THE INFLUENCE OF ISOSPIN CONSERVATION ON PION MULTIPLICITY DISTRIBUTION IN INDEPENDENT EMISSION MODELS

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We discuss two models for multiple pion production in which strict conservation of isospin generates strong correlations in multiplicity distribution. The first model is a particular version of the uncorrelated jet model in which pions are produced in an eigenstate of total isospin and charge, the second one is the hadronic bremsstrahlung model of pion emission by an isospinor nucleon current. The predictions of these models concerning multiplicity distribution are almost identical: 1. linear rise of the dispersion of the negative pion multiplicity distribution with the average multiplicity which agrees with the data very well, and 2. strong negative correlations between charged and neutral pions which is in disagreement with the data.

1. Introduction

The influence of isospin conservation on the particle number distribution in multiparticle production has been discussed in various aspects. In models with weak, or without any dynamical correlations such as multiperipheral models [1] or the uncorrelated jet model (UJM) [2] with isospin conservation included by means of the statistical weights of Cerulus [3], the main effect of isospin conservation consist in negative correlations due to charge conservation.

Recently Ruijgrok, and Dadić, Martinis and Pisk have constructed models with positive correlations between equally charged pions arising from isospin conservation [4, 5, 6]. The isospin conservation does not affect the total multiplicity distribution, which remains roughly Poissonian, but broadens the charged multiplicity distribution, giving an excellent fit to the experimental data, which show linear growth of the dispersion of charged pion multiplicity distribution with the average charged multiplicity [7].

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In this paper we investigate the model of Ref. [5] in some details taking into account the energy and momentum conservation. We also propose another model with exactly conserved isospin, which gives similar predictions.

Graphs describing the isospin structure of investigated models are presented on Figs 1a and 1b.

2. *The uncorrelated jet model with isospin conservation*

The isospin structure of the first model we consider is depicted in Fig. 1a in which also part of our notation is explained. This model corresponds to the production of one cluster of pions. We assume furthermore that the dependence on the nucleon momenta and isospin factorizes. Thus, the model is a special case of the model developed by Ruijgrok and Schlitt [5].

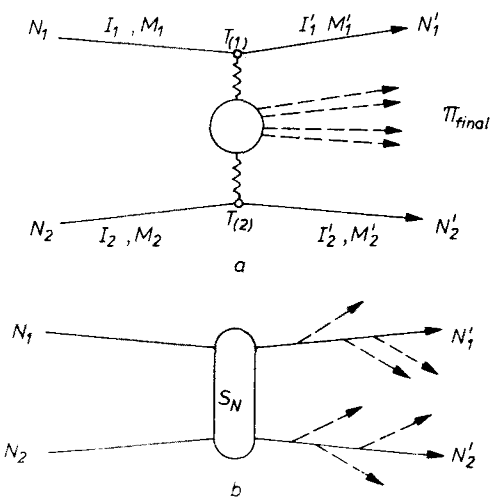

Fig. 1. a) The diagram presenting the isospin structure of the model discussed in Section 2. b) The diagram presenting the bremsstrahlung model

In this model it is assumed that pions are the only particles produced. Let I, M (I', M') denote the isospin and its third component for the initial (final) nucleons and π_{final} the configuration of the pions in the final state. The production amplitude takes the following form:

$$A = \langle I_1', M_1', I_2', M_2', \pi_{\text{final}}|S|I_1, M_1, I_2, M_2\rangle. \tag{1}$$

The S operator must be a scalar in isospin space due to isospin invariance. It is written in the factorized form:

$$S = \sum_{k,\mu} S_\mu^k \cdot \pi_{-\mu}^k \cdot \frac{(-1)^{k-\mu}}{\sqrt{2k+1}}, \tag{2}$$

where S^k_μ describes the nucleon part of the scattering process — the skeleton, and $\pi^k_{-\mu}$ the creation of mesons. Our S operator is not unitary.

For pion production amplitude Ruijgrok and Schlitt have proposed

$$\pi^k_\mu|0\rangle \equiv |k, \mu\rangle = \int d\Omega Y^k_\mu(\Omega) \exp\{g \sum_m D^1_{m0}(\Omega)a^+_m\} |0\rangle, \tag{3}$$

where a^+_m can be interpreted as a multimode creation operator

$$a^+_m = \int \frac{d^3p}{2E} \varphi(p) a^+_m(p), \tag{4}$$

$\varphi(p)$ denoting the one pion wave function which is assumed to be independent of pion isospin, $Y^k_\mu(\Omega)$ is the spherical harmonics and $D^1_{m0}(\Omega)$ the D matrix.

The state $|k, \mu\rangle$ is an eigenstate of isospin, constructed by the projection of an uncorrelated (coherent) pion state

$$\exp\{\sum_m g_m \cdot a^+_m\} |0\rangle, \qquad m = +1, 0, -1, \tag{5}$$

on the corresponding Hilbert subspace with definite isospin. (The normalization factor has been omitted.) Actually, in numerical calculations we have imposed the energy and momentum conservation by multiplying the cross-section for each channel by an appropriate phase space integral with transverse momentum cut-off. This has not changed the general predictions of the model, the main effect consists on damping the tail of the multiplicity distribution.

Using the factorization property of amplitude (2) and the Wigner-Eckart theorem we rewrite (1) in terms of the reduced invariant amplitudes:

$$A = \sum_{I,I',k} \langle \pi_{\text{final}}|k, \mu\rangle \frac{(-1)^{k+I-I'}}{\sqrt{(2k+1)(2I+1)}} (I'_1, M'_1, I'_2, M'_2|I', M') \times$$

$$\times (I_1, M_1, I_2, M_2|I, M)(I', M', k, M-M'|I, M) S_k(I', I). \tag{6}$$

I, M (I', M') denote the isospin and its third component of the two nucleon system in the initial (final) state, k, μ, the isospin of the produced pion cluster. Obviously $\mu = M-M'$. The six invariant amplitudes $S_k(I', I)$ describe all final state charge configuration of the nucleons in the NN collision. In the model of Ref. [5] these amplitudes are left unspecified.

In what follows we assume that the nucleon part of the S matrix can be factorized in two parts describing each nucleon separately. Then the number of the arbitrary functions in our version of the model is strongly reduced (see Fig. 1a). The factorization assumption means that S^k_μ can be written in the following form:

$$S^k_\mu = \sum_{\substack{k_1,k_2\\ \mu_1,\mu_2}} (k_1, \mu_1, k_2, \mu_2|k, \mu) T^{k_1}_{\mu_1}(1) T^{k_2}_{\mu_2}(2), \tag{7'}$$

where $T^{k_i}_{\mu_i}(i)$ acts on the i-th nucleon state ($i = 1, 2$) and indices k_i, μ_i denote isospin and its third component exchanged in the i-th vertex.

Using (7′) we can rewrite matrix elements of S^k_μ as follows:

$$\langle I'_1M'_1I'_2M'_2|S^k_\mu|I_1M_1I_2M_2\rangle = \sum_{k_1,k_2}(k_1,\mu_1,k_2,\mu_2|k,\mu)\times$$
$$\times\langle I'_1,M'_1|T^{k_1}_{\mu_1}(1)\,|I_1,M_1\rangle\cdot\langle I'_2,M'_2|T^{k_2}_{\mu_2}(2)\,|I_2,M_2\rangle. \tag{7}$$

Decomposing (7) into invariant amplitudes and comparing (6) and (7), we can express six amplitudes $S_k(I', I)$ by four quantities $T_k(i)$:

$$S_0(1,1) = \frac{\sqrt{3}}{2}\,T_0(1)T_0(2) - \tfrac{1}{6}\,T_1(1)T_1(2),$$

$$S_1(1,1) = \frac{1}{\sqrt{2}}\,(T_0(1)T_1(2) + T_1(1)\cdot T_0(2)),$$

$$S_2(1,1) = \frac{\sqrt{5}}{3}\,T_1(1)\cdot T_1(2),$$

$$S_0(0,0) = \frac{1}{2}\left(T_0(1)T_0(2) + \frac{1}{\sqrt{3}}\,T_1(1)T_1(2)\right), \tag{8}$$

where $T_k(i) = (\tfrac{1}{2}||T^{k_i}_{\mu_i}(i)||\tfrac{1}{2})$ is the reduced invariant amplitude describing i-th vertex ($k = 0, 1$). Due to identity of the nucleons $T_k(1) = T_k(2)$ and we are left with two complex parameters only.

In general, the amplitudes T_0 and T_1 depend also on nucleon momentum. Assuming that the momentum dependence of the amplitudes T_0 and T_1 is the same, the amplitudes $S_1(0,1)$ and $S_1(1,0)$ identically vanish after the antisymmetrization of nucleon amplitudes.

Integrating over the phase space and summing over the pion multiplicity distribution we obtain the following expressions for the total cross-sections for proton-proton and proton-neutron scattering:

$$\sigma_{\rm tot}(\mathrm{pp}) = \tfrac{1}{3}\,(|S_0(1,1)|^2\cdot C_0 + \tfrac{1}{3}\,|S_1(1,1)|^2\cdot C_1 + \tfrac{1}{5}\,|S_2(1,1)|^2\cdot C_2), \tag{9}$$

$$\sigma_{\rm tot}(\mathrm{pn}) = \tfrac{1}{2}\,\sigma_{\rm tot}(\mathrm{pp}) + \tfrac{1}{2}\,|S_0(0,0)|^2\cdot C_0, \tag{10}$$

$$C_I = \sum_{n/0}^{\infty}\frac{|g|^{2(I+2n)}R_{I+2n}}{2^n\cdot n!(2I+2n+1)!!}, \qquad I = 0, 1, 2. \tag{11}$$

R_n denotes the $n+2$ particle phase space integral.

The experimentally observed approximate equality of the cross-sections for pp and pn scattering yields the following condition on parameter $\alpha = T_0/T_1$:

$$\mathrm{Re}\,(\alpha^2) = \frac{1}{\sqrt{3}}\left(|\alpha|^2 - \frac{1}{6}\right), \tag{12}$$

which would be exactly fulfilled if the phase space factors C_I were independent of I. At high energies this is a reasonable approximation.

The probability of finding a given final configuration of pions in a state with a definite isospin can be calculated from (3), taking for $|\pi_{\text{final}}\rangle$ the state with definite number of pions: $|n_+, n_-, n_0\rangle$

$$p(n_+, n_-, n_0, k, \mu) = |\langle n_+, n_-, n_0|\, k, \mu\rangle|^2. \tag{13}$$

Analytic expressions for the correlation parameters can also be obtained in a straightforward but tedious way. The formulae for $\langle n_+, n_-, n_0|k,\mu\rangle$, for the multiplicity distributions and the asymptotic evaluation of the correlation parameters are given in Ref. [5] without taking into account the energy and momentum conservation. We have computed

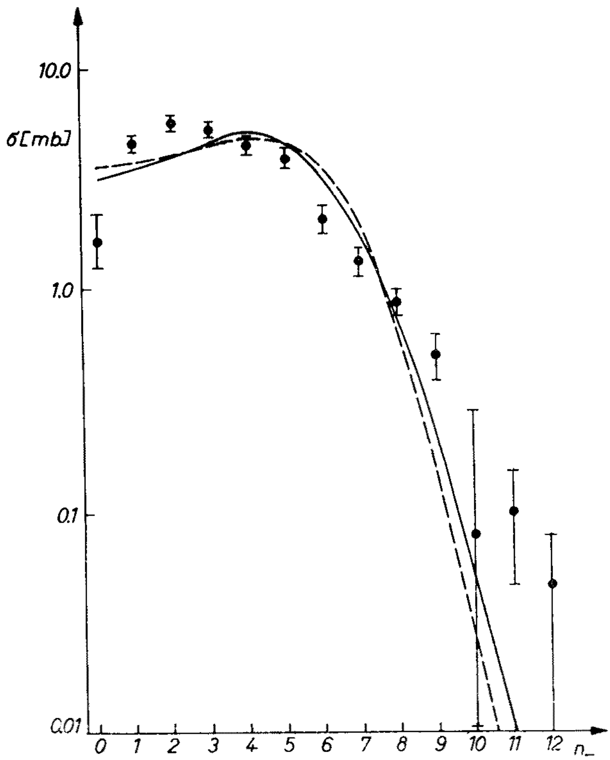

Fig. 2. Negative pion multiplicity distribution at $p_{\text{lab}} = 303$ GeV. The data were taken from Ref. [14] The solid line presents the prediction of UJM with isospin conservation, the dashed one — the prediction of the bremsstrahlung model

numerically the dependence of the dispersion of negative pion multiplicity distribution on the average multiplicity and the negative pion multiplicity distribution at $p_{\text{lab}} = 303$ GeV for pp scattering. The results are plotted on Figs 2 and 3. For $|\alpha|^2$ we have chosen the value of 0.75, which reasonably reproduces the average number of neutrons observed in pp scattering $\langle n_N\rangle \cong 0.5$. $|g|^2$ was adjusted at each energy to reproduce the experimental average charged multiplicity. The experimental data are taken from Refs [13–17].

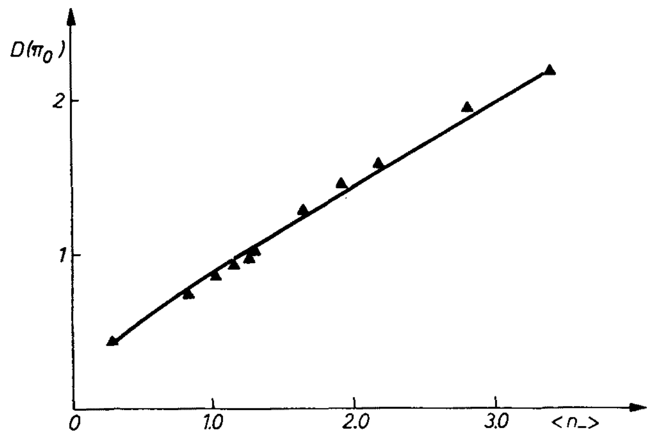

Fig. 3. The dependence of the dispersion of negative pion multiplicity distribution on the negative pion average multiplicity. The data are taken from Refs [13, 14, 16, 17]

In order to show how the isospin conservation distorts the Poissonian multiplicity distribution

$$p(n_+, n_-, n_0) = \frac{|g_+|^{2n_+}|g_-|^{2n_-}|g_0|^{2n_0}}{n_+!n_-!n_0!} \tag{14}$$

which we would expect from the uncorrelated pion production (expression (5)), we quote one formula (without normalization factor)

$$p(n_+, n_-, n_0; 0, 0) = \frac{|g|^{2n} \cdot R_n}{(n+1)!!^2} \cdot \frac{(n_0-1)!!}{n_0!!}, \quad n = n_+ + n_- + n_0. \tag{15}$$

A similar result was obtained by Horn and Silver [8] which algebraically constructed an $I = 0$, n-pion state from isoscalar pairs of pions. In comparison with (14) the distribution of charged and neutral pions for a given total multiplicity is very broad, all charge branching ratios being comparable. We discuss the effects of this fact in Section 4.

3. *The bremsstrahlung model*

Instead of coupling a separately constructed pion state with definite isospin to the nucleon skeleton, we can assure the isospin invariance of the production amplitude by modifying the hadronic bremsstrahlung model in which pions are emitted by an isospin current. In the following we will neglect the kinematic aspects of the model. The S operator takes the following form:

$$S = S_N \cdot e^{\vec{J} \cdot \vec{a}^+}, \tag{16}$$

where S_N denotes the part responsible for the nucleon skeleton and which for our purposes is only a normalization factor. The operator (16) can be formally unitarized (provided S_N

is unitary) by substracting from the term in exponent its hermitian adjoint, but then the influence of the isospin structure on the multiplicity distribution would be less transparent. In numerical calculations we have imposed the energy and momentum conservation exactly as in the previous section.

Dadić, Martinis and Pisk have constructed the current $\vec{J}$ by coupling both nucleons in one system obtaining a four dimensional representation of the isospin current which was rather difficult to handle. We make assumption that each nucleon acts as an independent source of pions.

$$S = S_N \cdot e^{\vec{J}_1 \cdot \vec{a}^+} \cdot e^{\vec{J}_2 \cdot \vec{a}^+}, \tag{17}$$

$$\vec{J}_i = g N_i^{+\prime} \vec{\sigma} N_i, \tag{18}$$

$$\vec{\sigma} \cdot \vec{a}^+ \equiv \sigma_+ a_-^+ - \sigma_- a_+^+ + \sigma_0 a_0^+, \tag{19}$$

where N denotes the nucleon annihilation operator, $\vec{\sigma}$ is the set of Pauli matrices and g is the coupling constant. The contribution from one nucleon to the production amplitude can be written in the form

$$N^+ \left\{ \cosh (gA^+) + \frac{\vec{\sigma} \cdot \vec{a}^+}{A^+} \sinh (gA^+) \right\} N, \tag{20}$$

where by A^+ is denoted the isoscalar creation operator

$$A^+ = \sqrt{a_0^{+2} - 2a_+^+ a_-^+}\,. \tag{21}$$

Expanding (20) in series of creation operators a_+^+, a_-^+, a_0^+ it is easy to obtain the full set of expressions for the probability distribution of the final states. For pp scattering we obtain the following formulae (unnormalized):

$$p_{pp}(n_+, n_-, n_0) = \begin{cases} \dfrac{1}{4} \dfrac{z^n (n_0 - 1)!!}{n_0!!(n+1)!!^2} \left(1 + \dfrac{n_0}{n}\right)^2 & \text{for } n \text{ even,} \\[2ex] \dfrac{z^n n_0!!}{(n_0-1)!!n!!^2} & \text{for } n \text{ odd,} \end{cases} \tag{22}$$

$$p_{pn}(n_+, n_-, n_0) = \begin{cases} \dfrac{z^n \cdot n_+ \cdot n_0!!}{(n_0-1)!!n^2(n-1)!!^2} & \text{for } n \text{ even,} \\[2ex] \dfrac{z^n \cdot n_+ (n_0-1)!!}{n_0!!n!!^2} & \text{for } n \text{ odd,} \end{cases} \tag{23}$$

$$p_{nn}(n_+, n_-, n_0) = \begin{cases} \dfrac{z^n \cdot n_+(n_+-1)\,(n_0-1)!!}{n_0!!n^2(n-1)!!^2} & \text{for } n \text{ even,} \\[2ex] 0 & \text{for } n \text{ odd,} \end{cases} \tag{24}$$

where $z = |2g|^2$ and subscripts pp, pn, nn denote nucleons in the final state.

These expressions are not very different from those derived by Ruijgrok and Schlitt. The general predictions concerning the charged multiplicity distribution are almost identical (see Fig. 2). The plot of the dispersion of negative pion distribution *versus* $\langle n_- \rangle$ practically cannot be distinguished from the prediction of the previous model, although it can be shown that for asymptotic energies the ratio $D(\pi^-)/\langle n_- \rangle$ approaches $1/\sqrt{5} = 0.447$ which is smaller than the observed slope. Dadić *et al.* also have obtained the same value. At low energies, however, the energy and momentum conservation seems to improve the agreement with the data.

4. *Correlations between charged and neutral pions*

Both models give similar predictions concerning the dependence of the average neutral pion multiplicity on the number of negative pions produced. The tendency is opposite compared to the experimentally observed one (see Fig. 4). It is actually much worse than

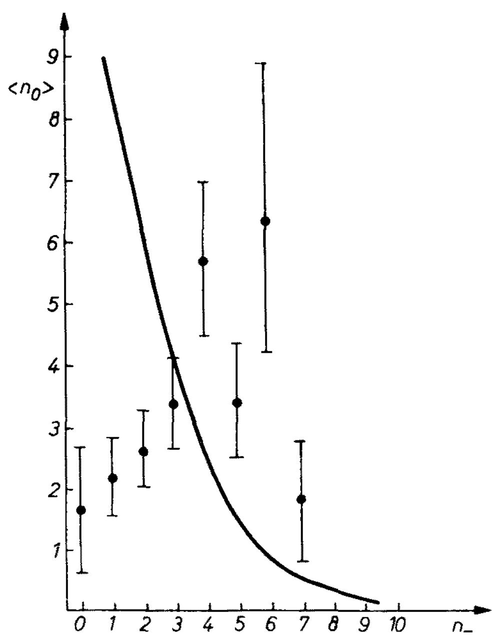

Fig. 4. The average neutral pion multiplicity as a function of number of the negative pions. The experimental data at $p_{\mathrm{lab}} = 205$ GeV from Ref. [15]

in the simplest UJM-expression (14) which predicts no correlations between these two quantities.

To understand qualitatively the reason of this discrepancy, let us note that in the considered models, the probability distribution $p(n_+, n_-, n_0)$ depends mainly on the total multiplicity $n = n_+ + n_- + n_0$. For a given n the neutral pion distribution is very broad which yields a strong contribution to low charged multiplicity channels, even if the total average multiplicity is big. Thus, the cross-sections for channels with small number of charged pions collect contributions from the whole multiplicity spectrum, and are strongly enhanced. The predictions of analysed models are shown on Fig. 2. It is clearly shown that the large dispersion of the negative pions comes from the left part of the distribution. The same mechanism is responsible for the broadening of the neutral pion multiplicity distribution. The correlation parameters f_2 for two negative (or two charged) or for two neutral pions asymptotically behave as const $\cdot \langle n_{ch} \rangle^2$ and const $\cdot \langle n_0 \rangle^2$ respectively. From the simple identity (*cf.* Ref. [5]):

$$f_2^{(\mathrm{ch,ch})} + f_2^{(0,0)} + 2f_2^{(0,\mathrm{ch})} = f_2, \tag{25}$$

where

$$f_2^{(0,\mathrm{ch})} = \langle n_0 \cdot n_{ch} \rangle - \langle n_0 \rangle \langle n_{ch} \rangle, \tag{26}$$

$$f_2 = \langle n(n-1) \rangle - \langle n \rangle^2, \tag{27}$$

($f_2^{(\mathrm{ch,ch})}$ and $f_2^{(0,0)}$ are given by the last formula after replacing n by n_{ch} and n_0 respectively), we see that correlations between charged and neutral pions must be negative, because the distribution of the total multiplicity has an approximate Poisson-like shape, which asymptotically gives a constant value for f_2. This argument indicates also that if we try to improve the predictions concerning the charged-neutral correlations by allowing the production of ϱ or σ mesons in place of pions [9] without changing appreciably the behaviour of f_2 we obtain discrepancy in $f_2^{(\mathrm{ch,ch})}$ (and in $f_2^{(--)}$).

Good fits of the multiplicity distribution for pp scattering were obtained in the frame of the two component model in which positive correlations arise from the interplay of diffraction dissociation and pionization (Refs [10, 11]). In the models we consider, the dispersion of the negative pion multiplicity distribution agrees very well with the data. Consequently the amount of the diffraction which could be added is rather limited (below 10%) and thus will not influence significantly our results. It is interesting to note that the energy dependence of the cross-sections for low multiplicity channels is weak — the cross-section for two prongs behave as $p_{\mathrm{lab}}^{-0.32}$ and for four prongs as $p_{\mathrm{lab}}^{-0.35}$ — due to the growing contribution from channels with large number of π^0's. Thus, the isospin conservation "simulates" the existence of diffraction. On the other side, the tail of the charged multiplicity distribution in the discussed models falls down too rapidly. It seems to be in disagreement with the data, which suggests that large multiplicities also exhibit some positive correlations [12].

5. *Conclusions*

We have discussed two independent emission models for multiple pion production in nucleon-nucleon scattering with strict conservation of isospin. Both models predict: 1) strong positive correlations between equally charged pions, and 2) strong negative correlations between charged and neutral pions, the last prediction being in violent disagreement with the data. Our results suggest that the isospin conservation may be a decisive factor in determining the shape of the multiplicity distribution in a large class of models.

The discussion above shows that the phenomenological fits of multiplicity distributions based on formulae which do not take into account the internal quantum numbers may be misleading. It is particularly important for the attempts of extracting the values of the diffraction cross-section from the charged multiplicity distribution data.

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