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REVIEW

Comment on: "Absorption and scattering of massless scalar wave from Regular Black Holes"

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Absorption and scattering properties of matter fields in the background of black hole (BH) spacetimes have been investigated in several standard scenarios since the 1960s (see, e.g., Refs. [1–6] and references therein). In the last decade, some attempts to improve the comprehension of these properties in the background of regular BH (RBH) geometries, considering mainly test scalar fields, have been performed (see, e.g., Refs. [7–12]). The authors of the recent work "Absorption and scattering of massless scalar wave from Regular Black Holes" [12] addressed the absorption and scattering cross sections of massless test scalar fields in the background of several well-known RBH solutions. They considered the partial-waves approach and computed the absorption and scattering cross sections through a numerical method for arbitrary values of the frequency ω and scattering angle θ of the scalar wave.

Nevertheless, the absorption and scattering results obtained by the authors of Ref. [12] using the Numerov numerical method [13] combined with the Pöschl-Teller

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potential approximation [14] present some inconsistencies. In this comment, we highlight these inaccuracies, by taking into account some previous results available in the literature [7–10].

Concerning the absorption cross section (ACS), the authors of Ref. [12] obtained that the total ACS decreases as we consider higher BH charge values, and it oscillates around the corresponding geometric cross section, in agreement with some results presented in the literature. However, in the low-frequency regime, the total ACS does not tend to the BH area, as expected from Ref. [15].

Now let us turn our attention to the scattering case. The authors of Ref. [12] obtained that the differential scattering cross section (SCS) of massless test scalar fields in the background of the considered RBH geometries is finite for small scattering angles. This result is incorrect since for scattering potentials with infinite range, the differential SCS diverges in the forward direction [16]. From analytical approximations, for small scattering angles, we also know that the BH charge does not contribute to the dominant term of the differential SCS, in this regime (see, e.g., Refs. [17–20] for Ayón-Beato-García, Bardeen, Bronnikov, and Dymnikova RBHs). Therefore, as $\theta \to 0$, we do not expect to see considerable differences in the differential SCSs as we vary the BH charge. Notwithstanding, the results obtained by the authors show considerable differences in this regime as we increase the BH charge values.

We also emphasize that light rays in the background of RBH geometries associated with nonlinear electrodynamics models follow null geodesics of an effective geometry [21], which is different from the geometry of the spacetime itself. Consequently, the classical results for the absorption and scattering cross section of massless test scalar fields in the background of nonlinear electrodynamics-based RBHs correspond to the trajectory of massless particles with nature other than electromagnetic. Therefore, the geodesic analysis exhibited by the authors of Ref. [12] is not valid for photons, in contrast with what they have stated.

In summary, regarding the absorption and scattering results obtained by M.-Y. Wan and C. Wu using the Numerov numerical method and the Pöschl-Teller potential approximation [12], we conclude that the approach, as applied by them, leads to erroneous results for small values of the frequency and of the scattering angle. Furthermore, the oscillatory pattern of the SCS is unclear. The correct results concerning the Ayón-Beato-García and Bardeen RBHs can be found in Refs. [7–10].

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Declarations

Conflict of interest The author declares to have no conflicts of interest.

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