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A STUDY ON WHETHER THE WOODS-SAXON OR THE WOODS-SAXON SQUARE PARAMETRISATION IS APPROPRIATE FOR THE PHENOMENOLOGICAL REPRESENTATION OF DIFFERENT α-PARTICLE NUCLEUS FOLDING POTENTIALS*

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The Woods-Saxon (WS) and the squared Woods-Saxon $(WS)^2$ parametrisations for the single and double folding potentials were tested. We showed that the $(WS)^2$ form is appropriate for single as well as double folding approaches.

In many works the scattering of the alpha-particles from nuclei has been described by simple microscopic models. The double-folding formula and two different single-folding formulas, presented below, have been used by different authors to generate the real part of the alpha-nucleus interaction.

The double-folding potential [1] is given by:

$$V^{\mathrm{D}}_{\alpha-\mathrm{T}}(\vec{r}) = \iint d\vec{r}'d\vec{r}''\varrho_{\mathrm{T}}(\vec{r}')\varrho_{\alpha}(\vec{r}'')t(\vec{r}, \vec{r}', r''), \tag{1}$$

where $t(r, r', r'')$ is the effective nucleon-nucleon interaction, ϱ_{T} and ϱ_{α} are the point matter density distributions of the target nucleus and of the alpha-particle. The double-folding microscopic model was used for the scattering of 27 MeV alpha-particles from a number of nuclei in the region from ^{45}Sc up to ^{208}Pb [2]. It was also used for the ^{90}Zr (α, α) scattering in a broad range of energies and scattering angles [3]. The double-folding potential was recently successfully applied also for the heavy ion scattering [4].

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Due to the strong binding of the alpha-particle and the possibility to use the available information from the alpha-particle nucleon scattering, the real part of the alpha-nucleus potential can be calculated in terms of single-folding (target folding):

$$V^{\mathrm{ST}}_{\alpha-\mathrm{T}}(\vec{r}) = \int d\vec{r}'\varrho_{\mathrm{T}}(\vec{r}')V^{(1)}_{\mathrm{eff}}(\vec{r}-\vec{r}'), \tag{2}$$

where $V^{(1)}_{\mathrm{eff}}$ is the effective alpha-particle nucleon interaction usually assumed to be in the Gaussian form:

$$V^{(1)}_{\mathrm{eff}}(r) = V_0 \exp(-r^2/\mu^2). \tag{3}$$

On the other hand, one may fold the alpha-particle density distribution with the target--nucleon effective interaction and obtain another form of the single-folding (projectile folding):

$$V^{\mathrm{SP}}_{\alpha-\mathrm{T}}(\vec{r}) = \int d\vec{r}''\varrho_{\alpha}(\vec{r}'')V^{(2)}_{\mathrm{eff}}(\vec{r}-\vec{r}''). \tag{4}$$

The target–nucleon potential, $V^{(2)}_{\mathrm{eff}}$, has in this case a Woods–Saxon shape.

The single-folding form (2) was used by many authors [5, 6]. The single-folding potential (4) have been applied by Singh et al. [7]. It is interesting to note that both single--folding formulas fail to describe heavy ion scattering [8].

From the formal point of view formulas (1), (2) and (4) are completely equivalent. However, due to the different kinds of effective interactions $t(r, r', r'')$, $V^{(1)}_{\mathrm{eff}}$ or $V^{(2)}_{\mathrm{eff}}(r)$, the resulting potentials can differ considerably.

All folding potentials presented in this paper need a renormalization factor N decreasing slightly the depth. This renormalization factor is treated as an additional degree of freedom in the comparison of the optical model predictions with experimental data. Such procedure may be justified by the fact that the simple folding model represents the direct term of the real potential only neglecting higher order contributions resulting from the antisymmetrization and the density dependence of the effective nucleon-nucleon interaction. N is close to 0.8 for the double folding potential and close to 0.9 in the single folding case. This is due to the fact that in the single folding case the higher order effects are partially included in the nucleon-target (or nucleon-projectile) effective interaction.

Finally we would like to underline that the double folding potential with the density dependence included provides a good description of experimental scattering data at higher scattering energies where the antisymmetrization effect can be neglected [9].

The purpose of this work was to investigate the shape of the form factors obtained from formulas (1), (2) and (4) by comparing them to the traditional Woods–Saxon form factor and to the Woods–Saxon squared factor. The optical potential with the form factor having the shape of the square of the Woods–Saxon function was first used by Budzanowski et al. [10] for low incident energies and by Goldberg [11] for high incident energies. Recently, it has been used for a number of nuclei and in a wide range of scattering angles and incident energies.

Formulas (1), (2) and (4) were used in this work to generate the real part of the microscopic potential for ^{90}Zr(α, α) ^{90}Zr scattering. Since the only purpose of this work was to investigate the shape of the form factors of the potential, we used here the normalization factor $N = 1$. The ^{90}Zr target was selected for this purpose because the elastic alpha-particle scattering from this nucleus exhibits regular angular distributions without any special effects as, i.e., anomalous large angle scattering [12].

The target nucleus density matter distribution was assumed to have a Fermi shape with a half-way radius of $R = 1.12\,A^{1/3}$ fm and a diffuseness parameter $a = 0.545$ fm.

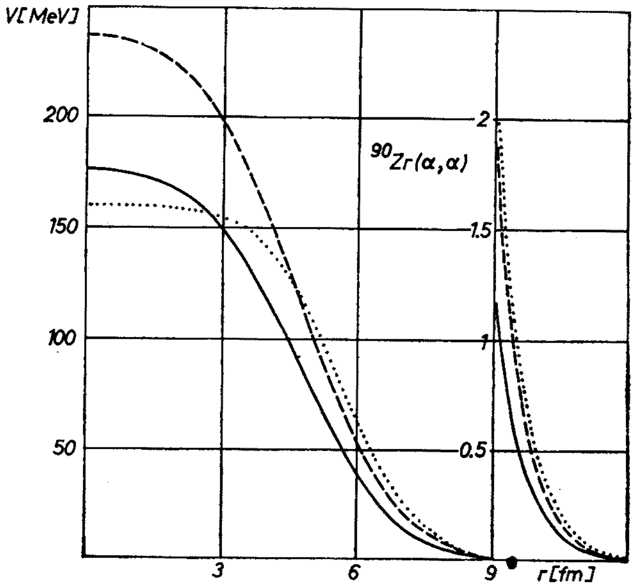

Fig. 1. The double-folding (solid line), the target-folding (dashed line) and the projectile-folding (dotted line) potentials

The numbers above were taken from measurements of the photoproduction of rho-mesons [13]. The alpha-particle density distribution had a Gaussian form with $\mu = 1.18$ fm [14].

The Slanina–McManus [15] effective nucleon—nucleon interaction was taken for $t(r, r', r'')$. This kind of interaction was used by Budzanowski et al. [1, 2] and Majka et al. [3].

The effective interaction $V_{\text{eff}}^{(1)}$ was calculated from formula (3) with $V_0 = -37$ MeV and $\mu = 2$ fm. This interaction was used by Bernstein [6].

Parameters of the effective potential, $V_{\text{eff}}^{(2)}$: $V_0 = -40$ MeV, $r_0 = 1.28$ fm and $a = 0.65$ fm were selected and these agree with Singh et al. [7].

The form factors of the microscopic potentials (1), (2) and (4) were fitted with the function of the Woods–Saxon type:

$$(\text{WS}) = U_1\left(1+\exp\left(\frac{r-R}{a}\right)\right)^{-1} \tag{5}$$

and also with the function of the Woods–Saxon square type:

$$(\mathrm{WS})^2 = U_2\left(1+\exp\left(\frac{r-D}{d}\right)\right)^{-2}. \tag{6}$$

The MINUITS searching routine [16] was used in the fitting procedure with the error function, γ, defined as:

$$\gamma = \sum_i \frac{(V_{\mathrm{MIC}}(r_i)-V_{\mathrm{FIT}}(r_i))^2}{(V_{\mathrm{MIC}}(r_i))^2}, \tag{7}$$

where $V_{\mathrm{MIC}}(r_i)$ and $V_{\mathrm{FIT}}(r_i)$ are the microscopic and fitted values of the potentials.

The double- and single-folding potentials (1), (2) and (4) are shown in Fig. 1. As was mentioned earlier, potentials (1), (2) and (4) are completely different although formulas

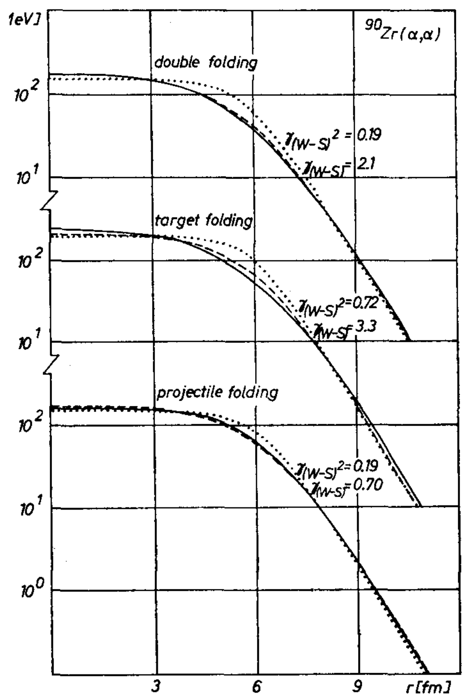

Fig. 2. Three different microscopic potentials (1), (2) and (4) together with the best fit (WS) (dotted line) and $(\mathrm{WS})^2$ (dashed line) functions

(1), (2) and (4) are mathematically equivalent. A comparison of the microscopic potentials (1), (2) and (4) with the best fit (WS) and $(\mathrm{WS})^2$ functions is shown in Fig. 2. Corresponding γ values are also given.

As one can see from Fig. 2 and the corresponding γ values, all the microscopic potentials have a shape which is better described by $(\mathrm{WS})^2$ than by the traditional (WS) function.

This was found previously for the double-folding potential calculations [3, 17], stimulated in a number of recent optical model analyses performed with the $(WS)^2$ phenomenological form factor [18]. The double-folding and the single-alpha folding formulas provide form factors much more similar to the $(WS)^2$ function than the single-target folding formula.

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