Stability Analysis for Retrial Queue with Collisions and r-Persistent Customers

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Abstract. We consider a single server retrial queue with general distribution of service times, collisions and r-persistent customers. The last phenomena describes the behaviour of customers that are leaving the system immediately if the server is busy upon arrival. We consider the system with customers, which leave the system without servicing with constant probability r. We provide the numerical stability analysis in such system using the following approach. First, we build the diffusion limit for the number of customers in the orbit and then analyze its drift coefficient. For different system parameters, we have different stability conditions.

Keywords: retrial queue · collisions · r-persistent customers · diffusion approximation

Introduction

Retrial queues arose as models of communication systems. The basic phenomenon of such systems is the retrial behavior of customers: if the server is busy upon arrival, the customer enters the orbit and repeats the attempt to access the server after a random amount of time.

There are several modifications of retrial queues that reflect the system features such as collisions and non-persistent customers, which appear in various switching communication systems and CSMA-based networks [1]. In recent years, queueing systems with collisions are of interest due to the reborn of IEEE 802.11 wireless LANs. In papers [8,9], authors describe the markovian retrial queue with collisions and shows applications of persistence to modeling CSMA-CD protocols. In paper [6], the author consider similar markovian model and takes into account the impatience of customers.

Nazarov and Sztrik with their research group have considered several models of finite-source retrial queues with collisions [7,13,14,17–19]. The phenomena of non-persistent customers in retrial queues was considered by [4,5]. Lakaour and his colleagues have considered markovian models with collisions, transmission errors and unreliable server [10,11].

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Retrial queues with collisions and impatient customers were considered in [2,3,16]. The phenomena of impatient customers is similar to the r-persistence due to the fact that a customer, which have not received the service upon arrival can leave the system. However, there is some difference, because non-persistent customers leave the system immediately with some probability and never join the orbit.

Another model of queueing system with collisions is considered by Phung-Duc and Fiems [15]. The model is markovian and has two phases of service. The authors study how the division into phases affects queueing performance.

We consider retrial queue with arbitrary distribution of service times, collisions and r-persistent customers. We build diffusion approximation for the number of customers in the orbit and construct the approximation of its probability distribution under the limit condition of growing delay in the orbit. Considering different sets of parameters, we show the numerical examples of system stability using the obtained approximation.

The rest of the paper is organized as follows. In Sect. 1, we describe the model structure and derive the equations for the probability distribution of system states. Section 2 is devoted to the asymptotic-diffusion analysis of the system under consideration. The approach is described in the paper [12]. After that, we show the results of numerical experiments in Sect. 3. Section 4 is dedicated to the conclusion.

1 Model Description and Problem Definition

We consider a retrial queue with an arbitrary distribution of service times defined by the distribution function $B(x)$. The input is stationary Poisson process with rate λ. If the server is idle upon arrival, the incoming customer occupies it for service. Otherwise, the collision occurs and one of the customers joins the orbit. The other customer can also join the orbit with probability r or leave the system with probability $(1 - r)$.

At the orbit, a customer waits for some random time and tries again to occupy the server. The duration of delay follows an exponential distribution with rate σ.

Let $k(t)$ denote the state of the server at instant t: 0, if the server is idle; 1, if the server is busy. Let $i(t)$ denote the number of customers in the orbit at instant t. We also introduce process $z(t)$, which represents the residual service time. Thus, process $\{k(t),\ i(t),\ z(t)\}$ has variable number of components and exhaustively describes the system state. We denote the probability distribution of process $\{k(t),\ i(t),\ z(t)\}$ as follows:

$$P_0(i,t) = P\{k(t) = 0, i(t) = i\},\ P_1(i,z,t) = P\{k(t) = 1, i(t) = i, z(t) < z\},$$

and introduce the partial characteristic functions

$$H_0(u,t) = \sum_{i=0}^{\infty} e^{jui} P_0(i,t),\ H_1(u,z,t) = \sum_{i=0}^{\infty} e^{jui} P_1(i,z,t),$$

where j is the imaginary unit. The Kolmogorov system of differential equations for the partial characteristic functions has the following form:

$$\begin{aligned}
\frac{\partial H_0(u,t)}{\partial t} &= -\lambda H_0(u,t) + j\sigma \frac{\partial H_0(u,t)}{\partial u} + \frac{\partial H_1(u,0,t)}{\partial z} \\
&+ \lambda e^{ju}(1 + r(e^{ju} - 1))H_1(u,t) - j\sigma(1 + r(e^{ju} - 1))\frac{\partial H_1(u,t)}{\partial u}, \\
\frac{\partial H_1(u,z,t)}{\partial t} &= \frac{\partial H_1(u,z,t)}{\partial z} - \frac{\partial H_1(u,0,t)}{\partial z} - \lambda H_1(u,z,t) \\
&+ j\sigma \frac{\partial H_1(u,z,t)}{\partial u} + \lambda H_0(u,t)B(z) - j\sigma e^{-ju}\frac{\partial H_0(u,t)}{\partial u}B(z).
\end{aligned} \tag{1}$$

After that, we sum up the equations of system (1). Taking the limit by $z \to \infty$, we obtain

$$\begin{aligned}
\frac{\partial H(u,t)}{\partial t} &= (e^{ju} - 1) \\
&\times \left\{ j\sigma e^{-ju}\frac{\partial H_0(u,t)}{\partial u} + \lambda(1 + re^{ju})H_1(u,t) - j\sigma r \frac{\partial H_1(u,t)}{\partial u} \right\}.
\end{aligned} \tag{2}$$

Solving system (1) and equation (2) in the limit by $\sigma \to 0$, we derive drift and diffusion coefficients of approximating diffusion process.

2 Asymptotic-Diffusion Analysis

In system (1) and equation (2), we introduce the following notations:

$$\begin{aligned}
\sigma = \varepsilon,\ u = \varepsilon w,\ \tau = \varepsilon t, \\
H_0(u,t) = F_0(w,\tau,\varepsilon),\ H_1(u,z,t) = F_1(w,z,\tau,\varepsilon),
\end{aligned} \tag{3}$$

and obtain the system of equations

$$\begin{aligned}
\varepsilon\frac{\partial F_0(w,\tau,\varepsilon)}{\partial \tau} &= -\lambda F_0(w,\tau,\varepsilon) + j\frac{\partial F_0(w,\tau,\varepsilon)}{\partial w} + \frac{\partial F_1(w,0,\tau,\varepsilon)}{\partial z} \\
&+ \lambda e^{jw\varepsilon}(1 + r(e^{jw\varepsilon} - 1))F_1(w,\tau,\varepsilon) - j(1 + r(e^{jw\varepsilon} - 1))\frac{\partial F_1(w,\tau,\varepsilon)}{\partial w}, \\
\varepsilon\frac{\partial F_1(w,z,\tau,\varepsilon)}{\partial \tau} &= \frac{\partial F_1(w,z,\tau,\varepsilon)}{\partial z} - \frac{\partial F_1(w,0,\tau,\varepsilon)}{\partial z} - \lambda F_1(w,z,\tau,\varepsilon) \\
&+ j\frac{\partial F_1(w,z,\tau,\varepsilon)}{\partial w} + \lambda F_0(w,\tau,\varepsilon)B(z) - je^{-jw\varepsilon}\frac{\partial F_0(w,\tau,\varepsilon)}{\partial w}B(z), \\
\varepsilon\frac{\partial F(w,\tau,\varepsilon)}{\partial \tau} &= (e^{jw\varepsilon} - 1) \\
&\times \left\{ je^{-jw\varepsilon}\frac{\partial F_0(w,\tau,\varepsilon)}{\partial w} + \lambda(1 + re^{jw\varepsilon})F_1(w,\tau,\varepsilon) - jr\frac{\partial F_1(w,\tau,\varepsilon)}{\partial w} \right\}.
\end{aligned} \tag{4}$$

We solve system (4) in the limit by $\varepsilon \to 0$ and formulate the following theorem.

Theorem 1. *In considered retrial queue, under the limit condition $\sigma \to 0$, the following equality holds:*

$$\lim_{\sigma\to 0} \mathbb{E}e^{jw\sigma i\left(\frac{\tau}{\sigma}\right)} = e^{jwx(\tau)},$$

where $x(\tau)$ is a solution of differential equation

$$x'(\tau) = -x(\tau)r_0 + [\lambda + (\lambda + x(\tau))r]r_1, \tag{5}$$

values r_0, r_1 have the following form:

$$r_0 = \frac{1}{2 - B^*(\lambda + x)},\ r_1 = \frac{1 - B^*(\lambda + x)}{2 - B^*(\lambda + x)}. \tag{6}$$

Here $B^*(s)$ is the Laplace-Stieltjes transform (LST) of the distribution function of the service times $B(x)$.

Proof. We assume that $\lim_{\varepsilon\to 0} F_k(w, z, \tau, \varepsilon) = F_k(w, z, \tau)$ and consider system (4) in the limit by $\varepsilon \to 0$. After that, we seek the solution in the form

$$F_0(w, \tau) = r_0 e^{jwx(\tau)},\ F_1(w, z, \tau) = r_1(z)e^{jwx(\tau)},$$

which give us the following system:

$$\begin{aligned} -(\lambda + x)r_0 + r_1'(0) + (\lambda + x)r_1 = 0,\\ r_1'(z) - r_1'(0) - (\lambda + x)r_1(z) + (\lambda + x)r_0 B(z) = 0,\\ x'(\tau) = -x(\tau)r_0 + [\lambda + (\lambda + x(\tau))r]r_1. \end{aligned} \tag{7}$$

Here $r_1 = r_1(\infty)$. The last equation of system (7) coincides with (5). From the first equation of system (7), we have

$$r_1'(0) = (\lambda + x)(r_0 - r_1).$$

Substituting the equality into the second equation yields

$$r_1'(z) - (\lambda + x)(r_0 - r_1) - (\lambda + x)r_1(z) + (\lambda + x)r_0 B(z) = 0.$$

We apply the Laplace-Stieltjes transform to the obtained differential equation and obtain

$$r_1^*(s)(\lambda + x - s) = (\lambda + x)r_1 - (\lambda + x)r_0(1 - B^*(s)).$$

If we set $s = \lambda + x$ in the last equation, we can write

$$(\lambda + x)r_1 - (\lambda + x)r_0(1 - B^*(\lambda + x)) = 0,$$

which we finally consider as system together with the normalization condition $r_0 + r_1 = 1$. We have

$$r_0 = \frac{1}{2 - B^*(\lambda + x)},\ r_1 = \frac{1 - B^*(\lambda + x)}{2 - B^*(\lambda + x)},$$

which coincides with (6).

We note that r_0 and r_1 depend on τ since they depend on x. We omit the arguments to simplify the expressions.

From (5), we denote function

$$a(x) = -xr_0 + (\lambda + (\lambda + x)r)r_1. \tag{8}$$

For the second step of analysis, we make the following substitutions in equations (1)–(2):

$$H_0(u,t) = e^{j\frac{u}{\sigma}x(\sigma t)} H_0^{(2)}(u,t),\ H_1(u,z,t) = e^{j\frac{u}{\sigma}x(\sigma t)} H_1^{(2)}(u,z,t).$$

Thus, we obtain the equations for the partial characteristic functions of centered number of customers in the orbit. After that, we introduce the following substitutions:

$$\begin{gathered}\sigma = \varepsilon^2,\ u = w\varepsilon,\ \tau = t\varepsilon^2,\\ H_0^{(2)}(u,t) = F_0^{(2)}(w,\tau,\varepsilon),\ H_1^{(2)}(u,z,t) = F_1^{(2)}(w,z,\tau,\varepsilon),\end{gathered} \tag{9}$$

and obtain the system of equations

$$\begin{gathered}
\varepsilon^2 \frac{\partial F_0^{(2)}(w,\tau,\varepsilon)}{\partial \tau} + jw\varepsilon a(x) F_0^{(2)}(w,\tau,\varepsilon) = -(\lambda + x) F_0^{(2)}(w,\tau,\varepsilon)\\
+ j\varepsilon \frac{\partial F_0^{(2)}(w,\tau,\varepsilon)}{\partial w} + \frac{\partial F_1^{(2)}(w,0,\tau,\varepsilon)}{\partial z}\\
+ (\lambda e^{jw\varepsilon} + x)(1 + r(e^{jw\varepsilon} - 1)) F_1^{(2)}(w,\tau,\varepsilon)\\
- j\varepsilon(1 + r(e^{jw\varepsilon} - 1)) \frac{\partial F_1^{(2)}(w,\tau,\varepsilon)}{\partial w},\\
\varepsilon^2 \frac{\partial F_1^{(2)}(w,z,\tau,\varepsilon)}{\partial \tau} + jw\varepsilon a(x) F_1^{(2)}(w,z,\tau,\varepsilon) = \frac{\partial F_1^{(2)}(w,z,\tau,\varepsilon)}{\partial z}\\
- \frac{\partial F_1^{(2)}(w,0,\tau,\varepsilon)}{\partial z} - (\lambda + x) F_1^{(2)}(w,z,\tau,\varepsilon) + j\varepsilon \frac{\partial F_1^{(2)}(w,z,\tau,\varepsilon)}{\partial w}\\
+ (\lambda + x e^{-jw\varepsilon}) F_0^{(2)}(w,\tau,\varepsilon) B(z) - j\varepsilon e^{-jw\varepsilon} \frac{\partial F_0^{(2)}(w,\tau,\varepsilon)}{\partial w} B(z),\\
\varepsilon^2 \frac{\partial F^{(2)}(w,\tau,\varepsilon)}{\partial \tau} + jw\varepsilon a(x) F^{(2)}(w,\tau,\varepsilon)\\
= (e^{jw\varepsilon} - 1) \left\{ j\varepsilon e^{-jw\varepsilon} \frac{\partial F_0^{(2)}(w,\tau,\varepsilon)}{\partial w} - x e^{-jw\varepsilon} F_0^{(2)}(w,\tau,\varepsilon) \right.\\
\left. + (\lambda + r(\lambda e^{jw\varepsilon} + x)) F_1^{(2)}(w,\tau,\varepsilon) - j\varepsilon r \frac{\partial F_1^{(2)}(w,\tau,\varepsilon)}{\partial w} \right\}.
\end{gathered} \tag{10}$$

Solving system (10) in the limit by $\varepsilon \to 0$, we present Theorem 2.

Theorem 2. *Function* $\lim_{\varepsilon\to 0} F_k^{(2)}(w,\tau,\varepsilon) = F_k^{(2)}(w,\tau)$ *has the following form:*

$$F_k^{(2)}(w,\tau) = \Phi(w,\tau) r_k,$$

where r_k *is given by* (6)*, function* $\Phi(w,\tau)$ *is the solution of equation*

$$\frac{\partial \Phi(w,\tau)}{\partial \tau} = w\frac{\partial \Phi(w,\tau)}{\partial w} a'(x) + \frac{(jw)^2}{2}\Phi(w,\tau) b(x). \tag{11}$$

Function $a(x)$ *is defined by* (8)*,* $b(x)$ *is determined as follows:*

$$b(x) = a(x) + 2[-(\lambda+x)(1+r)g_0 + xr_0 + r\lambda r_1], \tag{12}$$

where

$$g_0 = \frac{(a(x)+x)(1-B^*(\lambda+x)) + (\lambda+x)a(x)B^{*\prime}(\lambda+x)}{(\lambda+x)(2-B^*(\lambda+x))^2}.$$

Proof. Making the following substitutions in the system (10):

$$F_0^{(2)}(w,\tau,\varepsilon) = \Phi(w,\tau)\{r_0 + jw\varepsilon f_0\} + O(\varepsilon^2),$$

$$F_1^{(2)}(w,z,\tau,\varepsilon) = \Phi(w,\tau)\{r_1(z) + jw\varepsilon f_1(z)\} + O(\varepsilon^2), \tag{13}$$

we obtain the system of equations for f_0 and $f_1(z)$.

$$\begin{gathered} -(\lambda+x)f_0 + f_1'(0) + (\lambda+x)f_1 \\ = a(x)r_0 - \frac{\partial\Phi(w,\tau)/\partial w}{w\Phi(w,\tau)}r_0 - (\lambda + r(\lambda+x))r_1 + \frac{\partial\Phi(w,\tau)/\partial w}{w\Phi(w,\tau)}r_1, \\ f_1'(z) - f_1'(0) - (\lambda+x)f_1(z) + (\lambda+x)f_0B(z) \\ = a(x)r_1(z) - \frac{\partial\Phi(w,\tau)/\partial w}{w\Phi(w,\tau)}r_1(z) + xr_0B(z) + \frac{\partial\Phi(w,\tau)/\partial w}{w\Phi(w,\tau)}r_0B(z), \end{gathered} \tag{14}$$

We solve system (14) using the following substitutions:

$$f_0 = Cr_0 + g_0 - \frac{\partial\Phi(w,\tau)/\partial w}{w\Phi(w,\tau)}\varphi_0,$$

$$f_1(z) = Cr_1(z) + g_1(z) - \frac{\partial\Phi(w,\tau)/\partial w}{w\Phi(w,\tau)}\varphi_1(z),$$

which yield three systems of equations. The first system coincide with the system for r_0 and $r_1(z)$. It is easy to see that the second system for φ_0 and $\varphi_1(z)$ can be obtained by differentiating of system (7). Thus, we can conclude that $\varphi_k = r_k'(x)$. The last system is given by

$$\begin{gathered} -(\lambda+x)g_0 + g_1'(0) + (\lambda+x)g_1 = a(x)r_0 - (\lambda + r(\lambda+x))r_1, \\ g_1'(z) - g_1'(0) - (\lambda+x)g_1(z) + (\lambda+x)g_0B(z) = a(x)r_1(z) + xr_0B(z). \end{gathered} \tag{15}$$

We add an additional condition $g_0+g_1 = 0$ and obtain the solution of the system in the following form:

$$g_0 = \frac{(a(x)+x)(1-B^*(\lambda+x)) + (\lambda+x)a(x)B^{*\prime}(\lambda+x)}{(\lambda+x)(2-B^*(\lambda+x))^2}, g_1 = -g_0.$$

During the analysis, we also obtain equation for $\Phi(w,\tau)$:

$$\frac{\partial\Phi(w,\tau)}{\partial\tau} = w\frac{\partial\Phi(w,\tau)}{\partial w}a'(x) + \frac{(jw)^2}{2}\Phi(w,\tau)b(x),$$

which coincide with (11). Here $a(x)$ and $b(x)$ are given by (8) and (12), respectively.

Here equation (11) is the Fourier transform of the Fokker-Planck equation for the process approximating the number of customers in the orbit of considered retrial queue. If we make the inverse Fourier transform, we can see that the drift coefficient of the obtained diffusion limit is $a(x)$ and diffusion coefficient if $b(x)$.

Discrete function $PD(i)$ is the approximation of the probability distribution of the number of customers in the orbit and has the following form:

$$PD(i) = \frac{D(i\sigma)}{\sum\limits_{n=0}^{\infty} D(n\sigma)}, \tag{16}$$

where

$$D(z) = \frac{1}{b(z)} \int\limits_0^z \frac{2}{\sigma}\frac{a(x)}{b(x)}dx.$$

We have briefly prooven theorems 1 and 2. The approach is widely described in [12]. In this paper, we concentrate at analysis of drift coefficient of the diffusion limit $a(x)$, which is given by (8).

3 Numerical Examples

3.1 Bistability Case

For the numerical examples, we show the analysis of the drift coefficient $a(x)$. Based on the number of roots of the equation $a(x) = 0$, we can consider several modes of stability. The first case occurs (Fig. 1) when the parameters of the system are as follows:

$$\lambda = 0.258,\ \ \alpha = 2,\ \ \beta = \alpha, \sigma = 0.1, r = 0.98,$$

where α and β are the shape and scale parameters of Gamma distribution of the service times. We note that in all cases we show graphics of $a(\sigma x)$, because the number of calls in the orbit is normalized by σ. When $a(x) > 0$ the number of customers in the orbit grows. On the other hand, if $a(x) < 0$, the number of

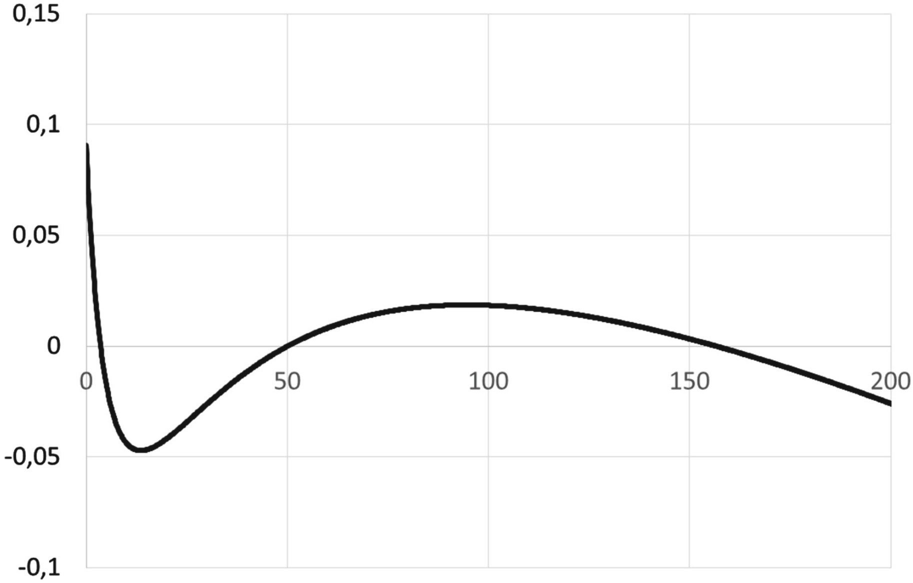

Fig. 1. Drift coefficient $a(x)$

customers in the orbit decreases. Here we have two stability areas around roots of equation $a(x) = 0$, when the sigh of $a(x)$ turns from plus to minus. In such case, the distribution of the number of customers in the orbit is bimodal (Fig. 2). We also note that if $a(x) < 0$ when $x \to \infty$, then the system is stable. If not, the steady state does not exist for the current set of parameters.

3.2 Standard Stability Case

The next case occurs when the parameters of the system are as follows:

$$\lambda = 0.258,\ \ \alpha = 1.8,\ \ \beta = \alpha, \sigma = 0.1, r = 0.98,$$

where α and β are the shape and scale parameters of Gamma distribution. In Fig. 3, we show that equation $a(x) = 0$ have only one root. Here we have the standard distribution with only one mode (Fig. 4) and $a(x) < 0$ when x grows to the infinity. Thus, the system is stable with such set of parameters.

3.3 Mixed Bistability Case

Another case occurs when the parameters of the system are as follows:

$$\lambda = 0.312,\ \ \alpha = 2,\ \ \beta = \alpha, \sigma = 0.1, r = 0.96,$$

where α and β are the shape and scale parameters of Gamma distribution. Here we also can observe the bistability phenomena (Fig. 5), but the modes are too close and affect on each other (Fig. 6).

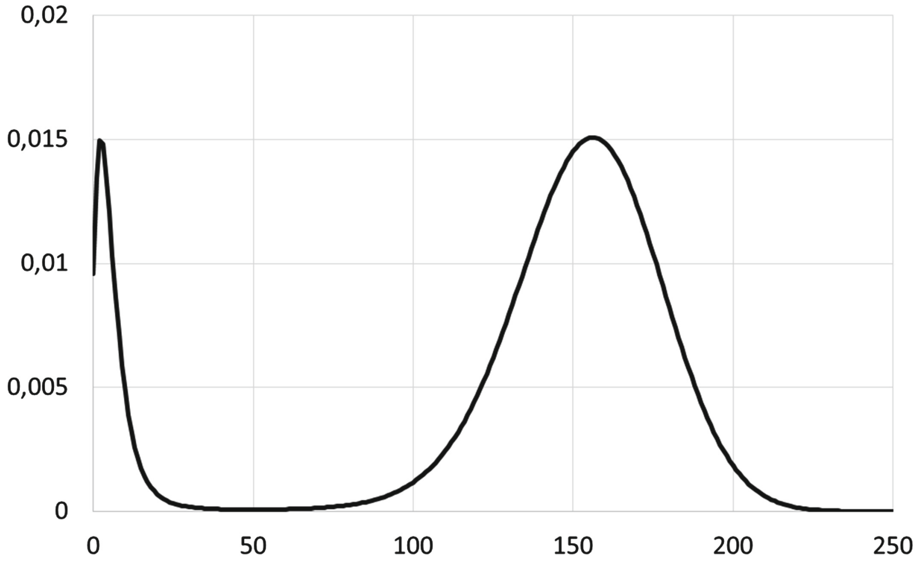

Fig. 2. Diffusion approximation of distribution of the number of customers in the orbit

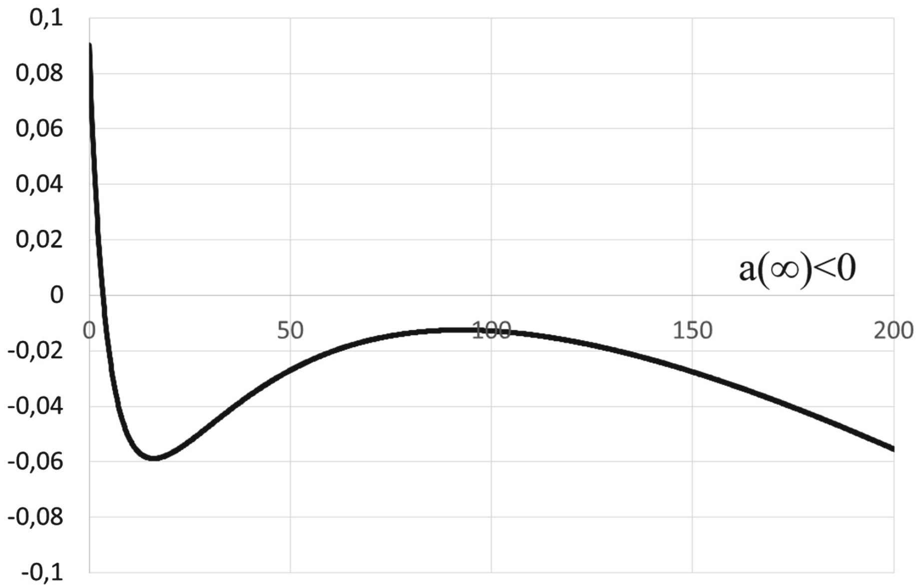

Fig. 3. Drift coefficient $a(x)$

3.4 Stabilization Area in Unstable System

The last case (Fig. 7) arise when the parameters of the system are given by

$$\lambda = 0.2,\ \ \alpha = 2,\ \ \beta = \alpha, \sigma = 0.1, r = 1,$$

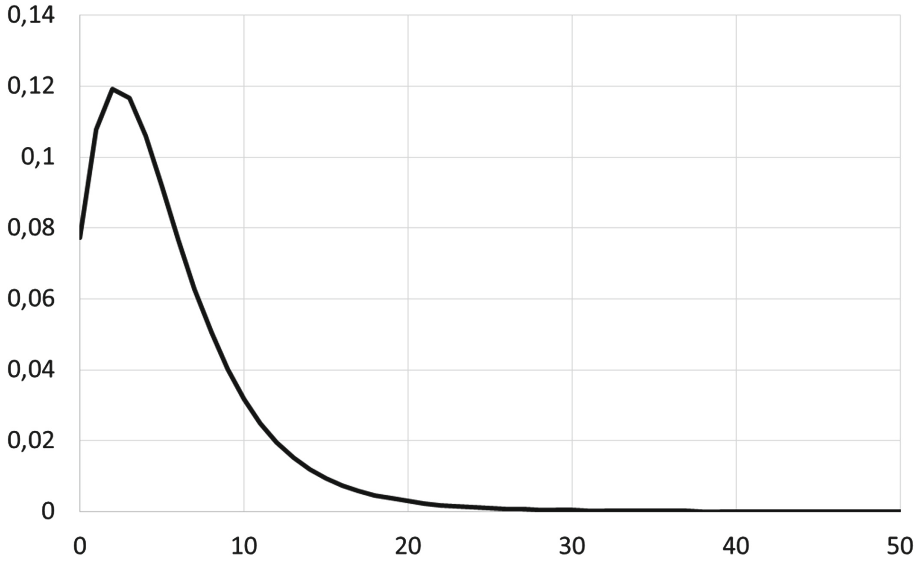

Fig. 4. Diffusion approximation of distribution of the number of customers in the orbit

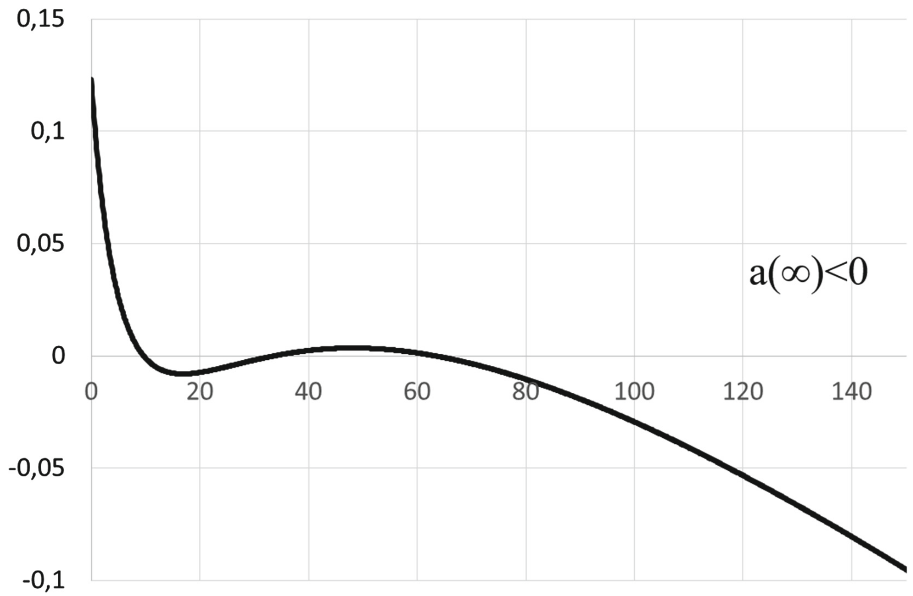

Fig. 5. Drift coefficient $a(x)$

where α and β are the shape and scale parameters of Gamma distribution. Even if $a(x) > 0$ when x grows to the infinity, the distribution has a stability area around the point where $a(x) = 0$. The process can spend a lot of time

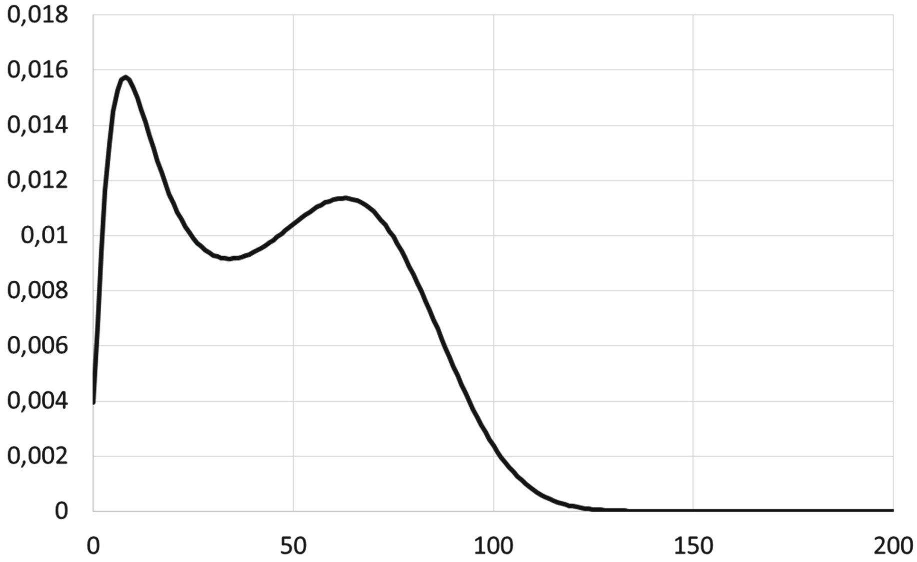

Fig. 6. Diffusion approximation of distribution of the number of customers in the orbit

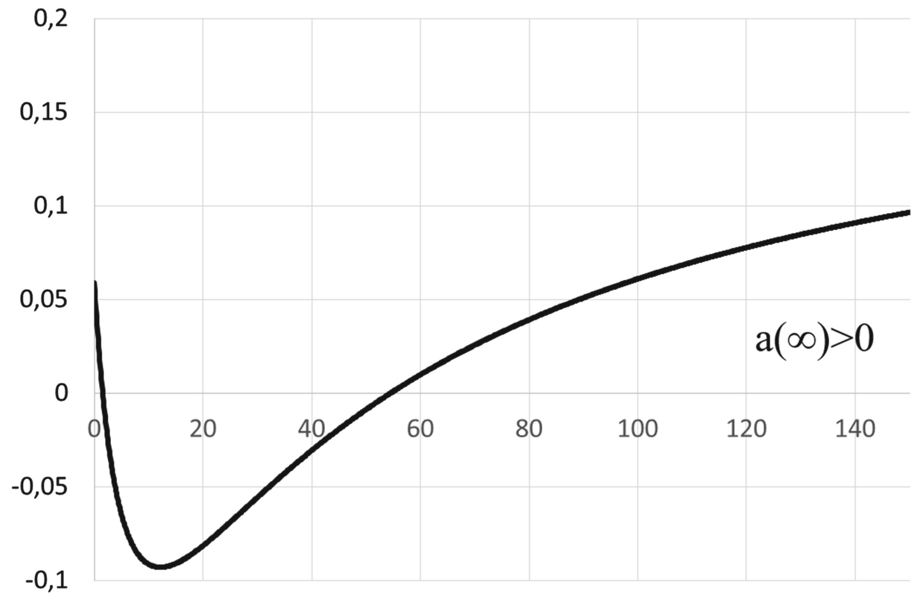

Fig. 7. Drift coefficient $a(x)$

before leaving the stability area. Thus, if we use zero of the function $a(x)$ as the truncation point, we can build an approximation (Fig. 8) for the distribution of the number of customers in the orbit using formula (16).

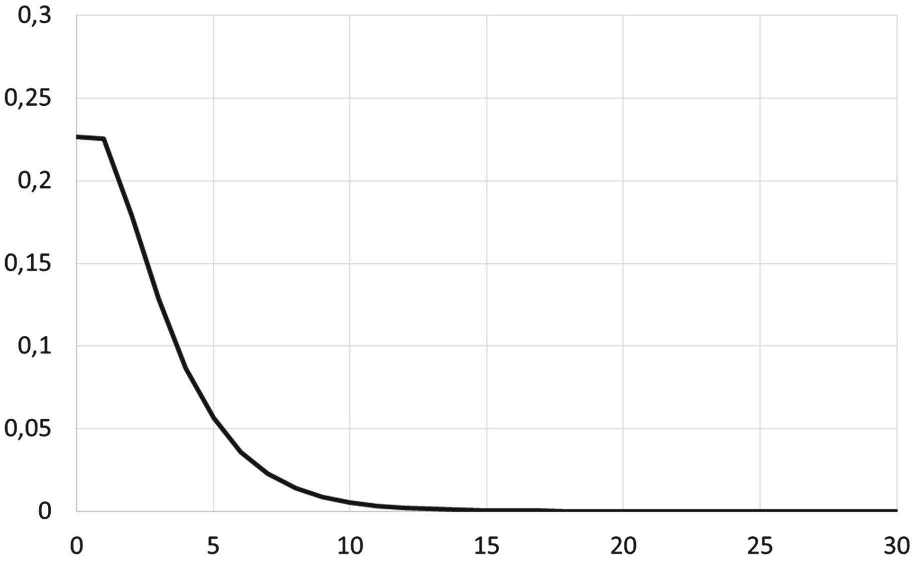

Fig. 8. Diffusion approximation of distribution of the number of customers in the orbit

4 Conclusion

We have considered the retrial queue with collisions and r-persistent customers. For the number of customers in the orbit, we have derived the approximation of the probability distribution (16). The analysis was prepared to show that there are several stability phenomena arise in such system. We show the numerical examples and the cases of stability for some sets of parameters based on the analysis of drift coefficient $a(x)$ of the obtained diffusion limit. For the future study, we plan to investigate the transition time between stability points in bistable retrial queue with collisions.

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