STATISTICAL ANALYSIS OF EXCITATION FUNCTIONS FOR ELASTIC AND INELASTIC SCATTERING OF α-PARTICLES ON Mg AND Si NUCLEI

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The excitation functions for inelastic α-scattering leading to the low lying excited states in ^{24}Mg and ^{28}Si were measured at $\Theta_{LAB} = 170°$, 175° and 179° in the LAB energy range 22.75–28.40 MeV. Statistical analysis of these excitation functions and those previously measured for elastic scattering was performed. The direct interaction contribution y_D to the reaction studied was obtained from probability distributions of cross sections and from correlation coefficients. Cross correlation coefficients between different reaction channels were calculated.

1. *Introduction*

Our recent data on elastic scattering of α-particles of Mg and Si nuclei show a strong energy dependence of the cross section measured at extremely backward angles (179°) in the energy range 23–28 MeV [1]. A similar structure of the excitation functions was observed at smaller angles by other authors [2].

The reaction mechanism responsible for the appearance of resonance-like structures in the excitation curves is not yet well known. It therefore seems interesting to examine the energy dependence of the cross section in other reaction channels.

2. *Experiment*

In the present experiment the excitation functions for inelastic α-scattering leading to the low lying excited states of ^{24}Mg and ^{28}Si were measured at $\Theta_{LAB} = 170°$, 175°, and 179°. The energy resolution was good enough to separate the first and the sum of the second and third excited states in ^{24}Mg and ^{28}Si.

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The measurements were performed in the energy range from 22.75 to 28.40 MeV in steps of about 250 keV, using α-particles from the 120 cm cyclotron of the Institute of Nuclear Physics, Cracow. The energy spread of the beam was about 200 keV. The experimental arrangement and procedure were the same as those described in [1].

The results together with elastic data are presented in Fig. 1 and 2 for ^{24}Mg and ^{28}Si respectively. The absolute errors of all cross sections are around 10%.

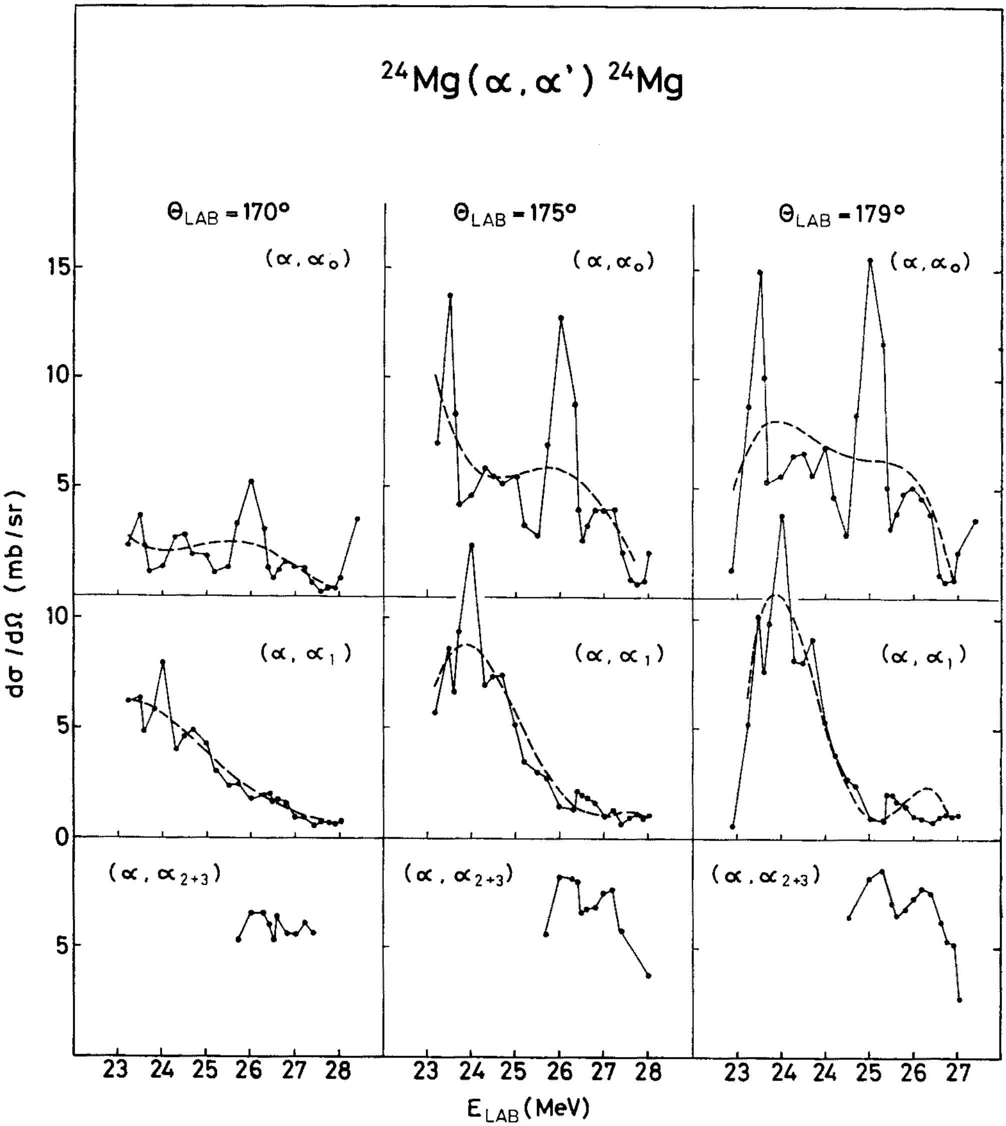

Fig. 1. Excitation functions for the $^{24}Mg(\alpha, \alpha')^{24}Mg$ reactions. Dotted line indicates the averaged cross section

3. *Analysis*

A rich resonance-like structure has been found in all excitation functions. It has been suggested by various authors that this kind of structure may be connected with some intermediate resonances [3].

In order to understand the reaction mechanism responsible for these structures it seems interesting to examine the energy dependence of the cross sections in terms of

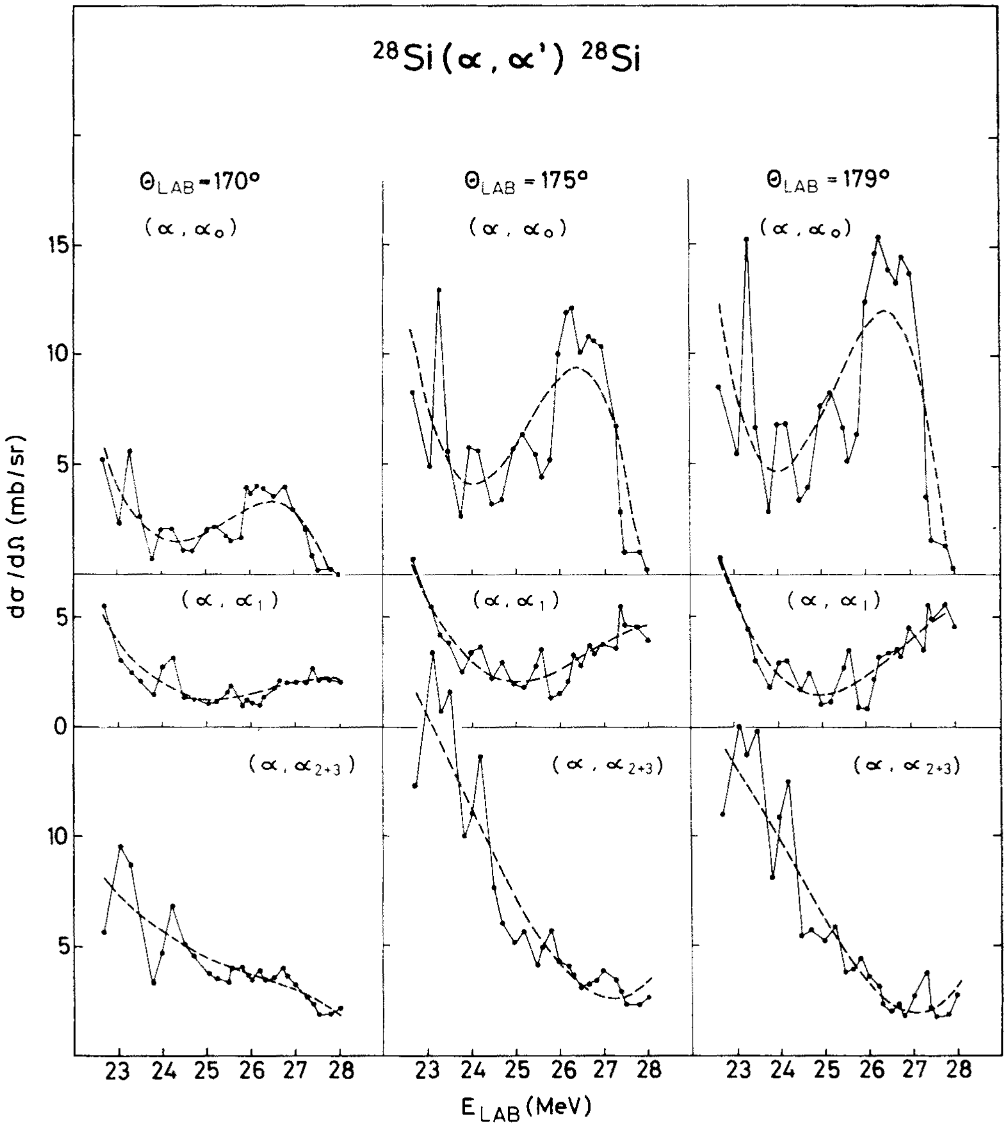

Fig. 2. Excitation functions for the $^{28}Si(\alpha, \alpha')^{28}Si$ reactions. Dotted line indicates the averaged cross section

Ericson's theory of cross section fluctuations and to separate in this way possible contributions from nonstatistical effects.

The simple fluctuation theory was developed for excitation functions with constant averaged cross section. In our case, however, this condition seems not to be fulfilled. In order to eliminate the nonstatistical modulations from the excitation functions these

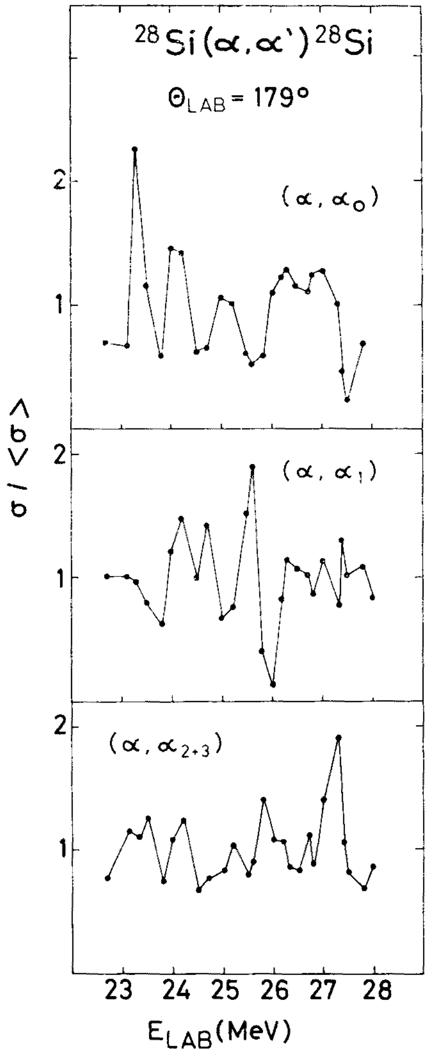

Fig. 3. Reduced excitation functions for the $^{28}Si(\alpha,\alpha')^{28}Si$ reactions at 179°

functions were reduced according to the procedure given by Rubbino [4]. The averaged cross sections were evaluated by fitting the polynomial of the order 8 in the case of ^{24}Mg and 4 in the case of ^{28}Si to the data points. The averaged cross sections and examples of reduced excitation functions are shown in Figs 1, 2 and Fig. 3 respectively.

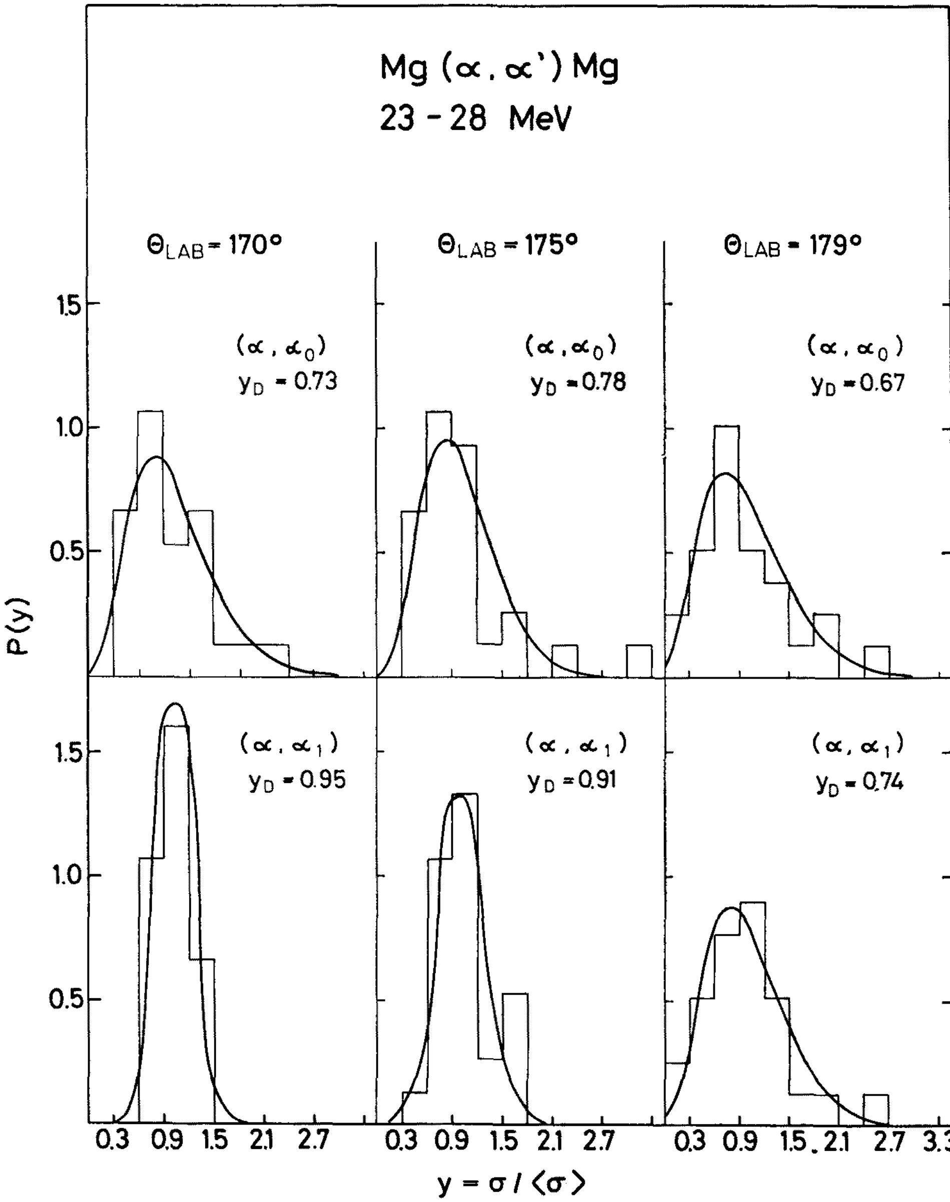

Fig. 4. Probability distributions of the cross sections for the $^{24}Mg(\alpha, \alpha')^{24}Mg$ reactions

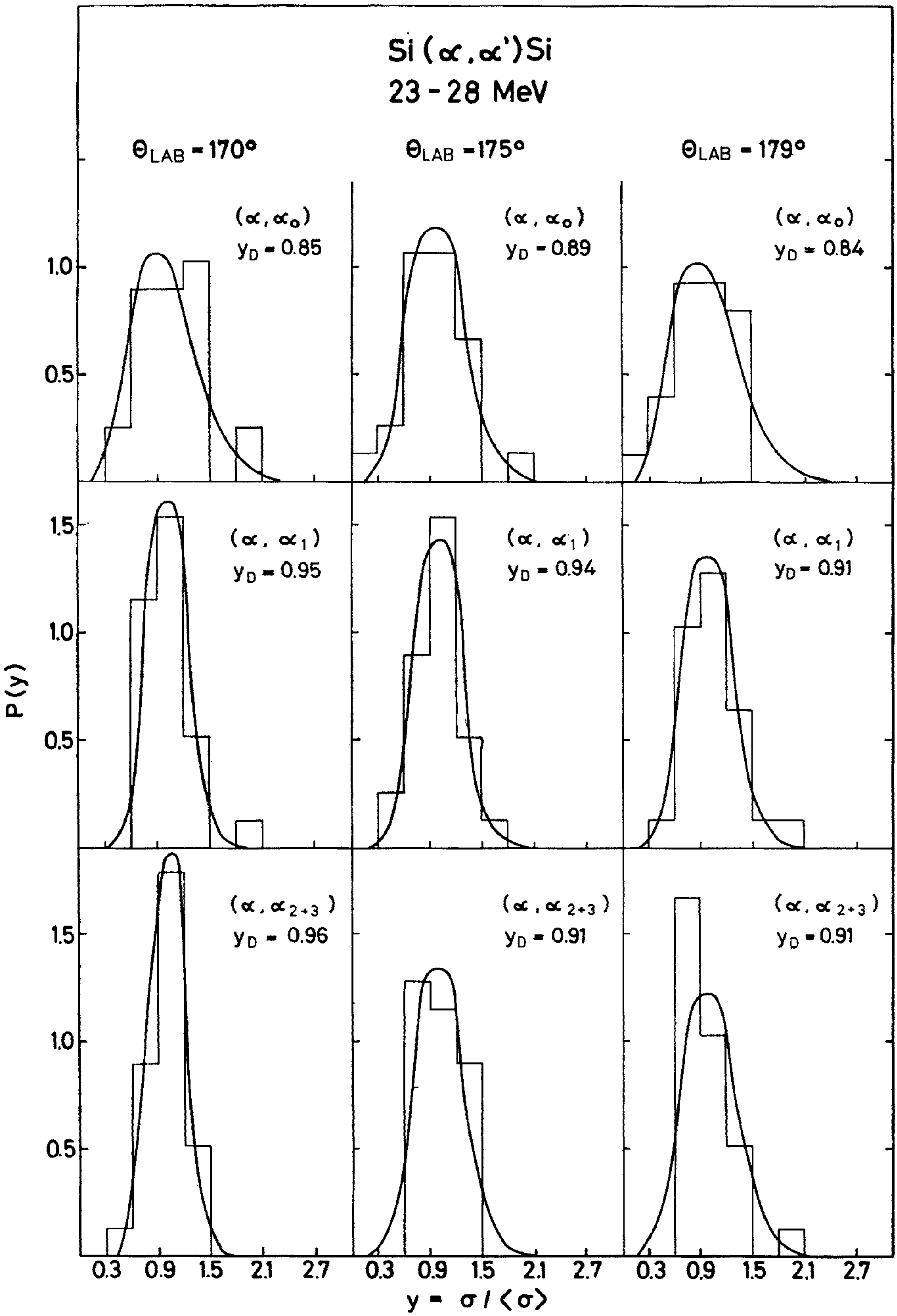

Fig. 5. Probability distributions of the cross sections for the $^{28}Si(\alpha, \alpha')^{28}Si$ reactions

The experimental probability distributions $P\left(\frac{\sigma}{\langle\sigma\rangle}\right)$ were obtained for the ground state, first excited state, and the sum of the second and third excited states of both nuclei. In order to determine the relative contribution of direct interaction y_D these histograms were fitted by the following distribution function [5]:

$$P(y;\, y_D, N) = \left(\frac{N}{1-y_D}\right)^N \exp\left\{-\frac{N(y+y_D)}{1-y_D}\right\} \frac{J_{N-1}(iz)}{(iz)^{N-1}}$$

where: $y_D = \frac{\sigma_D}{\langle\sigma\rangle}$, $y = \frac{\sigma}{\langle\sigma\rangle}$, $z = \frac{2N}{1-y_D}\sqrt{yy_D}$, $J_{N-1}\,(iz)$ is the cylindrical Bessel function of order $N-1$, and N is the number of independent reaction channels.

The number of independent reaction channels N in our case was equal to one, due to the extreme backward angles of our measurements and zero spins of α-particles and target nuclei [6]. However, the correction for the finite energy resolution ΔE had to be taken into account. According to Fessenden *et al.* [7], this correction is equivalent to increasing the number of independent channels by a factor of $n = \frac{\Delta E}{\pi\Gamma}+1$, where Γ indicates the coherence width. In our case, assuming $\Delta E = 250$ keV and $\Gamma \approx 80$ keV [8], n was equal to 2.

The histograms of $P\left(\frac{\sigma}{\langle\sigma\rangle}\right)$ and fitted theoretical probability distributions are presented in Figs 4 and 5. The values of direct interaction contributions y_D obtained in this way are shown in Table I.

TABLE I

Values of direct interaction contributions

Target	Reaction \ Angle	170°	175°	179°
Mg	(α, α_0)	0.73	0.78	0.67
	(α, α_1)	0.95	0.91	0.74
Si	(α, α_0)	0.85	0.89	0.84
	(α, α_1)	0.95	0.94	0.91
	(α, α_{2+3})	0.96	0.91	0.90

It is also possible to find the direct interaction contribution y_D from the formula for the autocorrelation coefficient [9]:

$$C(0) \equiv \frac{\langle\sigma^2(E)\rangle - \langle\sigma\rangle^2}{\langle\sigma\rangle^2} = \frac{1-y_D^2}{N}$$

These coefficients were calculated for all measured excitation functions except for the sum of the second and third excited states in ^{24}Mg. The values of y_D obtained from these coefficients are presented in Table II.

TABLE II

Values of direct interaction contributions

Target	Reaction \ Angle	170°	175°	179°
Mg	(α, α_0)	0.74	0.52	0.55
	(α, α_1)	0.97	0.91	0.52
Si	(α, α_0)	0.80	0.75	0.70
	(α, α_1)	0.93	0.95	0.90
	(α, α_{2+3})	0.97	0.95	0.93

It can be seen that these values are in good agreement with those found from probability distributions.

The cross correlation coefficients between different reaction channels may, in principle, give us information about possible intermediate resonances. The normalized cross correlation coefficients:

$$C_{\alpha_i\alpha_j}(0) = \frac{1}{\sqrt{C_{\alpha_i}(0)C_{\alpha_j}(0)}}\left\{\frac{\langle\sigma_{\alpha_i}(E)\sigma_{\alpha_j}(E)\rangle}{\langle\sigma_{\alpha_i}(E)\rangle\,\langle\sigma_{\alpha_j}(E)\rangle} - 1\right\}$$

were calculated for all pairs of excitation functions. The values of these coefficients are shown in Table III.

TABLE III

Values of cross correlation coefficients

Target	Channels \ Angle	170°	175°	179°
Mg	(α_0, α_1)	−0.14	−0.06	+0.29
Si	(α_0, α_1)	+0.07	+0.12	−0.06
	(α_0, α_{2+3})	+0.40	+0.46	+0.32
	(α_1, α_{2+3})	+0.13	−0.11	−0.23

As can be seen from this table, the values of cross correlation coefficients are rather small, except for the ground and the sum of the second and third excited states in ^{28}Si. These values are independent of the scattering angle.

It should be remembered that results of statistical calculations contain various kinds of error. According to Ericson and Mayer-Kuckuk [6], the most important errors in a fluctuation analysis are produced by "finite-range deviations" (FRD). They are due to the finite energy range of the sample from which our cross correlation coefficients were calculated. The number of statistically independent points n in the excitation function is given by the measured energy range I divided by Γ. The FRD error is equal roughly to $\pm\left(\frac{\pi}{2nN_iN_j}\right)^{1/2} \cong \pm 0.1$.

4. Conclusions

As can be seen from Tables I and II direct interaction processes dominate in the backward elastic and inelastic scattering of α-particles on ^{24}Mg and ^{28}Si nuclei in our energy range.

Fig. 2 for ^{28}Si demonstrates the existence of some gross structure in the averaged elastic cross section.

Most conspicuous is the maximum with a width of about 2 MeV at the energy 26.5MeV. It may indicate the existence of some intermediate structure. Indeed, it was shown in the analysis of angular distributions measured in the energy range of the maximum that the inclusion of Regge Pole [10] or Breit-Wigner term [11] near or to the $l = 10$ partial wave, respectively, can improve the fits significantly.

On the other hand, it was shown recently [12] that the maxima of similar width in excitation curves at backward angles can be explained as interference effects between surface waves in scattering from a strongly absorbing potential.

The obtained values of cross correlation coefficients are small in most cases, with the exception of the $C_{\alpha_0\alpha_{2+3}}(0)$ for ^{28}Si. This indicates that most of the observed fine structure has a statistical character.

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