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Research article

Distributed adaptive control for nonlinear multi-agent systems with nonlinear parametric uncertainties

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Abstract: This paper considers the distributed tracking control problem for a class of nonlinear multi-agent systems with nonlinearly parameterized control coefficients and inherent nonlinearities. The essential of multi-agent systems makes it difficult to directly generalize the existing works for single nonlinearly parameterized systems with uncontrollable unstable linearization to the case in this paper. To dominate the inherent nonlinearities and nonlinear parametric uncertainties, a powerful distributed adaptive tracking control is presented by combing the algebra graph theory with the distributed backstepping method, which guarantees that all the closed-loop system signals are global bounded while the range of the tracking error between the follower's output and the leader's output can be tuned arbitrarily small. Finally, a numerical example is provided to verify the validity of the developed methods.

Keywords: distributed adaptive tracking; nonlinear multi-agent systems; nonlinear parametric uncertainties; unstable linearization; global bounded; arbitrarily small

1. Introduction

Uncertainty is an important consideration when discussing a control system with satisfactory performance [1, 2]. As the most common uncertainty, parametric uncertainty exists in various practical control problems, for which a large number of integrated theories have been obtained for compensating this uncertainty over past decades, such as [3–8]. Unlike the parametric uncertainty in nonlinearities, the case in the control coefficients means that the range of the controller can not be determined.

Cooperative control of multi-agent systems (MASs) with parametric uncertainty has received considerable attention in recent years for constructing distributed controllers to ensure that all agent reaches consensus on each other (leaderless consensus) or the leader (leader-following consensus) in [9, 10] and [11–14]. Specifically, the global full-state synchronization of networks of nonidentical Euler-Lagrange systems with a linear parametrization is achieved in [9]. By employing the Artstein model reduction method, robust consensus control is completed in [10], where the parametric uncertainty is illustrated by the uncertain time-varying system matrices with known bound. By the distributed model reference adaptive control, the consensus problem is investigated for a group of linear subsystems with unknown parameters in [11]. Especially, the bounds of the unknown parameters are not required. In [12–14], the backstepping-based consensus tracking control schemes are presented for parametric strict-feedback systems. However, these results are inapplicable for the case that the system with inherent nonlinearities, i.e., the system with uncontrollable unstable linearization. To our knowledge, no investigation is available for distributed tracking control of nonlinear multi-agent systems with inherent nonlinearities and nonlinear parametric uncertainties both in the control coefficients and in the system nonlinearities.

This paper is devoted to the powerful compensation for nonlinear parametric uncertainties and to the distributed adaptive tracking control for a class of nonlinear multi-agent systems with rather inherent nonlinearities. The generality of the systems and the challenge of the control design distinguishing this paper from existing techniques are illustrated by that: (1) The nonlinear parametric uncertainties appear in the control coefficients, which, however, is removed in the related works [12–21]. Besides, the unknown parameter vector is without known bounds rather than that in [10]. How to skillfully separate the parametric uncertainty from the control coefficients is an essential step, and how to cancel the influence of the parametric uncertainty for the control is a significant work. (2) The considered systems are inherent nonlinearities unlike those in [12–14, 22], which renders us to search for a powerful tracking control scheme to deal with the inherent nonlinearities. (3) Different from the results for single systems such as [6, 7, 23, 24], the MASs motivate us to design a distributed tracking control based on not only the individual dynamics of the systems but also the interconnection pattern among them. A direct result is more technical Lyapunov function construction and more complex calculation.

In this paper, to compensate the nonlinear parametric uncertainties and realize the tracking aim for a class of nonlinear MASs, a distributed adaptive tracking control scheme is raised by combing the algebra graph theory with the distributed integrator backstepping method. Specifically, a parameter separation technique is adopted first, based on which we separate out the unknown parameter vector from unknown control coefficients and unknown nonlinearities. Then, on the basis of the algebra graph theory, we generate a new variable from the tracking error with desirable properties, which uses the relative state information between the leader and the agents, and is a foundation for the control design and performance analysis later. Finally, by the distributed integrator backstepping method, we give the update law to accurate estimate the unknown parameter and the explicit controller for the MASs in an iterative manner. Note that during the design scheme, the most important is to cancel the effects of the unknown coefficients and the inherent nonlinearities.

The rest of the paper is organized as follows. Section 2 presents some preliminary knowledge and the problem formulation. Section 3 provides the distributed tracking controllers. Section 4 summarizes the verification for the performance of the closed-loop system. Section 5 offers a numerical example, and Section 6 gives some concluding remarks.

2. Preliminary knowledge and problem formulation

This paper aims to the distribute tracking control design for a class of high-order nonlinear MASs. In this section, we will give the specific MASs structure and problem formulation. Before that, we first introduce some graph theories and technical lemmas which are foundational for understanding the

MASs and play an important role for the control design of this paper.

2.1. Graph theory and technical lemmas

Let a weighted digraph of order n be $\mathcal{G} = (\nu, \varepsilon, A)$, where the set of nodes, the set of arcs and a weighted adjacency matrix are respectively defined as $\nu = \{1, 2, \ldots, n\}$, $\varepsilon \subset \nu \times \nu$ and $A = (a_{ij})_{n\times n}$ with nonnegative elements. The agent j directly sends information to agent i is represented by $(j, i) \in \varepsilon$, in which case, j is called the parent of i, while i is called the child of j. $\mathcal{N}_i = \{j \in \varepsilon : (j, i) \in \varepsilon, i \neq j\}$ denotes the set of neighbors of vertex x. $a_{ii} > 0$ if node j is a neighbor of i, and otherwise, $a_{ii} = 0$. If node i has neither parent nor child, it is called an isolated node, and if it has no parents but children, then the node is called a source. Denote the sets composed by all sources and isolated nodes in ν as $\nu_s = \{j \in \nu \big| \mathcal{N}_j = \emptyset, \emptyset \text{ is the empty set}\}$. To avoid the trivial cases, $\nu - \nu_s \neq \emptyset$ is always assumed in this paper. A sequence $(i_1, i_2), (i_2, i_3), \ldots, (i_{k-1}, i_k)$ of edges is called a directed path from node i_1 to i_k. A directed tree is a digraph that every node (except the root) has exactly one parent and the root is a source. A spanning tree of $\mathcal{G}$ is a directed tree with the node set being ν and the edge set being a subset of ε. The diagonal matrix $D = diag(\kappa_1, \kappa_2, \ldots, \kappa_n)$ is the degree matrix with $\kappa_i = \sum_{j\in\mathcal{N}_i} a_{ij}$. The Laplacian of a weighted digraph $\mathcal{G}$ is defined as $L = D - A$.

In this paper, we consider a MAS with N agents and a leader (labeled by 0), which is depicted as $\bar{\mathcal{G}} = (\bar{\nu}, \bar{\varepsilon})$ with $\bar{\nu} = \{0, 1, 2, \ldots, n\}$ and $\bar{\varepsilon} \subset \bar{\nu} \times \bar{\nu}$. If $(0, i) \in \bar{\varepsilon}$, then $0 \in \mathcal{N}_i$. $B = diag(b_1, b_2, \ldots, b_N)$ is the leader adjacency matrix associated with $\bar{\mathcal{G}}$, where $b_i > 0$ if node 0 is a neighbor of i, and $b_i = 0$ otherwise.

We next cite two lemmas which are frequently used in the later sections. In fact, the proofs of the two lemmas can be found in [25] and [23] with detailed proof, respectively.

Lemma 2.1. *If $p > 0$, $q > 0$ and $c > 0$, then for $\forall x, y \in \mathrm{R}$,*

$$|x|^p|y|^q \le c|x|^{p+q} + \frac{q}{p+q}\Big(\frac{p}{c(p+q)}\Big)^{\frac{p}{q}}|y|^{p+q}. \tag{2.1}$$

Furthermore, if $p \in \mathrm{R}^{odd}_{\ge 1} = \{\frac{m_1}{m_2}\big| m_1$ and m_2 are odd positive integers and $m_1 \ge m_2\}$, then

$$|x^p - y^p| \;\le\; p|x - y|(x^{p-1} + y^{p-1}). \tag{2.2}$$

Lemma 2.2. *For any real-valued continuous function $f(x, y)$ with $x \in \mathrm{R}^m$ and $y \in \mathrm{R}^n$, there exist smooth scalar functions $a(x) \ge 1$ and $b(y) \ge 1$, such that*

$$|f(x, y)| \;\le\; a(x)b(y). \tag{2.3}$$

Notably, Lemma 2.2 provides an effective coupling of a parameter separation technique, which plays an important role to cope with unknown control coefficients and unknown system nonlinearities as shown later.

2.2. Problem formulation

This paper consider the tracking problem of the following nonlinear MASs with N followers and one leader (labeled by 0), and the i-th agent being described as:

$$\begin{cases} \dot{x}_{ij} \;=\; d_{ij}(\bar{x}_{ij}, \theta_i)x_{i,j+1}^{p_{ij}} + f_{ij}(\bar{x}_{ij}, \theta_i), \quad 1 \le j \le n_i - 1, \\ \dot{x}_{i,n_i} \;=\; d_{i,n_i}(x_i, \theta_i)u_i^{p_{i,n_i}} + f_{i,n_i}(x_i, \theta_i) \\ y_i \;=\; x_{i1}, \end{cases} \tag{2.4}$$

where $\bar{x}_{ij} = (x_{i1}, x_{i2}, \ldots, x_{ij})^{\mathrm{T}} \in \mathrm{R}^j$; $x_i = (x_{i1}, x_{i2}, \ldots, x_{i,n_i})^{\mathrm{T}} \in \mathrm{R}^{n_i}$ is the system state of the i-th agent with the initial condition $x_i(0)$; $u_i \in \mathrm{R}$ and $y_i \in \mathrm{R}$ are the control input and system output of the i-th agent, respectively; $p_{ij} \in \mathrm{R}^{odd}_{\geq 1}$ are known numbers while $\theta_i \in \mathrm{R}^m$ and d_{ij}'s are unknown; the system nonlinearities f_{ij}'s are smooth with satisfying $f_{ij}(0, \ldots, 0, \theta_i) = 0$. The leader's output is denoted by y_0. Particularly, d_{ij}'s have known signs, and without loss of generality, assume that d_{ij}'s are positive..

System (2.4) is necessarily to be investigated, which is displayed in two aspects: (i) as mentioned in [23], nonlinear parameterization can be found in various practical control problems such as biochemical processes and machines with friction; (ii) there is very few results available about nonlinear MASs control due to the complex of the system structure, let alone those with parametric uncertainty.

Notably, unknown parameter vector θ_i exists both in the system coefficients and the nonlinearities, which means that serious nonlinear parameter uncertainties are allowed in the considered system. This makes system (2.4) essentially different from the systems considered in [14] and [16] with known control coefficients, and challenges the distributed control design of this paper.

This paper is devoted to design a distributed adaptive controller u_i for agent i to guarantee the globally bounded signals of all the closed-loop system with adjustable $|\delta_i(t)| = |y_i(t) - y_0(t)|$ being arbitrarily small.

In what follows, we make the following assumptions on system (2.4):

Assumption 2.1. *$d_{11} = d_{21} = \ldots = d_{N1}$ and $p_{11} = p_{21} = \ldots = p_{N1} = 1$.*

Assumption 2.2. *For $j = 2, \ldots, n_i$ and $i = 1, \ldots, N$, there exist smooth functions $\underline{d}_{ij}(\bar{x}_{ij})$ such that*

$$0 < \underline{d}_{ij}(\bar{x}_{ij}) \leq d_{ij}(\bar{x}_{ij}, \theta_i).$$

Assumption 2.3. *The leader's output $y_0(t) \in \mathrm{R}$ and $\dot{y}_0(t)$ are bounded, and there are available for the i-th agent satisfying $0 \in \mathcal{N}_i (i = 1, \ldots, N)$.*

Assumption 2.4. *The leader is the root of a spanning tree in $\bar{\mathcal{G}}$.*

$p_{i1} = 1$ in Assumptions 2.1 is satisfied in a class of mechanical systems as stated in [16], which means that system (2.4) is of practical value. $d_{11} = d_{21} = \ldots = d_{N1}$, as shown in the following control design, plays an important role to guarantee that x^*_{i2} is well-defined. Assumptions 2.3 and 2.4 are in common with the assumptions in [16] for designing the distributed tracking control of nonlinear MASs. By Assumption 2.4, we can furthermore arrive at $\sum_{s=1}^{N} a_{is} + b_i > 0$ and $L + B$ being positive stable as shown in [16] and [26], respectively. Besides, as shown in Assumption 2.2, the upper bound of d_{ij} is unknown, which means that the upper bound of d_{ij} should be replaced in other forms for the control design later if necessary. Inspired by Lemma 2.2, unknown control coefficients can be parameterized, that is, there are functions $\psi_{ij}(\bar{x}_{ij}) \geq 1$ and $\varrho_i(\theta_i) \geq 1$ such that

$$d_{ij}(\bar{x}_{ij}, \theta_i) \ \leq \ \psi_{ij}(\bar{x}_{ij})\varrho_i(\theta_i). \tag{2.5}$$

Similarly, there exist functions $\phi_{ij}(\bar{x}_{ij}) \geq 1$ and $\rho_i(\theta_i) \geq 1$ such that

$$|f_{ij}(\bar{x}_{ij}, \theta_i)| \ \leq \ \phi_{ij}(\bar{x}_{ij})\rho_i(\theta_i). \tag{2.6}$$

3. Distributed adaptive tracking control

This and the next sections will solve the control problem described above. A distributed tracking control and the update law for estimating the unknown parameter vectors are provided in this section, and then the control aim in the next section. Motivated by [16], we first introduce a coordinate transformation, by which the output tracking error of agent i is alternated into the other variable. Then we present a distributed adaptive tracking control for system (2.4) in this section.

Make a coordinate transformation:

$$\begin{cases} \xi_{i1} = \sum_{s=1}^{N} a_{is}(y_i - y_s) + b_i(y_i - y_0), \\ \xi_{i,j} = x_{i,j} - x^*_{i,j},\ j = 2, \dots, n_i, \\ x^*_{i2} = -\frac{1}{\lambda_i}\Big(k_{i1} + \varepsilon_i + \Phi_{i1}\hat{\Theta} + \Psi_{i1}\Big)\xi_{i1} + \frac{1}{\lambda_i}\sum_{s=1}^{N} a_{is}x^*_{s2}, \\ x^*_{ij} = -\Big(\frac{1}{\underline{d}_{i,j-1}}\Big)^{\frac{1}{p_{i,j-1}}}\Big(k_{i,j-1} + \Phi_{i,j-1}\hat{\Theta}_i + \Psi_{i,j-1} + \bar{\Phi}_{i,j-1} + \bar{\Psi}_{i,j-1} + \sum_{s=1}^{N}\beta_{s_{j-3},i}\Big)^{\frac{1}{p_{i,j-1}}}\xi_{i,j-1},\ j = 3, \dots, n_i. \end{cases} \tag{3.1}$$

Then, from the definition of ξ_{i1}, we can see that $\xi_1 = (L + B)\delta$ where $\xi_1 = [\xi_{11}, \xi_{21}, \dots, \xi_{N1}]^{\mathrm{T}}$ and $\delta = [\delta_1, \dots, \delta_N]^{\mathrm{T}}$. Since that $L + B$ is positive stable as previously stated, it is invertible and therefore, $\delta = (L + B)^{-1}\xi_1$. This implies that we can transform the proof of globally bounded for δ into that of ξ_1.

Besides,noting that $H = L + B$ is invertible, it is not hard to see that

$$\begin{pmatrix} x^*_{12} \\ \vdots \\ x^*_{N2} \end{pmatrix} = -H^{-1}\begin{pmatrix} \frac{1}{\lambda_1}\Big(k_{11} + \varepsilon_1 + \Phi_{11}\hat{\Theta}_1 + \Psi_{11}\Big)\xi_{11} \\ \vdots \\ \frac{1}{\lambda_N}\Big(k_{N1} + \varepsilon_N + \Phi_{N1}\hat{\Theta}_N + \Psi_{N1}\Big)\xi_{N1} \end{pmatrix}.$$

Therefore, x^*_{i2} is well-defined.

It should be mentioned that the virtual control x^*_{ij}, $j = 3, \dots, n_i$ is quite different from that in [14] in the following two aspects: (i) no information of $\xi_{i,j-2}$ is used in this paper, which makes it more easily to obtain the desired properties as shown later; (2) the cross-terms $-\sigma\frac{\partial x^*_{i,j-1}}{\partial\hat{\Theta}_i}\hat{\Theta}_i$ and $\frac{\partial x^*_{i,j-1}}{\partial\hat{\Theta}_i}\hat{\Theta}_i\tau_{i,j-1}$ are avoided in this paper, which guarantees the controller structure much more simple.

Then, we present the controller and the update law as:

$$\begin{cases} u_i = -\Big(\frac{1}{\underline{d}_{i,n_i}}\Big)^{\frac{1}{p_{i,n_i}}}\Big(k_{i,n_i} + \Phi_{i,n_i}\hat{\Theta}_i + \Psi_{i,n_i} + \bar{\Phi}_{i,n_i} + \bar{\Psi}_{i,n_i} + \sum_{s=1}^{N}\beta_{s_{n_i-2},i}\Big)^{\frac{1}{p_{i,n_i}}}\xi_{i,n_i}, \\ \dot{\hat{\Theta}}_i = \tau_{i,n_i} - r_i\sigma\hat{\Theta}_i, \end{cases} \tag{3.2}$$

with

$$\begin{cases} \tau_{i,n_i} = \tau_{i,n_i-1} + r_i\xi_{i,n_i}^{p_0+1}\Phi_{i,n_i}, \\ \beta_{i,n_i-2} = \frac{r_i}{2}\Phi_{i,n_i}\sum_{j=2}^{n_i-1}\Big(\xi_{ij}^{2(p_0-p_{ij}+1)} + (\frac{\partial x^*_{ij}}{\partial\hat{\Theta}_i})^2\Big), \\ k = min_{1\le i\le N, 1\le j\le n_i}\{\frac{k_{ij}(p_0-p_{ij}+2)}{g(\epsilon)}, r_i\sigma\}, \\ \eta = \sum_{i=1}^{N}\sum_{j=1}^{n_i}(\varepsilon_{ij} + \mu_{ij}) + \frac{N\sigma}{2}\Theta_i^2 > 0, \end{cases} \tag{3.3}$$

while ε_{i,n_i}, μ_{i,n_i} are positive design parameters, and Φ_{i,n_i} and Ψ_{i,n_i} are continuous functions.

By defining the Lyapunov function

$$V_{n_i} = \sum_{i=1}^{N}\Big(\sum_{j=1}^{n_i}\frac{1}{p_0 - p_{ij} + 2}\xi_{ij}^{p_0-p_{ij}+2} + \frac{1}{2r_i}\tilde{\Theta}_i^2\Big), \tag{3.4}$$

we can obtain

$$\dot{V}_{n_i} \leq -kV_{n_i} + \eta. \tag{3.5}$$

Remark 3.1. Notably, ignoring the term with $\hat{\Theta}_i$, there is no clear difference between the controller u_i in this paper and that in [16]. However, it should be mentioned that the construction of u_i here is much more difficult. In fact, since the different definition of V_i, more terms such as $\frac{1}{r_i}\tilde{\Theta}_i\dot{\hat{\Theta}}_i$ and $\sigma\xi_{il}^{p_0-p_{il}+1}\sum_{s=1}^{N} r_s\frac{\partial x_{il}^*}{\partial\hat{\Theta}_s}\hat{\Theta}_s$ appear at the estimation of $\dot{V}_i$. To achieve $\dot{V}_{n_i} \leq -kV_{n_i} + \eta$, we add the item $-\xi_{il}^{p_0-p_{il}+1}\sum_{s=1}^{N}\frac{\partial x_{il}^*}{\partial\hat{\Theta}_s}\tau_{sl}$ from the second step and estimate $-\sigma\tilde{\Theta}_i\hat{\Theta}_i$ in the last step, which brings numerous computational difficulties.

Remark 3.2. We have completed the distributed adaptive tracking control by combing the algebra graph theory with the distributed backstepping method. In fact, the distributed backstepping method is developed from the traditional backstepping method which means that the two methods are the same essentially. However, the two methods can be applied to different types of systems and achieve different control aim.

4. Main results

In this section, we first show the rationality of the distributed adaptive tracking control provided in the above section. This is verified in a recursive manner.

Step 1. From the definition of ξ_{i1} and (2.4), we have

$$\dot{\xi}_{i1} = \sum_{s=1}^{N} a_{is}(\dot{y}_i - \dot{y}_s) + b_i(\dot{y}_i - \dot{y}_0) = \lambda_i d_{i1}x_{i2} + \lambda_i f_{i1} - \sum_{s=1}^{N} a_{is}(d_{s1}x_{s2} + f_{s1}) - b_i\dot{y}_0, \tag{4.1}$$

where $\lambda_i = \sum_{s=1}^{N} a_{is} + b_i > 0$ as shown in the Section 2.

Construct a Lyapunov function

$$V_1 = \sum_{i=1}^{N}\Big(\frac{1}{p_0+1}\xi_{i1}^{p_0+1} + \frac{1}{2r_i}\tilde{\Theta}_i^2\Big), \tag{4.2}$$

where $p_0 = \max_{1\leq i\leq N, 1\leq j\leq N_i}\{p_{ij}\}$, $\tilde{\Theta}_i = \hat{\Theta}_i - \Theta_i$ with $\hat{\Theta}_i$ being the estimate of Θ_i by agent i to be designed later, and r_i is a positive design constant. Then along (2.4), V_1 satisfies

$$\dot{V}_1 = \sum_{i=1}^{N}\xi_{i1}^{p_0}\Big(\lambda_i d_{i1}x_{i2} + \lambda_i f_{i1} - \sum_{s=1}^{N} a_{is}(d_{s1}x_{s2} + f_{s1}) - b_i\dot{y}_0\Big) + \sum_{i=1}^{N}\frac{1}{r_i}\tilde{\Theta}_i\dot{\hat{\Theta}}_i. \tag{4.3}$$

Noting (2.5), (2.6) and Assumption 2.3, we can deduce by Lemma 2.1 that

$$\begin{cases}\xi_{i1}^{p_0}(\lambda_i f_{i1} - \sum_{s=1}^{N} a_{is}f_{s1}) \leq |\xi_{i1}|^{p_0}(\lambda_i\phi_{i1}\rho_i + \sum_{s=1}^{N} a_{is}\phi_{s1}\rho_s) \leq \varepsilon_{i1} + \xi_{i1}^{p_0+1}\Phi_{i1}(\xi_1)\Theta_i,\\ -b_i\dot{y}_0\xi_{i1}^{p_0} \leq \mu_{i1} + \xi_{i1}^{p_0+1}\Psi_{i1},\end{cases} \tag{4.4}$$

where ε_{i1} and μ_{i1} are positive design parameters, Θ_i is a positive constant depending on θ_i, Φ_{i1} is a smooth function depending on ξ_1 and ε_{i1}, and Ψ_{i1} is a positive constant depending on μ_{i1}.

Besides, from Assumption 2.1, Lemma 2.1 and the definition of ξ_{i2}, it follows that

$$\lambda_i d_{i1}\xi_{i1}^{p_0}(x_{i2}-x_{i2}^*)-\xi_{i1}^{p_0}\sum_{s=1}^{N}a_{is}d_{is}(x_{s2}-x_{s2}^*)\le \varepsilon_i\xi_{i1}^{p_0+1}+\sum_{s=1}^{N}\xi_{s2}^{p_0+1}, \tag{4.5}$$

where ε_i is a positive design parameter.

This, together with (4.3), (4.4) and the distributed virtual controller x_{i2}^* defined in (3.1), yields

$$\dot V_1 \le -\sum_{i=1}^{N}\Big(k_{i1}\xi_{i1}^{p_0+1}+(\varepsilon_{i1}+\mu_{i1})+\frac{1}{r_i}\tilde\Theta_i(\dot{\hat\Theta}_i-\tau_{i1})\Big)+N\sum_{i=1}^{N}\xi_{i2}^{p_0+1}, \tag{4.6}$$

where k_{i1}'s are some positive design parameters and $\tau_{i1}=r_i\Phi_{i1}\xi_{i1}^{p_0+2}$.

Step 2. Define

$$V_2=V_1+\sum_{i=1}^{N}\frac{1}{p_0-p_{i2}+2}\xi_{i2}^{p_0-p_{i2}+2}. \tag{4.7}$$

Noting from system (2.4) that $\dot\xi_{i2}=d_{i2}x_{i3}^{p_{i2}}+F_{i2}-\frac{\partial x_{i2}^*}{\partial\hat\Theta}\dot{\hat\Theta}-\frac{\partial x_{i2}^*}{\partial y_0}\dot y_0$ with $F_{i2}=f_{i2}-\sum_{s=1}^{N}\frac{\partial x_{i2}^*}{\partial x_{s1}}(d_{s1}x_{s2}^{p_{s1}}+f_{s1})$, we have

$$\begin{aligned}\dot V_2 \le{}& \sum_{i=1}^{N}\Big(-k_{i1}\xi_{i1}^{p_0+1}+\frac{1}{r_i}\tilde\Theta_i(\dot{\hat\Theta}_i-\tau_{i1})+d_{i2}\xi_{i2}^{p_0-p_{i2}+1}x_{i3}^{p_{i2}}-\xi_{i2}^{p_0-p_{i2}+1}\frac{\partial x_{i2}^*}{\partial\hat\Theta_i}\dot{\hat\Theta}_i+(\varepsilon_{i1}+\mu_{i1})\\ &+\xi_{i2}^{p_0-p_{i2}+1}F_{i2}-\xi_{i2}^{p_0-p_{i2}+1}\frac{\partial x_{i2}^*}{\partial y_0}\dot y_0\Big)+N\sum_{i=1}^{N}\xi_{i2}^{p_0+1}.\end{aligned} \tag{4.8}$$

Similar to the deduction of (4.4), it is easy to obtain that

$$\begin{cases}\xi_{i2}^{p_0-p_{i2}+1}F_{i2}\le\frac{1}{2}\varepsilon_{i2}+\xi_{i2}^{p_0+1}\Phi_{i2}(\xi_2)\Theta_i,\\ -\xi_{i2}^{p_0-p_{i2}+1}\frac{\partial x_{i2}^*}{\partial y_0}\dot y_0\le\frac{1}{2}\mu_{i2}+\xi_{i2}^{p_0+1}\Psi_{i2}(\xi_1),\end{cases} \tag{4.9}$$

where ε_{i2} and μ_{i2} are positive design parameters while Φ_{i2} and Ψ_{i2} are smooth functions with $\xi_2=(\xi_1^T,x_{12},\ldots,x_{N2})^{\mathrm T}$.

Submit the inequality (4.9) and

$$\begin{cases}-\frac{\partial x_{i2}^*}{\partial\hat\Theta_i}\tau_{i2}\xi_{i2}^{p_0-p_{i2}+1}\le\frac{1}{2}\varepsilon_{i2}+\xi_{i2}^{p_0+1}\bar\Phi_{i2}(\xi_2),\\ r_i\sigma\frac{\partial x_{i2}^*}{\partial\hat\Theta_i}\hat\Theta_i\xi_{i2}^{p_0-p_{i2}+1}\le\frac{1}{2}\mu_{i2}+\xi_{i2}^{p_0+1}\bar\Psi_{i2}(\xi_1)\end{cases} \tag{4.10}$$

into (4.8). By x_{i3}^* in (3.1) with $\sum_{s=1}^{N}\beta_{s_0,i}=N$, $\tau_{i2}=\tau_{i1}+r_i\xi_{i2}^{p_0+1}\Phi_{i2}$ and k_{i2}, σ being positive design constants, and noting that

$$\begin{cases}-\frac{d_{i2}}{\underline d_{i2}}\frac{1}{2\mu_{i2}}\Big(\frac{\partial x_{i2}^*}{\partial\hat\Theta_i}\tau_{i2}\Big)^2\xi_{i2}^2\le\frac{1}{2}\mu_{i2}-\Big|\xi_{i2}\frac{\partial x_{i2}^*}{\partial\hat\Theta_i}\tau_{i2}\Big|\le\frac{1}{2}\mu_{i2}+\xi_{i2}\frac{\partial x_{i2}^*}{\partial\hat\Theta_i}\tau_{i2},\\ -\frac{d_{i2}}{\underline d_{i2}}\frac{1}{2\varepsilon_{i2}}\Big(r_i\sigma\frac{\partial x_{i2}^*}{\partial\hat\Theta_i}\hat\Theta_i\Big)^2\xi_{i2}^2\le\frac{1}{2}\varepsilon_{i2}-r_i\sigma\frac{\partial x_{i2}^*}{\partial\hat\Theta_i}\hat\Theta_i\xi_{i2},\end{cases} \tag{4.11}$$

there must be

$$\dot{V}_2 \leq \sum_{i=1}^{N}\Big(-k_{i1}\xi_{i1}^{p_0+1} - k_{i2}\xi_{i2}^{p_0+1} + d_{i2}\xi_{i2}^{p_0-p_{i2}+1}\Big(x_{i3}^{p_{i2}} - (x_{i3}^*)^{p_{i2}}\Big) + (\tau_{i2} - \dot{\hat{\Theta}}_i)\frac{\partial x_{i2}^*}{\partial\hat{\Theta}_i}\xi_{i2}^{p_0-p_{i2}+1} - \sum_{j=1}^{2}(\varepsilon_{ij} + \mu_{ij}) - r_i\sigma\frac{\partial x_{i2}^*}{\partial\hat{\Theta}_i}\hat{\Theta}_i\xi_{i2}^{p_0-p_{i2}+1} + \frac{1}{r_i}\tilde{\Theta}_i(\dot{\hat{\Theta}}_i - \tau_{i2})\Big). \tag{4.12}$$

Recursive Step *l* ($l = 3, \ldots, n_i - 1$)**.** Suppose that the previous $l-1$ steps have been completed, that is, there are the Lyapunov functions

$$V_{l-1} = V_{l-2} + \sum_{i=1}^{N}\frac{1}{p_0 - p_{i,l-1} + 2}\xi_{i,l-1}^{p_0-p_{i,l-1}+2} \tag{4.13}$$

to satisfy

$$\dot{V}_{l-1} \leq \sum_{i=1}^{N}\Big(-\sum_{j=1}^{l-1}k_{ij}\xi_{ij}^{p_0+1} + d_{i,l-1}\xi_{i,l-1}^{p_0-p_{i,l-1}+1}\Big(x_{il}^{p_{i,l-1}} - (x_{il}^*)^{p_{i,l-1}}\Big) + \sum_{j=1}^{l-1}(\varepsilon_{ij} + \mu_{ij}) + \frac{1}{r_i}\tilde{\Theta}_i(\dot{\hat{\Theta}}_i - \tau_{i,l-1}) - r_i\sigma\hat{\Theta}_i\sum_{j=2}^{l-1}\frac{\partial x_{ij}^*}{\partial\hat{\Theta}_i}\xi_{ij}^{p_0-p_{ij}+1} + (\tau_{i,l-1} - \dot{\hat{\Theta}}_i)\sum_{j=2}^{l-1}\frac{\partial x_{ij}^*}{\partial\hat{\Theta}_i}\xi_{ij}^{p_0-p_{ij}+1}\Big), \tag{4.14}$$

where k_{ij}'s, ε_{ij}'s, μ_{ij}'s are some positive design constants.

At step l, define

$$V_l = V_{l-1} + \sum_{i=1}^{N}\frac{1}{p_0 - p_{il} + 2}\xi_{il}^{p_0-p_{il}+2}. \tag{4.15}$$

Then we can conclude from (2.4), (4.14) and the definition $F_{il} = f_{il} - \sum_{j=1}^{l-1}\sum_{s=1}^{N}\frac{\partial x_{il}^*}{\partial x_{sj}}(d_{sj}x_{s,j+1}^{p_{sj}} + f_{sj})$ that

$$\dot{V}_l \leq \sum_{i=1}^{N}\Big(-\sum_{j=1}^{l-1}k_{ij}\xi_{ij}^{p_0+1} + d_{i,l-1}\xi_{i,l-1}^{p_0-p_{i,l-1}+1}\Big(x_{il}^{p_{i,l-1}} - (x_{il}^*)^{p_{i,l-1}}\Big) + \frac{1}{r_i}\tilde{\Theta}_i(\dot{\hat{\Theta}}_i - \tau_{i,l-1}) - \sigma\sum_{j=2}^{l-1}\xi_{ij}^{p_0-p_{ij}+1}\sum_{s=1}^{N}r_s\frac{\partial x_{ij}^*}{\partial\hat{\Theta}_s}\hat{\Theta}_s + \sum_{j=2}^{l-1}\xi_{ij}^{p_0-p_{ij}+1}\sum_{s=1}^{N}\frac{\partial x_{ij}^*}{\partial\hat{\Theta}_s}(\tau_{s,l-1} - \dot{\hat{\Theta}}_s) + \sum_{j=1}^{l-1}(\varepsilon_{ij} + \mu_{ij}) + d_{il}\xi_{il}^{p_0-p_{il}+1}x_{i,l+1}^{p_{il}} + \xi_{il}^{p_0-p_{il}+1}F_{il} - \xi_{il}^{p_0-p_{il}+1}\sum_{s=1}^{N}\frac{\partial x_{il}^*}{\partial\hat{\Theta}_s}\dot{\hat{\Theta}}_s - \xi_{il}^{p_0-p_{il}+1}\frac{\partial x_{il}^*}{\partial y_0}\dot{y}_0\Big). \tag{4.16}$$

Besides, there holds

$$\begin{cases}\xi_{il}^{p_0-p_{il}+1}F_{il} \;\leq\; \frac{1}{4}\varepsilon_{il} + \frac{1}{2}\xi_{il}^{p_0+1}\Phi_{il}(\xi_l)\Theta_i,\\ -\xi_{il}^{p_0-p_{il}+1}\frac{\partial x_{il}^*}{\partial y_0}\dot{y}_0 \;\leq\; \frac{1}{2}\mu_{il} + \xi_{il}^{p_0+1}\Psi_{il}(\xi_{l-1}),\\ d_{i,l-1}\xi_{i,l-1}^{p_0-p_{il}+1}(x_{il}^{p_{i,l-1}} - (x_{il}^*)^{p_{i,l-1}}) \;\leq\; \frac{1}{2}\varepsilon_{il} + \frac{1}{2}\xi_{il}^{p_0+1}\Phi_{il}(\xi_l)\Theta_i,\\ -\xi_{il}^{p_0-p_{il}+1}\sum_{s=1}^{N}\frac{\partial x_{il}^*}{\partial\hat{\Theta}_s}\tau_{sl} \;\leq\; \frac{1}{2}\mu_{il} + \xi_{il}^{p_0+1}\bar{\Phi}_{il}(\xi_l),\\ \sigma\xi_{il}^{p_0-p_{il}+1}\sum_{s=1}^{N}r_s\frac{\partial x_{il}^*}{\partial\hat{\Theta}_s}\hat{\Theta}_s \;\leq\; \frac{1}{4}\varepsilon_{il} + \xi_{il}^{p_0+1}\bar{\Psi}_{il}(\xi_2).\end{cases} \tag{4.17}$$

With $\tau_{il} = \tau_{i,l-1} + r_i \xi_{il}^{p_0+1} \Phi_{il}$, and noting

$$\sum_{j=2}^{l-1} \xi_{ij}^{p_0-p_{ij}+1} \sum_{s=1}^{N} \frac{\partial x_{ij}^*}{\partial \hat{\Theta}_s}\Big(\tau_{s,l-1} - \tau_{sl}\Big) \leq \sum_{s=1}^{N} \frac{r_s}{2} \xi_{sl}^{p_0+1} \Phi_{sl} \sum_{j=2}^{l-1} \Big(\xi_{ij}^{2(p_0-p_{il}+1)} + \Big(\frac{\partial x_{ij}^*}{\partial \hat{\Theta}_s}\Big)^2\Big) \triangleq \sum_{s=1}^{N} \beta_{i_{l-2},s} \xi_{sl}^{p_0+1}, \quad (4.18)$$

we see that by $x_{i,l+1}^*$ defined above, there holds

$$\begin{aligned} \dot{V}_l \leq \sum_{i=1}^{N} \Big(&- \sum_{j=1}^{l} k_{ij} \xi_{ij}^{p_0+1} + d_{il} \xi_{il}^{p_0-p_{il}+1} \Big(x_{i,l+1}^{p_{il}} - (x_{i,l+1}^*)^{p_{il}}\Big) + \frac{1}{r_i} \tilde{\Theta}_i (\dot{\hat{\Theta}}_i - \tau_{il}) \\ &- \sigma \sum_{j=2}^{l} \xi_{ij}^{p_0-p_{ij}+1} \sum_{s=1}^{N} r_s \frac{\partial x_{ij}^*}{\partial \hat{\Theta}_s} \hat{\Theta}_s + \sum_{j=2}^{l} \xi_{ij}^{p_0-p_{ij}+1} \sum_{s=1}^{N} \frac{\partial x_{ij}^*}{\partial \hat{\Theta}_s} (\tau_{s,l-1} - \dot{\hat{\Theta}}_s) + \sum_{j=1}^{l} (\varepsilon_{ij} + \mu_{ij}) \Big). \end{aligned} \quad (4.19)$$

Step n_i. With the definition V_{n_i}, we can arrive at from (2.4), (4.19) for $l = n_i - 1$ and $F_{i,n_i} = f_{i,n_i} - \sum_{j=1}^{n_i-1} \sum_{s=1}^{N} \frac{\partial x_{i,n_i}^*}{\partial x_{sj}} (d_{sj} x_{s,j+1}^{p_{sj}} + f_{sj})$ that

$$\begin{aligned} \dot{V}_{n_i} \leq \sum_{i=1}^{N} \Big(&- \sum_{j=1}^{n_i-1} k_{ij} \xi_{ij}^{p_0+1} + d_{i,n_i-1} \xi_{i,n_i-1}^{p_0-p_{i,n_i-1}+1} \Big(x_{i,n_i}^{p_{i,n_i-1}} - (x_{i,n_i}^*)^{p_{i,n_i-1}}\Big) + \frac{1}{r_i} \tilde{\Theta}_i (\dot{\hat{\Theta}}_i - \tau_{i,n_i-1}) \\ &- \sigma \sum_{j=2}^{n_i-1} \xi_{ij}^{p_0-p_{ij}+1} \sum_{s=1}^{N} r_s \frac{\partial x_{ij}^*}{\partial \hat{\Theta}_s} \hat{\Theta}_s + \sum_{j=2}^{n_i-1} \xi_{ij}^{p_0-p_{ij}+1} \sum_{s=1}^{N} \frac{\partial x_{ij}^*}{\partial \hat{\Theta}_s} (\tau_{s,l-1} - \dot{\hat{\Theta}}_s) + \sum_{j=1}^{n_i-1} (\varepsilon_{ij} + \mu_{ij}) \\ &+ d_{i,n_i} \xi_{i,n_i}^{p_0-p_{i,n_i}+1} u_i^{p_{i,n_i}} + \xi_{i,n_i}^{p_0-p_{i,n_i}+1} F_{i,n_i} - \xi_{i,n_i}^{p_0-p_{i,n_i}+1} \sum_{s=1}^{N} \frac{\partial x_{i,n_i}^*}{\partial \hat{\Theta}_s} \dot{\hat{\Theta}}_s - \xi_{i,n_i}^{p_0-p_{i,n_i}+1} \frac{\partial x_{i,n_i}^*}{\partial y_0} \dot{y}_0 \Big). \end{aligned} \quad (4.20)$$

In addition, it results from Lemma 2.1, (2.5) and (2.6) that

$$\begin{cases} \xi_{i,n_i} F_{i,n_i} \ \leq \ \frac{1}{4} \varepsilon_{i,n_i} + \frac{1}{2} \xi_{i,n_i}^{p_0+1} \Phi_{i,n_i} \Theta_i, \\ -\xi_{i,n_i}^{p_0-p_{i,n_i}+1} \frac{\partial x_{i,n_i}^*}{\partial y_0} \dot{y}_0 \ \leq \ \frac{1}{2} \mu_{i,n_i} + \xi_{i,n_i}^{p_0+1} \Psi_{i,n_i}, \\ d_{i,n_i-1} \xi_{i,n_i-1}^{p_0-p_{i,n_i-1}+1} \Big(x_{i,n_i}^{p_{i,n_i-1}} - (x_{i,n_i}^*)^{p_{i,n_i-1}}\Big) \ \leq \ \frac{1}{2} \varepsilon_{i,n_i} + \frac{1}{2} \xi_{i,n_i}^{p_0+1} \Phi_{i,n_i} \Theta_i. \end{cases} \quad (4.21)$$

From the definition of τ_{i,n_i} and β_{i,n_i-2} in (3.3), it is clear that

$$\sum_{j=2}^{n_i-1} \xi_{ij}^{p_0-p_{ij}+1} \sum_{s=1}^{N} \frac{\partial x_{ij}^*}{\partial \hat{\Theta}_s} (\tau_{s,n_i-1} - \dot{\hat{\Theta}}_s) \leq \sum_{j=2}^{n_i-1} \xi_{ij}^{p_0-p_{ij}+1} \sum_{s=1}^{N} \frac{\partial x_{ij}^*}{\partial \hat{\Theta}_s} (\tau_{s,n_i} - \dot{\hat{\Theta}}_s) + \sum_{s=1}^{N} \beta_{i_{l-1},s} \xi_{sl}^{p_0+1}. \quad (4.22)$$

Noting

$$\begin{cases} -\xi_{i,n_i}^{p_0-p_{i,n_i}+1} \sum_{s=1}^{N} \frac{\partial x_{i,n_i}^*}{\partial \hat{\Theta}_s} \tau_{s,n_s} \leq \frac{1}{2} \mu_{i,n_i} + \xi_{i,n_i}^{p_o+1} \bar{\Phi}_{i,n_i}, \\ \sigma \xi_{i,n_i}^{p_0-p_{i,n_i}+1} \sum_{s=1}^{N} r_s \frac{\partial x_{i,n_i}^*}{\partial \hat{\Theta}_s} \hat{\Theta}_s \leq \frac{1}{4} \varepsilon_{i,n_i} + + \xi_{i,n_i}^{p_o+1} \bar{\Psi}_{i,n_i}, \\ \xi_{ij}^{p_0-p_{ij}+2} \leq \epsilon + \frac{p_0-p_{ij}+2}{p_0+1} \Big(\frac{p_0+1}{p_{ij}-1} \epsilon\Big)^{-\frac{p_{ij}-1}{p_0-p_{ij}+2}} \xi_{ij}^{p_0+1} \triangleq \epsilon + g(\epsilon) \xi_{ij}^{p_0+1}, \\ -\sigma \tilde{\Theta}_i \hat{\Theta}_i \leq -\frac{\sigma}{2} \tilde{\Theta}_i^2 + \frac{\sigma}{2} \Theta_i^2, \end{cases} \quad (4.23)$$

we can easily obtain (3.5) by submitting (4.21), (3.2) and (4.22) into (4.20).

Then, we can immediately obtain the main results of this paper, which is summarized into the following theorem.

Theorem 4.1. *Consider system* (2.4) *under Assumptions* 2.1 *and* 2.2*, while the leader's output satisfies Assumption* 2.3 *and the digraph topology* $\bar{\mathcal{G}}$ *satisfies Assumption* 2.4*. The distributed controller and the update law defined in* (3.2) *guarantee that for any initial conditions* x_0*, there are*

(*i*) *all the closed-loop system signals* x_{ij}*,* u_i *and* $\hat{\Theta}_i$ *with* $i = 1, \ldots, N$ *and* $j = 1, \ldots, n_i$ *are global bounded;*

(*ii*) *the range of the tracking error* $||\delta(t)||$ *can be tuned arbitrarily small.*

Proof. Directly solving (3.5), we have

$$V_{n_i}(t) \leq e^{-kt}V_{n_i}(0) + \int_0^t e^{-k(t-\tau)}\eta d\tau \leq e^{-kt}V_{n_i}(0) + \frac{\eta}{k}(1 - e^{-kt}). \tag{4.24}$$

From this we can see that, $\lim_{t\to+\infty} V_{n_i}(t) \leq \frac{\eta}{k}$. This, together with the definition of V_{n_i}, implies that ξ_{ij}'s, $\tilde{\Theta}_i$'s and $\hat{\Theta}_i$'s are global bounded, and so are x_{ij}'s and u_i.

On the other hand, from Lemma 2.2 in [27] and (3.5) that a finite time $t^* = \frac{1}{k}ln\frac{|V_{n_i}(0)-\eta/k|}{\eta/k}$ must exist such that

$$0 \leq V_{n_i}(t) \leq \frac{2\eta}{k}, \quad \forall t > t^* \geq 0. \tag{4.25}$$

This, together with the definition of V_{n_i} and Remark 3.1, implies that

$$||\delta(t)|| = \frac{||\xi_1(t)||}{\xi_{min}(L+B)} \leq \frac{MV_{n_i}(t)}{\xi_{min}(L+B)} \leq \frac{2M\eta}{k\xi_{min}(L+B)}, \quad \forall t > t^* \geq 0, \tag{4.26}$$

where M is a positive constant depending on p_0. Therefore, the range of the tracking error $||\delta(t)||$ can be tuned arbitrarily small.

Thus far, we complete the proof the Theorem 4.1. □

Remark 4.1. As shown in Theorem 4.1, the range of the tracking error $||\delta(t)||$ can be arbitrarily small instead of converging to the origin or an arbitrarily pre-given small neighborhood of zero such as [16]. Even though, by choosing the design parameters k_{ij}'s, r_i's and σ large enough, we can see from (3.3) that k is also large enough. This, together with (4.26), keeps the range of the tracking error as small as expected.

5. A simulation example

Consider the digraph topology $\bar{\mathcal{G}}$ with $a_{12} = a_{13} = a_{21} = a_{23} = a_{31} = b_3 = 0$, $a_{32} = b_1 = b_2 = 1$. Clearly, the digraph satisfies Assumption 2.4. The i-th agent of the nonlinear MASs with 3 followers and one leader (labeled by 0) is:

$$\begin{cases} \dot{x}_{i1} &= d_{i1}x_{i,1}^{p_{i1}} + f_{i1}, \\ \dot{x}_{i2} &= d_{i2}u_i^{p_{i2}} + f_{i2} \\ y_i &= x_{i1}, \end{cases} \tag{5.1}$$

where $p_{ij} = 1, d_{ij} = 1, i = 1, \ldots, 3, j = 1, 2$, $f_{11} = \theta sinx_{11}$, $f_{21} = f_{31} = f_{12} = f_{22} = f_{32} = 0$, θ is an unknown constant. Apparently, this system satisfies Assumptions 2.1 and 2.2. The leader's output is $y_0 = \frac{1}{1+t}$, which means that Assumption 2.3 is satisfied.

Define $\Theta = \theta^2$. By the design process in Section 3, we can get the distributed controllers as

$$\begin{cases} u_1 = -\Big(\frac{3}{2} + \frac{1}{2}(\frac{\partial x_{12}^*}{\partial x_{11}}x_{12} - \frac{\partial x_{12}^*}{\partial y_0}\frac{1}{(1+t)^2} + \frac{\partial x_{12}^*}{\partial \hat{\Theta}}\tau)^2 + \frac{1}{2}(\frac{\partial x_{12}^*}{\partial x_{11}}sinx_{11})^2\hat{\Theta}\Big)\xi_{12}, \\ u_2 = -(2 + \frac{1}{2}(\frac{\partial x_{22}^*}{\partial x_{21}}x_{22} - \frac{\partial x_{22}^*}{\partial y_0}\frac{1}{(1+t)^2})^2)\xi_{22}, \\ u_3 = -(\frac{3}{2} + \frac{1}{4}(\frac{\partial x_{32}^*}{\partial x_{31}}x_{32} + \frac{\partial x_{32}^*}{\partial x_{21}}x_{22} - \frac{\partial x_{32}^*}{\partial y_0}\frac{1}{(1+t)^2})^2)\xi_{32}, \end{cases} \tag{5.2}$$

and the update law for the unknown parameters as

$$\dot{\hat{\Theta}} = \tau - \hat{\Theta}, \tag{5.3}$$

where

$$\begin{cases} \xi_{11} = x_{11} - \frac{1}{1+t}, \quad \xi_{21} = x_{21} - \frac{1}{1+t}, \quad \xi_{31} = x_{31} - x_{21}, \\ \xi_{12} = x_{12} - x_{12}^*, \quad x_{12}^* = -(\frac{1}{2}\hat{\Theta}_1 + 2)\xi_{11}, \\ \xi_{22} = x_{22} - x_{22}^*, \quad x_{22}^* = -2\xi_{21}, \\ \xi_{32} = x_{32} - x_{32}^*, \quad x_{32}^* = -2\xi_{31} + x_{22}^*, \\ \tau = \frac{1}{2}\xi_{11}^2 + \frac{1}{4}\xi_{12}^2(\frac{\partial x_{12}^*}{\partial x_{11}}sinx_{11})^2. \end{cases} \tag{5.4}$$

Choose $\theta = 1$, and the initial conditions $x_{11}(0) = 1, x_{12}(0) = -2, x_{21}(0) = -1, x_{22}(0) = -1, x_{31}(0) = 1, x_{32}(0) = -1, \hat{\theta}(0) = 0.3$. We can obtain Figures 1 and 2, which illustrate that the signals of the closed-loop system x_{ij}, u_i and $\hat{\Theta}_i$ are global bounded while the range of the tracking errors is arbitrarily small. Thus, the effectiveness of the distributed adaptive control in this paper for nonlinear multi-agent systems with nonlinear parametric uncertainties is verified.

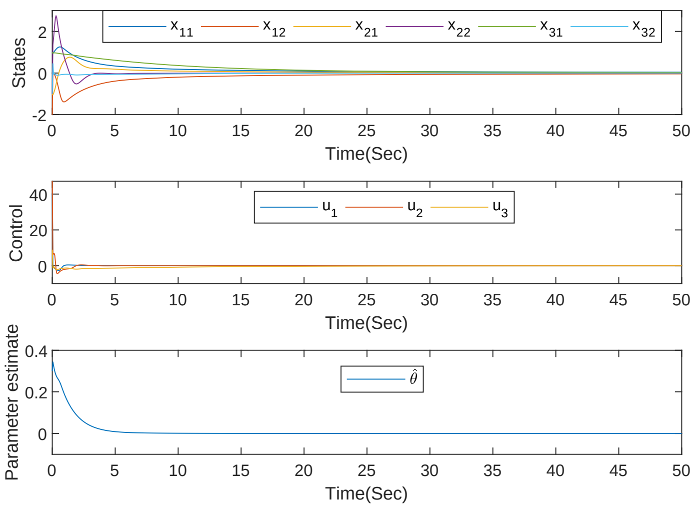

Figure 1. The responses of x_{ij}, u_i and $\hat{\Theta}_i$.

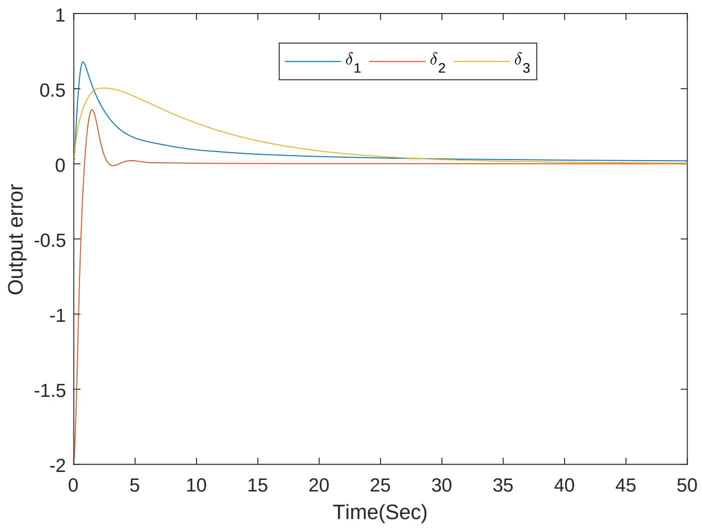

Figure 2. The responses of δ_i.

Remark 5.1. Compared to the simulation example in the related work [16], the distributed controllers (5.2) is much more powerful since that no unknown parameter is contained in the simulation example in [16]. Besides, although sharing the same digraph topology and system powers with [14], the structure of distributed controllers here is much more simple.

6. Concluding remarks

In this paper, a distributed adaptive controller has been developed for the tracking problem of a class of inherent nonlinear multi-agents systems with serious uncertainties from the control coefficients and the system nonlinearities. By employing backstepping and adaptive technology for the control design, we overcome the unstable linearization and the parameter uncertainty in the considered system. Comparing with the related literature, we provide a different distributed adaptive controller structure and construct a different Lyapunov function for the analysis. However, this paper doesn't consider the case that the control directions (that is, the sign of d_{ij}'s) are unknown as in [28] or the nonlinear MASs is with a stochastic process such as [29], which is our further research.

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Conflict of interest

The authors declare there is no conflict of interest.

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