A MATHEMATICAL MODELLING APPROACH TO ANALYZE THE DYNAMICS OF MATH ANXIETY

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University of Missouri-Kansas City, 2022

ABSTRACT

The main objective of this study is to develop a mathematical modeling framework for a deeper understanding of dynamics of math anxiety as a contagious process. Borrowing from theories of the spread of infectious disease, we develop two classes of mathematical models representing the spread of math anxiety in math gateway classes. The first mathematical model does not entirely fit with our collected data of math anxiety (n=53, Calculus II & III summer of 2020). However, the second mathematical model, which is a generalization of the first model, can exhibit periodic solutions as observed in the collected data. In addition to the mathematical modeling framework, we have applied a variety of statistical methods and models to analyze the survey data. This includes descriptive analysis of the data, correlation and hypothesis testing, and a machine learning approach, which utilizes the classification and regression tree models to identify key factors associated with math anxiety. These regression tree models include factors such as gender, academic level, number of hours studied, motivation, and confidence. In conclusion, the present work lays the foundation for applying mathematical models to measure the spread of math anxiety in gateway STEM courses.

APPROVAL PAGE

The faculty listed below, appointed by the Dean of the School of Graduate Studies, have examined a dissertation titled "A Mathematical Modelling Approach to Analyze the Dynamics of Math Anxiety", presented by Dilek Soysal, candidate for the Doctor of Philosophy degree, and certify that in their opinion it is worthy of acceptance.

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DEDICATION

To my son, Murat, and my husband, Soner.

CHAPTER 1

INTRODUCTION TO MATH ANXIETY

1.1 Overview

Math Anxiety and negative attitudes towards math are significant challenges for students at all stages of their academic careers (Geist, 2010). Researchers have conducted multiple studies to investigate math anxiety and related topics for decades. Chapter 1 provides a definition of math anxiety and literature review of variables (such as confidence, motivation, gender, academic levels, and COVID-19 pandemic) associated with math anxiety. Since we began this study during the COVID-19 pandemic, we examine COVID-19 as another potential factor contributing to student math anxiety. Interestingly, the review of literature revealed that math anxiety can spread among students or transfer from teacher to student or parent to student. This chapter covers a detailed literature review of the above-mentioned factors associated with math anxiety, explains the study's research questions, and provides a statement of significance.

A substantial part of this chapter has been published in the *International Electronic Journal of Mathematics Education* and in the journal *Pedagogical Research* (Soysal et al., 2022).

1.2 Definition of Math Anxiety

Students who pursue degrees in Science, Technology, Engineering, and Mathematics (STEM) are continually presented with very hard and rigorous degree programs, which may cause them to become overconfident in their ability to achieve their academic goals (Hsu & Goldsmith, 2021). Students may be motivated to make decisions or focus on their learning and achievement based on emotions associated with STEM courses. Because math anxiety is such a relevant factor for student success, it has become an important topic of inquiry for researchers and teachers, as well (Warwick, 2008; Williams,1988; Greenwood, 1984; Ramirez et al., 2018). The earliest study of math anxiety began in the 1950s (Ashcraft & Moore, 2009), and since then, math anxiety is defined as "a feeling of tension and anxiety that interferes with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations" (Richardson & Suinn, 1972). Also, Tobias and Weissbrod (1980) expressed math anxiety as "the panic, helplessness, paralysis, and mental disorganization that arises among some people when they are required to solve a mathematical problem" and claimed that this affects a large percentage of the population (p.65).

Based on my experience as a math instructor, I have observed firsthand math anxiety in higher education. The following excerpt from an email sent by a former student demonstrates his experience with math anxiety:

> Well I managed to do it again... I have never had this happen to me before, but for some reason I am getting extremely nervous and anxiety overwhelms me instantly... I do not have this same experience while taking tests in chemistry this semester so it if very odd...for some reason I get so anxious and nervous that I cant think clearly.

In this email, my student describes his course-related anxiety and how he chose to seek help. I understand and am aware of how worried my students are. They also have talked about how confident they were in math classes. This is math anxiety, not an anxiety towards their physical study or availability. This feeling appears inside of the individual student and affects their success, confidence, and ability to solve problems.

1.3 Spread of Math Anxiety

Notably, there was a reduction in anxiety symptoms and an improvement in learning achievement among students who had teachers with better math abilities and confidence (Geist, 2015). Geist (2015) investigated how teachers deliver a lesson in the classroom and found that teacher math anxiety affects their teaching techniques and tactics. Teacher practices, then, can impact student math anxiety through a teacher's lack of lesson preparation for class, as well as a lack of presentation skills, misrepresentation, and an inability to link content to the real world (Beilock et al., 2010; White, 1997). Students may have math anxiety as a result of the teacher's in-class behavior, which may include gender stereotypes and bias (Jackson & Leffingwell, 1999). Teachers are thus more likely to pass math anxiety on to their students (Beilock et al., 2010; Rubinsten, 2017; R. Martinez, 1987).

Teachers are not the only ones who might cause math anxiety, though; parents can also pass on their negative attitudes toward math to their children (Maloney et al., 2015; Soni & Kumari, 2015; Foley et al., 2017). When parents have poor math skills or fears about math, they often spread it to their children. Because of parent math anxiety, their children's ability to learn math during the school year is often affected. When parents empathize with their children's math troubles, they reinforce this anxiety. Instead, parents should think of learning mathematics as similar to teaching their children to ride a bicycle or read, rather than showing their children their own mathematical weaknesses (Sisto, 2014).

Besides teacher and parent behaviors, peer pressure (Buckley & Ribordy, 1982; Beilock & Carr, 2005) and insufficient mathematical backgrounds (Betz, 1978; Ashcraft & Moore, 2009) are some possible causes of math anxiety (Beilock & DeCaro, 2007; Ashcraft & Moore, 2009). Negative school experiences, family pressure, insensitive and inadequate field teachers, and a passive classroom setting are all environmental influences (Naderi Dehsheykh et al., 2021; Chang & Beilock, 2016; Ashcraft & Moore, 2009). Ashcraft and Moore (2009) found that all math anxiety had an effective reduction in student performance.

In terms of its impact on student confidence and engagement, however, anxiety is seen as an engaging emotion (Hsu & Goldsmith, 2021). As a result, math anxiety is determined by the motivation and interests of individual students. Institutions and instructors have devised a variety of learning platforms and ways to manage students' anxiety in courses while also limiting the spread of the math anxiety. These techniques are intended to inspire students and parents, as well as foster confidence in them (Snelling & Fingal, 2020).

In some ways, math anxiety is contagious (Gurin et al., 2017; Morris, 1981; Beilock et al., 2010). Teachers and parents who suffer from math anxiety might pass it on to their students and children. Some research suggests there might be a genetic link between parents and their children related to math performance and anxiety (Wang et al., 2014). Maloney et al., (2015) showed how poor math success and high math anxiety may have intergenerational effects, including impacts on children's arithmetic achievement and anxiety. Math anxiety in parents predicts math self-efficacy, GPA, behavior, math attitudes, and math devaluation in children (Casad et al., 2015).

1.4 Factors Associated with Math Anxiety

To model math anxiety's relationship to confidence, previous researchers have utilized traditional statistical methods for decades. For example, math anxiety has been inversely correlated to high school students' self-confidence (i.e., self-esteem, self-efficacy) (Akin and Kurbanoglu, 2011; Naderi Dehsheykh et al., 2021). Likewise, Tapia and Marsh (2004) found that students with lower math anxiety had significantly more confidence than students with high math anxiety. Rozgonjuk et al. (2020) concluded that reducing students' math anxiety could be helpful in boosting their mathematics confidence. Broadly speaking, the literature indicates that confidence in mathematics has a significant impact on math anxiety.

Confidence is not the only factor, though; motivation can also impact math anxiety. Having a positive attitude might help to minimize anxiety about learning and increase motivation to succeed (Chen et al., 2018). The relationship between math anxiety and motivation has been frequently studied, and a negative correlation between the two has been observed (Zakaria & Nordin, 2008; Wang et al., 2015; Chang & Beilock, 2016; Gunderson et al., 2018). However, in some cases, the relationship between math anxiety and motivation remains ambiguous (Wang et al., 2018). Wang et al. (2018) discovered eight unique profiles in a sample of 927 high school students (13–21 years old) defined by varying combinations of math anxiety and motivation. Simply put, they found some math-anxious students are highly motivated, while the current understanding in the literature is anxious students often have low motivation.

Researchers have also found a connection between gender and math anxiety. The relationship between math anxiety and gender has sparked debate and controversy (Rubinsten et al., 2012). Studies have found distinct gender disparities in certain populations. Hembree's (1990) meta-analysis analyzed 151 studies, including 49 journal articles, 23 ERIC documents, 75 Ph.D. dissertations, and 4 reports from additional investigations. According to the author's results, females had higher levels of math anxiety than males at the collegiate level. Other researchers have found similar results (Betz, 1978; Woodard, 2002). Xie et al. (2019) revealed women showed a higher level of math anxiety compared to men at a high school level, as well. On the other hand, Tapia and Marsh’s (2004) study shows math anxiety is unrelated to gender in their sample of university students, though the other literature shows a significant relationship between the two.

Several other factors may contribute to math anxiety including a fear of making mistakes, insufficient mathematical background, teacher and parent comments, attitudes, behaviors, and parental education levels (Sevindir et al., 2014). As the world continues to come to terms with the COVID-19 pandemic, the main challenge many schools are facing is teaching advanced subjects such as calculus online.

1.5 Math Anxiety During the COVID-19 Pandemic

The COVID-19 pandemic has had a huge impact on people across the world, requiring people to change their habits to safer ones to avoid the disease from spreading. Education is one of the most affected industries, with millions of students dependent on new health measures. Even though the curriculum remained unchanged during the pandemic, several components of the curriculum's delivery changed, and these changes helped students greatly, making studying easier and allowing them to maintain good academic performance. One of the key lessons learned from the pandemic is that we live in a modern society defined by numerous questions and a scarcity of solid answers (Paraskeva, 2022). Most individuals in schools and many professions have been unable to propose credible answers that would allow life to resume normally without the limits imposed by the pandemic.

Similarly, cultural, and technological inquiries can have a big influence on how individuals cope with some of the problems that have arisen as a result of the crisis. An e-learning-based culture has altered educational provisions, with most learners throughout the world relying on digital resources for their studies and communication (Brunelli & Macirella, 2021). The practice reflects a cultural movement away from traditional learning to a new one in which learners and instructors are not expected to be in the same place at the same time. STEM students whose curriculum includes cultural inquiry are more likely to be comfortable with change in this scenario. The reason for this is that, in contrast to students whose education does not focus on cultural inquiry, the education and abilities obtained from the cultural inquiry make it simple for learners to adjust to different cultural situations and practices.

Technological inquiry in STEM education has been critical throughout the pandemic since the knowledge supports the learning processes of students. However, policies such as remote working and learning assume that all students have reliable internet connections and computers, making it impossible for all citizens to have equal opportunity in fulfilling their tasks (Paraskeva, 2022). The policies have created hierarchies in which those who have the knowledge, skills, and resources to support the use of technology in their educational and professional activities have an advantage over others. Students' lives have been disrupted because of the pandemic, and some students, particularly those with expertise, have been able to employ current technology to increase their opportunities. During the pandemic, some students are negatively affected by technology use in their courses, while others are positively influenced. Students' anxiety levels have also risen, as a result of this. However, the positive side of technology is that it increases students' confidence and motivation.

The disruption of accessible education caused by COVID-19's emphasis on technological inquiry has impacted university students, including STEM students. School closures, together with the associated public health and economic crisis, present great challenges for students (García & Weiss, 2020). With the increase in the number of COVID-19 infections, most university lessons take place virtually, which is especially challenging for STEM (science, technology, engineering, and math) students who are used to hands-on, interactive learning (Hensley et al., 2020). Students were affected because their classes were interrupted, and they were expected to rapidly adapt to the unique expectations of online learning. Forakis et al., (2020) asked their students by responding to a survey to describe the way this transition has affected their academic plans to provide an in-depth look into their intentions. The challenges of COVID-19 impacted the university students' graduations (Saw et al., 2020). Saw et al. (2020) found that nearly 10% of STEM students had not decided or would not enroll in fall 2020 due to the pandemic, while 35.5% of doctoral STEM students, 18% of master's students, and 7.6% of undergraduate students delayed their graduation.

The rapidly changing educational landscape caused students to experience extreme stress, emotional exhaustion and frustration, average absenteeism, a somatic burden, and laziness while learning Calculus (Casinillo & Casinillo, 2021). Mendoza et al. (2021) analyzed the levels of anxiety in university mathematics students studying at the Universidad Nacional de Chimborazo (UNACH) during the fall 2020 academic semester. The results of their study showed a statistically significant difference in students' understanding of virtually presented content, which correlated to increased levels of math anxiety during the COVID-19 pandemic. Therefore, the pandemic has contributed to mounting anxiety and stress among many STEM students (Alemany-Arrebola et al., 2020).

Online learning has disrupted the educational environment many students are accustomed to. Serhan (2020) found that many students were not satisfied with their learning experience during the initial transition to online learning in March 2020. His results indicated that students negatively perceive the use of Zoom and believe it has a negative effect on their learning outcomes and their motivation to learn. Online learning might demotivate students and disincentivize them from watching lectures synchronously; however, student anxiety levels are likely to increase if they miss classes because it is challenging to follow up on difficult calculations independently. Additionally, students' Internet connections may be unreliable, making it impossible to continue watching their lessons. Notably, Internet connectivity problems disproportionately affect rural students and negatively impact their learning experience (Hampton et al., 2020). Sabates et al. (2021) estimated the learning loss during the three-month transition period from face-to-face learning to online learning in Ghana was 66% of previous learning gains in foundational numeracy. According to their estimates, the expanding gaps in learning losses are caused by a lack of both home learning support and home learning resources.

Serhan's (2020) finding that synchronous learning decreased student's academic motivation may explain the increased incidence of math anxiety during the COVID-19 pandemic. Mubeen and Reid (2014) explored how motivation relates to self-efficacy, self-concept, confidence, and self-esteem in an educational setting. The researchers found that motivation positively correlates with strong feelings of self-efficacy and self-esteem – two factors that protect against academic anxiety. Thus, low motivation is associated with poor self-efficacy and increased academic anxiety. However, these studies do not measure the level of motivation and confidence as a function of time. The main objective of the present study is to investigate temporal variations in students' levels of motivation, anxiety, and confidence during the COVID-19 pandemic. We also analyze the effects of COVID-19 on students throughout the semester.

Poignantly, student's development of motivation, self-efficacy, and confidence depends on their teachers' capacity to understand, anticipate, and deal with students' ideas about mathematics and mathematical thinking within the context of the world at large (Voica et al., 2020). When the pandemic struck, teachers quickly adapted their teaching methods to be as effective as possible (Midcalf & Boatwright, 2020). In a course on differential calculus, Jungic (2021) found that the level of anxiety student's experienced decreased when facts and statistics about the pandemic were integrated into course exercises. Therefore, to bolster student's self-efficacy and decrease feelings of academic anxiety, Jungic (2021) suggested a new approach that incorporates acknowledgment of the current pandemic in mathematics exams.

Math anxiety can affect people of any age, especially when their situation changes drastically. Many students are affected by anxiety because they lack the proper channels to divert their stress. The pandemic has made it more difficult to redirect stress while simultaneously introducing a number of new challenges (Marpa, 2020). Within the scope of online learning, some of these challenges include communication difficulties that make it easier to misunderstand information and more difficult to ask questions. Reinhold et al. (2021) found that maintaining a positive attitude towards synchronous learning is key to buffering students' stress during this pandemic. A positive attitude corresponds to the more productive redirection of stress and less academic anxiety.

While existing research confirms an increase in math anxiety associated with the current COVID-19 pandemic, a number of questions remain unanswered. This study seeks to understand how the motivation, anxiety, and confidence of STEM students changed during the summer 2020 term, how these variables defer with respect to academic level and gender, and how the COVID-19 pandemic has influenced STEM students' learning environment and utilization of supplemental resources.

If we draw a comparison between the COVID-19 pandemic and the Spanish flu in 1918-1920, real-time communication tools have made it easier to continue life as normal. Recent technological developments have made synchronous learning accessible for many students, making learning mathematics online easier. There are various tools and resources that make it easier to teach calculus and pay special attention to the needs of students during this period of isolation. Real-time communication tools such as Zoom ensure that students are engaged. Contact time is precious and synchronous learning ensures maximum participation in lessons. The pandemic has created feelings of isolation in students used to the collaborative approach used in higher-level math classes (Ng et al., 2020). Fortunately, real-time learning platforms make it possible to reinforce this collaborative teaching approach. Additionally, online resources like YouTube and Kahn Academy offer additional instruction that may temporarily stand in for in-person resources, like office hours and tutoring. Amidst the rapid transition to online learning at the start of the pandemic, mathematics support/tutoring centers also moved their services online (Johns & Mills, 2021). Furthermore, concerning the future of online tutoring, roughly 13% of mathematics center administrators claimed they would not continue due to limited student utilization and a personal preference for face-to-face tutoring (Johns & Mills, 2021).

1.6 Statement of The Problem

Research on math anxiety (Chen, 2019; Gurin et al., 2017; Richardson & Suinn, 1972) focused on how math anxiety develops and spreads through statistical analysis of relevant data. Furthermore, the Math Anxiety Rating Scale was developed to quantify the level of anxiety that each student experiences (Richardson & Suinn, 1972).

There is currently no theoretical framework regarding the spread of math anxiety in a classroom or a learning environment. This is a crucial issue, particularly for STEM students, who may drop or fail their gateway courses and change their major as the result of math anxiety. Given that previous studies have established the contagiousness of math anxiety

(Amani et al., 2021; Gurin et al., 2017; Morris, 1981; Beilock et al., 2010), the main goal of this work is to apply the theory of infectious disease modeling to investigate the spread of my anxiety and its corresponding dynamics using mathematical and statistical models.

In this study, we explore the spread of math anxiety as an epidemiological contagion. To deepen our understanding of math anxiety and provide a modeling framework to quantify the dynamics of math anxiety, we employ epidemiological and mathematical tools such as the force of infection (Reiner et al., 2014; Diekmann et al., 1995) and basic reproduction number (Bani-Yaghoub et al., 2012). This project is a qualitative and quantitative modeling approach rather than an explanatory modeling approach. In other words, rather of looking for the causes of anxiety and how it spreads among students, we will develop and study a series of mathematical and statistical models to deepen our understanding of anxiety dynamics. Using the collected data, we will also examine the correlation between math anxiety and the COVID-19 pandemic. COVID-19 has had an impact on college students' coursework, stress levels, and health perceptions (Perz et al., 2022; Sabates, 2021). Therefore, students may be less able to function at their peak and perform adequately in their academics during the COVID-19 outbreak.

In sum, the significance of this study can be explained in two parts. Using mathematical modeling (1) we can identify the patterns (shift, spikes, periodicity, concavity, rates, monotonicity, etc.) of math anxiety and its correlations to student performance. (2) Borrowing from Mathematical Epidemiology (e.g., susceptibility of students, transmissibility of anxiety, force of anxiety, chain of anxiety, anxiety endemic status, epidemic wave affecting majority students, exposed level, active immunity of students, odds ratio of infection, prevalence of infection, virulence of anxiety) we define and quantify dynamics of math anxiety, especially during the COVID-19 pandemic. Therefore, we developed a conceptual framework based on mathematical epidemiology, which is used to quantify and better understand the dynamics of math anxiety in higher education.

1.7 Research Questions

These questions were divided into two parts, modeling and developing theory and data collection, data analysis, and testing the models:

1. Modeling and Developing Theory

Building on research that finds math anxiety to be contagious, can theory of infectious disease transmission and control be extended to complete the following tasks?

- investigate the spread and dynamics of math anxiety in college students
- measure the severity of math anxiety based on factors such as the gender, academic level and course level
- understand the underlying mechanisms (such as student-student, teach-student, and parent-student interactions, exams, assignments) governing math anxiety, and
- measure the effectiveness of curricular and extracurricular strategies to reduce math anxiety

2. Data Collection, Data Analysis and Testing the Models

Using the collected data, we would like to answer the following questions:

- How does COVID-19 affect student’s performance, motivation, and confidence?

- What are the time series patterns in students' anxiety, motivation, or confidence change during the semester?
- What are the relations between student demographics, students' efforts, and factors such as math anxiety, motivation, and confidence?
- Will the developed theory and mathematical results obtained in part 1 coincide with the observed dynamics present in the collected data?

1.8 Framework of The Chapters

Chapter 1 includes the background and history of math anxiety as a scholarly conversation. This chapter also covers a definition and explanation of the problem and research questions which are the heart of this dissertation. Chapter 2 contains statistical analysis such as descriptive statistical analysis, hypothesis testing, and time series analysis that we utilized in this dissertation. Chapter 3 contains machine learning analysis and Pearson correlation analysis of math anxiety and factors (i.e., confidence, motivation, genders, and academic levels). Chapter 4 presents the developed extended SIR model for math anxiety. In the proposed model, we apply techniques using MATLAB to estimate the parameters of the model. We also analyze and validate the model analytically and numerically. Chapter 5 contains Hopf bifurcation theorem which we utilized to the mathematical model. Lastly, we will conclude the dissertation by discussing and suggesting ideas for future study in Chapter 6.

CHAPTER 2

INFERENCE AND MATH ANXIETY DATA

2.1 Overview

The current COVID-19 pandemic has largely impacted the academic performance of several college students. The present study is concerned with the effects of the COVID-19 pandemic on students pursuing a STEM (science, technology, engineering, and math) degree. We collected weekly survey data (w = 9) of students (n = 53) taking calculus courses during the COVID-19 pandemic (specifically, the summer of 2020). Using the self-reported survey data, we investigated the temporal variations in the levels of anxiety, motivation, and confidence of STEM students. Studies on temporal changes to math anxiety are scarce. The present work aims to fill this gap by analyzing longitudinal survey data associated with math anxiety. Furthermore, using descriptive and inferential statistical methods such as one-way ANOVA, we analyze the data with respect to gender and academic level. As a conclusion of this chapter, we found that the time series analysis of the data indicated that the levels of motivation and confidence significantly dropped toward the end of the semester, whereas the level of anxiety increased in all groups.

This chapter has been published in the *International Electronic Journal of Mathematics Education* (Soysal et al., 2022).

2.2 Materials and Methods

2.2.1 Participants

Every week, we surveyed n=53 STEM students enrolled in Calculus II and Calculus III courses during the COVID-19 pandemic (total of 9 weeks, see Table 1). The participants were STEM students of the Math Department at the University of Missouri-Kansas City (UMKC) in the summer 2020 semester.

Students met four times a week (total of 7 hours and 20 minutes per week) in Calculus II and Calculus III courses.

Table 1. The demographic characteristics of Calculus II and Calculus III students.

	Calculus II			Calculus III			
	Female	Male	Total	Female	Male	Total	Grand Total
Freshman/Sophomore	11	19	30	3	2	5	35
Junior/Senior	2	4	6	3	9	12	18
Grand Total	13	23	36	6	11	17	53

2.2.2 Course Format

COVID-19 became a worldwide pandemic in mid-March 2020. Consequently, UMKC suspended in-person class sessions and resumed them in an online synchronous format at their regular meeting times. Due to the lack of online teaching experience among faculty, the school had a dedicated team of e-Learning and Information Systems personnel working closely with faculty to facilitate a speedy transition so that the students could finish the semester successfully under these unusual COVID-19 circumstances.

Students were given housing and food discounts at UMKC. Discount and refund policies also applied to other areas of the school, such as parking, the recreation center, cancellation and late penalties, and financial charges. Students with extenuating circumstances will be allowed to challenge the decisions of school administration through a formal process. Because the situation was so serious, the school advised students to wear masks, socially isolate, and avoid small and big groups.

Campus and academic restrictions – campus closures and online learning, remained in place for the whole of Summer 2020. Thus, Calculus II and III were taught for the first time in the Summer of 2020 in an online synchronous style. Previously these classes had only been available in person at UMKC.

Calculus II and III classes took place Monday through Thursday in two, 50-minute periods with a 10-minute break in between. Zoom was used for all classes and office hours. Each of these classes was worth four credits. The passing grade in the previous grading method was 60%, however due to COVID-19, it was reduced to 55%. In the drop-fail-withdraw (DFW) rate, 17.7% of Calculus III students failed, while 0% dropped and withdrew. 29.4 % of Calculus II students failed, while 5.9 % dropped out and 0% withdrew.

Many courses moved to online and were assessed online, which could have influenced students' opportunities to cheat (Lancaster & Cotarlan, 2021). The exams were timed and designed in a way that reduced the chance of academic dishonesty. Their exam solutions were to be uploaded to an online education site (CANVAS) by the student. Exams and homework assignments were often graded within a week. The multiple-choice online quizzes were instantly graded by the Pearson platform My Math Lab.

In Calculus II, students studied techniques of integration, applications of the definite integral, improper integrals, sequences and series, power series, Taylor series and convergence, and analytic geometry in calculus. In Calculus III, students studied vectors, solid analytic geometry, vector functions and multiple variable functions, partial derivatives, multiple integrals, line and surface integrals with their applications.

For the first time, these two classes were offered as online synchronous courses. Some students came from different States in the United States, and some were attending these classes from their native country. For a few students who did not live in the United States, the time zone was particularly problematic.

2.2.3 Instruments

We utilized a questionnaire inspired by chapter six of Creswell's (2011) Educational Research and applied temporal and statistical analysis to the collected responses. The survey's items were created using best practices as guided by Dillman et al. (2009) and DeVellis and Thorpe (2017). We designed an online survey accessible through the "Quiz" tab of students' Calculus II and Calculus III canvas sites. The survey included a series of six questions (see Appendix A) developed by us that assessed students' anxiety in relation to their assignments, exams, Zoom meetings, level of motivation and confidence, and usage of additional resources like supplement instruction, tutoring, etc. during COVID-19. Student responses were collected via Canvas once a week for the eight-week duration of the 2020 summer semester and one week prior to the beginning of the semester. The survey results were collected and complied with using Canvas each week. The results were then given to an expert in mathematics education for determination of the validity and clarity of the data.

2.2.4 Data Preparations

At the end of the summer semester, we prepared the data for time series analysis, descriptive statistics, and hypothesis testing. Due to the low number of participants, we combined data of both calculus classes for all statistical analyses performed in this study.

The weekly survey asked students to rank experiences on a numerical scale. For example, question 5 asked "How confident are you today that you will pass this class with a grade B or better? (1 = not confident at all, 7 = very confident)." After compiling nine weeks of student responses in Qualtrics, we prepared and organized the data for statistical analysis by selecting and using MS Excel software. Data were cleaned (non-essential data, such as timestamps and duplicate information from across the nine weeks, e.g., student numbers were removed) and we assigned numeric codes to each response option as needed. While coding student responses to complex questions, we broke responses into categories. For example, for question 5 we created three categories - low confidence (scores 1-2), medium confidence (scores 3-5), and high confidence (scores 6-7). Students' responses to yes or no questions were coded dichotomously such that "yes" was coded as 1 and "no" as 0. Then we summed the coded data within groups and weeks and organized these values in tables. Next, we calculated these values as percentages and graphed them according to the time series.

Next, we methodically entered the data into an MS Excel spreadsheet and descriptive statistical analyzes were performed. After statistical analysis, the results were reported as figures and demographic data like gender and the academic level was plotted against confidence, motivation, and the effects of COVID-19 on self-reported math anxiety. Next, the practical meaning of these results was elucidated and discussed in further detail.

2.2.5 Statistical methods

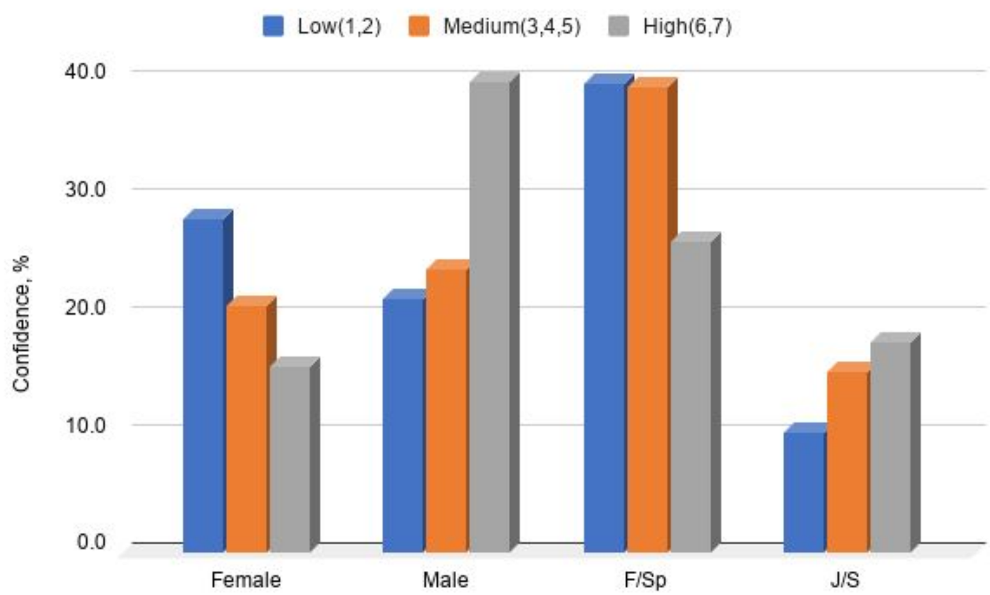
This study used weekly survey data collected from 53 (34 male and 19 female) undergraduate students enrolled in Calculus II and Calculus III courses during summer 2020 to investigate rates of math anxiety in the widths of the COVID-19 pandemic. Student responses were collected via weekly Canvas surveys each week of the eight-week summer semester and one week prior to the beginning of the semester. Thus, we had a nine-week data set. This data set allowed us to visually inspect temporal changes in the student's anxiety levels throughout to nine-week study duration.

The prepared data was then analyzed using Statistical MS Excel for conducting descriptive analysis of the data, inferential statistical methods, like one-way ANOVA, were further used, as well as time series analysis.

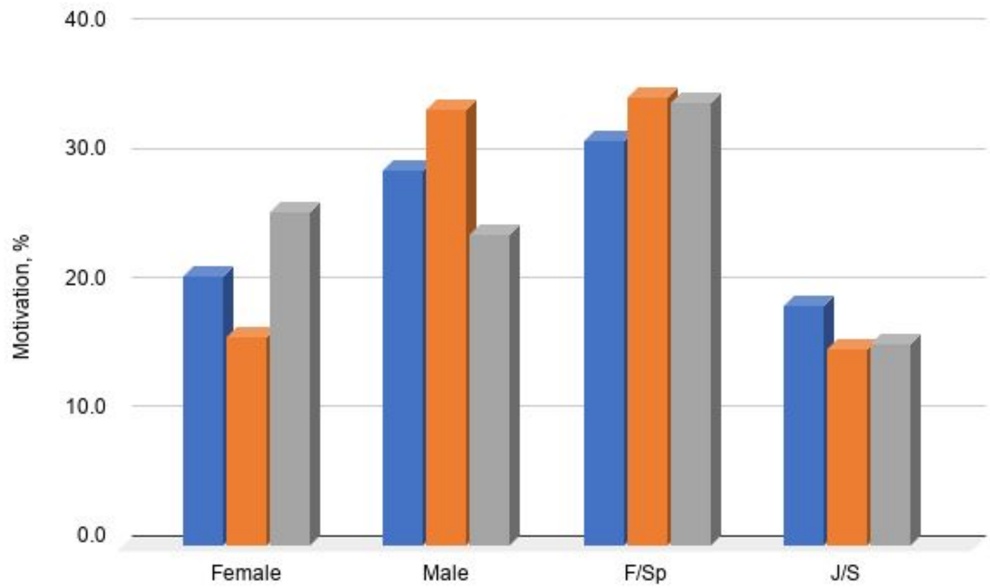
Descriptive statistics were used to generate tables and graphs of data collected in this study (in Figures 1-4). The analyses were conducted using the ANOVA test for statistical computing; the results of which are presented in Tables 2-5. To determine whether the hypotheses were supported, we examined the significance of individual paths and the global fit of the model to the observed data.

2.3 Descriptive Statistics

Descriptive statistical analysis suggested that the COVID-19 pandemic may have contributed to increased levels of math anxiety. Figure 1 (a) showed that male and F/Sp STEM students had higher levels of confidence within their groups. Figure 1 (b) showed that female and F/Sp STEM students had higher levels of motivation within their groups. Figure 1 (c) showed that more male and F/Sp STEM students expressed an increase in math anxiety due to COVID-19.



(a)



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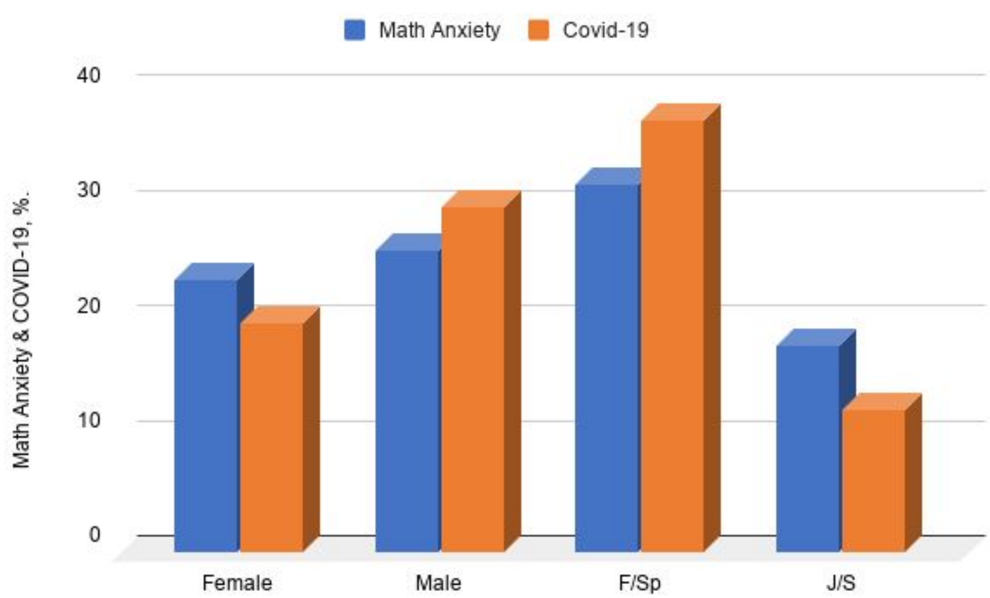


(c)

Figure 1. Measures of confidence, motivation, and anxiety with respect to gender and level of education. (a) The proportions of male and F/Sp STEM students with high levels of confidence are substantially higher than those with medium or low confidence. (b) The

proportion of female STEM students with high levels of motivation are more than males with low or medium levels of motivation. (c) There are higher proportions of male and F/Sp STEM students who expressed increased math anxiety due to COVID-19.

2.4 Hypothesis Testing

The one-way ANOVA test applied to the nine-week data set helps us understand temporal variations in students' levels of motivation, anxiety, and confidence during the COVID-19 pandemic. Students' questionnaire responses indicate a relationship between gender and self-reported anxiety levels throughout the semester, see Table 2. The inferential analysis based on the gender variable showed significant differences ($p < 0.05$) between male and female students.

Table 2. One-way ANOVA test results indicate a significant difference in the mean level of math anxiety with respect to gender.

	SV	SS	df	MS	F	P-value	FC
Did you have any math anxiety this week?	Between Groups	2062.96	1	2062.96	7.79	0.01	4.49
	Within Groups	4238.64	16	264.915			
	Total	6301.61	17				

SV= Source of Variation, SS= Sum of Squares, df= degrees of freedom, MS= Mean Squares, F= F value, FC= F critical value.

Table 3 summarizes the results of one-way ANOVA with respect to the effect of COVID-19 on math anxiety. Specifically, the inferential analysis based on the academic level showed significant differences ($p<0.05$) between the average responses of F/Sp and J/S students to the question "Did COVID-19 increase your math anxiety?"

Table 3. One-way ANOVA test results indicate that the COVID-19 pandemic has significantly increased the mean level of math anxiety.

	SV	SS	df	MS	F	P-value	FC
Did COVID-19 increase your math anxiety?	Between Groups	368.11	1	368.11	9.17	0.01	4.49
	Within Groups	642.47	16	40.15			
	Total	1010.58	17				

SV= Source of Variation, SS= Sum of Squares, df= degrees of freedom, MS= Mean Squares, F= F value, FC= F critical value.

Similarly, Table 4 is a summary of ANOVA results with respect to gender and student motivation. The inferential analysis showed significant differences ($p<0.05$) between male and female students for the average levels of motivation. This was demonstrated by variable responses to the question "How would you rate your desire to study this week?" by gender.

Table 4. One-way ANOVA test results indicate a significant difference among groups with regard to motivation.

	SV	SS	df	MS	F	P-value	FC
How would you rate your desire to study this week?	Between Groups	25.85	1	25.85	9.46	0.002	3.87
	Within Groups	1010.45	370	2.73			
	Total	1036.30	371				

SV= Source of Variation, SS= Sum of Squares, df= degrees of freedom, MS= Mean Squares, F= F value, FC= F critical value.

Finally, Table 5 demonstrates the ANOVA results with respect to the level of confidence and gender. There was a significant difference between the male and female students and their responses to the question "How confident are you today that you will pass this class with a grade B or better?" Summary of means and standard deviations related to Tables 2-5 are given in the supplementary document (see Table B1).

Table 5. One-way ANOVA test results indicate a significant difference among groups with regard to confidence.

	SV	SS	df	MS	F	P-value	FC
How confident are you today that you will pass this class with a grade B or better?	Between Groups	15.92	1	15.92	5.71	0.02	3.87
	Within Groups	1034.82	371	2.79			
	Total	1050.74	372				

SV= Source of Variation, SS= Sum of Squares, df= degrees of freedom, MS= Mean Squares, F= F value, FC= F critical value.

2.5 Time Series Analysis

Based on the time series descriptive statistical analysis shown in figure 2 (a), it was found that the prevalence of students with low confidence increased a week before the midsemester exam. The prevalence of low confidence increased in all groups, reaching an average of 9.4% end of the semester. Serhan (2020) found that 9.68 % of participants said that using Zoom helped them gain confidence in the subject, which is in line with our findings. Freshman and sophomore students experienced the greatest loss of confidence during the semester, reaching 13.2%. Also, self-reported low confidence increased from week four onward.

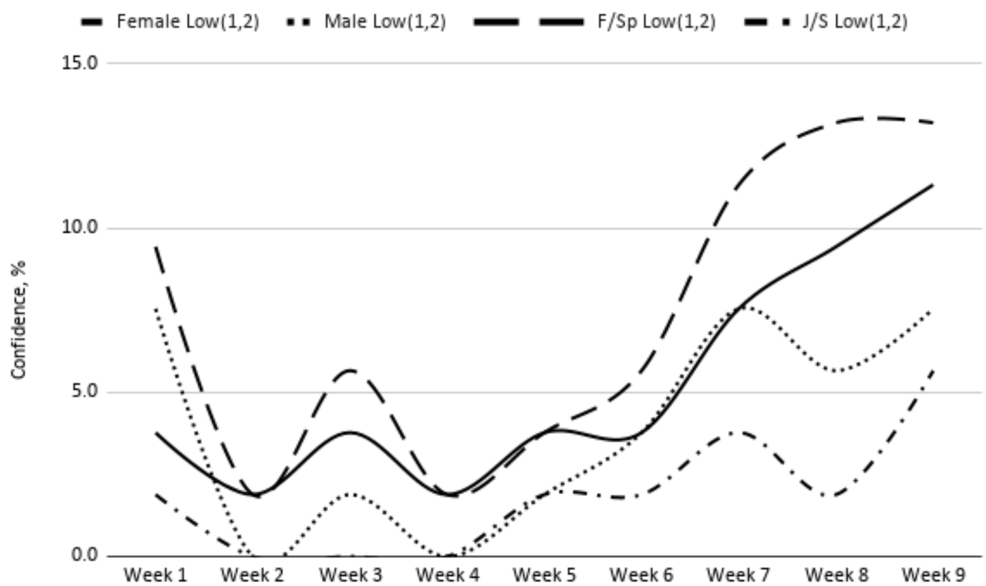


Figure 2(a). Percentage of the STEM students with low levels of confidence. The prevalence of students with low confidence increased a week before the exam.

Figure 2 (b) showed that the prevalence of low motivation increased in all groups reaching an average of 6.6%. According to the findings of Serhan's (2020) study, students reported a negative influence on their learning experience and motivation. Freshman and sophomore students lost their motivation more than other groups, reaching 9.4%. The descriptive analysis of the relationship between gender and low motivation showed significant differences between male and female students. Male students started with lower motivation than their female peers.

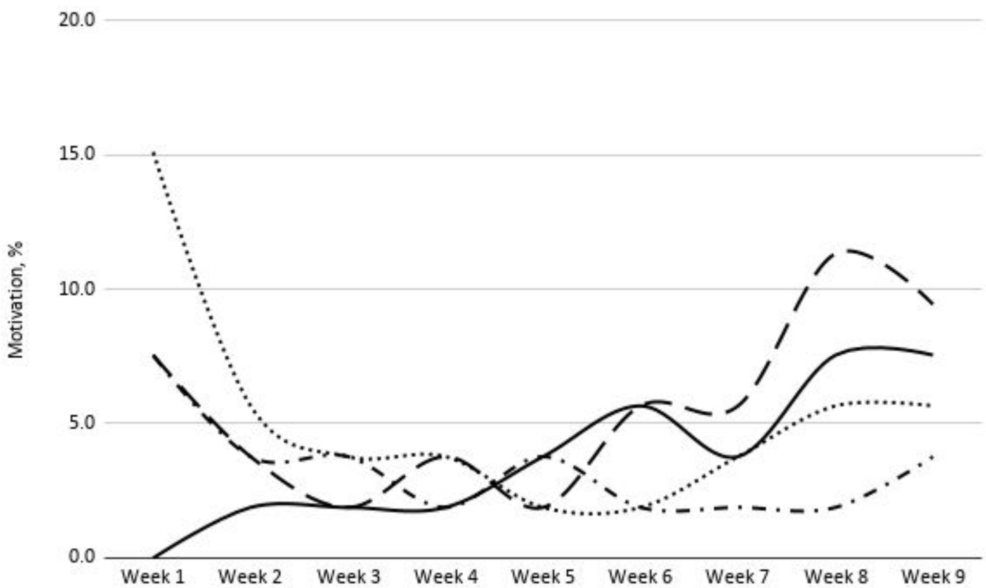


Figure 2(b). Percentage of the STEM students with low levels of motivation. Students started and finished the semester with low motivation.

As shown in Figure 3 (a), approximately 20% of students reported math anxiety during the semester and the prevalence of math anxiety was much higher at the beginning of the semester, where 25% to 50% of students reported math anxiety. The prevalence of math anxiety in male students flattened to around 17%. However, in female students, it decreased to 13% halfway through the semester, then bounced back to 24%. Freshman and sophomore students reported higher math anxiety than their junior and senior counterparts, averaging 27.4% and 15.5% respectively.

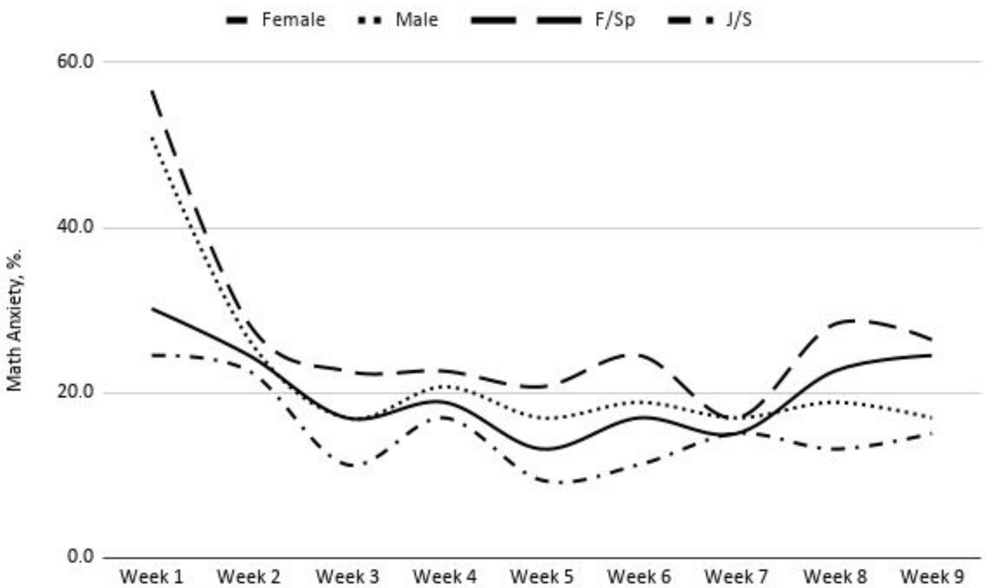


Figure 3(a). Percentage of the STEM student's math anxiety. Freshman and sophomore students had much higher math anxiety compared to junior and senior students

According to Figure 3 (b), an average of 8.7% of female students reported an increased level of math anxiety due to the COVID-19 pandemic. Poignantly, COVID-19 affected male students more than female students. Male students reported an increased level of math anxiety, 13.2%. An average of 16.7% of freshman and sophomore students reported increased math anxiety due to COVID-19, while an average of 5.5% of junior and senior students reported this. This result is consistent with Mendoza et al. (2021). Mendoza et al. (2021) claimed that the levels of math anxiety increased due to the COVID-19 pandemic. Also, Ludwig (2021) concluded that high COVID-19 despair had a negative impact on student performance even though they found no correlation between performance and general anxiety towards learning math or remote learning.

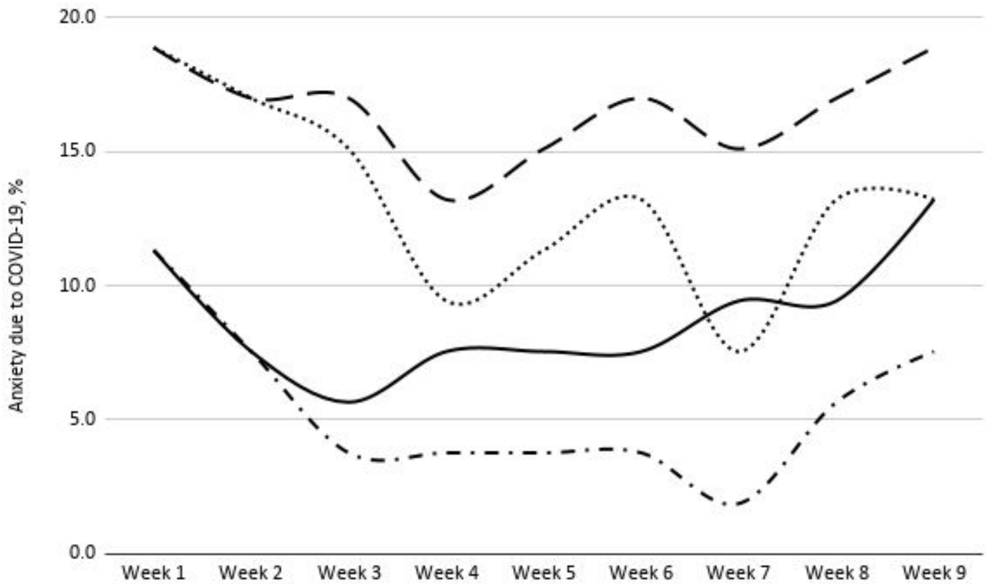
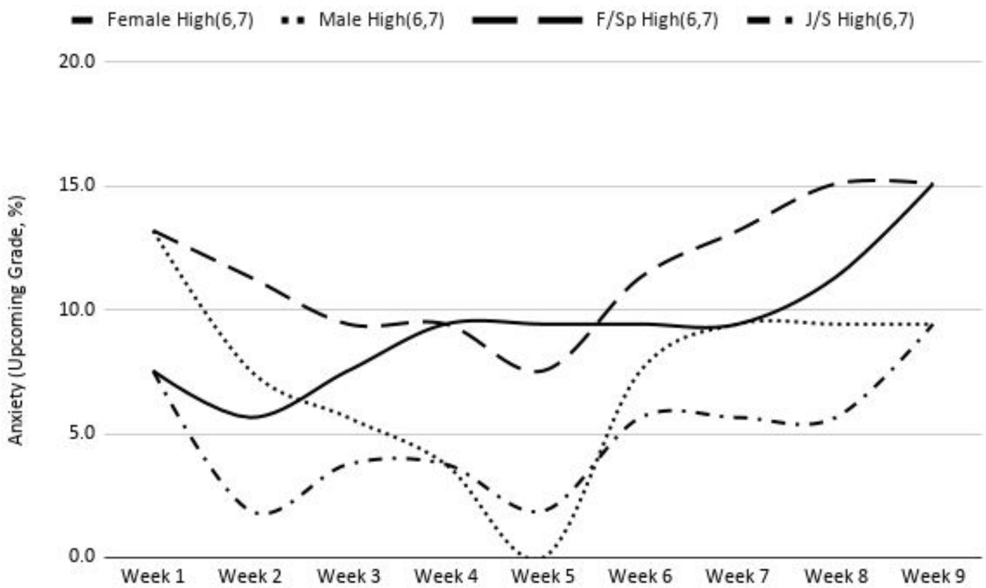
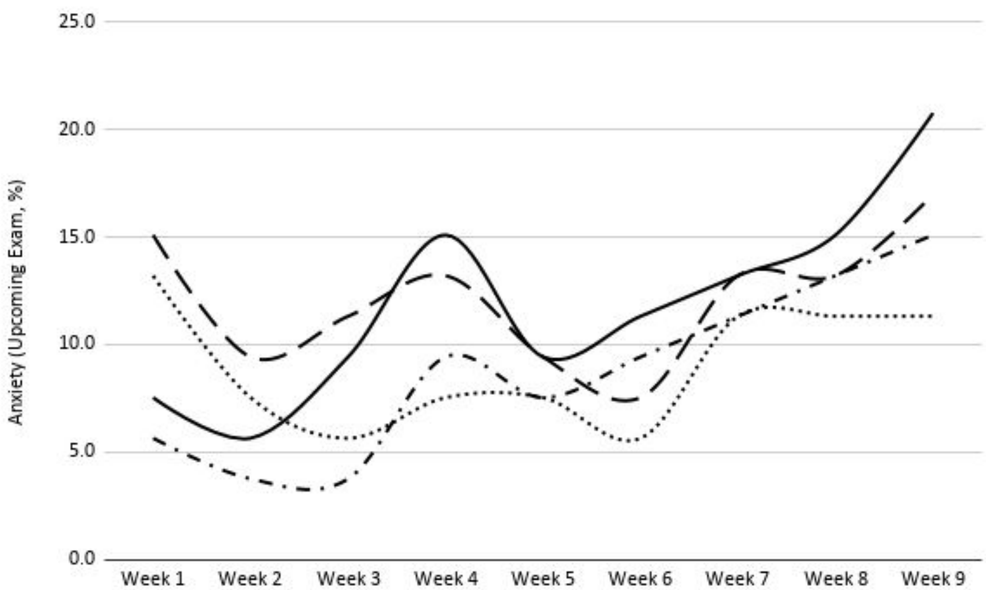


Figure 3(b). Percentage of the STEM student's math anxiety due to COVID-19. F/Sp students were more affected than other groups.

We began surveying students a week before the start of the eight-week summer semester. So, students had two midterm exams during weeks 4 and 7 and their final exam took place during week 9. Thus, there are two corresponding increases in exam anxiety in Figure 4 (b). The female population reported the highest overall anxiety. Though female students initially reported less anxiety than their male counterparts, as the semester progressed female students' anxiety level surpassed that of the male students reaching 20.8%. After analyzing Figure 4 (a), we also observed that female and freshman/sophomore STEM students were more anxious about upcoming grades than other groups.



(a)



(b)

Figure 4. Percentage of the STEM students with high levels of math anxiety for the upcoming grades and exams. F/Sp students were more anxious for their grades than other

groups (a), and female students had more anxiety for their two midterm exams on weeks 4 and 7 than other groups.

Further analysis of time series data showed that the COVID-19 pandemic has substantially decreased the use of supplemental instruction (see Figure B1(a) in the supplementary document). The number of STEM students using UMKC supplemental instruction (SI) dropped from 14.4% to less than 8% as the semester progressed. More freshmen and sophomore students sought help from SI than junior and senior students, averaging 11.5% and 4.2% respectively. Notably, student utilization of SI surged at week 4 because of the upcoming exam but decreased afterward perhaps because students did not find it useful. Similarly, Figure B1(b) in the supplementary document showed that the COVID-19 pandemic decreased student utilization of UMKC tutoring services. About 8.2% of STEM students used UMKC tutoring services before the COVID-19 pandemic, but less than 3% used this resource during the pandemic. Only 1.9% (3.8%) of male (female) STEM students used UMKC tutoring during summer 2020. No junior or senior STEM students used UMKC tutoring services during summer 2020. Few students sought help from private tutoring, the highest incidence of which was in freshmen and sophomore students (about 5.7%).

The COVID-19 pandemic decreased the use of office hours. While there is no data that compares both pre COVID-19 and post COVID-19 data when it comes to math anxiety, some researchers (Candarli & Yuksel, 2012; Serhan, 2020) found that the use of Zoom had a negative impact on student learning. Notably, most of the students responded that they would have been more comfortable in a traditional classroom setting (Roy et al., 2020; Doggett & Mark, 2008). Using Zoom tutoring sessions, on the other hand, dropped considerably professors' workload by 25% (Sayem et al., 2017). These findings agree with the results of our study. Particularly, only a small percentage of students (2%-6%) used Zoom office hours to supplement their learning during the summer 2020 semester (see Figure B2(a) in the supplementary document). Other than the initial high level of anxiety in freshman/sophomore and male STEM students, all groups tended to have low anxiety with respect to upcoming assignments according to Figure B2(b) with the prevalence of high anxiety between 4% and 7%.

The COVID-19 pandemic may have increased cooperative learning within the student population. Figure B3(a) in the supplementary shows that a relatively large percentage of students sought help from their classmates during the pandemic. Namely, an average of 12.1% of female students, 18.9% of male students, 24.3% of freshman/sophomore students, and 6.7% of junior/senior students sought help from their classmates throughout the semester. Serhan (2020) also found that the usage of Zoom increased students' contact with their classmates, with a mean response range of 1.97 out of 5 regarding classroom interaction. During the semester we also observed that freshman/sophomore STEM students spent less time studying by calculating the weekly average hours of study. Female students spent 14.8 hours a week studying calculus, male students spent 15.8 hours, freshman/sophomore students spent 13.8 hours, and junior/senior students spent 19.7 hours (see Figure B3(b) in the supplementary document).

2.6 Discussion

The current COVID-19 pandemic has further complicated the math anxiety of many college students due to changes in coursework, course modality, and poor perceived health (Perz et al., 2022). Researchers have argued that mathematically anxious students tend to move away from studying STEM subjects (Warwick, 2008). The increased anxiety due to COVID-19 may affect their entire trajectory. While the impact of COVID-19 in the field of education is very diverse, less is known about the effects of this crisis on STEM students specifically. This paper investigated time-series changes in each variable (i.e., motivation, anxiety, and confidence) with respect to academic level and gender to understand the effects of the pandemic on STEM students. We investigated the temporal variations in the levels of confidence, motivation, and anxiety during the COVID-19 pandemic. Studies on temporal changes to math anxiety are scarce. The present work aims to fill this gap by analyzing longitudinal survey data associated with math anxiety.

The time series analysis and the inferential statistics of the survey data showed that: (i) levels of motivation and confidence dropped towards the end of the semester (see Figure 2) which is consistent with Ashcraft (2019). In the mentioned study, Ashcraft (2019) found that the math anxiety correlates negatively with enjoyment of math, self-confidence in math and motivation to learn math (Desender & Sasanguie, 2022; Grežo & Sarmány-Schuller, 2018). Additionally, female and F/Sp student was slightly more motivated than their peers (see Figure 1); (ii) freshman/sophomore and female students had lower confidence compared to male and junior/senior students (see Figure 2) which is consistent with Bhowmick et al. (2017); (iii) COVID-19 significantly increased the level of math anxiety in all groups of students surveyed (see Figure 3); (iv) level of math anxiety is highly influenced by upcoming exams and upcoming grades (see Figure 4) which is consistent with Ashcraft's (2002) study. Ashcraft (2002) found strong interrelation with math anxiety and test anxiety; (v) freshman/sophomore and female students exhibited more anxiety due to an upcoming grade (see Figure 4). These findings were not directly similar but agree with the findings in some studies that female students tend to be more anxious than males in mathematics (Ashcraft, 2002; Bhowmick et al. 2017; Delage et al., 2022; Rahe & Quaiser-Pohl, 2021).

There are many questionnaires that claim to measure motivation, self-efficacy, and anxiety. We chose to model our questionnaire after chapter six of Creswell's (2011) Educational Research and applied time series descriptive statistical analysis to our survey data. The key to establishing causality is to analyze each variable on its own and compare the pattern of one population of respondents to that of another. This is illustrated here in relation to gender and academic level.

Our research results indicate that the utilization of math resources, such as tutoring and supplemental instruction, significantly fell due to the COVID-19 pandemic which is consistent with Serhan (2020). Further, the pandemic decreased the use of office hours, but it may have increased cooperative learning between calculus students.

With respect to the general trends, STEM students' math anxiety was highest the week before exams. The prevalence of students who reported low motivation increased from week four onward in freshman and sophomore students and female students. Notably, the prevalence of students with low confidence increased a week before exams. According to the findings of Xie et al. (2019), students' math anxiety might be alleviated by boosting confidence (i.e., self-esteem) and lowering test and general anxiety.

2.7 Limitations of Study

Although this study followed a small population (n = 53), it presents apparently consistent data from two different groups of calculus students. That being said, this study has some limitations that should be taken into account when drawing conclusions or generalizing results. One limitation is that we did not collect survey responses before the pandemic, so we could not compare values before and after. The second limitation is that for a small number of students who did not live in the United States, the time zone might become a significant issue that affected their academic performance. We had no control over that aspect of their participation, whether it was due to their time zone, (possibly being in their native country) or the fact that they were residing in a different State within the United States. But this study provides some insights for further research and practice. Moving forward, further research should be conducted on the effect of parameters like ethnicity and age on motivation, self-efficacy, and anxiety. Further study of such parameters would allow educators to better serve students as online learning continues to be a fixture of university education.

CHAPTER 3

MODELLING MATH ANXIETY USING MACHINE LEARNING

3.1 Overview

The relationships between math anxiety and other variables such as students' motivation and confidence have been extensively studied. The main purpose of the present study was to employ a machine learning approach to provide a deeper understanding of variables associated with math anxiety. Specifically, we applied classification and regression tree models to weekly survey data of science, technology, engineering, and mathematics (STEM) students enrolled in calculus. The tree models accurately identified that the level of confidence is the primary predictor of math anxiety. Students with low levels of confidence expressed high levels of math anxiety. The academic level of students and the number of weekly hours studied were the next two predictors of math anxiety. The junior and senior students had lower math anxiety. Also, those with a higher number of hours studied were generally less anxious. Weekly tree diagrams provided a detailed analysis of the interrelations between math anxiety and variables including academic level, number of hours studied, gender, motivation, and confidence. We noticed that the nature of such interrelations can change during the semester. For instance, in the first week of the semester, confidence was the primary factor, followed by academic level and then motivation. However, in the third week, the order of the interrelation changed to confidence, academic level, and course level, respectively. In summary, decision tree models can be used to predict math anxiety and to provide a more detailed analysis of data associated with math anxiety.

This chapter has been published in the journal *Pedagogical Research* (Soysal et al., 2022).

3.2 Introduction to Machine Learning Methods

Machine learning is an approach to data analysis that involves the use and development of computer systems that can learn and adapt without being given explicit instructions by evaluating and deriving conclusions from patterns in data using algorithms and statistical models. A fundamental scientific goal of machine learning is the exploration of alternative learning processes, such as the scientific breakthrough of various intervention points, the research framework of certain methods, the knowledge that must be accessible to the learner, the concern of dealing with unsatisfactory datasets, and the development of general methods applicable in many task domains (Carbonell et al., 1983). Thus, machine learning assists researchers and educators to interpret outcomes of data, especially for large dataset.

Machine learning involves two types of techniques: supervised learning and unsupervised learning. In this study, we used supervised learning, which includes training a model on known input and output data in order to predict future outcomes. Thus, machine learning is utilized for large amounts of complex data or tasks with multiple variables. For example, healthcare researchers use machine learning to analyze enormous amounts of deidentified patient data. This enables easy evaluation of patient compliance as well as early detection of heart and lung problems (Sharmila et al., 2017). Identifying patients with COVID-19 who are at risk of deterioration during hospitalization is critical for appropriate resource allocation and disease management. When compared to other prediction methods, machine-learning models have been particularly successful in predicting critical COVID-19 cases (Assaf et al., 2020).

Every day, medical practitioners deal with vast amounts of data. Users' outcomes will suffer if they manage data in the traditional manner (Reddy, 2021). Likewise, the educational field is particularly hampered by using traditional strategies. For example, big-scale educational exams capture massive volumes of incredibly rich cognitive and contextual information from large populations of students (Bentayeb & Darmont, 2007). Machine learning applications for educational projects (also known as academic data mining) are a relatively new field that attempts to create ways to analyze data from computerized educational environments and identify significant trends (Kotsiantis, 2012).

There is an increasing prevalence of online learning and digital course materials in collegiate settings, especially during the COVID-19 crisis. This has enabled researchers to gather data, analyze, and predict student's unique learning preferences using machine learning algorithms. Researchers have used machine learning techniques to investigate how sensor data and online learning methods influence student attitudes toward mathematics (Roberts-Mahoney et al., 2016). Machine learning techniques have recently demonstrated impressive advances in data analysis and prediction. They have, however, been far less commonly applied in the assessment of learning quality. Ciolacu et al. (2017) conducted an analysis based on neural networks, support vector machines, decision trees, and cluster analyses to estimate students' performance on examinations and foster Industry 4.0 proficiency in the next generation.

3.3 Machine Learning in the Field of Education

Statistical models have been traditionally used for data analysis and forecasting different outcomes. In the past two decades, a significant increase in computational power has cleared the way for automated analytical model building techniques, which are known as machine learning methods. A major goal of machine learning is to use the ever-growing computational power to extract information, make predictions, and ultimately make informed decisions applicable to a variety of task domains (Carbonell et al., 1983). Reddy (2021) provided a concise survey of various machine learning methods including support vector machines, decision trees, Bayes classifiers, and K-Nearest Neighbors techniques. These machine learning methods have been used in healthcare, science, engineering, and education.

In recent years machine learning has been increasingly used in the field of education. Machine learning approaches for evaluating large amounts of educational data can provide vital information, potentially with large impacts on future education (Bienkowski et al., 2012). For example, highly accurate machine learning models have been constructed to predict the time students are required to generate a response and to estimate the likelihood that the student's response was correct (Inaba et al., 2000). Saarela et al. (2016) used a combination of unsupervised and supervised learning algorithms to predict student performance on math scores, which is a unique way of learning directly from large-scale educational assessment studies' (LSAs) data. In another research, Lezhnina and Kismihók (2022) employed random forest algorithm applications to provide a more complete view of the connections between attitudes toward information and communication technology (ICT) and mathematical and scientific literacy, with an emphasis on the multilayered structure of the data.

In addition, machine learning techniques have lately made significant progress in data analysis and prediction, as well as in evaluating learning quality. Assessing students' academic progress is challenging, but machine learning techniques can aid both students and instructors in this process. Wang and Zhang (2020) investigated the application of machine learning algorithms in an educational quality evaluation model. In their investigation, the machine learning technique was successful in tackling tough challenges such as classification, fitting, and pattern identification. This technique may be used to evaluate university instructors' classroom teaching performance in a more thorough, reasonable, and efficient way. The goal of developing a teaching quality assessment index is to establish a link between the learning quality evaluation index and the teaching effect.

Predictive student performance statistical models have been created with the goal of forecasting mathematics performance. Because a single tool cannot be easily scaled from one circumstance to another, a variety of learning approaches have been investigated and compared to determine the best prediction model (Sokkhey & Okazaki, 2019). In this chapter, we utilize the CRT machine learning method known as decision tree models to provide a deeper analysis of factors (gender, confidence, motivation, academic level, etc.) associated with math anxiety. We chose decision tree models because they have a tree-like structure and are easy to grasp, handle predictions, classification, and factor importance (Al-batah, 2014).

3.4 Materials and Methods

This section describes the study's participants, data sources, and analyses. The classification and regression tree (CRT) is a decision tree machine learning algorithm that is used to classify students into subgroups to better understand the modulating effects of independent variables influencing whether students become anxious about mathematics.

3.4.1 Participants

This study's data were automatically collected by Canvas, a learning management system used at the University of Missouri-Kansas City (UMKC). The sample for the study included students' participating in fully online semester-length mathematics courses offered in the summer of 2020. Course topics included "Calculus II" and "Calculus III". The final sample included all 45 students who participated in both classes. Among the two courses included in the analysis, Calculus II contained 30 (66.7%) total participants—12 (40.0%) females and 18 (60.0%) males, 25 (83.3%) F/S and 5 (16.7%) J/S. Calculus III contained 15 (33.3%) total participants—6 (40.0%) females and 9 (60.0%) males, 5 (33.3%) F/S and 10 (66.7%) J/S. All analyses were conducted using SPSS (Version 27.0). 45 students participated in the nine-week study and contributed to 405 individual observations.

3.4.2 Context

Due to COVID-19, the University of Missouri-Kansas City (UMKC) suspended in-person classes and continued them in an online synchronous format at their normal hours beginning in March 2020. For the whole summer of 2020, campus closures and online learning were in place. Thus, in the summer of 2020, Calculus II and III were taught for the first time in an online synchronous format—previously, these programs were exclusively offered in person at UMKC. Some students were enrolled from various states in the United States, while others were participating in similar programs as part of their home country.

Monday through Thursday, Calculus II and III sessions were split into two 50-minute sections with a 10-minute break in between. All classes and office hours were conducted through Zoom. These classes were worth four credits each. In the former grading process, a passing grade of 60% was required; however, due to COVID-19, it was cut to 55%.

During the summer semester, students learned about integration techniques, definite integral applications, improper integrals, sequences and series, power series, Taylor series and convergence, and analytic geometry in Calculus II class. Additionally, students learned about vectors, solid analytic geometry, vector functions, and multiple variable functions, partial derivatives, multiple integrals, line and surface integrals, and their applications in Calculus III class.

3.4.3 Measures

We developed an instrument to measure students' self-reported math anxiety across a number of factors including their math assignments, upcoming exams, course meetings, grades, etc. (see Appendix A). The instrument further asked students to report usage of math and emotional supports (e.g., supplemental instruction, math tutoring, and the counseling center, among other options, and no options). Finally, it inquired about students' math confidence, their desire to study, and their weekly experience with COVID-19 as related to their math anxiety or ability to succeed in class. The six-question instrument was created based on the researchers' experience with prior survey creation and guidance from Fowler's principles for good survey practice (2013).

This instrument was created as an online survey in the Canvas learning module that students could access via the "Quiz" tab on their Calculus II and Calculus III Canvas course sites. During the COVID-19 pandemic, students answered the same survey questions one week prior to the beginning of the semester and during each of the eight weeks of the semester. Thus, survey data was used to extract levels of students' anxiety as it related to examinations, motivation, confidence, and weekly hours studied. For the eight-week summer semester of 2020, students' answers were gathered once a week through Canvas and one week prior to the start of the semester. The results were then presented to a mathematical education specialist who evaluated the data for validity and intelligibility.

3.4.4 Data Preparation

We pooled data from both calculus classes for all statistical analyses performed in this study due to the small number of participants. To minimize unintended effects (such as students not taking the survey or providing inaccurate self-assessments), the surveys were given in the form of a weekly quiz with bonus points. During the semester, we randomly monitored the survey data to identify human error and asked some participants to provide answers within the given range of values (e.g., level of anxiety should be between 1 and 7). Weekly reminders were sent to students which encouraged them to take the survey. Consequently, we had a high participation rate of 84.9% (45 out of 53 students).

Students were asked to rate their experiences on a numerical scale in a weekly survey. For example, question 4 stated, “How would you rate your desire to study this week?” (1 = no desire to study, 7 = very motivated to study).” We selected and used MS Excel software to compile and organize the data for statistical analysis after gathering nine weeks of student answers in Canvas. We cleaned the data (removing non-essential information such as timestamps and duplicate information from across the nine-week study duration, such as student identification numbers) and gave numeric codes to each response choice as appropriate.

We divided student replies to complicated questions into groups when coding them. For example, we defined three categories for question 4: low motivation (scores 1-2), medium motivation (scores 3-5), and high motivation (scores 6-7). Students' replies to yes or no questions were categorized in a dichotomous manner, with "yes" equaling 1 and "no" equaling 0. The data was then totaled by groups and weeks, and the results were organized.

Following that, we carefully put the data into SPSS (Version 27.0) and ran classification statistical analyses. Demographic variables including gender and academic level were plotted against weekly hours studied, confidence, motivation, and the effects of COVID-19 on self-reported math anxiety. The practical implications of these findings were then clarified and addressed in further depth.

3.4.5 Classification and Regression Tree

Decision tree models, which have an easy to grasp tree-like structure, perform prediction and classification, and evaluate factor importance (Alkhasawneh et al., 2014).

Decision tree analysis, a non-parametric approach for expressing how examples from a sample are categorized into increasingly smaller subgroups until reaching the final or terminal child nodes in a tree diagram, was used to perform this study. Because of data sparsity among collected responses, this technique was chosen.

The regularity in which participants modify their predictions to match those of the model, as well as their self-reported levels of trust in the model, show that claimed accuracy has a substantial influence on people's trust in a model (Yin et al., 2019).

In this study, we used the IBM SPSS (Version 27.0) software as one of a variety of machine learning techniques. This tool assisted us in creating decision tree models that will present data in an easy-to-understand format. We imported all of the cleaned data from MS Excel into the SPSS software to perform the analysis. Data cleaning entails determining the optimal method for dealing with missing information. SPSS gave us a few algorithmic choices to model the decision trees. A classification and regression tree were used (CRT) in this study. The CRT divides instances into forecast-dependent variable values based on independent variable values. We applied this method for each individual week of data in addition to all the data collected over the entire nine-week study.

Regarding the dependent variables, the CRT divides the data into segments that are as homogenous as possible (IBM, 2012). The independent variables can be continuous, ordinal, nominal, or scale. For example, we identified the dependent variables of interest as “yes” (anxious) and “no” (non-anxious). Also, we used this category for the classification of independent variables. This approach was used for each week of study as well as the entirety of the nine-week period.

Each parent node divides into just two child nodes as the tree grows. We used one case for parent nodes and one case for child nodes in the default value category. Occasionally, CRT produced trees without any nodes below the root node. This helped us to produce more useful results.

Once a model has been adapted to a specific set of data, it might have a worse predictive value when applied to other data sets. This concern addresses a model’s specificity and correctness. The capacity of a model to anticipate a positive result and be right in its forecast is known as correctness. To assess the models' correctness, we used a 10-fold cross-validation technique. Our model's overall percentage accuracy indicates how effectively it predicted if there would be a yes (anxious) or a no (non-anxious) in Table 8.

3.4.6 Pearson Correlation Analysis

During the summer semester of 2020, the surveyed data were collected from 45 students enrolled in calculus II and calculus III courses. The classification and regression tree techniques were performed during preliminary statistical analysis to better understand the anxiety levels of students as they related to the studied independent variables. Participants were split into groups based on gender and academic level in the second portion of the analysis. The relationship between the dependent and independent variables was determined using a Pearson correlation. Only the strength of the linear relationship between the two variables is measured by correlations and the Pearson correlation implies that both variables have normal distributions (DeCoster & Claypool, 2004).

Table 7 displays the correlation coefficients and their related significance levels. SPSS software was used to conduct this analysis. The main goal of this study is to see if there is a relationship between math anxiety as a dependent variable and independent variables such as exam anxiety, confidence, motivation, and weekly hours studied.

3.5 Outcomes of Machine Learning Approach

We began surveying students one week prior to the start of the eight-week summer semester in 2020. Table 6 shows average values in reported hours studied (varying between 0 and 96 hours), confidence, motivation, and exam anxiety. Students were asked to rate their confidence, motivation, and exam anxiety on a scale of 1 to 7, where 1 represented low confidence/motivation/exam anxiety and 7 represented high confidence/motivation/exam anxiety. The first column demonstrates that on average male students tended to study more hours than female students surveyed.

Data in Table 6 are formatted as follows. For each entry, the first two numbers are the average values reported by male and female students, respectively, and the numbers within parenthesis correspond to average values among freshman, sophomore, junior, and senior students. Observe that in most cases, the junior and senior students studied longer hours compared to freshman and sophomore students. Also, in most cases, male students had slightly more confidence than female students. Furthermore, junior and senior students exhibited mainly higher levels of confidence than freshman and sophomore students. In all groups, the average exam anxiety went up during exam weeks 4, 7, and 9.

Table 6. Average values of data for each variable based on gender (Male, Female) and academic level (Freshman, Sophomore, Junior, and Senior).

	Hours Studied	Confidence	Motivation	Exam anxiety
W1	10.4, 4.4, (7.8, 7.2, 3.4, 13)	4.7, 5.2, (4.8, 4.6, 5.7, 5.1)	4.2, 5.6, (4.9, 4.4, 5, 4.7)	3.9, 3.8, (3.9, 4.9, 2.4, 4.3)
W2	14, 12.3, (10.6, 14.9, 16.3, 15)	5.3, 5.3, (4.9, 5.1, 6.2, 5.6)	4.3, 5.4, (4.8, 5, 4.3, 4.6)	3.7, 4.2, (4.3, 5.1, 1.8, 3.7)
W3	17.6 ,15.7, (12.3, 19.6, 25.2, 18.3)	5, 4.7, (4.8, 4.6, 6, 4.7)	4.4, 5, (4.6, 5.1, 4.5, 4.4)	4.1, 4.6, (4.5, 4.4, 3.4, 4.5)
W4	16.9, 15.2, (10.9, 20.2, 25.7, 16.4)	5, 4.8, (4.6, 5.1, 5.5, 5)	4.5, 4.9, (4.2, 5.6, 4.3, 4.9)	5.1, 5.1, (5.2, 5.8, 4.7, 5)
W5	15.4, 17.6, (12, 19.2, 21, 18.4)	5, 4.9, (4.8, 5.1, 5.4, 4.6)	4.4, 5, (4.5, 5, 4, 4.7)	3.9, 4.6, (4, 4.8, 3.6, 4.3)
W6	16.8, 16.6, (11.9, 19.4, 24.2, 18.7)	4.8, 4.6, (4.5, 4.7, 5.4, 4.9)	4.5, 4.8, (4.3, 5.7, 3.6, 4.7)	4.3, 5, (4.2, 4.5, 4.4, 5.5)
W7	16.2, 16.3, (11, 19.3, 20.4, 23.4)	4.6, 4.2, (4.3, 4.9, 4.8, 4.1)	4.4, 4.7, (4.3, 5, 4.4, 4.7)	4.8, 5.5, (5.1, 4.5, 4.6, 5.8)
W8	18.6, 17.1, (12.7, 16.6, 25, 26.9)	4.6, 3.8, (4.1, 3.9, 5, 4.7)	4.6, 4.3, (4.1, 4.9, 4.2, 5)	5.1, 5.7, (5.4, 5, 4.8, 6.1)
W9	16.9, 17.4, (11.9, 16.8, 22.3, 26.1)	4.8, 3.6, (4.3, 4, 4.3, 4.7)	4.7, 4.6, (4.3, 5, 4.2, 5.6)	5, 6, (5.1, 5.5, 5.2, 6.3)

The correlation analysis of the survey data has been summarized in Table 7. As expected, math anxiety is directly correlated with exam anxiety in most subpopulations. We also noted that the number of hours studied and being a junior is negatively correlated with math anxiety. In other words, the higher number of hours studied, the lower the level of math anxiety. Surprisingly the correlation between the number of hours studied and math anxiety in the sophomore population was positively correlated. This suggests that the more hours sophomore students studied the more confused and anxious they became. We also noticed a positive correlation between motivation and anxiety in the sophomore students. Whereas in senior students, lower motivation was correlated with higher levels of anxiety. In all subpopulations, confidence and anxiety were negatively correlated. The COVID-19 pandemic disproportionately affected female, junior, and senior students. In both subpopulations (senior and junior), COVID-19 was positively correlated with math anxiety.

Table 7. Person Correlation between model variables and math anxiety within each subpopulation during the summer semester of 2020.

	Exam Anxiety	Hour Studied	Motivation	Confidence	COVID-19
Female	0.337**	0.142	0.1	-0.277**	0.246**
Male	0.291**	-0.085	-0.059	-0.172*	-0.076
Freshman	0.378**	-0.083	-0.114	-0.304**	-0.082
Sophomore	0.609**	0.289**	0.262*	-0.097	0.075
Junior	0.336*	-0.384**	-0.031	-0.069	0.318*
Senior	-0.017	-0.021	-0.204	-0.342**	0.198
F/SP	0.451**	0.109	0.02	-0.214**	-0.034
J/S	0.174	-0.142	-0.108	-0.215*	0.246**
All Students	0.322**	-0.012	-0.021	-0.219**	0.053

F/S: Freshman &Sophomore, J/S: Junior &Senior.
**: Correlation is significant at the 0.01 level(2-tailed).
*: Correlation is significant at the 0.05 level(2-tailed).

Next, we applied the CRT method to further analyze the survey data. We constructed decision tree models for each week of survey data as well as the entire data. For each decision tree model, the dependent variable was math anxiety, and the independent variables were confidence, motivation, weekly hours studied, gender (F/M), academic level (F/SP/J/S), the relative effects of COVID-19, and course level. Table 8 provides a summary of the model accuracies and main predictors of decision tree models for weeks 1-9 and all weeks. As it can be seen all CRT models have high levels of overall accuracies. Observe that confidence is the main predictor for most cases (i.e., weeks 1, 3, 6, 7, 9, and all weeks). Whereas academic level and the number of hours studied are the second most important predictors of math anxiety (see the columns primary and secondary nodes). Table 9 is a summary of the importance of variables to each CRT model. Again, note that confidence, number of hours studied, and academic level are the top three variables in the prediction of math anxiety.

Table 8. Model accuracy and main predictors of math anxiety for each week 1-9, as well as the entire semester. Model accuracy is summarized by percentage correct for overall (O), no anxiety (N), those with anxiety (Y).

Week	Model Accuracy	Primary Nodes	Secondary Nodes
W#1	93.3%(O), 88.9% (N), 94.4% (Y)	Confidence (4)	Academic Level (F/SP/J/S)
W#2	86.7% (O), 62.5% (N), 100% (Y)	Motivation (3)	Academic Level (F/SP/J/S) &Hours Studied (22)
W#3	93.3% (O), 75% (N), 100% (Y)	Confidence (6)	Academic Level (F/SP/J/S) &Hours Studied (28.5)
W#4	93% (O), 66.7% (N), 100% (Y)	Hours Studied (47)	Academic Level (F/SP/J/S) &COVID
W#5	97.8% (O), 92.9% (N), 100% (Y)	Calculus 2 or 3	Confidence (6) &COVID
W#6	95.6% (O), 91.7% (N), 97% (Y)	Confidence (4)	Hours Studied (10) &Gender (F/M)
W#7	88.9% (O), 50% (N), 100% (Y)	Confidence (3)	Motivation (2)
W#8	97.8% (O), 87.5% (N), 100% (Y)	Motivation (4)	Academic Level (F/SP/J/S) &Hours Studied (11)
W#9	95.6% (O), 75% (N), 100% (Y)	Confidence (4)	Hours Studied (37) &Hours Studied (4.5)
W#1-9	86.2% (O), 49% (N), 98% (Y)	Confidence (4)	Calculus 2 or 3

F: Freshman, SP: Sophomore, J: Junior, S: Senior, F: Female, and M: Male.

Table 9. Weekly Importance of Factors (I.F.) associated with math anxiety determined by the decision tree models. The number of hours studied (HS), confidence (C), and motivation (M) were the top three predictors of math anxiety throughout the semester.

Week #	1st I.F.	2nd I.F.	3rd I.F.	4th I.F.	5th I.F.	6th I.F.	7th I.F.
W#1	HS	C	M	AL	C2/3	G	C-19
W#2	HS	M	C	AL	G	C2/3	C-19
W#3	M	HS	C-19	C2/3	C	AL	G
W#4	HS	C	AL	C2/3	C-19	M	G
W#5	HS	C	AL	M	C-19	C2/3	G
W#6	HS	M	C	C-19	G	AL	C2/3
W#7	HS	C	M	G	AL	C-19	C2/3
W#8	HS	C	C2/3	M	AL	G	C-19
W#9	HS	C	M	AL	G	C-19	C2/3
W#1-9	HS	M	C	AL	C-19	G	C2/3

I.F: Important Factor, HS: Hours Studied, M: Motivation, C: Confidence, C2/3: Calculus 2or3, AL: Academic Level, G: Gender, C-19: COVID.

The tree diagram of each CRT model has been included in the supplementary document. In the following paragraphs, we will summarize the results of tree diagrams for weeks 1-9 and the diagram modeled the entire semester. See Table 8 for the accuracy and key outputs of each decision tree model.

In the first week, all students who reported confidence levels with less than 4 also reported anxiety. Whereas students with confidence greater than 4 had different results based on the academic level. Namely 50% of J/S students were anxious as compared to 83% of F/SP students (See Figure B4 in the supplementary document).

During the second week, 22% of students who reported a level of motivation less than 3 were anxious. Within this group, F/SP students reported they were not anxious, while 50% of J/S students reported they were. Among those students with a level of motivation of more than 3, 75% of students reported they were anxious. Under the same category, 72% of students who studied less than 22 hours were anxious, while 100% of students who studied more than 22 hours were anxious (See Figure B5 in the supplementary document).

In week 3, 78% of students with a level of confidence less than 6 reported that they were anxious. By contrast, 40% of students who reported with a level of confidence more than 6 were anxious. Among those students with a level of confidence less than 6, all S students were anxious compared to 73% of F/SP/J students. All students who reported a level of confidence with more than 6 and who studied less than 28.5 hours were not anxious, while all students who studied more than 28.5 hours were anxious (see Figure B6 in the supplementary document).

In week 4, the students' exam week, 83% of students have reported anxiety if they studied less than 47 hours that week. By contrast, 33% of students who studied for more than 47 hours were anxious. Among those students who studied less than 47 hours, all S students were anxious versus 80% of F/SP/J students. Of those who studied more than 47 hours, all students who reported experiencing adverse effects due to the COVID-19 pandemic were anxious. By contrast, all students who reported no adverse effects of the COVID-19 pandemic reported no anxiety (see Figure B7 in the supplementary document).

In week 5, 80% of calculus 2 students reported feeling anxious, as compared to 47% of calculus 3 students. Among those calculus 2 students, 88% of students who reported with a level of confidence less than 6 were anxious. By contrast, 40% of students who reported with a level of confidence of more than 6 were anxious. 42% of calculus 3 students reported feeling anxious if they reported no adverse effects due to the COVID-19 pandemic, while

67% of students who experienced effects were anxious (see Figure B8 in the supplementary document).

In week 6, 90% of students who reported with a level of confidence less than 4 were anxious, while only 60% of students who reported with a level of confidence more than 4 were anxious. Among those students with a level of confidence of less than 4, 71% were anxious if they reported less than 10 weekly hours studied. By contrast, all students who studied for more than 10 hours were anxious. Among those with a level of confidence greater than 4, 73% of M students were anxious as compared to 40% of F students (see Figure B9 in the supplementary document).

During the second exam week, week 7, all students who reported with a level of confidence less than 3 were anxious while only 70% of students who reported with a level of confidence more than 3 were anxious. Within the level of confidence with more than 3, all students who reported a level of motivation less than 2 were not anxious as compared to the 72% of students who were anxious (see Figure B10 in the supplementary document).

Throughout week 8, 95% of students who reported with a level of motivation less than 4 were anxious, while only 73% of students who reported with a level of motivation more than 4 were anxious. 50% of J students who reported a level of motivation with less than 4 were anxious as compared to 100% of F/SP/S students. Among those students who reported a level of motivation of more than 4, 50% of students who studied less than 11 hours were anxious. By contrast, 88% of students who studied for more than 11 hours were anxious in the level of motivation with more than 4 (see Figure B11 in the supplementary document).

During finals week, week 9, 95% of students who reported with a level of confidence less than 4 were anxious. By contrast, 72% of students who reported with a level of confidence more than 4 were anxious. Within the level of confidence less than 4, all students who reported were anxious if they reported less than 37 weekly hours studied. By contrast, 50% of students who studied for more than 37 hours were anxious. Among those students who reported a level of confidence of more than 4, all students who studied less than 4.5 hours were not anxious. By contrast, 78% of students who studied for more than 4.5 hours were anxious (see Figure B12 in the supplementary document).

Throughout the semester, 88% of students who reported with a level of confidence less than 4 were anxious (see Figure B13(a) in the supplementary document). Within this group, 90% of F/SP/S students reported they were anxious, while 64% of J students reported they were anxious. By contrast, 70% of students who reported with a level of confidence more than 4 were anxious (see Figure B13(b) in the supplementary document). Among those students who reported with a level of confidence more than 4, 75% of calculus 2 students reported feeling anxious, versus 58% of calculus 3 students (see Figure B13(b) in the supplementary document).

3.6 Discussion

The purpose of the present study was to utilize the power of machine learning to have a deeper understanding of the factors associated with math anxiety in college students. Applying the CRT method to the survey data of summer 2020 calculus students, we identified the interrelationships between math anxiety and factors including students' motivation, confidence, weekly hours studied, academic level, and gender. The tree diagrams of the CRT method revealed temporal variations of these factors over the course of the semester. Specifically, from weeks 1-9, there was an interchange between the level of confidence and motivation (less than 3 or 4) as the significant predictors of math anxiety which is consistent with the results of previous research (Akin & Kurbanoglu, 2011; Rozgonjuk et al., 2020; Tapia & Marsh, 2004; Zakaria & Nordin, 2008). There was an exception for weeks 4 and 5, where the number of hours studied, and course level became the primary predictors of math anxiety. This could be due to the fact that the students had a midterm exam on the fourth week. This study distinguishes itself from other studies on math anxiety in two important aspects. First, we collected longitudinal survey data to analyze the temporal changes to math anxiety affected by the abovementioned variables. Secondly, it illustrates the capabilities of machine learning methods over the traditional statistical models to extract crucial information about education and the level of math anxiety in college students.

The list of primary and secondary nodes and the accuracies of weekly CRT models are summarized in Table 8. Note that all CRT models have reasonably good accuracies that suggest that confidence is the first primary factor involved in the mediation of math anxiety. Students who report a confidence level of less than a 4 out of 7 were significantly more anxious than those with higher levels of confidence in the proposed model. Another finding of the present study showed the academic level and the number of hours studied were the top factors involved in the production or negation of math anxiety in half of our CRT models. In addition, our study indicates that J/S students tend to study more hours than F/SP students and therefore are more confident and less anxious (Table 6). Interestingly, the first column of Table 6 demonstrates that on average, male students tend to study more hours than female students surveyed, which is inconsistent with the results obtained by Smail (2016). Smail (2016) mentioned that female students are more likely to have math anxiety and study math more than male students.

Furthermore, the adverse effects of the COVID-19 pandemic had the least impact among other factors on math anxiety in calculus students. The effects of the COVID-19 pandemic as a secondary factor causing math anxiety only occurred in two of the ten CRT models which is consistent with the results of previous research (Derling et al., 2021; Ludwig, 2021; Soysal et al., 2022). For example, Derling et al., 2021 claims that the level of math anxiety increased due to COVID- 19. However, one study measuring the magnitude of math anxiety in students found relatively low levels during COVID-19 (Ariapooran & Karimi 2021). Ariapooran and Karimi (2021) found that 67.21 % of students showed minimal mathematics anxiety in the COVID-19 pandemic.

In addition to machine learning methods, and for the purpose of drawing connections to earlier studies on math anxiety, we performed a correlation analysis of the survey data. There was significant correlation between math anxiety and the factors mentioned on Pearson's correlation analysis in Table 7. We also observed that confidence was negatively correlated with math anxiety based on Pearson's correlation analysis in Table 7, which is consistent with the results obtained from previous research (Akin & Kurbanoglu, 2011; Hembree, 1990; Samuel & Warner, 2021; Ashcraft & Ridley, 2005; Naderi Dehsheykh et al., 2021).

In this study, the population of interest was a random population of UMKC students who were taking a calculus course in a given semester. In particular, the population (n = 45) consisted of those who were STEM majors and enrolled in an online synchronous Calculus II (or Calculus III) course during the summer of 2020. Our findings are generalizable to public institutions and to students who are taking online calculus classes. However, there could be some limitations to generalizability perhaps due to the effects of the COVID-19 pandemic on the circumstances at the time, although we did control for the effects of participants' COVID-19 anxiety in our analysis.

The limitations of the present work are as follows: first, it should be noted that the collected data related to the number of hours studied and levels of anxiety, motivation and confidence are self-reported data. Although we eliminated incorrect data, we could not validate the full accuracy of self-reported data. Secondly, the participants in this study were students enrolled in a summer course rather than fall or spring courses. This may create a slight selection bias in the collected data. For example, some students may not have enrolled because of the COVID-19 pandemic or perhaps did not enroll because it was an online summer course. Thirdly, there may have also been other unexplained biases in the data related to this particular institution, and thus there is some sampling/selection bias present. Lastly, a limitation of this study is that environmental factors (such as parents, teachers, ethics, etc.) may enhance the association between math anxiety and the factors which we used in this study. However, we believe that these findings offer considerable insight into strengthening academic achievement in higher education. Moving forward, the results of this study should be validated using larger samples of STEM students across different college-level math courses. It is our profound hope that the results of this study will serve as a foundation to build upon as other researchers continue to develop the field. In addition, the present work shows that researchers in the field of education can use machine learning methods to provide a deeper analysis of data associated with math anxiety and attitudes toward math education. In conclusion, the present study underlines the importance of machine learning methods to extract detailed and accurate information from survey data in the field of mathematics education.

CHAPTER 4

MATHEMATICAL MODELLING OF MATH ANXIETY

4.1 Overview

Mathematical models enable the study of infectious disease dynamics over time and may benefit research design. In science, medicine, and infectious disease progression, mathematical models have been widely used to predict the future behavior of diseases. Mathematical modeling is covered in Chapter 4 to better understand the dynamics of math anxiety. In this chapter, we start from the assumption that math anxiety may be transmitted among students; then we build a model that is an extension of the SIR infectious disease model that makes use of ordinary differential equations. We discuss the nonnegative and bounded of the system (4.1) of the model mathematically. We also demonstrate the existence and stability of equilibria such as anxiety-free and endemic equilibrium. We further find a fundamental reproduction number, which describes the rate math anxiety spreads or dissipates within the student population. Finally, we use MATLAB ODE45 to do a numerical simulation of the system in this study. Interestingly, we could not observe any periodic or oscillatory behaviors of the system (4.1) of the first model. Lastly, we discuss about the nonexistence of limit cycles in this chapter.

4.2 Mathematical Models of Infectious Diseases

The study of epidemiology has evolved as a topic of significant importance to modern society. In epidemiology, mathematical models have a long history as a valuable resource for understanding how infectious illnesses spread. Beginning in 1760, Bernoulli (1760) was the first to use mathematical methods to study the effects of smallpox vaccination (Bernoulli, 1760). In recent years, the mathematical foundations of epidemiology have garnered attention. The use of mathematical modeling as a research tool in the study of the progression of illnesses can benefit both mathematicians and epidemiologists (Kermack & McKendrick, 1932). The spread of an illness through a population over time can be described in a simplified form using mathematical models. Most epidemic models begin by segmenting the population into a small number of distinct areas. Each of these groups has people who are similar in terms of how they are impacted by the disease under investigation. The SIR model, which was established in the early twentieth century by Kermack and McKendrick is based on a set of three connected nonlinear ordinary differential equations (Kermack & McKendrick, 1932; Kermack & McKendrick, 1933; Kermack & McKendrick, 1991). Disease transmission models are useful tools for understanding epidemics and guiding public health planning.

Traditional models of disease transmission among populations, such as the SIR model, generally, take the form of nonlinear ordinary differential equations such as:

$$\dot{S} = \frac{dS}{dt}(t) = -\beta S(t)I(t)$$

$$\dot{I} = \frac{dI}{dt}(t) = \beta S(t)I(t) - \gamma I(t)$$

$$\dot{R} = \frac{dR}{dt}(t) = \gamma I(t),$$

where S represents the number of susceptible, I represent the number of infectious, and R represents the number of recovered. Negative values reflect flows out whereas positive values reflect flows into the compartments.

The SIR model considers a variety of important aspects of population dynamics, such as death rate, immigration or birth rate, recovery, and immunity, which makes is easy to modify. Even the most basic model has significant implications for public health. Kermack and McKendrick (1991) made the following assumptions: the disease's rate of spread is proportionate to population size; infection occurs instantly; no one is immune to the disease at the outset; immunity, once attained, is permanent; and the illness does not result in death. To make the SIR model work, each member of the population is allocated to one of three compartments at any given time: those who are susceptible to illness, those who are already infected, and those who have been removed. The eliminated compartment includes those who are not diseased and are not susceptible to infection; in other words, those who are immune, in quarantine, or have died. The class is significant because it may account for permanent or transient immunity gained from vaccination or disease experience in other forms of the SIR model. Individuals can transfer from one compartment to another. For example, a person who recovers from an infection may transfer from the infected class to the recovered compartment (see Figure 5). Thus, the model considers the interdependence that occurs among the various compartments within the population.

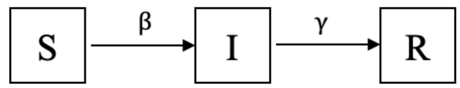


Figure 5. The flow of individuals between compartments in the SIR model from left to right. The compartments of the SIR disease model include a susceptible individual (S) who contacts disease becomes infectious (I) and then recovers (R) to become recovered. The parameters of the SIR model are the rate at which susceptible hosts become infected (β) and the rate at which Infectious individuals recover (γ).

When it comes to mathematical modeling of epidemiology, like with most other types of mathematical modeling, there is often a trade-off between basic or strategic models, which ignore the majority of information and are intended mainly to emphasize qualitative behavior at the broadest level, and comprehensive or operational models, which are intended for more particular scenarios and sometimes include quantitative forecasts for the near future (Anastassopoulou et al., 2020; Costa et al., 2021). Detailed models are notably challenging or even impossible to solve analytically; as a result, their applicability to theoretical endeavors is restricted, despite the possibility that their relevance to strategic endeavors is significant (Nwankwo & Okuonghae, 2022). For instance, extremely simple models for epidemics predict that an epidemic would end after a certain amount of time (see Figure 6), leaving a portion of the population unaffected by the illness. Mathematical models used to make management policy suggestions must have quantitative outcomes, and the models required in a public health environment require a large level of information to adequately explain the issue (Balcan et al., 2009). For instance, if the issue at hand is to make a recommendation regarding which age group or groups need to be the primary focus of attention in the event of a global pandemic, it is vital to adopt a model that divides the population into an adequate number of age categories and acknowledges the relationship between the various age categories. In recent years, there has been a surge in the advent of high computation, which has enabled the application of such models.

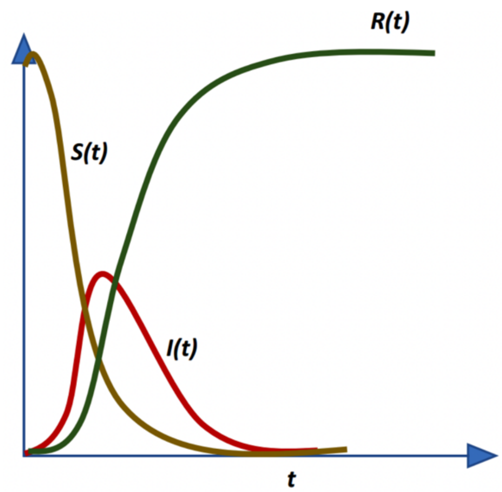


Figure 6. A sample of basic dynamics of SIR (susceptible S(t), infectious I(t), and recovered R(t)) model. Adapted from (Abou-Ismail, 2020).

When a person recovers from illness, the SIR model implies that the person carries lifelong immunity to that disease; this is the case for a variety of different diseases (Brauer et al., 2019). An individual's immunity to a distinct category of airborne illnesses, such as influenza, which circulates at different times of the year, may deteriorate over time. Gai et al. (2020) argued that the SIRS model should be applied to let recovered people revert back to a vulnerable condition. The SIRS model can be used if there is sufficient inflow to the sensitive population; however, once equilibrium is attained, the dynamics will be in an endemic state with oscillations. Some SIRS models use a delayed exponential distribution to depict diminishing immune response on epidemiological modeling software (Keeling & Danon, 2009). Immunity persists in individuals for a certain amount of time, after which it begins to decrease according to an exponential distribution.

Additionally, stochastic mathematical models of infectious diseases provide a more realistic approach to pandemics since they allow for the detection of early patterns in an outbreak as well as the analysis of the geographical distribution of incidence rates in a given location. This improves the accuracy of stochastic mathematical models of infectious diseases in representing how epidemics work. According to Brauer (2017), stochastic mathematical models allow for predictions about the length of an epidemic by accounting for disparities among people in a population. The models consider not only geographical and socioeconomic characteristics, but also environmental factors that influence disease transmission. Every stochastic model assumes that there are probabilities in the shift between the structure's compartments, as well as that there is at least one disease-free state (Lin et al., 2020; Huppert & Katriel, 2013; Kretzschmar & Wallinga, 2009). Because stochastic models allow for uncertainty, they provide a more accurate picture of disease dynamics for modeling infectious diseases. A wide range of parameters impacting the epidemiological behavior of an infectious illness can be included into stochastic models, which subsequently provide a range of possible outbreak scenarios.

Finally, the SIR family of epidemiological models provides an effective and adaptable set of tools for understanding the spread of illness and organizing public health interventions at various phases even another fields. The most basic of these models, such as the one (SIR) examined here, can provide us with useful numerical information. Researchers can predict whether or not an epidemic will occur, the maximum number of infected people at any given time, and consequently, the severity of the epidemic. For example, despite the challenging concepts and terms, there can be a considerable number of students who suffer significant math anxiety experience significant math anxiety as defined by Gurin et al. (2017). Since students can develop math anxiety at any age Gurin et al. (2017) developed a mathematical model to describe how peer and teacher relationships interact to transmit math anxiety to identify the most effective time for support. Similarly, Amani et al. (2021) designed an epidemiological model to predict the level of math anxiety in students to help educators have a better understanding of their students. According to Amani et al. (2021), a high percentage of students, approximately 70%, will suffer from high mathematics anxiety in the near future. In this chapter, we build a mathematical model to capture the dynamics of transmission of math anxiety between students by using a system of nonlinear ordinary differential equations.

4.3 First Mathematical Model of Math Anxiety

4.3.1 Mathematical Model Formulation

We develop a model based on non-linear interactions to investigate how math anxiety spreads among students. To create this model, we divide the N-students total population into four sub-populations (i.e., compartments): susceptible S(t), anxious A(t), untreated U(t), and treated T(t) over time. Figure 7 illustrates the modified SIR model of math anxiety transmission between students. The SIR model is an epidemiological model that computes the number of people infected with a contagious illness over time in a closed population.

In this study, we consider math anxiety as an epidemiological disease that spreads through student interactions. These student interactions may have an impact on both anxious and non-anxious peers (Gurin et al., 2017). Therefore, the mathematical model is designed to study the dynamics of students' math anxiety. In the SATU model, the set of ordinary differential equations (ODEs) is given by:

$$\begin{aligned}
\dot{S} &= (1 - p)\Lambda - \beta AS + \alpha A - m_s S - \sigma S \\
\dot{A} &= \beta AS - (\alpha + \delta + m_a + \gamma)A + \sigma S \qquad (4.1) \\
\dot{T} &= p\Lambda + \gamma A - m_t T \\
\dot{U} &= \delta A - m_u U
\end{aligned}$$

where S(t) is the number of students susceptible to math anxiety, A(t) is the number of students who have developed math anxiety, T(t) is the number students treated for math anxiety (by advising, tutoring, or mentoring programs), and U(t) be the number of untreated students. Also, time (t) is considered via the number of days in a semester.

Given the abovementioned set of ordinary differential equations representing the SATU model (4.1), a compartmental diagram of the transmission of math anxiety among students is seen in Figure 7, and a summary of parameters and variables is given in Table 10. We describe the flow chart of the model, where anxious students have a probability of affecting susceptible students and vice-versa daily. The parameter p is the proportion of the students who are not susceptible to anxiety. The parameter β is the anxiety transmission rate. Particularly, β is the rate at which susceptible students get anxiety from interactions with anxious students.

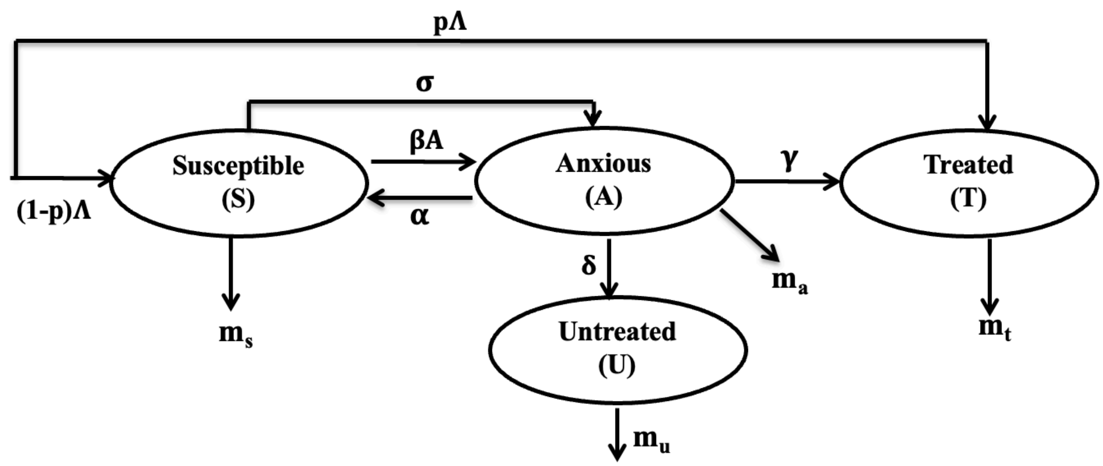


Figure 7. Flow chart describing the interaction between students in different compartments as math anxiety is transmitted for the first model.

The parameter σ is the rate of students who can leave the anxious state without any treatment. According to the mass action law (Ross 1910; Anderson & May 1992; Diekmann & Heesterbeek, 2000), when an infected student makes contact, they are able to transmit anxiety with βA to others per unit of time. Parameter m is the rate at which the students are dropping the course. For example, m_u is the rate of untreated students dropping the course. And Λ is the rate of college students who are taking gateway math courses.

Table 10. Parameters, units, and descriptions for the SATU model.

Parameters	Units	Descriptions
Λ	Students/semester	Students entering gateway math courses
β	1/days	Anxiety transmission rate (through contact)
σ	1/days	Anxiety transmission rate (Self-induced)
α	1/days	Recovery rate
γ	1/days	Treated rate
δ	1/days	Untreated rate
m_s	1/days	Rate of students dropping course while susceptible
m_t	1/days	Rate of students dropping course after treated
m_u	1/days	Rate of students dropping course who are untreated
m_a	1/days	Rate of students dropping course while anxious
p	Proportion	Proportion of students who are not susceptible to Math anxiety
S	State	Susceptible
A	State	Anxious
T	State	Treated
U	State	Untreated

4.3.2 Model Well-Posedness Theorem

In the first step for validating the SATU model, we prove that the model solutions with nonnegative initial values are always nonnegative and bounded.

Theorem 4.1. If the parameter values and the initial condition of the model (4.1) are nonnegative, then the solutions of model (4.1) are nonnegative and bounded in $\dot{\Gamma}$, where

$$\Gamma = \{(S, A, T, U) \in \mathbb{R}_+^4 \,|\, S + A + T + U \leq \Lambda/m \}$$

Proof 4.1: Here we show that for the model (4.1), all solutions with non-negative initial values remain non-negative and bounded in $\dot{\Gamma}$. Let N(t) = S(t)+A(t)+T(t)+U(t). By adding equations (4.1), we have that $N' \leq \Lambda$-mN where m = min $\{m_s\ m_a\ \ m_t\ \ m_u\}$.

Then, lim $\sup_{t\to\infty} N(t)\ \leq \Lambda/$m. Hence, the feasible region

$$\Gamma = \{(S(t), A(t), T(t), U(t)\ ) \in \mathbb{R}_+^4\ |S(t) + A(t) + T(t) + U(t) \leq \Lambda/m\}$$

is positively invariant with respect to model (4.1). This completes the proof.

4.4 Analysis of The Model

4.4.1 Existence of Equilibria

Existence of Anxiety-Free Equilibrium

The Anxiety-Free Equilibrium (AFE) is the state where anxiety disappears. Therefore, all anxious compartments will be zero except susceptible and treated compartments.

Theorem 4.2. A unique Anxiety-Free Equilibrium (AFE) exists at the point (S*, 0, T*, 0) if $\sigma = 0$ and all parameters are nonnegative.

Proof 4.2. We denote AFE with (S*, 0, T*, 0). The initial step is to solve ODEs of the system (4.1) to find equilibria. To remove anxiety from the system, we set the right side of the ODEs of the system (4.1), i.e.,

$$\dot{S} = (1 - p)\Lambda - \beta AS + \alpha A - m_s S - \sigma S = 0$$

$$\dot{A} = \beta AS - (\alpha + \delta + m_a + \gamma)A + \sigma S = 0$$

$$\dot{T} = p\Lambda + \gamma A - m_t T = 0$$

$$\dot{U} = \delta A - m_u U = 0$$

and $A = 0$. Thus, we have Anxiety-Free Equilibrium (AFE):

$$AFE = (S^*, A^*, T^*, U^*) = \left(\frac{(1-p)\Lambda}{m_s},\ 0,\ \frac{p\Lambda}{m_t},\ 0\right) \tag{4.2}$$

Basic Reproduction Number, R_0

A basic reproduction number (R_0) is defined as the expected number of secondary infections caused by a single infection introduced into a total student population composed entirely of susceptible individuals (Bani-Yaghoub et al., 2012; van den Driessche & Watmough, 2002). Note that the R_0 is a threshold number which is presented for a compartmental model based on a system of ordinary differential equations. R_0 is used in epidemiology to determine whether diseases spread or disappear. There will be an outbreak when $R_0 > 1$, whereas the infection will gradually disappear when $R_0 < 1$.

Specifically, we use the Next Generation Matrix (NGM) to generate the reproduction number (van den Driessche & Watmough, 2002). This approach is most useful for an epidemiological model to calculate the reproduction number provided by Diekmann et al. (1990) and van den Driessche & Watmough (2002).

Theorem 4.3. The basic reproduction number for the system of ODEs is:

$$R_0 = \frac{\beta(1-p)\Lambda}{m_s(\alpha+\delta+m_a+\gamma)}$$

when $R_0 > 1$, the AFE is unstable, whereas $R_0 < 1$ implies that the AFE is locally asymptotically stable.

Proof 4.3. By applying the NGM to find the reproduction number, we use the following form: $\dot{f}$ = "entry" − "exit" (Gurin et al., 2017). Then, we define the vector of $X = \begin{bmatrix} A \\ U \end{bmatrix}$. We introduce $F = \frac{df}{dX_{AFE}}$ which is the derivative of f and contains the anxious term; in a similar way, we introduce $V = \frac{dv}{dX_{AFE}}$ which contains transfer terms.

Constructing the NGM and anxiety-free equilibrium is

$$X = \begin{bmatrix} A \\ U \end{bmatrix} = \begin{bmatrix} \beta AS + \sigma S \\ \delta A \end{bmatrix} - \begin{bmatrix} (\alpha + \delta + m_a + \gamma)A \\ m_u U \end{bmatrix}$$

Then, we have

$$F = \frac{df}{dX_{AFE}} = \begin{bmatrix} \beta S & 0 \\ \delta & 0 \end{bmatrix} \text{ and } V = \frac{dv}{dX_{AFE}} = \begin{bmatrix} \alpha + \delta + m_a + \gamma & 0 \\ 0 & m_u \end{bmatrix}$$

Hence the NGM is:

$$FV^{-1} = \begin{bmatrix} \frac{\beta(1-p)\Lambda}{m_s(\alpha + \delta + m_a + \gamma)} & 0 \\ \frac{\delta}{\alpha + \delta + m_a + \gamma} & 0 \end{bmatrix}$$

To get the basic reproductive number R_0, we select the maximum eigenvalue of the matrix shown in the equation $\frac{\beta(1-p)\Lambda}{m_s(\alpha+\delta+m_a+\gamma)}$. The second part of the theorem is a direct implication of theorem 2 of the work by van den Driessche and Watmough, (2002).

Existence of Endemic Equilibrium

The Endemic Equilibrium (EE) is a steady state where the anxiety cannot disappear, which means the anxiety persist in the population. In this condition, all model compartments are different from zero at the endemic equilibrium point, i.e.,

$$EE = (S^*, A^*, T^*, U^*) \neq (0, 0, 0, 0) \tag{4.3}$$

Theorem 4.4. A unique Endemic Equilibrium (EE) exists at the point

$$EE = (S^*, A^*, T^*, U^*) = \left(\frac{(1-p)\,\Lambda - \alpha a}{m_s + \sigma + \beta a}, a , \frac{p\Lambda + \gamma a}{m_t}, \frac{\delta a}{m_u} \right)$$

where “a” is the positive root of

$$\beta(\delta + m_a + \gamma)A^2 + ((\sigma(\delta + m_a + \gamma) + m_s(\alpha + \delta + m_a + \gamma) - \beta(1 - p)\,\Lambda))\,A - \sigma(1 - p)\,\Lambda = 0$$

Proof 4.4. If $A \neq 0$, we set the right side of the equations (4.1) to find an endemic equilibrium point. By adding the first two ordinary differential equations in the system (4.1) we have

$$S^* = \frac{(1-p)\Lambda - (\delta + m_a + \gamma)A}{m_s}$$

Substituting S* into the first ordinary differential equation in the system (4.1), we obtain the quadratic equation for A:

$$\beta(\delta + m_a + \gamma)A^2 + ((\sigma(\delta + m_a + \gamma) + m_s(\alpha + \delta + m_a + \gamma) - \beta(1 - p)\,\Lambda))\,A - \sigma(1 - p)\,\Lambda = 0.$$

It is important to note that the roots are real: $\Delta > 0$. Therefore, we let ‘a’ represent the positive root of the quadratic equation. Plugging it back into the set of equations, we have an endemic equilibrium point satisfied at:

$$EE = (S^*, A^*, T^*, U^*) = \left(\frac{(1-p)\,\Lambda - \alpha a}{m_s + \sigma + \beta a},\ a\ ,\ \frac{p\Lambda + \gamma a}{m_t},\ \frac{\delta a}{m_u} \right) \tag{4.4}$$

4.4.2 Stability of The Equilibria

We proposed a mathematical model of real-world phenomena using nonlinear differential equation systems. Because these equations are difficult to solve, we employ linearization, a qualitative algebraic approach, to the analysis of nonlinear dynamic system solutions (4.1). In this section, we utilize linearization to solve the nonlinear equation mathematically and then we employ the stability analysis to observe the asymptotic behavior of the solution for nonlinear dynamic system equilibria (anxiety-free and endemic equilibria).

Stability of Anxiety-Free Equilibrium

Theorem 4.5. The Anxiety-Free Equilibrium (AFE) of system (4.1) is locally asymptoticly stable with monotonically converging solutions, if $\beta \frac{(1-p)\Lambda}{m_s} < (\alpha + \delta + m_a + \gamma)$.

Proof 4.5. To determine the stability of the system at AFE, we first calculate the Jacobian matrix to determine the stability of the anxiety-free equilibrium. The Jacobian matrix is evaluated at the anxiety-free equilibrium to get the eigenvalues of the linearized system, which is given by:

$$J_{AFE} = \begin{bmatrix} -m_s & \alpha - \beta(\frac{(1-p)\Lambda}{m_s}) & 0 & 0 \\ 0 & \beta(\frac{(1-p)\Lambda}{m_s}) - (\alpha + \delta + m_a + \gamma) & 0 & 0 \\ 0 & 0 & -m_t & 0 \\ 0 & 0 & 0 & -m_u \end{bmatrix} \tag{4.5}$$

From (4.5), we see that the eigenvalues are $\lambda_1 = -m_t$, $\lambda_2 = -m_u$, $\lambda_3 = -m_s$, and

$\lambda_4 = \beta(\frac{(1-p)\Lambda}{m_s}) - (\alpha + \delta + m_a + \gamma)$. Then, we find all eigenvalues are real and negative

provided $\beta\frac{(1-p)\Lambda}{m_s} < (\alpha + \delta + m_a + \gamma)$. Hence, the anxiety-free equilibrium of the system

(4.1) is locally asymptoticly stable with monotonically converging solutions, if

$\beta\frac{(1-p)\Lambda}{m_s} < (\alpha + \delta + m_a + \gamma)$.

Stability of Endemic Equilibrium

Theorem 4.6. If the Endemic Equilibrium (EE) of system (4.1) exists, then its stability is as follows:

If $\Delta = b^2 - 4c < 0$	If $\Delta = b^2 - 4c > 0$	If $\Delta = b^2 - 4c = 0$
• $b < 0$ Unstable oscillatory	• $b > 0, c > 0$ Stable	• $b < 0$ Degenerate unstable
• $b > 0$ Stable oscillatory	• $b < 0, c > 0$ Unstable	• $b > 0$ Degenerate unstable
• $b = 0$ inconclusive	• $c < 0$ Unstable	

where $b = m_s + \sigma + \beta a + \gamma + \delta + m_a + \alpha - \beta(\frac{(1-p)\Lambda - \alpha a}{m_s + \sigma + \beta a})$,

and $c = (m_s + \sigma + \beta a)(\alpha + \delta + m_a + \gamma - \beta\frac{(1-p)\Lambda - \alpha a}{m_s + \sigma + \beta a}) - (\beta a + \sigma)(\alpha - \beta\frac{(1-p)\Lambda - \alpha a}{m_s + \sigma + \beta a})$,

and “a” is the positive root of the quadratic equation:

$\beta(\delta + m_a + \gamma)A^2 + ((\sigma(\delta + m_a + \gamma) + m_s(\alpha + \delta + m_a + \gamma) - \beta(1-p)\Lambda))A - \sigma(1-p)\Lambda = 0$

Proof 4.6. To determine the stability of the system at EE, we first calculate the Jacobian matrix to determine the stability of the endemic equilibrium. The Jacobian matrix is evaluated at the endemic equilibrium to get the eigenvalues of the linearized system, which is given by:

$$J_{EE} = \begin{bmatrix} -m_s - (\sigma + \beta a) & \alpha - \beta \frac{(1-p)\Lambda - \alpha a}{m_s + \sigma + \beta a} & 0 & 0 \\ \sigma + \beta a & \beta \frac{(1-p)\Lambda - \alpha a}{m_s + \sigma + \beta a} - (\alpha + \delta + m_a + \gamma) & 0 & 0 \\ 0 & \gamma & -m_t & 0 \\ 0 & \delta & 0 & -m_u \end{bmatrix} \quad (4.6)$$

From (4.6), we see that the eigenvalues are $\lambda_1 = -m_t$, $\lambda_2 = -m_u$, and the other eigenvalues (λ_3, λ_4) are the roots of the quadratic equation:

$$\lambda^2 + \left(m_s + \sigma + \beta a + \gamma + \delta + m_a + \alpha - \beta \frac{(1-p)\Lambda - \alpha a}{m_s + \sigma + \beta a}\right)\lambda + (m_s + \sigma + \beta a)\left(\alpha + \delta + m_a + \gamma - \beta \frac{(1-p)\Lambda - \alpha a}{m_s + \sigma + \beta a}\right) - (\beta a + \sigma)\left(\alpha - \beta \frac{(1-p)\Lambda - \alpha a}{m_s + \sigma + \beta a}\right) = 0.$$

Hence, the stability of the Endemic Equilibrium depends on the roots of this quadratic equation using the linear stability theorem.

4.5 Parameter Estimations

Data collection has started by surveying students taking the gateway entry courses (Math-220 and 250) at UMKC. The survey was made available on Canvas and students completed it every Friday during the summer semester (see Appendix A). Based on the collected data, we estimate the number of students suffering from math anxiety and the number of students that drop because of it. We also use data from UMKC's gateway entry courses (Math 110, 120, 125, 210, 220, and 250) to estimate pre-covid parameters from two-year sequences. For example, we assume that the majority of pre-calculus (Math 120) students enroll in calculus I. We also assume that the students take the above Math series in the fall or spring semesters. Summer semesters were excluded due to low enrollment. The model's parameter values are estimated before and during COVID-19.

The SATU model includes a flow chart and four ODEs (see figure 7). To begin, we will compute m (m_s, m_u, m_a, m_t) values that represent dropping rates using ordinary differential equations of the system (4.1). By adding equations of the system (4.1), we get

$$(S+A+T+U)' = \Lambda - m_s S - m_a A - m_t T - m_u U \tag{4.7}$$

Let m = min (m_s, m_u, m_a, m_t)= m_t because we assume the drop rate of treated students is minimal.

The net flow is defined as N= S+A+T+U. Then (4.7) indicates that $N' \leq \Lambda - mN$ is true. Hence, the system is invariant (see theorem 1), and the total student population is bounded by Λ/m. We also assume that $m_t < m_s < m_a < m_u$ and $N' = \Lambda - \mu N$ with μ representing the overall drop rate. By using an integrating factor to solve for N, we get

$$N(t)=\Lambda e^{-\mu t}\left(\frac{1}{\mu}e^{\mu t} + c\right) \tag{4.8}$$

If t = 0, the number of students at the beginning of the semester is N (0) = N_0. And $N(t)=\Lambda e^{-\mu t}\left(\frac{1}{\mu}e^{\mu t} + \frac{N_0}{\Lambda} - \frac{1}{\mu}\right)$ is obtained using equation (4.8).

By doing so, we can estimate the parameters and use MATLAB's Curve Fitting Toolbox. The main idea is to use the Curve Fitting Toolbox, which contains a function for fitting curves to data. To do regression analysis, we developed our own customized equations. We evaluate confidence intervals after establishing a fit to achieve the best estimates for these parameters (see Tables 11, 12, 13, and 14). We evaluate confidence intervals after establishing a fit to achieve the best estimates for these parameters (see Tables 11, 12, 13, and 14). We also used the MATLAB curve fitting toolbox to measure the goodness of fit (adjusted squared R and sum of the squared residuals) (see Tables 11, 12, 13, and 14). In the linear regression model fitting, the coefficient of determination (R-squared) represents the proportionate amount of variance in the response variable N (total population) explained by the independent variables t (weeks).

Table 11. Estimated parameter values of model (4.1), confidence intervals, and the goodness of fit.

	μ	mu	ms	ma	mt	Λ	Confidence Interval (95%)	Goodness of Fit
SP 2014 M110	0.07757557962948	0.15515115925896	0.07757557962948	0.11636336944422	0.03878778981474	11.375	(0.077575579629480, 0.077575579629481)	(SSE:7.27e-27) (RMSE:8.53e-14)
SP 2014 M125	0.07523196529741	0.15046393059482	0.07523196529741	0.11284794794612	0.03761598264871	2	(0.075231965297412, 0.075231965297413)	(SSE:4.56e-27) (RMSE:6.75e-14)
SP 2014 M120	0.08597917878462	0.17195835756924	0.08597917878462	0.12896876817693	0.04298958939231	1.625	(0.085979178784616, 0.085979178784622)	(SSE:4.09e-27) (RMSE:6.40e-14)
FS 2014 M210	0.19611264171930	0.39222528343859	0.19611264171930	0.29416896257895	0.09805632085965	9.25	(0.196112641719297, 0.196112641719298)	(SSE:2.91e-26) (RMSE:1.71e-13)
SP 2015 M220	0.07523196529741	0.15046393059482	0.07523196529741	0.11284794794612	0.03761598264871	4.75	(0.075231965297412, 0.075231965297412)	(SSE:8.08e-28) (RMSE:2.84e-14)
FS 2015 M250	0.07619621125398	0.15239242250796	0.07619621125398	0.11429431688097	0.03809810562699	2.1875	(0.076196211253918, 0.076196211254040)	(SSE:3.11e-24) (RMSE:1.76e-12)

SP: Spring Semester; FS: Fall Semester; RMSE: Root Mean Square Error; SSE: Sum of Squares Error

Table 12. Estimated parameter values of model (4.1), confidence intervals, and the goodness of fit.

	μ	mu	ms	ma	mt	Λ	Confidence Interval (95%)	Goodness of Fit
FS 2014 M110	0.075317878373703	0.15063575674741	0.07531787837370	0.11297681756055	0.03765893918685	11	(0.075317878373703, 0.075317878373703)	(SSE:8.08e-28) (RMSE:2.84e-14)
FS 2014 M125	0.071930929256964	0.14386185851393	0.07193092925696	0.10789639388545	0.03596546462848	3.75	(0.071930929256960, 0.071930929256968)	(SSE:3.41e-26) (RMSE:1.85e-13)
FS 2014 M120	0.078525816132281	0.15705163226456	0.07852581613228	0.11778872419842	0.03926290806614	2.4375	(0.078525815980442, 0.078525816284120)	(SSE:2.40e-17) (RMSE:4.90e-09)
SP 2015 M210	0.072757744948079	0.14551548989616	0.07275774494808	0.10913661742212	0.03637887247404	8.25	(0.072757744948075, 0.072757744948084)	(SSE:2.34e-25) (RMSE:4.83e-13)
FS 2015 M220	0.068420083589926	0.13684016717985	0.06842008358993	0.10263012538489	0.03421004179496	5.8125	(0.068420083589924, 0. 0.068420083589929)	(SSE:2.91e-26) (RMSE:1.71e-13)
SP 2016 M250	0.071530403895309	0.14306080779062	0.07153040389531	0.10729560584296	0.03576520194765	2.3125	(0.071530403891322, 0.071530403899297)	(SSE:1.44e-20) (RMSE:1.20e-10)

SP: Spring Semester; FS: Fall Semester; RMSE: Root Mean Square Error; SSE: Sum of Squares Error

Table 13. Estimated parameter values of model (4.1), confidence intervals, and the goodness of fit.

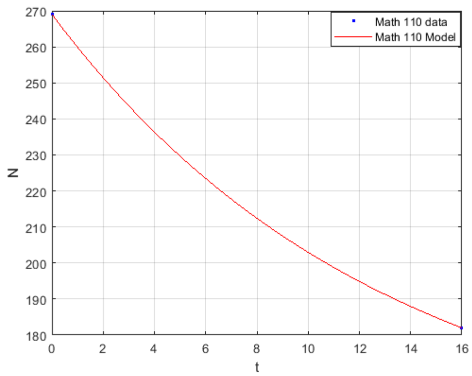
	μ	mu	ms	ma	mt	Λ	Confidence Interval (95%)	Goodness of Fit
SP 2015 M110	0.077827245050336	0.15565449010067	0.07782724505034	0.11674086757550	0.03891362252517	9.75	(0.077827245050329, 0.077827245050343)	(SSE:7.27e-25) (RMSE:8.53e-13)
SP 2015 M125	0.071034747832322	0.14206949566464	0.07103474783232	0.10655212174848	0.03551737391616	3.6875	(0.071034747830386, 0.071034747834259)	(SSE:8.63e-21) (RMSE:9.30e-11)
SP 2015 M120	0.083578741492542	0.16715748298508	0.08357874149254	0.12536811223881	0.04178937074627	1.5	(0.083578741492531, 0.083578741492553)	(SSE:4.70e-26) (RMSE:2.17e-13)
FS 2016 M210	0.069699414225640	0.13939882845128	0.06969941422564	0.10454912133846	0.03484970711282	10.625	(0.069699414225587, 0.069699414225694)	(SSE:5.46e-23) (RMSE:7.40e-12)
SP 2016 M220	0.073311768296190	0.14662353659238	0.07331176829619	0.10996765244429	0.03665588414810	4.9375	(0.073311768296189, 0.073311768296191)	(SSE:3.43e-27 (RMSE:5.86e-14)
FS 2016 M250	0.076371388263103	0.15274277652621	0.07637138826310	0.11455708239465	0.03818569413155	1.4375	(0.076371388263102, 0.076371388263104)	(SSE:4.54e-28) (RMSE:2.13e-14)

SP: Spring Semester; FS: Fall Semester; RMSE: Root Mean Square Error; SSE: Sum of Squares Error

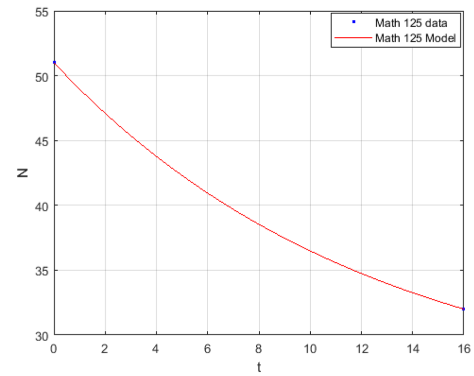
Table 14. Estimated parameter values of model (4.1), confidence intervals, and the goodness of fit.

	μ	mu	ms	ma	mt	Λ	Confidence Interval (95%)	Goodness of Fit
SP 2018 M110	0.071272350424948	0.14254470084990	0.07127235042495	0.10690852563742	0.03563617521247	7.875	(0.071272350387320, 0.071272350462577)	(SSE:1.49e-17) (RMSE:3.86e-09)
SP 2018 M125	0.071675762817314	0.14335152563463	0.07167576281731	0.10751364422597	0.03583788140866	2.5	(0.071675762817312, 0.071675762817316)	(SSE:4.14e-27) (RMSE:6.43e-14)
SP 2018 M120	0.077630037418966	0.15526007483793	0.07763003741897	0.11644505612845	0.03881501870948	1.5625	(0.077630037418965, 0.077630037418968)	(SSE:8.07e-28) (RMSE:2.841e-14)
FS 2018 M210	0.078554730822062	0.15710946164412	0.07855473082206	0.11783209623309	0.03927736541103	9	(0.078554730822059, 0.078554730822064)	(SSE:6.54e-26) (RMSE:2.56e-13)
FS 2019 M250	0.064288580663932	0.12857716132786	0.06428858066393	0.09643287099590	0.03214429033197	2.5	(0.064288580663909, 0.064288580663954)	(SSE:5.36e-25) (RMSE:7.32e-13)

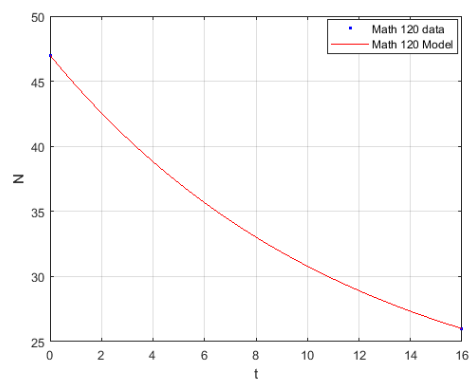
SP: Spring Semester; FS: Fall Semester; RMSE: Root Mean Square Error; SSE: Sum of Squares Error



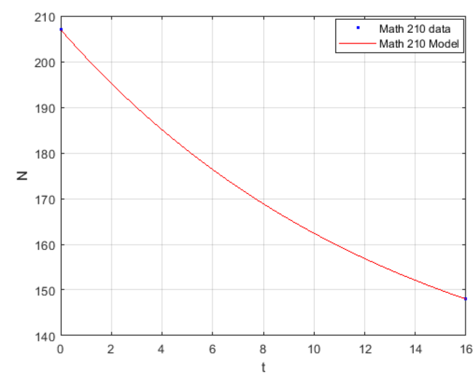
(a)



(b)



(c)



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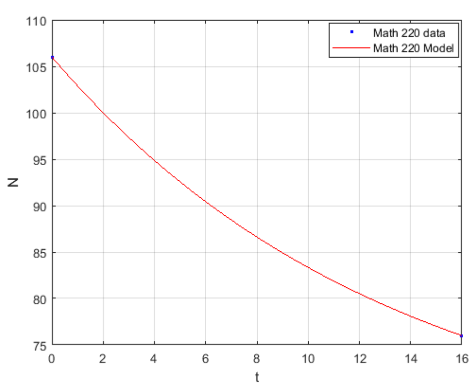
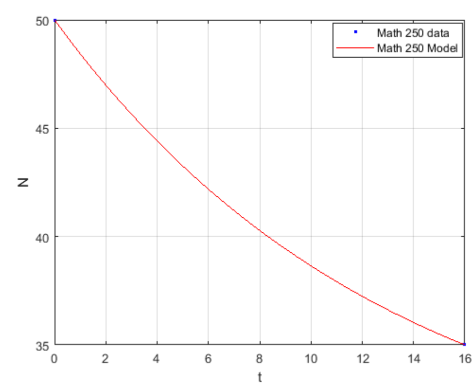
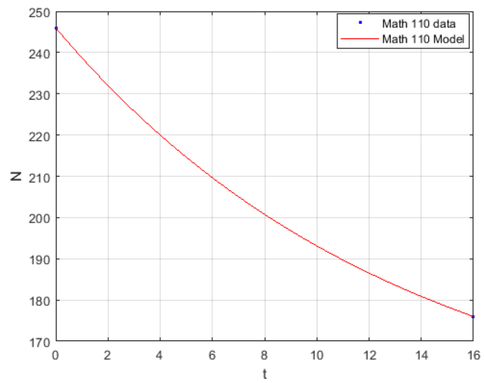


(e)

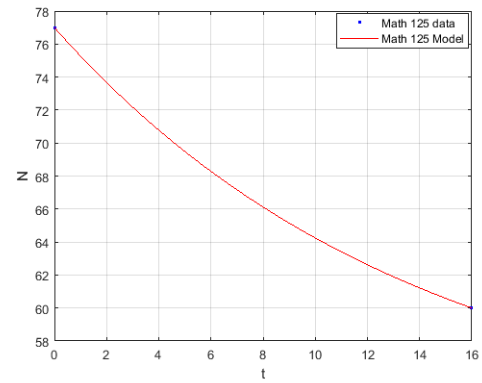


(f)

Figure 8. The model (4.1) fitted to DFW data a) Spring semester 2014 math 110, b) Spring semester 2014 math 125, c) Spring semester 2014 math 120, d) Fall semester 2014 math 210 e) Spring semester 2015 math 220 f) Fall semester 2015 math 250.



(a)



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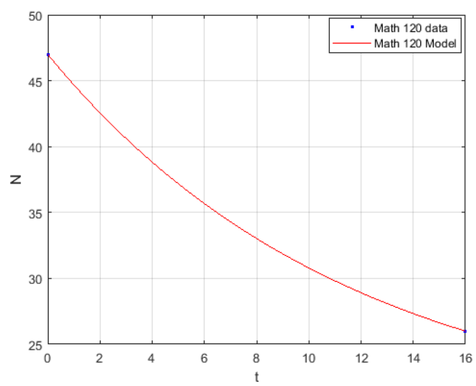

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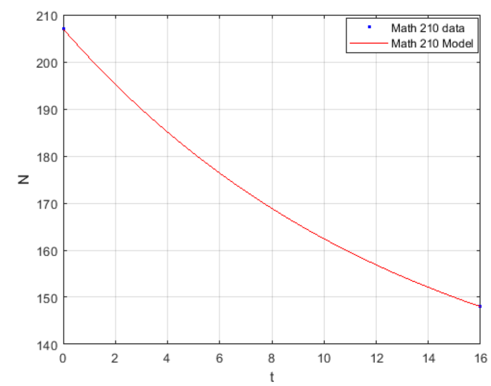

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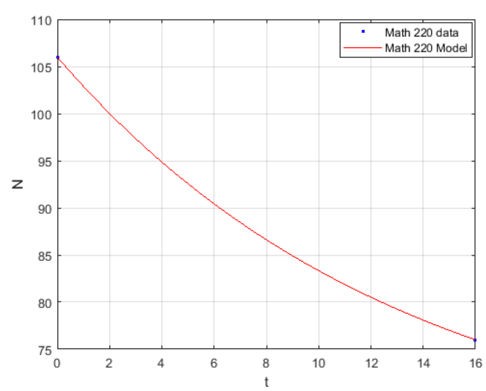

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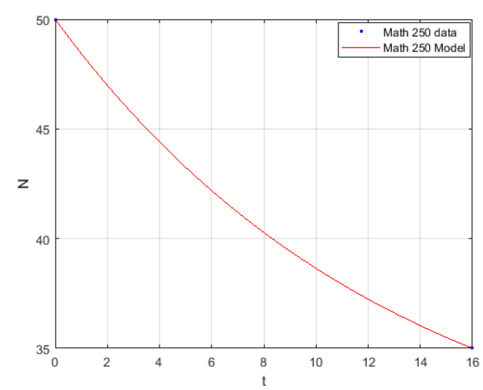


(f)

Figure 9. The model (4.1) fitted to DFW data a) Fall semester 2014 math 110, b) Fall semester 2014 math 125, c) Fall semester 2014 math 120, d) Spring semester 2015 math 210 e) Fall semester 2015 math 220 f) Spring semester 2016 math 250.

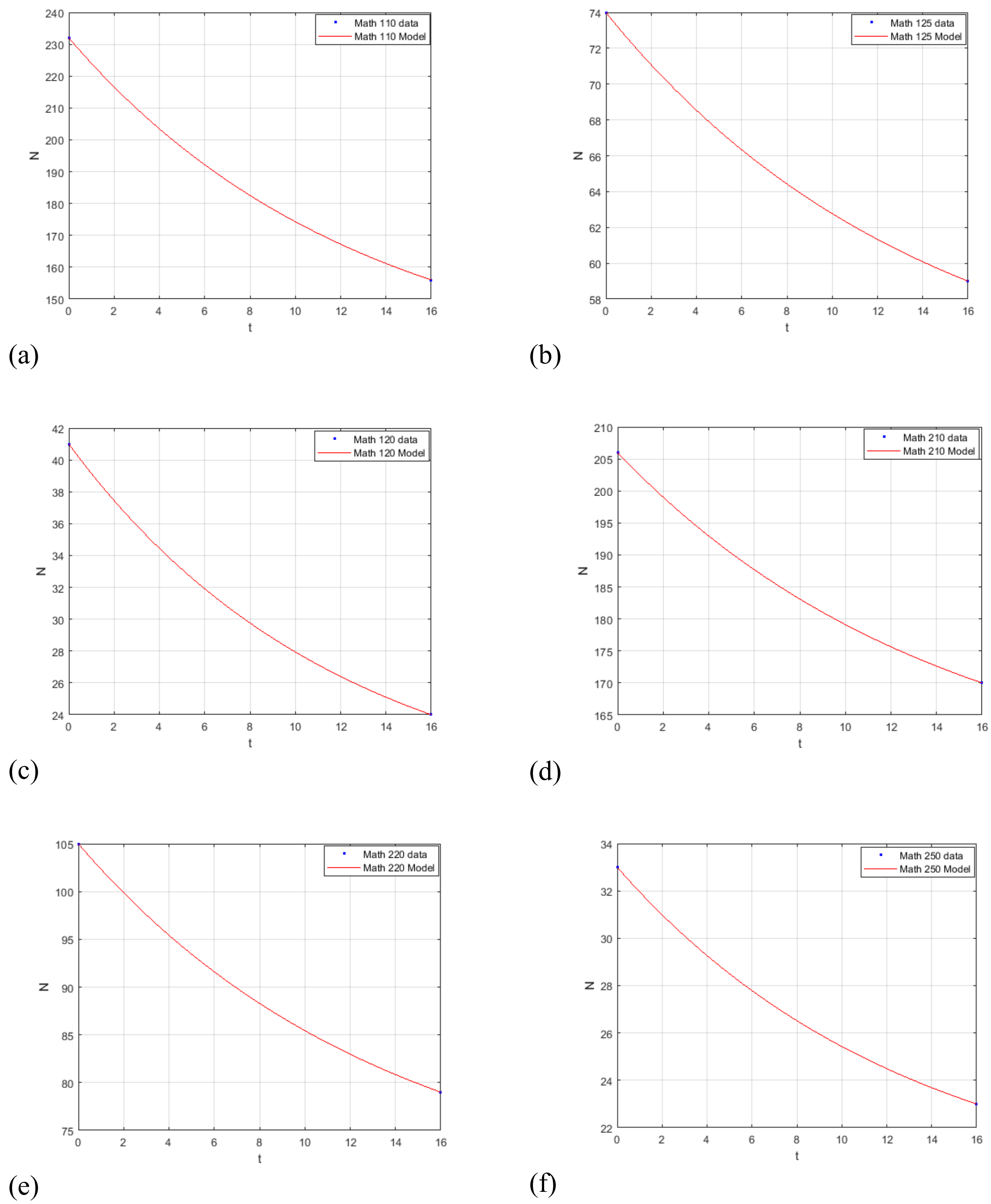
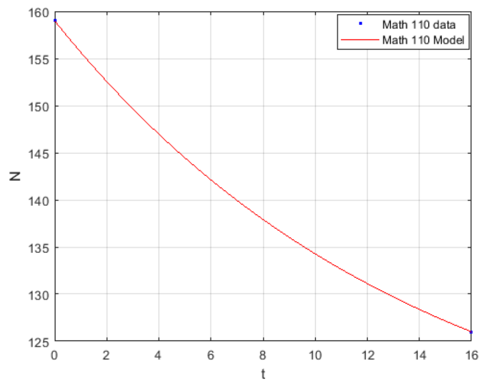
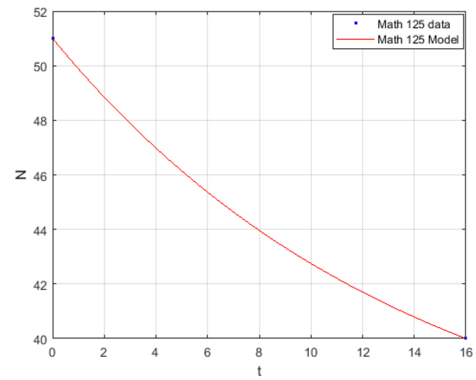


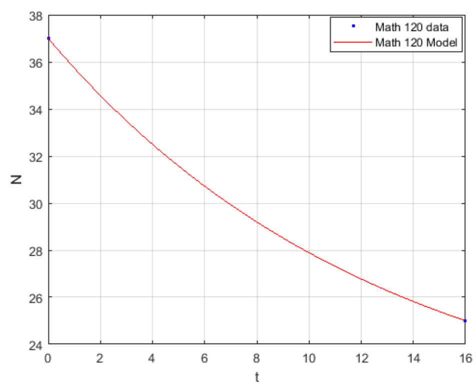
Figure 10. The model (4.1) fitted to DFW data a) Spring semester 2015 math 110, b) Spring semester 2015 math 125, c) Spring semester 2015 math 120, d) Fall semester 2016 math 210 e) Spring semester 2016 math 220 f) Fall semester 2016 math 250.



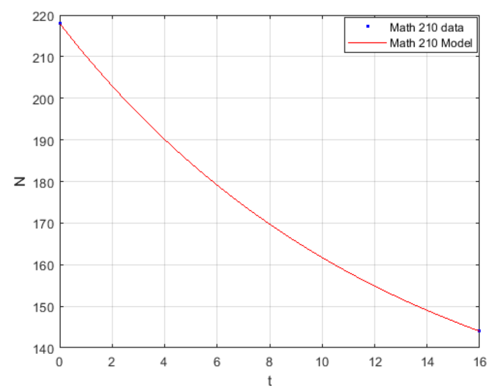
(a)



(b)



(c)



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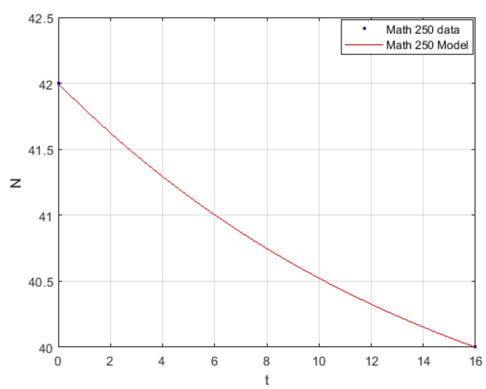


(e)

Figure 11. The model (4.1) fitted to DFW data a) Spring semester 2018 math 110, b) Spring semester 2018 math 125, c) Spring semester 2018 math 120, d) Fall semester 2018 math 210 e) Spring semester 2019 math 220 f) Fall semester 2019 math 250.

4.6 Limitation of the model: Nonexistence of Limit Cycles

Although we have estimated the parameter values of model (4.1), in this section we show that model is not capable of generating any limit cycles. Therefore, no matter how hard we try to fit the model to the collect data, the model will have a poor predictive ability and low goodness of fit.

In this section, we show that the model (4.1) cannot have a limit cycle. To have a limit cycle, the model should have a pair of pure imaginary eigenvalues. However, in the following we show that the Jacobian matrix evaluated at the endemic equilibrium cannot have a pair of imaginary eigenvalues and therefore the existence of a limit cycle is not possible.

The reason for this is that we found that the sum of $a_{11} + a_{22}$ in the Jacobian matrix at endemic equilibrium will never become zero. Therefore, the quadratic characteristic equation: $\lambda^2 - \mathrm{Tr}\,(J_{EE})\,\lambda + \det\,(J_{EE}) = 0$ will never give eigenvalues with zero real parts (i.e., pure imaginary parts). Hence, a necessary condition for the existence of the Hopf bifurcation is violated, and the model cannot generate periodic solutions. This is a strong limitation of the model because the data clearly shows existence of periodic and oscillatory solutions. Hence, the model cannot resemble the real-world data, and we in the next chapter we will modify the model to address this issue.

Theorem 4.7. (Nonexistence of periodic solutions) The model (4.1) does not have any limit cycle and therefore no periodic solutions.

Proof 4.7. Here, we show that the model cannot have a pair of pure imaginary eigenvalues. We already showed that the endemic equilibrium is given by:

$$EE = (S^*, A^*, T^*, U^*) = \left(\frac{(1-p)\,\Lambda - \alpha a}{m_s + \sigma + \beta a},\ a\ ,\ \frac{p\Lambda + \gamma a}{m_t},\ \frac{\delta a}{m_u}\right).$$

We also calculated the Jacobian matrix evaluated at EE:

$$J_{EE} = \begin{bmatrix} -m_s - (\sigma + \beta a) & \alpha - \beta\frac{(1-p)\,\Lambda - \alpha a}{m_s + \sigma + \beta a} & 0 & 0 \\ \sigma + \beta a & \beta\frac{(1-p)\,\Lambda - \alpha a}{m_s + \sigma + \beta a} - (\alpha + \delta + m_a + \gamma) & 0 & 0 \\ 0 & \gamma & -m_t & 0 \\ 0 & \delta & 0 & -m_u \end{bmatrix}$$

where, two eigenvalues are always negative: $\lambda_1 = -m_t$ and $\lambda_2 = -m_u$. Hence, the stability of the Endemic Equilibrium depends upon the roots of this quadratic equation:

$$\lambda^2 + \left(m_s + \sigma + \beta a + \gamma + \delta + m_a + \alpha - \beta\frac{(1-p)\,\Lambda - \alpha a}{m_s + \sigma + \beta a}\right)\lambda + (m_s + \sigma + \beta a)\left(\alpha + \delta + m_a + \gamma - \beta\frac{(1-p)\,\Lambda - \alpha a}{m_s + \sigma + \beta a}\right) - (\beta a + \sigma)\left(\alpha - \beta\frac{(1-p)\,\Lambda - \alpha a}{m_s + \sigma + \beta a}\right) = 0.$$

To have a pair of pure imaginary eigenvalues, we must have the following two conditions:

1. $\left(m_s + \sigma + \beta a + \gamma + \delta + m_a + \alpha - \beta\frac{(1-p)\,\Lambda - \alpha a}{m_s + \sigma + \beta a}\right) = 0$
2. $(m_s + \sigma + \beta a)\left(\alpha + \delta + m_a + \gamma - \beta\frac{(1-p)\,\Lambda - \alpha a}{m_s + \sigma + \beta a}\right) - (\beta a + \sigma)\left(\alpha - \beta\frac{(1-p)\,\Lambda - \alpha a}{m_s + \sigma + \beta a}\right) > 0.$

where “a” is the positive root of

$\beta(\delta + m_a + \gamma)A^2 + ((\sigma(\delta + m_a + \gamma) + m_s(\alpha + \delta + m_a + \gamma) - \beta(1 - p)\Lambda))A - \sigma(1 - p)\Lambda = 0.$

We show that the first condition can never be satisfied because the first expression is always negative. By doing algebra in the first condition we have,

$$C_1 + C_2 - \frac{\beta(1-p)\Lambda - \alpha a}{C_1} < 0 \text{ where} \tag{4.9}$$

$C_1 = m_s + \sigma + \beta a > 0,$

$C_2 = \gamma + \delta + m_a + \alpha > 0$ since all parameters and “a” are nonnegative. Divided by “ C_2”

both side of the inequality (4.9) we have $\frac{C_1}{C_2} + 1 < \frac{\beta(1-p)\Lambda - \alpha a}{C_1 C_2}$.

Assume $(\beta(1 - p)\Lambda - \alpha a) \neq 0$, then we will have

$$\frac{C_1}{C_2} + 1 < \frac{1}{1 + \frac{C_1C_2 - (\beta(1-p)\Lambda - \alpha a)}{(\beta(1-p)\Lambda - \alpha a)}}$$

- If $(\beta(1 - p)\Lambda - \alpha a) > 0$, then we have

$(\frac{C_1}{C_2} + 1)(1 + e) < 1$, where $e = (\frac{C_1C_2 - (\beta(1-p)\Lambda - \alpha a)}{(\beta(1-p)\Lambda - \alpha a)})$. Hence,

$(\frac{C_1}{C_2} + e\frac{C_1}{C_2} + e) < 0.$

Similarly,

- If $(\beta(1 - p)\Lambda - \alpha a) < 0$, then we have

$$\frac{C_1}{C_2} + 1 < \frac{1}{1 - \frac{C_1C_2 + (\beta(1-p)\Lambda - \alpha a)}{(\beta(1-p)\Lambda - \alpha a)}}, \text{ and}$$

$(\frac{C_1}{C_2} + 1)(1 - d) < 1$, where $d = \frac{C_1C_2 + (\beta(1-p)\Lambda - \alpha a)}{(\beta(1-p)\Lambda - \alpha a)}$. Hence,

$(\frac{C_1}{C_2} + d\frac{C_1}{C_2} - d) < 0.$

Now, we have

$\{(\delta + m_a + \gamma)(m_s + \sigma + \beta a) + (\beta(1 - p)\Lambda - \alpha a) + m_s\alpha\} - \{(\beta(1 - p)\Lambda - \alpha a) +$

$\alpha(m_s + \sigma + \beta a) + (\beta\frac{(1-p)\Lambda - \alpha a}{m_s + \sigma + \beta a})m_s\} \leq 0$ for the second condition for the Theorem 4.7

if $\{(\delta + m_a + \gamma)(m_s + \sigma + \beta a) + (\beta(1 - p)\Lambda - \alpha a) + m_s\alpha\} \leq \{(\beta(1 - p)\Lambda - \alpha a) +$

$\alpha(m_s + \sigma + \beta a) + (\beta\frac{(1-p)\Lambda - \alpha a}{m_s + \sigma + \beta a})m_s\}.$

The proof is completed for all nonnegative parameters and "a" is the positive root of

$\beta(\delta + m_a + \gamma)A^2 + ((\sigma(\delta + m_a + \gamma) + m_s(\alpha + \delta + m_a + \gamma) - \beta(1 - p)\Lambda))A - \sigma(1 - p)\Lambda = 0.$

4.7 Numerical Simulations of The Model

The goals at this point are to confirm the analytical results of chapter 4. We utilize computer simulations via MATLAB ODE45 to estimate the various parameters it contains. Also, we conduct a stability analysis for the parameters and develop intervention strategies to effectively reduce the anxiety among students. This work on the simulations of the dynamic system is developed with the help of the function ODE45 and the parameter settings shown in Table 10 with the initial conditions (S, A, T, U).

Table 15. Estimated parameter values of the system (4.1).

Parameters	Case 1	Case 2	Case 3
Λ	6	6	5
β	0.03	0.015	0.001
σ	0.005	0.05	0.001
α	0.08	0.08	0.08
γ	0.1	0.1	0.04
δ	0.01	0.01	0.02
m_s	0.01	0.01	0.1
m_t	0.01	0.01	0.03
m_u	0.06	0.06	0.05
m_a	0.9	0.9	0.7
R_0	15.0275	7.5138	0.0565
p	0.09	0.09	0.05
S	40	4	4
A	2	40	50
T	3	5	5
U	1	1	1
t	50	80	50

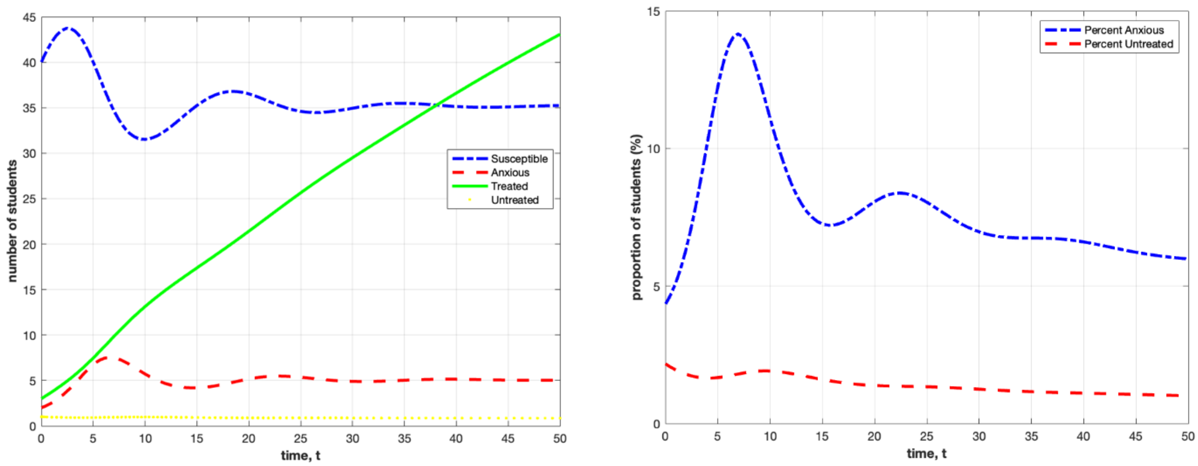


Figure 12. The solution of the system (4.1) with the initial conditions for case 1 (see Table 15).

If we consider the set of the parameters for case 1 (see Table 15), we can see that the system (4.1) has a unique endemic equilibrium:

EE = (S*, A*, T*, U*)≈ (35.1742, 5.0577, 104.5768, 0.8429)

The solutions are initially oscillating; they will eventually reach constant numbers of anxious and susceptible students, which is not good because our data is showing otherwise. Put simply, the simulation explained in chapter 4 shows something different than the simulation from chapter 2. Indeed, the coefficient of the quadratic equation becomes b = 0.2015 > 0, c = 0.0013 > 0, and $\Delta = -0.085 < 0$ which does not satisfy the conditions of Theorem 4.6. After the initial oscillations, a portion of the students will remain anxious during the semester (oscillation for the case $R_0 > 1$).

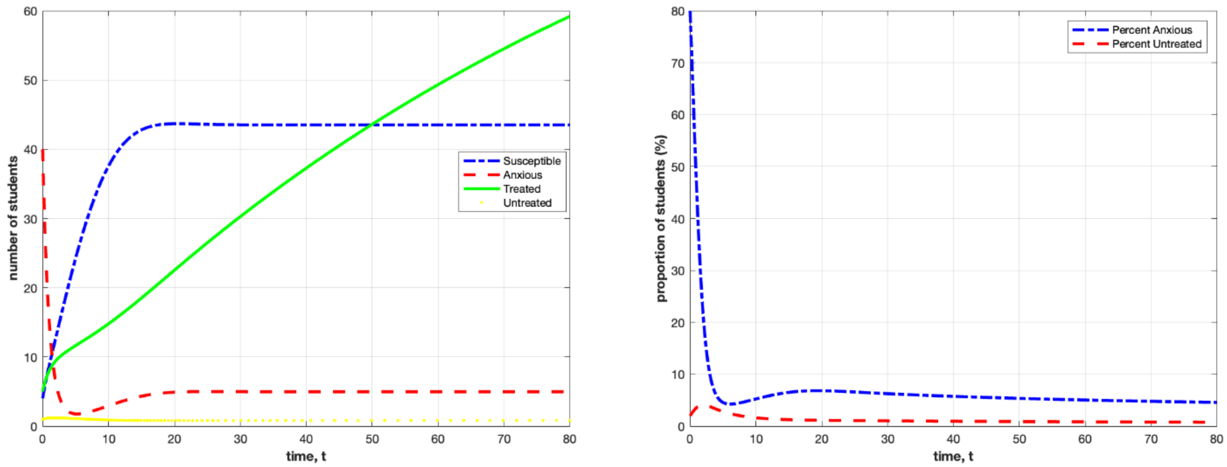


Figure 13. The solution of the system (4.1) with the initial conditions for case 2 (see Table 15).

If we consider the set of parameters for case 2 (see Table 15), we see that the system (4.1) has a unique endemic equilibrium

EE = (S*, A*, T*, U*)≈ (43.5130, 4.9751, 103.7512, 0.8292)

Figure 13 shows that the solutions are reaching to Endemic Equilibrium without oscillations which is not good. Indeed, the coefficient of the quadratic equation becomes b = 0.5719 > 0, c = 0.0685 > 0, and $\Delta = 0.0532 > 0$ which does not satisfy the conditions of Theorem 4.6. Without any oscillations, a portion of the students will remain anxious during the semester (Monotonicity for the case $R_0 > 1$).

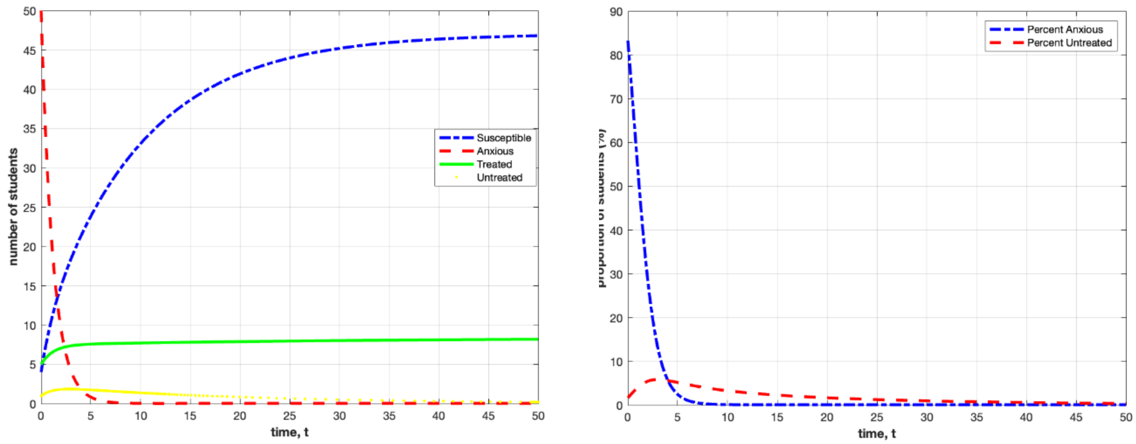


Figure 14. The solution of the system (4.1) with the initial conditions for case 3 (see Table 15).

If we consider the set of parameters for case 3 (see Table 15), we see that the system (4.1) has a unique Anxiety-Free Equilibrium

AFE = (S*, A*, T*, U*)≈ (47.0491, 0.0593, 8.4124, 0.0237)

Figure 14 shows that the red line approaches zero. Ideally, we want to reach an anxious-free equilibrium, but we know that this will not happen in a real-world situation. However, the coefficient of the quadratic equation becomes b = 0.8940 > 0, c = 0.0801 > 0, and $\Delta = 0.4787 > 0$; we also find all eigenvalues are real and negative, which does satisfy the conditions of Theorem 4.5. Hence, all students will eventually become anxious free when $R_0 < 1$.

CHAPTER 5

ADVANCED MATHEMATICAL MODELLING AND ANALYSIS

5.1 Overview

In this chapter, we modify the first mathematical model (4.1) to generate limit cycles which can be used for both epidemiologic modeling and in-host disease modeling. The endemic equilibrium will be stable when there are no limit cycles. However, the mathematical result in chapter 4 shows that we should have limit cycles. We already discussed and proved the nonexistence of limit cycles of the system (4.1) in Chapter 4. Chapter 5 provides analytical proofs and numerical simulations of the existing periodic solutions. We use the theorem of Hopf bifurcation to have limit cycles for the new system. In addition, we analyze the new model mathematically by studying the local stability of the anxious-free equilibrium and endemic equilibrium. Also, we derive the basic reproduction number (R_0) using the method of the Next Generation Matrix (NGM). Moreover, we do numerical simulation to verify the new model meets the conditions of Hopf bifurcation to understand the dynamics of math anxiety among students over time.

5.2 Second Mathematical Model

Although the first mathematical model exhibits robust dynamics including the existence of anxiety free and endemic steady states, it fails to capture periodic behaviors of math anxiety as observed in the data (see figure 4(b)). There were two semester exams in week 4 and 7, and the final exam was in week 9. We observed that there were two increases in anxiety corresponding to two exams (see figure 4(b)). In this section, we address this issue and modify the first mathematical model (4.1) using the existing literature (van Gaalen & Wahl, 2009; Yu et al., 2016; Yu et al., 2020) on in-host disease models (HIV). Epidemiological models and in-host disease models (HIV) share similarities in that they divide a population of individuals or cells, respectively, into discrete classes and describe their dynamics using a system of ordinary differential equations. Therefore, we consider the classic SIR (Susceptible-Infectious-Recovered) model (van Gaalen & Wahl, 2009; Yu et al., 2020) in place of the first model.

Note that T(t) and U(t) are not present in the first two equations in the first model (4.1). Therefore, T(t) and U(t) will never affect the dynamics of S(t) and A(t). However, A(t) can affect the values of T(t) and U(t). We generalize the first model (4.1) in the same way that has been proposed in van Gaalen & Wahl (2009) and Yu et al. (2020).

$$\begin{aligned} \dot{S} &= (1-p)\Lambda - AF(A,S) - m_s S \\ \dot{A} &= AF(A,S) - (\delta + m_a + \gamma)A \\ \dot{T} &= p\Lambda + \gamma A - m_t T \\ \dot{U} &= \delta A - m_u U \end{aligned} \tag{5.1}$$

Remark: If F (A, S) = $\beta S + \frac{\sigma S}{A} - \alpha$ then the general model (5.1) is reduced to the original model (4.1) in the previous section.

We assume that F (A, S) = S G(A), where G(A) = $\beta + \frac{kA^{p-1}}{c+A^{p-1}}$ for c > 0 and p > 1. Here, we used a modified Michaelis-Menten saturation curve (i.e., G(A) = $\beta + \frac{kA^{p-1}}{c+A^{p-1}}$ equation, which is a well-known equation in biochemistry) to model transmission of anxiety from an anxious student to susceptible students (Michaelis, L., & Menten (1913); Cherayil, 2013; van Gaalen & Wahl, 2009). This approach has also been used in other models such as the HIV model (van Gaalen & Wahl, 2009) and other studies (Yu et al., 2016; Yu et al., 2020)

Note that, F (A, 0) = 0 and ∂F/∂S > 0 does satisfy for all A.

We let p = 2 then we have

$\lim_{A\to 0} G(A) = \beta$ and $\dot{G}(A) > 0$ for p > 1, $\lim_{A\to\infty} G(A) = \beta + \frac{k}{c}$. The system can be rewritten as:

$$\dot{S} = (1 - p)\Lambda - AS(\beta + \frac{kA}{c+A}) - m_s S$$

$$\dot{A} = AS(\beta + \frac{kA}{c+A}) - (\delta + m_a + \gamma)A$$

$$\dot{T} = p\Lambda + \gamma A - m_t T \tag{5.2}$$

$$\dot{U} = \delta A - m_u U$$

The first two equations of model (5.2) are similar to the model proposed by Yu et al. (2016). Also, the flow chart of the above-mentioned model (5.2) is seen in Figure 15, and all parameters are positive, real values.

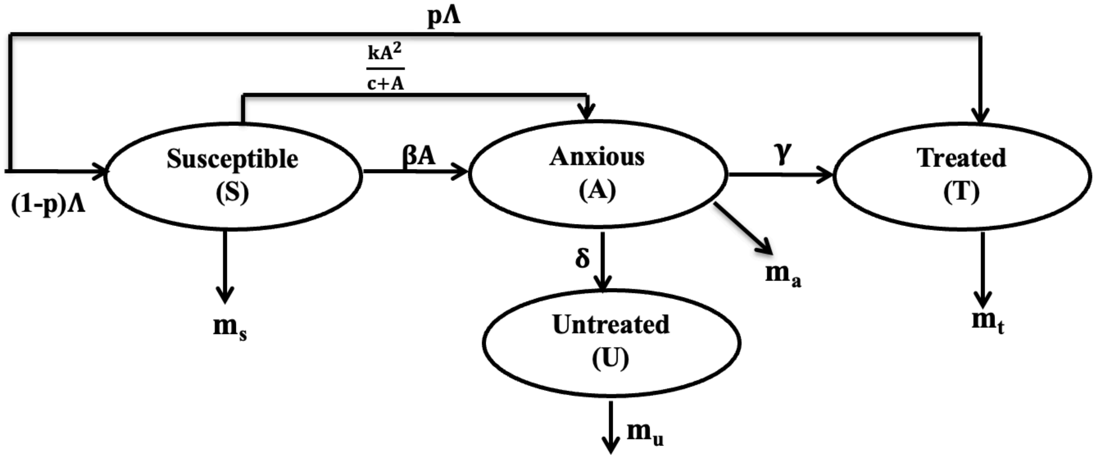


Figure 15. Flow chart describing the interaction between students in different states as math anxiety is transmitted for the second model.

5.3 Analysis of The Model

5.3.1 Existence of Equilibria

Existence of Anxiety-Free Equilibrium

Theorem 5.1. A unique Anxiety-Free Equilibrium (AFE) exists at the point (S*, 0, T*, 0) if all parameters are nonnegative.

Proof 5.1. We denote AFE with (S*, 0, T*, 0). To remove anxiety from the system, we set the right side of the ODEs (5.2) to zero, i.e.,

$$\dot{S} = (1-p)\Lambda - AS(\beta + \frac{kA}{c+A}) - m_s S = 0$$

$$\dot{A} = AS(\beta + \frac{kA}{c+A}) - (\delta + m_a + \gamma)A = 0$$

$$\dot{T} = p\Lambda + \gamma A - m_t T = 0$$

$$\dot{U} = \delta A - m_u U = 0$$

and $A = 0$. Thus, we have Anxiety-Free Equilibrium (AFE):

$$\text{AFE} = (S^*, A^*, T^*, U^*) = \left(\frac{(1-p)\Lambda}{m_s},\ 0,\ \frac{p\Lambda}{m_t},\ 0\right) \tag{5.3}$$

Basic Reproduction Number, R_0

Theorem 5.2. The basic reproduction number for the system of ODEs is:

$$R_0 = \frac{\beta(1-p)\Lambda}{m_s(\delta+m_a+\gamma)}$$

when $R_0 > 1$, the AFE is unstable, whereas $R_0 < 1$ implies that the AFE is locally asymptotically stable.

Proof 5.2. By applying the NGM to find the reproduction number, we use the following form: $\dot{f}$ = "entry" − "exit" (Gurin et al., 2017). Then, we define the vector of $X = \begin{bmatrix} A \\ U \end{bmatrix}$. Now, we introduce $F = \frac{df}{dX_{AFE}}$ which is the derivative of f and contains anxious term, in a similar way, we introduce $V = \frac{dv}{dX_{AFE}}$ which contains transfer terms.

Constructing the NGM and anxiety-free equilibrium is

$$X = \begin{bmatrix} A \\ U \end{bmatrix} = \begin{bmatrix} AS(\beta + \frac{kA}{c+A}) \\ \delta A \end{bmatrix} - \begin{bmatrix} (\delta + m_a + \gamma)A \\ m_u U \end{bmatrix}$$

Then we have

$$F = \frac{df}{dX_{AFE}} = \begin{bmatrix} S\beta + (\frac{2SkcA+SkA^2}{(c+A)^2}) & 0 \\ \delta & 0 \end{bmatrix} \text{ and } V = \frac{dv}{dX_{AFE}} = \begin{bmatrix} \delta + m_a + \gamma & 0 \\ 0 & m_u \end{bmatrix}$$

Hence the NGM is:

$$FV^{-1} = \begin{bmatrix} \frac{\beta(1-p)\Lambda}{m_s(\delta + m_a + \gamma)} & 0 \\ \frac{\delta}{\delta + m_a + \gamma} & 0 \end{bmatrix}$$

To get the basic reproductive number R_0, we select the maximum eigenvalue of the matrix shown in the equation is $\frac{\beta(1-p)\Lambda}{m_s(\delta+m_a+\gamma)}$ **.** The second part of the theorem is a direct implication of theorem 2 of the work by van den Driessche, P., & Watmough, J. (2002).

Existence of Endemic Equilibrium

Theorem 5.3. If $ML - m_sN > 0$, $p\Lambda NL + \gamma ML - \gamma m_sN > 0$, and $\delta ML - \delta m_sN > 0$ then the system (5.2) has a unique Endemic Equilibrium (EE)

$$EE = (S^*, A^*, T^*, U^*) = \left(\frac{N}{L}, \frac{ML-m_sN}{NL}, \frac{p\Lambda NL+\gamma ML-\gamma m_sN}{NL\, m_t}, \frac{\delta ML-\delta m_sN}{NL\, m_u} \right) \quad (5.4)$$

where $M = (1-p)\Lambda$, $N = (\delta + m_a + \gamma)$, and $L = \left(\beta + \frac{kA}{c+A}\right)$.

Proof 5.3. If $A \neq 0$, we set the right side of the equations (5.2) to zero to find an endemic equilibrium point. By adding the first two ordinary differential equations in the system (5.2) we have

$$S^* = \frac{(1-p)\Lambda-(\delta+m_a+\gamma)A}{m_s}$$

Substituting S* into the first ordinary differential equation in the system (5.2), we obtain the quadratic equation for A:

$$(NL)A^2 + (m_sN - ML)A = 0$$

Then we have $A_1 = 0$ and $A_2 = \frac{ML - m_s N}{NL}$. Plugging A_2 back into the set of equations, we have an endemic equilibrium point satisfied at:

$$EE = (S^*, A^*, T^*, U^*) = \left(\frac{N}{L}, \frac{ML - m_s N}{NL}, \frac{p\Lambda NL + \gamma ML - \gamma m_s N}{NL\, m_t}, \frac{\delta ML - \delta m_s N}{NL\, m_u} \right)$$

where $M = (1 - p)\Lambda$, $N = (\delta + m_a + \gamma)$, and $L = \left(\beta + \frac{kA}{c+A}\right)$

Now we need to show that $ML - m_s N > 0$. We can rewrite the inequality $ML - m_s N > 0$ as:

$(1 - p)\Lambda\left(\beta + \frac{kA}{c+A}\right) - m_s(\delta + m_a + \gamma) > 0$. Applying the distribution property then we have,

$(1 - p)\Lambda\, \beta + \frac{(1-p)\Lambda kA}{c+A} > m_s(\delta + m_a + \gamma)$. Dividing by "$\frac{(1-p)\Lambda kA}{c+A} > 0$" both side of the inequality then we have, $\frac{\beta}{kA} + 1 > \frac{D}{(1-p)\Lambda kA}$, where $D = m_s(\delta + m_a + \gamma)(c + A)$, $(1 - p)\Lambda kA \neq 0$, and $0 < p < 1$. Then

$\frac{\beta}{kA} + 1 > \frac{1}{1 + \frac{(1-p)\Lambda kA - D}{D}}$ and

$\left(\frac{\beta}{kA} + 1\right)\left(1 + \frac{(1-p)\Lambda kA - D}{D}\right) > 1$. Hence $\left(\frac{\beta}{kA} + \frac{(1-p)\Lambda kA - D}{D} + \frac{(1-p)\Lambda kA - D}{D}\frac{\beta}{kA}\right) > 0$.

By using $ML - m_s N > 0$, it is clearly to see $p\Lambda NL + \gamma ML - \gamma m_s N > 0$ and $\delta ML - \delta m_s N > 0$, where $M = (1 - p)\Lambda$, $N = (\delta + m_a + \gamma)$, and $L = \left(\beta + \frac{kA}{c+A}\right)$.

The proof is completed for nonnegative parameters.

5.3.2 Stability of The Equilibria

Stability of Anxiety-Free Equilibrium

Note that the anxious-free equilibrium (AFE) of the new model (5.2) is the same as the anxious free equilibrium of the original model (4.1). In the following we see that the same condition as theorem 4.5 is required for stability of the AFE of system (5.2)

Theorem 5.4. The Anxiety-Free Equilibrium (AFE) of system (5.2) is locally asymptoticly stable with monotonically converging solutions, if $\beta\frac{(1-p)\Lambda}{m_s} < (\delta + m_a + \gamma)$.

Proof 5.4. The proof is similar with theorem 4.5 and therefore is omitted here.

Stability of Endemic Equilibrium

Theorem 5.5. If the Endemic Equilibrium (EE) of system (5.2) exists, then its stability is as follows:

If $\Delta = b^2 - 4c < 0$	If $\Delta = b^2 - 4c > 0$	If $\Delta = b^2 - 4c = 0$
• $b < 0$ Unstable oscillatory	• $b > 0$, $c > 0$ Stable	• $b < 0$ Degenerate unstable
• $b > 0$ Stable oscillatory	• $b < 0$, $c > 0$ Unstable	• $b > 0$ Degenerate unstable
• $b = 0$ inconclusive	• $c < 0$ Unstable	

where $b = \frac{L^2MNc + L^2M^2 - LMNm_s - N^2Lc}{N^2Lc + LNM - N^2m_s}$,

$$c = \frac{N^3{m_s}^2 - 2N^3Lcm_s - L^2Mc + 2L^2N^2Mc - 2LMN^2m_s - N^2Lc}{MLc + LM - Nm_s},$$

and $M = (1-p)\Lambda$, $N = (\delta + m_a + \gamma)$, and $L = \left(\beta + \frac{kA}{c+A}\right)$.

Proof 5.5. To determine the stability of the system at EE, we first calculate the Jacobian matrix to determine the stability of the endemic equilibrium. The Jacobian matrix is evaluated at the endemic equilibrium to get the eigenvalues of the linearized system, which is given by:

$$J_{EE} = \begin{bmatrix} -\frac{LM}{N} & -\frac{2N^2Lc+NLM-N^2m_s}{NLc+LM-Nm_s} & 0 & 0 \\ \frac{LM-m_sN}{N} & \frac{NLc}{NLc+LM-Nm_s} & 0 & 0 \\ 0 & \gamma & -m_t & 0 \\ 0 & \delta & 0 & -m_u \end{bmatrix} \tag{5.5}$$

From (5.5), we see that the eigenvalues are $\lambda_1 = -m_t$, $\lambda_2 = -m_u$, and the other eigenvalues (λ_3, λ_4) are the roots of the quadratic equation:

$$\lambda^2 + \left(\frac{L^2MNc+L^2M^2-LMNm_s-N^2Lc}{N^2Lc+LNM-N^2m_s}\right)\lambda$$

$$+ \left(\frac{N^3{m_s}^2-2N^3Lcm_s-L^2Mc+2L^2N^2Mc-2LMN^2m_s-N^2Lc}{MLc+LM-Nm_s}\right) = 0.$$

where $M = (1-p)\Lambda$, $N = (\delta + m_a + \gamma)$, and $L = \left(\beta + \frac{kA}{c+A}\right)$

Hence, the stability of the Endemic Equilibrium depends on the roots of this quadratic equation using the linear stability theorem. It is straightforward to verify the conditions stated in the above table using the quadratic formula. This completes the proof.

Hopf Bifurcation and Limit Cycles

Hopf bifurcation occurs around an equilibrium when the Jacobian matrix has a pair of eigenvalues and no other eigenvalues with zero real parts. In this case, we prove the presence of limit cycles around the endemic equilibrium point to claim the existence of Hopf bifurcation. Hopf Bifurcation theorem can be considered a tool to prove the existence of limit cycles. We will apply Hopf bifurcation theorem as seen in Theorem 5.6.

Theorem 5.6 (Hopf, Perko (2013) p. 353): Suppose that the C^4-system $\dot{x} = Af(x)$ with $x \in R^n$ and $\mu \in R$ has a critical point x_0 for $\mu = \mu_0$ and that $Df(x_0, \mu_0)$ has a simple pair of pure imaginary eigenvalues and no other eigenvalues with zero real part. Then there is a smooth curve of equilibrium points $x(\mu)$ with $x(\mu_0) = x_0$ and the eigenvalues, $\lambda(\mu)$ and $\overline{\lambda}(\mu)$ of $Df(x(\mu), \mu)$, which are pure imaginary at $\mu = \mu_0$, vary smoothly with μ. Furthermore, if

$$\frac{d}{d\mu}[\mathrm{Re}\,\lambda(\mu)]_{\mu=\mu_0} \neq 0$$

then there is a unique two-dimensional center manifold passing through the point (x_0, μ_0) and a smooth transformation of coordinates such that the system $\dot{x} = Af(x)$ on the center manifold is transformed into the normal form

$$\dot{x} = -y + ax(x^2 + y^2) - by(x^2 + y^2) + O(|x^4|)$$

$$\dot{y} = x + bx(x^2 + y^2) + ay(x^2 + y^2) + O(|x^4|)$$

in a neighborhood of the origin which, for $a \neq 0$, has a weak focus of multiplicity one at the origin and

$$\dot{x} = \mu x - y + ax(x^2 + y^2) - by(x^2 + y^2)$$

$$\dot{y} = x + \mu y + bx(x^2 + y^2) + ay(x^2 + y^2)$$

is a universal unfolding of this normal form in a neighborhood of the origin on the center manifold.

Proof 5.6. The proof can be found in the book (Guckenheimer, J., & Holmes, P., 2013).

Using the Hopf bifurcation theorem to have a limit cycle, we must have a pair of complex eigenvalues $\lambda = a \pm bi$ so that "a" changes signs from a negative to a small positive number. Note that, $a = (-\frac{1}{2})\frac{L^2MNc+L^2M^2-LMNm_s-N^2Lc}{N^2Lc+LNM-N^2m_s}$. Hence, we have the following theorem.

Theorem 5.7. System (5.2) exhibits a Hopf bifurcation at the endemic equilibrium, if

1. $L^2MNc + L^2M^2 - LMNm_s - N^2Lc = 0,$
2. $\frac{N^3{m_s}^2-2N^3Lcm_s-L^2Mc+2L^2N^2Mc-2LMN^2m_s-N^2Lc}{MLc+LM-Nm_s} > 0$

where $M = (1-p)\Lambda$, $N = (\delta + m_a + \gamma)$, and $L = \left(\beta + \frac{kA}{c+A}\right)$

Proof 5.7. Here, we show that the model of system (5.2) has a pair of pure imaginary eigenvalues to have a limit cycle. We already showed that the endemic equilibrium is given by:

$$EE = (S^*, A^*, T^*, U^*) = \left(\frac{N}{L}, \frac{ML - m_s N}{NL}, \frac{p\Lambda NL + \gamma ML - \gamma m_s N}{NL\, m_t}, \frac{\delta ML - \delta m_s N}{NL\, m_u}\right)$$

where $M = (1-p)\Lambda$, $N = (\delta + m_a + \gamma)$, and $L = \left(\beta + \frac{kA}{c+A}\right)$.

We also calculated the Jacobian matrix evaluated at EE:

$$J_{EE} = \begin{bmatrix} -\frac{LM}{N} & -\frac{2N^2Lc + NLM - N^2 m_s}{NLc + LM - Nm_s} & 0 & 0 \\ \frac{LM - m_s N}{N} & \frac{NLc}{NLc + LM - Nm_s} & 0 & 0 \\ 0 & \gamma & -m_t & 0 \\ 0 & \delta & 0 & -m_u \end{bmatrix}$$

From Jacobian matrix at EE, we see that the eigenvalues are $\lambda_1 = -m_t$, $\lambda_2 = -m_u$, and the other eigenvalues (λ_3, λ_4) are the roots of the quadratic equation:

$$\lambda^2 + \left(\frac{L^2MNc + L^2M^2 - LMNm_s - N^2Lc}{N^2Lc + LNM - N^2 m_s}\right)\lambda$$

$$+ \left(\frac{N^3 m_s{}^2 - 2N^3Lcm_s - L^2Mc + 2L^2N^2Mc - 2LMN^2m_s - N^2Lc}{MLc + LM - Nm_s}\right) = 0$$

where $M = (1-p)\Lambda$, $N = (\delta + m_a + \gamma)$, and $L = \left(\beta + \frac{kA}{c+A}\right)$.

Clearly, two of the eigenvalues will be pure imaginary numbers, if conditions (1) and (2) are satisfied. Through extensive calculations, it can be algebraically shown that these two conditions are satisfied for a range of parameter values. This completes the proof.

Remark: Although theorem 5.7 provides conditions for existence of a limit cycle, it should be noted that the limit cycle does not exist in all situations. The following theorem provides one of those situations.

Theorem 5.8. A sufficient condition for non-existence of limit cycle if $m_s = m_a$, $\delta = 0$, and $\gamma = 0$ then the system (5.2) cannot have a stable limit cycle.

Proof 5.8. Suppose that the system (5.2) has a limit cycle S (t) = S (t +T), where T > 0 is the period the limit cycle from the first two equations of the system (5.2) we have that

$$\dot{S} = (1 - p)\Lambda - AS(\beta + \frac{kA}{c+A}) - m_s S$$

$$\dot{A} = AS(\beta + \frac{kA}{c+A}) - (\delta + m_a + \gamma)A$$

where $m_s = m_a$. Let X = S + A and we have $\dot{X} = \dot{S} + \dot{A}$ then by adding these two equations we get that

$$\dot{X} = (1 - p)\Lambda - 2\, m_a S \tag{5.6}$$

Note that the system (5.6) has a globally stable equilibrium

$$X^* = \frac{(1 - p)\Lambda}{2\, m_a}$$

Therefore, for all X (0) we have $\lim_{t \to \infty} X(t) = X^*$ or equivalently $\lim_{t \to \infty} S(t) = S^*$ and

$\lim_{t \to \infty} A(t) = A^*$, where $X^* = S^* + A^*$

This is a contradiction to existence of a stable limit cycle.

5.4 Numerical Simulations of Second Model

In this section, we present the limit cycles of the system (5.2) using simulation of the Hopf bifurcation. We utilized computer simulations via MATLAB ODE45. There are three sets of parameter values with specific initial conditions (S, A, T, U) as shown in Table 16.

Table 16. Estimated parameter values of the system (5.2).

Parameters	Case 1	Case 2	Case 3	Case 4	Case 5
$(1\text{-}p)\ \Lambda$	1	1	1	1	1
β	0.057	0.060	0.060	0.057	0.08
$(\delta + m_a + \gamma)$	1	1	1	1	1
k	0.01846287	0.364	0.364	0.1	0.42
c	0.11969000	0.352	0.352	1.55	0.52
m_s	0.057	0.057	0.057	0.057	0.057
m_t	0.2	0.2	0.2	0.2	0.2
m_u	0.3	0.3	0.3	0.3	0.3
S	15	20	10	15	15
A	0.25	0.8	0.25	0.25	0.25
T	0	0	0	0	0
U	0	0	0	0	0
t	200	200	50	200	200

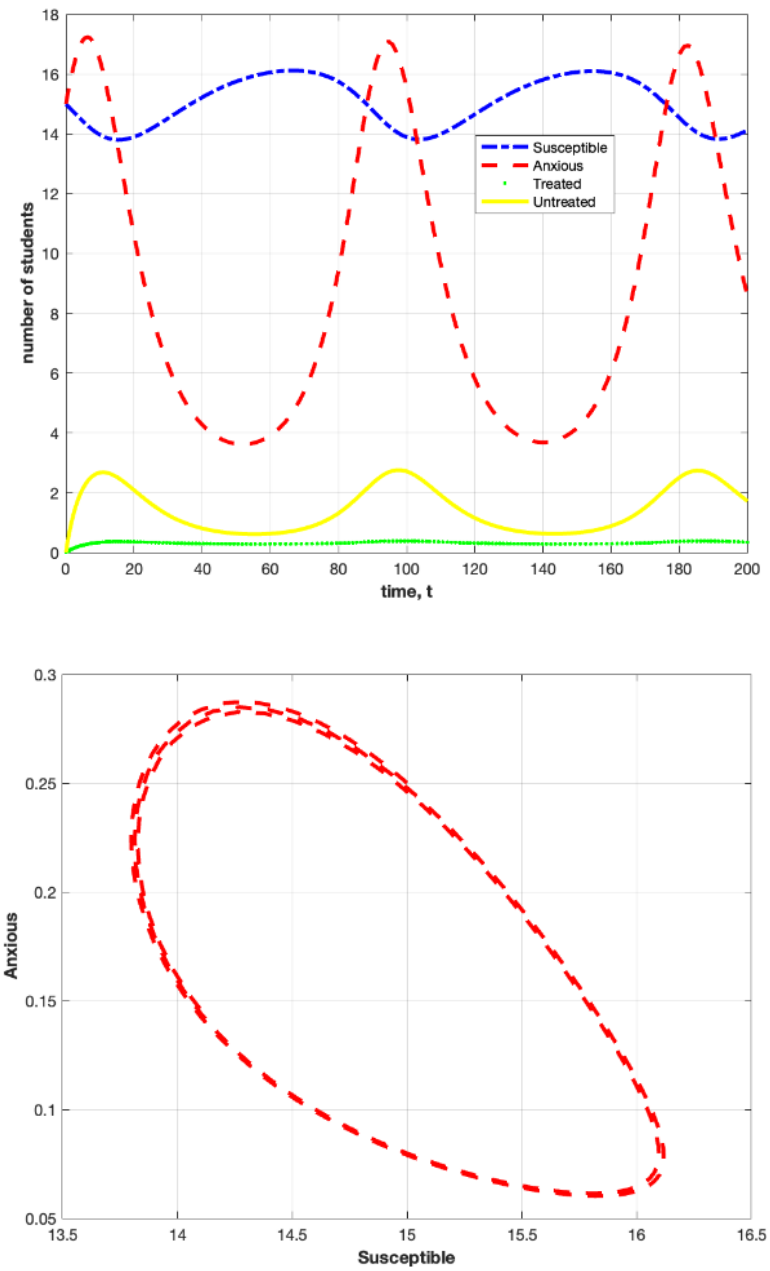
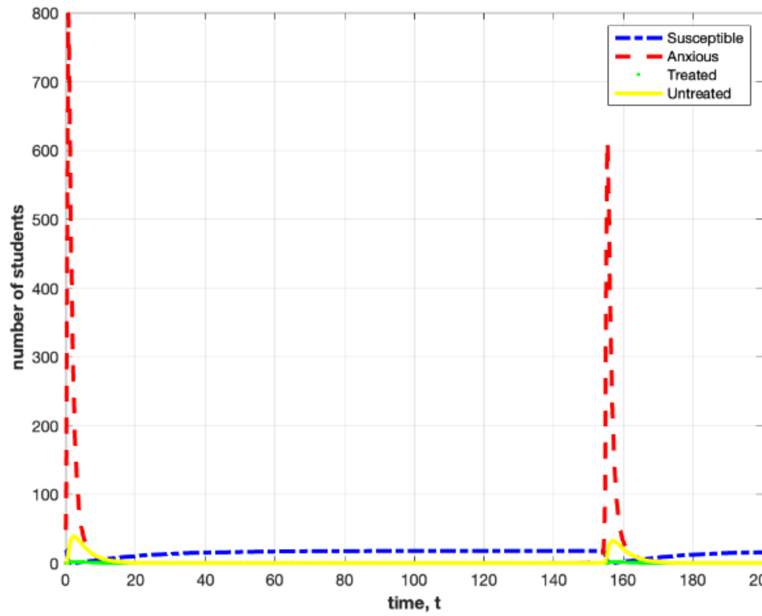


Figure 16. The solution of the system (5.2) with the initial conditions for case 1 (see Table 16). In the Figure 16, we see the existence of a limit cycle and the dimensions of anxious and susceptible population. We also see that the math anxiety becomes endemic in the student's population. Note that the treated and untreated population are functions of this limit cycle. Students will periodically become anxious due to midterm exams (existence of periodic solutions).



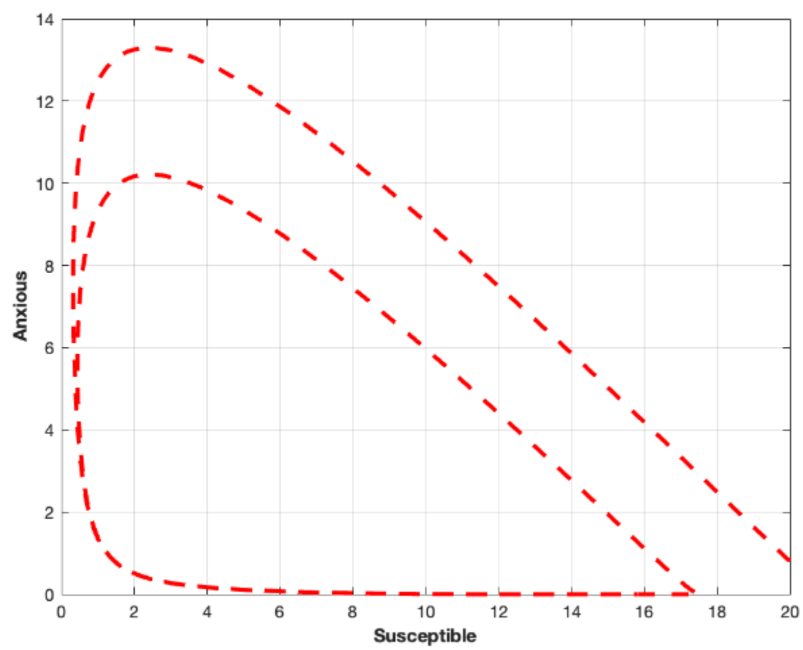


Figure 17. The solution of the system (5.2) with the initial conditions for case 2 (see Table 16). In the Figure 17, we observe a homoclinic orbit connecting a steady state to itself. The equilibrium has about 17 susceptible individuals. Inside the homoclinic orbit there exists a stable spiral. There are spikes of anxiety outbreaks possibly due to a difficult topic presented in a lecture (existence of a homoclinic orbit).

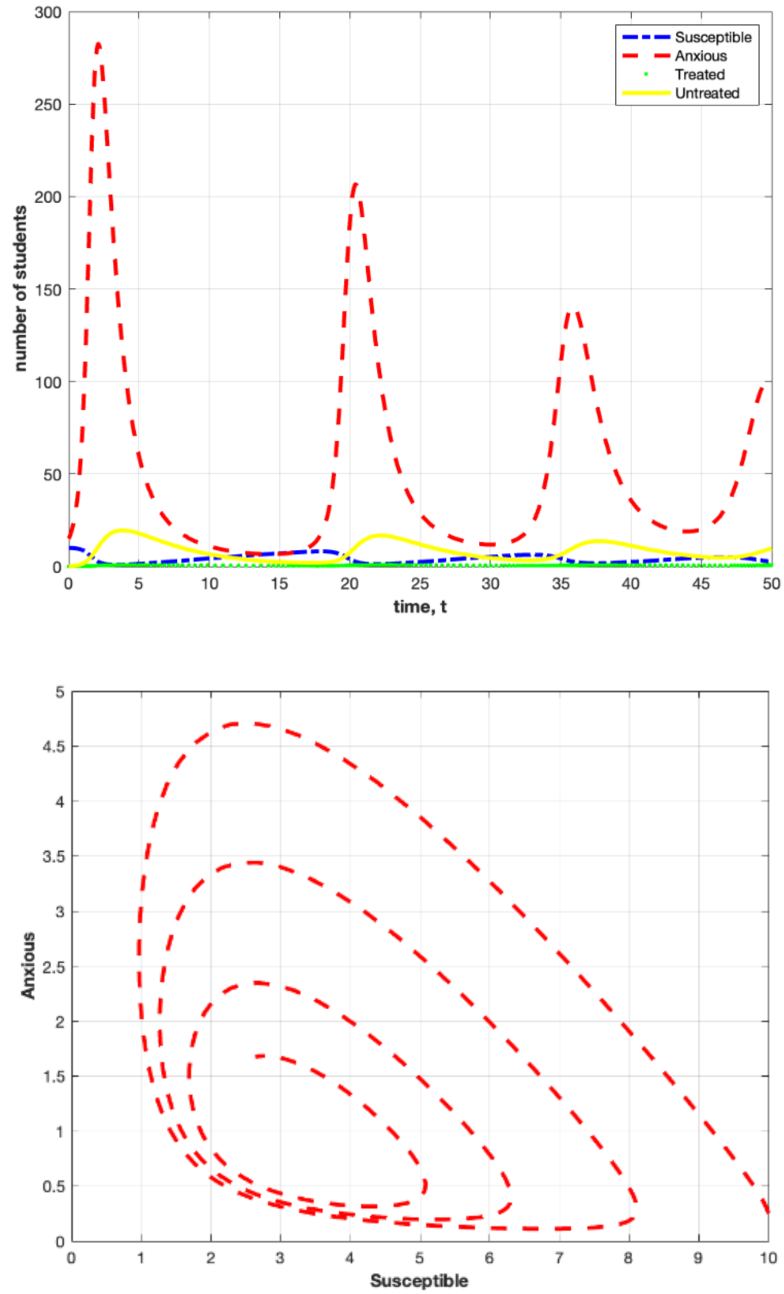


Figure 18. The solution of the system (5.2) with the initial conditions for case 3 (see Table 16). If we consider the set of parameters for case 3, we can have a better understanding of the stable spiral steady state by only changing the initial condition. The students will periodically get anxious, but eventually the anxiety reduces over time (existence of a decaying oscillation).

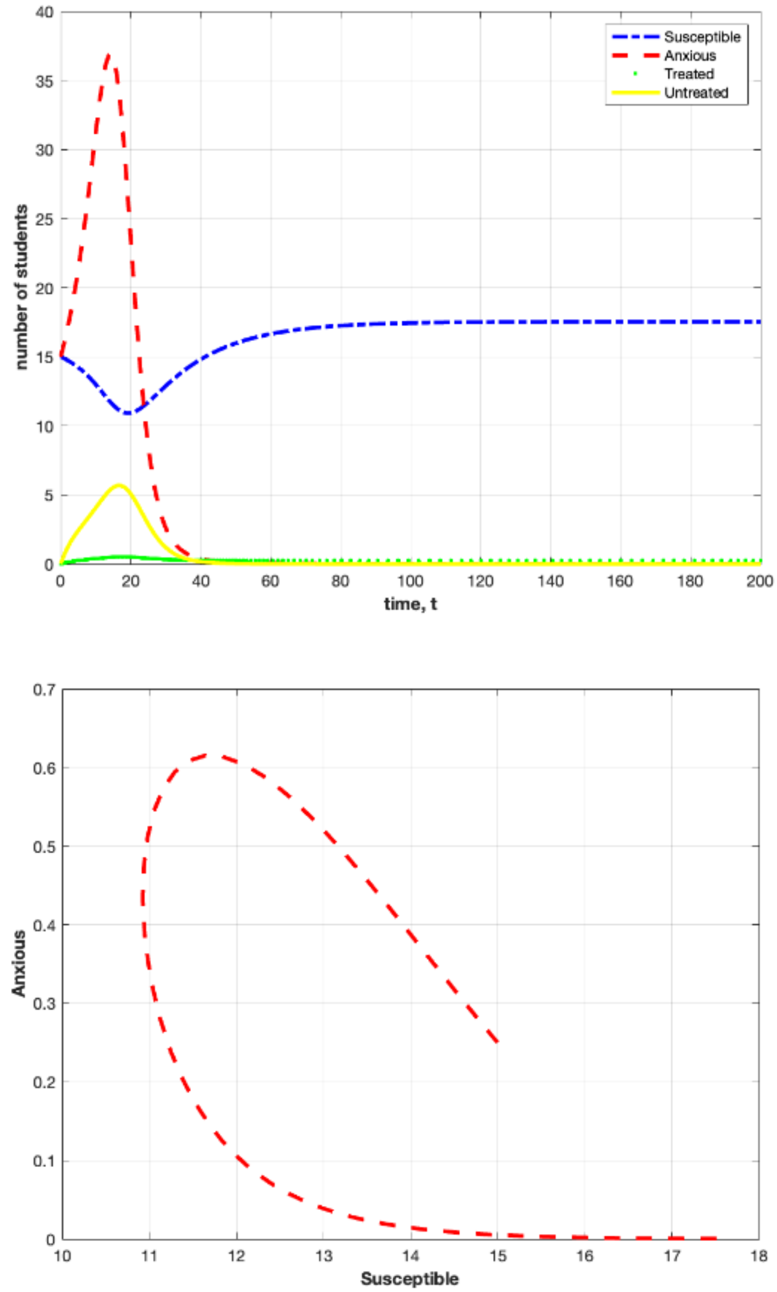


Figure 19. The solution of the system (5.2) with the initial conditions for case 4 (see Table 16). In the Figure 19, we may get a case that the limit cycle as its stability and the system will converge to stable equilibrium by changing the parameter values. Eventually all students will become anxious free (convergence to the stable AFE).

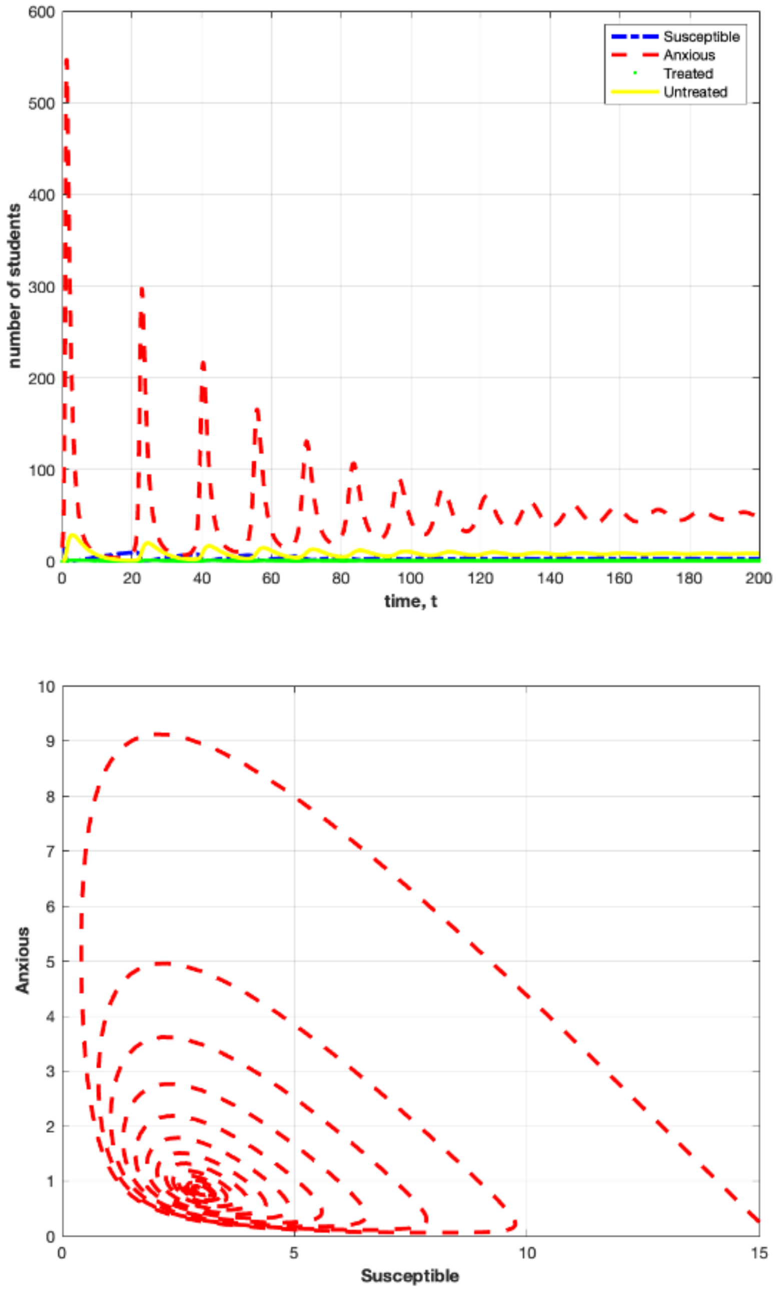


Figure 20. The solution of the system (5.2) with the initial conditions for case 5 (see Table 16). Similarly, the change of parameter values can lead to other forms of steady states as presented in figure 20. The students will periodically get anxious, but eventually the anxiety reduces over time (existence of a decaying oscillation).

5.5 Center Manifold Theory

A center manifold theory is crucial to mathematical modeling which is proposed to determine the stability of degenerate equilibria (Hamzi, 2005). The center manifold dynamical system is based on the system's equilibrium point. The nearby orbits that neither decay nor increase exponentially quickly form a central manifold of the equilibrium.

When investigating the equilibrium points of dynamical systems mathematically, the first step is to linearize the system and then compute its eigenvalues and eigenvectors. Linearization is a typical technique to nonlinear issues, although it might be difficult to examine characteristics like irreversibility, which are closely linked to nonlinearity. If one or more of the objective functions or constraints in a mathematical model are represented by a nonlinear equation, the model is called a nonlinear model.

The stable eigenspace is built on the eigenvectors, if they exist, corresponding to eigenvalues with negative real parts. The unstable eigenspace is corresponding to eigenvalues with positive real part. If the equilibrium has eigenvalues with a real fraction of zero, the eigenvectors form the center eigenspace. The center manifold theory is used for non-hyperbolic dynamical system if one possibility eigenvalue becomes zero. Because linear stability analysis will not work, it will become a problem.

Therefore, the center manifold theorem is utilized to mathematically model the qualitative behavior of a non-linear system near its hyperbolic equilibrium points by linearizing the system at that equilibrium point (Perko, 2013).

A general approach to reorganize and analyze the manifolds at a non-hyperbolic equilibrium is by assuming that there is a dynamical system with an equilibrium point (x_0). Let $\dot{x} = f(x)$ and assume that $f \in C^r(E)$, $E \subset R^n$, $f(x_0) = 0$, $A = Df(x_0) = diag[C, P, Q]$ where A is a Jacobian matrix at x_0 and

- $C \rightarrow Re(\lambda) = 0$
- $P \rightarrow Re(\lambda) < 0$
- $Q \rightarrow Re(\lambda) > 0$

Thus, the non-linear system $\dot{x} = f(x)$ is rewritten as

- $\dot{x} = Cx + F(x, y, z)$
- $\dot{y} = Py + G(x, y, z)$
- $\dot{z} = Qz + H(x, y, z)$ where $(x, y, z) \in R^c x\, R^s x\, R^u \cong R^n$, $F(x_0) = 0$, $G(x_0) = 0, H(x_0) = 0, D(F(x_0)) = 0, D(G(x_0)) = 0, D(H(x_0)) = 0.$

If we assume that z=0, Q=0, and u=0, we also assume that $(F, G) \in C^r(E)$, let $C = \{(x, y) \in R^c x\, R^s, y = h(x) \text{ for } |x| < \delta\}$ where $\delta > 0$ and $h \epsilon C^r(N_\delta(x_0)), h(x_0) = 0, D(h(x_0)) = 0$ is always true (Perko, 2013).

Theorem 5.9. (The Local Center Manifold, Perko (2013) p. 155): Let $f \in C^r(E)$, where E is an open subset of R^n containing the origin and $r \geq 1$. Suppose that $f(0) = 0$ and $D(f(0)) = 0$ has c eigenvalues with negative real parts, where $c + s = n$. The system $\dot{x} = f(x)$ then can be written in diagonal form is:

- $\dot{x} = Cx + F(x, y)$
- $\dot{y} = Py + G(x, y)$

where $(x, y) \in R^c x R^s$, C is a square matrix with c eigenvalues having zero real parts, P is a square matrix with s eigenvalues with negative real parts, and

$F(0) = 0, G(0) = 0, D\big(F(0)\big) = 0, D\big(G(0)\big) = 0$; furthermore, there exists a $\delta > 0$ and a function $h(E) \in C^r(N_\delta(0))$, that defines the local center manifold and satisfies

$Dh(x)\big[Cx + F\big(x, h(x)\big)\big] - Ph(x) - G\big(x, h(x)\big) = 0$ for $|x| < \delta$; and the flow on the center manifold W (0) is defined by the system of differential equations is $\dot{x} = Cx + F(x, h(x))$ for all $x \in R^c$ with $|x| < \delta$.

Corollary 5.10. Under the assumptions of theorem 5.9 and theorem 5.7, the system (5.2) can be rewritten as

$\dot{x} = Cx + F(x, y)$

$\dot{y} = Py + G(x, y)$

Proof 5.10.

Let $x = [S\ A]^T$,

let $y = [T\ U]^T$,

let $C = \begin{bmatrix} -\frac{LM}{N} & -\frac{2N^2Lc+NLM-N^2m_s}{NLc+LM-Nm_s} \\ \frac{LM-m_sN}{N} & \frac{NLc}{NLc+LM-Nm_s} \end{bmatrix}$, and let $P = \begin{bmatrix} -m_t & 0 \\ 0 & -m_u \end{bmatrix}$ then it can be easily shown that

$$F = \begin{bmatrix} (1 - p)\Lambda - AS\left(\beta + \frac{kA}{c+A}\right) - m_s S + \frac{LM}{N} S + \frac{2N^2Lc + NLM - N^2 m_s}{NLc + LM - Nm_s} A \\ AS\left(\beta + \frac{kA}{c+A}\right) - (\delta + m_a + \gamma)A - \frac{LM - m_s N}{N} S - \frac{NLc}{NLc + LM - Nm_s} A \end{bmatrix}$$

and

$$G = \begin{bmatrix} p\Lambda + \gamma A \\ \delta A \end{bmatrix}$$

The completes the proof.

5.6 Normal Form Theory

A normal form theory is a method for analytic translating nonlinear dynamical systems' ordinary differential equations into standard forms. The inessential part of higher-order nonlinearities can be removed using a coordinate transformation by preserving the essential features of the system.

The main goal is to create a way for using center manifold theory that is as simple as possible. In the previous sub-section, we discussed center manifold theory, which has many difficulties and is difficult to implement into a mathematical system. The normal form theory for autonomous differential equations near an equilibrium point was established by Henry Poincare and he used it in his thesis from 1879 (Siegmund, 2003).

Poincare developed a technique known as near identity transformation, in which he applied a concept known as normal quadratic forms. This technique is useful to study systems that are undergoing non-hyperbolic equilibrium solutions, in order to determine stability analysis of the system (Siegmund, 2003).

To obtain the necessary and sufficient conditions for the occurrence of a normal form theory around the equilibrium point, we need to do the following steps:

1. We consider the general non-linear system (5.2) in the form of

   $\dot{x} = f(x^*)$, $f(x^*) = 0$ (x^* is equilibrium solution).

2. We rewrite the system as a linearized part; it can be brought into Jordan normal form by applying an analytical coordinate transformation,

Note that,

- If $\mathrm{Re}(\lambda i) \neq 0$, for $1 < i < n$, the stability is determined using the Hartman-Grobman theorem.
- If $\mathrm{Re}(\lambda i) = 0$, for $1 < i < n$, the stability can be determined using the center manifold theorem.

The center manifold theorem in the section 5.5 showed us to determine the qualitative behavior of

$$\dot{x} = Jx + F(x) \tag{5.7}$$

the nonlinear system on the center manifold. Since the dimension of the center manifold is typically less than n, this simplifies the problem of determining the qualitative behavior of the system (5.2) near a nonhyperbolic critical point. However, analyzing this system still may be a difficult task. The normal form theory allows us to simplify the nonlinear part, F(x), of (5.7) to make this task as easy as possible. This is accomplished by making a nonlinear, analytic transformation of coordinates of the form

$x = y + h(y)$, where $h(y) = O(|y^2|)$ as $|y| \to 0$ (Perko, 2013 pp. 164) and where h (y) = $\begin{pmatrix} a_{20}y_1^2 + a_{11}y_1y_2 + a_{02}y_2^2 \\ b_{20}y_1^2 + b_{11}y_1y_2 + b_{02}y_2^2 \end{pmatrix}$ (Perko, 2013 pp. 165).

Although normal form theory provides guidance to transform the system and determine the stability of the non-hyperbolic equilibrium solution, our efforts to determine and specify the transformation has been unfruitful.

CHAPTER 6

CONCLUSION AND FUTURE WORK

6.1 Conclusion

In this study, we have utilized statistical and mathematical tools essential to analyzing math anxiety and factors associated with it. We did a good prediction by using statical analyses such as an ANOVA test and time series analysis. By using statical analysis, we observed the level of math anxiety in students during nine weeks, and we were able to observe gender and academic-level differences. Also, we had a chance to observe COVID-19 effects on the level of math anxiety in students. The time series analysis discussed in Chapter 2 helps us understand math anxiety, upcoming exam anxiety, upcoming grade anxiety, and the usage of math resources (e.g., supplemental instructions, tutoring, office hours, and other resources).

Furthermore, we applied one of the machine learning approaches, a classification and regression tree model, to predict math anxiety and to provide a more detailed analysis of data associated with math anxiety and variables (i.e., confidence, motivation, exam anxiety, hours studied, genders, and academic levels). We identified the interrelationships between math anxiety and factors including students' motivation, confidence, weekly hours studied, academic level, and gender. In addition, we performed a Pearson's correlation analysis of the survey data. For example, confidence was negatively correlated with math anxiety, as seen in Chapter 3.

Significantly, the development of math anxiety in students also carries over to the next generations, as those students' become teachers and parents with math anxiety. This creates a vicious and compounding cycle. This study also focused on peer-to-peer contagion rather than from teachers, parents, and other environmental factors, utilizing epidemiological analysis to demonstrate the vast contagion and need to broaden the scope of interventions for math anxiety. We have developed mathematical models to understand the possible dynamics of the math anxiety. We tested our model and were able to conclude that it is a good representation of how math anxiety is transferred among students. We have given a detailed dynamical study for epidemiologic modeling in Chapter 4 and for in-host disease modeling in Chapter 5. Chapter 5 provided Hopf bifurcation theorem to study the oscillatory behaviors of the system (5.2). In both models, we rigorously proved and numerically simulated the numerous dynamics. The main attention of this dissertation was given to analysis of the systems ((4.1) and (5.2), including stability analysis, Hopf bifurcations, and limit cycles (5.2). Using result of the stability analysis we can develop intervention strategies to effectively reduce the anxiety of students. One of the important steps after this would be to identify risk factors associated with anxiety spikes. Also, we can identify where to intervene throughout the semester to reduce Drop/Fail/Withdraw (DFW) rates.

We believe that these findings offer considerable insight into strengthening academic achievement in higher education. It is our profound hope that the results of this study will serve as a foundation to build upon, as other researchers continue to develop the field. In addition, the present work shows that researchers in the field of education can use machine learning methods to provide a deeper analysis of data associated with math anxiety and attitudes toward math education. However, a limitation of the present study is that environmental factors (such as parents, teachers, ethics, etc.) may enhance the association between math anxiety and the factors which we used in this study. A second limitation is that we used a self-reported, small data set. Moving forward, the results of this study should be validated using a larger sample of students across more math gateway courses.

6.2 Future Work

From this point onward, there are various important and relevant next steps to consider. First, we would like to apply the perturbation theory in the system (5.2) to study the effects of small variations to the new model, which is an approach to prove the existence of multiple limit cycles. Further, we would like to extend the bifurcation analysis for the system (5.2) to Bogdanov–Takens bifurcation, which occurs when an equilibrium point in ordinary differential equations has a zero eigenvalue of multiplicity two. The Bogdanov-Takens bifurcation provides a new mechanism for generating anxiety recurrence among the students. The real-world implications of the new systems are interesting and may provide explanations for several observed phenomena. In other words, students may feel nervous a second time, but less or more than the first time, or they may feel anxious without any significant observation.

APPENDIX

SUPPLEMENTARY DOCUMENTS

APPENDIX A

QUESTION 1:

Please rank the level of math anxiety you are feeling today: 1-7 (1 = the lowest level of anxiety, 7= the highest level of anxiety)

a) Upcoming assignment

b) Upcoming midterm or final exam

c) Upcoming Zoom meeting

d) Your anticipated grade in this course

e) Other reasons (specify)

f) I am not feeling anxious about math today (enter Yes if this is the case. Otherwise, leave blank)

Copy this and paste it in the following box to enter your answers:

(b) (c) (d) (e) (f)

QUESTION 2:

What type of help did you seek for your math anxiety this week? (Enter 1 for all that apply and 0 for those that don't)

(a) Supplemental Instruction

(b) UMKC Tutoring

(c) Office hours

(d) Online Tutoring (Net Tutor)

(e) Classmates

(f) Private Tutor

(g) UMKC Counseling Center (https://info.umkc.edu/counseling-services/)

(h) Other (specify)

(i) I did not seek any help

(j) I did not have any math anxiety this week

Copy this and paste it in the following box to enter your answers:

(a) (b) (c) (d) (e) (f) (g) (h) (i) (j)

QUESTION 3:

Aside from Zoom meetings and My Math Lab videos, how many hours have you approximately spent studying for this class this week?

QUESTION 4:

How would you rate your desire to study this week? (1-7) 1 = no desire to study, 7 = very motivated to study).

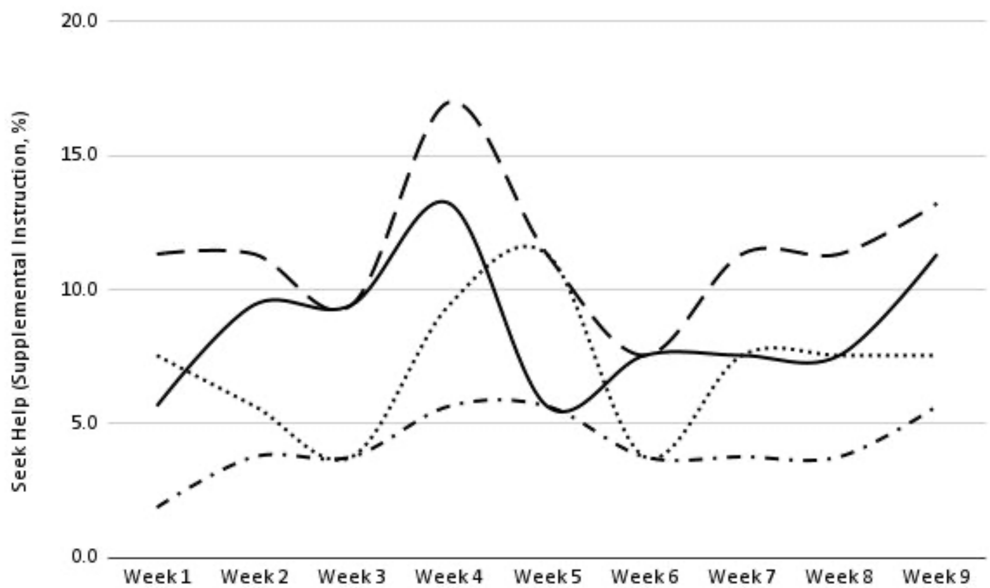
QUESTION 5:

How confident are you today that you will pass this class with a B or better? (1 = not confident at all, 7 = very confident).

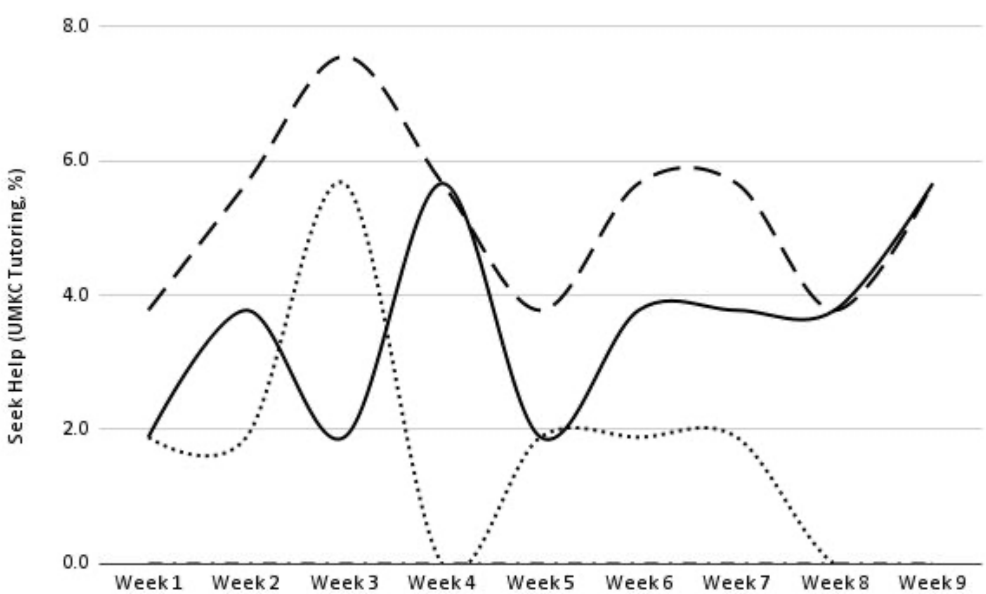
QUESTION 6:

Are you or any of your family members currently affected by Covid-19 in a way that is influencing your ability to succeed in this class or increasing your math anxiety? Enter Yes/No, Also, include any comments if you would like to.

B.1 Supplementary Figures of Chapter 2 and Chapter 3

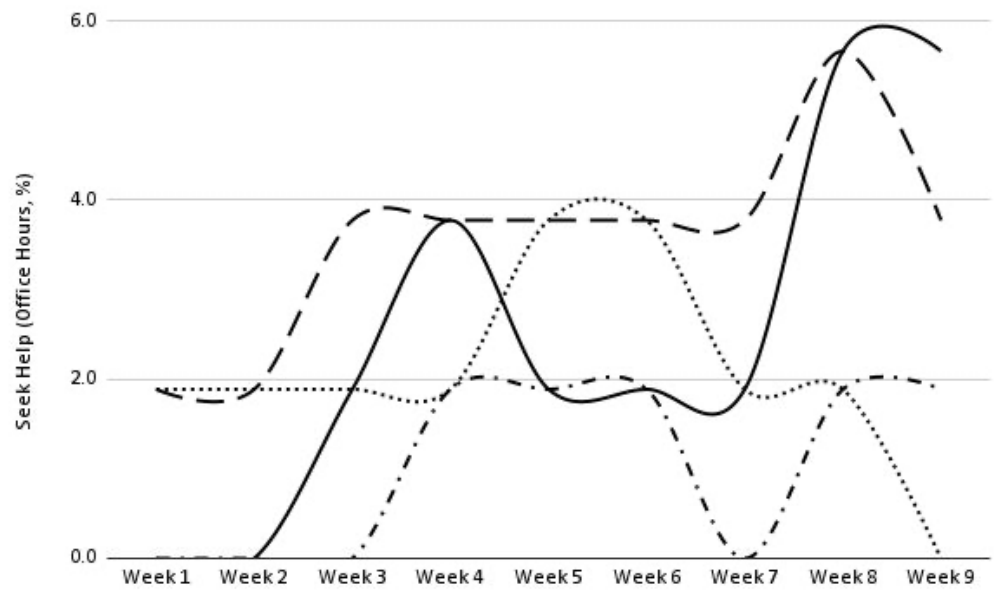


(a)

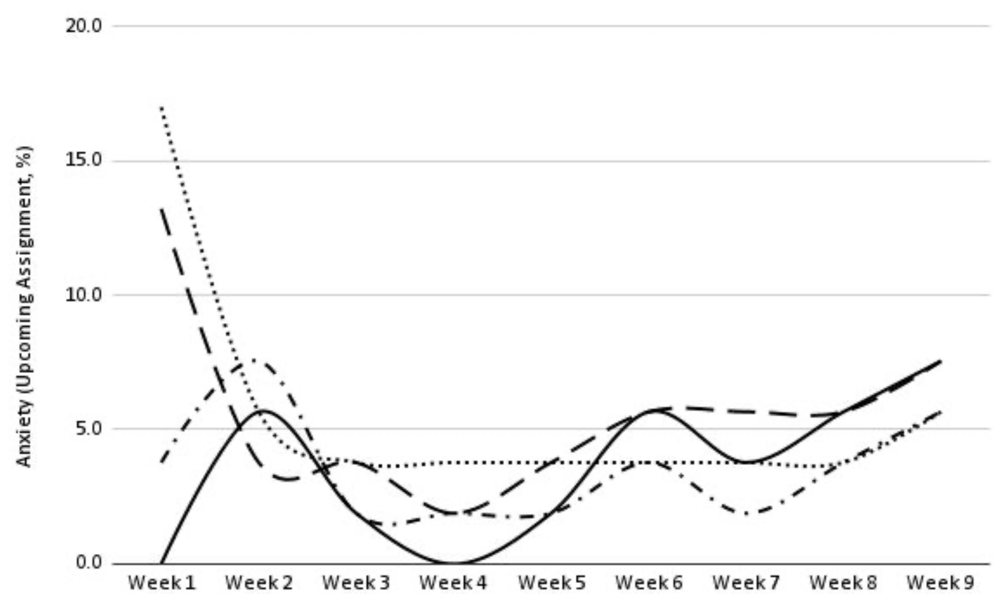


(b)

Figures B1. Prevalence of the STEM students seek help from supplemental instruction (a) and UMKC tutoring (b). Students sought help mostly on week 4 (a) and one week before (b). J/S STEM students less interested than other groups (b). And F/Sp more sought help from Supplemental instruction (a) and UMKC tutoring (b).

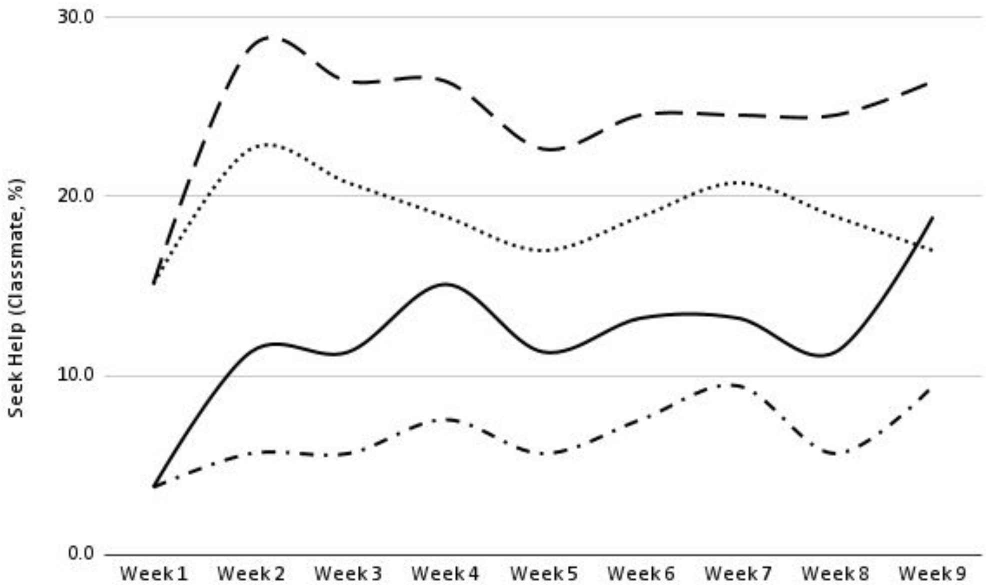


(a)

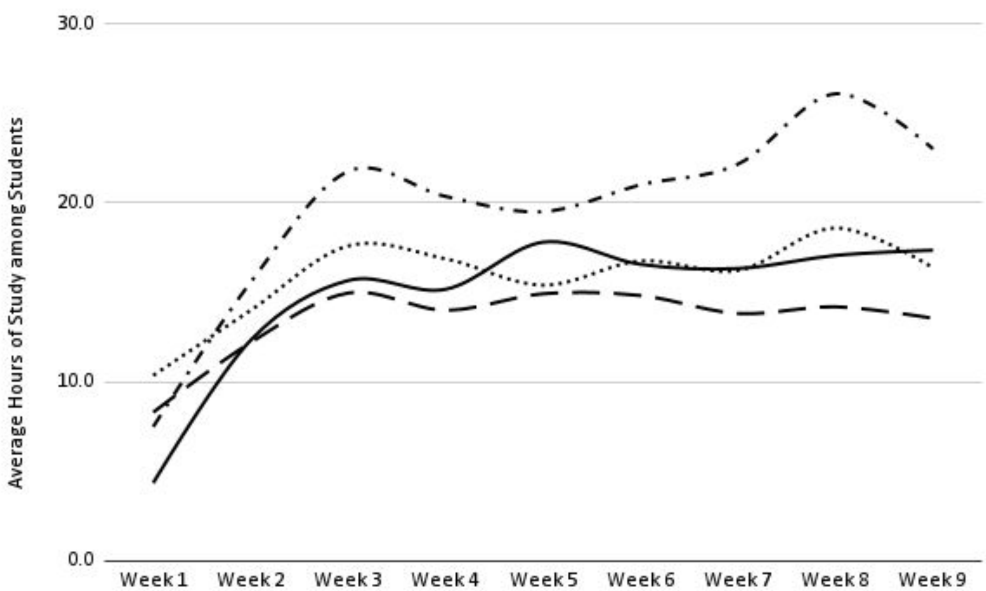


(b)

Figure B2. Prevalence of the STEM students seek help from office hours (a), and percentage of the STEM students with high levels of math anxiety for upcoming assignment (b). All group interested to seek help from office hours around exam weeks (a), and male students were more anxious for their upcoming assignment on the first week of the semester and F/Sp followed the male groups.



(a)



(b)

Figure B3. Prevalence of the STEM students seek help from classmates (a), and students average study hours (b) for calculus during the semester. All group preferred to study with their peers except J/S (a). J/S students studied on their calculus more than other groups.

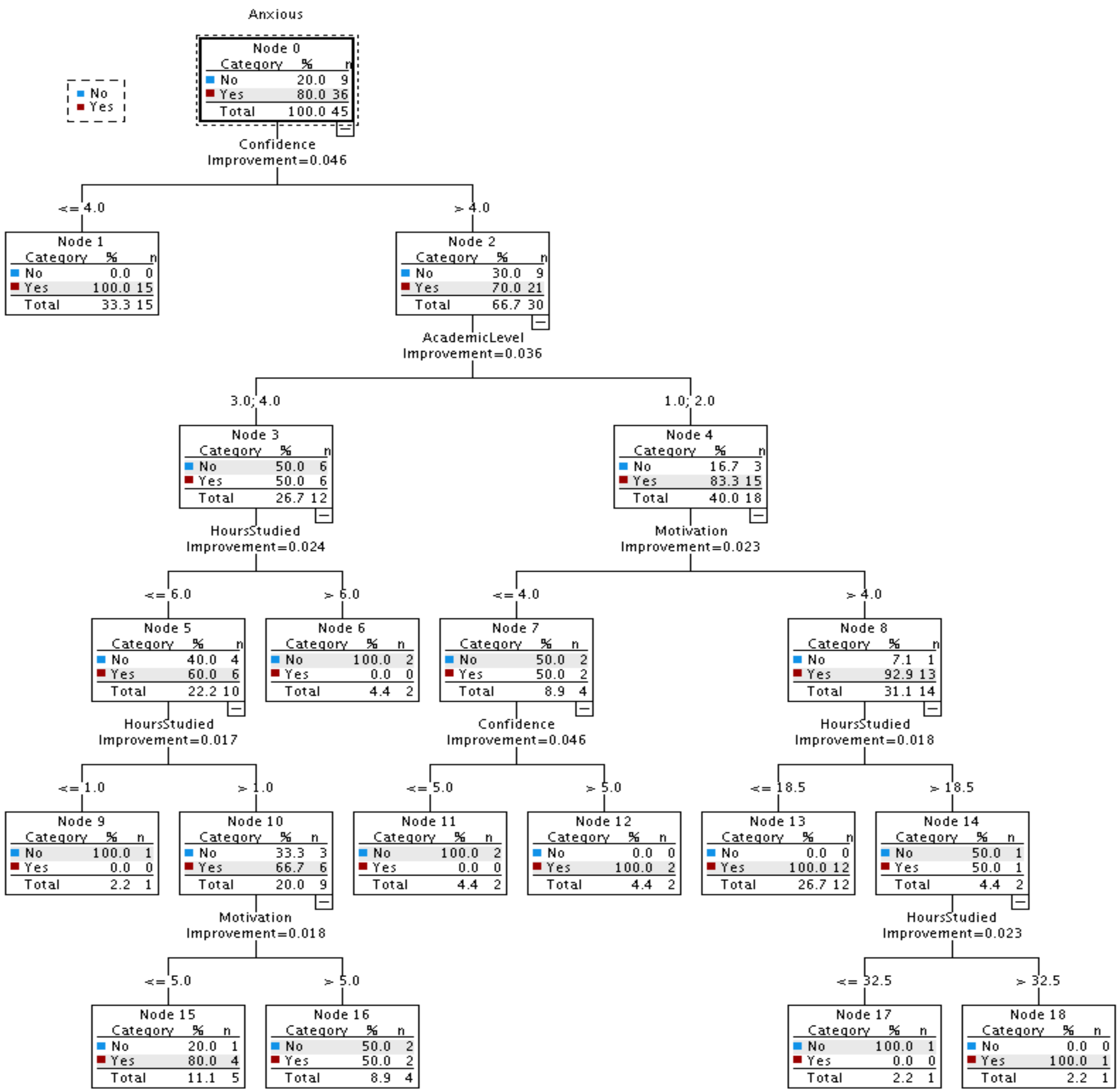


Figure B4. The output tree diagram of the CRT model fitted to survey data of the first week.

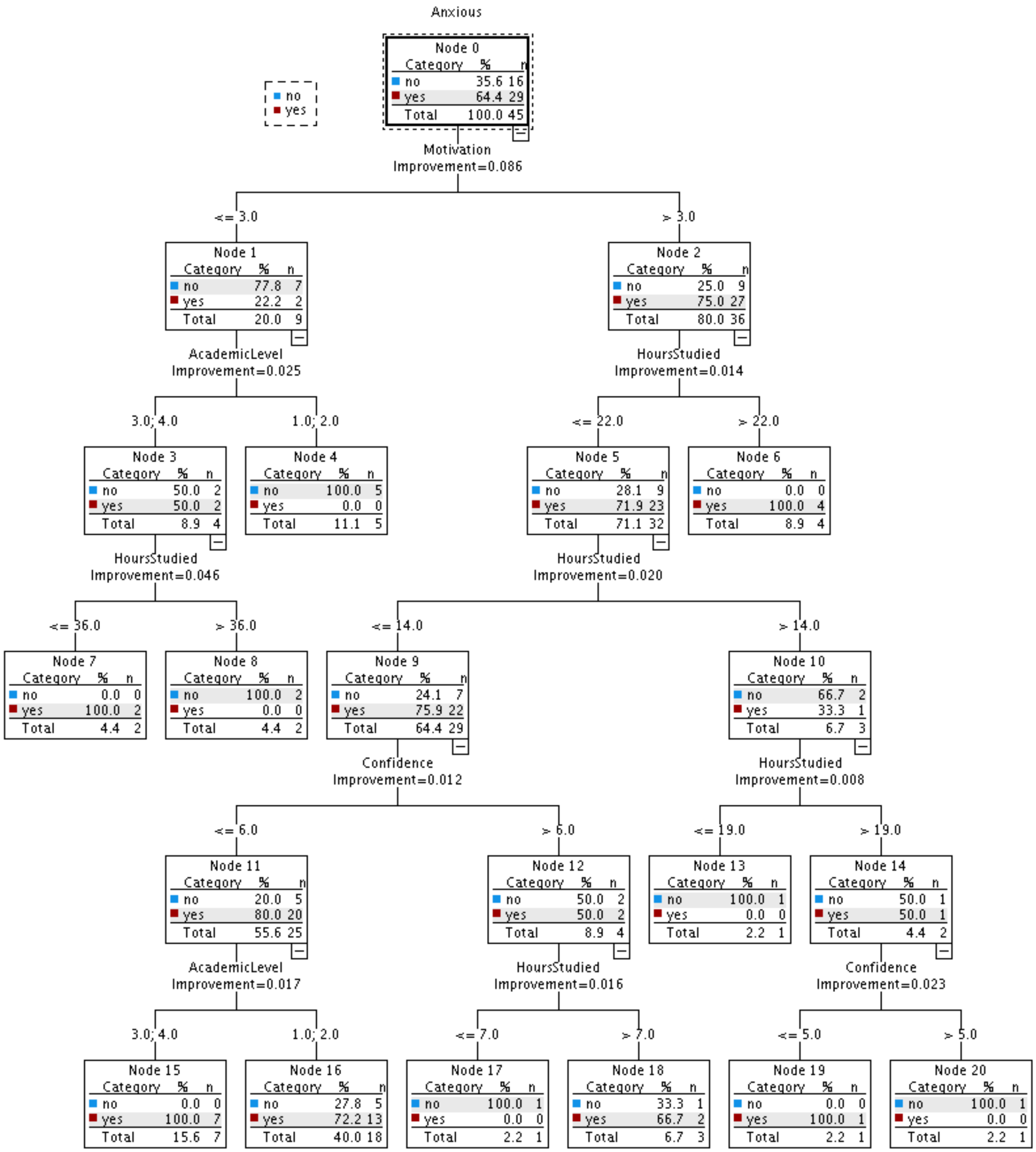


Figure B5. The output tree diagram of the CRT model fitted to survey data of the second week.

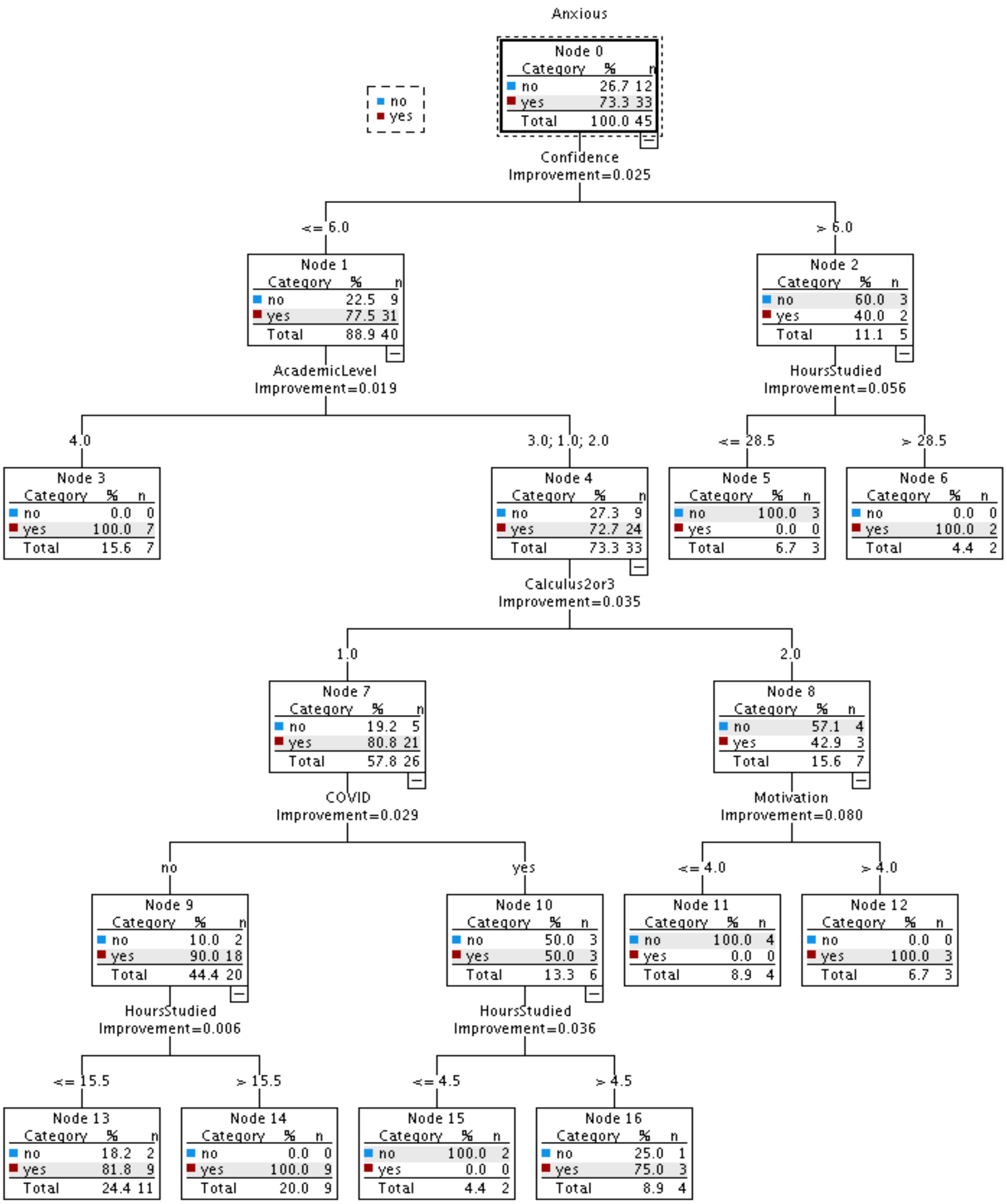


Figure B6. The output tree diagram of the CRT model fitted to survey data of the third week.

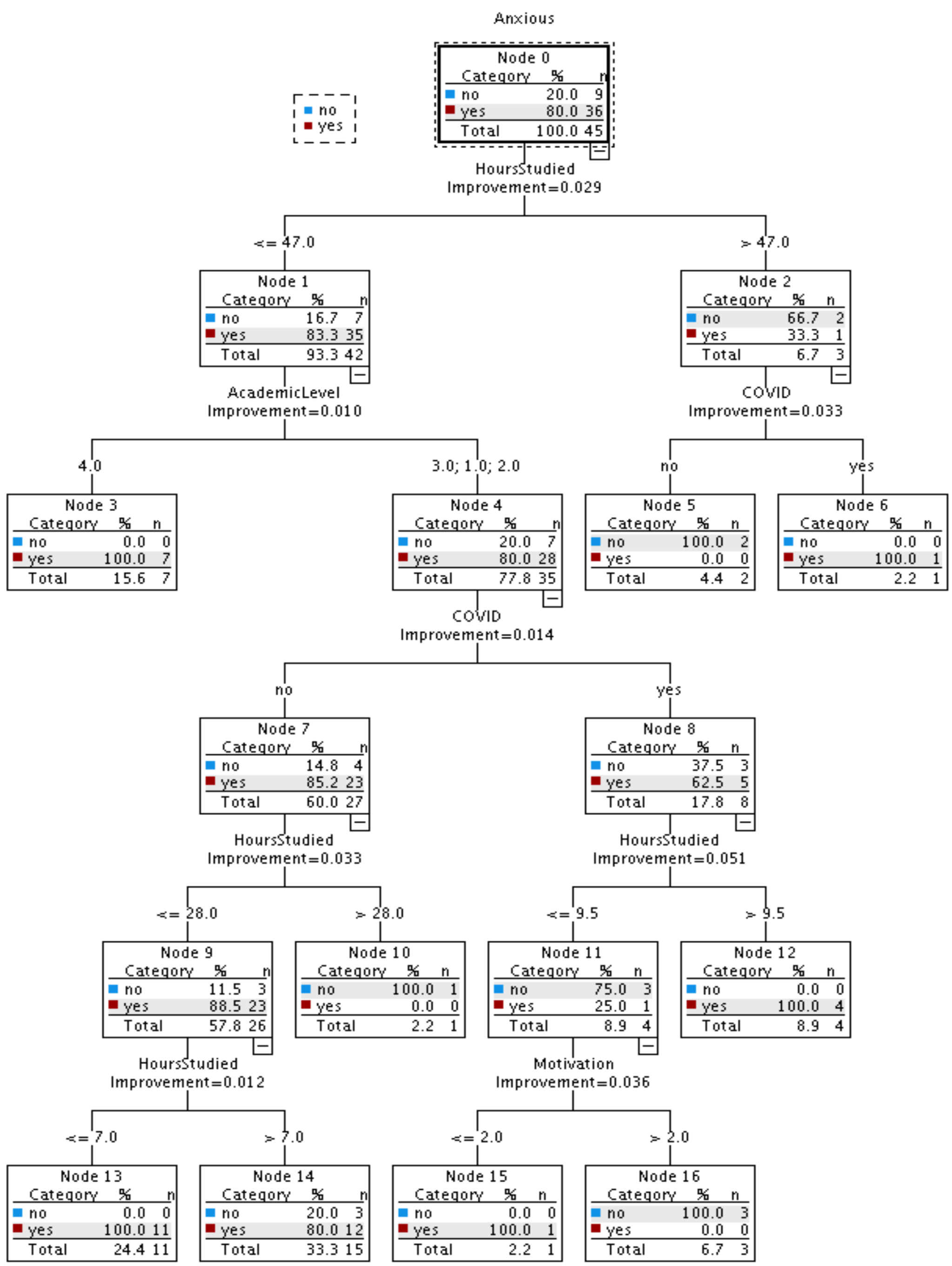


Figure B7. The output tree diagram of the CRT model fitted to survey data of the fourth week.

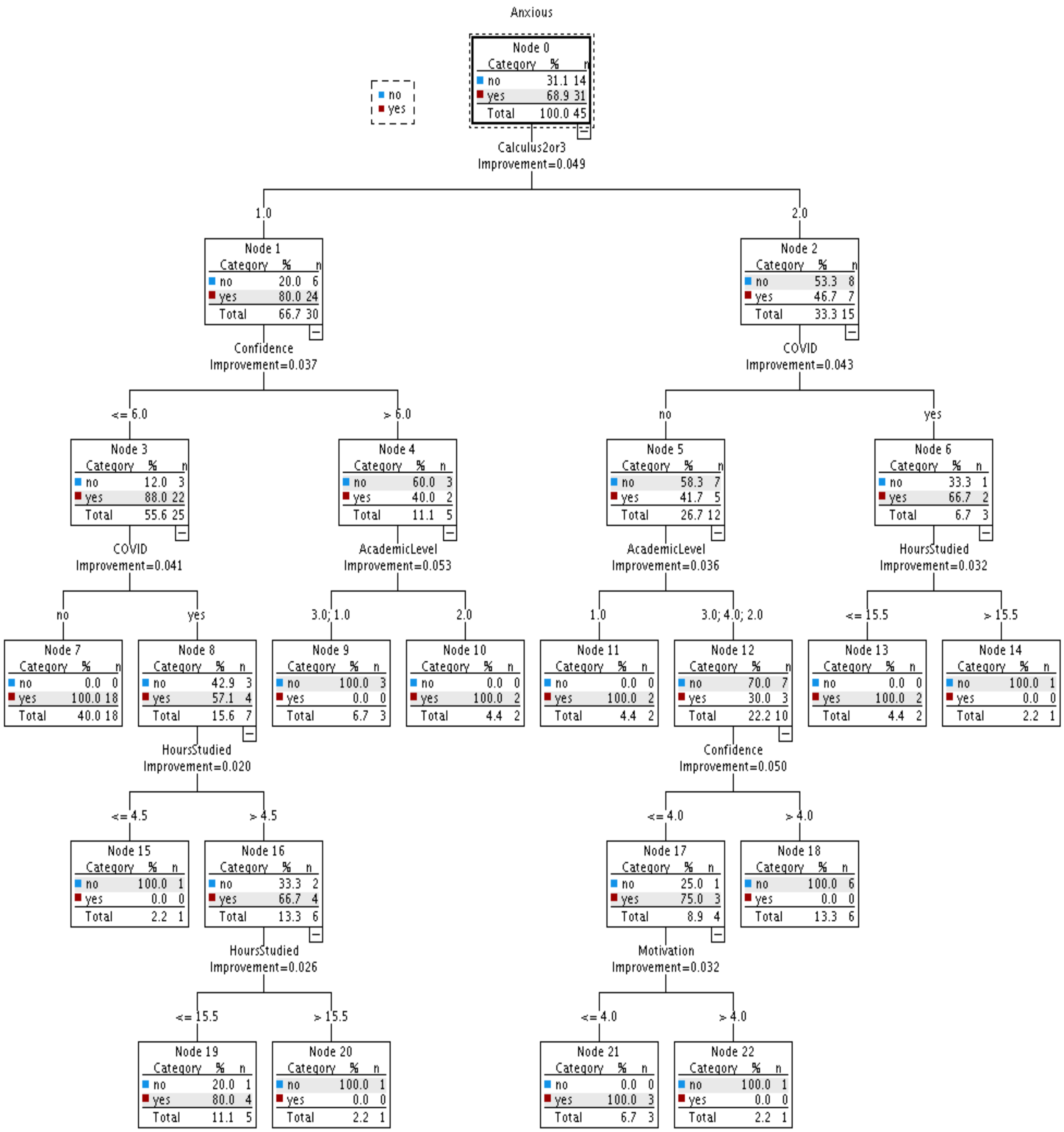


Figure B8. The output tree diagram of the CRT model fitted to survey data of the fifth week.

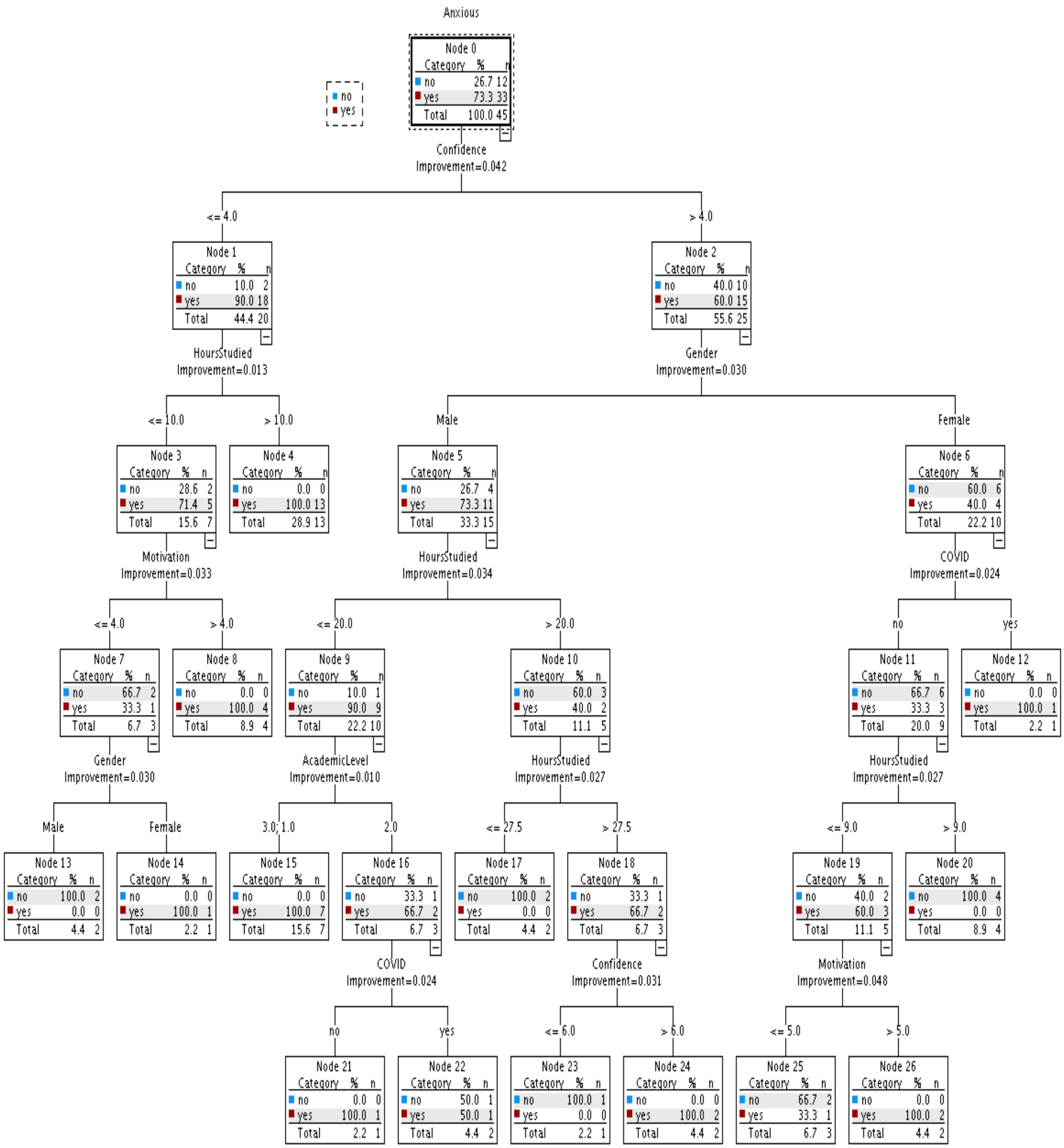


Figure B9. The output tree diagram of the CRT model fitted to survey data of the sixth week.

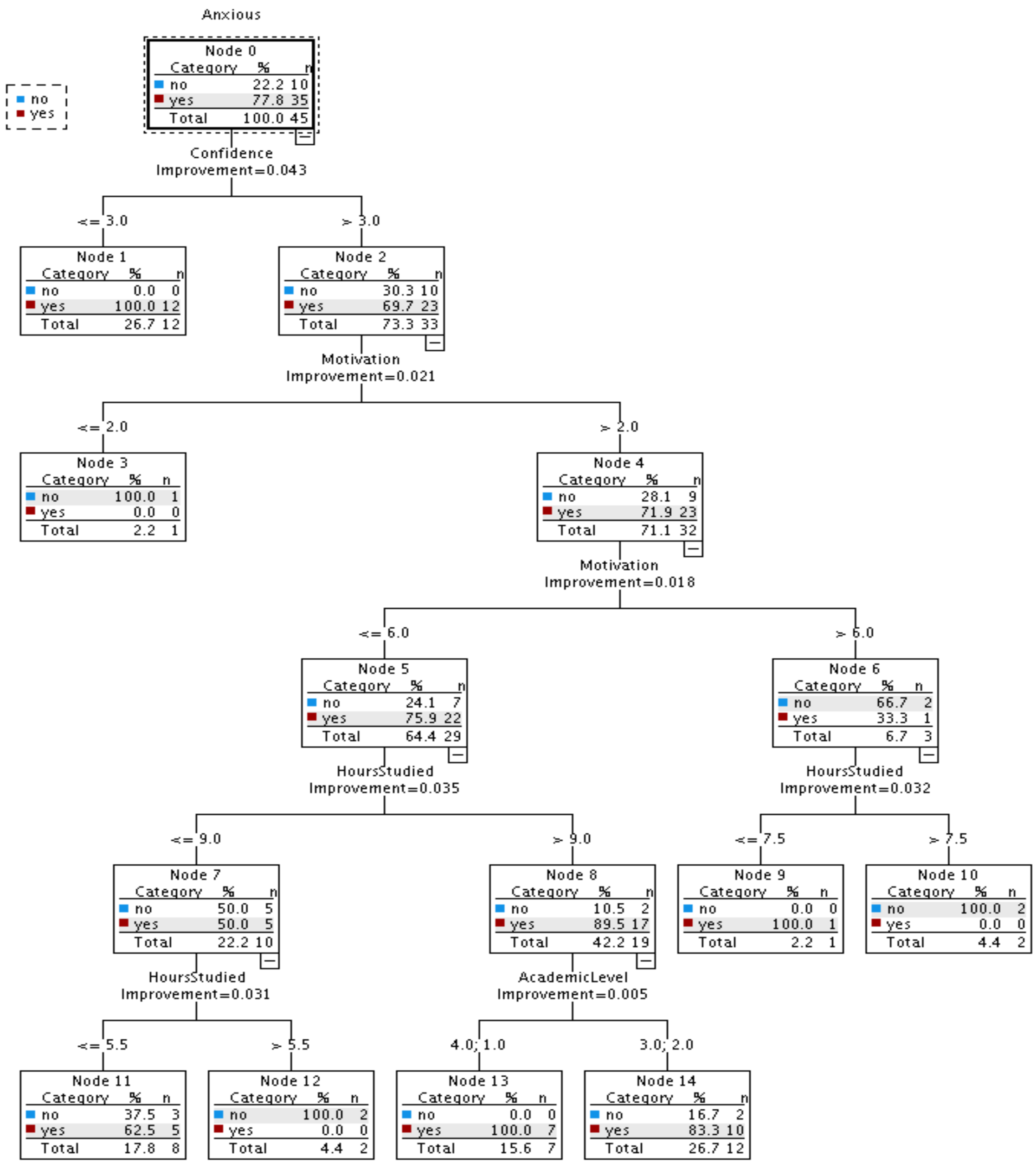


Figure B10. The output tree diagram of the CRT model fitted to survey data of the seventh week.

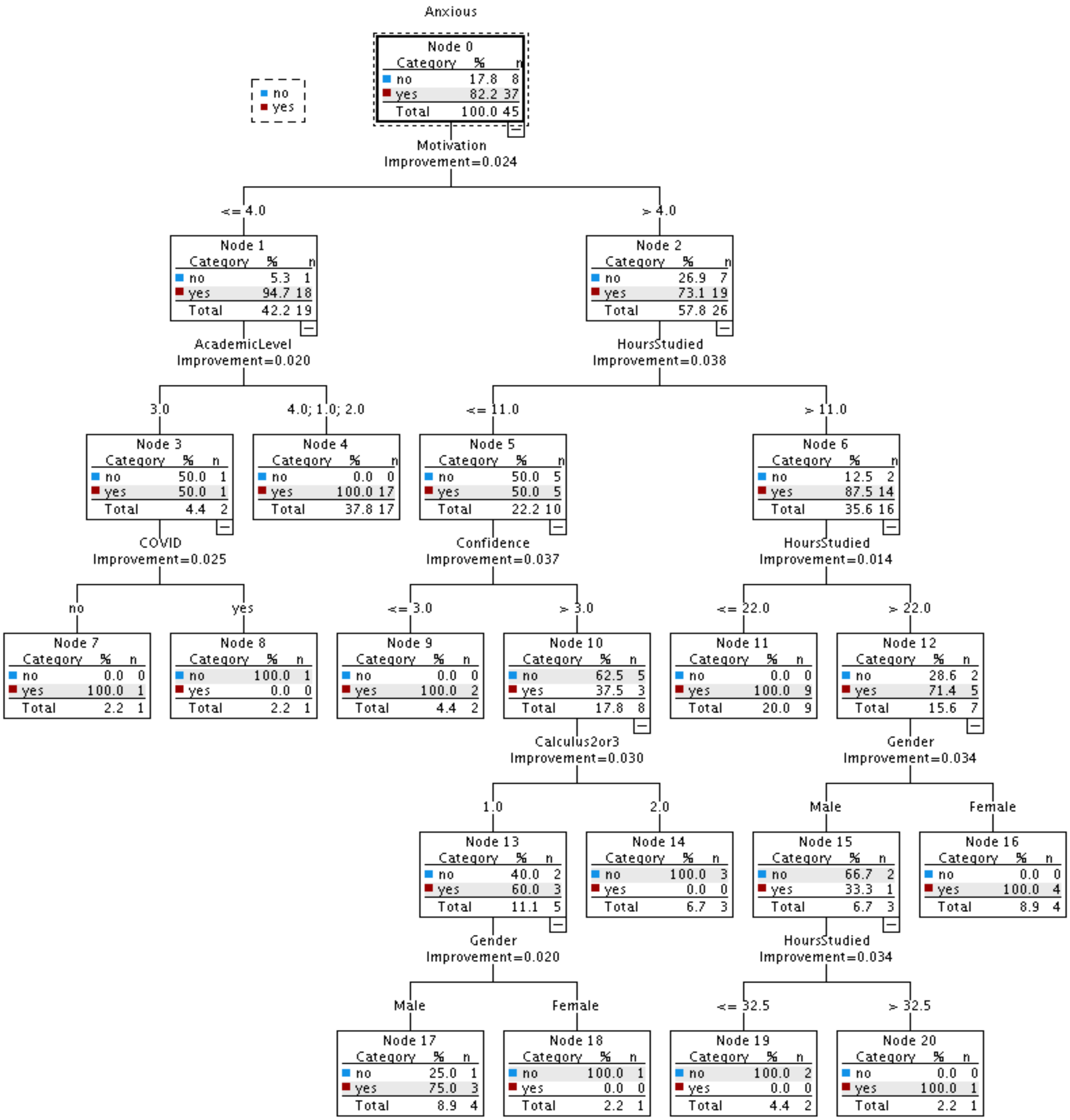


Figure B11. The output tree diagram of the CRT model fitted to survey data of the eighth week.

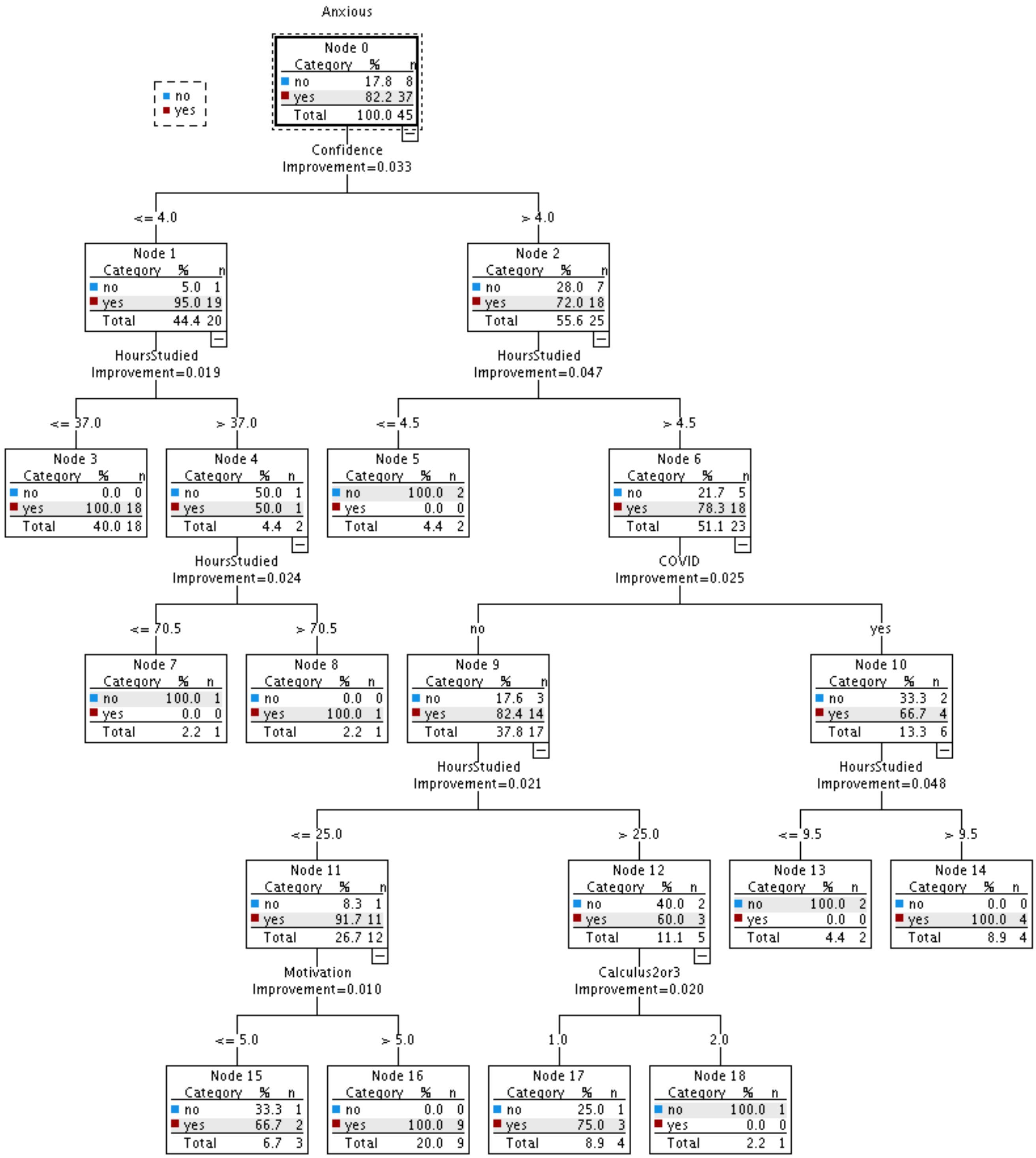


Figure B12. The output tree diagram of the CRT model fitted to survey data of the ninth week.

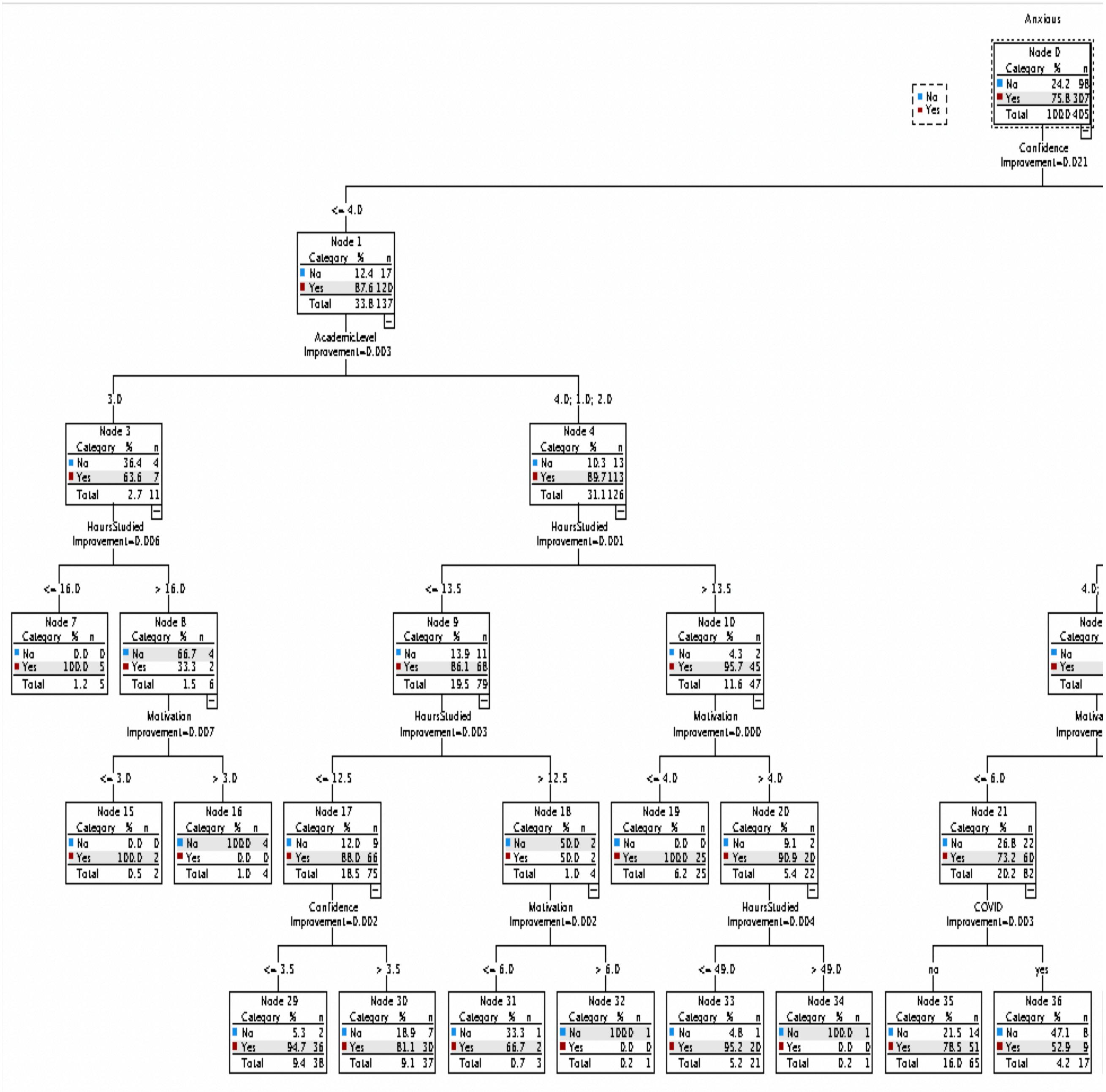


Figure B13(a). The output tree diagram of the CRT model fitted to survey data of the one to ninth week.

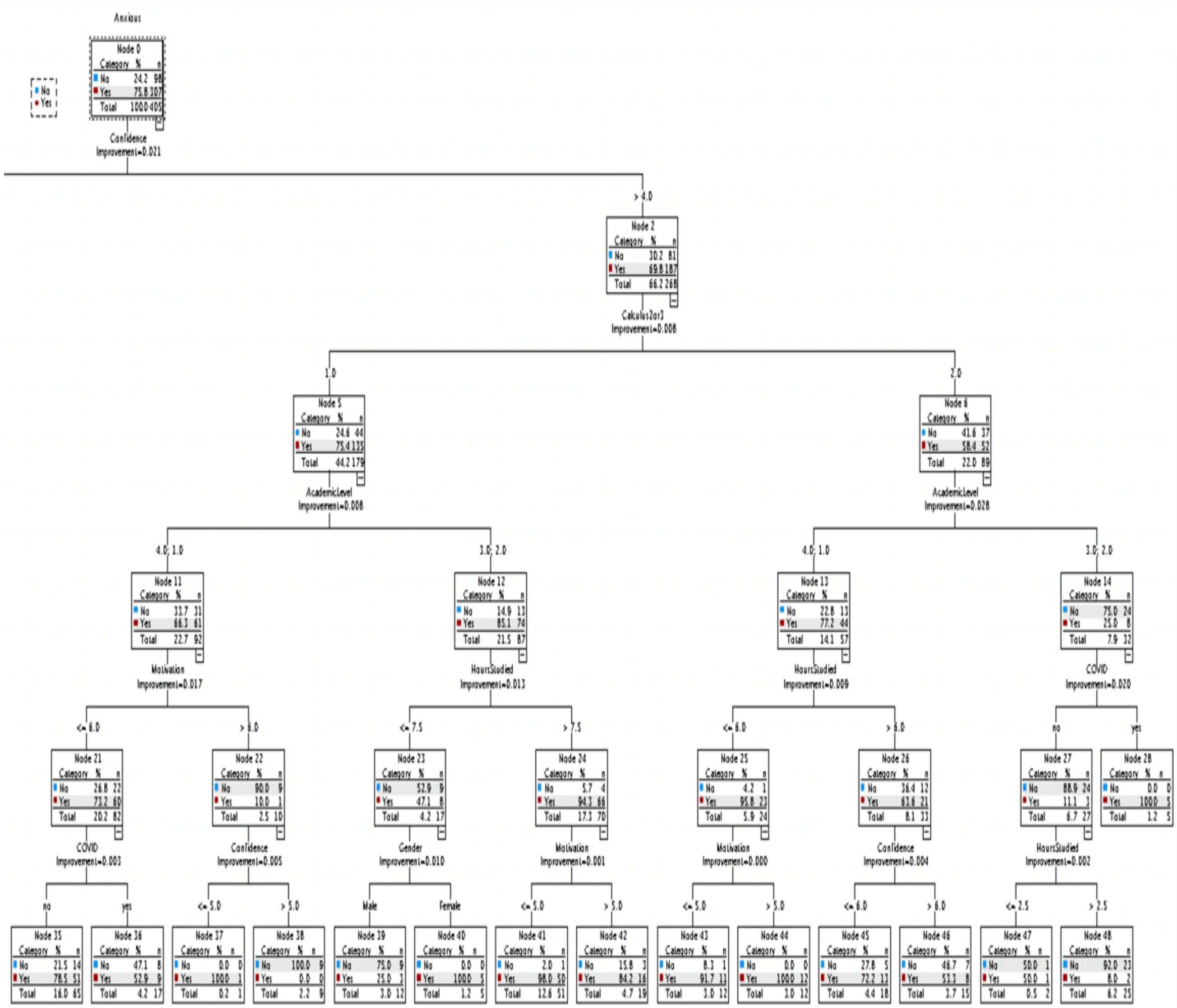


Figure B13(b). The output tree diagram of the CRT model fitted to survey data of the one to ninth week.

B.2 Supplementary Tables of Chapter 2

The following Table is the supporting result for the ANOVA testing.

Table B1. Means and standard deviations for survey items and performance variables.

	Means	Standard Deviations	
Female	56.72	15.29	for Table 2
Male	35.31	17.2	for Table 2
Freshman/Sophomore	25.9	2.78	for Table 3
Junior/Senior	16.04	8.52	for Table 3
Female	4.94	1.75	for Table 5
Male	4.4	1.59	for Table 5
Freshman/Sophomore	4.62	1.67	for Table 4
Junior/Senior	5.06	1.67	for Table 4

B.3 Supplementary MATLAB Codes for Numerical Simulation of Chapter 4

```
clear;
clc
    if caseN==1
lambda=6;
rho=.09;
ms=.01;
alpha=.08;
beta=.03;
sigma=.005;
delta=.01;
mu=.06;
ma=.9;
gamma=.1;
mt=.01;
    elseif caseN==2
lambda=6;
rho=0.701234;
ms=.01;
alpha=.000001;
beta=.02;
sigma=.000001;
```

```
delta=.01;
mu=.06;
ma=.9;
gamma=0.601;
mt=.01;
        end
CT2=beta*(1-rho) *lambda/ms -(alpha+delta+ma+gamma)
if CT2<0
   display ('condition of theorem 2 is satisfied')
   CT2
   sigma
else
   display ('condition of theorem 2 is NOT satisfied')
   CT2
   sigma
end
A= roots([beta*(delta+ma+gamma) sigma*(delta+ma+gamma)
+ms*(alpha+delta+ma+gamma)-beta*(1-rho) *lambda -sigma*(1-rho) *lambda])
a=sort(A)
a=a (2)
b=ms+sigma+beta*a+gamma+delta+ma+alpha-beta*((1-rho) *lambda-
alpha*a)/(ms+sigma+beta*a)
```

```
c=(ms+sigma+beta*a) *((alpha+delta+ma+gamma) -(beta*((1-rho) *lambda-
alpha*a))/(ms+sigma+beta*a)) -(beta*a+sigma) *((alpha)-(beta*((1-rho) *lambda-
alpha*a))/(ms+sigma+beta*a))
DDelta=b^2-4*c
syms y1 y2 y3 y4 lambda2;
dy1=(1-rho) *lambda+alpha*y2-sigma*y1-beta*y1*y2-ms*y1;
dy2=sigma*y1+beta*y1*y2-(alpha+delta+gamma+ma) *y2;
dy3=gamma*y2+rho*lambda-mt*y3;
dy4=delta*y2-mu*y4;
eq = solve (dy1, dy2,dy3,dy4,y1,y2,y3,y4);
equil1 = double (subs ([eq. y1(1), eq.y2(1), eq.y3(1) , eq.y4(1)]))
equil2 = double (subs ([eq. y1(2), eq.y2(2), eq.y3(2), eq.y4(2)]))
 if (equil1(1)<0) ||(equil1(2)<0) ||(equil1(3)<0)|| (equil1(4)<0)
     equil1=equil2;
     equil1
 end
 J=jacobian([dy1; dy2; dy3; dy4], [y1 y2 y3 y4]); % this will generate the Jacobian Matrix
 Jeval1= subs (J, {y1, y2, y3, y4},{equil1(1), equil1(2),equil1(3),equil1(4)});
 JE=double (Jeval1)
 charEQ1=det (Jeval1-lambda2*eye (4))
eigen1=solve (charEQ1, lambda2)
eigenvalues=double(eigen1)
```

```
p= [lambda rho ms alpha beta sigma delta mu ma gamma mt];

options=odeset ('RelTol',1e-4,'AbsTol’, [1e-4 1e-5 1e-6 2e-6]);

[T, Y] =ode45(@mamodel, [0 50],[100 50 2 4],options, p);

plot(T, Y(:,1),'-.b',T,Y(:,2),'--r',T,Y(:,3),'-g',T,Y(:,4),'.y','linewidth',2.5);

xlabel('time, t','FontWeight','bold')

ylabel('number of students','FontWeight','bold')

grid on

legend1=legend ('Susceptible', 'Anxious','Treated', 'Untreated');

figure

plot(T,100*Y(:,2)./(Y(:,1)+Y(:,2)+Y(:,3)+Y(:,4)),'-

.b',T,100*Y(:,4)./(Y(:,1)+Y(:,2)+Y(:,3)+Y(:,4)),'--r','linewidth',2.5)

xlabel('time, t','FontWeight','bold')

ylabel('proportion of students (%)','FontWeight','bold')

grid on

legend2=legend ('Percent Anxious', 'Percent Untreated');
```

B.4 Supplementary MATLAB Codes for Numerical Simulation of Chapter 5

```
clear;
clc
    if caseN==1
A=0.01846287;
B=0.057;
C=0.11969000;
D=0.057;
rho=0.5;
L=0.1;
gamma=0.1;
mt=0.2;
mu=0.3;
delta=0.05;
IC= [10 2 0 0];
TS= [0 200];
    elseif caseN==2
A=0.364;
B=0.060;
C=0.352;
D=0.057;
rho=0.5;
```

```
L=0.1;
gamma=0.1;
mt=0.2;
mu=0.3;
delta=0.05;
IC= [20 0.8 0 0];
TS= [0 200];
          elseif caseN==3
A=0.364;
B=0.060;
C=0.352;
D=0.057;
rho=0.5;
L=0.1;
gamma=0.1;
mt=0.2;
mu=0.3;
delta=0.05;
IC= [10 0.25 0 0];
TS= [0 50];
            elseif caseN==4
A=0.1;
```

```
B=0.057;
C=1.55;
D=0.057;
rho=0.5;
L=0.1;
gamma=0.1;
mt=0.2;
mu=0.3;
delta=0.05;
IC= [15 0.25 0 0];
TS= [0 200];
        elseif caseN==4
A=0.42;
B=0.08;
C=0.52;
D=0.057;
rho=0.5;
L=0.1;
gamma=0.1;
mt=0.2;
mu=0.3;
delta=0.05;
```

```
IC= [15 0.25 0 0];

TS= [0 200];

end

p= [A B C D rho L gamma mt mu delta];

options=odeset ('RelTol',1e-4,'AbsTol’, [1e-6 2e-6 1e-6 2e-6]);

[T, Y] =ode45(@YuExtendedModel, TS, IC, options, p);

plot (T, Y(:,1),'-.b',T,60*Y(:,2),'--r', T,Y(:,3),'.g',T,60*Y(:,4),'-y','linewidth',2.5);

xlabel ('time, t','FontWeight','bold')

ylabel ('number of students','FontWeight','bold')

grid on

legend1=legend ('Susceptible', 'Anxious', 'Treated', 'Untreated');

figure

plot(Y(:,1),Y(:,2),'--r','linewidth',2.5);

grid on

xlabel('Susceptible','FontWeight','bold')

ylabel('Anxious','FontWeight','bold')
```

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VITA

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