

Quantity Flexibility Contract Model for Emergency Procurement Considering Supply Disruption

Bin Wu*, Shuangwei Bai, Bijina Rajbhandari, Bangyuan Li, and Kesheng Wang

Abstract: Supply chain disruption risk usually poses a serious challenge to the management of emergency supplies procurement between the government and enterprises in cooperation. To research the impact of supply chain disruption on the supply and demand sides of emergency supplies for disaster relief, the emergency procurement model based on quantity flexibility contract is constructed. The model introduces a stockout disruption to measure the degree of supply chain disruption and uses per unit of material relief value to quantify government disaster relief benefits. Further, it analyzes the basic pricing strategy and the agreed order quantity between the government and enterprises, focusing on the negative impact of supply disruption on the government and enterprises. The model deduction and data analysis results show that supply disruption creates a “lose-lose” situation for governments and enterprises, reducing their benefits and willingness to cooperate. Finally, a sensitivity analysis is conducted on the case data to explain the decision-making changes in the contract price and flexibility parameters between the government and enterprises before and after the supply disruption.

Key words: emergency procurement; supply chain disruption; quantity flexibility contract; sensitivity analysis

1 Introduction

The Government and humanitarian relief organizations play an important role in relief activities and are responsible for the bulk of emergency supplies^[1]. A number of social contracts indicate that government and disaster relief organizations in different regions shall sign agreements with relevant enterprises according to the actual situation of the region to ensure the

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production security and stockpiling security of emergency relief supplies, necessary items and emergency equipment^[2]. The importance of emergency supplies as a component of emergency management is fundamentally obvious^[3], and the occurrence of public emergencies often elevates emergency supplies to a higher level, such as flood, epidemics, etc. Emergency supplies are characterized by uncertainties, sudden demand, and short delivery times^[4], and are accompanied by the risk of supply chain disruption. For example, when public health events occur, the supply to the disaster areas may be disrupted for a short period of time in some places, which results not only in local governments having no supplies for disaster relief but also in enterprises facing huge losses and social instability. Therefore, studying the impact of supply chain disruption on the decision-making of the government and enterprises is an important emergency management issue.

In the government procurement process of disaster relief materials, supplier selection is a strategic step

that requires selecting appropriate suppliers and establishing stable procurement cooperation. Hu et al.^[5] reviewed relevant literature on supplier selection problems, and they put forward that the selection of suppliers should be made according to the characteristics of suppliers and the uncertainty of disasters with scientific evaluation methods. Flexibility is a key consideration in emergency procurement^[6]. At present, when conducting emergency supply procurement, the government mostly uses the supply chain contract with enterprises in the form of option contracts and quantity flexibility contracts, among others. Ding and Liu^[7] presented an option contract into the pricing model for the physical procurement of government emergency supplies, comparing the pricing model in two cases with and without a contract. Liang et al.^[8] designed a pricing model for the two-step delivery of supplies based on the study of option contract models and obtained a feasible range of approximate regression solutions for option pricing. Tian et al.^[9] proposed the spot market and the probability of disasters to make the emergency supply procurement model based on the option contract more realistic, and they analyzed the decisions of suppliers and the government with the maximization of supply chain benefits. In addition, the government will take the form of production capacity reserves. Furthermore, Tian et al.^[10] considered the production capacity option in the option contract model, which means that the government would buy a certain production capacity option from suppliers to reduce the number of physical reserves. However, the production capacity option has a high delivery risk, which lacks flexibility for its fixed quantities. Hence, the enterprise should convert production capacity into a promised quantity in disasters. The quantity flexibility contract has been further applied due to its flexibility. Balcik and Ak^[11] presented the quantity flexibility contract as a procurement agreement between one relief organization and multiple suppliers to research the procurement problem between relief organizations and suppliers, with minimization of the total expected agreement and procurement costs within the scope of the agreement. Nikkhoo et al.^[12] proposed the quantity flexibility contract into supply chains that have a single supplier and single relief organization. They investigated the influence mechanisms of the procurement quantity and procurement elasticity on procurement price in the quantity flexibility contract.

Zhang et al.^[13] studied the quantity flexibility contract in the cooperative relationship between the government and enterprises about stockpiling of emergency supplies. The Stackelberg model based on the quantity flexibility contract was constructed between the government and enterprise. It was essentially a shift from a production capacity option to quantity flexibility, which allows the government to be more flexible in stockpiling supplies in response to emergencies and reduces risks. Collaboration between the government and enterprises can effectively reduce costs and improve overall efficiency. Hu et al.^[14] considered the procurement pricing under joint reserves problem. The purpose is to maximize the overall supply chain profits to achieve a win-win situation in which the probability of disasters and spot-market purchase prices play an important role in the decision-making for the government and enterprises. Then the impact of the two factors on government procurement costs and enterprise profits was also studied^[15]. Torabi et al.^[16] combined a pre-disaster procurement model based on the quantity flexibility contract with post-disaster material prepositioning, and applied it to develop disaster relief plans which further demonstrate that the quantity flexibility contract is well suited in emergency management practice. Thus, the quantity flexibility contract has a more significant effect on the government preparedness flexibility, risk reduction, and cost optimization, and has a wide application value.

The supply chain disruption risk refers to the unintended risk caused by natural or man-made factors which leads to the collapse and disruption of the supply chain. He et al.^[17] studied the supply chain risk and its vulnerability, which only manifest themselves when a disruptive outcome occurred as a result of the supply chain disruption. DuHadway et al.^[18] proposed that the types and sources of risks need to be considered in order to develop risk management strategies. The key to effective risk management lies in risk mitigation and disruption recovery. In terms of specific disruption countermeasures, Khalilpourazari and Hashemi^[19] studied the blood supply chain under the background of the COVID-19 pandemic, which led to the reduction of the blood supply and network disruption. They established a mathematical model considering risk aversion and robust optimization to scientifically guide supply chain decision-making. Nikkhah et al.^[20] designed a hierarchical prediction model for optimal

scheduling in the case of high penetration and unexpected interruption of the energy supply. Chen^[21] investigated the procurement strategy under a manufacturer’s risk-averse preference with a two-tier supply chain of primary and backup suppliers. The procurement strategy of both parties under the disruption information asymmetry from a game theory perspective was presented. Yang et al.^[22] discussed supplier reliability and supply chain disruption risk under information asymmetry, and suggested that backup production could be considered to prevent manufacturer disruption. Lei et al.^[23] considered the supply chain disruption risk and studied the optimal supplier contract problem in a state of information asymmetry due to demand disruption. F. Liu and J. G. Liu^[24] summarized the specific processes and addressing strategies for dealing with supply chain disruption from three perspectives: disruption processing time, response theme, and processing path.

However, few studies have considered supply disruption in the management of emergency supplies, while supply disruption frequently occurs in emergency events and has a significant impact on emergency relief. Accordingly, present study has established an emergency procurement model of the quantity flexibility contract, considering associated disruption risks when disasters happen. Then, it introduces the number of disruption shortages, focusing on analyzing the negative impact of disruption on governments and businesses to the extent that it can provide some

management conclusions.

The remainder of this paper is organized as follows. Section 2 describes the problem, where assumptions are made, reasonable variables are set up, and general explanations are given. Section 3 establishes and presents solutions to the procurement mathematical model considering disruptions and non-disruptions, as well as propositions that can be used to guide practices. Section 4 sets up a numerical example to conduct simulation experiments on MATLAB and sensitivity analysis. Section 5 ends the paper with conclusions and suggestions for future works.

2 Problems Statement

2.1 Emergency procurement process

Emergency supply procurement is a typical buyer’s market. As shown in Fig. 1, in this study, a single government and a single supplier establish a quantity flexibility contract relationship for emergency supplies, forming a secondary supply chain. Procurement activities consist of three stages: Firstly, before a disaster, the government issues tender documents and seeks reliable suppliers to sign a procurement contract in order to prepare for disaster responses and reduce disaster uncertainty costs, while suppliers participate in competitive bidding to seek a stable partnership and increase corporate profits. Eventually quantity flexibility contract relationship is established between the government and the supplier, That is, the supplier

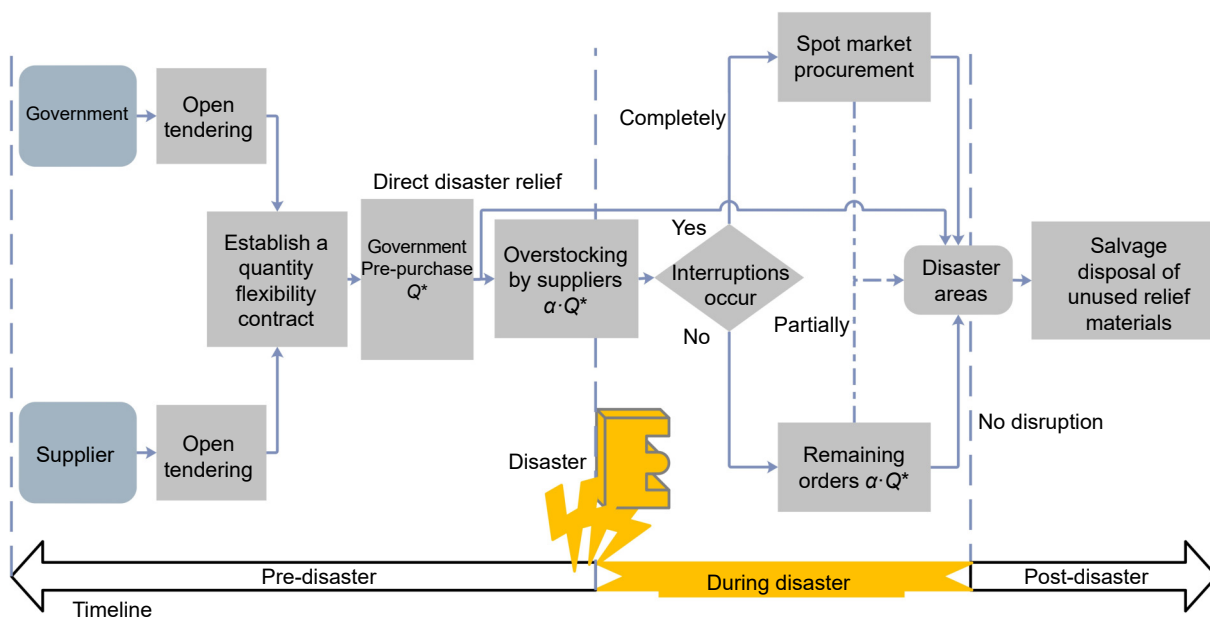


Fig. 1 Schematic diagram of the emergency supply procurement process.

commits to provide no less than the quantity $Q+q$, which is the quantity of emergency supplies procured as determined by a quantity flexibility contract between the government and the supplier of emergency supplies during one cycle T , i.e., government-supplier contract cycle, in which the government commits to procure no less than the quantity q of emergency supplies. Second, during disasters, the government uses pre-purchased emergency supplies for disaster relief, and flexible reserves of suppliers will be used if there is a sudden demand. Meanwhile, disaster relief activities are delivery disruption risk at this time, which occurs in two scenarios, namely, partial and complete disruptions. Urgent supplemental purchases in the spot market are required. This study assumes that partial and complete disruptions are continuous processes, with stockouts λ ($0 < \lambda \leq q$) following a uniform distribution, represents complete disruption. In particular, the government still needs to urgently procure emergency supplies from the spot market to meet the excess demand when the demand is sufficiently high. Finally, after disasters, any surplus emergency supplies reserved by the government and

the supplier should be treated as salvage value.

2.2 Assumptions

(1) The period when the government and the supplier establish a quantity flexibility contract relationship is equal to the shelf life of the emergency supplies.

(2) In this study, the two-tier supply chain structure consists of a government and a supplier, and transportation costs are included in the unit price of procurement.

(3) Emergency supplies are broadly defined as supplies that are in high demand when disasters occur, such as food and other household goods.

(4) The government and supplier risk preferences are neutral.

2.3 Parameters

Table 1 presents all the variables in the model, including their indications and their constraints.

3 Mathematical Model and Deduction

Based on the problem description, the decision-making process between the government and the supplier can

Table 1 Parameter description.

Parameter	Explanation
T	Government-supplier contract cycle
c_p	Unit production cost
c_{sup}^h, c_{gov}^h	Government unit cost of inventory and supplier unit cost of inventory, respectively
p_w	Unit material agreement price under quantity flexibility contract (decision variable)
p_m	Spot market unit purchase price
v_{sup}, v_{gov}	Unit residual value, while v_{sup} indicates unit residual value on the supplier's side, and v_{gov} indicates unit residual value on the government's side, typically $v_{sup} \geq v_{gov}$
S	Unit material disaster relief value
B	Unit penalty cost of stock-out
α	Flexibility parameter for supplier's choice of overstock ($0 < \alpha \leq 1$) (decision variable)
Q, q	Emergency supplies procurement quantities determined by the government and suppliers based on quantity flexibility contract are $Q+q$, while the minimum quantity of emergency supplies promised by the government is Q and the flexibility quantity of emergency supplies promised by suppliers is q (decision variable)
θ	Probability of a disaster occurring, which is related to each region's environment, population, etc., $0 < \theta \leq 1$
x	Stochastic quantity of emergency supplies in demand at the time of a disaster event, with a cumulative distribution function $F(x)$, continuous and derivable, and a distribution density function $f(x)$, and it is assumed that $x \sim D(0, U)$
λ	Shortage due to supply chain disruption, $0 < \lambda \leq q$ and $\lambda \sim D(0, q)$. Its cumulative distribution function is $G(\lambda)$, and the distribution density function is $g(\lambda)$
$\Pi_{gov}(Q, \alpha)$	Expectation function of social benefits of disaster relief for the government before the disruption
$D\Pi_{gov}(Q, \alpha)$	Expectation function of social benefits of disaster relief for the government after the disruption
$\Pi_{sup}(Q, \alpha)$	Profit expectation functions for suppliers before disruption
$D\Pi_{sup}(Q, \alpha)$	Profit expectation functions for suppliers after disruption

Note: Without loss of generality, it is assumed that $c_p > v_{sup} > v_{gov} > 0, S > B > p_m > p_w > (c_p + c_{sup}^h) > v_{sup} > v_{gov} > 0$.

be divided into three stages according to the disaster timeline, and the sequence of decisions conducted is as follows:

(1) Before a disaster, the government seeks a reliable supplier to establish a quantity flexibility contract relationship based on the regional environment, population, and historical disaster situation. The government can procure a minimum purchase quantity Q during the contractual cycle, referred to as the agreement purchase quantity. Meanwhile, the supplier adds additional reserves q to meet sudden demands in the event of a disaster.

(2) During a disaster, the government directly uses the reserve quantity Q emergency supplies for disaster relief. When it is not sufficient to meet the demand of the affected area, the government uses the remaining undelivered flexible supplies from the supplier for disaster relief and makes supplementary purchases according to the real need. However, at this time, the delivery of emergency supplies will face risk situations with and without disruptions.

(3) After a disaster, any surplus materials held by the government and the supplier will be treated as salvage value.

3.1 Procurement model without disruption

If no disaster occurs in cycle T , the pre-stocks of the government referring to the base purchase quantity of supplies will not be used for disaster relief, and no additional purchases will be made. The stockpiled supplies will eventually be disposed of at a residual value. The supplier stockpiles the remaining orders without delivery until the end of the period when they will be disposed at the residual value. The profit function of the supplier is

$$\Pi_{sup}(Q)_1 = (p_w - c_p)Q + [v_{sup} - (c_{sup}^h + c_p)]q \quad (1)$$

Hence, the benefit of the supplier consists of the sales revenue, production and inventory costs, and residual value of supplies. Meanwhile, the government cost functions is

$$\Pi_{gov}(Q)_1 = (v_{gov} - p_w - c_{gov}^h)Q \quad (2)$$

The cost of the government consists of the inventory cost and residual value of supplies. If a disaster occurs in cycle T , the government gains social benefits by saving the lives of the affected people and reducing economic losses as a result of disaster relief. As a result, the following three scenarios will occur due to the uncertainty of demand.

(1) When $0 < x \leq Q$, the government meets the disaster relief demand with the agreement purchase quantity, and the supplier disposes of the remaining materials according to the residual value.

(2) When $Q < x \leq Q + q$, the emergency supplies stocked by the government are not sufficient to meet the disaster relief demand, and further flexible procurement is required from the supplier to provide for the disaster relief. Surplus supplies are disposed according to the residual value by both parties.

(3) When $Q + q < x \leq U$, the total amount of contractual procurement established between the government and the supplier is still insufficient to meet the uncertain demand arising from the occurrence of a disaster. The government needs to procure emergency supplies through the spot market.

In combination with the above analysis, for the supplier,

$$\Pi_{sup}(x)_2 = \begin{cases} (p_w - c_p)Q - (c_{sup}^h + c_p)q + v_{sup}q, & 0 \leq x \leq Q; \\ (p_w - c_p)Q - (c_{sup}^h + c_p)q + p_w(x - Q) + v_{sup}(Q + q - x), & Q < x \leq Q + q; \\ (p_w - c_p)Q - (c_{sup}^h + c_p)q + p_wq, & Q + q < x \leq U \end{cases} \quad (3)$$

Above all, the supplier expectation profit function is

$$\begin{aligned} \Pi_{sup}(Q) &= (1 - \theta)\Pi_{sup}(Q)_1 + \theta\Pi_{sup}(Q)_2 = \\ &= (p_w - c_p)Q - (c_{sup}^h + c_p)q + (1 - \theta)v_{sup}q + \\ &= \theta \left\{ p_w \left[q - \int_Q^{Q+q} F(x)dx \right] + v_{sup} \int_Q^{Q+q} F(x)dx \right\} \quad (4) \end{aligned}$$

Considering the complexity of the model and the need for subsequent analysis, this study assumes that the demand x and the stockout λ at the time of the disaster obey a uniform distribution, an assumption that was applicable and reasonable^[8–10, 12–15]. Therefore, this study has set up with two hypotheses, $x \sim D(0, U)$, $\lambda \sim D(0, q)$. Substituting the assumption into the supplier profit expectation function, it is reduced to

$$\begin{aligned} \Pi_{sup}(Q) &= \theta(v_{sup} - p_w) \frac{\alpha^2 + 2\alpha}{2U} Q^2 + \\ &= [p_w - c_p + \theta p_w \alpha - v_{sup} \alpha(1 - \theta) - \alpha(c_p + c_{sup}^h)]Q \quad (5) \end{aligned}$$

Referring to John et al.^[25], who innovatively included the economic value generated by the government disaster relief in the government revenue approach for studying the option contract, they also introduced per unit of material revenue S while the government meets the disaster demand. Thus, the

government's profit function is

$$\Pi_{gov}(x)_2 = \begin{cases} Sx - [p_w Q + c_{gov}^h Q - v_{gov}(Q - x)], & 0 \leq x \leq Q; \\ Sx - [p_w Q + c_{gov}^h Q + p_w(x - Q)], & Q < x \leq Q + q; \\ Sx - [p_w Q + c_{gov}^h Q + p_w q + p_m(x - Q - q)], & Q + q < x \leq U \end{cases} \quad (6)$$

Substitute the parameters,

$$\begin{aligned} \Pi_{gov}(Q) &= (1 - \theta)\Pi_{gov}(Q)_1 + \theta\Pi_{gov}(Q)_2 = \\ &S \left[\int_Q^{Q+q} xf(x)dx + \int_{Q+q}^U (Q+q)f(x)dx \right] - \\ &p_w Q + c_{gov}^h Q - (1 - \theta)v_{gov}Q - \\ &\theta \left\{ p_w \left[q - \int_Q^{Q+q} F(x)dx \right] + \right. \\ &\left. p_m \left[U - Q - q - \int_{Q+q}^U F(x)dx \right] - v_{gov} \int_0^Q F(x)dx \right\} \quad (7) \end{aligned}$$

Simplify to

$$\begin{aligned} \Pi_{gov}(Q) &= -\theta \left[(p_m - p_w) \frac{(\alpha^2 + 2\alpha)}{2U} + \right. \\ &\left. \frac{(p_m - v_{gov})}{2U} + \frac{S(\alpha + 1)^2}{2U} \right] Q^2 + \\ &[S\theta(\alpha + 1) - c_{gov}^h + v_{gov}(1 - \theta) - p_w + \\ &\theta(p_m(1 + \alpha) - p_w\alpha)]Q - \frac{Up_m\theta}{2} \quad (8) \end{aligned}$$

The second order derivative of the supplier's profit expectation function on α and Q is

$$\begin{aligned} \frac{\partial^2 \Pi_{sup}(Q)}{\partial \alpha^2} &= \theta(v_{sup} - p_w) \frac{Q^2}{U} < 0, \\ \frac{\partial^2 \Pi_{sup}(Q)}{\partial Q^2} &= \\ \frac{\theta}{U} &[(p_m - p_w)(\alpha^2 + 2\alpha) + (p_m - v_{gov}) + S(\alpha + 1)^2] < 0. \end{aligned}$$

Thus, the function has the maximum value, and then derivatives. When it is taken to the maximum value,

$$\begin{aligned} \alpha_{sup}^* &= \frac{U(c_{sup}^h + c_p - v_s - p_w\theta + \theta v_{sup}) + Qp_w\theta - Q\theta v_{sup}}{\theta(p_w - v_{sup})Q}, \\ Q_{sup}^* &= \frac{U[p_w - c_p + \theta p_w\alpha + v_{sup}\alpha(1 - \theta) - \alpha(c_p + c_{sup}^h)]}{\theta(p_w - v_{sup})(\alpha^2 + 2\alpha)} \quad (9) \end{aligned}$$

Proposition 1 When the agreed price meets

$$p_w > \frac{c_p + \alpha[c_p + c_{sup}^h - v_{sup}(1 - \theta)]}{1 + \theta\alpha},$$

suppliers will then be willing to enter into a

procurement contract relationship with the government and have the optimal quantity $Q_{sup}^*(\alpha)$ of the contract.

Proof When $Q_{sup}^*(\alpha) > 0$, then

$$\frac{U[p_w - c_p + \theta p_w\alpha + v_{sup}\alpha(1 - \theta) - \alpha(c_p + c_{sup}^h)]}{\theta(p_w - v_{sup})(\alpha^2 + 2\alpha)} > 0,$$

$\theta > 0$, $p_w - v_{sup} > 0$, $\alpha > 0$, and $U > 0$. Therefore,

$$p_w - c_p + \theta p_w\alpha + v_{sup}\alpha(1 - \theta) - \alpha(c_p + c_{sup}^h) > 0,$$

and

$$p_w > \frac{c_p + \alpha[c_p + c_{sup}^h - v_{sup}(1 - \theta)]}{1 + \theta\alpha}.$$

The proposition is proven. ■

Proposition 2 When the government's agreed order quantity Q to suppliers is met,

$$Q < U \frac{\theta p_w - [c_p + c_{sup}^h - (1 - \theta)v_{sup}]}{\theta p_w}.$$

Suppliers will then be willing to stockpile additional supplies and the amount ordered is negatively related to the negotiated pricing and positively related to the probability of a disaster.

Proof From the supplier's expected profit function, the expected profit is a quadratic function on α . In practice, $\alpha > 0$ has practical economic significance, such that the quadratic function symmetry axis is greater than zero, that is

$$Qv_s - Qc_{sup}^h - Qc_p + Qp_w\theta - Q\theta v_{sup} - \frac{Q^2 p_w\theta}{U} + \frac{Q^2 \theta v_{sup}}{U} > 0,$$

it is simplified to

$$Q < U \frac{\theta p_w - [c_p + c_{sup}^h - (1 - \theta)v_{sup}]}{\theta(p_w - v_{sup})}$$

Thus, if the government wishes to enter a flexible purchasing relationship with suppliers, it cannot indefinitely increase the base purchase volume agreed with suppliers, and increasing the agreed price can induce suppliers to hold additional reserves. Hence, the proposition is proven. For the government's cost expectation function, the second order derivative function is

$$\begin{aligned} \frac{\partial^2 \Pi_{gov}(Q)}{\partial Q^2} &= \\ -2\theta &\left[(p_m - p_w) \frac{(\alpha^2 + 2\alpha)}{2U} + \frac{(p_m - v_{gov})}{2U} \right] < 0. \end{aligned}$$

The function has a maximum value, then the first order derivative is obtained as

$$Q_{gov}^*(\alpha) = U[S\theta(\alpha+1) - c_{gov}^h + v_{gov}(1-\theta) - p_w(1+\theta\alpha) + \theta(p_m(1+\alpha))]/\{\theta[(p_m - p_w)(\alpha^2 + 2\alpha) + (p_m - v_{gov})] + S\theta(\alpha+1)^2\} \quad (10)$$

Proposition 3 When the contract price

$$p_w < \frac{\theta(S + p_m)(1 + \alpha) - c_{gov}^h + v_{gov}(1 - \theta)}{(1 + \theta\alpha)},$$

the government will then be willing to reach cooperation with the supplier, the maximum agreed price acceptable to the government is positively correlated with the social benefits of disaster relief units, and the government has the optimal order quantity for $Q_{gov}^*(\alpha)$. In practical terms, Proposition 3 also suggests that the government should maximize disaster mitigation and relief to maximize the saving of people's lives and minimize economic losses.

Proof When $Q_{gov}^*(\alpha) > 0$, as shown in Eq. (10), and $\theta > 0, p_m - p_w > 0, p_m - v_{gov} > 0, \alpha > 0, U > 0$, and $S > 0$, therefore,

$$U[S\theta(\alpha+1) - c_{gov}^h + v_{gov}(1-\theta) - p_w(1+\theta\alpha) + \theta(p_m(1+\alpha))] > 0,$$

then

$$S\theta(\alpha+1) - c_{gov}^h + v_{gov}(1-\theta) - p_w(1+\theta\alpha) + \theta(p_m(1+\alpha)) > 0.$$

So

$$p_w < \frac{\theta(S + p_m)(1 + \alpha) - c_{gov}^h + v_{gov}(1 - \theta)}{(1 + \theta\alpha)}.$$

The proposition is proven. ■

3.2 Procurement model with disruption

During a disaster in cycle T , if a supply chain disruption occurs, three scenarios will occur due to the uncertainty of demand.

(1) When $0 < x \leq Q$, the government meets the disaster relief demand with the agreement purchase quantity and is not affected by the disruption.

(2) When $Q < x \leq Q + q$, suppliers are exposed to the disruption risk, and the stockout quantity is due to disaster disruption. Hence, the government needs to procure extra emergency supplies from the spot market because the demand is not yet met. However the uncertainty of the spot market and the long delivery period make the government face penalty costs as well.

(3) When $Q + q < x \leq U$, the shortage quantity is introduced as λ . The total amount of contractual procurement established between the government and the supplier is insufficient to cope with the uncertain

demand arising from the occurrence of a disaster.

Putting the above analysis together, when a supplier disruption occurs, the supplier profit function becomes

$$D\Pi_{sup}(x) = \begin{cases} (p_w - c_p)Q - (c_{sup}^h + c_p)q + v_{sup}(q - \lambda), & 0 \leq x \leq Q; \\ (p_w - c_p)Q - (c_{sup}^h + c_p)q + p_w(x - Q) \times [x + \lambda | \text{if } x + \lambda - Q - q < 0] + v_{sup} \max\{Q + q - \lambda - x, 0\} - B \max\{x + \lambda - Q - q, 0\}, & Q < x \leq Q + q; \\ (p_w - c_p)Q - (c_{sup}^h + c_p)q + p_w(q - \lambda) - B\lambda, & Q + q < x \leq U \end{cases} \quad (11)$$

It is assumed that $x \sim D(0, U)$ and $\lambda \sim D(0, q)$. According to the convolution formula for the distribution of the sum function of two random variables,

$$G(Z) = \begin{cases} \frac{Z}{Uq}, & 0 < Z < q; \\ \frac{1}{U}, & q < Z < U; \\ \frac{U + q - Z}{Uq}, & U < Z < U + q \end{cases} \quad (12)$$

We substitute the parameters,

$$D\Pi_{sup}(Q) = \theta \left(\frac{v_{sup}}{6U} + \frac{B}{3U} - \frac{p_w}{4U} \right) Q^2 \alpha^2 + \left[\theta \frac{B + v_{sup} - p_w}{2U} Q + v_{sup}(1 - \theta) - (c_p + c_{sup}^h) - \frac{\theta}{2}(B - p_w) \right] Q\alpha + (p_w - c_p)Q \quad (13)$$

Correspondingly, the social benefit function of the government is changed to

$$D\Pi_{gov}(x) = \begin{cases} Sx - [p_w Q + c_{gov}^h Q - v_{gov}(Q - x)], & Q < x \leq Q + q; \\ Sx - \{p_w Q + c_{gov}^h Q + p_w(x - Q) \times [x + \lambda | \text{if } x + \lambda - Q - q < 0] + p_m \max\{x + \lambda - Q - q, 0\} + B \max\{x + \lambda - Q - q, 0\}\}, & 0 \leq x \leq Q; \\ Sx - \{p_w Q + c_{gov}^h Q + p_w[q - \lambda] + p_m[x - Q - q] + p_m \lambda + B\lambda\}, & Q + q < x \leq U \end{cases} \quad (14)$$

We simplified it to

$$\begin{aligned}
 D\Pi_{gov}(Q) = & -\theta \left[(2p_m - p_w) \frac{\alpha^2 + 2\alpha}{4U} + (B - p_m) \times \right. \\
 & \left. \frac{2\alpha^2 + 3\alpha}{6U} + \frac{p_m - v_{gov}}{2U} + \frac{S(\alpha + 1)^2}{2U} \right] Q^2 + \\
 & \left\{ S\theta(\alpha + 1) - c_{gov}^h + v_{gov}(1 - \theta) - p_w + \right. \\
 & \left. \frac{1}{2}\theta[p_m(2 + \alpha) - p_w\alpha - B\alpha] \right\} Q - \frac{Up_m\theta}{2} \quad (15)
 \end{aligned}$$

In summary, the supplier’s profit function and the government’s social benefit function remain a quadratic function with respect to Q and α , when there is a shortage of stockouts $\lambda \sim D(0, q)$ due to a supply disruption during a disaster. Accordingly,

$$\begin{aligned}
 Q_{gov}^{**}(\alpha) = & U \left\{ S\theta(\alpha + 1) - c_{gov}^h + v_{gov}(1 - \theta) + \right. \\
 & \left. - p_w + \frac{1}{2}\theta[p_m(2 + \alpha) - p_w\alpha - B\alpha] \right\} / \\
 & \left[\frac{1}{2}\theta(2p_m - p_w)(\alpha^2 + 2\alpha) + \frac{1}{3}\theta(B - p_m) \times \right. \\
 & \left. (2\alpha^2 + 3\alpha) + \theta(p_m - v_{gov}) + \theta S(\alpha + 1)^2 \right] \quad (16)
 \end{aligned}$$

Thus, the concavity of the supplier’s expected profit function changes after a supply disruption occurs and the concavity of the government’s social benefit function remains unchanged. The government has the optimal order quantity

Proposition 4 After a supply disruption, when the contract price

$$p_w < \frac{\theta(S + p_m)(1 + \alpha) - c_{gov}^h + v_{gov}(1 - \theta)}{(1 + \theta\alpha)},$$

the government will then be willing to cooperate with the supplier, the maximum agreed price acceptable to the government is positively related to the social benefits of the relief unit, the government has the optimal order quantity $Q_{gov}^{**}(\alpha)$.

Proof When $Q_{gov}^{**}(\alpha) > 0$, and $\theta > 0, p_m - p_w > 0, p_m - v_{gov} > 0, \alpha > 0, U > 0, S > 0$, then,

$$\begin{aligned}
 & U \left\{ S\theta(\alpha + 1) - c_{gov}^h + v_{gov}(1 - \theta) - p_w + \right. \\
 & \left. \frac{1}{2}\theta[p_m(2 + \alpha) - p_w\alpha - B\alpha] \right\} > 0.
 \end{aligned}$$

So

$$\begin{aligned}
 & S\theta(\alpha + 1) - c_{gov}^h + v_{gov}(1 - \theta) - p_w + \\
 & \frac{1}{2}\theta[p_m(2 + \alpha) - p_w\alpha - B\alpha] > 0.
 \end{aligned}$$

That is

$$p_w < \frac{S\theta(\alpha + 1) - c_{gov}^h + v_{gov}(1 - \theta) + \frac{1}{2}\theta[p_m(2 + \alpha) - B\alpha]}{1 + \frac{1}{2}\theta\alpha}.$$

Hence, the proposition is proven. ■

Proposition 5 After a supply disruption, when the contract price

$$p_w \geq \frac{(c_p + c_{sup}^h) - v_{sup} + \frac{\theta}{2}B(1 - \frac{Q}{U}) - \frac{\theta}{2}v_{sup}(2 - \frac{Q}{U})}{\frac{\theta}{2}(1 - \frac{Q}{U})},$$

suppliers will be inclined to offer flexible procurement quantities. When

$$p_w < \frac{(c_p + c_{sup}^h) - v_{sup} + \frac{\theta}{2}B(1 - \frac{Q}{U}) - \frac{\theta}{2}v_{sup}(2 - \frac{Q}{U})}{\frac{\theta}{2}(1 - \frac{Q}{U})},$$

suppliers will then be reluctant to offer flexible procurement quantities. Thus, suppliers are less willing to participate in the contract when prices are high, so supply disruption reduces the willingness of both partners to collaborate to some extent.

Proof Based on the previous analysis, the supplier profit function is a quadratic function with respect to α . Its second-order derivative is solved with respect to α ,

$$\frac{\partial^2 D\Pi_{sup}(Q)}{\partial \alpha^2} = 2\theta \left(\frac{v_{sup}}{6U} + \frac{B}{3U} - \frac{p_w}{4U} \right) Q^2 > 0.$$

The supplier profit function is a concave function, such that when the axis of symmetry is greater than zero, the proposition is proven and the details are omitted. Hence, the proposition is proven. ■

4 Simulation and Analysis

Simulation is the process of experimentally studying a real system by building a model of the an actual system^[26, 27]. In view of the complexity of the model, the following numerical examples and sensitivity analyses will be used to explain the relationship between material stockpiles and profits, government procurement costs and social benefits, and the pricing strategies of governments and enterprises. Moreover, they will be used to explore the impact of supply chain disruption on the benefits of both parties.

4.1 Case description

A local government and a local material enterprise established a flexible contractual relationship with the following details. Two case data were set up as shown in [Table 2](#).

Table 2 Case data.

Item	Population affected	Parameter									
		U (pack)	S (yuan/pack)	p_m (yuan/pack)	c_p (yuan/pack)	v_{sup} (yuan/pack)	v_{gov} (yuan/pack)	B (yuan/pack)	c_{sup}^h (yuan/pack)	c_{gov}^h (yuan/pack)	θ
A	742	1500	200	70	30	5	4	90	5	4	0.3
B	1571	3000	400	160	65	13	12	190	10	9	0.3

Note: Case data are from Ref. [13].

Combined with the procurement model in Section 3, to promote the willingness of the government and the suppliers, $\alpha = 0.2$ [13] was taking as an example. The results show the price range for supply A is (34.25, 91.23) yuan/pack, and the pricing range for supply B is (73.74, 190.28) yuan/pack, after considering the disruption, the price range for supply A is (34.25, 88.54) yuan/pack. The pricing range for supply B is (73.74, 184.95) yuan/unit.

Setting up $\alpha = 0.2$, $p_{wA} = 50$, and $p_{wB} = 130$ (p_{wA} means p_w of item A , p_{wB} means p_w of item B , similarly hereinafter), Fig. 2 shows the expected profits of suppliers (Fig. 2a) and the government (Fig. 2b) when items A and B are set at different prices without disruptions. When the procurement price of emergency supplies is very low, as shown in Fig. 2a, for items A and B , the supplier's profit expectation function is a monotonic minus function. At this point, the supplier is unprofitable and has no intention of entering into a quantity flexibility contract with the government. When the procurement price of emergency supplies is very high, as shown in Fig. 2b for item A ($p_{wA} = 95$) and item B ($p_{wB} = 195$), the government's benefit expectation function is a monotonic minus function. At this point, the willingness of the government to establish a quantity flexibility contract with suppliers is low. Therefore, the government and enterprises should price reasonably during the negotiation stage with a view to establishing a good cooperative relationship. In addition, Fig. 2 shows that the government's optimal order quantity decision is generally smaller than the enterprise's optimal readiness decision. That is to say, under the condition of satisfying profitability, the enterprise is willing to sell as many supplies as possible to meet the government's need. For the government, disaster relief is humanitarian activity. As the quantity ordered increases, the cost of disaster relief gradually increases, and the government's expected benefit decreases. There is an upper limit to disaster relief needs. Hence, when the agreed procurement quantity is big, the government's expected benefit is generally a

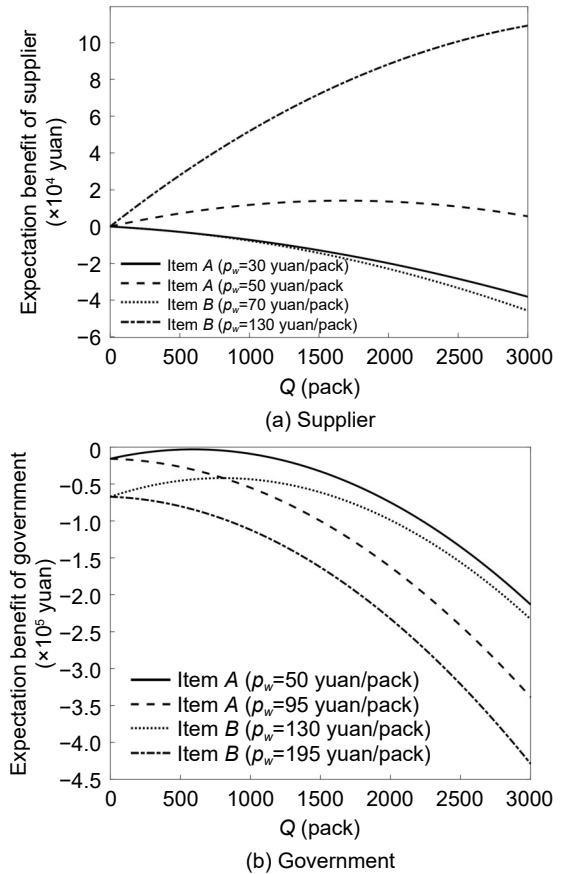


Fig. 2 Purchase volume and profit of supplier and government agreement under different prices ($\alpha=0.2$).

negative return value.

When a supply disruption occurs, the expected profits of the government and suppliers are modified. Figure 3 shows an image of the expected supplier profit and the expected government social benefit function before and after a supply chain disruption during a disaster. It can be seen that when a supply disruption exists, there is a different degree of reduction in the benefits of the government and the profits of suppliers at the same order quantity for different materials.

For suppliers, as shown in Figs. 3a and 3b, the supplier is willing to sign the quantity flexibility contract with the government to sell as many supplies as possible on the basis of meeting the profitability condition, and the profit function is an increasing

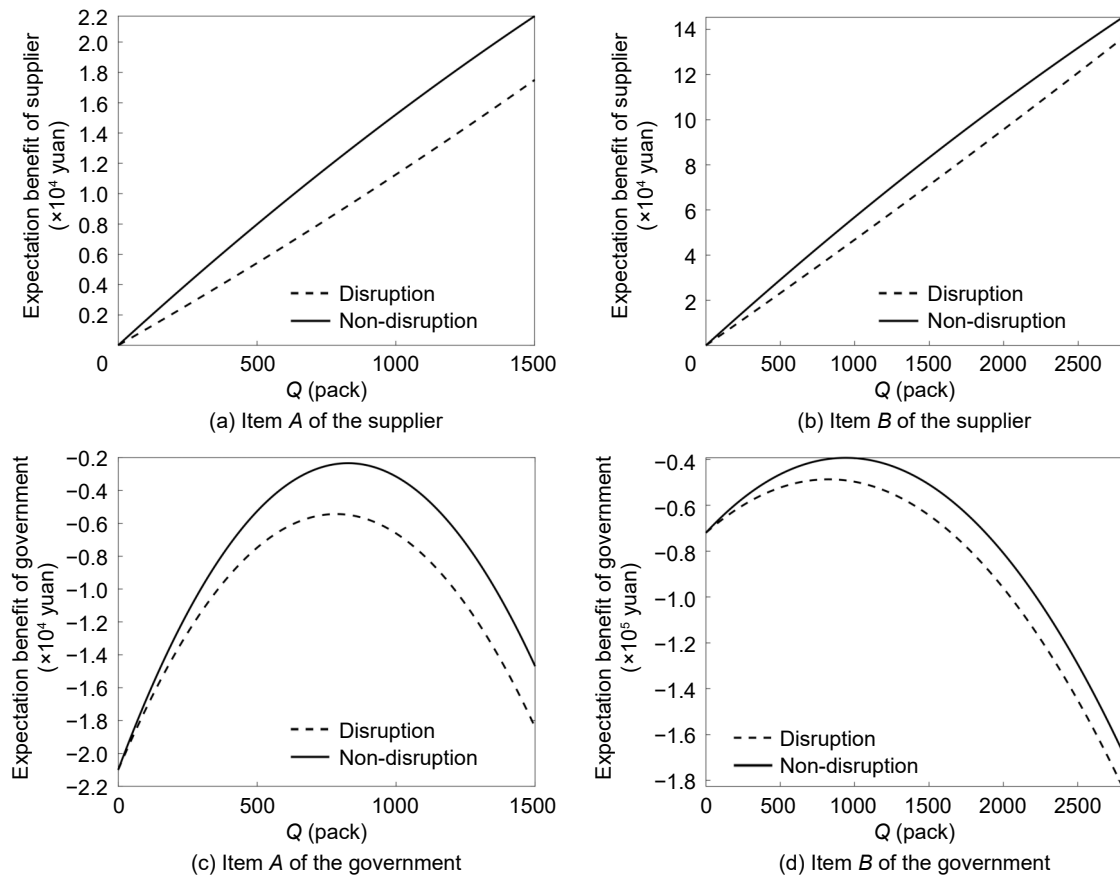


Fig. 3 Government and supplier profit functions before and after supply chain disruption.

function. When supply interruptions are considered, the profit obtained decreases for the same ordered quantity. As the ordered quantity increases, the penalty cost due to stock-out increases and the profit loss increases.

For the government, as shown in Figs. 3c and 3d, the government's profit function is convex, and there is an optimal order quantity that maximizes social benefits. The optimal procurement quantities before the disruption occurs are 851 packs for material *A* and 1105 packs for material *B*, and the optimal procurement quantities after the disruption are 732 packs for material *A* and 928 packs for material *B*. Thus, when a supply disruption occurs, the government's optimal order quantity and total efficiency both decrease. Then, as the order quantity increases, the government's relief cost increases and the social benefit gradually decreases. Therefore, supply chain disruptions have a negative effect on the government and enterprises. The supply chain disruption means increasing costs for the government and reducing profits for the suppliers, and it will reduce the optimal stock and optimal order quantity for both parties and reduce their willingness to cooperate.

4.2 Sensitivity analysis

Based on the analysis in Section 2, flexible contractual procurement pricing, ordering quantity, and parameter decisions affect the expected profit of the enterprise and the expected benefit of the government. This study conducted a sensitivity analysis for the optimal procurement quantity of the government and the enterprise with respect to the agreed price and flexibility parameters. The data used in this section are the parameters of item *A*.

4.2.1 Sensitivity analysis of the unit material disaster relief value and unit contract price

For the government, the unit material disaster relief value and unit contract price are the two main exogenous parameters that affect the order quantity. Hence, this study conducted a sensitivity analysis of the two parameters.

As shown in Fig. 4a, the government's optimal order quantity is positively correlated with the unit material disaster relief value. For materials with a significantly higher unit material disaster relief value, the more the affected people's immediate survival needs and life and health category are at stake, the more they tend to

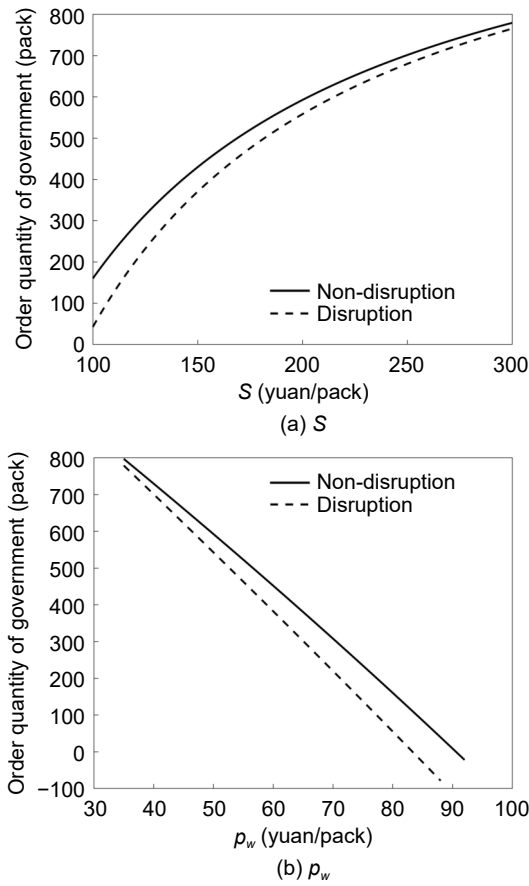


Fig. 4 Sensitivity analysis of unit material disaster relief value S and unit contract price p_w to government order quantity ($\alpha=0.2$) for Item A.

stockpile more materials. However, when a supply disruption occurs, the government’s optimal stockpile quantity decreases for the same unit material disaster relief value. As shown in Fig. 4a, the higher the pack material disaster relief value is, the less the disruption factor causes a reduction in the optimal order quantity. Hence, the government tends to ignore the disruption factor when the relief value of supplies is high enough. As shown in Fig. 4b, the government optimal order quantity decreases as the unit contract price increases. The government’s willingness to seek cooperation with external companies’ decreases when unit contract price increases. While a supply disruption occurs, the government’s optimal reserve quantity decreases at the same unit contract price. Moreover as shown in Fig. 4b, the disruption factor has a greater reduction in the optimal order quantity when the agreed price is higher. Hence, the government attaches more importance to the disruption factor when the unit contract price is higher.

4.2.2 Sensitivity analysis of the flexibility parameter α

Setting up $p_{wA} = 50$ yuan/pack, the flexibility parameter $0 < \alpha \leq 1$. For suppliers, as shown in Fig. 5a, when the flexibility parameter increases, the supplier’s optimal order quantity decreases before and after the supply disruption. Compared with the case without interruption, the influence of the increase of the unit flexibility parameters on the decrease of the optimal order quantity under interruption is significantly weakened.

For the government, as shown in Fig. 5b, the government flexibility parameter is a quadratic function about with an optimal value without supply disruption while the government’s optimal order quantity decreases when a supply disruption occurs. However, as the flexibility parameter increases, the reduction in the government’s optimal order quantity gradually decreases.

5 Conclusion

For emergency supplies that are in high demand during disasters, such as food and other types of household

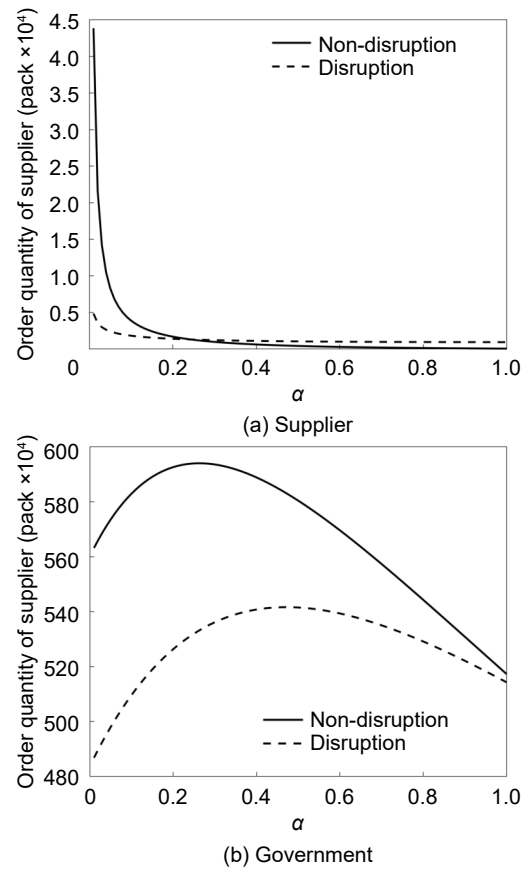


Fig. 5 Sensitivity analysis of the flexibility parameters α to the order quantity for Item A.

goods, this study analyses the problem of emergency supply procurement by establishing a quantity flexibility contract between the government and enterprises. It considers the supply chain disruption during disaster relief. A mathematical model was built to quantify the mechanism of the impact of supply disruption on the decision-making of the government and enterprises. Through numerical analysis, the validity of the quantity flexibility contract relationship was verified, the basic decisions of both parties in terms of the pricing strategy and agreed order quantity were derived, and the impact of supply disruptions on the decisions of both parties was analyzed. Through a sensitivity analysis, the impact of the unit material disaster relief value and unit contract price on the government's optimal expected order quantity was analyzed, and the impact of flexibility parameters on the government and enterprises' decisions on agreement order quantity was demonstrated. The following are some management insights in this paper.

(1) A reasonable contract price is a prerequisite for the establishment of a flexible contractual relationship between the government and enterprises. The contract price is too high or too low to reduce the willingness of both sides to cooperate. The occurrence of supply chain disruptions will reduce the government's social relief benefits and the suppliers' profit, which is a "lose-lose" scenario. Then, there will be different degrees of decline in pricing decisions and the agreed order quantity, and the government-enterprise cooperation will be reduced. Therefore, it is necessary for the government and enterprises to take certain measures to avoid the risk of supply chain disruptions.

(2) Before and after the disruptions, the relationship between the relief value, the contract price, and the government's order quantity remain unchanged, and the government's optimal order quantity increases with the increase in the relief value and decreases with the increase in the contract price. An occurrence of disruption will reduce the overall optimal order quantity. When the unit relief value of a material is high, and its unit contract price is low, the reduction of the disruption factor on the optimal order quantity is low. The government focuses on the relief value and the contract price, and tends to disregard the interruption factor and maintain a cooperative relationship with suppliers at that time. Before and after interruptions, the relationship between the

flexibility parameter and the influence of supplier reserves and government ordering quantity remains the same, and the optimal supplier reserves decrease with the increase in the flexibility parameter. Accordingly, the agreement ordering quantity decreases when the interruption occurs.

Further research can be conducted on the game behavior of governments and enterprises and their decision-making strategies after taking measures to avoid the risk of supply chain disruptions^[23]. In addition, the demands and disruption shortages in this thesis are based on the assumption of having a uniform distribution, but other probability distribution models that may be more suitable for actual situations can also be utilized in future research.

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