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Comparison of data from fall-cone and laboratory vane tests to investigate undrained shear strength for some fine-grained materials

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Abstract. To investigate the variation of undrained shear strength with changes in water content, determination of a value of undrained shear strength at the liquid limit is needed. There is debate as to the extent of the typical range of values of undrained shear strength at the liquid limit. In this paper, the undrained shear strength is measured using the miniature laboratory shear vane apparatus and compared with values back-analysed from British Standard fall-cone test data. Four geomaterials: Kaolin, Bentonite, Chalk and Bothkennar clay were selected for testing. The variation of the undrained shear strength values from both testing approaches with changing water content was also studied and compared with the results of previous work. The paper reports a narrow range of undrained shear strength values at the liquid limit for the geomaterials tested.

Keywords: Fall-cone; Miniature laboratory shear vane; Undrained shear strength; Water content; Liquidity index

1 INTRODUCTION

1.1 Fall-cone testing for soil classification and undrained shear strength determination

The liquid limit (w_L) of fine-grained materials can be determined using the fall-cone device i.e. the fall-cone liquid limit ($w_{L,FC}$) (BSI 1990a, 2018). The fall-cone can also be used to determine fall-cone undrained shear strength ($c_{u,FC}$) using Hansbo's equation (Hansbo 1957) shown as Eq. (1):

$$c_{u,FC} = K(mg/d^2) \quad (1)$$

where $c_{u,FC}$ is the fall-cone undrained shear strength, m is the cone mass, g is the acceleration due to gravity, d is the fall-cone penetration and K is a cone factor.

To use Eq. (1) to estimate undrained shear strength a value of K is needed. Theoretical values are proposed in Koumoto and Houlsby (2001) and experimental values are given in Wood (1985). In this paper K is taken as 0.867. This K value is derived for the BSI (1990a, 2018) standard cone (30º, 80g) taking $w_{L,FC}$ at 20mm fall-cone penetration (as required by the code) and assuming a c_u at w_L (c_L) of 1.7kPa following Wroth and Wood (1978). This cone factor was also used in the studies of Vardanega and Haigh (2014) and Vardanega *et al.* (2019). For a detailed review of the use of fall-cones for soil classification see O'Kelly *et al.* (2018). The accuracy of the $c_{u,FC}$ values estimated using Eq. (1) depends on the validity of the K-value used and hence, for the $c_{u,FC}$ values presented in this paper, is reliant on c_L = 1.7kPa being a reasonable assumption.

There is debate on the range of c_L values (e.g. Nagaraj *et al.* 2012, Haigh and Vardanega 2012 and O'Kelly 2019). The review of O'Kelly (2019) confirmed that the range of c_L values is relatively narrow (around 1 to 3kPa) (a similar range to that given in Wroth and Wood 1978): however, this range is based on shear strength measurements taken at the percussion cup w_L ($w_{L,PC}$). (O'Kelly *et al.* (2018) presented correlations linking $w_{L,FC}$ to $w_{L,PC}$: the difference being generally not significant up to w_L values of about 120%.) Given the debate regarding the range of c_L values, laboratory shear vane testing was also undertaken alongside the fall-cone testing in this study. In this paper, the c_L values taken from the laboratory shear vane testing are denoted as $c_{L,v}$.

1.2 Undrained shear strength variation with changing water content

Wroth and Wood (1978) and Wood (1990) proposed an equation for c_u with changing liquidity index (I_L) of the form given in Eq. (2):

$$c_u = c_L(R_{MW})^{(1-I_L)} \quad (2)$$

where R_{MW} is the ratio of the implied c_u values at the w_L and plastic limit (w_p). I_L is given by Eq. (3):

$$I_L = (w - w_P)/(w_L - w_P) \quad (3)$$

where w is the water content. Wroth and Wood (1978) (following the recommendation of Schofield and Wroth 1968) suggested that R_{MW} can be taken as 100. Vardanega and Haigh (2014) used a fall-cone test database to show that an average value of R_{MW} of about 34.3 for fine-grained materials may be more appropriate for $0.2 < I_L < 1.1$. Vardanega *et al.* (2019) calculated $R_{MW} \approx 21.9$ for some peat derived soils.

Koumoto and Houlsby (2001) suggested the use of logarithmic liquidity index (I_{LN}) (Eq. 4) to capture changes of c_u with w.

$$I_{LN} = \ln(w/w_P)/\ln(w_L/w_P) \quad (4)$$

Vardanega and Haigh (2014) showed a modified form of Eq. (2) with an analogous R_{MW} value of 83.5 for the database of fall-cone tests on clays and silts (Eq. 5) (Vardanega *et al.* 2019 found an analogous R_{MW} of about 30.7 for some peat derived soils).

$$c_u = c_L(83.5)^{(1-I_{LN})} \quad (5)$$

Federico (1983), Berilgen *et al.* (2007) and Kuriakose *et al.* (2017) used the water content ratio (w/w_L) as a predictor of c_u. While such formulations have the advantage of not requiring w_P to be determined (as pointed out in Kuriakose *et al.* 2017), Vardanega and Haigh (2017) and Vardanega *et al.* (2019) found that I_L and I_{LN} yielded stronger fits to the examined data. Vardanega *et al.* (2019) found the following equation (Eq. 6) linking w/w_L for some peat derived soils and this form of the equation will be used in the analysis presented in this paper.

$$w/w_L = 1 - 0.102\ln\left(c_{u,FC}/c_L\right) \quad (6)$$

1.3 Study aims

This study has two main aims: (1) measure the $c_{L,v}$ values for four geomaterials and (2) examine the use of I_L, I_{LN} and w/w_L for describing changes of $c_{u,v}$ and $c_{u,FC}$ with changing w for the tested materials.

2 METHODOLOGY

In this study four geomaterials were tested: Bentonite, Kaolin, Bothkennar clay and Chalk (see Gan 2020 for more details on the testing). The Kaolin and Bentonite were selected as they are standard laboratory soils of different plasticity levels. Characterisation of the Bothkennar clay has been reported in various publications (e.g. Hight *et al.* 1992, Nash *et al.* 1992). Chalk is a soft, fine-grained, friable limestone (e.g. Meigh and Early 1957). For more geotechnical information on the Chalk tested in this study see e.g. Bialowas *et al.* (2018). For the statistical analysis in this paper: R^2 is the coefficient of determination, n is the number of data-points and p is the p-value (see Montgomery *et al.* 2004 and Vardanega and Haigh 2014 for further details on the statistical definitions and approaches used in this work).

2.1 Fall-cone testing and plastic limit determination

The BSI 30°, 80g cone was used to determine the w_L values (BSI 1990a, 2018). BSI (2018) does permit the use of a 60°, 60g cone (with w_L taken at 10mm fall-cone penetration) which was not the case in the previous version of the code (BSI 1990a). For the fall-cone used in this study w_L corresponds to the w taken at 20mm of fall-cone penetration. Fig. 1 shows the collected fall-cone data from this study along with the derived w_L values. The thread-rolling procedure outlined in BSI (1990a, 2018) was used to determine the w_p values. The w_P values quoted on Fig.1 are the average of three thread rolling tests in the case of Bentonite and four thread rolling tests for the other three materials. Fall-cone penetrations at values lower than the 15 – 25mm range recommended in BSI (1990a, 2018) for the 30°, 80g cone were also taken to study the c_u variation with w in the plastic range. The data shown on Fig. 1 for Chalk is the average of tests conducted before and after the vane testing (see Fig. 2).

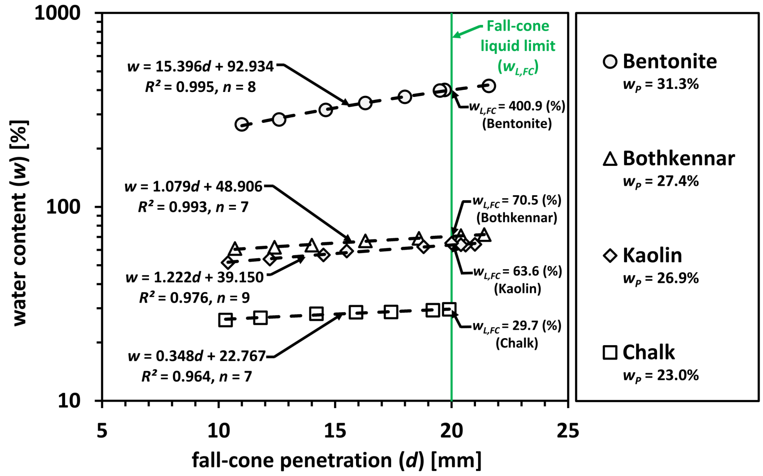

Figure 1. Water content (w) versus fall-cone penetration (d) for the tested materials

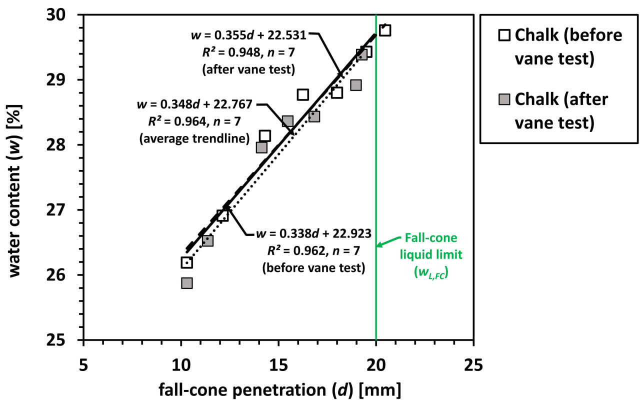

Figure 2. Water content (w) versus fall-cone penetration (d) for the Chalk tests

When carrying out the fall-cone testing for the Chalk there was a concern that the Chalk was drying out faster during the sample preparation than the other tested materials. Therefore fall-cone tests were taken before and after the vane testing had been carried out in the sample mould (see Fig. 2). While some variation in the trendlines can be observed it was judged by the authors that the difference was sufficiently small such that the average trendline could be used to describe the Chalk test series. To plot the ‘before vane test’ series on Fig. 2 two w measurements were averaged (i.e. one reading after the fall-cone test and the other just prior to vane testing). A third w measurement taken after the vane testing was used to plot the ‘after vane test’ series shown on Fig. 2.

2.2 Laboratory vane testing

The miniature laboratory shear vane tests were carried out in accordance with BSI (1990b) (referred to as the ‘laboratory vane method’ in BSI 1990b). The test device is hand-cranked and measures the angular rotation of the torsional spring which is used to compute the $c_{u,v}$ of the sample using Eq. (7):

$$c_{u,v} = 1000(m/V) \tag{7}$$

where $c_{u,v}$ is the undrained shear strength in vane shear, m is the applied torque (product of maximum angular rotation and the spring calibration factor) and V is the vane constant which depends on the dimensions of the vane. Kravitz (1970) compared both the fall-cone and laboratory vane tests and showed that the standard deviation (SD) of $c_{u,v}$ (for a motorised laboratory vane device) was about double that of the $c_{u,FC}$ values (obtained with a 60° cone). While the devices from Kravitz’s study (Kravitz 1970) are not identical to those used in this study they are sufficiently similar and therefore allow the postulation that the fall-cone data should show less spread than the laboratory vane data. The higher SD values for the vane device may explain (at least in part) some of the range of the $c_{L,v}$ values encountered (see also Haigh and Vardanega 2012).

3 UNDRAINED SHEAR STRENGTH AT LIQUID LIMIT

Fig. 3 shows the $c_{u,v}$ data plotted against the $c_{u,FC}$ data. Power laws were fitted to the data as they pass through the origin. The w_L occurs on this plot at $c_{u,FC}$ = 1.7kPa as the $c_{u,FC}$ values were determined using K = 0.867 as previously discussed. Substituting 1.7kPa into the fitted power equations on Fig. 3 gives values for $c_{L,v}$ of 0.72kPa for Chalk, 1.21kPa for Bothkennar, 1.51kPa for Kaolin and 2.14kPa for Bentonite. These values are (apart from Chalk which is a weak rock) within the range of 1 to 3kPa given in the review of O’Kelly (2019) for c_L at the $w_{L,PC}$.

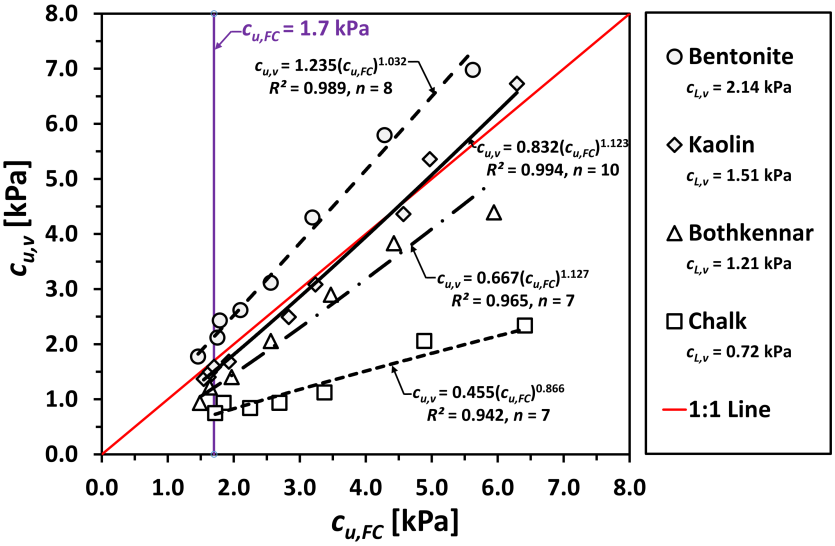

Figure 3. $c_{u,v}$ versus $c_{u,FC}$

4 VARIATION OF UNDRAINED SHEAR STRENGTH WITH WATER CONTENT

Fig. 4 shows the $c_{u,v}$ and $c_{u,FC}$ values plotted against I_L. The following sub-sections will now examine the use of the I_L, I_{LN} and w/w_L parameters to predict values of $c_{u,v}$ and $c_{u,FC}$ for the tested materials.

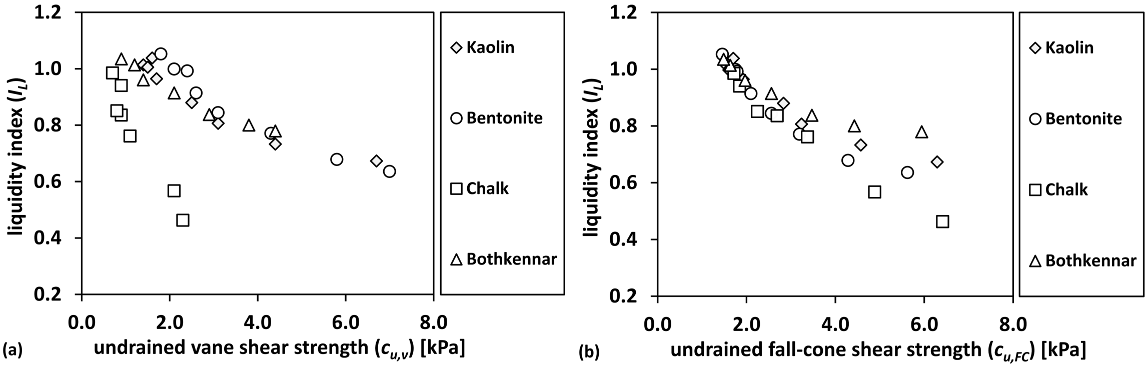

Figure 4. (a) I_L versus $c_{u,v}$ and (b) I_L versus $c_{u,FC}$

4.1 Liquidity index versus normalised undrained shear strength

Fig. 5 shows I_L plotted against normalised c_u. Fig. 5a shows the data of $c_{u,v}$ normalised with $c_{L,v}$ and Fig. 5b shows the data of $c_{u,FC}$ normalised with 1.7kPa. The data on Fig. 5 have been fitted with equations of the form shown as Eq. 8 below (the statistical measures are shown on Fig. 5):

$$I_L = 1 - \alpha \ln(c_u/c_L) \tag{8}$$

The value of α for the entire dataset is 0.278 (n = 30) for the vane shear data and 0.299 (n = 31) for the fall-cone data. (These values equate to R_{MW} values (from Eq. 2) of 36.5 and 28.3 respectively.) The values correspond well with the value from Vardanega and Haigh (2014) (i.e. R_{MW} = 34.3). The value of α computed with the Chalk data excluded is 0.236 (n = 23) for the vane shear data and 0.261 (n = 24) for the fall-cone data. The values equate to R_{MW} values (from Eq. 2) of 69.2 and 46.1 respectively.

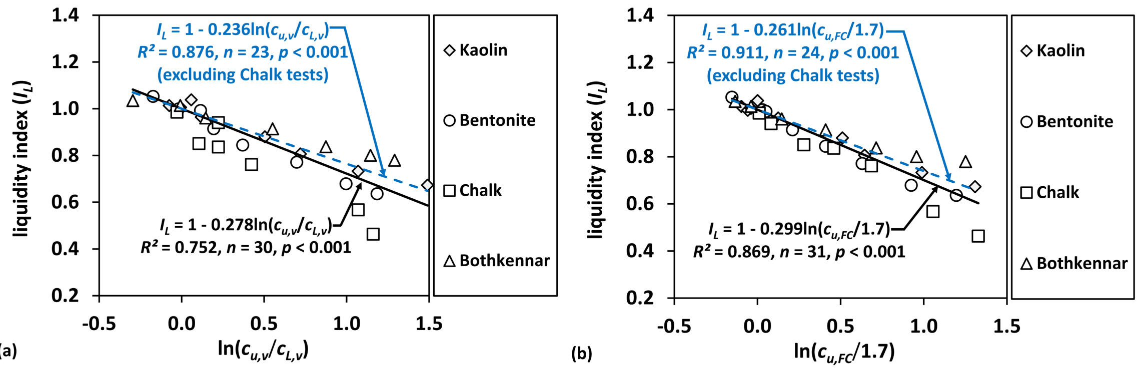

Figure 5. (a) I_L versus ln($c_{u,v}$/$c_{L,v}$) and (b) I_L versus ln($c_{u,FC}$/1.7)

4.2 Logarithmic liquidity index versus normalised undrained shear strength

Fig. 6 shows I_{LN} plotted against normalised c_u. Fig. 6a shows the data of $c_{u,v}$ normalised with $c_{L,v}$ and Fig. 6b shows the data of $c_{u,FC}$ normalised with 1.7kPa. The data on Fig. 6 have been fitted with equations of the form shown as Eq. 9 below (the statistical measures are shown on Fig. 6):

$$I_{LN} = 1 - \beta \ln(c_u/c_L) \tag{9}$$

The value of β for the entire dataset is 0.197 ($n = 30$) for the laboratory vane shear data and 0.216 ($n = 31$) for the fall-cone data. (These values correspond to analogous R_{MW} values of 160.2 and 102.5 respectively.) The values are higher than those from Vardanega and Haigh (2014) and Vardanega *et al.* (2019). The value of β computed with the Chalk data excluded is 0.143 ($n = 23$) for the vane shear data and 0.157 ($n = 24$) for the fall-cone data. These values correspond to analogous R_{MW} values of 1089 and 584 respectively. These values are much higher than those from Vardanega and Haigh (2014) and Vardanega *et al.* (2019) which may be due to lack of data at lower I_{LN} values in this dataset.

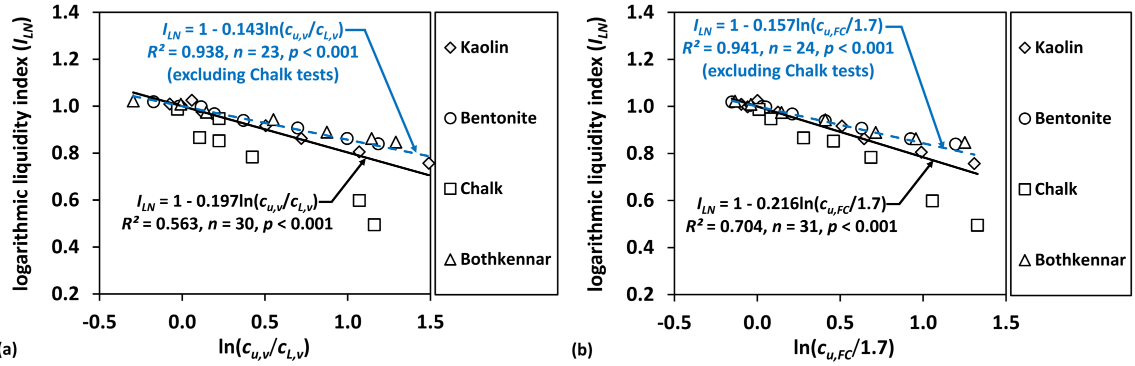

Figure 6. (a) I_{LN} versus ln($c_{u,v}/c_{L,v}$) and (b) I_{LN} versus ln($c_{u,FC}$/1.7)

4.3 Water content ratio versus normalised undrained shear strength

Fig. 7 shows w/w_L plotted against normalised c_u. Fig. 7a shows the data of $c_{u,v}$ normalised with $c_{L,v}$ and Fig. 7b shows the data of $c_{u,FC}$ normalised with 1.7kPa. The data on Fig. 7 have been fitted with equations of the form shown as Eq. 10 below (the statistical measures are shown on Fig. 7):

$$w/w_L = 1 - \eta \ln(c_u/c_L) \tag{10}$$

The value of η for the entire dataset is 0.155 ($n = 30$) for the vane shear data and 0.161 ($n = 31$) for the fall-cone data: both values are greater than the 0.102 value from Eq. (6). The value of β with the Chalk data excluded is 0.168 ($n = 23$) for the vane shear data and 0.188 ($n = 24$) for the fall-cone data.

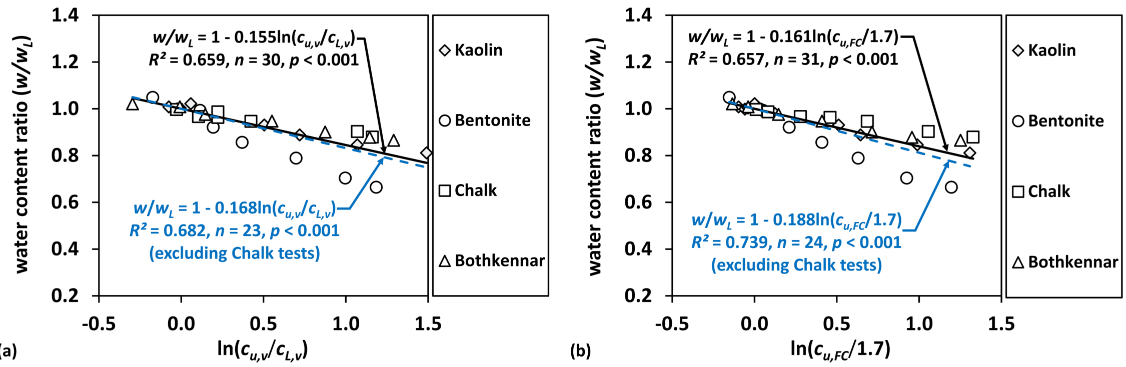

Figure 7. (a) w/w_L versus ln($c_{u,v}/c_{L,v}$) and (b) w/w_L versus ln($c_{u,FC}$/1.7)

4.4 Comparison of fitting methods

Table 1 summarises the statistical fitting parameters from Figs. 5 to 7. For the full dataset the I_L provides the formulation with the highest R^2 values. (When the Chalk tests are excluded the I_{LN} formulation has the highest R^2 values.) Fig. 8 shows the I_L versus normalised c_u data along with the fitted equations as well as those from Vardanega and Haigh (2014) and Vardanega et al. (2019) for comparison.

Table 1. Comparison of fitting parameters from Fig. 5 to Fig. 7 (strongest fits by c_u series shown in **bold** font)

Fitting parameter	$c_{u,v}$	$c_{u,v}$ (no Chalk)	$c_{u,FC}$	$c_{u,FC}$ (no Chalk)
	$n = 30$	$n = 23$	$n = 31$	$n = 24$
α (I_L correlation)	**0.278**	0.236	**0.299**	0.261
R^2	**0.752**	0.876	**0.869**	0.911
β (I_{LN} correlation)	0.197	**0.143**	0.216	**0.157**
R^2	0.563	**0.938**	0.704	**0.941**
η (w/w_L correlation)	0.155	0.168	0.161	0.188
R^2	0.659	0.682	0.657	0.739

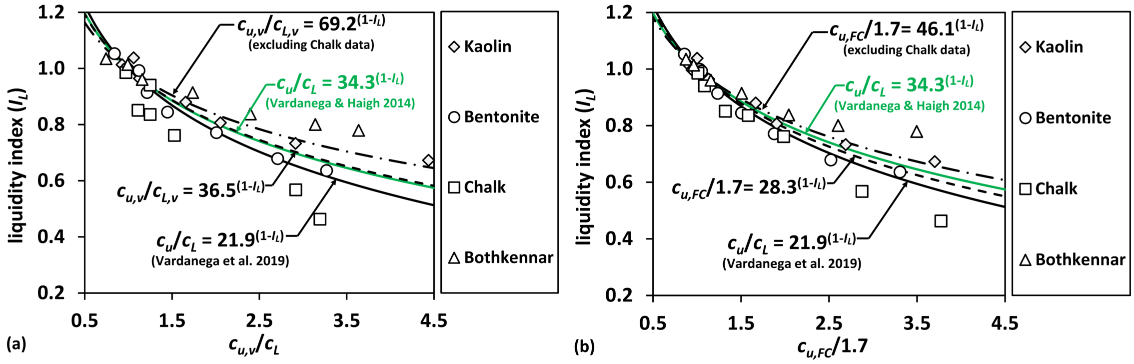

Figure 8. Eq. 2 calibrated with (a) the laboratory vane data and (b) the fall-cone data

5 CONCLUDING REMARKS

Fall-cone tests on Kaolin, Bentonite, Bothkennar clay and Chalk have been presented with accompanying miniature laboratory shear vane data. The following conclusions are drawn:

- The c_u values measured at $w_{L,FC}$ using the miniature laboratory vane test give values in the range 0.72 to 2.14 kPa (similar to the range given in O'Kelly 2019 and Wroth and Wood 1978).
- The I_L was found to offer the best predictor for normalised c_u for both the laboratory vane and fall-cone data when the entire dataset is considered. While I_{LN} (with the Chalk tests removed) does offer somewhat stronger regression fits (higher R^2 values) than those which use I_L the analogous R_{MW} values produced are very high and probably unrealistic. This may be due to the lack of data at lower values of w.

Future investigations should involve testing with both the fall-cone and laboratory vane on samples with lower w values nearer to the w_P. This will probably involve challenges with sample preparation (cf. Haigh *et al.* 2013).

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