs as shown in Figure 5.12.
The waveform that is obtained from this circuit (ATP simulation), with this value of L (concentrate electric parameter), is:

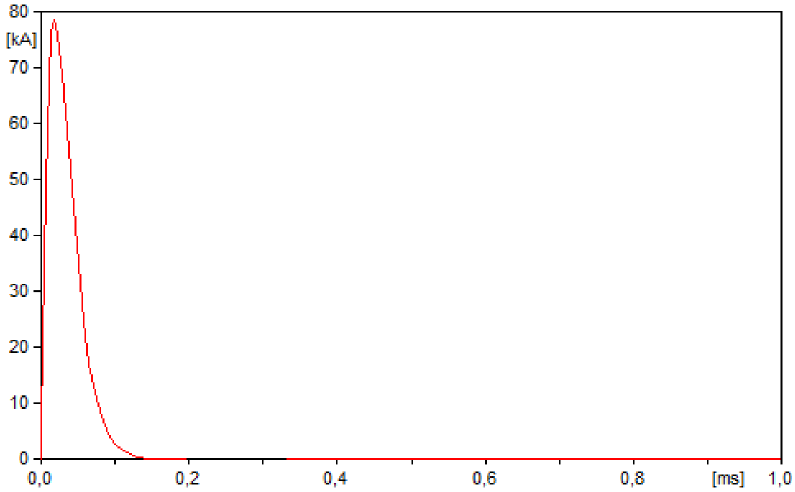


Figure 5.11 – Current's plot by ATP simulation for the D-bank's circuit.

The final comparison with excel is the following:

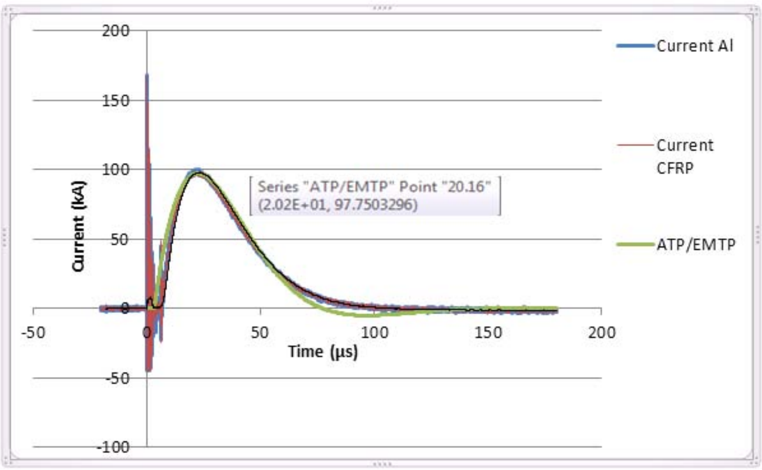


Figure 5.12 – Excel graphic comparison between ATP (with L=4,4μF) and the sampling values form the testing of the aluminum's material

5.3 CHARGING OF C-BANK (TEST)

During the charging of the C-bank the switches of the C-bank's circuit, that are used to shut down (to the ground) the current, are open and so the circuit is not working (is open the connection to the test cabin), while is closed the power switch (closing of the switch in figure 5.13). This make a RC transitory, solved using the Kirchhoff law's equations applied to this circuit (considering the constitutive equations of the components during the transient, the equation becomes a differential equations). During the test of the material the switches are switched exactly in the opposite position that they assume during the charging. The C-bank is connected with a circuit of discharging with dissipative and inductive characteristic (RLC filter), so that at the end the total circuit works like an RLC transitory during the discharging, for arrive to a level of 200-800 (A) of current in the test cabin.

Instead during the charging, as has been said, the capacitor bank are open to the rest of the circuit (used during the test) so the total inductance in this case is included in the wires that connect each component (the resistances and the capacitors) and is so low that can be neglected. For this reason the circuit can be simplified as one total resistance and as one total capacitor. This is drawn down here:

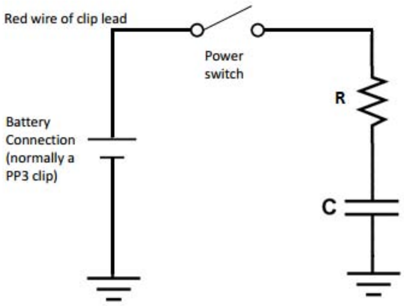


Figure 5.13 - Scheme of the capacitor's charging circuit

The Kirchhoff laws valid for the transitory case, discussed here, are the follow (time=0_- switch, for connection to the generator, open/ time=0_+ switch shut):

$$i_C = C\frac{\partial v}{\partial t}$$
$$V_r = Ri$$
$$V_C = \int \frac{i\delta t}{C}$$
$$RC\frac{\partial v}{\partial t} + v = V_{batt} \tag{5.2}$$

The solution analytically is:
Considering a solution like: *($v = Ae^{st}$)*
Is obtained:

$$ARCse^{st} + Ae^{st} = 0$$
$$Ae^{st}(RCs+1) = 0$$
$$s = -\frac{1}{RC}$$

And so the solution (homogeneous solution) of the differential equation (5.2) is:

$$v = Ae^{-\frac{1}{RC}t}$$

The particular solution of the differential equation (5.2) is:

$$v = V_{batt}$$
$$v = Ae^{-\frac{1}{RC}t} + V_{batt}$$

and so (when t=0, *v*=0)

$$A = -V_{batt}$$
$$v = V_{batt}\left(1 - e^{-\frac{1}{RC}t}\right)$$
$$ic = C\frac{\partial v}{\partial t} = \frac{V_{batt}}{R}e^{-\frac{1}{RC}t}$$

Series of two resistance of R = 9 Ω, plus one resistance (one above each capacitor) of R = 3 Ω is:

$$R = 9 + 9 + 3 = 21\,\Omega$$

Parallel of thirteen capacitor each one of C_1 = 16700 μF is:

$$C = 13C1 = 250\,\mu F$$

And so the time constant of the transitory circuit is:

$$\tau = \frac{1}{RC} = 190.4762\ s$$
$$RC = 5{,}25 \cdot 10^{-3} \quad s^{-1}$$

This circuit can be analyzed with ATP program and with matlab program.

5.4 ATP CAPACITORS CHARGING'S SIMULATION

The simplified circuit that is been analyzed is written in ATP as:

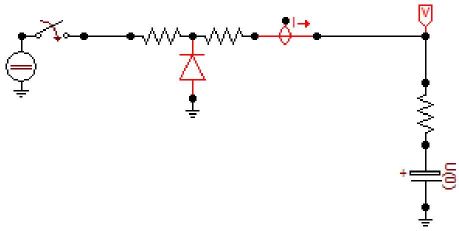


Figure 5.14 - ATP capacitor’s charging scheme

And the waveform of current and tension obtained with ATP plot is:

Current Tension

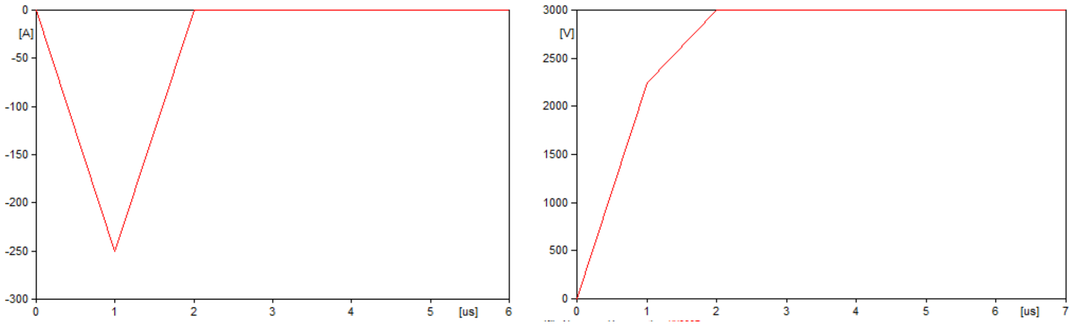


Figure 5.15 – ATP plot of current and tension in the capacitor of the C-bank during the charging

5.5 MATLAB SIMULATION

By using in matlab the “code 1.2”, in appendix I, is obtained the following shape of tension and current:

Current Tension

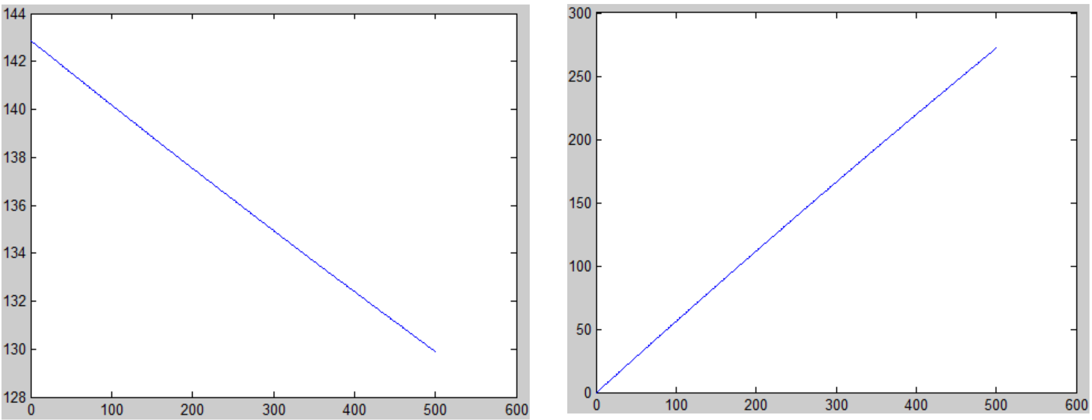


Figure 5.16 - Matlab plot of current and tension in the capacitor of the C-bank during the charging

These are the tension’s and the current’s shape inside the C-bank during the charging. In both of them in the vertical axis there is time in “µs” and in the horizontal axis there is the Ampere (in “A”) and the voltage (in “V”).
The same graphics can be easily obtained in an excel file by putting the values of the two previous functions *i(t)* and *v(t)* obtained for different times (with a specific time's step, $\Delta t=1\mu s$). This is obtained in the following file in which can be put together the two waveforms (*i(t)* and *v(t)*):

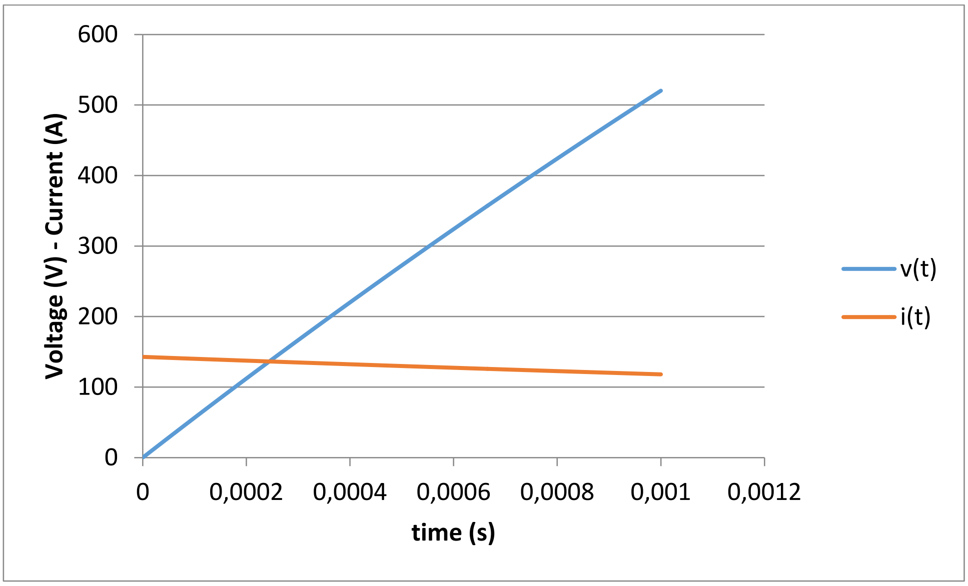


Figure 5.17 - Excel plot of current and tension in the capacitors of the C-bank during the charging

In table 5.5 there are the main electric component values used in the graphic 5.17

Table 5.5 –Main electric components values of figure 5.17

R:	21 (Ω)
C:	0,00025 (F)
t:	0,00525 (s)
V_{batt}:	3000 (V)

5.6 PROBLEMS AND CONSIDERATIONS ENCOUNTERED IN PRACTICE

In the reality there is a noise during the charging made in the DC-generator of 3kV by the RC circuit (the circuit C-bank’s, the circuit where the inductances are so low that can be neglected and there are only resistances and capacities). The C-bank's circuit is in fact a RC circuit who is working in a transitory way, distant from the continuous regime in which the direct current (DC) is produced by the generator. This generate these noise. So at the end there will be, in the real course of the current, a peak made by the noise produced by putting, in a transitory RC circuit, a DC generator (who is working in a continuous regime). The charging of the C-bank is not working and for this reason is impossible to use the C-bank's circuit and its waveform for testing the airplane’s materials. Substantially, during the charging of the "C" capacitors with a DC generator of 3kV, the big value of the impedance “Z” of the "C" bank's circuit (composed, in the generator, by a series and by a parallel of resistances and capacities components) make a level of noise, in the ideal waveform of the C-bank's charging (shown before with the ATP simulation), so high that the real waveform is in many point during the charging at a level two time higher than the level wanted in the idealist result. (In fact is possible to see an increase of 1,5 kV in the point where should be a voltage value of 3kV, the voltage wanted at the end of the charging’s waveform). So, for this peak, the level of the voltage is put too high for what the capacitor could contain (the "C" capacitors are designed for almost a voltage of 3kV) and the charging is shut down.

Figure (5.18) is a graphic where are plotted the values (tested with these wrong components) and, in this way, it's shows the failure to charge the capacitors' bank "C".

During the test are considered only six of the thirteen capacitors with 40% of the voltage, so at the end with this test the voltage arrive to something like V = 1,2 kV while the noise arrive to V = 2,4 kV.

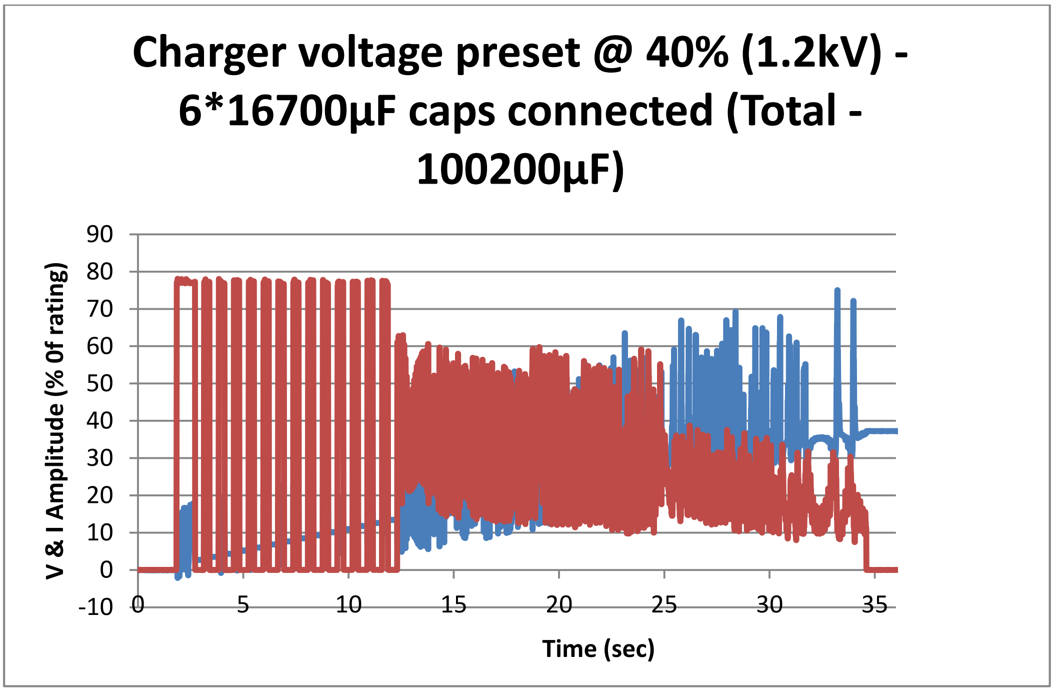


Figure 5.18 – Shape of current and tension (in 6 of the 13 capacitors) during the charging of the C-Bank (oscilloscope sampling)

As we can see the courses of voltage and of current are really disturbed by the noise
For solve this problem could be put in parallel to the DC generator a capacitor (as is showed in the picture (5.10)).

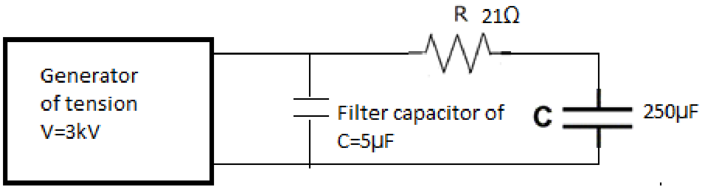


Figure 5.19 - Scheme of a possible filter for the capacitor's charging

This capacitor of C = 5 µF is used as a filter ("C" compensator) for filter this noise too much high. Therefore this capacitor works to take a part of the peak, made by the noise, and to put this peak down enough to not shut down the charging of the capacitors (so that in the end you get a level of Voltage of 200 V, very close to the ideal shape, a level that gives no problems for charging). This is a way to reduce the noise at an harmless level for the charging. The sperimental results are shown in the graphic of the values that come from the test did with the capacitor filter.

For the Voltage

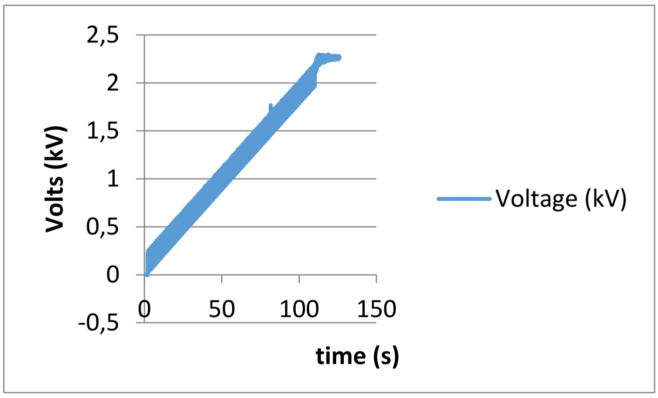


Figure 5.20 – Shape of tension (in 6 of the 13 capacitors) during the charging of the C-Bank with the capacitor filter (oscilloscope sampling)

For the current

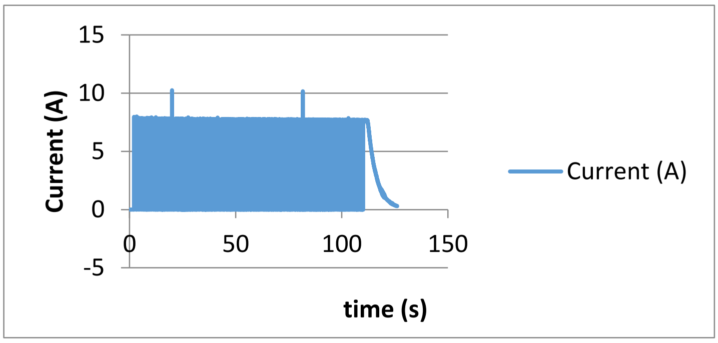


Figure 5.21 - Shape of current (in 6 of the 13 capacitors) during the charging of the C-Bank with the capacitor filter (oscilloscope sampling)

As can be see these waveform are very close to the ones simulated.
Actually the DC generator is broken, but with a new one and with this “C” filter, in the future it will be possible to use also the C-bank for testing the airplane’s materials.

Chapter 6

6.1 INDUCTANCE AND RESISTANCE ESTIMATION OF THE D-BANK CIRCUIT

To estimate the inductance and resistance values in the real D-bank's circuit is necessary to follow more ways of calculus of these value. In this way from the comparison of the result obtained in these different typological ways of calculus, can be given the final confirmation of the value or of the number of values in which the real electric elements, that you are looking for, belong. This way of estimation of electric components is usually adopted also for other engineering's field.
In Chapter 2 (paragraph 5.2) has been described the first graphic way to get an estimate of the value of inductance. Now there are other three ways to measuring the L and R values (of the inductance and of the resistance) implicit in the circuit:

(a) An "analytical method" that starts by solving the differential equation which comes from the Kirchhoff's laws, applied to the simplified D-bank's circuit. This simplified circuit includes all the components in only one resistance, one capacitor and one inductance, for obtained the course of *i(t)*. By this course of *i(t)*, knowing experimentally the peak of the current and the time in which it's reached, can be solved with matlab a non linear system, for obtained the two value of R and L.
(b) A "direct method" that uses the physical equations for calculus the L and R values, just by the physical dimensions of the wires, directly measured in the laboratory.
(c) A "measurement method" that consist in take the measures of resistance and inductance detected by a specific instrument, used in this last case.

6.2 (a) - CIRCUIT ANALYSIS METHOD

Measurement of L, C and R overall values of the D-bank:
Starting by the resolution of the circuit R, L, C you get to a shape of current very close to the one request for the D-bank's.
The ideal shape of current wanted in the lightning, is:

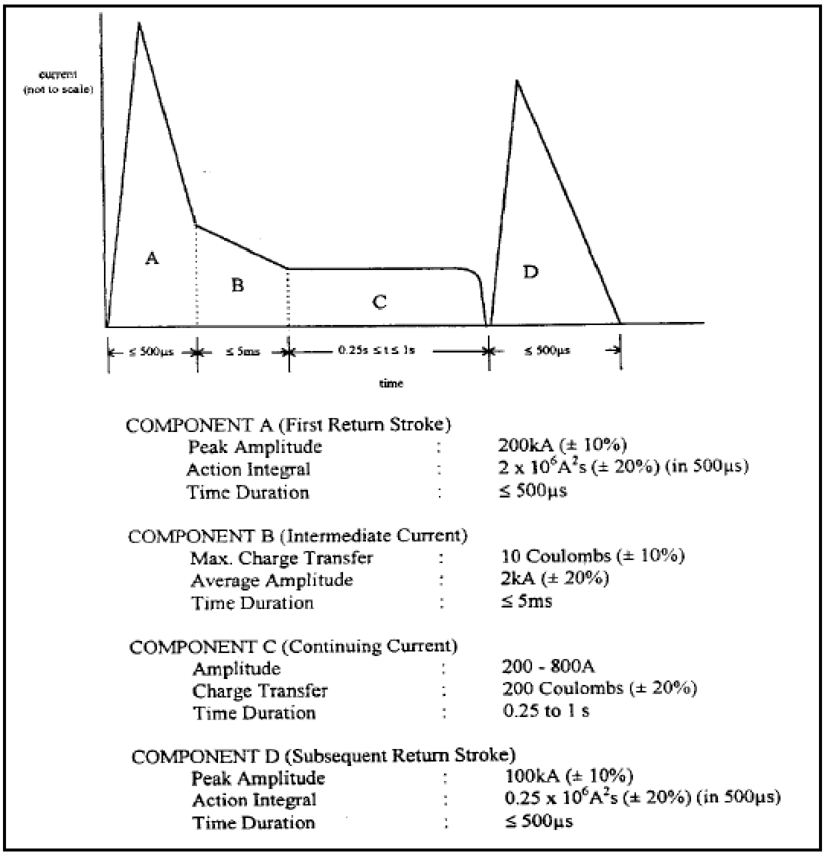


Figure 6.1 - Ideal overall waveform

And the equivalent circuit, from which derives the discharge in the test cabin, is:

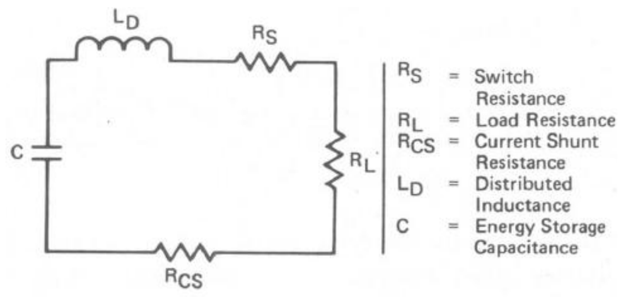


Figure 6.2 - Equivalent RLC circuit

This circuit is the equivalent one obtained for describe the real D-bank's circuit where C, L_D and R including in them all the equivalent electric components.
It can be analyzed as the following similar circuit:

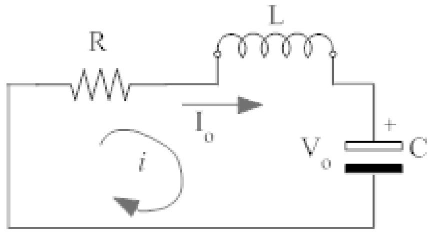


Figure 6.3 - Simplify RLC equivalent circuit

Where it must be obtained the value of current, knowing the initial value of current in the inductance "Io" and the initial one of the voltage in the capacitor "Vo". For obtain it we have to resolve the second grade's differential equation. The Kirchhoff equation for the current and the voltage of the above circuit are:

$$V_r + V_l + V_c = 0$$

$$i = C\frac{\partial v}{\partial t}$$

$$V_l = L\frac{\partial i}{\partial t}$$

$$V_c = \int \frac{i\delta t}{C}$$

$$Ri + L\frac{\partial i}{\partial t} + \int \frac{i\delta t}{C} = 0$$

$$R\frac{\partial i}{\partial t} + L\frac{\delta^2 i}{\delta t^2} + \frac{i}{C} = 0$$

$$\frac{\delta^2 i}{\delta t^2} + \frac{R}{L}\frac{\partial i}{\partial t} + \frac{i}{LC} = 0 \qquad (6.1)$$

The analytical solution is
Considering a solution like " $i = Ae^{st}$ " is obtained:

$$As^2 e^{st} + As\frac{R}{L}e^{st} + \frac{A}{LC}e^{st} = 0$$

$$Ae^{st}(s^2 + \frac{R}{L}s + \frac{1}{LC}) = 0$$

$$S_{1,2} = \frac{-\frac{R}{L} \pm \sqrt{\frac{R}{L}^{2} - 4\frac{1}{LC}}}{2}$$

and so the solution is:

$$i(t) = A1e^{\left(-\frac{R}{2L}+\sqrt{\frac{R}{4L}^{2}-\frac{1}{LC}}\right)t} + A2\, e^{\left(-\frac{R}{2L}-\sqrt{\frac{R}{4L}^{2}-\frac{1}{LC}}\right)t} \tag{6.2}$$

If

$$\alpha = -\frac{R}{2L} + \sqrt{\frac{R}{4L}^{2} - \frac{1}{LC}} \quad and \quad \beta = -\frac{R}{2L} - \sqrt{\frac{R}{4L}^{2} - \frac{1}{LC}}$$

$$i = A1e^{(\alpha)t} + A2\, e^{(\beta)t}$$

$$Vc = \int \frac{i\delta t}{C} = \frac{A1}{\alpha C}\, e^{(\alpha)t} + \frac{A2}{\beta C}\, e^{(\beta)t}$$

The value of A_1 and A_2 is given by initial condition:

$$Io = A1 + A2$$

$$Vo = \frac{A1}{\alpha C} + \frac{A2}{\beta C}$$

Solving this system is obtained:

$$A1 = \frac{VoC\alpha\beta}{\beta - \alpha} - \frac{Io\alpha}{\beta - \alpha}$$

$$A2 = \frac{Io\beta}{\beta - \alpha} - \frac{VoC\alpha\beta}{\beta - \alpha}$$

The previous differential equation could be solved in easier way with the Laplace transformations. Considering the same initial condition, with the Laplace transformations, the equation (6.1) becomes:

$$\mathcal{L}(\frac{\partial i}{\partial t}) = sI - Io$$

$$\mathcal{L}(\frac{\delta^2 i}{\delta t^2}) = s^2 I - sIo - i'(o)$$

But:

$$Vl(0) = -(RIo - Vo) \quad and\ so \quad i'(0) = -\frac{RIo - Vo}{L}$$

By putting these values in the (6.1) we have:

$$s^2 I - sIo + \frac{RIo - Vo}{L} + \frac{R}{L}\,(sI - Io) + \frac{I}{LC} = 0$$

$$I(s^2 + \frac{R}{L}\, s + \frac{I}{LC}) = \frac{Vo}{L} + sIo$$

$$I = \frac{\frac{Vo}{L} + sIo}{s^2 + \frac{R}{L}\, s + \frac{I}{LC}} \tag{6.3}$$

An other easier way to write the Kirchhoff's law is to overwrite the circuit with the following, valid for the Laplace transformations:

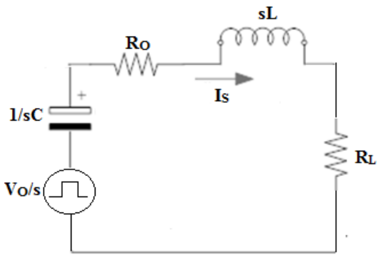


Figure 6.4 - Equivalent RLC circuit with Laplace-Transformation for each electric component

Where are put a voltage step generator, for explain the initial condition "*Vo*", and a resistance "*Ro*", for the initial condition "*Io*", (where *Ro·I(s)=Io*). In this way you can arrive directly to the same final solution written above (in the differential equation's solution) (6.3).

In fact with this circuit the current is:

$$\frac{\mathrm{Vo}}{s} = \frac{I(s)}{sC} + (\mathrm{Ro} + \mathrm{Rl})\mathrm{I(s)} \ + \mathrm{sLI(s)}$$

$$I(s) = \frac{\frac{Vo}{s}}{Ro + Rl + sL + \frac{1}{sC}} = \frac{\frac{V0}{L}}{s^2 + \frac{(Ro + Rl)}{L}\ s + \frac{1}{LC}}$$
$$= \frac{\frac{V0}{L}}{\left(s + \frac{(Ro + Rl)}{2L} - \sqrt{\Delta}\right)\left(s + \frac{(Ro + Rl)}{2L} + \sqrt{\Delta}\right)}$$

Where:

$$\Delta = \frac{(Ro + Rl)^2}{4L^2} - \frac{1}{LC}$$

(Now knowing that: $\Delta = 0 \ \rightarrow \frac{1}{LC} = \frac{(Ro+Rl)^2}{4L^2} \rightarrow \frac{C(Ro+Rl)^2}{4}$ with the overall value of "R", "C" of the electric circuit, previously showed (5.1), we have L = *2,53512675* μH)

Putting $Ro * I(s) = Io$ you arrive at the same (6.3) final equation:

$$I(s) = \frac{\frac{Vo}{L} + sIo}{s^2 + \frac{R}{L}\ s + \frac{I}{LC}}$$

.
The value of each element is obtained by the sum of the components of the D-bank's scheme (see figure 5.3 ATP) while R is obtained by the following expression:

$$RI^2 = \frac{1}{2}CV^2$$

$$I^2 = \int_0^{\infty} i(t)^2 \partial t \ = Io^2(-\frac{e^{-\alpha t}}{2\alpha} - \frac{e^{-\beta t}}{2\beta} - \frac{2e^{-(\alpha+\beta)t}}{-(\alpha + \beta)})|_0^{\infty} = \ -Io^2(-\frac{1}{2\alpha} - \frac{1}{2\beta} - \frac{2}{-(\alpha + \beta)})$$

(If $i(t) = Io(e^{-\alpha t} - e^{-\beta t})$ where Io = 109,405 A; α = 22,708 s^{-1}; β = 1,294,530 s^{-1} like the ideal waveform of the current for the D-bank).

So is obtained:

$$R = \frac{\frac{1}{2}CV^2}{-Io^2(-\frac{1}{2\alpha} - \frac{1}{2\beta} - \frac{2}{-(\alpha+\beta)})}$$

$C = 66.67\ \mu F,$

$V = 54\ kV,$

$L(ideal\ start\ value) = 3.1\ \mu H,$

$$R = \frac{\frac{1}{2}CV^2}{-Io^2(-\frac{1}{2\alpha} - \frac{1}{2\beta} - \frac{2}{-(\alpha+\beta)})} = 0.39\ \Omega$$

By putting these values, "Δ" results a negative number and you have an imaginary solution of the differential equation. So the solution results, (by Euler's formula), are an exponential sinusoidal function damped.

The following expression could be implement with Matlab:

$$I(s) = \frac{(\frac{V0}{IoL} + s)Io}{\left(s + \frac{(Rl)}{2L} - \sqrt{\Delta}\right)(s + \frac{(Rl)}{2L} + \sqrt{\Delta})}$$

Where:

$$\Delta = \frac{(\mathrm{Rl})^2}{4L^2} - \frac{1}{LC}$$

The initial condition for the current is "Io = 0" and you have:

$$Is = \frac{\frac{V0}{L}}{\left(s + \frac{(Rl)}{2L} - \sqrt{\Delta}\right)(s + \frac{(Rl)}{2L} + \sqrt{\Delta})} \tag{6.4}$$

(where there are the values of: *Rl = 0.39 Ω*, *L = 3.1 μH*, *C = 66.67 μF*, *V = 54 kV* and consequentially Δ is a negative value *Δ = −881651937.2*)

The expression (6.4) could be factorized and is obtained:

$$I(s) = \frac{V0}{L}\left(\frac{A1}{\left(s + \frac{(Rl)}{2L} - \sqrt{\Delta}\right)} + \frac{A2}{\left(s + \frac{(Rl)}{2L} + \sqrt{\Delta}\right)}\right)$$

$$A1 = \frac{1}{2\sqrt{\Delta}};\quad A2 = -\frac{1}{2\sqrt{\Delta}}$$

so the equation becomes:

$$I(s) = \frac{V0}{2L\sqrt{\Delta}}\left(\frac{1}{\left(s + \frac{(Rl)}{2L} - \sqrt{\Delta}\right)} - \frac{1}{\left(s + \frac{(Rl)}{2L} + \sqrt{\Delta}\right)}\right)$$

Knowing that the antilaplace transformation of "$\frac{1}{(s+a)}$" is "e^{-at}", the antilaplace of "I(s)" becomes the final shape of "*I(s)*" in time function (the same function obtained in other way previously (6.2)):

$$is(t) = \frac{V0}{2L\sqrt{\Delta}}\left(e^{-\left(\frac{Rl}{2L} - \sqrt{\Delta}\right)t} - e^{-\left(\frac{(Rl)}{2L} + \sqrt{\Delta}\right)t}\right) \tag{6.5}$$

"Δ" is a negative number and so it could be written $\sqrt{\Delta} = \sqrt{-(-\Delta)} = i\sqrt{-\Delta}$. The same shape of electric current could be written as (using the Euler expression $e^{\pm iat} = \cos(at) \pm isen(at)$):

$$is(t) = \frac{V0}{2L\sqrt{\Delta}} e^{-\left(\frac{(\mathrm{Rl})}{2L}\right)t} \left(e^{(i\sqrt{-\Delta})t} - e^{-(i\sqrt{-\Delta})t}\right) \rightarrow$$
$$\frac{V0}{2L\sqrt{\Delta}} e^{-\left(\frac{(\mathrm{Rl})}{2L}\right)t} \left(\cos(\sqrt{-\Delta}t) + isen(\sqrt{-\Delta}t) - \cos(\sqrt{-\Delta}t) + isen(\sqrt{-\Delta}t)\right) \rightarrow$$
$$\frac{V0}{2L\sqrt{\Delta}} e^{-\left(\frac{(\mathrm{Rl})}{2L}\right)t} \left(2isen(\sqrt{-\Delta}t)\right) \rightarrow$$
$$\frac{V0}{2Li\sqrt{-\Delta}} e^{-\left(\frac{(\mathrm{Rl})}{2L}\right)t} \left(2isen(\sqrt{-\Delta}t)\right) \rightarrow$$
$$is(t) = \frac{V0}{L\sqrt{-\Delta}} e^{-\left(\frac{(\mathrm{Rl})}{2L}\right)t} \left(sen(\sqrt{-\Delta}t)\right)$$

This solution could be implemented in excel for obtained the following graphic of *is(t)*:

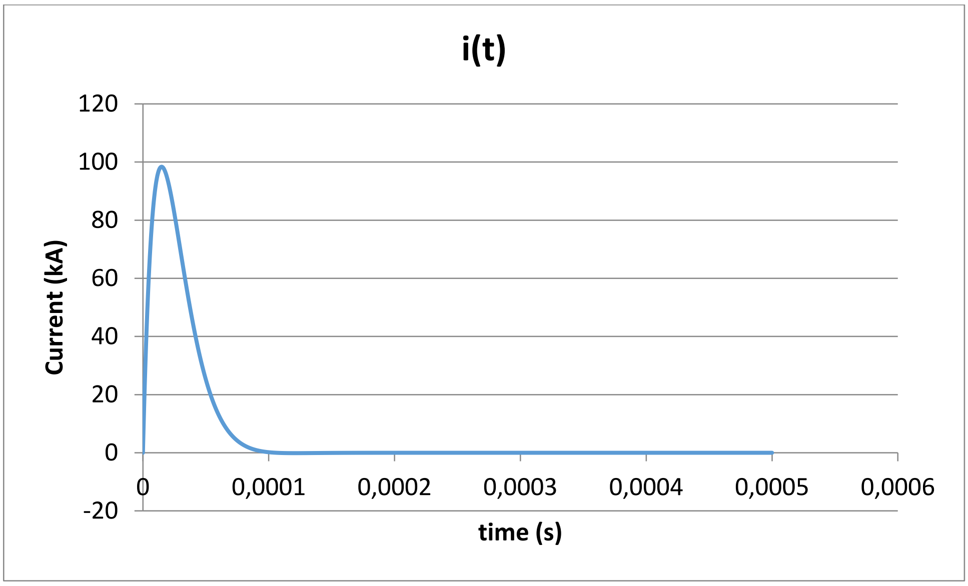


Figure 6.5 – Excel plot of *is(t)* for a negative "Δ" value in the equation (6.5)

In the table 6.1 there are the main electric components used in the graphic 6.5:

Table 6.1 - Main electric components used in the graphic 6.5

L:	0,0000031 (H)
C:	0,0000667 (F)
R:	0,39 (Ω)
V:	54 (kV)
Δ:	-8,79E+08

If is considered again the Euler expression ($sen(\sqrt{-\Delta}t) = \frac{e^{i\sqrt{-\Delta}t} - e^{-i\sqrt{-\Delta}t}}{2i} = \frac{e^{\sqrt{\Delta}t} - e^{-\sqrt{\Delta}t}}{2i}$) you return at this result previously obtained:

$$is(t) = \frac{V0}{L\sqrt{-\Delta}} e^{-\left(\frac{(\mathrm{Rl})}{2L}\right)t} \left(\frac{e^{\sqrt{\Delta}t} - e^{-\sqrt{\Delta}t}}{2i}\right) = \frac{V0}{Li\sqrt{-\Delta}} e^{-\left(\frac{(\mathrm{Rl})}{2L}\right)t} \left(\frac{e^{\sqrt{\Delta}t} - e^{-\sqrt{\Delta}t}}{2}\right)$$

$$is(t) = \frac{V0}{2L\sqrt{\Delta}} e^{-\left(\frac{(\mathrm{Rl})}{2L}\right)t} \left(e^{\sqrt{\Delta}t} - e^{-\sqrt{\Delta}t}\right)$$

This result can be obtained also by in matlab “code 1.3” (in appendix I). (considering the value of “R”, ”L” and “C” of above) and is obtained the same shape of electric current (see “code 1.3”).

For the calculus of the resistance of the mash for each material subject at these level of electric current measured:
For the carbon a resistance of: $r_c = 0.008\ \Omega$;
For the aluminum is used the formula ($r = \rho \frac{l}{Area}$) in this case $Area = \frac{\pi}{4} d^2$, $l = 2.5\ mm$, $d = 550\ mm$
$\rho(20^0 C) = 28{,}3 \frac{mm^2}{km} \Omega$

If is considered $\rho = \rho_o[1+\alpha(T-T_o)]$ ($\alpha=0.004\ C^{-1}$) and the laboratory temperature is around $T = 16\ ℃$ so you have $\rho(16^0 C) = 27{,}8472 \frac{mm^2}{km} \Omega$

And so $r_{all}=2.930260847*10^{-7}\ \Omega$

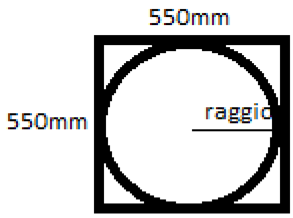


Figure 6.6 – Geometrical main dimension of the aluminum platform

In figure 6.7 there’s a representation of a line with the passive components (in the constant concentrate model).

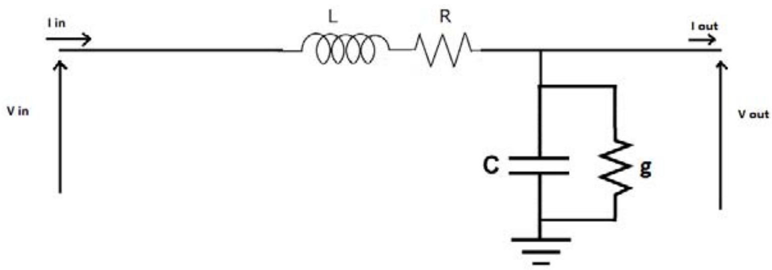


Figure 6.7 - Wires of connection to the test cabin representation with distributes electric component

In this case the g and c components are really low and could be neglected ($c = 0$ and $g = 0$).
It must be calculate only the inductance, "L" component (if we assume the previous value for the resistance “R”) or could be, in other way, estimated analytically the values of “R” and “L” by solving a not linear system of two equation, starting by the knowing of the peak of the current and the time in which is reach.

6.2.1 1° Method

Knowing the Max value of current and the time to reach that peak of current ($I_p = 100\ KA$, $T_{max} = 25\ \mu s$) could be calculated the max of the function is(t). By putting its derivate equal to zero ($\frac{\delta i(t)}{\delta t} = 0$) in the equation that comes out you can place the current maximum value of i(t) and the time in which is reach Tmax , where now the only unknown is L, $R = \frac{\frac{1}{2}\mathrm{CV}^2}{-\mathrm{Io}^2\left(-\frac{1}{2\alpha} - \frac{1}{2\beta} - \frac{2}{-(\alpha+\beta)}\right)} = 0{,}39\ \Omega$ is the value obtained before and you put the value of $I_p = 100\ KA$, $T_{max} = 25\ \mu s$ in the equations obtained.
The mathematical steps are:

Doing now the derivate of the function *is(t)* and placing it equal to zero is obtained the max value of current (I_p) and so (for $\Delta > 0$):

$$\frac{\partial is(t)}{\partial t} = 0$$

$$\frac{V0}{2L\sqrt{\Delta}} e^{-\frac{Rl}{2L}t} \left\{ \left[\left(\frac{Rl}{2L} + \sqrt{\Delta} \right) e^{-\sqrt{\Delta}t} \right] - \left[\left(\frac{Rl}{2L} - \sqrt{\Delta} \right) e^{\sqrt{\Delta}t} \right] \right\} = 0$$

This is the equation $T_{max} = f(L,Rl)$ where now the unknowns are become L and R. By placing in it the hypothetical value of $T_{max} = 25\ \mu s$ can be obtained the function $Rl = f(L)$ and considering a value of $Rl \approx 0.39\ \Omega$ you can find the value of L by resolving the equation. For resolving it are done the following passages:

$$\frac{V0}{2L\sqrt{\Delta}} e^{-\frac{Rl}{2L}t} \left\{ \left[\left(\frac{Rl}{2L} + \sqrt{\Delta} \right) e^{-\sqrt{\Delta}t} \right] - \left[\left(\frac{Rl}{2L} - \sqrt{\Delta} \right) e^{\sqrt{\Delta}t} \right] \right\} = 0 \ \rightarrow$$

$$\left[\left(\frac{Rl}{2L} + \sqrt{\Delta} \right) e^{-\sqrt{\Delta}t} \right] - \left[\left(\frac{Rl}{2L} - \sqrt{\Delta} \right) e^{\sqrt{\Delta}t} \right] = 0 \ \rightarrow$$

$$\left(\frac{Rl}{2L} + \sqrt{\Delta} \right) e^{-\sqrt{\Delta}t} = \left(\frac{Rl}{2L} - \sqrt{\Delta} \right) e^{\sqrt{\Delta}t} \ \rightarrow$$

$$\ln \left[\left(\frac{Rl}{2L} + \sqrt{\Delta} \right) e^{-\sqrt{\Delta}t} \right] = \ln \left[\left(\frac{Rl}{2L} - \sqrt{\Delta} \right) e^{\sqrt{\Delta}t} \right] \ \rightarrow$$

$$\ln \left(\frac{Rl}{2L} + \sqrt{\Delta} \right) + \ln \left(e^{-\sqrt{\Delta}t} \right) = \ln \left(\frac{Rl}{2L} - \sqrt{\Delta} \right) + \ln \left(e^{\sqrt{\Delta}t} \right) \ \rightarrow$$

$$\ln \left(\frac{Rl}{2L} + \sqrt{\Delta} \right) + \ln \left(e^{-\sqrt{\Delta}t} \right) = \ln \left(\frac{Rl}{2L} - \sqrt{\Delta} \right) + \ln \left(e^{\sqrt{\Delta}t} \right) \ \rightarrow$$

$$\ln \left(\frac{Rl}{2L} + \sqrt{\Delta} \right) - \sqrt{\Delta}\, t = \ln \left(\frac{Rl}{2L} - \sqrt{\Delta} \right) + \sqrt{\Delta}t \ \rightarrow$$

$$2\sqrt{\Delta} = \ln \left(\frac{Rl}{2L} + \sqrt{\Delta} \right) - \ln \left(\frac{Rl}{2L} - \sqrt{\Delta} \right) \ \rightarrow$$

$$2\sqrt{\Delta} = \ln \frac{\frac{Rl}{2L} + \sqrt{\Delta}}{\frac{Rl}{2L} - \sqrt{\Delta}} \ \rightarrow$$

$$2\sqrt{\Delta} = \ln \frac{\frac{Rl}{2L} + \sqrt{\Delta}}{\frac{Rl}{2L} - \sqrt{\Delta}} \ \rightarrow$$

$$e^{2\sqrt{\Delta}} = e^{\ln \frac{\frac{Rl}{2L} + \sqrt{\Delta}}{\frac{Rl}{2L} - \sqrt{\Delta}}} \ \rightarrow$$

$$e^{2\sqrt{\Delta}} = \frac{\frac{Rl}{2L} + \sqrt{\Delta}}{\frac{Rl}{2L} - \sqrt{\Delta}} \tag{6.6}$$

From this equation you can bring out the *Tmax = f(L,Rl)* equation and is obtained exactly:

$$t = \frac{\ln \frac{\frac{Rl}{2L} + \sqrt{\Delta}}{\frac{Rl}{2L} - \sqrt{\Delta}}}{2\sqrt{\Delta}} \tag{6.7}$$

Taking the previous final result (6.6) you arrive to the equation:

$$\left(\frac{Rl}{2} - L\sqrt{\Delta}\right) e^{2\sqrt{\Delta}t} = \frac{Rl}{2} + L\sqrt{\Delta}$$

For solving this could be used the iteration methods like “punto fisso” or “Newton Rapshon”. The solution is obtained by a excel work paper.

For “punto fisso” solution could be used the following expression:

$$L = \frac{1}{\sqrt{\Delta}}\left[\left(\frac{Rl}{2} - L\sqrt{\Delta}\right) e^{2\sqrt{\Delta}t} - \frac{Rl}{2}\right]$$

The Newton Raphson method consist in $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ where now the x_n unknown is L and so *f(L)* is:

$$\mathrm{f(L)} = \left(L\sqrt{\Delta} - \frac{Rl}{2}\right) e^{2\sqrt{\Delta}t} + \frac{Rl}{2} + L\sqrt{\Delta}$$

$$\Delta = \frac{(Ro + Rl)^2}{4L^2} - \frac{1}{LC}$$

$$\frac{\partial\Delta}{\partial L} = \frac{1}{CL^2} - 2\,\frac{(\mathrm{Ro} + \mathrm{Rl})^2}{4L^3}$$

$$\frac{\partial L\sqrt{\Delta}}{\partial L} = \sqrt{\Delta} + \frac{1}{2}\mathrm{L}\frac{\partial\Delta}{\partial L}(\Delta)^{\frac{3}{2}}$$

$$\frac{\partial e^{2\sqrt{\Delta}t}}{\partial L} = 2\mathrm{t}\frac{\partial\Delta}{\partial L} e^{2\sqrt{\Delta}t}$$

$$\frac{\partial(L\sqrt{\Delta}e^{2\sqrt{\Delta}t})}{\partial L} = (\sqrt{\Delta} + \frac{1}{2}\mathrm{L}\frac{\partial\Delta}{\partial L}(\Delta)^{\frac{3}{2}})\, e^{2\sqrt{\Delta}t} + 2\mathrm{t}L\sqrt{\Delta}\frac{\partial\Delta}{\partial L} e^{2\sqrt{\Delta}t}$$

So finally is:

$$\mathrm{f'(L)} = \frac{\partial \mathrm{f(L)}}{\partial L} = (\sqrt{\Delta} + \frac{1}{2}\mathrm{L}\frac{\partial\Delta}{\partial L}(\Delta)^{\frac{3}{2}})\, e^{2\sqrt{\Delta}t} + 2\mathrm{t}L\sqrt{\Delta}\frac{\partial\Delta}{\partial L} e^{2\sqrt{\Delta}t} - \left(\frac{(\mathrm{Rl})}{2}\right) 2\mathrm{t}L\sqrt{\Delta}\frac{\partial\Delta}{\partial L} e^{2\sqrt{\Delta}t} + \sqrt{\Delta} + \frac{1}{2}\mathrm{L}\frac{\partial\Delta}{\partial L}(\Delta)^{\frac{3}{2}}$$

For $\Delta<0$ is known that $\sqrt{\Delta} = \mathrm{i}\sqrt{-\Delta}$ and the equation becomes the following:

$$is(t) = \frac{V0}{L\sqrt{-\Delta}} e^{-\frac{Rl}{2L}t} \sin(\sqrt{-\Delta}t)$$

So now is:

$$\frac{\partial is(t)}{\partial t} = 0$$

$$\frac{V0\sqrt{-\Delta}}{L\sqrt{-\Delta}} e^{-\frac{Rl}{2L}t} cos(\sqrt{-\Delta}t) - \frac{\frac{Rl}{2L}V0}{L\sqrt{-\Delta}} e^{-\frac{Rl}{2L}t} sen(\sqrt{-\Delta}t) = 0 \rightarrow$$

$$\sqrt{-\Delta}e^{-\frac{Rl}{2L}t} cos(\sqrt{-\Delta}t) - \frac{Rl}{2L} e^{-\frac{Rl}{2L}t} sen(\sqrt{-\Delta}t) = 0 \rightarrow$$

$$\sqrt{-\Delta}e^{-\frac{Rl}{2L}t} cos(\sqrt{-\Delta}t) = \frac{Rl}{2L} e^{-\frac{Rl}{2L}t} sen(\sqrt{-\Delta}t) \rightarrow$$

$$\sqrt{-\Delta}cos(\sqrt{-\Delta}t) = \frac{Rl}{2L}sen(\sqrt{-\Delta}t) \rightarrow$$

$$\sqrt{-\Delta}= \frac{Rl}{2L}tan(\sqrt{-\Delta}t) \rightarrow$$

$$\Delta= \left(-\frac{Rl}{2L}tan(\sqrt{-\Delta}t)\right)^2$$

For Newton Raphson implementation $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ you can consider the following function and you have:

$$f(L) = \Delta + \left(\frac{Rl}{2L}tan(\sqrt{-\Delta}t)\right)^2$$

$$\Delta= \frac{(\mathrm{Ro} + \mathrm{Rl})^2}{4L^2} - \frac{1}{LC}$$

$$\frac{\partial\Delta}{\partial L} = \frac{1}{CL^2} - 2\frac{(\mathrm{Ro} + \mathrm{Rl})^2}{4L^3}$$

$$\mathrm{f}'(\mathrm{L}) = \frac{\partial \mathrm{f(L)}}{\partial L} = \frac{\partial\Delta}{\partial L} + 2\frac{Rl}{2L}tan(\sqrt{-\Delta}t)\left\{-\frac{Rl}{2L^2}tan(\sqrt{-\Delta}t) + \left(\frac{\partial\Delta}{\partial L}\right)\frac{Rl}{2L}\left\{1 + \left[tan(\sqrt{-\Delta}t)\right]^2\right\}\right\}$$

For “punto fisso” implementation can be used the following expression:

$$\frac{1}{LC} = \frac{(\mathrm{Ro} + \mathrm{Rl})^2}{4L^2} + \left[\frac{Rl}{2L}tan(\sqrt{-\Delta}t)\right]^2$$

$$\frac{1}{\left\{\frac{(\mathrm{Ro} + \mathrm{Rl})^2}{4L^2} + \left[\frac{Rl}{2L}tan(\sqrt{-\Delta}t)\right]^2\right\}C} = L$$

6.2.2 2° Method

Another way could be that of put in Matlab the two function:

- $T_{max} = f(L, Rl)$ (where *is(t)= I_p = 100 KA*)
- is (t) =f(L, Rl) (where t = T_{max} = 25 μs)

is(t) is obtained from the solution of the differential equation:

$$\frac{\partial^2 i}{\partial t^2} + \frac{R}{L}\frac{\partial i}{\partial t} + \frac{i}{LC} = 0$$

T_{max} is obtained by the solution of:

$$\frac{\partial is(t)}{\partial t} = 0$$

So finally the system becomes:

$$\begin{cases} T_{max} = f(L, Rl) \\ \mathrm{is}\ (\mathrm{t}) = f(L, Rl) \end{cases}$$

$$\begin{cases} t = \dfrac{\ln \dfrac{\frac{Rl}{2L} + \sqrt{\Delta}}{\frac{Rl}{2L} - \sqrt{\Delta}}}{2\sqrt{\Delta}} \\ is(t) = \dfrac{V0}{2L\sqrt{\Delta}} e^{-\frac{Rl}{2L}t} \left(e^{\sqrt{\Delta}t} - e^{-\sqrt{\Delta}t} \right) \end{cases} \tag{6.8}$$

and the intersection of the two graphics gives the correct value of *Rl* and L (the system of two equation in two unknowns can be solved analytically but is very difficult and it request the implementation with Newton Raphson's or Gauss's progression)

In fact by placing the value (*I_p = 100KA*, *T_{max} = 25 μs*) is obtained:

- $T_{max} = f(L, Rl) \to 0{,}000025 = \dfrac{\ln \dfrac{\frac{Rl}{2L} + \sqrt{\Delta}}{\frac{Rl}{2L} - \sqrt{\Delta}}}{2\sqrt{\Delta}}$
- $I_p = f(L.Rl) \to is(t) = \dfrac{V0}{2L\sqrt{\Delta}} e^{-0{,}000025\frac{Rl}{2L}} \left(e^{0{,}000025\sqrt{\Delta}} - e^{-0{,}000025\sqrt{\Delta}} \right) \to$

  $100000 = \dfrac{V0}{2L\sqrt{\Delta}} e^{-0{,}000025\frac{Rl}{2L}} \left(e^{0{,}000025\sqrt{\Delta}} - e^{-0{,}000025\sqrt{\Delta}} \right)$

This is the system in two variable *(Rl,L)* and two equation but not so easy to solve because the two equation are non linear.

For simplify the equation could be called $\left(\frac{Rl}{2L} + \sqrt{\Delta}\right) = b$ *and* $\left(\frac{Rl}{2L} - \sqrt{\Delta}\right) = \mathrm{a}$ and so you have

$2\sqrt{\Delta} = \mathrm{b} - \mathrm{a}$ and $\mathrm{a} + \mathrm{b} = \frac{Rl}{L}$ so the equations becomes:

$$100000 = \frac{V0}{2L\sqrt{\Delta}} \left(e^{-0.000025\left(\frac{Rl}{2L} - \sqrt{\Delta}\right)} - e^{-0.000025\left(\frac{Rl}{2L} + \sqrt{\Delta}\right)} \right)$$

And so you have:

$$100000 = \frac{V0}{L(b-a)} \left(e^{-0.000025(a)} - e^{-0.000025(b)} \right)$$

$$0.000025 = \frac{ln\left(\frac{b}{a}\right)}{b-a} \to 0.000025(b-a) = ln\left(\frac{b}{a}\right) \to \mathrm{e}^{0.000025(b-a)} = \left(\frac{b}{a}\right) \to$$

$$\mathrm{e}^{0.000025b} e^{-0.000025a} = \left(\frac{b}{a}\right)$$

For solving this equation is useful linearizing it with the Taylor series for exponential function $\mathrm{e}^{0.000025(a)}$ and $\mathrm{e}^{0.000025(b)}$ (for the small value of the factor above the exponential: *T1=25 μs*)

Taylor progression:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \qquad per\ ogni\ x$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots\ldots\ldots$$

$$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots\ldots\ldots$$

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} x^n \qquad per \quad |x| < 1$$

So considering the peak of current (t⟶0) and stopping at second order of progression could be written

$$\left(x = \left(\frac{Rl}{2L} + \sqrt{\Delta}\right) t\right)$$

$$e^{(-\frac{Rl}{2L}+\sqrt{\Delta})t} - e^{-(\frac{Rl}{2L}+\sqrt{\Delta})t} = 1 + \left(-\frac{Rl}{2L} + \sqrt{\Delta}\right) t - 1 + \left(\frac{Rl}{2L} + \sqrt{\Delta}\right) t = 2\sqrt{\Delta} t$$

And so:

$$is(t)\frac{2L\sqrt{\Delta}}{V0} = 2\sqrt{\Delta} t$$

$$t = is(t)\frac{L}{V0}$$

By stopping at an higher level of progression (the third order of progression) the equation becomes, considering again $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots\ldots\ldots / \, e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots\ldots\ldots$

For the equation *is(t)* =$f(L, Rl)$:

$$100000 = \frac{V0}{L(b-a)}\left(1 - 0.000025a + \frac{0.000025a^2}{2} - \left(1 - 0.000025b + \frac{0.000025b^2}{2}\right)\right) \rightarrow$$

$$100000 = \frac{V0}{L(b-a)}\left(0.000025b - 0.000025a - 0.000025^2\frac{b^2 - a^2}{2}\right) \rightarrow$$

$$100000 = \frac{V0}{L(b-a)}\left(0.000025(b-a) - 0.000025^2\frac{(b-a)(b+a)}{2}\right) \rightarrow$$

$$100000 = \frac{V0}{L(b-a)}\left((b-a)0.000025 - 0.000025^2\frac{(b-a)(b+a)}{2}\right) \rightarrow$$

$$100000 = \frac{V0}{L(b-a)}\left((b-a)\left(0.000025 - 0.000025^2\frac{(b+a)}{2}\right)\right) \rightarrow$$

$$100000 = \frac{V0}{L}\left(0.000025 - 0.000025^2\frac{(b+a)}{2}\right) \rightarrow$$

$$100000 = \frac{V0}{L}\left(0.000025 - 0.000025^2\frac{(b+a)}{2}\right) \rightarrow$$

$$100000 = \frac{V0}{L}\left(0.000025 - 0.000025^2\frac{Rl}{2L}\right) \rightarrow$$

$$2L\frac{0.000025 - 100000\frac{L}{V0}}{0.000025^2} = Rl$$

With the same linearization for the other equation *(T_{max} =*$f(L, Rl)$*)*:

$$e^{0.000025b}e^{-0.000025a} = \left(\frac{b}{a}\right)$$

Is obtained:

$$1+0.000025(b-a)=\left(\frac{b}{a}\right)\rightarrow$$

$$1+0.000025\left(2\sqrt{\Delta}\right)=\frac{\frac{Rl}{2L}+\sqrt{\Delta}}{\frac{Rl}{2L}-\sqrt{\Delta}}$$

So placing " $l=2L\frac{0.000025-100000\frac{L}{V0}}{0.000025^2}$ " (previously obtained) you have:

$$1+0.000025\left(2\sqrt{\Delta}\right)=\frac{\frac{0.000025-100000\frac{L}{V0}}{0.000025^2}+\sqrt{\Delta}}{\frac{0.000025-100000\frac{L}{V0}}{0.000025^2}-\sqrt{\Delta}}$$

Where now:

$$\Delta=\left(\frac{0.000025-100000\frac{L}{V0}}{0.000025^2}\right)^2-\frac{1}{LC}$$

By implementing of this equation with Newton Rapshon's or Gauss's implementation is found the solution for L. Putting this solution in the other equation is obtained the *Rl* value. This is what is done in the excel work paper.

"Punto Fisso" way:

$$\left\{[1+0.000025\left(2\sqrt{\Delta}\right)]\left(\frac{100000\frac{L}{V0}-0.000025}{0.000025^2}-\sqrt{\Delta}\right)\right\}-\sqrt{\Delta}=\left(\frac{100000\frac{L}{V0}-0.000025}{0.000025^2}\right)\rightarrow$$

$$\left\{\left[\left(1+0.00005\sqrt{\Delta}\right)\left(\frac{100000\frac{L}{V0}-0.000025}{0.000025^2}-\sqrt{\Delta}\right)\right]-\sqrt{\Delta}\right\}0.000025^2$$
$$=100000\frac{L}{V0}-0.000025\rightarrow$$

$$\left\{\left\{\left[\left(1+0.00005\sqrt{\Delta}\right)\left(\frac{100000\frac{L}{V0}-0.000025}{0.000025^2}-\sqrt{\Delta}\right)\right]-\sqrt{\Delta}\right\}0.000025^2+0.000025\right\}\frac{V0}{10^5}=L$$

And so could be implemented this equation.

6.3 RESULT OBTAINED WITH MATLAB AND MATLAB CODE

1° Method

For this solution are needed a code to derivate the function i(t) in time ($\frac{\partial i}{\partial t}$) this is made in Matlab's "code 2.1" (in Appendix II).

The $df=\frac{\partial i}{\partial t}$ obtained by this code is a very long expression written down:

df=(27000*exp(t*((49/(1600*L^2) - 18446744073709551616/(1230397829716427*L))^(1/2) - 7/(40*L)))*((49/(1600*L^2) - 18446744073709551616/(1230397829716427*L))^(1/2) -

7/(40*L)))/(L*(49/(1600*L^2) - 18446744073709551616/(1230397829716427*L))^(1/2)) + (27000*((49/(1600*L^2) - 18446744073709551616/(1230397829716427*L))^(1/2) + 7/(40*L)))/(L*exp(t*((49/(1600*L^2) - 18446744073709551616/(1230397829716427*L))^(1/2) + 7/(40*L)))*(49/(1600*L^2) - 18446744073709551616/(1230397829716427*L))^(1/2))

We can call this expression "F" and is the same one used in the subfunction called "basicfun" of the "code 2.2" (in Appendix II).

Letting run the main code 2.2 you arrive to the result of L = 4.40011e^{-006}

2° Method
First of all we could see graphically the solution of the system:

- $T_{max} = f(L, Rl)$ (where *is(t)= is^(t)=100KA*)
- is (t) $= f(L, Rl)$ (where $t = T_{max} = 25\,\mu s$)
-

Using Matlab "code 2.3" (in appendix II) you arrive to the following graphic:

For the function *is(t)* $= f(L, Rl)$ the graphic is: For the function $T_{max} = f(L, Rl)$ the graphic is:

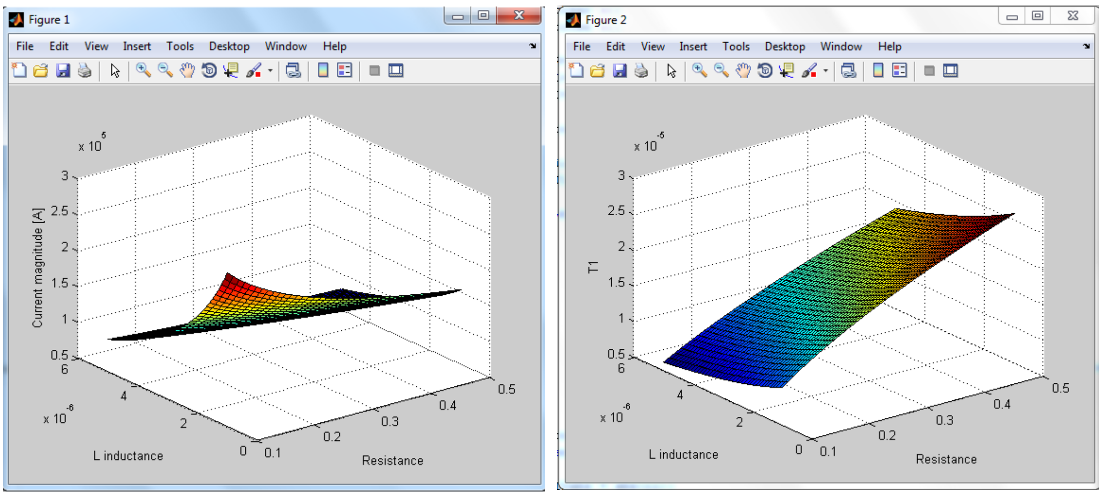


Figure 6.8 – 3D Matlab plot of *is(t)* $= f(L, Rl)$ and $T_{max} = f(L, Rl)$ functions

These are the 3-D graphics shape of the current's function and of the time's function both of them in two unknowns. Their graphic intersection give us the value of R and L.

The analytical solution is given by the "code 2.4" (in appendix II).

The solution, obtained by letting run the main program in "code 2.4", is:

$R = 0.349997\ \Omega$
$L = 4.4001\ \mu H$

So with the two methods you arrive at the same solution of the inductance "L".

The system as was already said can be simplify with the use of Taylor expression and you can arrive at the end to only one simplify equation. This implementation is maid in "code 2.5":

Letting run the main code 2.4 is obtain the value of $L = 6.14276\ \mu H$.

This value is a little different from the previous one probably for the simplifications includes in using Taylor progression.

6.4 (b) - PHYSICAL-ANALYTICAL METHOD

6.4.1 Direct methods to find R value

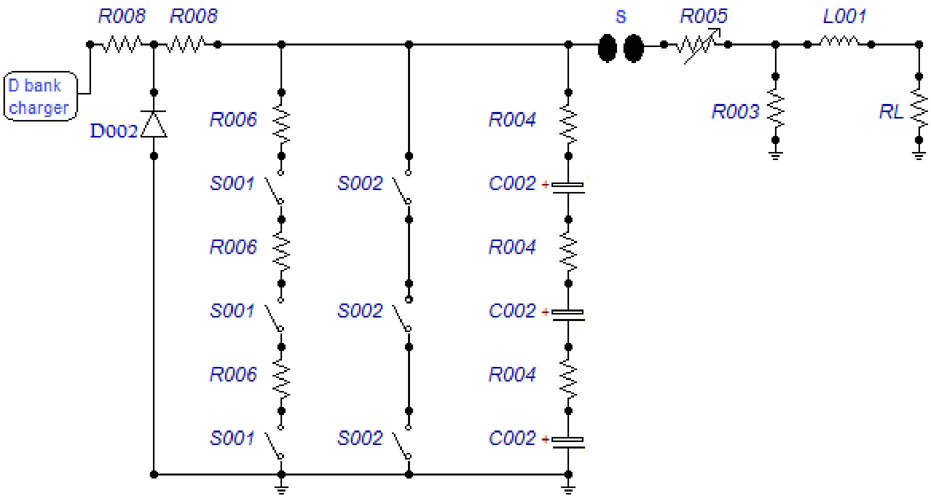


Figure 6.9 - D-bank electric scheme with evidence of the fundamental dissipative component

In this paragraph can be included a different approach used to determine the resistance of the aluminum panel. Because an ohm meter could not read its very low resistance (since aluminum is a good conductor) this value must be estimate analytically. If the assumption is made that the lightning current is discharged through the center of the aluminum panel then the current will flow uniformly towards the four edges (of length l). Hence, for simplicity, the corners can be neglected and the resistance can be approximated for a circular panel by equation (6.8) using the geometry shown in figure (6.10).

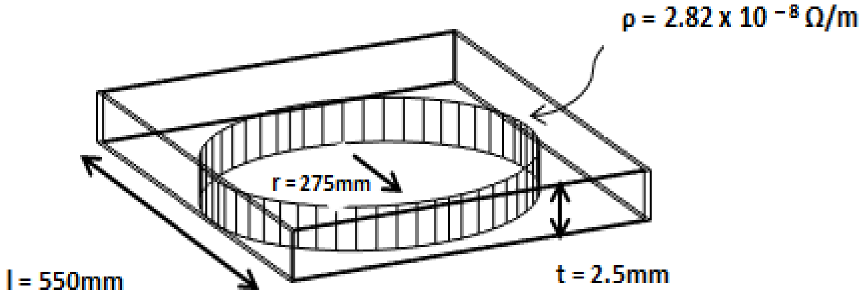


Figure 6.10 - Geometry of the aluminum panel

$$R_{al} = \frac{\rho}{\pi r^2}\, t \qquad (6.9)$$

Using equation (6.9) and the values for resistivity (ρ), radius (r) and thickness (t) of the panel from Figure 6.10, the resistance (R_{al}) was calculated to be 0.297 nΩ. The resistance is negligible and hence can be neglected.
The total resistance (R), of the D-component circuit, using the resistor values from Figure 6.9 and the carbon composite panel resistance (RL = 0.03Ω), is given as:

$R = 3 \cdot R004 + R005_{min} + RL = 3 \cdot 0{,}09375\Omega + 0{,}0625\Omega + 0{,}03\Omega = 0{,}37\ \Omega$

The total resistance (R), of the D-component circuit, using the resistor values from Figure 5-3 and neglecting the aluminum panel resistance (R_{al}), is given as:
$R = 3 \cdot R004 + R005_{min} = 3 \cdot 0{,}09375\Omega + 0{,}0625\Omega = 0{,}34\ \Omega$

[The resistance of the cables are not estimate because, in the RLC Transitory regime here considerate (where $T_{peak} = 2$ μs/$I_{peak} = 100$ kA), the voltage component linked to resistance (dissipative) is very low in comparison to that linked to the electromagnetic induction, where there is the term $\frac{\partial i}{\partial t}$ (very high considering this transitory).]

6.4.2 Direct methods to find L value

An other way to calculate the inductance can be a direct method that consist in consider the physics laws (that concerning the electromagnetic fields).
For used the electromagnetic laws are necessary the geometrical values of the wires and their positions in the circuit. These values were already measured in the laboratory and are now placed down here in a simplified outline of the real electric wires' disposition.

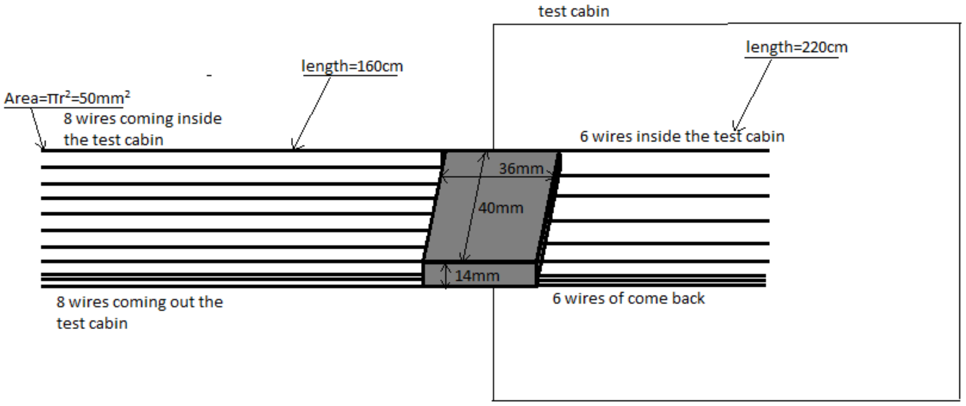


Figure 6.11 - Geometry of the eight/six wires of connection to the test cabin

The figure 6.12 is a photo of the eight wires going in and the corresponding eight wires coming out of the test room:

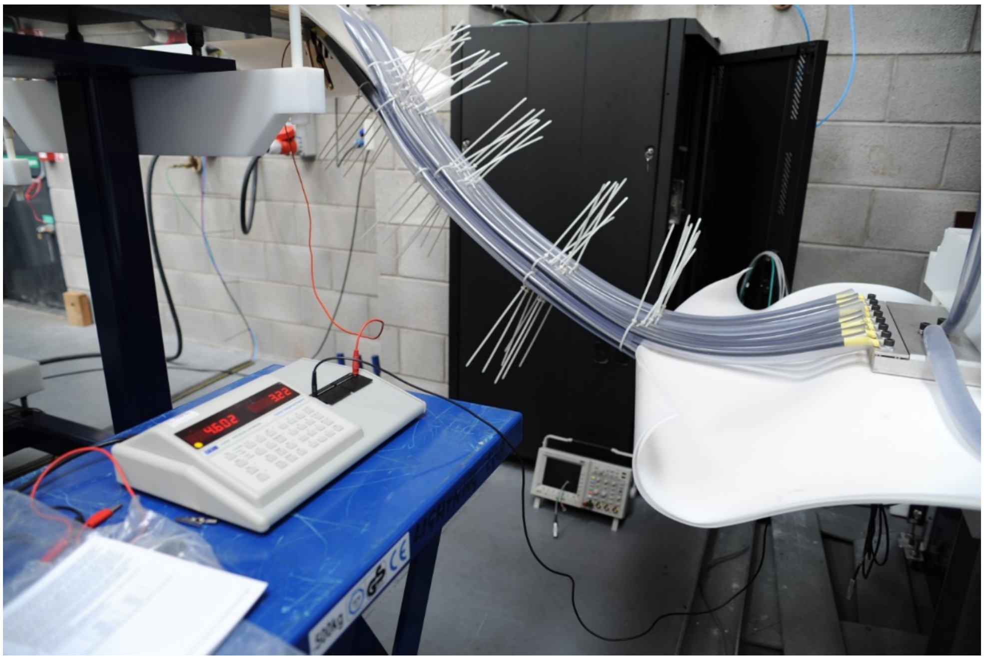


Figure 6.12 - Picture of the eight wires coming inside the test cabin

These are the fundamental geometrical values necessary for using the electromagnetic laws.

With these laws you can arrive to the conclusion that the mutual inductance of two equal parallel straight filaments is:

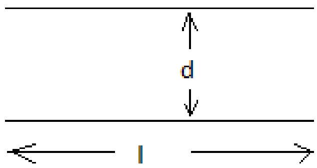


Figure 6.13 - Wires disposition for the mutual induction

$$M = 0.002l * \left[log\left(\frac{l}{d} + \sqrt{1 + \frac{l^2}{d^2}}\right) - \sqrt{1 + \frac{d^2}{l^2}} + \frac{d}{l} \right] \qquad (6.10)$$

Where “l” (in figure (6.13)) is the lengths of the filaments and “d” is their distance apart.
[For estimate the distance are necessary a lot of simplification. Therefore the final value that will be placed in the formula will be no the exact one.]
For do the estimation of the distance “d” from the center of two consequent wires is known the diameter of the outside isolation, 30mm for each wires. For the eight wires outside 30mm is the distance “d” for most of the length of the wires, because of the very close way in which are placed together these conductors for most of their length, except at the end before to enter in the test cabin where the distant “d” from each one can be assumed of 50mm. Inside the test cabin the distance of each of the six conductor is bigger at the beginning of the wires, there is in fact a distance of 50mm between each wires for two group of three conductor and a distance of 150mm between these two group.
Taking into account of these consideration can be considered a medium distance “d” of 31mm for the eight conductors outside and a medium distance “d” of 45 mm for the six conductors inside (coming from the consideration of d = 75 mm at the beginning of the wires (on average for each one) and d=30mm for all the extension of the remaining wires).
Figure (6.14) and (6.15) are pictures of the wires that show better their real geometrical disposition:

For the eight wires outside the test cabin:

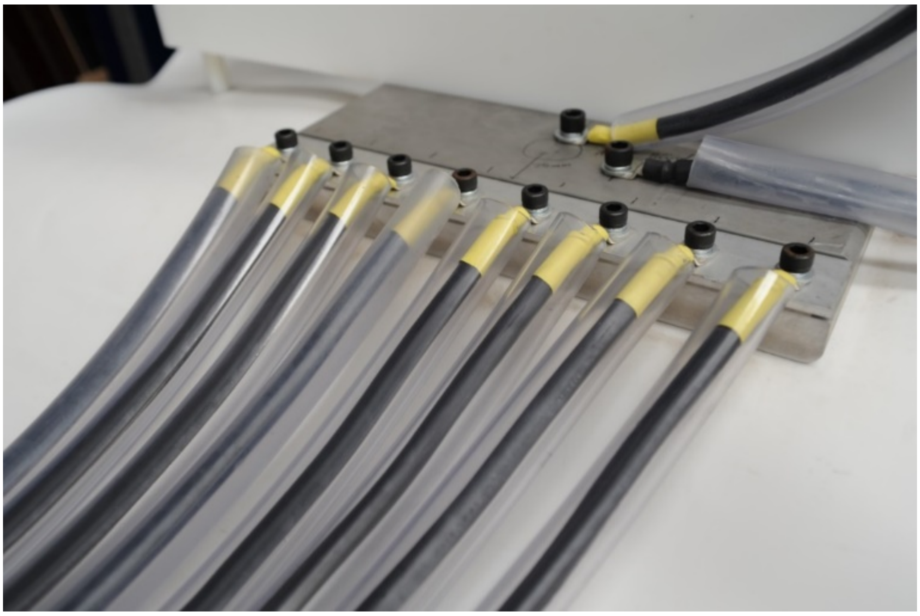


Figure 6.14 - Picture of the eight wires outside the test cabin

For the six wires inside the test cabin:

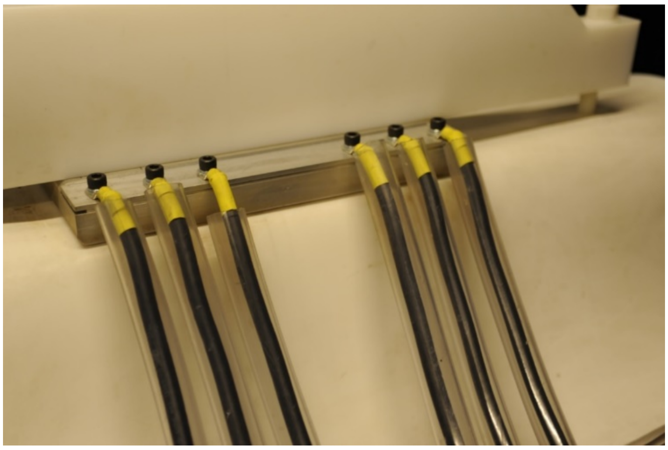


Figure 6.15 - Picture of the six wires inside the test cabin

For the self-inductance of straight conductor for a round wire of radius "r" you have the following expression (neglecting the last term that consider the permeability μ):

$$L = 0{,}002l \cdot \left[log\left(\frac{2l}{r} - \frac{3}{4}\right)\right] \tag{6.11}$$

Where "l" is the length of the conductor.
Using these formulas for the eight conductor that are dispose like in figure (6.16)

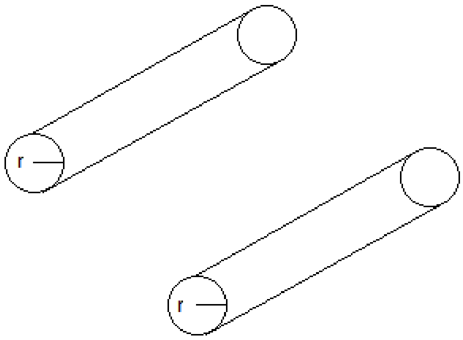


Figure 6.16 - Disposition of the wires

For the reason that the wires are very close each other can be considered the distance "d" as d=2*r.
Using the electromagnetic law for the two wire you have (applying the (6.11) and the (6.10) for the eight wires):

$$\phi 1 = (L * i1) + (M * i2) \tag{6.12}$$

(where ϕ is the electromagnetic flux, i1 is the current of the wire 1 and i2 is the current of the wire 2).
For the eight wires outside the test cabin you have (applying the (6.11) for the eight wires):

$$\begin{cases} \Phi\, 1 = L * I1 + M * I2 + M * I3 + M * I4 + M * I5 + M * I6 + M * I7 + M * I8 \\ \phi\, 2 = M * I1 + L * I2 + M * I3 + M * I4 + M * I5 + M * I6 + M * I7 + M * I8 \\ \phi\, 3 = M * I1 + M * I2 + L * I3 + M * I4 + M * I5 + M * I6 + M * I7 + M * I8 \\ \phi\, 4 = M * I1 + M * I2 + M * I3 + L * I4 + M * I5 + M * I6 + M * I7 + M * I8 \\ \phi\, 5 = M * I1 + M * I2 + M * I3 + M * I4 + L * I5 + M * I6 + M * I7 + M * I8 \\ \phi\, 6 = M * I1 + M * I2 + M * I3 + M * I4 + M * I5 + L * I6 + M * I7 + M * I8 \\ \phi\, 7 = M * I1 + M * I2 + M * I3 + M * I4 + M * I5 + M * I6 + L * I7 + M * I8 \\ \phi\, 8 = M * I1 + M * I2 + M * I3 + M * I4 + M * I5 + M * I6 + M * I7 + L * I8 \end{cases} \tag{6.13}$$

If this system (6.12) is written in a compact way is obtained a simple vector equation:

$$\begin{pmatrix} \phi 1 \\ \vdots \\ \phi 8 \end{pmatrix} = \begin{bmatrix} L & \cdots & M \\ \vdots & \ddots & \vdots \\ M & \cdots & L \end{bmatrix} * \begin{pmatrix} \mathrm{I}1 \\ \vdots \\ I8 \end{pmatrix} \tag{6.14}$$

$$(\Phi) = [L] * (I) \tag{6.15}$$

Where $[L]$ is:

$$[L] = \begin{bmatrix} L\ M_{12}\ M_{13}\ M_{14}\ M_{15}\ M_{16}\ M_{17}\ M_{18} \\ M_{12}\ L\ \ M_{12}\ M_{13}\ M_{14}\ M_{15}\ M_{16}\ M_{17} \\ M_{13}\ M_{12}\ L\ \ M_{12}\ M_{13}\ M_{14}\ M_{15}\ M_{16} \\ M_{14}\ M_{13}\ M_{12}\ L\ M_{12}\ M_{13}\ M_{14}\ M_{15} \\ M_{15}\ M_{14}\ M_{13}\ M_{12}\ L\ \ M_{12}\ M_{13}\ M_{14} \\ M_{16}\ M_{15}\ M_{14}\ M_{13}\ M_{12}\ L\ \ M_{12}\ M_{13} \\ M_{17}\ M_{16}\ M_{15}\ M14\ M_{13}\ M_{12}\ L\ \ M_{12} \\ M_{18}\ M_{17}\ M_{16}\ M_{15}\ M_{14}\ M_{13}\ M_{12}\ L \end{bmatrix}$$

For the Faraday law can be written:

$$(V) = \frac{\partial(\phi)}{\partial t} = [L]\frac{\partial(\mathrm{I})}{\partial t} \tag{6.16}$$

The eight conductor are disposed in parallel in fact they have the same tension at their extremes and so you have a system of eight equation as the following:

$$\begin{cases} v = L * \frac{\partial I1}{\partial t} + M12 * \frac{\partial I2}{\partial t} + M13 * \frac{\partial I3}{\partial t} + M14 * \frac{\partial I4}{\partial t} + M15 * \frac{\partial I5}{\partial t} + M16 * \frac{\partial I6}{\partial t} + M17 * \frac{\partial I7}{\partial t} + M18 * \frac{\partial I8}{\partial t} \\ v = M12 * \frac{\partial I1}{\partial t} + L * \frac{\partial I2}{\partial t} + M12 * \frac{\partial I3}{\partial t} + M13 * \frac{\partial I4}{\partial t} + M14 * \frac{\partial I5}{\partial t} + M15 * \frac{\partial I6}{\partial t} + M16 * \frac{\partial I7}{\partial t} + M17 * \frac{\partial I8}{\partial t} \\ v = M13 * \frac{\partial I1}{\partial t} + M12 * \frac{\partial I2}{\partial t} + L * \frac{\partial I3}{\partial t} + M12 * \frac{\partial I4}{\partial t} + M13 * \frac{\partial I5}{\partial t} + M14 * \frac{\partial I6}{\partial t} + M15 * \frac{\partial I7}{\partial t} + M16 * \frac{\partial I8}{\partial t} \\ v = M14 * \frac{\partial I1}{\partial t} + M13 * \frac{\partial I2}{\partial t} + M12 * \frac{\partial I3}{\partial t} + L * \frac{\partial I4}{\partial t} + M12 * \frac{\partial I5}{\partial t} + M13 * \frac{\partial I6}{\partial t} + M14 * \frac{\partial I7}{\partial t} + M15 * \frac{\partial I8}{\partial t} \\ v = M15 * \frac{\partial I1}{\partial t} + M14 * \frac{\partial I2}{\partial t} + M13 * \frac{\partial I3}{\partial t} + M12 * \frac{\partial I4}{\partial t} + L * \frac{\partial I5}{\partial t} + M12 * \frac{\partial I6}{\partial t} + M13 * \frac{\partial I7}{\partial t} + M14 * \frac{\partial I8}{\partial t} \\ v = M16 * \frac{\partial I1}{\partial t} + M15 * \frac{\partial I2}{\partial t} + M14 * \frac{\partial I3}{\partial t} + M13 * \frac{\partial I4}{\partial t} + M12 * \frac{\partial I5}{\partial t} + L * \frac{\partial I6}{\partial t} + M12 * \frac{\partial I7}{\partial t} + M13 * \frac{\partial I8}{\partial t} \\ v = M17 * \frac{\partial I1}{\partial t} + M16 * \frac{\partial I2}{\partial t} + M15 * \frac{\partial I3}{\partial t} + M14 * \frac{\partial I4}{\partial t} + M13 * \frac{\partial I5}{\partial t} + M12 * \frac{\partial I6}{\partial t} + L * \frac{\partial I7}{\partial t} + M12 * \frac{\partial I8}{\partial t} \\ v = M18 * \frac{\partial I1}{\partial t} + M17 * \frac{\partial I2}{\partial t} + M16 * \frac{\partial I3}{\partial t} + M15 * \frac{\partial I4}{\partial t} + M14 * \frac{\partial I5}{\partial t} + M13 * \frac{\partial I6}{\partial t} + M12 * \frac{\partial I7}{\partial t} + L * \frac{\partial I8}{\partial t} \end{cases} \tag{6.17}$$

Also this system (6.16) can be written in a compact way:

$$\begin{bmatrix} V \\ V \\ V \\ V \\ V \\ V \\ V \\ V \end{bmatrix} = \begin{bmatrix} L & M_{12} & M_{13} & M_{14} & M_{15} & M_{16} & M_{17} & M_{18} \\ M_{12} & L & M_{12} & M_{13} & M_{14} & M_{15} & M_{16} & M_{17} \\ M_{13} & M_{12} & L & M_{12} & M_{13} & M_{14} & M_{15} & M_{16} \\ M_{14} & M_{13} & M_{12} & L & M_{12} & M_{13} & M_{14} & M_{15} \\ M_{15} & M_{14} & M_{13} & M_{12} & L & M_{12} & M_{13} & M_{14} \\ M_{16} & M_{15} & M_{14} & M_{13} & M_{12} & L & M_{12} & M_{13} \\ M_{17} & M_{16} & M_{15} & M14 & M_{13} & M_{12} & L & M_{12} \\ M_{18} & M_{17} & M_{16} & M_{15} & M_{14} & M_{13} & M_{12} & L \end{bmatrix} * \begin{bmatrix} \frac{\partial \mathrm{I1}}{\partial t} \\ \frac{\partial \mathrm{I2}}{\partial t} \\ \frac{\partial \mathrm{I3}}{\partial t} \\ \frac{\partial \mathrm{I4}}{\partial t} \\ \frac{\partial \mathrm{I5}}{\partial t} \\ \frac{\partial \mathrm{I6}}{\partial t} \\ \frac{\partial \mathrm{I7}}{\partial t} \\ \frac{\partial \mathrm{I8}}{\partial t} \end{bmatrix} \tag{6.18}$$

Considering the system (6.17 - 6.18) you can write an equivalent circuit like below (figure 6.17) where there are two inductances (one that considered the self-inductance and the other that considered the mutual inductance):

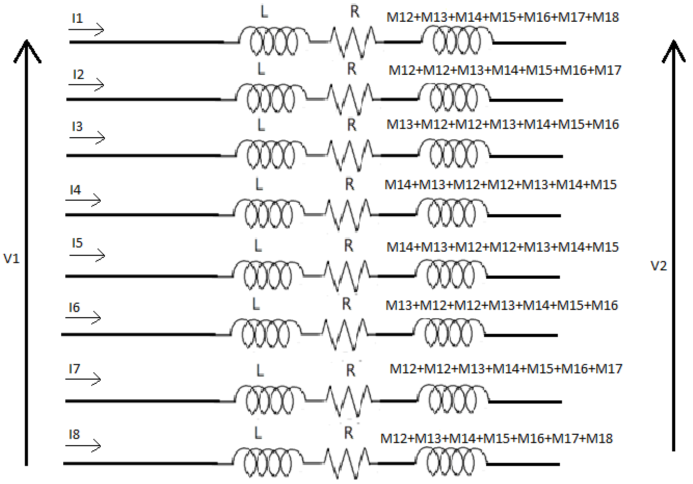


Figure 6.17 - Simplify equivalent electric scheme for the eight wires taken from the equation (6.17-6.18)

With this simplified electrical scheme, where all the potential on the left is at V1 tension value and all the potential on the right is at V2 tension value, can be written the following equation:

$$\frac{V}{Ltot} = \frac{\partial \mathrm{I1}}{\partial t} + \frac{\partial \mathrm{I2}}{\partial t} + \cdots + \frac{\partial \mathrm{I8}}{\partial t} = \frac{\partial (\mathrm{I1} + \mathrm{I2} + \cdots + \mathrm{I8})}{\partial t} = \frac{\partial \mathrm{Itot}}{\partial t} \tag{6.19}$$

[Where is neglected the tension's terms refers to the resistance "V = RI", dissipative terms. This because the resistance of each cable is very low (R≈0), considering a length of (d = 3,8 m). This is even more valuable considering that are treated extremely fast electrical transients (T_{peak} = 2 μs/I_{peak} = 100 kA), where the derivative of the current $\frac{\partial I}{\partial t} \gg 1$ and therefore the induced voltage assumes a decisive and relevant value. This is like to consider the cables as purely inductive elements]

And for each wire you have:

$$V = Ltot1\frac{\partial \mathrm{I1}}{\partial t} = Ltot2\frac{\partial \mathrm{I2}}{\partial t} = Ltot3\frac{\partial \mathrm{I3}}{\partial t} = Ltot4\frac{\partial \mathrm{I4}}{\partial t} = Ltot5\frac{\partial \mathrm{I5}}{\partial t} = Ltot6\frac{\partial \mathrm{I6}}{\partial t} = Ltot7\frac{\partial \mathrm{I7}}{\partial t} = Ltot8\frac{\partial \mathrm{I8}}{\partial t}$$

Where:

$$Ltot8 = Ltot1 = L + M12 + M13 + M14 + M15 + M16 + M17 + M18$$

$$Ltot2 = Ltot7 = L + M12 + M12 + M13 + M14 + M15 + M16 + M17$$

$$Ltot3 = Ltot6 = L + M12 + M12 + M13 + M13 + M14 + M15 + M16$$

$$Ltot4 = Ltot5 = L + M12 + M12 + M13 + M13 + M14 + M14 + M15$$

So you have for each wires:

$$\frac{V}{Ltot1} = \frac{\partial I1}{\partial t}; \quad \frac{V}{Ltot2} = \frac{\partial I2}{\partial t}; \quad \frac{V}{Ltot3} = \frac{\partial I3}{\partial t}; \quad \frac{V}{Ltot4} = \frac{\partial I4}{\partial t}$$

$$\frac{V}{Ltot5} = \frac{\partial I5}{\partial t}; \quad \frac{V}{Ltot6} = \frac{\partial I6}{\partial t}; \quad \frac{V}{Ltot7} = \frac{\partial I7}{\partial t}; \quad \frac{V}{Ltot8} = \frac{\partial I8}{\partial t}$$

and so is obtained:

$$\frac{V}{Ltot} = \frac{V}{Ltot1} + \frac{V}{Ltot2} + \frac{V}{Ltot3} + \frac{V}{Ltot4} + \frac{V}{Ltot5} + \frac{V}{Ltot6} + \frac{V}{Ltot7} + \frac{V}{Ltot8} \tag{6.20}$$

$$\frac{1}{Ltot} = \frac{1}{Ltot1} + \frac{1}{Ltot2} + \frac{1}{Ltot3} + \frac{1}{Ltot4} + \frac{1}{Ltot5} + \frac{1}{Ltot6} + \frac{1}{Ltot7} + \frac{1}{Ltot8}$$

At the same conclusion you can arrive with the solution of the system (6.16) rewritten here:

$$(V) = [L]\frac{\partial(\mathrm{I})}{\partial t} \tag{6.21}$$

By solving the (6.21) you have:

$$(\mathrm{X}) = \frac{\partial(\mathrm{I})}{\partial t} = (V) * inv([L]) \tag{6.22}$$

And this vector of solution (X) can be placed in the equation (6.19):

$$\mathrm{X}(1,1) + \mathrm{X}(1,2) + \mathrm{X}(1,3) + \mathrm{X}(1,4) + \mathrm{X}(1,5) + \mathrm{X}(1,6) + \mathrm{X}(1,7) + \mathrm{X}(1,8) = \frac{V}{Ltot}$$

And so you find the L_{tot} value.

In the same way, for the six wires inside the test cabin, you have:

$$\begin{bmatrix} V \\ V \\ V \\ V \\ V \\ V \end{bmatrix} = \begin{bmatrix} L & M_{12} & M_{13} & M_{14} & M_{15} & M_{16} \\ M_{12} & L & M_{12} & M_{13} & M_{14} & M_{15} \\ M_{13} & M_{12} & L & M_{12} & M_{13} & M_{14} \\ M_{14} & M_{13} & M_{12} & L & M_{12} & M_{13} \\ M_{15} & M_{14} & M_{13} & M_{12} & L & M_{12} \\ M_{16} & M_{15} & M_{14} & M_{13} & M_{12} & L \end{bmatrix} * \begin{bmatrix} \frac{\partial I1}{\partial t} \\ \frac{\partial I2}{\partial t} \\ \frac{\partial I3}{\partial t} \\ \frac{\partial I4}{\partial t} \\ \frac{\partial I5}{\partial t} \\ \frac{\partial I6}{\partial t} \end{bmatrix} \tag{6.23}$$

So, as previously, from the system (6.23) you have a simplified outline (figure (6.18)) of the six wires:

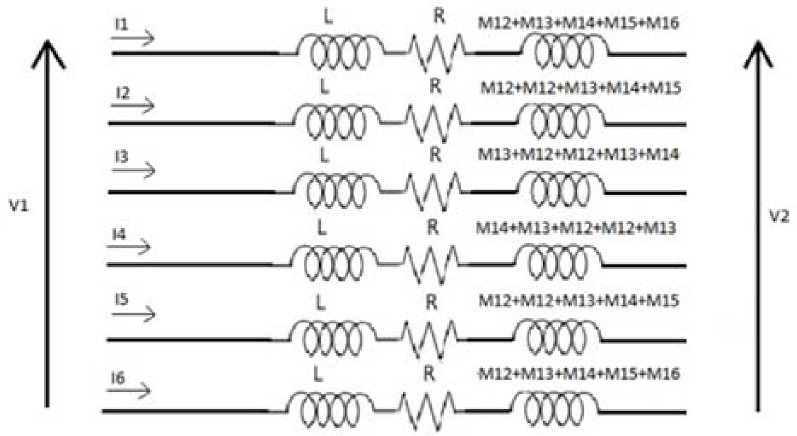


Figure 6.18 - Simplify scheme of the six wires that came from the equation (6.22)

So, from the circuit drawn above, can be written the following expression (with the same consideration of neglecting the resistance's contribution):

$$\frac{V}{Ltot} = \frac{\partial \mathrm{I1}}{\partial t} + \frac{\partial \mathrm{I2}}{\partial t} + \frac{\partial \mathrm{I3}}{\partial t} + \frac{\partial \mathrm{I4}}{\partial t} + \frac{\partial \mathrm{I5}}{\partial t} + \frac{\partial \mathrm{I6}}{\partial t} = \frac{\partial(\mathrm{I1} + \mathrm{I2} + \mathrm{I3} + \mathrm{I4} + \mathrm{I5} + \mathrm{I6})}{\partial t} = \frac{\partial \mathrm{Itot}}{\partial t} \qquad (6.24)$$

For each wire:

$$V = Ltot1\frac{\partial \mathrm{I1}}{\partial t} = Ltot2\frac{\partial \mathrm{I2}}{\partial t} = Ltot3\frac{\partial \mathrm{I3}}{\partial t} = Ltot4\frac{\partial \mathrm{I4}}{\partial t} = Ltot5\frac{\partial \mathrm{I5}}{\partial t} = Ltot6\frac{\partial \mathrm{I6}}{\partial t}$$

Where:

$Ltot6 = Ltot1 = L + M12 + M13 + M14 + M15 + M16$
$Ltot2 = Ltot5 = L + M12 + M12 + M13 + M14 + M15$
$Ltot3 = Ltot4 = L + M12 + M12 + M13 + M13 + M14$

$$\frac{V}{Ltot1} = \frac{\partial \mathrm{I1}}{\partial t}; \quad \frac{V}{Ltot2} = \frac{\partial \mathrm{I2}}{\partial t}; \quad \frac{V}{Ltot3} = \frac{\partial \mathrm{I3}}{\partial t}; \quad \frac{V}{Ltot4} = \frac{\partial \mathrm{I4}}{\partial t}; \quad \frac{V}{Ltot5} = \frac{\partial \mathrm{I5}}{\partial t}; \quad \frac{V}{Ltot6} = \frac{\partial \mathrm{I6}}{\partial t}$$

and so is obtained:

$$\frac{V}{Ltot} = \frac{V}{Ltot1} + \frac{V}{Ltot2} + \frac{V}{Ltot3} + \frac{V}{Ltot4} + \frac{V}{Ltot5} + \frac{V}{Ltot6}$$

$$\frac{1}{Ltot} = \frac{1}{Ltot1} + \frac{1}{Ltot2} + \frac{1}{Ltot3} + \frac{1}{Ltot4} + \frac{1}{Ltot5} + \frac{1}{Ltot6}$$

As was done for the eight wires, you can also solve, with the matlab help, the equation (6.15) now applied for the six wires, inside the test cabin:

$$(\mathrm{V}) = [L]\frac{\partial(\mathrm{I})}{\partial t}$$

And placing in the equation below (6.24), the vector of solution $(X) = \frac{\partial(\mathrm{I})}{\partial t}$:

$$\mathrm{X}(1,1) + \mathrm{X}(1,2) + \mathrm{X}(1,3) + \mathrm{X}(1,4) + \mathrm{X}(1,5) + \mathrm{X}(1,6) + \mathrm{X}(1,7) + \mathrm{X}(1,8) = \frac{V}{Ltot}$$

The value of the mutual induction, caused by the current of coming back that crosses the wires (in the other side), can be estimated in the following way.
First of all can be estimated the distance D, as you can see from the figure 6.19, by the Pythagoras's Theorem $D = \sqrt{d1^2 + d2^2}$ and (calling all these mutual inductions, valid for the first wire "1", m11,

m12, m13, m14, m15, m16, m17, m18) is obtained a negative contribute by the other self-inductance and mutual inductance, because of the side of the current. The same can be made also for the other wires "2, 3, 4, …"
So now the terms "L, M12, M13, …." are considering also the mutual inductance in the wires "1, 2, 3, 4, …" generated by the coming back current that crosses the first, the second, the third,... wire and so are the values written in the new system (6.25) (different for this reason from the system (6.18):

$$\begin{bmatrix} V \\ V \\ V \\ V \\ V \\ V \\ V \\ V \end{bmatrix} = \begin{bmatrix} L\ M12\ M13\ M14\ M15\ M16\ M17\ M18 \\ M12\ L\ \ M12\ M13\ M14\ M15\ M16\ M17 \\ M13\ M12\ L\ \ M12\ M13\ M14\ M15\ M16 \\ M14\ M13\ M12\ L\ M12\ M13\ M14\ M15 \\ M15\ M14\ M13\ M12\ L\ \ M12\ M13\ M14 \\ M16\ M15\ M14\ M13\ M12\ L\ \ M12\ M13 \\ M17\ M16\ M15\ M14\ M13\ M12\ L\ \ M12 \\ M18\ M17\ M16\ M15\ M14\ M13\ M12\ L \end{bmatrix} * \begin{bmatrix} \frac{\partial I1}{\partial t} \\ \frac{\partial I2}{\partial t} \\ \frac{\partial I3}{\partial t} \\ \frac{\partial I4}{\partial t} \\ \frac{\partial I5}{\partial t} \\ \frac{\partial I6}{\partial t} \\ \frac{\partial I7}{\partial t} \\ \frac{\partial I8}{\partial t} \end{bmatrix} \tag{6.25}$$

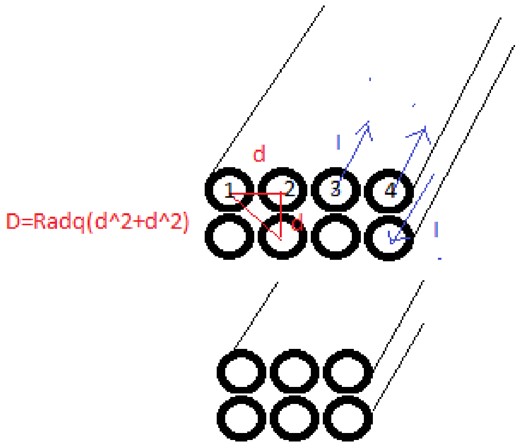


Figure 6.19 - Geometrical representation of the eight wires

By doing these operations will be found a system with a small difference from the previous system. This can be explained because in total the values of "m11, m12, m13, m14, m15, m16, m17, m18" are compensate with the correspondent mutual inductance values of the other wires "2, 3, 4, …5, 6, 7, 8" and overall there isn't any new contribution to the total inductance. This is easier to understand if we consider the Ampere's Law (Law circuitry) where is known that $\oint_{\partial s} H * dr = Ii$ and where "Ii" is the current that concatenated the surface "S" (figure 6-15) you have:

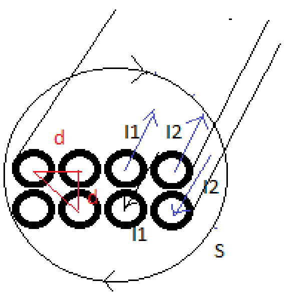


Figure 6.20 - Ampere Low apply for the eight wires

And the Ampere Law becomes $\oint_{\partial s} H * dr = \sum_{i=1}^{8} I\, i$

$$\sum_{i=1}^{8} I\, i = I1 - I1 + I2 - I2 \ldots\ldots = 0 \tag{6.26}$$

And for the zero total value of current in (6.26) is known that $\oint_{\partial s} H * dr = \sum_{i=1}^{8} I\, i = 0$ and so the magnetic field is $B = \mu H = 0$ so isn't necessary to consider the coming back's current in the calculation of the total inductance.

6.4.3 Result obtained by the use of Matlab code and excel file work:

For find the solution in excel can be followed two way:

a) This way use “code 3.1” (in appendix III) for find the solution of the system V= $\frac{\partial(\phi)}{\partial t}$=$[L]\,\frac{\partial(\mathrm{I})}{\partial t}$

The code 3.1 refers to the 8 wire before the test cabin.
This code 3.2 gives us ztot= 72057594037927936/7690855272850109=0,93692562766 (that consider the distance “d” in “cm” measurement’s unit), the same ztot used in the excel file.
The “code 3.2” (in appendix III) is refers to the 6 wire in the test cabin is:
This code 3.2 gives us *ztot*= 72057594037927936/4918576186372705=1,465009208103131(ztot= 72057594037927936/7690855272850109=0,93692562766 that consider the distance “d” in “cm” measurement’s unit), the same *ztot* used in the excel file.

b) This way is easier because use only an equivalent circuit obtained by the equation of the system.
The excel file with the final solution is:

Area	0,5	cm^2	Mutual inductance for the 8 wires				Mutual inductance for the 6 wires:			Inductance of		Inductance of	
N1	8		distance(cm)	Mutual inductance			distance(cm)	Mutual inductance		the 8 wires:		the 6 wires:	
N2	8		3,1	1,169984	M12		4,5	1,585341	M12	L1		L2	
Ntot	16		6,2	0,954287	M13		9	1,289218	M13	1,899923		2,752514	
n1	6		9,3	0,830588	M14		13,5	1,119583	M14	Total inducance obtained considering			
n2	6		12,4	0,74452	M15		18	1,001682	M15	neglectable the mutual inductance			
ntot	12		15,5	0,679044	M16		22,5	0,912085	M16	value:			
L	160	cm	18,6	0,626572	M17					L1parallelo		L2parallelo	
l	220	cm	21,7	0,583055	M18					0,2374904		0,458752	
r	0,39894228	cm								Ltot	1,3924854		
Solution made by solving the semplify cicuit:				Solution made by Matlab (solving the system V=[L] * ∂(I)/∂t:									
8 wires:				Impedence for the 8 wires									
Ltot8=Ltot1	7,487971897			ztot	9,369256	(mm)							
Ltot2=Ltot7	8,074901283			0,936926	(cm)								
Ltot3=Ltot6	8,40261642			Impedence for the 6 wires									
Ltot4=Ltot5	8,554160355			ztot2	14,65009	(mm)							
Z	0,986601527			1,465009	(cm)								
ztot	1,01358043	(cm)											
6 wires:				ztotale	4,80387	μH							
Ltot6=Ltot1	8,660422693												
Ltot2=Ltot5	9,333678424												
Ltot3=Ltot4	9,62121488												
Z	0,653087358												
ztot	1,531188726	(cm)											
ztotale	5,089538313	μH											

Where the “Ztotale” is the L value that is wanted by the two way (the upper one by matlab way “Z = 4,8039 μH”, the down one by the equivalent circuit solution “Z = 5,08953 μH”)

6.5 (c) - MEASUREMENT METHOD

The last way to estimate the inductance and the resistance is the measurement of them with a particular instrument, whose description will be given down here.
This instrument works by introducing inside the circuit a current in different level of frequency. This circuit (the D-bank’s circuit), that before was grounded for download any eventual current (so for the security of the people who are working around), is not charged as when is working for testing material. Therefore the cables’ end (on which the measurements are made) have the potential floating. So now the only current that go through the circuit is the current given by the instrument. In the laboratory were

made many measurement with each of the three kind of frequency (100/120Hz-1kHz-10kHz) but a quick vision of the result shows that only the two frequency, 1kHz and 10 kHz, are acceptable for their result. Therefore here are considered only this two frequency and greater attention should be given to the 10 kHz frequency (maybe the best one, considering the result very near compared to what is expected). Infact this is the highest frequency, the nearest to the one which is probably required to work in conditions close to those in here you have the phenomenon of lightning $\left(\mathrm{t_{peak}} = 2\mu s\ and\ \Delta\mathrm{t_{sampling}} = 0{,}02\mu s \Rightarrow \frac{1}{\mathrm{t_{sampling}}} = f_{sampling} = 50MHz\right)$.

Is given more importance to the higher frequency because, even if the d-bank is loaded by a generator at the voltage of 60 kV and at the frequency of 50Hz (the same for the mains), one is focusing on the inductance's value and on the resistance's value in the current peak's time (*T = 25 μs*), the time in which is reached the peak of current (I_{peak} = *100 kA*)). The final shape of the current is a periodic exponential damped and for it can be considered f$=\frac{1}{T} = \frac{1}{0{,}000025} = 40000Hz$.[Really the pulsation "damping constant" is of $\omega_0 = 2\pi f = \frac{1}{\sqrt{LC}}$ for a RLC transitory circuit and so with around L = 4,4 μH and C = 66,667 μF there is for sure a low value of "damping constant" and a frequency of $\omega_0 = 2\pi f = \frac{1}{\sqrt{LC}} = 58385{,}96\ [rad/s]$. Therefore the frequency can be assumed as $f = \frac{\omega_0}{2\pi} = 9292{,}414\ [Hz]$. By these considerations is necessary to consider between the frequencies "100/120Hz-1kHz-10kHz" only the highest one.

6.5.1 Measurement

Measurement along the whole extension of the cables (including parts on the outside of the testing cabin and on the inside of the testing cabin):
The figures (6.21) are the pictures that show the position of the contact used for the measure:

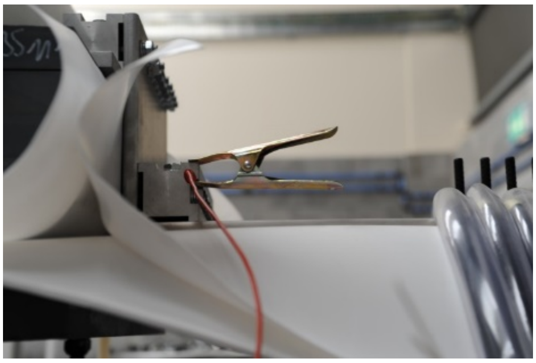


Figure 6.21 - Pictures of the contact used for measure

Table 6.2 - Resistance's and inductance's values for each current's frequency applied to the power line (whole extension of the cables)

Frequency "f"	Resistance "R" (Ω)	Inductance "L" (μH)
100/120 Hz	0.107	16
1kHz	0.1854	9
10kHz	0.2236	6.013

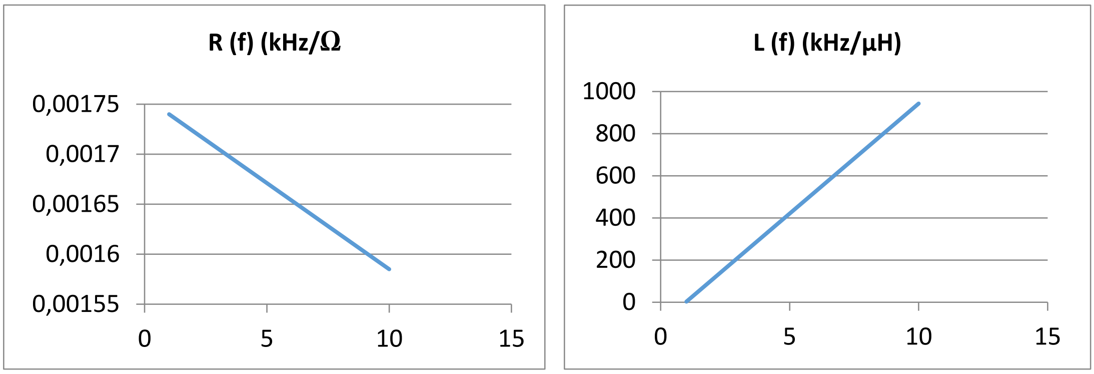
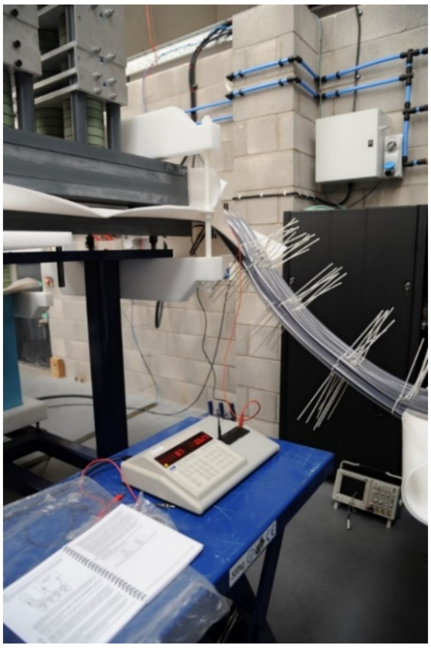


Figure 6.22 – Graphic representation of resistance’s and inductance’s values in function of the current’s frequency applied to the power line (whole extension of the cables)

The pictures (6.23) show the instrument for the measurement along the whole extension of the cables:



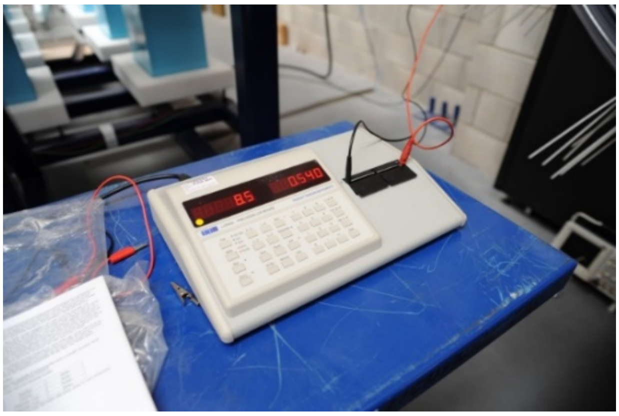


Figure 6.23 - Picture of the instrument used for the measurement of the eight wires.

Measure along the cables, inside the test cabin

The collocation of the instrument’s contacts is shown in the picture (6.24):

Figure 6.24 - Pictures of the contact of the instrument

Table 6.3 - Resistance’s and inductance’s values for each current’s frequency applied to the power line (inside the test cabin)

Frequency “f”	Resistance “R” (Ω)	Inductance “L” (μH)
100/120 Hz		
1kHz	0.0854	6.45
10kHz	0.1038	5.345

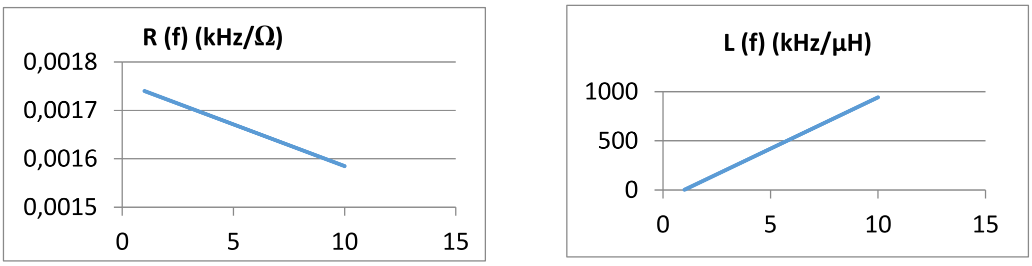


Figure 6.25 - Graphic representation of resistance’s and inductance’s values in function of the current’s frequency applied to the power line (inside the test cabin)

Measure along the cables, outside the test cabin
1kHz:

Table 6.4 - Resistance’s and inductance’s values for each current’s frequency applied to the power line (outside the test cabin)

Frequency “f”	Resistance “R” (Ω)	Inductance “L” (μH)
100/120 Hz	0.0820	5.8
1kHz	0.0823	4.82
10kHz	0.0639	4.589

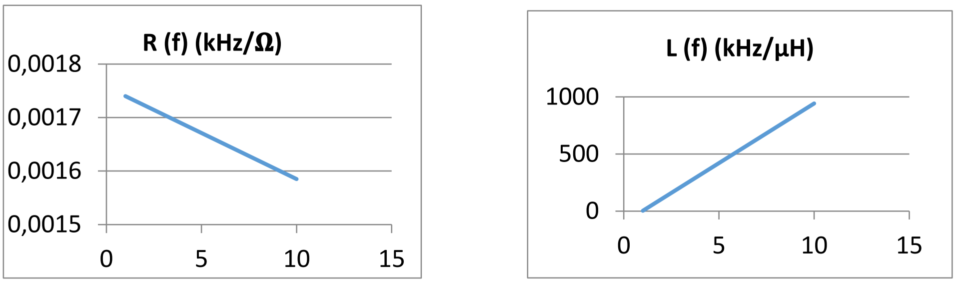


Figure 6.26 – Graphic representation of resistance’s and inductance’s values in function of the current’s frequency applied to the power line (outside the test cabin)

Measurement along the cables of coming back, outside the test cabin
The collocation of the instrument’s contacts is shown in the picture (6.18):

Figure 6.27 - Picture of the instrument’s contact for the comeback wires outside the test cabin

Table 6.5 - Resistance's and inductance's values for each current's frequency applied to the power line (cables of coming back, outside the test cabin)

Frequency "f"	Resistance "R" (Ω)	Inductance "L" (μH)
100/120 Hz		
1kHz	0.0823	4.9
10kHz	0.0891	4.603

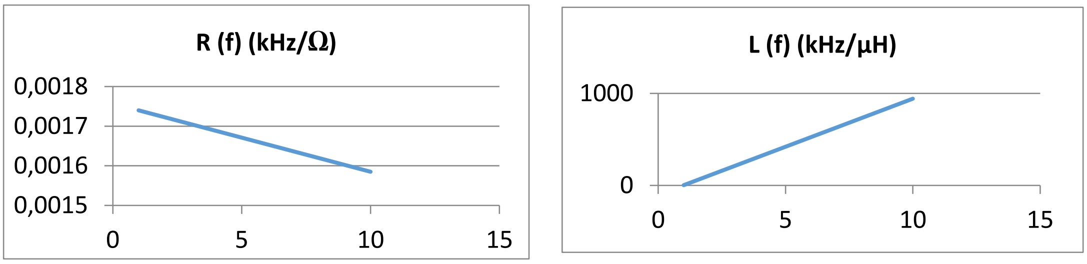


Figure 6.28 – Graphic representation of Resistance's and inductance's values in function of the current's frequency applied to the power line (cables of coming back, outside the test cabin)

In the figure (6.29) are show the wires of coming back, outside the test cabin.
An other measure taken was the one about resistance's components before these cables, where there is the connection with the switch for charging the circuit and with the switch for grounding the circuit.
Here (figure 6.24) there is a picture of the instrument contact's position:

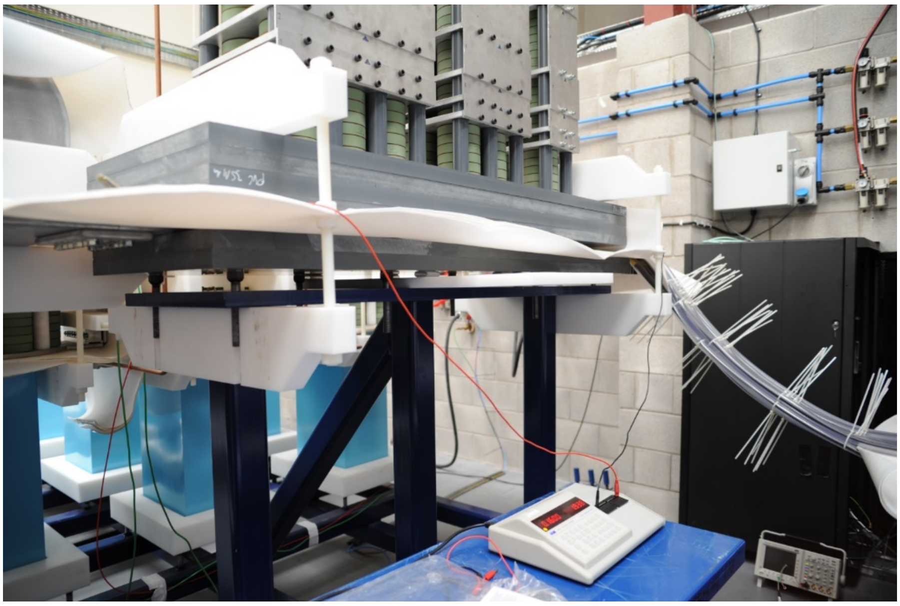
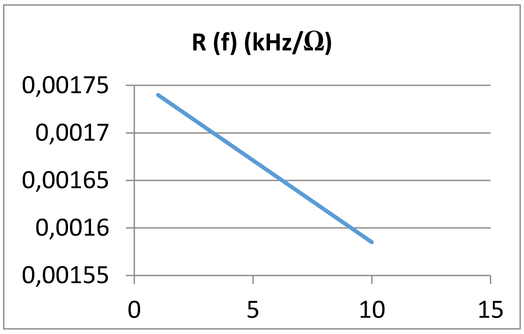


Figure 6.29 - Picture of the instrument contact position

Table 6.6 - Resistance's and inductance's values for each current's frequency applied to the power line (resistance's components before the cables)

Frequency "f"	Resistance "R" (Ω)	Inductance "L" (μH)
100/120 Hz		
1kHz	0.1740	2.88
10kHz	0.1585	943



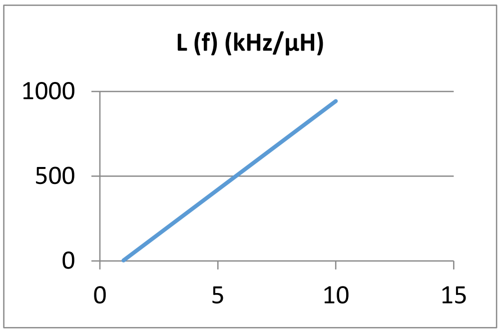


Figure 6.30 – Graphic representation of resistance's and inductance's values in function of the current's frequency applied to the power line (resistance's components before the cables)

From these last results has been found a too high value for the inductance (for 10kHz L=943µH absolutely too much high), so can be said that only the resistance value can be considered acceptable in these last measurements.

6.5.2 Final consideration with the measured values

Inductance

10 kHz
Direct measure of the total inductance:
Ltot = 6,013 µH
Inductance obtained with the sum of each measured part of circuit:
Ltot = 5,345+4,589+4,603=14,537 µH

1kHz
Direct measure of the total inductance:
Ltot = 9µH
Inductance obtained with the sum of each measured part of circuit:
Ltot = 6,45+4,82+4,9=16,17 µH

Resistance

10 kHz
Direct measure of the total resistance:
Rtot = 0,2236 Ω
Resistance obtained with the sum of each measured part of circuit:
Rtot= 0,1038+0,0639+0,0891=0,2568 Ω

1kHz
Direct measure of the total resistance:
Rtot = 0,154 Ω
Resistance obtained with the sum of each measured part of circuit:
Rtot=0,0854+0,0823+0,1740=0,3417 Ω

Measure's results of only cable component (without the resistance's components of the rest of the circuit):

For 10Khz

R = 0,1585 Ω

So in total for 10kHz you have (with a direct measure of the total resistance):

Rtotal = 0,1585 Ω +0,2236 Ω =0,3821 Ω

With the measure of each part:

Rtotal = 0,2568 Ω +0,1585 Ω = 0,4153 Ω

At the end these result make no sense for the reason that are too distant from each other and also some of these results are too distant from reality.
Can be found the reason of these wrong value in the too much low frequency (as has been said before, the frequency must be around f = 9292,414 Hz ÷ 40 kHz)
However, by these results, can be plotted the graphics of R(f) and of L(f), both of them function of the frequency.
These graphics show that the resistance go down with the increase of the frequency while the inductance growing up with the increase of the frequency. So resistance and inductance are not constant with the frequency and for this reason is necessary to put the exact frequency in which we have the peak of current $f = \frac{1}{T} = \frac{1}{0,000025} = 40\ kHz$ for have the exactly value of resistance and of inductance (as can be see in the graphic, the resistance change is less than the inductance's one with the variation of frequency and for these reason the final results of resistance are more acceptable than the inductance's ones).

6.5.3 Instument: LCR400

a) Instrument's specifications:

- Parameters Measured: R, L, C, D & Q.
- Measurement Modes: Series or parallel equivalent circuit.
- Measurement Functions: Fully autoranging including selection between L, C and R. The Zero C
- function nulls out up to 100pF of stray capacitance in the test fixture.
- Measurement Frequency: User selectable to be 100Hz, 1kHz or 10kHz; frequency accuracy
- ± 0.01%. 120Hz instead of 100Hz by factory option for 60Hz operation.
- Measurement Ranges and Resolution(Parameter Range):

Table 6.7 - Measurement Ranges

R:	0.1mΩ ÷ 990MΩ
L:	0.001μH ÷ 9900H
C:	0.001pF ÷ 99000μF
D:	0.001 ÷ 999
Q:	0.001 ÷ 999

This instrument operated with an alternating current. The operating voltage of the instrument can change from 230V to 115V or vice-versa. This instrument has been designed for indoor use and in a environment in the temperature range 5°C to 40°C, 20% −80% RH (non−condensing). It may occasionally be subjected to temperatures between +5°C and −10°C without degradation of its safety. Shouldn't be operated while condensation is present.

b) Advertaising:

This instrument must be earthed. Any interruption of the mains earth conductor inside or outside the instrument will make the instrument dangerous. Intentional interruption is prohibited. The protective action must not be negated by the use of an extension cord without a protective conductor. When the instrument is connected to its supply, terminals may be live and opening the covers or removal of parts is likely to expose live parts. The apparatus shall be disconnected from all voltage sources before it is opened for any adjustment, replacement, maintenance or repair.

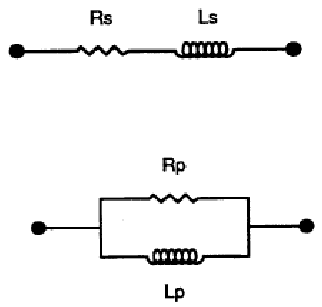
c) Display's function:

In normal use the left–hand 5–digit display shows the value of the major parameter (L, C or R) and the right–hand display shows the value of the minor parameter (Q, D or R). The parameters being displayed are indicated above their respective numeric values and the units of the parameter are shown to the right of the value itself. (Basic measurement accuracy is 0.1%.)

d) Circuit models:

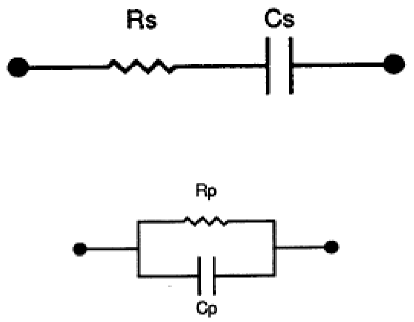
Resistors, capacitors and inductors can all be represented at a given frequency by a simple series or parallel equivalent circuit. It must be stressed that this is a *simple* equivalent circuit and as such will only be representative over a limited frequency range. The effects of a wide frequency range are discussed later.

The Models used by the LCR400 are as follows:



$$Z_s = R_s + j\omega L_s$$
$$Z_p = \frac{j\omega L_p R_p}{R_p + j\omega L_p}$$
$$Q = \frac{R_p}{\omega L_p} = \frac{\omega L_s}{R_s}$$
$$L_s = \frac{Q^2}{1+Q^2} L_p \quad R_s = \frac{\omega L_s}{Q} \quad R_p = Q\omega L_p$$

Figure 6.31 - Resistance's and inductance's model used by the LCR400



$$Z_s = R_s - j\frac{1}{\omega C_s}$$
$$Z_p = \frac{R_p}{1 + j\omega L_p C_p}$$
$$D = \omega R_s C_s = \frac{1}{\omega R_P C_p} \qquad D = \frac{1}{Q}$$

(D is also known as tanδ)

$$C_s = (1 + D^2) C_p \quad R_s = \frac{D^2}{1 + D^2} R_p$$
$$where\ \omega = 2\pi f$$

Figure 6.32 - Resistance's and capacitor's model used by the LCR400.

e) Inductors

All inductors have resistive losses, parasitic capacitance and an external coupled magnetic field.

The resistive losses are the resistance equivalent to losses in the core and the resistance of the conductive wire making up the turns of the inductor. There is capacitance between each turn of conductor and every other turn. The magnetic field of an inductor can extend outside the physical package of the component. In its simplest form the resistance can be represented as a resistor in series with the inductance, and the capacitance as a capacitor in parallel. The effect of an inductor's self capacitance and inductance at any given frequency combine to produce net inductance below the resonant frequency or capacitance above the resonant frequency. Above the self-resonant frequency these inductors will appear as a lossy capacitor. Due to the distributed nature of these parasitics, the equivalent values of the resistance and capacitance change with frequency. The inductance of a device with a 'leaky' magnetic field can vary considerably depending upon the characteristics of any conducting or magnetic material close to the device. Any conductive material within the device's field will contain induced currents that can in turn have the effect of reducing the apparent inductance of the component. Conversely any ferro-magnetic material in the immediate area of the component can have the effect of increasing the apparent inductance. In extreme cases the inductance of a component can appear to vary depending upon its distance above the connectors and steel case of the LCR400. Low value inductors (<100 μH) are best measured at 10 kHz whilst high values >25 H should be measured at 100 Hz.

f) Resistors

All resistors have parasitic impedances, both inductance and capacitance and distributed effects of both. Fortunately, however, in normal use these parasitic effects are usually very small compared with the resistance. The LCR 400 provides the opportunity to evaluate the series and parallel components of resistors at 100Hz and 1 kHz and 10 kHz. Some types of resistor have more prominent parasitic effects than others. There is also always capacitance between the end cap connections - on metal film resistors it is typically around 0,25 pF. This usually only becomes significant on high value resistors or/and at high frequencies. Bifilar wound resistors may have low inductance but the close proximity of the windings can introduce significant capacitance – distributed along the resistance. To predict the performance of such a component at high frequencies requires a more complex equivalent circuit than the simple two component series or parallel circuits discussed here. In practice the solution is to select component types to match the frequency range of the application. For the majority of resistors, where inductive and capacitive parasitics are minimal, both series and parallel circuits will give identical results for resistance. Normally R+Q should be selected for resistors; the Q of a resistor will usually be very low – especially at the low measurement frequencies used. However if the series and parallel resistances at 10 kHz differ significantly to those at 100 Hz or 1 kHz, the Q will be significant. Either the inductance or capacitance of the resistor is producing an effect. Selecting either C+R or L+Q will quantify the parasitic capacitance or inductance.

g) Series / Parallel connection

The LCR400 provides the capability of measuring the series or parallel equivalent circuit parameters of resistors, capacitors and inductors.
In Auto mode the bridge uses the following models.

Table 6.8 - Models in auto mode for LCR400

Resistor	Series
Inductor	Series
Capacitor	<1μF Parallel
Capacitor	>1μF Series

6.5.4 Theory behind the LCR400 instrument

This instrument used the theory refers to the study of electrical current’s and voltage’s transmission along the electric lines (which leading to the equations telegrapher). Applied a sinusoidal current in a cable can be considered the following considerations:
The general equation for describe the current’s and the tension’s course in a power line (considering current and tension variable in time and space “I(x,t)”, “V(x,t)”) are:

$$\begin{cases} \partial \mathrm{V}(\mathrm{x},\mathrm{t}) = \mathrm{r}\,\partial \mathrm{x}\mathrm{I}(\mathrm{x},\mathrm{t}) + \mathrm{l}\,\partial \mathrm{x}\,\dfrac{\partial \mathrm{I}(\mathrm{x},\mathrm{t})}{\partial \mathrm{t}} \\ \partial \mathrm{I}(\mathrm{x},\mathrm{t}) = \mathrm{g}\,\partial \mathrm{x}(\mathrm{V}(\mathrm{x},\mathrm{t}) + \partial \mathrm{V}(\mathrm{x},\mathrm{t})) + \mathrm{c}\,\partial \mathrm{x}\,\dfrac{\partial(\mathrm{V}(\mathrm{x},\mathrm{t}) + \partial \mathrm{V}(\mathrm{x},\mathrm{t}))}{\partial \mathrm{t}} \end{cases}$$

In which the concentrate electric elements of the longitudinal impedance and of the transversal admittance for an infinitesimal element “∂x” are:

$$(r\partial x|l\,\partial \mathrm{x})$$

$$(g\partial x|c\,\partial \mathrm{x})$$

By ignoring the second order of infinitesimal term “∂V(x,t)c∂x ” and the term “∂V(x,t)g∂x” you arrive to:

$$\begin{cases} \partial \mathrm{V(x,t)} = \mathrm{r}\,\partial \mathrm{xI(x,t)} + \mathrm{l}\,\partial \mathrm{x}\,\dfrac{\partial \mathrm{I(x,t)}}{\partial \mathrm{t}} \\ \partial \mathrm{I(x,t)} = \mathrm{g}\,\partial \mathrm{xV(x,t)} + \mathrm{c}\,\partial \mathrm{x}\,\dfrac{\mathrm{V(x,t)}}{\partial \mathrm{t}} \end{cases} \tag{6.27}$$

Starts by considering a monophasic model that has, for each km of length, a resistance "r", an inductance "l", an transversal conductance and a capacitor (where there is a comeback power line without resistance), can be written the following line for an infinitesimal distance "∂x":

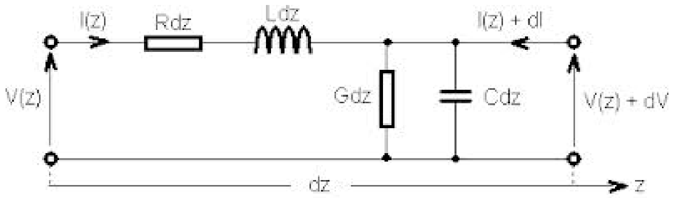


Figure 6.33 - Electrical elements of a power line (represented by concentrate constants)

For sinusoidal regime can be considered complex representation of current and tension instead of time one. So the current and the tension are function only of space and as is known for sinusoidal regime:

$$i(t) = \tilde{\mathrm{I}}$$

$$v(t) = \tilde{\mathrm{V}}$$

At the same time also the ż and ẏ elements of the power line becomes easier to study with a complex representation:

$$\begin{cases} \dot{\mathrm{z}}\,\partial \mathrm{x} = (\mathrm{r} + \mathrm{j\omega l})\,\partial \mathrm{x} \\ \dot{\mathrm{y}}\,\partial \mathrm{x} = (\mathrm{g} + \mathrm{j\omega c})\,\partial \mathrm{x} \end{cases} \tag{6.28}$$

So considering the line under a sinusoidal regime of frequency "*f*" (pulsation "*ω=2πf*") the system 6.23 becomes:

$$\begin{cases} \partial \tilde{\mathrm{V}} = \mathrm{r}\,\partial \mathrm{x}\tilde{\mathrm{I}} + \mathrm{j\omega l}\,\partial \mathrm{x}\,\tilde{\mathrm{I}} \\ \partial \tilde{\mathrm{I}} = \mathrm{g}\,\partial \mathrm{x}\tilde{\mathrm{V}} + \mathrm{j\omega c}\,\partial \mathrm{x}\,\tilde{\mathrm{V}} \end{cases}$$

By using the notation (6.28), you arrive to the system of differential equation:

$$\begin{cases} \dfrac{\partial \tilde{\mathrm{V}}(x)}{\partial x} = \tilde{\mathrm{I}}(x)\dot{\mathrm{z}} \\ \dfrac{\partial \tilde{\mathrm{I}}(x)}{\partial \mathrm{x}} = \tilde{\mathrm{V}}(x)\,\dot{\mathrm{y}} \end{cases} \tag{6.29}$$

From derivate the second equation of the (6.29) and use the first one you arrive to only one differential equation in current unknown:

$$\frac{\partial^2 \tilde{\mathrm{I}}(x)}{\partial \mathrm{x}^2} = \frac{\partial \tilde{\mathrm{V}}(x)}{\partial x}\,\dot{\mathrm{y}} = \tilde{\mathrm{I}}(x)\dot{\mathrm{z}}\,\dot{\mathrm{y}}$$

$$\frac{\partial^2 \tilde{\mathrm{I}}(x)}{\partial \mathrm{x}^2} - \tilde{\mathrm{I}}(x)\dot{\mathrm{z}}\,\dot{\mathrm{y}} = 0 \tag{6.30}$$

Can be solved this differential equation directly by putting the I(x) current's expression function of space or by using the integration with the Laplace-transformation.
This second one start by the Laplace-Transformation:

$$\mathrm{L}\left(\frac{\partial \tilde{\mathrm{I}}(X)}{\partial x}\right) = s\tilde{\mathrm{I}}_x - \tilde{\mathrm{I}}(x=0)$$

$$\mathrm{L}\left(\frac{\partial^2 \tilde{\mathrm{I}}(X)}{\partial x^2}\right) = s^2\tilde{\mathrm{I}}_x - s\tilde{\mathrm{I}}(x=0) - \frac{\partial \tilde{\mathrm{I}}(x=0)}{\partial x}$$

And so with those transformation the differential equation becomes:

$$s^2\tilde{\mathrm{I}}_x - s\tilde{\mathrm{I}}(x=0) - \frac{\partial\tilde{\mathrm{I}}(x=0)}{\partial x} - \tilde{\mathrm{I}}_x\dot{\mathrm{z}}\,\dot{\mathrm{y}} = 0$$
$$\tilde{\mathrm{I}}_x = \frac{\frac{\partial\tilde{\mathrm{I}}(x=0)}{\partial x} + s\tilde{\mathrm{I}}(x=0)}{s^2 - \dot{\mathrm{z}}\dot{\mathrm{y}}}$$
$$\tilde{\mathrm{I}}_x = \frac{\frac{\partial\tilde{\mathrm{I}}(x=0)}{\partial x} + s\tilde{\mathrm{I}}(x=0)}{(s+\hbar)(\mathrm{s}-\hbar)}$$

Where $\hbar$=$\sqrt{\dot{\mathrm{z}}\,\dot{\mathrm{y}}}$

Solving the previous equation without the Laplace transformation you have:

For a solution like:

$$\tilde{\mathrm{I}}_x(x) = Ae^{sx}$$
$$\frac{\partial\tilde{\mathrm{I}}(x)}{\partial x} = sAe^{sx}$$
$$\frac{\partial^2\tilde{\mathrm{I}}(x)}{\partial \mathrm{x}^2} = s^2Ae^{sx}$$

And so the equation (6.30) becomes:

$$s^2Ae^{sx} - Ae^{sx}\hbar^2 = 0$$
$$Ae^{sx}(s^2 - \hbar^2) = 0$$

Without the banal solution (s=0) you have $s_{1/2} = \pm\hbar$:

$$\tilde{\mathrm{I}}_x(x) = A_1e^{\hbar x} + A_2e^{-\hbar x} \qquad 6.31$$

Solving this system can be found the constant A_1 and A_2

$$\begin{cases} \tilde{\mathrm{I}}_x(x) = A_1 + A_2 \\ \frac{\partial\tilde{\mathrm{I}}(x=0)}{\partial x} = A_1\hbar - A_2\hbar \end{cases}$$

These constant values are obtained by initial condition. You have:

$$\begin{cases} \sum_{i=1}^{\infty} I_i = A_1 + A_2 \\ \frac{\sum_{i=1}^{\infty} V_i}{\hbar}\dot{\mathrm{y}} = A_1 - A_2 \end{cases}$$

(where $\sum_{i=1}^{\infty} I_i$ and $\sum_{i=1}^{\infty} V_i$ are the value of $\tilde{\mathrm{I}}_x(x)$ and $\tilde{\mathrm{V}}_x(x)$ for (x=0) and considering a specific frequency “f” i.c.)

From the sum and the difference of the two equation you have:

$$\begin{cases} \sum_{i=1}^{\infty} I_i + \frac{\sum_{i=1}^{\infty} V_i}{\hbar}\dot{\mathrm{y}} = A_1 + A_1 \\ \sum_{i=1}^{\infty} I_i - \frac{\sum_{i=1}^{\infty} V_i}{\hbar}\dot{\mathrm{y}} = A_2 + A_2 \end{cases}$$

$$\begin{cases} A_{1=}\frac{1}{2}(\sum_{i=1}^{\infty} I_i + \frac{\sum_{i=1}^{\infty} V_i}{ħ}\dot{y}) \\ A_2 = \frac{1}{2}(\sum_{i=1}^{\infty} I_i - \frac{\sum_{i=1}^{\infty} V_i}{ħ}\dot{y}) \end{cases} \quad (6.32)$$

By calling $Z_0 = \sqrt{\frac{\dot{z}}{\dot{y}}}$ and by using the (6.32) in the equation (6.31) you have:

$$\tilde{I}_x(x) = \frac{1}{2}(\sum_{i=1}^{\infty} I_i + \frac{\sum_{i=1}^{\infty} V_i}{Z_0})\, e^{ħx} + \frac{1}{2}(\sum_{i=1}^{\infty} I_i - \frac{\sum_{i=1}^{\infty} V_i}{Z_0})e^{-ħx}$$

$$\tilde{I}_x(x) = \frac{1}{2}\sum_{i=1}^{\infty} I_i(e^{ħx} + e^{-ħx}) + \frac{\sum_{i=1}^{\infty} V_i}{2Z_0}(\, e^{ħx} - e^{-ħx})$$

(where $\sum_{i=1}^{\infty} V_i$ and $\sum_{i=1}^{\infty} I_i$ are the initial condition of current and tension, for each frequency considerate.)

Starting again from (6.27), by the derivation of the first equation and using the second one you can arrive to only one differential equation in tension unknown(can be skipped some passages that are very close to the previous one):

$$\frac{\partial^2 \tilde{V}(x)}{\partial x^2} = \frac{\partial \tilde{I}(x)}{\partial x}\dot{z} = \tilde{V}(x)\dot{z}\,\dot{y}$$

$$\frac{\partial^2 \tilde{V}(x)}{\partial x^2} - \tilde{V}(x)\dot{z}\dot{y} = 0 \quad (6.33)$$

Exactly the same kind of the current one.
We can solve this differential equation directly by putting the V(x) tension's expression function of space or by using the integration with the Laplace-transformation.
Solving the previous equation without the Laplace transformation you have:
For a solution like:

$$\tilde{V}_x(x) = Ae^{sx}$$

$$\frac{\partial \tilde{V}(x)}{\partial x} = sAe^{sx}$$

$$\frac{\partial^2 \tilde{V}(x)}{\partial x^2} = s^2Ae^{sx}$$

And so the equation 6.30 becomes:

$$s^2Ae^{sx} - Ae^{sx}ħ^2 = 0$$

$$Ae^{sx}(s^2 - ħ^2) = 0$$

Without the banal solution (s=0) you have $s_{1/2} = \pm ħ$:

$$\tilde{V}_x(x) = A_1e^{ħx} + A_2e^{-ħx} \quad (6.34)$$

Solving this system can be found the constant A_1 and A_2

$$\begin{cases} \tilde{V}_x(x) = A_1 + A_2 \\ \dfrac{\partial \tilde{V}(x=0)}{\partial x} = A_1 \hbar - A_2 \hbar \end{cases} \tag{6.35}$$

The initial condition came from the same value of current and tension at the beginning of the power line (x=0) in a specific frequency "f", previously used for the current (equation (6.32)). With these initial condition the system (6.35) becomes:

$$\begin{cases} \sum_{i=1}^{\infty} V_i = A_1 + A_2 \\ \dfrac{\sum_{i=1}^{\infty} I_i}{\hbar} \dot{z} = A_1 - A_2 \end{cases}$$

$$\begin{cases} A_1 = \dfrac{1}{2}\left(\sum_{i=1}^{\infty} V_i + \dfrac{\sum_{i=1}^{\infty} I_i}{\hbar} \dot{z}\right) \\ A_2 = \dfrac{1}{2}\left(\sum_{i=1}^{\infty} V_i - \dfrac{\sum_{i=1}^{\infty} I_i}{\hbar} \dot{z}\right) \end{cases} \tag{6.36}$$

By calling $Z_0 = \sqrt{\frac{\dot{z}}{\dot{y}}}$ and by using the (6.36) in the equation (6.34) you have:

$$\tilde{V}_x = \frac{1}{2}\left(\sum_{i=1}^{\infty} V_i + Z_0 \sum_{i=1}^{\infty} I_i\right) e^{\hbar x} + \frac{1}{2}\left(\sum_{i=1}^{\infty} V_i - Z_0 \sum_{i=1}^{\infty} I_i\right) e^{-\hbar x}$$

$$\tilde{V}_x = \frac{1}{2}\left(\sum_{i=1}^{\infty} V_i (e^{\hbar x} + e^{-\hbar x}) + Z_0 \sum_{i=1}^{\infty} I_i \,(e^{\hbar x} - e^{-\hbar x})\right)$$

The final system for both the equation $\tilde{V}_x(x)$ and $\tilde{I}_x(x)$ is:

$$\begin{cases} \tilde{V}_x(x) = \dfrac{1}{2}\left(\sum_{i=1}^{\infty} V_i (e^{\hbar x} + e^{-\hbar x}) + Z_0 \sum_{i=1}^{\infty} I_i \,(e^{\hbar x} - e^{-\hbar x})\right) \\ \tilde{I}_x(x) = \dfrac{1}{2}\sum_{i=1}^{\infty} I_i (e^{\hbar x} + e^{-\hbar x}) + \dfrac{\sum_{i=1}^{\infty} V_i}{2 Z_0} (e^{\hbar x} - e^{-\hbar x}) \end{cases} \tag{6.37}$$

With the use of the function:

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

The system 6.35 can be rewritten:

$$\begin{cases} \tilde{V}_x(x) = \sum_{i=1}^{\infty} V_i \cosh \hbar x + Z_0 \sum_{i=1}^{\infty} I_i \sinh \hbar x \\ \tilde{I}_x(x) = \sum_{i=1}^{\infty} I_i \cosh \hbar x + \dfrac{\sum_{i=1}^{\infty} V_i}{Z_0} \sinh \hbar x \end{cases} \tag{6.38}$$

Considering now the values of current and tension in the specific point in which the electrical terminals' instrument are placed (x=0 and x=d=3,8m) you can arrive to the value of Z_0 and $\hbar_0$ by the solution of the system (6.38) in the specific distance "d=3,8 m". At this distance the system (6.38) becomes:

$$\begin{cases} \tilde{V}_x(d) = \sum_{i=1}^{\infty} V_i \cosh \hbar d + Z_0 \sum_{i=1}^{\infty} I_i \sinh \hbar d \\ \tilde{I}_x(d) = \sum_{i=1}^{\infty} I_i \cosh \hbar d + \frac{\sum_{i=1}^{\infty} V_i}{Z_0} \sinh \hbar d \end{cases} \tag{6.39}$$

The solution of this system (6.39) gives Z_0 and $\hbar_0$ results, from whose can be found l, c, r, g parameters. These consideration can be made for each frequency of current and tension inserted in the power line (the LCR400 instrument use current of frequency: 100/120 Hz – 1 kHz – 10 kHz).
Is this the way in which operated the LCR400 instrument to find the "r" and "l" values for each frequency.

Chapter 7

7.1 STUDY OF THE WAVE IN THE WIRES WITH FAST FOURIER ANALYSIS

Starting again from the measurement obtained with the oscilloscope and with the ATP simulation, after some mathematical and physical considerations in the transitory in a wires (considerations that come from the theory of the "wave's equation"), can be obtained the waveform of the current along the wires during the transitory.

For doing these considerations are taken again the results obtained by the measurement with the oscilloscope (with a current probe applied in the end of the eight wires that goes inside the test cabin) for the testing of an aluminum panel ("scaled waveform of the aluminum"). This waveform of the current is shown again in figure 8-1. The oscilloscope gives a number of current values ($N_{sampling}$=9000) with an uniform sampling (a time step of "Δt"= 0,02 μs = 0,02·10^{-6} s). This uniform sampling can be shown in an excel work paper. The characteristics of this sampling is, as has been said, an uniform time step with a sampling's frequency of $f_c = \frac{1}{\Delta t} = \frac{1}{0{,}02*10^{-6}s} = 50\ MHz$. The number of sampling is N = 9000. Can be shown again this waveform in excel representation down here:

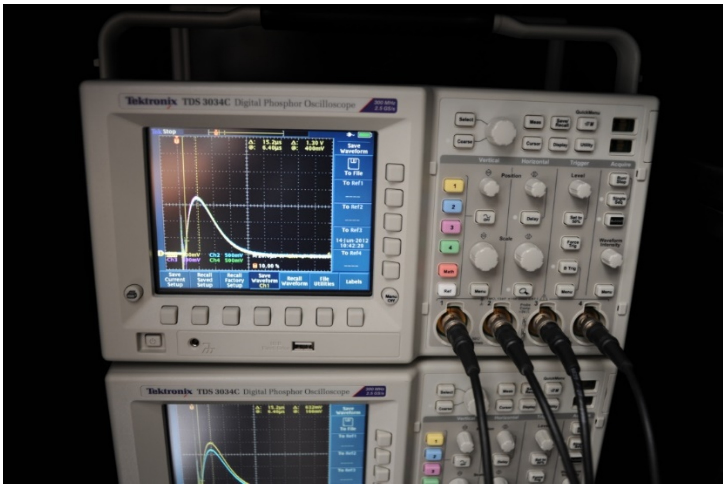


Figure 7.1 - Picture of the oscilloscope used for the measurement

Current and time step of the oscilloscope sampling				Δt sampl	2,00E-02	sec
		i(t) (kA)	t (sec)	N sampl	9000	
-2,00E-05	-0,028	0,36633	-2,00E+01	fc sampl	5,00E+01	Hz
-2,00E-05	-0,024	0,78073	-2,00E+01			
-2,00E-05	-0,02	1,19513	-2,00E+01			
-1,99E-05	-0,028	0,36633	-1,99E+01			
-1,99E-05	-0,028	0,36633	-1,99E+01			
-1,99E-05	-0,028	0,36633	-1,99E+01			
-1,99E-05	-0,032	-0,04807	-1,99E+01			
-1,99E-05	-0,032	-0,04807	-1,99E+01			
-1,98E-05	-0,044	-1,29127	-1,98E+01			
-1,98E-05	-0,024	0,78073	-1,98E+01			
-1,98E-05	-0,032	-0,04807	-1,98E+01			
-1,98E-05	-0,036	-0,46247	-1,98E+01			
-1,98E-05	-0,024	0,78073	-1,98E+01			
-1,97E-05	-0,032	-0,04807	-1,97E+01			
-1,97E-05	-0,016	1,60953	-1,97E+01			
-1,97E-05	-0,032	-0,04807	-1,97E+01			
-1,97E-05	-0,024	0,78073	-1,97E+01			
-1,97E-05	-0,02	1,19513	-1,97E+01			
-1,96E-05	-0,032	-0,04807	-1,96E+01			
-1,96E-05	-0,036	-0,46247	-1,96E+01			
-1,96E-05	-0,036	-0,46247	-1,96E+01			
-1,96E-05	-0,032	-0,04807	-1,96E+01			
-1,96E-05	-0,04	-0,87687	-1,96E+01			

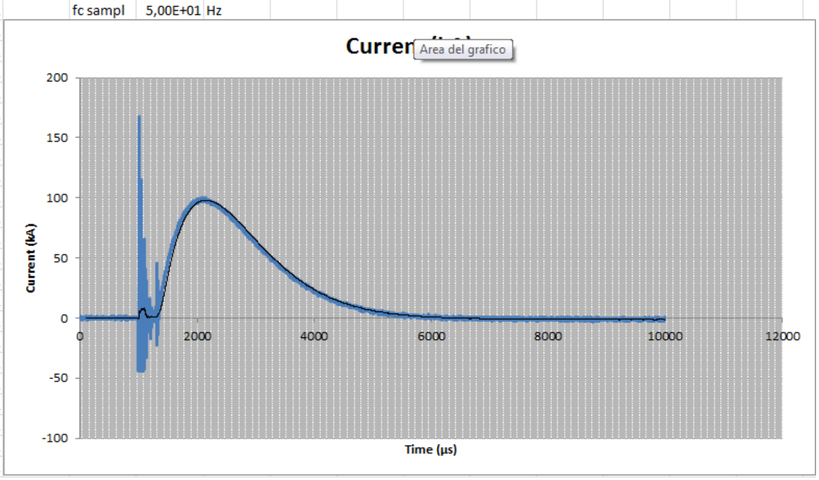


Figure 7.2 - Scaled waveform of the aluminum

These values can be implemented for an harmonic analysis by appealing to the discrete Fourier Transformation (calls Fast Fourier Transformation “FFT”).
The other waveform available is the tension, taken in the same point of the cable where were measured the current’s values. This waveform comes from the ATP simulation and it’s implemented in an excel work paper. The characteristic of this sampling is, as for the current, an uniform time step with a sampling’s frequency of $f_c = \frac{1}{\Delta t} = \frac{1}{10^{-3} s} = 1\ kHz$. The number of sampling is N = 1000. This waveform can be shown again in excel down here:

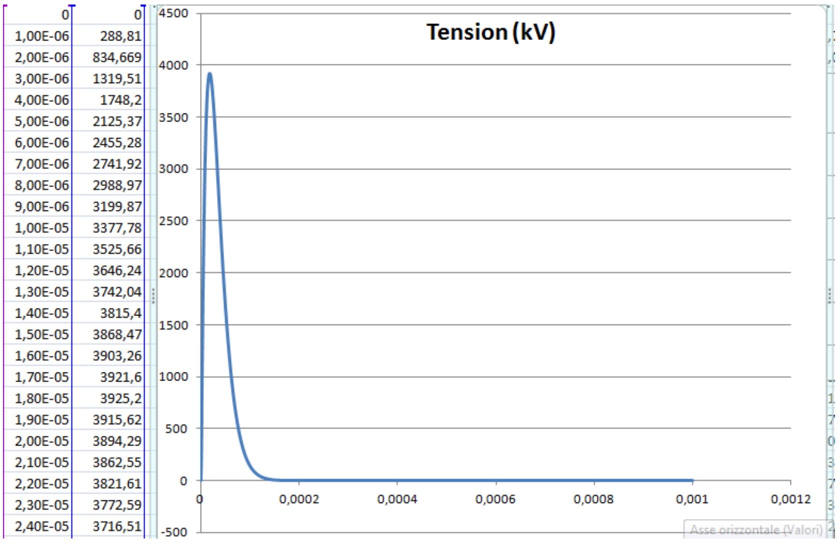


Figure 7.3 – Shape of tension at the end of the D-bank’s circuit by the sampling in excel of the ATP plot’s values

The Fast Fourier Transformation’s main equations, for a uniform sampling, are the following:

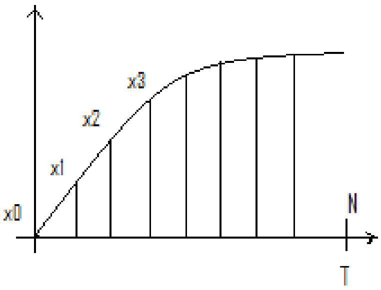


Figure 7.4 - Trapezoidal division (with equal bases of each trapezium) of a generic function to derive the FFT

$$a_0 = \frac{1}{N}\sum_{i=0}^{N-1} x_i$$

$$a_i = \frac{2}{N}\sum_{i=0}^{N-1} x_i \cos(\frac{2\pi i}{N})$$

$$b_i = \frac{2}{N}\sum_{i=0}^{N-1} x_i \mathrm{sen}(\frac{2\pi i}{N})$$

With an uniform sampling: $t_s = \frac{T}{N}$

$$i(t) = a_0 + a_1 \cos(\frac{2\pi}{T} t) + \mathrm{b}_1 \sin\left(\frac{2\pi}{T} t\right) + a_2 \cos(2\frac{2\pi}{T} t) + \mathrm{b}_2 \sin(2\frac{2\pi}{T} t)$$

Appling these equations in the same excel work paper for the current and for the tension sampling you obtained the complete harmonic analysis. The harmonic analysis brought to the following results:

- For the current (oscilloscope measurement) you have:

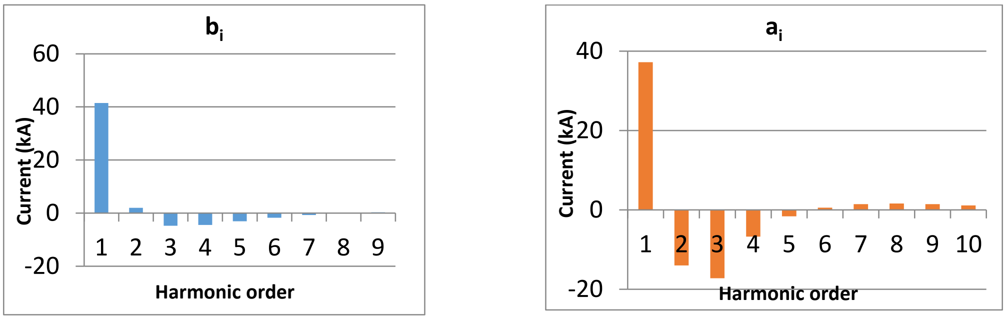


Figure 7.5 – Harmonics current's values' diagram for the "a_i" and for the "b_i" (FFT terms)

The representation of the current with the equivalent Fourier Series (stopping at the 9th, 6th harmonic) is:

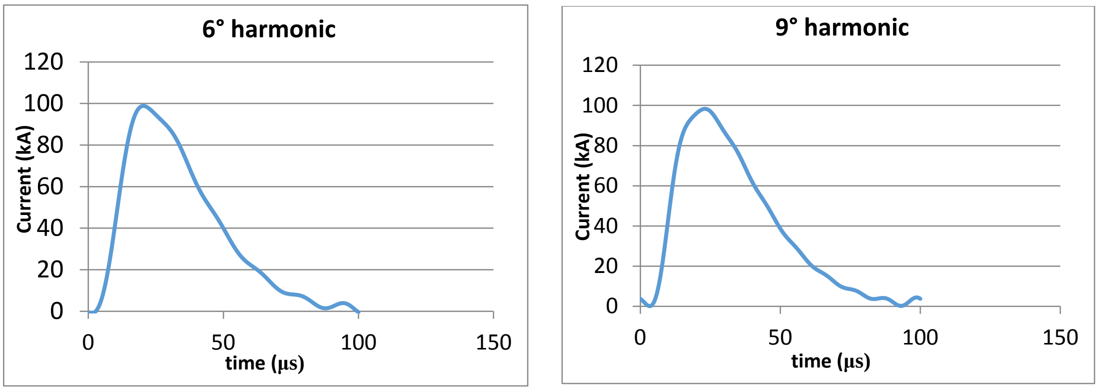


Figure 7.6 – Current's shape by using the FFT series representation (stopping at the 6° harmonic/9° harmonic, in excel)

- For the tension (ATP simulation) you have:

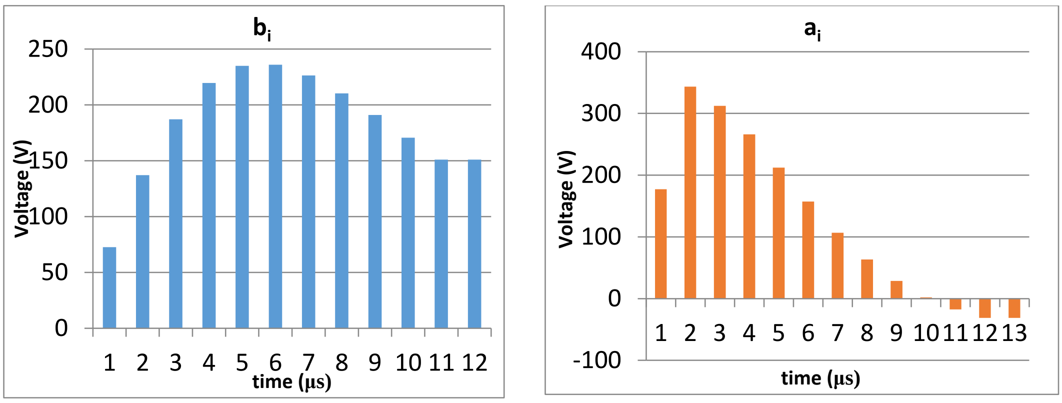


Figure 7.7 - Harmonics tension's values' diagram for the "a_i" and for the "b_i" (FFT terms)

So for the higher level of frequency fn =100 Hz is better to underline the harmonic with a step of 10 so that are taken the 10°, 20°, 40°,…., 90° harmonic like as is shown here:

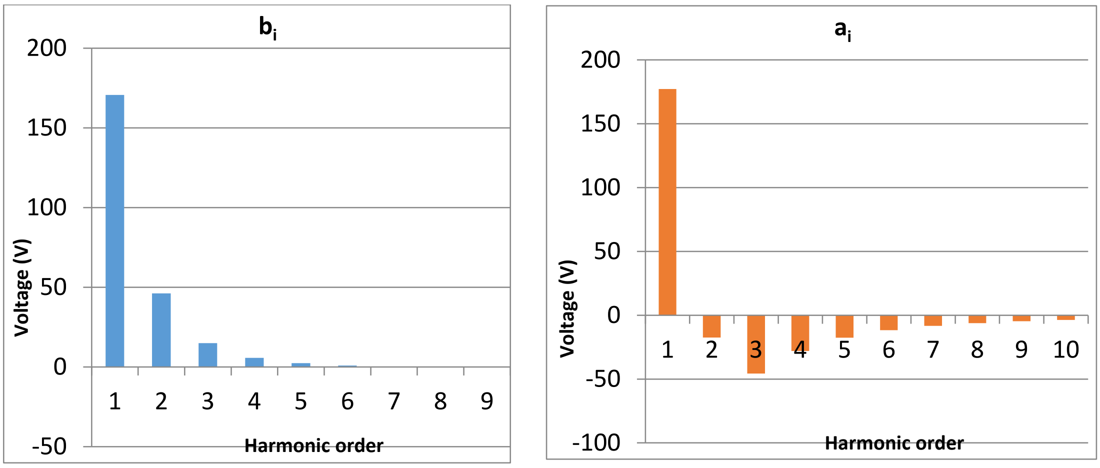


Figure 7.8 - Harmonics tension’s values’ diagram for the “a_i” and for the “b_i” (FFT terms) with a jump of 10 harmonics each step

The representation of the tension with the equivalent Fourier Series (stopping at the 90th, 60th harmonic) is:

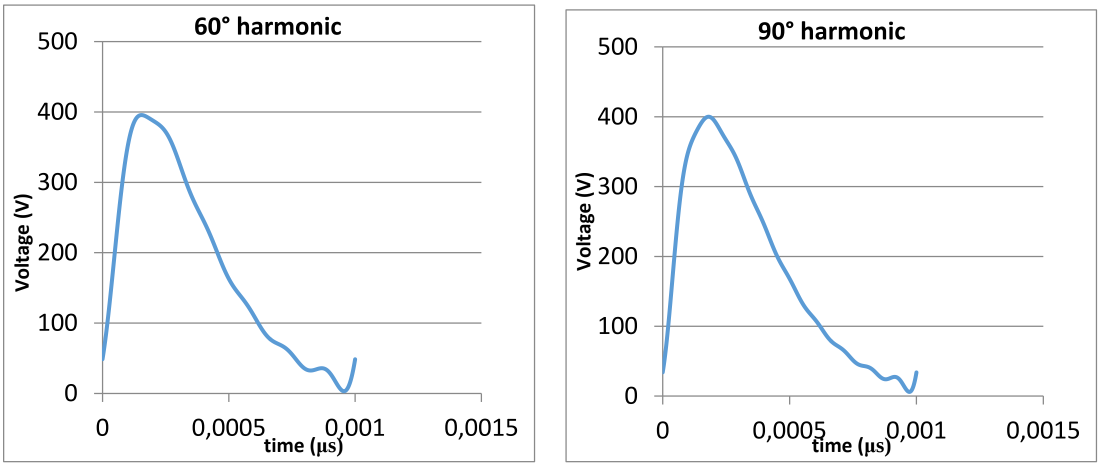


Figure 7.9 - Tension’s shape by using the FFT series representation (stopping at the 60° harmonic/90° harmonic, in excel, with a jump of 10 harmonics each step)

From this analysis you can see that, stopping until the 9°/6° harmonic for the current and until the 90°/60° harmonic for the tension, you have a good approximation of their shape and so to this waveform can be given an easier function’s representation i(t), as a composition of more sinusoidal functions different in frequency (the Fourier Series' decomposition of the current):

$$i(0,\mathrm{t}) = \sum_{i=1}^{\infty} a_i \sin(i\,\omega\mathrm{t}) + \sum_{i=0}^{\infty} b_i \cos(\,i\omega\mathrm{t}) = \sum_{i=0}^{\infty} I_i \,\mathrm{sen}(\,i\omega\mathrm{t} + \alpha_i)$$

and

$$v\,(\mathrm{o},\mathrm{t}) = \sum_{i=1}^{\infty} c_i \sin(i\,\omega\mathrm{t}) + \sum_{i=0}^{\infty} b_i \cos(\,i\omega\mathrm{t}) = \sum_{i=0}^{\infty} V_i \,\mathrm{sen}(\,i\omega\mathrm{t} + \alpha_i)$$

Referred to a position of the cable (x=0), the end of the cables, where the measurement are taken. As is known from the Theory of the Fourier Series’ decomposition of an electrical signal (in our case the current signal) it can be simplified by a series of current’s generators each one with the amplitude,

frequency and phase specific of each harmonic. Starting by the theory of concentrate constants for an electrical cable can be made the following passages:
It's considering a monophasic model that has, for each km of length, a resistance "r", an inductance "l", an transversal conductance and a capacitor. There is a coming back cable, supposed without resistance. So the line for an infinitesimal distance "∂x" is the one write in figure 6.30.
Each element of the cable is considered for an infinitesimal space:

$(r\partial x|l\,\partial$x)

$(g\partial x|c\,\partial$x)

Because it's considered a transitory regime in the cable, should be shown also the "t" variable for the "l" and "c" concentrate elements. As is known:

$$v = l\frac{\partial i}{\partial t}$$

$$i = c\frac{\partial v}{\partial t}$$

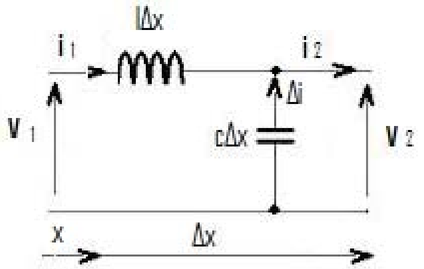


Figure 7.10 – Representation of a generic power line element

From now for simplify the accounts (especially for the low value of the dissipative component in a wire such short, d=3,8m) is considered "r=0" and "g=0". This is equivalent to assume a wire not dissipative. For such a wire, have values the equations:

$v_1 = v(x,t) \qquad v_2 = v(x+\Delta x, t) = v_1 + \Delta v$

$i_1 = i(x,t) \qquad i_2 = i(x+\Delta x, t) = i_1 + \Delta i$

So now can be written:

$$v_2 - v_1 = \Delta v = -\frac{\partial i_1}{\partial t}l\Delta x$$

$$i_2 - i_1 = \Delta i = -\frac{\partial v_2}{\partial t}c\Delta x = -\frac{\partial(v_1 + \Delta v)}{\partial t}c\Delta x$$

And so considering Δx infinitesimal and neglecting in the last equation the second order infinitesimal terms $\frac{\partial(c\Delta v\Delta x)}{\partial t}$, you arrive to the following expressions:

$$-\frac{\partial v(x,t)}{\partial x} = l\frac{\partial i(x,t)}{\partial t}$$

$$-\frac{\partial i(x,t)}{\partial x} = c\frac{\partial v(x,t)}{\partial t}$$

So this become, after some simple passage, a system of equation that are called "waves equation":

$$\begin{cases} \dfrac{\partial^2 v(x,t)}{\partial x^2} = cl\dfrac{\partial^2 v(x,t)}{\partial t^2} \\ \dfrac{\partial^2 i(x,t)}{\partial x^2} = cl\dfrac{\partial^2 i(x,t)}{\partial t^2} \end{cases} \tag{7.1}$$

Considering from now only the terms $\frac{\partial^2 i(x,t)}{\partial x^2} = cl\frac{\partial^2 i(x,t)}{\partial t^2}$. This is an equation similar to the one that comes from the problem of the "vibrating string" and so you can made the same kind of passages:

First of all there are the following initial conditions of the (7.1):

$$i(0,\mathrm{t}) = \sum_{i=1}^{\infty} a_i \sin(i\,\omega\mathrm{t}) + \sum_{i=0}^{\infty} b_i \cos(\,i\omega\mathrm{t}) = \sum_{i=0}^{\infty} I_i \operatorname{sen}(\,i\omega\mathrm{t} + \alpha_i)$$

If are taken the values of the tension measured in the same point (taken by the ATP circuit) you have:

$$v\,(\mathrm{o},\mathrm{t}) = \sum_{i=1}^{\infty} c_i \sin(i\,\omega\mathrm{t}) + \sum_{i=0}^{\infty} b_i \cos(\,i\omega\mathrm{t}) = \sum_{i=0}^{\infty} V_i \operatorname{sen}(\,i\omega\mathrm{t} + \alpha_i)$$

The initial condition are the values of current *i(0,t)* in time at the end of the wires and the values of tension in the same point (considering the orientation of “x” started from the end of the wires “x=0”). The values that came from oscilloscope’s measurement for the current and from ATP for the tension are the values at the end of the wires. With the decomposition of the current, so also of the tension, in Fourier Series you can arrive, in the end, at sum of sinusoidal current (everyone is a specify harmonic of the current), and for each of these harmonic can be estimate the final value *i(x=o,t)* in a given time. In the same time is considered the final value of tension for each harmonic tension. Those are the initial values that are considered for solving the differential equation.

$$\begin{cases} i(0,t) = \sum_{i=1}^{\infty} a_i \sin(i\,\omega\mathrm{t}) + \sum_{i=0}^{\infty} b_i \cos(\,i\omega\mathrm{t}) \\ \dfrac{\partial i(0,t)}{\partial x} = -c\dfrac{\partial v(0,t)}{\partial t} = \; -c\dfrac{\partial[\sum_{i=1}^{\infty} c_i \sin(i\,\omega\mathrm{t}) + \sum_{i=0}^{\infty} d_i \cos(\,i\omega\mathrm{t})]}{\partial t} = \omega c\left[-\sum_{i=1}^{\infty} ic_i \cos(i\,\omega\mathrm{t}) + \sum_{i=0}^{\infty} id_i \operatorname{sen}(\,i\omega\mathrm{t})\right] \end{cases}$$

The boundary conditions of 7.1 are (considering the time limited from t=0, initial time of sampling when wires are open, to t=Tc, final time of sampling when the current is completed extinct by the grounding circuit):

$i(x,o) = i(x,Tc) = 0$

Calling "$a = \sqrt{lc}$” and considering “$0 \le t \le Tc$” and “$0 \le x \le d$”

Can be considered a speed propagation of those waves that are coming from the “wave equation”:

$$v = \frac{1}{\sqrt{lc}} = \frac{1}{\sqrt{\mu\varepsilon}}$$

So the complete differential equation to solve with the “b.c.” and “i.c.” is:

$$\begin{cases} \dfrac{\partial^2 i(x,t)}{\partial x^2} = a^2\dfrac{\partial^2 i(x,t)}{\partial t^2} \\ i(x,o) = i(x,Tc) = 0 \\ i(0,t) = \sum_{i=1}^{\infty} a_i \sin(i\,\omega\mathrm{t}) + \sum_{i=0}^{\infty} b_i \cos(\,i\omega\mathrm{t}) \\ \dfrac{\partial i(0,t)}{\partial x} = \omega c\left[-\sum_{i=1}^{\infty} ic_i \cos(i\,\omega\mathrm{t}) + \sum_{i=0}^{\infty} id_i \operatorname{sen}(\,i\omega\mathrm{t})\right] \end{cases} \tag{7.2}$$

For solving this equation $\frac{\partial^2 i(x,t)}{\partial x^2} = a^2\frac{\partial^2 i(x,t)}{\partial t^2}$ (in (7.2)) can be assumed $i(x,t) = X(t)Y(x)$ where X(t) and Y(x) are two function dependent of only one variable and so you have:

$$\frac{\partial^2 i(x,t)}{\partial x^2} = a^2\frac{\partial^2 i(x,t)}{\partial t^2} \Rightarrow \; X(t)Y''(x) = a^2X''(t)Y(x)$$

$$\frac{Y''(x)}{Y(x)} = a^2\frac{X''(t)}{X(t)} \tag{7.3}$$

Because on the left there is a function not dependent of “t” and on the right a function not dependent of “x”, the equation (7.3) is valid if is considered a constant k^2 so that:

$$\frac{Y''(x)}{Y(x)} = a^2 \frac{X''(t)}{X(t)} = -k^2 < 0$$

(for other kind of constant "$k^2 \geq 0$" you have no solution of the previous equation with these b.c. and i.c.).
And so the equation becomes:

$$\begin{cases} X''(t) + \frac{k^2}{a^2} X(t) = 0 \\ i(x, o) = i(x, Tc) = 0 \end{cases}$$

So the solution is:

$$X(t) = Acos\left(\frac{k}{a}t\right) + Bsen\left(\frac{k}{a}t\right)$$

Using the boundary condition for find the constant value A and B:

$$\begin{cases} i(x, 0) = A = 0 \\ i(x, Tc) = Bsen\left(\frac{k}{a}Tc\right) = 0 \end{cases}$$

Because isn’t of interest the banal solution B=0 can be considered only the solution:

$$\frac{k}{a}Tc = n\pi$$

And so the *X(t)* function is:

$$X(t) = sen\left(\frac{n\pi}{Tc}t\right) \tag{7.4}$$

And so you have (by placing the (7.4) in the (7.3)):

$$\frac{Y''(x)}{Y(x)} = a^2 \frac{X''(t)}{X(t)} = -a^2 \frac{n^2\pi^2}{Tc^2} \frac{sen\left(\frac{n\pi}{Tc}t\right)}{sen\left(\frac{n\pi}{Tc}t\right)} == -a^2 \frac{n^2\pi^2}{Tc^2}$$

So the solution is:

$$Y(x) = Ccos\left(\frac{an\pi}{Tc}x\right) + Dsen\left(\frac{an\pi}{Tc}x\right)$$

$$i(x, t) = X(t)Y(x) = \sum_{n=1}^{\infty} \left(Ccos\left(\frac{an\pi}{Tc}x\right) + Dsen\left(\frac{an\pi}{Tc}x\right)\right) sen\left(\frac{n\pi}{Tc}t\right) \tag{7.5}$$

$$i(0, t) = \sum_{n=1}^{\infty} Csen\left(\frac{n\pi}{Tc}t\right) = C \sum_{n=1}^{\infty} sen\left(\frac{n\pi}{Tc}t\right)$$

$$\frac{\partial i(x, t)}{\partial x} = \sum_{n=1}^{\infty} \frac{an\pi}{Tc} \left(-Csen\left(\frac{an\pi}{Tc}x\right) + Dcos\left(\frac{an\pi}{Tc}x\right)\right) sen\left(\frac{n\pi}{Tc}t\right)$$

$$\frac{\partial i(0, t)}{\partial x} = \sum_{n=1}^{\infty} \frac{an\pi}{Tc} Dsen\left(\frac{n\pi}{Tc}t\right) = D \sum_{n=1}^{\infty} \frac{an\pi}{Tc} sen\left(\frac{n\pi}{Tc}t\right)$$

Using now the initial condition can be found the C and D values:

$$\begin{cases} i(0,t) = C\displaystyle\sum_{n=1}^{\infty} sen\left(\frac{n\pi}{Tc}t\right) = \sum_{i=1}^{\infty} a_i \sin(i\,\omega t) + \sum_{i=0}^{\infty} b_i \cos(\,i\omega t) \\ \dfrac{\partial i(0,t)}{\partial x} = D\displaystyle\sum_{n=1}^{\infty} \frac{an\pi}{Tc} sen\left(\frac{n\pi}{Tc}t\right) = \omega c\left[-\sum_{i=1}^{\infty} ic_i \cos(i\,\omega t) + \sum_{i=0}^{\infty} id_i\, \text{sen}(\,i\omega t)\right] \end{cases} \tag{7.6}$$

$$\begin{cases} C = \dfrac{\sum_{i=1}^{\infty} a_i \sin(i\,\omega t) + \sum_{i=0}^{\infty} b_i \cos(\,i\omega t)}{\sum_{n=1}^{\infty} sen\left(\frac{n\pi}{Tc}t\right)} \\ D = \dfrac{\omega c(-\sum_{i=1}^{\infty} ic_i \cos(i\,\omega t) + \sum_{i=0}^{\infty} id_i\, \text{sen}(\,i\omega t))}{\sum_{n=1}^{\infty} \frac{an\pi}{Tc} sen\left(\frac{n\pi}{Tc}t\right)} \end{cases}$$

By putting these values “C” and “D” in the equation (7.5) there is the following term:

$$\frac{sen\left(\frac{n\pi}{Tc}t\right)}{\sum_{n=1}^{\infty} sen\left(\frac{n\pi}{Tc}t\right)} \tag{7.7}$$

So if is considered a limited number of implementation of “n” (the implementation is stopped in the excel work paper at “n=9”) the term (7.7) can be simplified (considering N the number of terms at which stop the implementation, in the excel file N=9):

$$\frac{sen\left(\frac{n\pi}{Tc}t\right)}{\sum_{n=1}^{\infty} sen\left(\frac{n\pi}{Tc}t\right)} = \frac{1}{N}$$

So finally can be written the function:

$$\begin{aligned} (x,t) = \frac{1}{N}\sum_{n=1}^{\infty}\Bigg\{&\left[\sum_{i=1}^{\infty} a_i \sin(i\,\omega t) + \sum_{i=0}^{\infty} b_i \cos(\,i\omega t)\right] cos\left(\frac{an\pi}{Tc}x\right) \\ &+ \left\{\frac{\omega c Tc}{an\pi}\left[-\sum_{i=1}^{\infty} ic_i \cos(i\,\omega t) + \sum_{i=0}^{\infty} id_i\, \text{sen}(\,i\omega t)\right]\right\} sen\left(\frac{an\pi}{Tc}x\right)\Bigg\} \end{aligned} \tag{7.8}$$

(Where “N” is the number of $cos\left(\frac{an\pi}{Tc}x\right)$ and $sen\left(\frac{an\pi}{Tc}x\right)$ considerate, the implementation is stopped with a number “N” of "$cos\left(\frac{an\pi}{Tc}x\right)$" and "$sen\left(\frac{an\pi}{Tc}x\right)$" terms.)

In excel work paper is implemented the equation (7.8).

From the excel work paper are found the waveform of current after the distance of the wires d=3,8m. Its shape is shown down here:

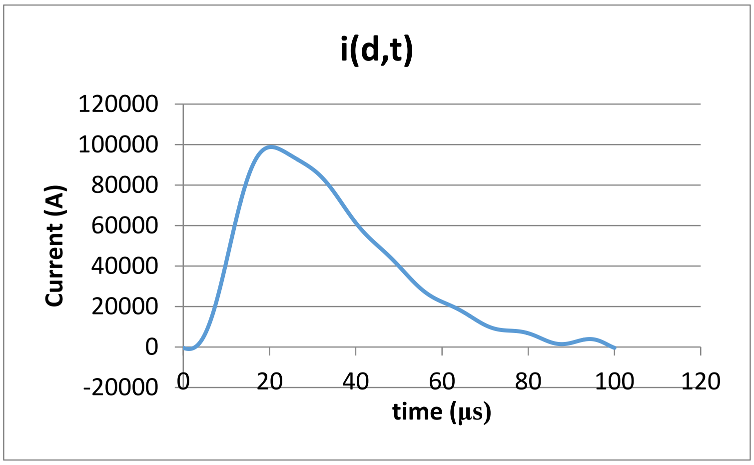
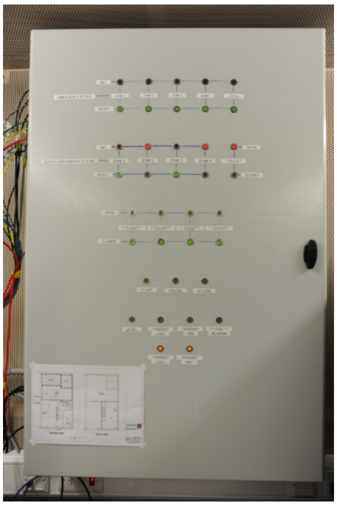


Figure 7.11 – *i(d,t)* Current’s shape by plotting in excel the equation 7.8 for the distance of the cable d=3,8 m

In This function i(t) (where d = 3,8 m) the vertical axis is in kA and the horizontal axis is in µs. As can be already imagined the value of current has a very low attenuation (at d = 3,8 m the peak of current is at i_p= 9 kA, instead of the i_p = 10 kA in d = 0 at the end of the wires where appeared the lighting). This can be easily accepted because of the very low length of the wires. As consequence the values of come-back current from the ark are very high and all the building (lightning laboratory), is provided of many protection for each electric component, and many system of grounding circuit for personal's safe, as has been already told in the description of the laboratory (Chapter 3). All the bank circuit is automatically close to the personal during the experiment by a system of alarm as is shown in the picture (figure 8.4). (Of course is absolutely forbidden any personal presence, in the room where is located the A,D,C/B-bank, during experiment for these same reasons). Every kind of operation and maneuver during the experiment is perforate outside by a computer that control directly each switch, charging and discharging component with a control system show next (figure 8.3)



b)

a)

c)

d)

Figure 7.12 – a) Electronic Control System for each component inside the laboratory - b), c), d) alarm and lock system in the "bank's room"

Finally is shown the shape of *i(x,t)* with a graphic where is placed the current's waveform *i(d,t)* for different value of the distance d. For have a complete shape and description the current's shape is varied until d = 38 km where the *i(t)* is became a negative function. The interpretation of this could be that the current's harmonic components rotate while moving across the wires until for such high distance d = 38 km they become overall a negative total value.

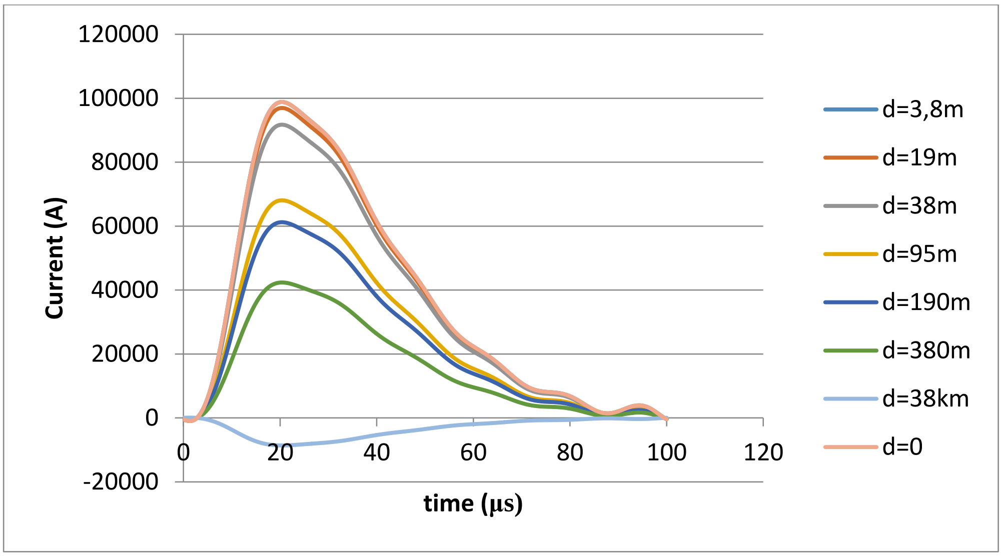


Figure 7.13 - *i(x,t)* Current's shapes by plotting in excel the equation 7.8 for different hypothetical distance (d, 5d, 10d, 25d, 50d, 100d, 10000d) in a cable ("d=3,8 m")

This graphic has some mistake made by the approximation of the i(t) until the 6th harmonic. In fact with an higher distance "d" the $cos\left(\frac{an\pi}{Tc}x\right)$ and $sen\left(\frac{an\pi}{Tc}x\right)$ terms have an higher values and so multiply them with the harmonic component of i(0,t) and v(0,t) give to the mistake inherent in the Fourier approximation an higher height. In this way we must see the graphic above, where we can observe that for this reason $i(x,0) = i(x,\infty) \neq 0$. We have so for high distance a translation of the current waveform caused by the amplification of the "Fourier's error" in high distance. Of course more harmonic we consider more precision we give to those graphic but we need an higher computational cost.
For have an idea of a more realistic graphic the previous one can be redrawn with a translation under for each waveform so that each waveform at each distance "d" started and finished with $i(x,0) = i(x,\infty) = 0$ (Is done in this way a kind of correction of the error told before).

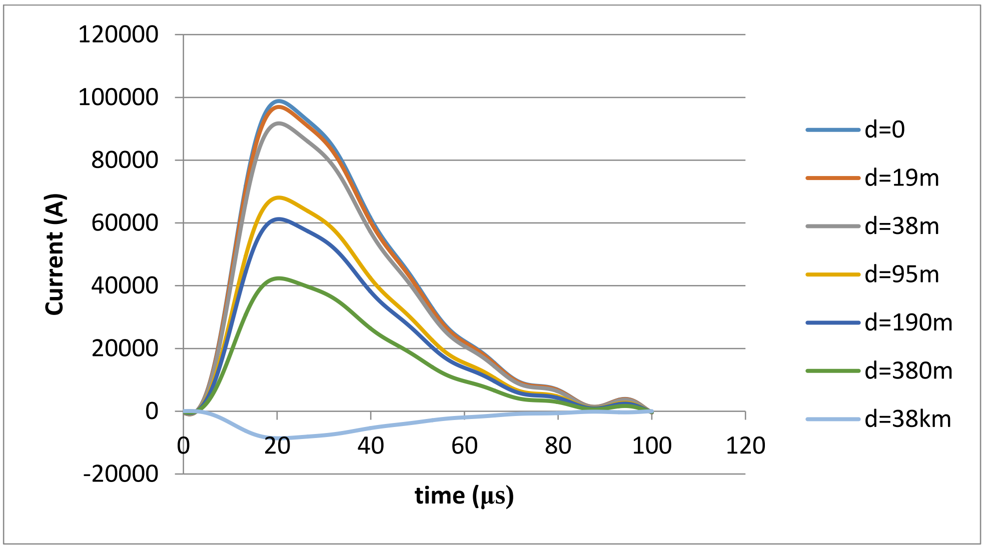


Figure 7.14 - *i(x,t)* Current's shapes for different hypothetical distance in a cable of the figure 7.13 corrected from the initial errors made by FFT approximation of the original current *i(t)* sampled

Using the function *i(x,t)* (in 7.8) and the function *v(x,t)* (not obtained in these reporter), can be found the "l" value from the equation 7.1 for all the extension of the wires (so for "x=d"):

$$-\frac{\partial v(x=d,t)}{\partial x}=l\frac{\partial i(x=d,t)}{\partial t}$$

From this equation you can see that the inductance "l" is variable in time l(t). Its very low value (L = 4,5 µH) is a consequence of the low length of the wires. This make very low voltage drops in it, allowing the formation of the arc.

7.1.1 Other way to find the function i(x.t)

Here is shown an other way to find the C and D values (which starts again from the equation 7.6 and obtains the i(x,t) function). This way gives a more precise function *i(x,t)* but needs much more implementation of Fourier series for arrive to a correct value of *i(x,t)* function. So this way isn't implemented in excel because of the too high computational cost. Here it's described only in theory.

As has been said, the C and D values can be find in other way by the following considerations:

- For i=0 in the equation 7.6 (knowing that Knowing that $\omega=2\pi f=\frac{2\pi}{Tc}$) you have:

$$\begin{cases} i(0,t)=C\sum_{n=1}^{\infty} sen\left(\frac{n\pi}{Tc}t\right)=b_0 \\ \frac{\partial i(0,t)}{\partial x}=D\sum_{n=1}^{\infty}\frac{na\pi}{Tc} sen\left(\frac{n\pi}{Tc}t\right)=0 \end{cases} \tag{7.9}$$

Integrating the (7.9):

$$\begin{cases} C\int_0^{Tc}\left[\sum_{n=1}^{\infty} sen\left(\frac{n\pi}{Tc}t\right)\partial t\right]=\int_0^{Tc} b_0\partial t=b_0 Tc \\ D=0 \end{cases}$$

$$\begin{cases} C\left\{\sum_{n=1}^{\infty}\frac{Tc}{n\pi}[-\cos(n\pi)+\cos(0)]\right\}=\int_0^{Tc} b_0\partial t=b_0 Tc \\ D=0 \end{cases}$$

For n = 1, 3, 5, 7, 9…, so for "n" odd, you have:

$$\begin{cases} C\left\{\sum_{n=1}^{\infty}\frac{2}{n\pi}\right\}=b_0 \\ D=0 \end{cases}$$

And finally (for "n" odd):

$$\begin{cases} C=\frac{b_0}{\sum_{n=1}^{\infty}\frac{2}{n\pi}} \\ D=0 \end{cases} \tag{7.10}$$

- For i=1 in the equation (7.6) (knowing that Knowing that $\omega=2\pi f=\frac{2\pi}{Tc}$) you have:

$$\begin{cases} i(0,t)=C\sum_{n=1}^{\infty} sen\left(\frac{n\pi}{Tc}t\right)=a_1\sin(\frac{2\pi}{Tc}t)+b_1\cos(\frac{2\pi}{Tc}t) \\ \frac{\partial i(0,t)}{\partial x}=D\sum_{n=1}^{\infty}\frac{an\pi}{Tc} sen\left(\frac{n\pi}{Tc}t\right)=\omega c\left[-c_1\cos(\frac{2\pi}{Tc}t)+d_1 sen(\frac{2\pi}{Tc}t)\right] \end{cases} \tag{7.11}$$

So integrate the equation (7.11):

$$\begin{cases} \frac{Tc}{2}C = \int_0^{Tc}\left[a_1 \sin(\frac{2\pi}{Tc}t) + b_1 \cos(\frac{2\pi}{Tc}t)\right] sen\left(\frac{n\pi}{Tc}t\right)\partial t \\ \frac{an\pi}{2}D = \int_o^{Tc}\omega c\left[-c_1 \cos(\frac{2\pi}{Tc}t) + d_1 \text{sen}(\frac{2\pi}{Tc}t)\right] sen\left(\frac{n\pi}{Tc}t\right)\partial t \end{cases} \tag{7.12}$$

The integration give result different from “0” only for “n=2” and the results are:

$$\begin{cases} \frac{Tc}{2}C = a_1\frac{Tc}{2} \\ \frac{an\pi}{2}D = \omega c d_1\frac{Tc}{2} \end{cases}$$

So finally (for “n=2”):

$$\begin{cases} C = a_1 \\ D = \frac{cd_1}{a} \end{cases} \tag{7.13}$$

In the same way:

- For “i=2” and “n=4”:

$$\begin{cases} C = a_1 \\ D = \frac{cd_1}{a} \end{cases}$$

- For “i=3” and “n=6”:

$$\begin{cases} C = a_1 \\ D = \frac{cd_1}{a} \end{cases}$$

And the same for “I = 4, 5, 6, 7, 8, 9” all give as C and D values the 7.13 considering $n = 2i$.

By putting these result in the 7.5 you arrive to (take attention to the 7.10 for the first term and the “n=2i” for the second one):

$$i(x,t) = \sum_{n=1,3,5,9}^{\infty}\left[\frac{b_0}{\sum_{n=1,3,5,9}^{\infty}\frac{2}{n\pi}}cos\left(\frac{an\pi}{Tc}x\right)\right]sen\left(\frac{n\pi}{Tc}t\right) + \sum_{n=2i}^{\infty}\left[\sum_{i=1}^{\infty}a_i\,cos\left(\frac{an\pi}{Tc}x\right) + \sum_{i=0}^{\infty}\frac{cd_i}{a}sen\left(\frac{an\pi}{Tc}x\right)\right]sen\left(\frac{n\pi}{Tc}t\right)$$

So finally:

$$i(x,t) = \sum_{n=1,3,5}^{\infty}\left[\frac{b_0}{\sum_{n=1,3,5}^{\infty}\frac{2}{n\pi}}cos\left(\frac{an\pi}{Tc}x\right)\right]sen\left(\frac{n\pi}{Tc}t\right) + \sum_{i=1}^{\infty}\left[a_i cos\left(\frac{2ai\pi}{Tc}x\right) + \frac{cd_i}{a}sen\left(\frac{2ai\pi}{Tc}x\right)\right]sen\left(\frac{2i\pi}{Tc}t\right)$$

Or remeber that $\omega = 2\pi f = \frac{2\pi}{Tc}$: .

$$i(x,t) = \sum_{n=1,3,5,9}^{\infty}\left[\frac{b_0 n\pi}{2}cos\left(\frac{an\pi}{Tc}x\right)\right] sen\left(\frac{n\pi}{Tc}t\right) + \sum_{i=1}^{\infty}\left[a_i cos(ai\omega x) + \frac{cd_i}{a}sen(ai\omega x)\right] sen(i\omega t) \tag{7.14}$$

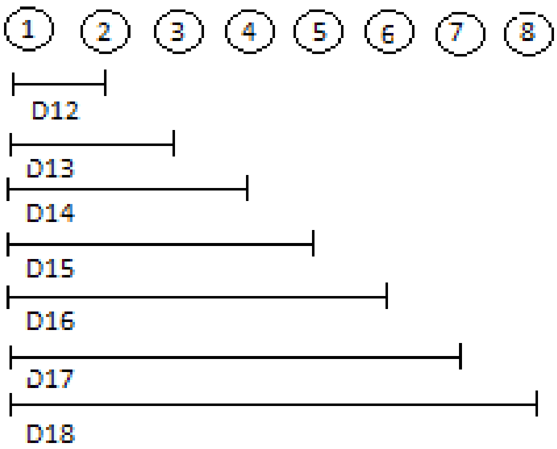


Figure 7.15 - Representation of the geometrical meaning of each "D_{ij}" terms' used in (7.15)

The value of "c" used in 7.8 and 7.14 is the general expression for the power line:

$$c = \frac{24,14}{\log_{10}\frac{2D}{d_{eq}}} 10^{-9} \quad [F/km] \qquad (7.15)$$

(where "d_{eq}" is the diameter of each cable and D is obtained by:

$$D = \sqrt[8]{D_{12}^7 D_{13}^6 D_{14}^5 D_{15}^4 D_{16}^3 D_{17}^2 D_{18}} \;)$$

Chapter 8

8.1 CONCLUSION

Since the result obtained in more way, we could assume as good value for L=4.5µF and for R=0,39Ω. Those are the identification of the total electric parameters hidden in the circuit of discharging for the D-bank. In the same way we could do the identification of the B-bank's fundamental electric parameters. We don't identify the capacity parameters "c" (it was made a simplify estimation only in the last paragraph, using the general expression for the electrical line (7.15)) and the conductance "g" hidden in the eight wires and inside each electric component (switch, D-bank…). This because their values are very low and considering them in our circuit has as consequence only a complication of every expression obtained for a power line. A particular attention could be made for the harmonic distribution in the measurement made since now:

- *i(t)* current measurement in the test cabin during the ark:

A=modulo	Spettro	frequenza
183892,36	89,79119	0,00E+00
88608,549	43,26589	1,22E-02
31266,265	15,26673	2,44E-02
14919,056	7,284695	3,66E-02
8718,962	4,257306	4,88E-02
5814,0712	2,838902	6,10E-02
4131,1836	2,017179	7,32E-02
3107,7141	1,517439	8,54E-02
2360,6268	1,15265	9,77E-02
1693,189	0,826752	1,10E-01
1202,2849	0,587053	1,22E-01
743,61369	0,363093	1,34E-01
320,70886	0,156596	1,46E-01
96,21866	0,046982	1,59E-01
268,8513	0,131275	1,71E-01
416,99869	0,203613	1,83E-01
600,85358	0,293386	1,95E-01
677,37653	0,33075	2,08E-01
702,20758	0,342875	2,20E-01
700,65697	0,342118	2,32E-01
584,24236	0,285275	2,44E-01
988,21793	0,482528	2,56E-01
684,91638	0,334432	2,69E-01

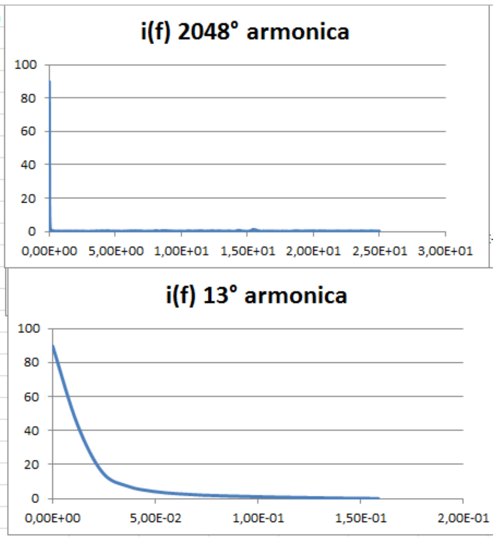


Figure 8.1 – Excel work paper for the harmonic analysis (FFT) of the oscilloscope sampling current's values

- *i(t)* current from the ATP simulation:

valore ass	spettro	frequenza
3544,2	1,38E+01	0
3359,918	1,31E+01	1,95E+03
2935,024	1,15E+01	3,91E+03
2470,209	9,65E+00	5,86E+03
2063,233	8,06E+00	7,81E+03
1732,177	6,77E+00	9,77E+03
1467,582	5,73E+00	1,17E+04
1255,488	4,90E+00	1,37E+04
1083,822	4,23E+00	1,56E+04
943,3146	3,68E+00	1,76E+04
827,0622	3,23E+00	1,95E+04
729,9267	2,85E+00	2,15E+04
648,0459	2,53E+00	2,34E+04
578,4781	2,26E+00	2,54E+04
518,9508	2,03E+00	2,73E+04
467,6862	1,83E+00	2,93E+04
423,2765	1,65E+00	3,13E+04
384,5957	1,50E+00	3,32E+04
350,7347	1,37E+00	3,52E+04
320,9533	1,25E+00	3,71E+04
294,6445	1,15E+00	3,91E+04
271,3072	1,06E+00	4,10E+04
250,525	9,79E-01	4,30E+04
231,9501	9,06E-01	4,49E+04

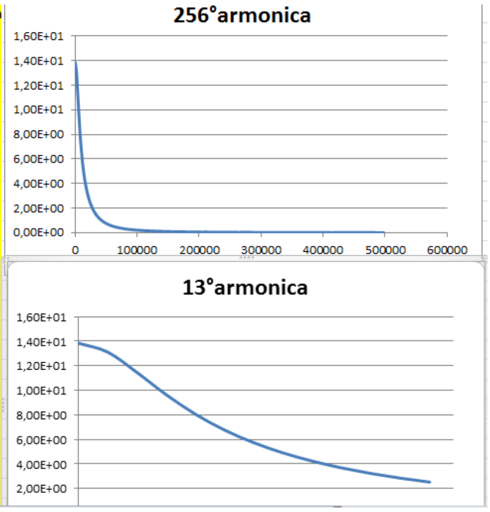


Figure 8.2 - Excel work paper for the harmonic analysis (FFT) of the current's values, taken by the ATP simulation

Those are made in an excel work paper, from which we have an indication of the noise and a possible value of the THD (Total Harmonic Distortion), from which we could estimate the inevitable mistake inherent in those measurement.
The THD is obtained by the expression (where I_{s1} is the value of the first harmonic I_s is the value of the total harmonic contribution and I_{sh} is the value of the total harmonic contribution without the first one):

$$THD\% = 100\frac{\sqrt{I_S^2 - I_{s1}^2}}{I_{s1}} = 100\sqrt{\sum_{h\neq1}\left(\frac{I_{sh}}{I_{s1}}\right)^2}$$

So we have:

- *i(t)* current measurement in the test cabin during the ark (*f_c=50MHz*; *N=4096*):

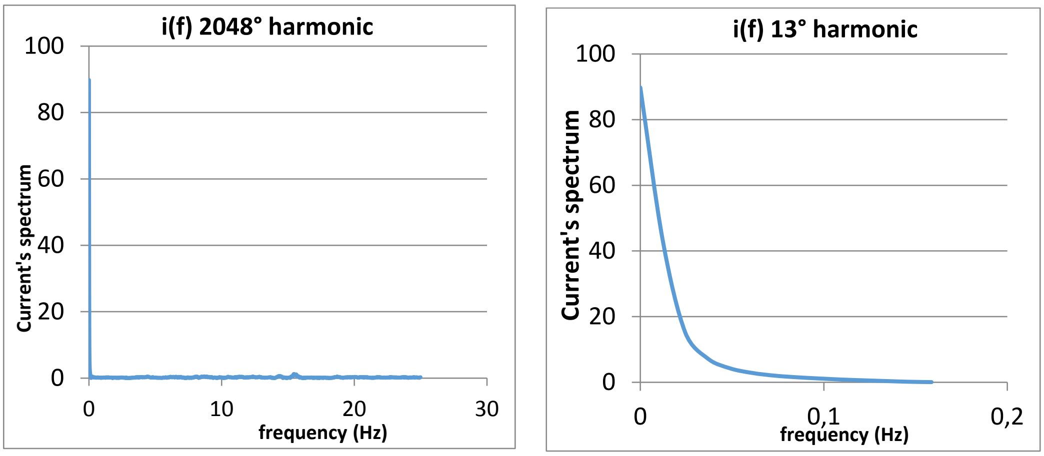


Figure 8.2 - Frequency spectrum of the oscilloscope sampling current

$$THD\% = 52{,}15802$$

- *i(t)* current from the ATP simulation (f_c=1MHz; N=512):

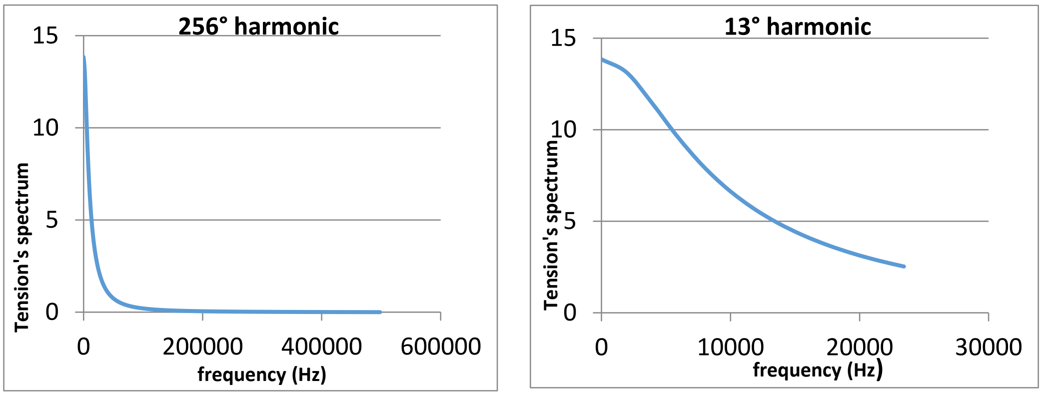


Figure 8.3 - Frequency spectrum of the current's values taken by the ATP simulation

$$THD\% = 179{,}9896$$

Acknowledgements

I want to begin by thanking Professor Manu Haddad sincerely for their valuable time dedicated, for his guidance and his support during the laboratory experience. I would also like to thank, the laboratory manager, Chris Stone, for its continuous availability, his continuous help in testing. Additional thanks to Professor Roberto Turri for availability and support during the rearrangement of the thesis. I also thank all the staff of the laboratory and my family for giving me the opportunity to undertake this experience very informative and of particular interest. Finally, I thank the University of Padua and Cardiff University who, with their cooperation (Erasmus exchange) have made this experience possible.

Appendix I

Matlab’s code 1.1:

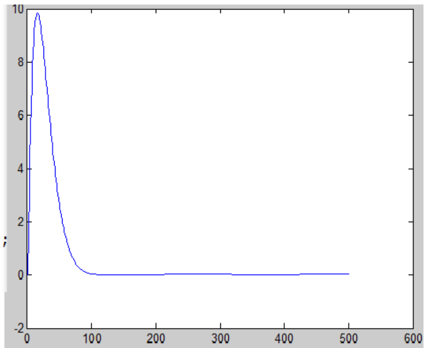
```
clear all;
clc;
close all;
  r=0.39;
  c=66.7e-6;
  l=4.4e-6;
  v=54000;
  t=0:1e-6:500e-6;
  d=((r/(2*l))^2)-(1/(l*c));
  i=v/(2*l*sqrt(d))*exp(-(r/(2*l)-sqrt(d))*t)-v/(2*l*sqrt(d))*exp(-(r/(2*l)
  plot(i);
```

Matlab’s code 1.2:

```
clear all;
clc;
close all;
  r=21;
  c=250e-6;
  T=r*c;
  Vb=3000;
  t=0:1e-6:500e-6;
  v=Vb*(1-(exp(-t/T)));
  i=(Vb/r)*(exp(-t/T));
  plot(i);
  plot(v);
```

Matlab’s code 1.3:

```
clc;
close all;
  r=0.39;
  c=66.7e-6;
  l=3.1e-6;
  v=54000;
  t=0:1e-6:500e-6;
  d=((r/(2*l))^2)-(1/(l*c));
  i=v/(2*l*sqrt(d))*exp(-(r/(2*l)-sqrt(d))*t)-v/(2*l*sqrt(d))*exp(-(r/(2*l)+sqrt(d))*t);
  plot(i);
```



Appendix II

Matlab’s code 2.1:

```
syms t L
C=66.7e-6;
R1=0.35;
I1=97.892E3;

k=-(1/(L*C))+(R1/(2*L))^2;
        V=54e3;
        io=(V/(2*L*sqrt(k)));
        a=(R1/(2*L))-sqrt(k);
        b=(R1/(2*L))+sqrt(k);
        ii=io*exp(-(a*t))-io*exp(-(b*t));
        df=diff(ii,t);
```

Matlab’s code 2.2:

Main code:

```
close all;
clear all;
x0=3.1e-6;
[x,fval]=fsolve(@basicfun,x0,optimset('Display','iter','MaxFunEvals',8200,'MaxIter',6000));
Xabs = abs(x);
fprintf('\n %g %g \n', Xabs )
```

“basicfun”(subfunction’s code):

```
function F=basicfun(x);
c=66.7e-6;
r=0.349997;
t=1.9220e-005; % [s]
F=(27000*exp(t*((49/(1600*x.^2) - 18446744073709551616/(x.*1230397829716427))^(1/2) - 7/(x.*40)))*((49/(1600*x.^2)
end
```

Where F is the derivate $\frac{\partial i}{\partial t}$ obtained by the code 2.1.

Matlab’s code 2.3:

```
clear all
close all
R=(0.1:0.01:0.5)';
L=linspace(1e-6,5e-6,length(R));
C=66.7e-6;
count1=1;
ii = zeros(length(R),length(L));
T1 = zeros(length(R),length(L));
while count1<=length(R)
    for count2=1:length(L)
        k=-(1/(L(count2)*C))+(R(count1)/(2*L(count2)))^2;
        V=54e3;
        io=(V/(2*L(count2)*sqrt(k)));
        a=(R(count1)/(2*L(count2)))-sqrt(k);
        b=(R(count1)/(2*L(count2)))+sqrt(k);
        ln=@log;
        T1(count1,count2)=ln(b/a)/(b-a);
        ii(count1,count2)=io*exp(-(a*T1(count1,count2)))-io*exp(-(b*T1(count1,count2)));


    end
count1=count1+1;
end
IIabs = abs(ii);
%figure1 = figure;
%axes1 = axes('Parent',figure1);
%view(axes1,[108.5 -16]);
%grid(axes1,'on');
%hold(axes1,'all');
surf(R,L,IIabs)
xlabel('Resistance')
ylabel('L inductance')
```

```
zlabel('Current magnitude [A]')
figure
surf(R,L,abs(T1))
xlabel('Resistance')
ylabel('L inductance')
zlabel('T1')
```

Matlab's code 2.4:

Main code:

```
clear all
close all

r0=0.3;      % [Ohm]
l0= 3.1e-006;  % [H]
x0 = [r0,l0];

%options = optimoptions('fsolve','Display','iter'); % Option to display output
[x,val]=fsolve(@myJfunction2,x0,optimset('Display','iter','MaxFunEvals',8200,'MaxIter',6000) )

Xabs = abs(x);
fprintf('\n %g %g \n', Xabs(1), Xabs(2) )
```

Subfunction "myJfunction2" code:

```
function F = myJfunction2(x)
C=66.7e-6; % [F]
V=54e3;    % [V]
T1=1.9220e-005; % [s]
ii=9.7892e+004;%   [A]
ln=@log;
F=[
(T1*(((x(1)/(2*x(2)))+sqrt(-(1/(x(2)*C))+(x(1)/(2*x(2)))^2))-((x(1)/(2*x(2)))-sqrt(-(1/(x(2)*C))+(x(1)/
 ii-((V/(2*x(2)*sqrt(-(1/(x(2)*C))+(x(1)/(2*x(2)))^2)))*exp(-(((x(1)/(2*x(2)))-sqrt(-(1/(x(2)*C))+(x(1)
```

Where in "F" are put the complete expression of the non linear system in two unknow (system 6.8).

Matlab's code 2.5:

Main code:

```
close all;
clear all;
x0=4e-6;
[x,fval]=fsolve(@bfun,x0,optimset('Display','iter','MaxFunEvals',8200,'MaxIter',6000));
Xabs = abs(x);
fprintf('\n %g %g \n', Xabs )
```

Subfunction "bfun" code:

```
function F=bfun(x);
C=66.7e-6;
V0=54e3;
t=1.9220e-005; % [s]
io=9.7892e+004;%   [A]
d=((t-(io*(x.*(1/V0))))/((t)^2) )^2-(1/(x.*C));
F=(((t-(io*(x.*(1/V0))))/((t)^2) )+sqrt(d))-((1+(t*(2*sqrt(d)))+(((t*(2*sqrt(d)))^2)/2))*(((t-(io*(x.*(1/V0))))/((t)^2)) -sqrt(d)));
end
```

In which "F" is the long simplify equation obtained by Taylor progression.

Appendix III

Matlab’s code 3.1:

```
syms V
M12=11,69984013;
M13=9,54286843;
M14=8,30587943;
M15=7,44519707;
M16=6,79044008;
M17=6,26571706;
M18=5,83054627;
L=18,99923049;
I=[L M12 M13 M14 M15 M16 M17 M18;M12 L  M12 M13 M14 M15 M16 M17;M13 M12 L  M12 M13 M14 M15 M16;M14 M13 M12 L M12 M13 M14 M15;
T=[V V V V V V V V];
M=inv(I);
M=M';
Y=T*M;
Z=Y(1,1)+Y(1,2)+Y(1,3)+Y(1,4)+Y(1,5)+Y(1,6)+Y(1,7)+Y(1,8);
ztot=(V/Z);
```

Matlab’s code 3.2:

```
syms V
M12=15,85340805;
M13=12,89218013;
M14=11,19583408;
M15=10,01681558;
M16=9,12085075;
L=27,52513834;
I=[L M12 M13 M14 M15 M16;M12 L  M12 M13 M14 M15;M13 M12 L  M12 M13 M14;M14 M13 M12 L M12 M13;M15 M14 M13 M12 L  M12;M16 M15 M14 M13 M12 L];
T=[V V V V V V];
M=inv(I);
M=M';
Y=T*M;
Z=Y(1,1)+Y(1,2)+Y(1,3)+Y(1,4)+Y(1,5)+Y(1,6);
ztot=(V/Z);
```

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