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An investigation of the object file system and the approximate number system in a non-symbolic arithmetic task

I sistemi “object file” e “approximate number” nella risoluzione di compiti aritmetici non simbolici

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Abstract

Humans may share with other animals a subset of non-verbal numerical skills, available soon after birth and considered evolutionary foundation of more complex numerical reasoning. Those skills are thought to be based on two systems: the object file system (number less than 3) and the approximate number system which processes larger number (greater than 4). Infants could discriminate 1 vs 2, 1 vs 3, 2 vs 3 but not 1 vs 4, thus it seems that the systems are independent, implying that the concept of continuous number processing is interrupted at or about the number four. Recent studies report how chicks are able to make discrimination regarding the borderline number. This thesis describes the experimental research carried out following this discovery and is based on testing this ability in infants, to assess if they share the skills with chicks.

CHAPTER 1

The object file system and the approximate number system

1.1 Introduction

Human adults typically solve mathematical problems using language and symbols. Pre-verbal numerical skills do exist in humans and can be compared with those found in other non-linguistic creatures such as preverbal infants and non-human animals. A variety of experimental studies has demonstrated that this mechanism is based on two separate systems: one regarding small values (≤3), the "Object file system" (OFS) and one concerning large numbers (≥4), the "Approximate number system" (ANS) (Trick 1994).

The OFS is an object-based attention mechanism according to which each element present in a real scene is represented by an "object file" that is stored in the working memory. The function of this system implies the capability of individuating each new object that is introduced into a scene to which a new file in the working memory is dedicated. Object files system plays a role in theories of singular reference, object individuation, perceptual memory, and the development of cognitive capacities.

The ANS is hypothesized to be a conceptual foundation for uniquely human symbolic mathematics skills. A host of recent studies have found that ANS acuity is correlated with symbolic mathematics skills. This relationship has been documented throughout the educational spectrum, starting from pre-schoolers who are just beginning formal mathematics education.

In the following paragraphs I will discuss the OFS and the ANS systems, their main differences and recent results reported in the literature on their use in interpreting how animals process data. (Houdè, 1997)

1.2 The OFS system

The Object File System is a system that deals with a numerosity of four or less. This system is specifically based on small numbers. We can consider this system as a form of attention. In feature integration theory, a spotlight of attention is volitionally directed to a particular spot on a master map of locations. This attentional spotlight enables the visual system to integrate features by bringing into register different feature maps at the location of interest. Object files provide a link from visual information to higher-order cognition, perceptual categories or knowledge about identified objects (Treisman 1998; Yi Lin 2021).

The OFS is an object based attention mechanism according to which each element present in a real scene is represented by a unique symbol called an ‘‘object file’’ that is stored in the working memory. That system may have originally evolved to represent objects, and its functioning implies the capability of individuating each new object that is introduced into a scene to which a new file in the working memory is dedicated.

Two cognitive systems, according to the two-system model, the object-file (OF) and physical-reasoning (PR) systems, work together to guide infants’ responses to these events. When an event begins, the OF system sends categorical information about the objects and their arrangements to the PR system. This system then categorizes the event, assigns event roles to the objects, and taps the OF system for information about features previously identified as causally relevant for the event category selected. These converging results provide strong support for the two-system model and for the claim that uncovering how the OF and PR systems represent and exchange information, this is essential for understanding how infants respond to physical events. (Rugani 2012 ; Yi Lin 2021)

1.3 The ANS system

The approximate number system is shared by adults, infants and non-human animals. This system allows to identify and memorize items in visual or auditory arrays without verbally counting, and animals use this capacity to guide everyday behaviour such as foraging. ANS produces numerical representations that grow increasingly imprecise as a linear function of the target array, with larger quantities represented less precisely than smaller quantities. This imprecision is given by a Weber fraction, expressing the minimum amount of extra stimulus (ΔI) that can be perceived for a stimulus I.

$$constant = k = \frac{\Delta I}{I}$$

Recent results showed, as reported in the paper by Mazzocco and Feigenson (2008), that individual differences in achievement in school mathematics are related to individual differences in the acuity of an evolutionarily ancient, unlearned approximate number sense. So, the sense of acuity affects later maths learning, while maths education enhances number sense acuity, and the extent to which tertiary factors can affect both. ANS is activated by group attention.

Brain imaging studies have identified the parietal lobe as being a key brain region for numerical cognition. Specifically within this lobe is the intraparietal sulcus which is "active whenever we think about a number, whether spoken or written, as a word or as an Arabic digit, or even when we inspect a set of objects and think about its cardinality" (Sousa 2010) . When comparing groups of objects, activation of the intraparietal sulcus is greater when the difference between groups is numerical rather than an alternative factor, such as differences in shape or size. This indicates that the intraparietal sulcus plays an active role when the ANS is employed to approximate magnitude.

Parietal lobe brain activity seen in adults is also observed during infancy during non-verbal numerical tasks, suggesting that the ANS is present very early in life. A neuroimaging technique, functional Near-Infrared Spectroscopy, was performed on infants revealing that the parietal lobe is specialized for number representation before the development of language. This indicates that numerical cognition may be initially reserved to the right hemisphere of the brain and becomes bilateral through experience and the development of complex number representation (Cantlon, 2006).

It has been shown that the intraparietal sulcus is activated independently of the type of task being performed with the number. The intensity of activation is dependent on the difficulty of the task, with the intraparietal sulcus showing more intense activation when the task is more difficult. In addition, studies in monkeys have shown that individual neurons can fire preferentially to certain numbers over others. For example, a neuron could fire at maximum level every time a group of four objects is seen but will fire less to a group three or five objects (Piazza, 2007). Memory and ANS seemed to influence early mathematics almost independently (Passolunghi, 2014)

1.4 Comparison between the OFS and the ANS systems

Beginning as infants, people have an innate sense of approximate number that depends on the ratio between sets of objects. Throughout life the ANS becomes more developed, and people are able to distinguish between groups having smaller differences in magnitude. The ratio of distinction is defined by Weber's law, which relates the different intensities of a sensory stimulus that is being evaluated. In the case of the ANS, as the ratio between the magnitudes increases, the ability to discriminate between the two quantities increases.

Today, some researchers theorize that the ANS lays the foundation for higher-level arithmetical concepts. Research has shown that the same areas of the brain are active during non-symbolic number tasks in infants and both non-symbolic and more sophisticated symbolic number tasks in adults. These results could suggest that the ANS contributes over time to the development of higher-level numerical skills that activate the same part of the brain.

These systems are further characterized by their contrasting limits (Feigenson et al. 2002). The imprecision of the approximate number system systematically increases as numerosity increases. As a result, the ability to estimate numerosity has no upper bound, but discrimination of any two numerical quantities follows Weber’s Law, as it is a function of the ratio between the two quantities to be compared. In contrast, the ability to simultaneously represent and track objects through parallel individuation is limited to only a few items. Representations through parallel individuation afford more fine-grained numerical discriminations than those of the approximate number system if numbers are within the range of this system, but comparisons fail when the limits of this system are surpassed.

The signature limits of ratio and capacity have allowed researchers to find evidence of the two numerical systems across age-groups and species, suggesting these systems are innate and arise very early in human infancy.

Other evidence, however, suggests that the approximate number system operates over both large and small numbers

To summarize the presently shared vision, most researchers agree that humans and many non-human animals possess both the ability to represent a set as an approximate numerical value or as distinct object files. Furthermore, there is accord that the ability to represent individual items in parallel is limited to only several items at a given moment. (Hyde, 2011)

Until recently, however, the field has disagreed as to the conditions that activate each system and whether these systems are specialized for small and large numbers. Recent work in psychophysics and, in particular, cognitive neuroscience, with non-human animals, human infants, and human adults has provided empirical evidence for the distinctness of these systems in the brain and a better understanding of the experimental conditions that elicit representations from each system. Furthermore, this recent work suggests a hybrid view of the "two systems" and "one system" views by delineating the conditions under which each system is engaged and the cognitive constraints that underlie this delineation.

1.5 Scientific evidence on animals

Many living organisms have two main systems, the object file and approximate number system to process numerosity.

A series of numerical comparisons, employing a procedure like the one already utilized and described elsewhere (Rugani, 2012) in which newly hatched domestic chicks were reared with five identical objects, were utilized in the study being described in (Rugani, 2012; Cantlon and Brannon 2006). Comparisons between 1 vs. 4 and 1 vs. 5 were assessed to evaluate the chicks' OFS storage capacity. To exclude the possibility that the chicks' performance could be explained by a mechanism distinguishing a singular versus a plural set, a comparison between 2 vs. 4 was also employed. Controls for quantitative cues were also performed to exclude the possibility that the discrimination could be based on non-numerical, continuous, physical variables, such as the overall amount of surface area or perimeter rather than on numerousness. The chicks underwent free-choice tests comparing two sets, each composed of a different number of objects. The chicks, as expected, preferred the larger set.

It is quite interesting and thought-provoking that human infants between 12 and 14 months (Feigenson, 2002) failed to discriminate between 1 vs. 4 crackers, while chicks can distinguish between sets of 1 vs. 4 objects. The diversity in performance cannot be explained by motivational factors linked to the kind of attractors that were used, since the chicks mastered the task when food as well as social attractors was utilized. The chicks' discrimination between 1 vs. 4 could suggest that an OFS is able to process larger groups, including up to 4 elements for each set. If that would be the case, the chicks should have failed when they were presented with the 1 vs. 5 comparisons. (Rugani, 2012; Feigenson, 2002)

The fact that the chicks were able to discriminate between 5 and 1 disproves this hypothesis, leaving two different alternative explanations: the discrimination could be supported by an OFS with a larger capacity capable of processing up to 5 elements per set. This seems unlikely because the signature limit of that system in animals has been reported at around 3 or, at most, 4 (Feigenson, 2005). Consistent with these findings, chicks have been found able to discriminate up to 3 items per set during a task in which discrimination was presumably based on OFS (Rugani, 2008).

Regarding the second possible explanation, the chicks' performance could be explained by a mechanism that discriminates between sets composed, on the one hand, by a singular entity and, on the other, by plural entities. A system that allows discrimination of numerical values only when singular and plural sets (1 vs. 2 and 1 vs. 5) and not when two plural sets (2 vs. 3, 2 vs. 4 and 2 vs. 5) are compared has been described in rhesus monkeys (Barner, 2008). The chicks, however, succeeded in comparing 2 vs. 4, thus disproving this hypothesis. The fact that the chicks' performance remained above chance levels even when continuous physical variables were being controlled indicates that this type of discrimination could be based on numerical cues. For what concerns the 2D stimuli, it should be noted that even if the performance remained statistically above chance, the percentage of correct responses are lower. This is due to the nature of the stimuli employed and not on a lack of discrimination.

A decrement of the performance was indeed found also in experiments carried out on arithmetic abilities in chicks (Rugani, 2009). Such evidence was also supported by similar data collected in these species. For example, it has been shown that chicks prefer two-dimensional pictures depicting possible three-dimensional objects rather than impossible versions of those same objects (Regolin et al. 2011). Neither the OFS nor the singular versus plural system seem to be capable of explaining all the literature data, suggesting that the ANS is involved in making the 2 vs. 4 discriminations. A significant collection of data on chicks (Rugani 2008, 2009, 2012; Garland 2012), non-human primates (Beran, 2001; Merritt et al. 2009), 7-month-old infants (Cordes, 2009), preschool and school children (De Hevia, 2009) supports this hypothesis.

CHAPTER 2

Methods and Results

2.1 Objectives

Recent studies reported how humans may share with others animals a subset of non-verbal numerical skills. Those skills are thought to be based on two systems: the object file system (≤3) and the approximate number system which processes larger numbers (≥4) REF. Infants could discriminate 1 vs 2, 1 vs 3, 2 vs 3 but not 1 vs 4, and large numerosities thus it seems that the systems are independent, implying that the concept of continuous number processing is interrupted at or about the number four (Rugani, 2012). Recent results demonstrate that chicks are able to make discrimination regarding the borderline number 4 (Rugani, 2012). This thesis describes the experimental research carried out following this discovery and is based on testing this ability in toddlers, to assess if they share these skills with chicks.

We started testing human infants with two types of what we call "games", divided in paper games (4 different subtypes) and computer-based games (4 other different subtypes). Paper games were identified and administered to achieve a better understanding of the numerical skills background of the child, while the 4 computer-based games, are the core tests that have been devised to target the purpose of the experiment.

Those types of tests are based on the types of test that where administered to the chicks in the lab (Rugani, 2009).

2.2 Methods

Both paper and computer-based games were administered to 66 children of age between 3 and 6 years, in four different schools. Before administering the tests parents were required to sign an informed consent declaration relative to the collected data handling. Participants underwent the experiment in two separate rooms, one devoted to paper-based games and the other to computer-based ones. Children were administered all the foreseen games by switching rooms. At the end of the daily testing session each participant underwent two paper games and two computer-based ones out of eight, for avoiding overloading their attention. The total procedure lasted two days for each child.

The paper games where administrated for a better comprehension of the numerical skills background of the child and for a deeper investigation of their cognitive capabilities.

I will report in the following a brief explanation of each paper game and follow with the computer-based ones.

Paper-based games:

1- *TNL (test neuropsicologico lessicale- neuropsychological lexical test)* a language task where the experimenter presents to the participant 50 paper sheets; each one has four images on it. The experimenter will pronounce the name of one the images represented on the sheet and note if the child could recognize the image associated with the word. The final score was calculated by giving one point for each correct answer and zero for the incorrect ones, giving a sum out of fifty (Cossu, 2013).

2- “Modify Bells Test”. This game consisted of four paper sheets with many little different images and among them there where the little bells. The experimenter calculated how many bells the child could detect in 30 seconds, in 60s, in 90s, and 120s that is the overall time (2m). This was repeated for each sheet. For the final score the experimenter summed all the result in the 30 seconds per all the four sheets and also all the results of the 120 seconds per each sheet, after having calculated all the results, he will compare the score to a statistic table, noticing if the child is on, above or below average (Biancardi, 1997).

The third and fourth games were the only games that had symbolic arithmetic tasks.

3 – “Protocollo memoria uditiva*” (*Auditory memory protocol) in the third game the experimenter asked to calculate from one to twenty and backwards from ten to one, the final score is based on how many numbers the child could count back and forward (D’Amore, 1993).

4 – “Batteria per la valutazione dell’Intelligenza Numerica” (BIN) in English this stands for “Numerical intelligence valuation Battery” The fourth game is based on 20 sheets where ten of them had represented three different numbers from 1 to 10 and the other sheets had two dices with different dot numbers. The participant has to answer which one is the biggest. On both games number 3 and 4, the final score is calculated based on how many sheets the child answers correctly. (Molin, 2007)

Computer-based games

1 -*Card game.* In this game the experimenter should tell the participant a background story for a better understanding and motivation of the main goal of the game. The focus of the game is to identify the target number and the position of card, if it is right or left. If the child chooses the correct card, as a prize a simulation of fireworks will start, appearing in the background. If he/she chooses the wrong one nothing will happen. At the end of the task the results are uploaded automatically in an excel document.

2- *Number line bisection.* In this game the participant had to solve different tasks based on positioning. By touching a ladybug when it appears on the screen, the ladybug will smile to create a bond with the child. After a few sequential appearances and touches, a line will appear, and the child should touch its middle. At first the middle of the line is highlighted on the screen, then the child must identify it on his own. If the child properly touches the middle of the line a smiling ladybug appears. The results of this tasks are uploaded to a file excel automatically.

3- *Train game*. This consisted in dragging two trains into the middle of the line like the game before but with the difference that they should drag the two objects halfway and not only touch them. The trains aren't really trains but have a shape of a head of a train. This game has been recently updated and changed and was tested only 20 children.

4 – *Arithmetic task*. Its focus is to select the right number of confetti that the participant sees entering in a box appearing onscreen. When two boxes appear with different numbers of confetti then the participants should choose the right box, the results are upload automatically in an excel file like the others.

I have personally taken part in administering all the above games, both the paper and the computer-based ones. In my thesis I will discuss in greater detail the computer-based arithmetic task, in the next paragraph.

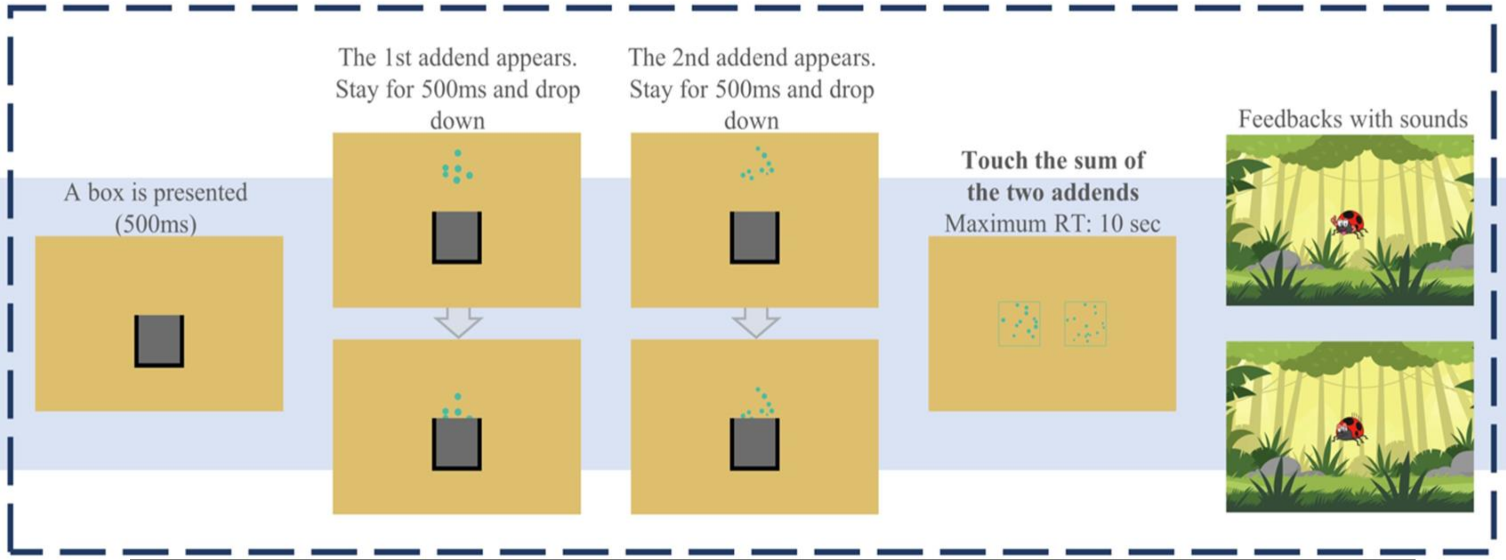
2.3 Procedure of the Arithmetic task (computer-based game number 4)

In this game the participant was asked to seat in front of a computer (average eye-screen distance is 30 cm), then the experimenter activates the game inserting the account number of the participant. At the beginning a short animation of confetti/dots appears to make the child familiarize with the concept and the animation (see Figure 1). The stimuli are created with PSHYCOPY software version 2022.2 (Peirce 2007). At the beginning of the task, a black box (2x2 cm^2) appears in the center of the screen, with its center located at the position (0,-2), center of the square is (0,0) that means the box its two centimetre below the center, for 500ms. The participant touches it, and by doing so it grows bigger (up to 6x6 cm^2). The shape and relative distance of confetti/dots are randomized; the total perimeter was kept the same (15 cm) for all the dots arrays area 5x5cm^2. The dots enter the box from the top of the screen in 500 ms, we will call this group "Add1".

Figure 1: Schematic representation of the Arithmetic task game: Screenshots that are shown to the child during the process of the game are illustrated. From left to right: the first black box, the entering of Add1 and Add2, the 2 boxes, one with Add1+Add2 and the other with the Distractor, finally the graphic images that the participant saw during the test

The RGD colours of the background and of the confetti/dots entering the box are reported in table I.

Table I



Dots color	**Background color**
[0.6549, -0.2549, 0.4353]	[0.9922, 0.9922, -0.2314]
[-0.2706, -0.5451, 0.2157]	[0.8040, 0.0559, -0.1510]
[-0.4980, 0.3804, 0.3020]	[0.7647, 0.4902, -0.1686]
[-0.9059, -0.0353, 0.7255]	[1.0000, 0.5216, -0.9216]

After 500ms a second group of confetti/dots will drop down into the box, we will call this group Add2. The child should try to identify how much confetti/dots are in the black box in total (Add1+Add2). Then, two boxes will automatically appear (5x5 cm^2 in size) located one next to the other, the right one at [4,0] and the left one and [-4,0]. One of the boxes contains the exact number of confetti present in the first box (Add1 +Add2) and the other a wrong one (Distractor). The child has 10 seconds to decide which one contains the total number of confetti seen in the first single box.

The numbers of confetti/dots dropping in Add1 and Add2 together with the relative Distractor are reported in Figure2 that reports the whole set of data relative to a typical complete test. An important parameter is the Ratio between the outcome (Add1+Add2) and the Distractor. The following ratios have been tested: 0.33- 0.5 - 0.67 - 0.8, by choosing appropriate yet different Add1, Add2 and Distractors values. The smaller the Ratio value the greater the difference between the number of confetti in the two boxes next to each other.

The experimenter will explain the game to the game to the child, structured by the following phases:

PHASE 1: This is a practice test, carried out to familiarize the child with the procedure of the game. This part of the game will have 8 trials, divided in two groups because in the middle there will be a little break to see if the child is tired and if he can keep the concentration. In this practice test only a single drop of confetti/dots (Add 1) will be shown.

PHASE 2: This is the actual test, consisting in 15+15 trials, carried out after the child had completed PHASE 1, the practice test. Basically, what changes with respect of PHASE 1 is that here both Add1 and Add2 actions will take place. As before, the child must identify which one of the two boxes appearing next to each other contains the same number of confetti as the first box. After this phase the game will finish giving a percentile of the final score obtained by the child.

add1	add2	outcome	distractor	ratio
5	5	10	30	0.333
5	9	14	42	0.333
6	7	13	39	0.333
6	8	14	28	0.5
5	7	12	24	0.5
8	9	17	34	0.5
7	9	16	24	0.667
9	9	18	27	0.667
7	5	12	18	0.667
6	6	12	15	0.8
7	9	16	20	0.8
5	8	12	15	0.8
7	5	12	4	0.333
9	9	18	6	0.333
8	7	15	5	0.333
5	5	10	5	0.5
7	5	12	6	0.5
8	6	14	7	0.5
7	5	12	8	0.667
5	4	9	6	0.667
9	9	18	12	0.667
8	7	15	12	0.8
9	9	18	14	0.8
9	6	15	12	0.8
1	1	2	3	Small Numb
1	2	3	2	Small Numb
3	2	5	4	Small Numb
2	4	6	8	Small Numb
3	4	7	9	Small Numb
4	1	5	3	Small Numb

Figure 2: Example of a simplified data set obtained from a test administered to a participant with the Add1, Add2, distractor, and ratio. See an example of complete data set in Figure 3

While the participant is playing the game, sometimes it could happen that he/she gets distracted. In these cases, the experimenter should use words like "good job, you are doing great, etc" to encourage him/her. Sometimes it also happened that the touch of the computer fails, and the child should be encouraged by saying "try again, good, nice work, go on". This game is also one of the longest of all the games described above and administered in this research. What I noticed, almost in all the tests I administered, is that near the end the children are not only very tired but also bored. The experimenter had to provide a significant amount of encouragement and support to have the child finish the task.

At the end of the task the participant had completed 38 trials (8 practice and 30 actual tests), and the final score will be automatically uploaded on an excel file that is here reported in Appendix 1. The data elaboration and the results obtained will be discussed in the next paragraph.

2.4 Results of Arithmetic task (computer game base number 4)

The data relative to the tests I have administered to 66 children are reported in Appendix 1. Here I show in Figure 3, as an example, the full set of data present in the excel file for a single child. The first 8 rows report the results of the 8 practice tests while the following 30 rows report the results of the actual 30 training tests. The full set of data provide information about the numerical data sets (Add1, Add2, distractor and ratio), the Response time (RT) the child gender and age and details if the child choice is correct or wrong.

participant	session	add1	add2	outcome	distractor	ratio	ratio2	trials.thisN	order_add	order_resp	RT	choice	data_file	gender	ageY	correct
1	practice	10	10	10	30	0.33	0.333333	0	1	[-1, 1]	7.105476	wrong	1_arithme	Male	3Y	0
1	practice	18	18	18	14	0.8	0.777778	1	1	[1, -1]	2.541343	correct	1_arithme	Male	3Y	1
1	practice	15	15	15	5	0.33	0.333333	2	2	[1, -1]	7.867103	wrong	1_arithme	Male	3Y	0
1	practice	17	17	17	34	0.5	0.5	3	2	[-1, 1]	2.284446	wrong	1_arithme	Male	3Y	0
1	practice	16	16	16	24	0.67	0.666667	4	1	[1, -1]	1.64175	wrong	1_arithme	Male	3Y	0
1	practice	12	12	12	15	0.8	0.8	5	2	[-1, 1]	9.742667	correct	1_arithme	Male	3Y	1
1	practice	9	9	9	6	0.67	0.666667	6	2	[-1, 1]	3.046069	wrong	1_arithme	Male	3Y	0
1	practice	14	14	14	7	0.5	0.5	7	1	[-1, 1]	3.323462	correct	1_arithme	Male	3Y	1
1	testing	7	5	12	4	0.33	0.333333	0	1	[-1, 1]	2.536002	wrong	1_arithme	Male	3Y	0
1	testing	7	9	16	20	0.8	0.8	1	1	[1, -1]	1.286056	wrong	1_arithme	Male	3Y	0
1	testing	1	1	2	3	SNS	None	2	2	[1, -1]	2.671769	correct	1_arithme	Male	3Y	1
1	testing	5	4	9	6	0.67	0.666667	3	2	[-1, 1]	2.397717	wrong	1_arithme	Male	3Y	0
1	testing	8	7	15	12	0.8	0.8	4	2	[-1, 1]	1.631055	wrong	1_arithme	Male	3Y	0
1	testing	9	9	18	27	0.67	0.666667	5	2	[-1, 1]	1.38913	correct	1_arithme	Male	3Y	1
1	testing	4	1	5	3	SNS	None	6	1	[1, -1]	2.110422	wrong	1_arithme	Male	3Y	0
1	testing	8	9	17	34	0.5	0.5	7	2	[1, -1]	1.687126	correct	1_arithme	Male	3Y	1
1	testing	9	9	18	6	0.33	0.333333	8	2	[-1, 1]	0.416157	correct	1_arithme	Male	3Y	1
1	testing	6	7	13	39	0.33	0.333333	9	1	[-1, 1]	1.885886	wrong	1_arithme	Male	3Y	0
1	testing	3	4	7	9	SNS	None	10	2	[-1, 1]	3.127884	correct	1_arithme	Male	3Y	1
1	testing	3	2	5	4	SNS	None	11	2	[-1, 1]	2.375481	wrong	1_arithme	Male	3Y	0
1	testing	5	5	10	30	0.33	0.333333	12	1	[1, -1]	2.201119	correct	1_arithme	Male	3Y	1
1	testing	6	8	14	28	0.5	0.5	13	1	[1, -1]	1.849171	correct	1_arithme	Male	3Y	1
1	testing	9	9	18	14	0.8	0.777778	14	1	[1, -1]	6.759949	correct	1_arithme	Male	3Y	1
1	testing	9	9	18	12	0.67	0.666667	15	1	[-1, 1]	3.804626	wrong	1_arithme	Male	3Y	0
1	testing	7	5	12	8	0.67	0.666667	16	1	[1, -1]	2.089485	correct	1_arithme	Male	3Y	1
1	testing	5	5	10	5	0.5	0.5	17	1	[-1, 1]	NA	no	1_arithme	Male	3Y	0
1	testing	5	8	12	15	0.8	0.8	18	1	[1, -1]	1.675208	wrong	1_arithme	Male	3Y	0
1	testing	2	4	6	8	SNS	None	19	1	[1, -1]	1.998609	correct	1_arithme	Male	3Y	1
1	testing	5	7	12	24	0.5	0.5	20	1	[-1, 1]	2.693782	correct	1_arithme	Male	3Y	1
1	testing	9	6	15	12	0.8	0.8	21	2	[-1, 1]	3.531247	correct	1_arithme	Male	3Y	1
1	testing	7	9	16	24	0.67	0.666667	22	1	[1, -1]	4.384328	wrong	1_arithme	Male	3Y	0
1	testing	7	5	12	6	0.5	0.5	23	2	[-1, 1]	2.214189	wrong	1_arithme	Male	3Y	0
1	testing	8	7	15	5	0.33	0.333333	24	1	[-1, 1]	1.618804	wrong	1_arithme	Male	3Y	0
1	testing	8	6	14	7	0.5	0.5	25	2	[1, -1]	6.114954	correct	1_arithme	Male	3Y	1
1	testing	1	2	3	2	SNS	None	26	2	[1, -1]	1.715225	correct	1_arithme	Male	3Y	1
1	testing	5	9	14	42	0.33	0.333333	27	2	[1, -1]	2.386968	correct	1_arithme	Male	3Y	1
1	testing	7	5	12	18	0.67	0.666667	28	2	[-1, 1]	4.46785	correct	1_arithme	Male	3Y	1
1	testing	6	6	12	15	0.8	0.8	29	2	[1, -1]	3.997976	wrong	1_arithme	Male	3Y	0

Figure 3: A complete example of the data set collected from one participant, also including the age and gender of the participant, the distractor position, the colour, the choice and the response time.

The actual data set I have elaborated with my research team for each child is reported in Figure 3. The meaning of the various columns and relative data is detailed in the following:

Add1: the number of confetti in the first drop into the box

Add2: the number of confetti in the second drop into the same box, adding up to the first drop

Outcome: the total number of confetti dropped in the first and second drop (the sum of Add1 and Add2)

Distractor: the number of confetti appearing into a second box next the one containing Add1+Add2

Ratio: the ratio between the outcome and the distractor. The following ratios have been tested: 0.33 - 0.5 - 0.67 - 0.8. The smaller the value the greater the difference between the number of confetti in the two boxes next to each other

Small numbers (SNS): tests where the number of confetti in Add1 and Add2 is ≤4 and the ratio is not relevant.

The data of all 66 tested children have been analysed, except for a total of 6.25% of the data, that have been removed for the following reasons: i) no response (28); ii) RT higher than the high inner fence of the whisker (Q3+1.5IQR; 8); iii) RT smaller than 500 (6)

We used the Kruskal-Wallis rank sum test and we analysed the effect of ratio and the effect of age, the P value adjustment method is the false discovery rate.

The Kruskal-Wallis test is a non-parametric procedure that can be used to compare more than two populations in a completely randomized design. It is used for testing whether samples originate from the same distribution and allows to compare their median. It can be used for comparing two or more independent samples of equal or different sample sizes (Kruskal–Wallis 1952) Since it is a nonparametric method, the Kruskal–Wallis test does not assume a normal distribution of the residuals, unlike the analogous one-way analysis of variance. If the researcher can make the assumptions of an identically shaped and scaled distribution for all groups, except for any difference in medians, then the null hypothesis is that the medians of all groups are equal, and the alternative hypothesis is that at least one population median of one group is different from the population median of at least one other group.

A large amount of computing resources is required to compute exact probabilities for the Kruskal–Wallis test. Our data have been computed by R-STUDIO software, version 3.3.3.

The Table II below reports the results:

	χ^2	**Df**	**p-value**	ε^2
Effect Of Ratio	17.926	4 (0.33-0.5-0.67-0.8)	0.001276	0.0553
Effect Of Age	16.535	3 (3Y-4Y-5Y)	0.000881	0.051

The results are plotted in Figure 4 that groups the results for the age of the children (3Y- to 6Y)

Figure 4A reports in each graph the accuracy of the response versus the 4 different ratios tested. The last point on the X-axis shows the results relative to the Small Numbers tests. A few major comments can be drawn by examining these graphs:

1- The accuracy lies roughly around 50% for all age groups (except 6Y)
2- *Effect of Age:* the accuracy of the responses is lower for group 3Y than for all the other groups (4>Y, 5Y and 6Y)

3- *Effect of Ratio*: The accuracy of the responses for ratio=0.33 have always a greater accuracy than those for ratios 0.67, 0.8 and also SNS

4- *Effect of Ratio:* The accuracy of the response for ratios=0.5 are always greater than those for ratio=0.67

5- *Effect of Ratio/Age*: The accuracy of the responses for ratio= SNS is always quite low, independently on the age group. Even for group 6Y it is lower than the smaller ratios reported (0.33 and 0.5)

Figure 4B reports in each graph the response time (RT) in milliseconds, i.e. the time it took each child to provide a response, either wrong or correct. Correct responses are indicated as light blue bars while wrong ones are the dark blue bars. The major comments that can be drawn from these graphs are:

1- *Effect of Age:* the group of 3Y has on average the longest response times.

2- *Effect of Age:* the response time is comparable for age groups 4Y, 5Y and 6Y.

3- *Effect of Ratio:* the response time for the ratio=SNS is roughly comparable to the other ratios for each age group,

4- *Effect of Ratio:* in the 3Y group the ratio=SNS has a predominancy of wrong responses, while in the 4Y, 5Y and 6Y groups correct answers dominate.

5- *Effect of Ratio* : in the 6Y group the RT for the ratio= SNS is higher than for the other teste ratios.

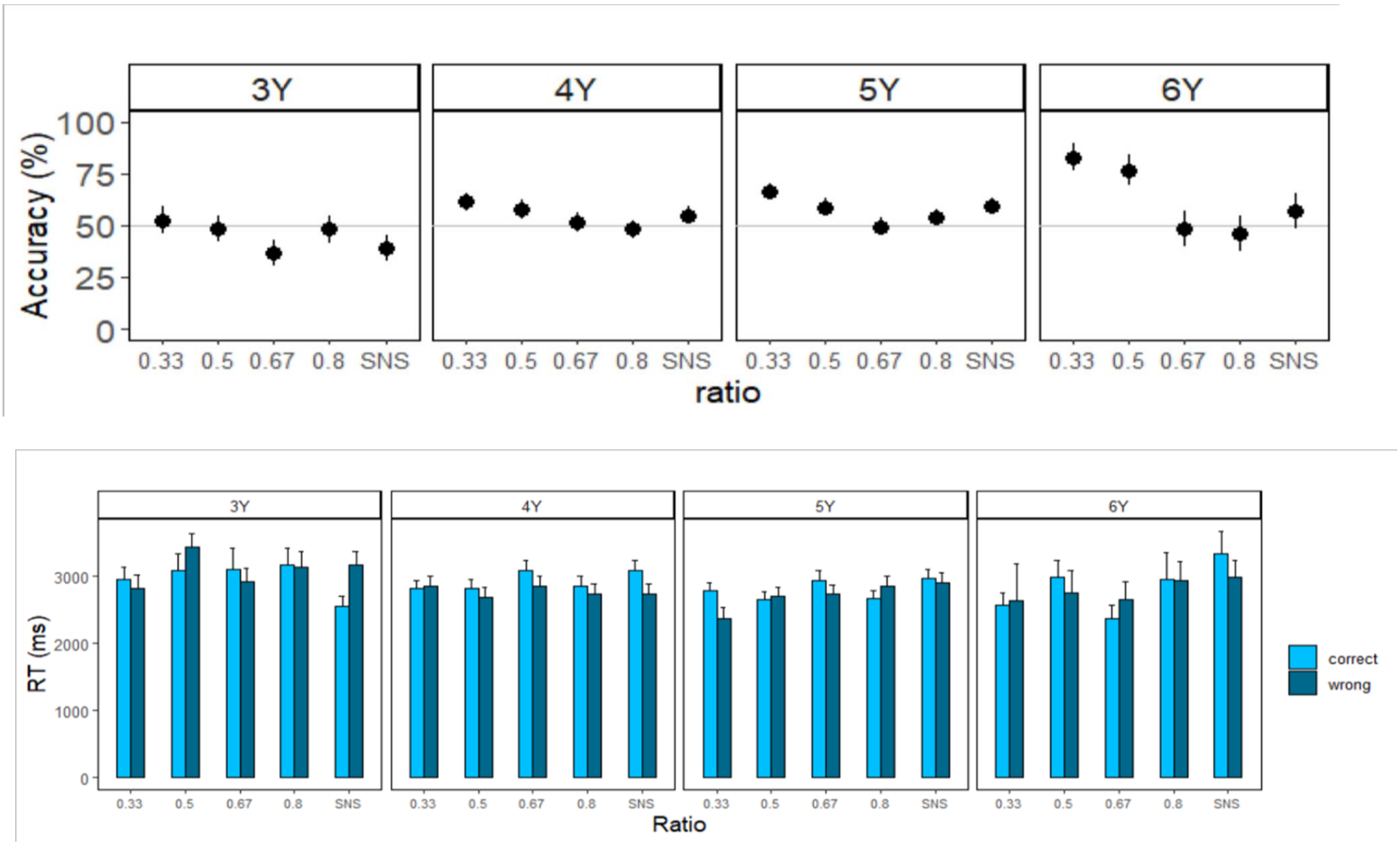


Figure 4: A) All the responses collected in the research campaign are represented in 4 graphs reporting the accuracy of the response as a function of the ratio. Each graph contains the results relative to a single age group (from 3Y to 6Y); B) The response time (RT) of the participant is reported as a function of the ratio, again in 4 graphs, each one for a single age group (from 3Y to 6Y).

CHAPTER 3

Discussion and Conclusions

The experimental research described in Chapter 2 allowed to collect original data relative to the response to a non-symbolic arithmetical task of over 60 children of age between 3 and 6 years.

Multiple tests were administered to each child both paper-based and computer game –based (as detailed in paragraph 2.2). In particular, the paper-based ones were administered to assess the cognitive capabilities of the child and to gain a better understanding of his/her mathematical background.

Each child had to complete 2 paper games and 2 computer games each day and he/she was engaged in two days in a row. This resulted to be quite a stressful condition for the children who ended up being very tired.

I personally administered the Arithmetic task game (see paragraph 2.3) which consists of 8 practices and 15 actual tests and is thus the longest one. I experienced by direct observation that it was often difficult to maintain the child attention during its execution, due to the long time needed to complete it. The experimenters had to exert great care and attention in encouraging and engaging the child to grant the reliable acquisition of consistent responses.

It is noteworthy that the children understood quite quickly the functioning and the flow of this test.

The collected data show a few interesting trends that can be correlated to the role and function of the object file system and approximate number system. Figure 4a shows how younger children on average provide responses whit lower accuracy then older ones (his could be a possibly related to the shorter concertation span of younger children).

The accuracy in the response for smaller ratios is always higher than for higher ratios (0.33>0.67,0.8) and also for SNS, for all age groups. Following the results recently reported and discussed in the literature (See chapter 1) I think that these results do not provide a particularly significant and novel insight on the different role and function of OFS and ANS.

On the contrary, I believe that the results reported in figure 4b are intriguing. The response time (RT) is consistently higher for the small numbers (SNS) for all age groups. The RT does not improve with age for SNS ratio. An interpretation of such an observation could invoke the more difficult activation of the OFS with respect to ANS at any age even for older children. This might suggest that humans can further develop the ANS system with age while the OFS maintains its operational capability almost constant up to at least six years of age.

This work was carried out as part of a larger research project led by Rosa Rugani and Silvia Benavides

Pietro Benfenati contribution is restricted to a part of the project adapted to meet the requirement criteria for the bachelor mandatory internship.

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